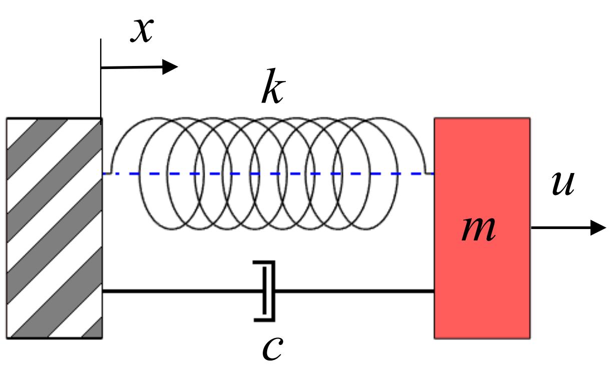

$$m\ddot{x} + c\dot{x} + kx = 0$$

$$\tilde{x} = [x, \dot{x}]^T$$

$$m, c, k = 1$$

$$\dot{\tilde{x}} = \underbrace{\begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{c}{m} \end{bmatrix}}_{A} \tilde{x} + \underbrace{\begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}}_{B} u$$

$$y = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C} \tilde{x}$$

$T_s = 0.05\,\text{sec}$

Discretization in Time $\longrightarrow$

$$x(t+1) = Ax(t) + Bu(t)$$

$$y(t+1) = Cx(t)$$

$$Y = \begin{bmatrix} y(t+1) \\ \vdots \\ y(t+N) \end{bmatrix} \quad U = \begin{bmatrix} u(t+1) \\ \vdots \\ u(t+N) \end{bmatrix}$$

$$G = \begin{bmatrix} CA \\ \vdots \\ CA^n \end{bmatrix} \quad H = \begin{bmatrix} 0 & 0 & \dots & 0 \\ CB & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{N-2}B & CA^{N-3}B & \dots & 0 \end{bmatrix} = \begin{bmatrix} h(1) & 0 & \dots & 0 \\ h(2) & h(1) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ h(N) & h(N-1) & \dots & h(1) \end{bmatrix} \quad F = \begin{bmatrix} h(2) \\ h(3) \\ \vdots \\ h(N+1) \end{bmatrix}$$

$$Y = Gx + Fu + HU$$

Solving The Optimization Problem $\longrightarrow$ $$J(Y,U) = Y^T Y + rU^T D^T DU$$

$$\min \quad J(Y,U)$$
$$sbj: \quad Y = Gx + Fu + HU$$

$$J = (Gx + Fu + HU)^T (Gx + Fu + HU) + rU^T D^T DU$$
$$J = \left((Gx + Fu)^T + U^T H^T\right)(Gx + Fu + HU) + rU^T D^T DU$$
$$J = (Gx + Fu)^T (Gx + Fu) + U^T H^T HU + (Gx + Fu)^T HU + U^T H^T (Gx + Fu) + rU^T D^T DU$$
$$J = \cancel{(Gx + Fu)^T (Gx + Fu)} + U^T \left(H^T H + rD^T D\right)U + 2(Gx + Fu)^T HU$$
$$J = \frac{1}{2} U^T \left(H^T H + rD^T D\right)U + (Gx + Fu)^T HU$$

$$J = \frac{1}{2} U^T QU + f^T U \rightarrow Q = H^T H + rD^T D, f = H^T (Gx + Fu)$$
$$\frac{dJ}{dU} = 0 \rightarrow QU + f^T = 0 \rightarrow U = -Q^{-1} f^T$$