Open Source Spatial Data Analytics in Python with GeostatsPy

Introduction to GeostatsPy

Lecture outline . . .

- Kriging
- Kriging Interactive Demo with GeostatsPy
- Kriging Workflow with GeostatsPy

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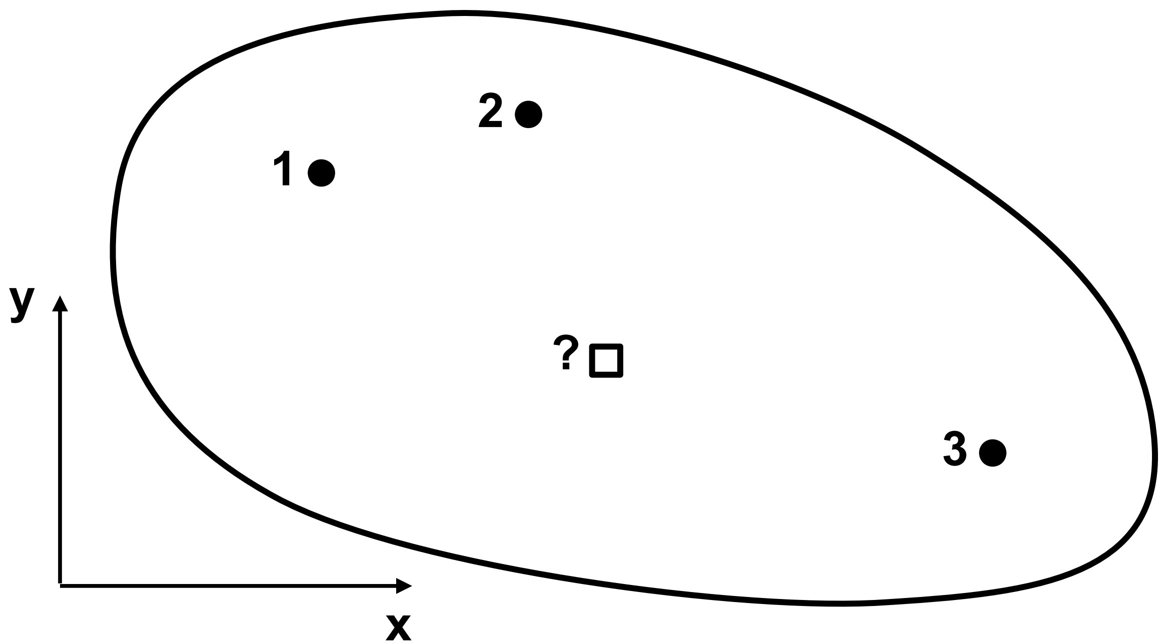
Motivation

We need to make predictions away from sampled locations.

- To determine where to sample next, find a resource, remediate the subsurface:



Recorded Lectures

12b Geostatistics Course: Kriging

GeostatsGuy Lectures

12c Data Analytics: Kriging in R

GeostatsGuy Lectures

12d Python Data Analytics: Simple Kriging

GeostatsGuy Lectures

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- **Kriging**

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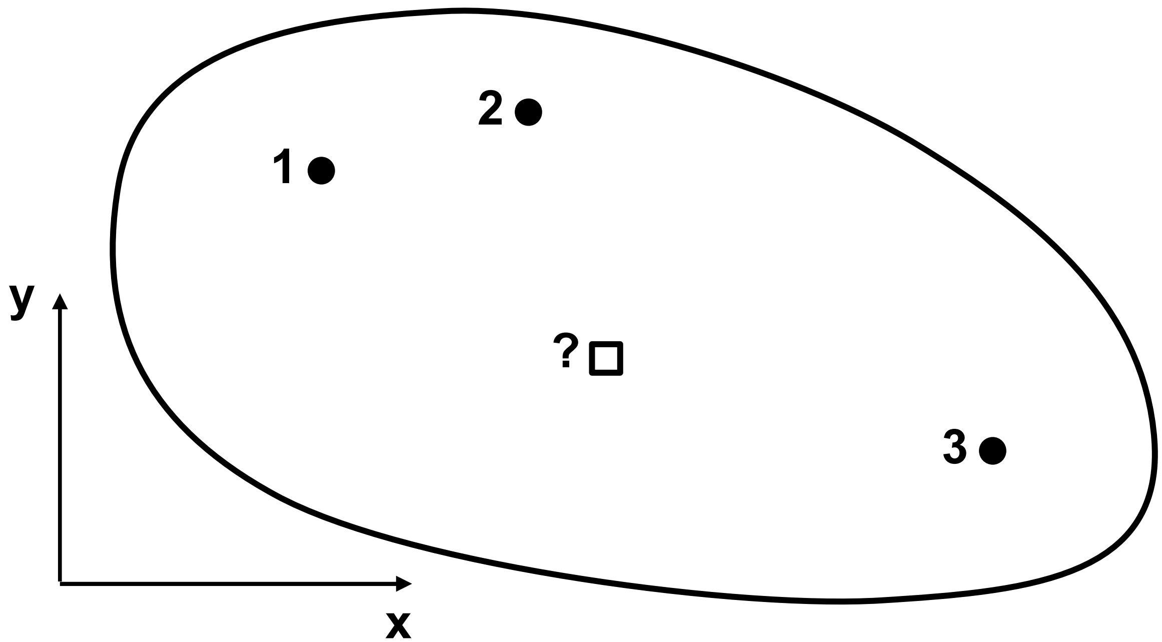
Spatial Estimation

- Consider the case of estimating at an unsampled location:



$\mathbf{z}(\mathbf{u}_\alpha)$ **is the data values**

$z^*(\mathbf{u}_0)$ **is an estimate**

λ_α **is the data weights**

m_z **is the global mean**

- How would you do this given data, $z(\mathbf{u}_1)$, $z(\mathbf{u}_2)$, and $z(\mathbf{u}_3)$?

$$z^*(\mathbf{u}_0) = \sum_{\alpha=1}^{n} \lambda_\alpha \mathbf{z}(\mathbf{u}_\alpha) + \boxed{\left(1 - \sum_{\alpha=1}^{n} \lambda_\alpha\right) m_z}$$

Unbiasedness Constraint Weights sum to 1.0.

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Simple Kriging System of Equations

We use kriging to calculate the best weights integrating:

1. **spatial continuity**: the variogram (expressed as covariance)
2. **closeness**: spatial correlation between samples and unknown location
3. **redundancy**: spatial correlation between samples and each other

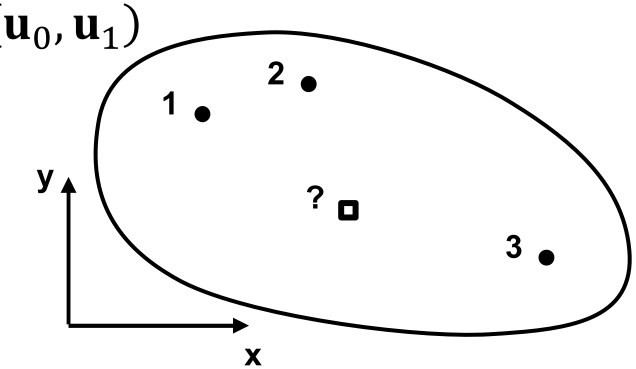
The kriging system of equations to determine the three weights:

$$\lambda_1 \cdot C(\mathbf{u}_1,\mathbf{u}_1) + \lambda_2 \cdot C(\mathbf{u}_1,\mathbf{u}_2) + \lambda_3 \cdot C(\mathbf{u}_1,\mathbf{u}_3) = C(\mathbf{u}_0,\mathbf{u}_1)$$

$$\lambda_1 \cdot C(\mathbf{u}_2,\mathbf{u}_1) + \lambda_2 \cdot C(\mathbf{u}_2,\mathbf{u}_2) + \lambda_3 \cdot C(\mathbf{u}_2,\mathbf{u}_3) = C(\mathbf{u}_0,\mathbf{u}_2)$$

$$\lambda_1 \cdot C(\mathbf{u}_3,\mathbf{u}_1) + \lambda_2 \cdot C(\mathbf{u}_1,\mathbf{u}_2) + \lambda_3 \cdot C(\mathbf{u}_1,\mathbf{u}_3) = C(\mathbf{u}_0,\mathbf{u}_1)$$

Note: $C(\mathbf{h}) = C(0) - \gamma(\mathbf{h})$



In matrix notation:

$$\begin{bmatrix} C(\mathbf{u}_1,\mathbf{u}_1) & C(\mathbf{u}_1,\mathbf{u}_2) & C(\mathbf{u}_1,\mathbf{u}_3) \\ C(\mathbf{u}_2,\mathbf{u}_1) & C(\mathbf{u}_2,\mathbf{u}_2) & C(\mathbf{u}_2,\mathbf{u}_3) \\ C(\mathbf{u}_3,\mathbf{u}_1) & C(\mathbf{u}_3,\mathbf{u}_2) & C(\mathbf{u}_3,\mathbf{u}_3) \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = \begin{bmatrix} C(\mathbf{u}_o,\mathbf{u}_1) \\ C(\mathbf{u}_o,\mathbf{u}_2) \\ C(\mathbf{u}_o,\mathbf{u}_3) \end{bmatrix}$$

redundancy **closeness**

Properties of Simple Kriging

- Solution exists and is unique of matrix $\left[C(v_i, v_j)\right]$ is positive definite

- Kriging estimator is unbiased: $E\left\{\left[Z - Z^*\right]\right\} = 0$

- Minimum error variance estimator (just try to pick weights, you won't bet it)

- Best Linear Unbiased Estimator

- Provides a measure of the estimation (or kriging) variance (uncertainty in the estimate):

$$\sigma_E^2(\mathbf{u}) = C(0) - \sum_{\alpha=1}^{n} \lambda_\alpha C(\mathbf{u} - \mathbf{u}_\alpha) \qquad \boldsymbol{\sigma_E^2} \rightarrow [\mathbf{0}, \boldsymbol{\sigma_x^2}]$$

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- **Kriging Interactive Demo with GeostatsPy**

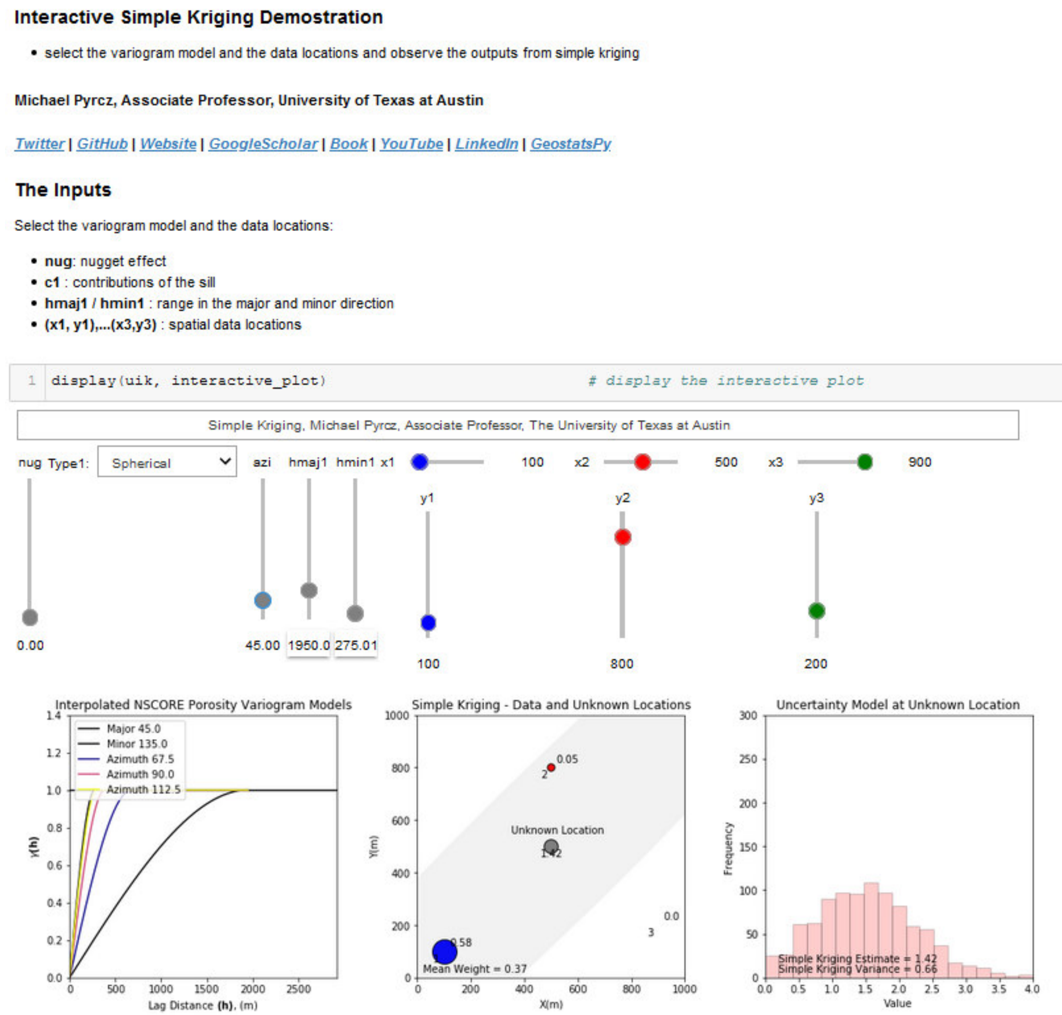
Interactive Variogram Calculation Demonstration with GeostatsPy

Let's calculate spatial estimates with kriging:

- we can move the data and change the variogram model and observe:
  - estimate and uncertainty
  - data weights

Observe the these for:

- 100% nugget effect
- isotropic with range of 9000m
- a data at the estimate location
- strong anisotropy
- two data very close to each other
- one data screened by another



Interactive Python Jupyter kriging (Interactive_Simple_Kriging.ipynb).

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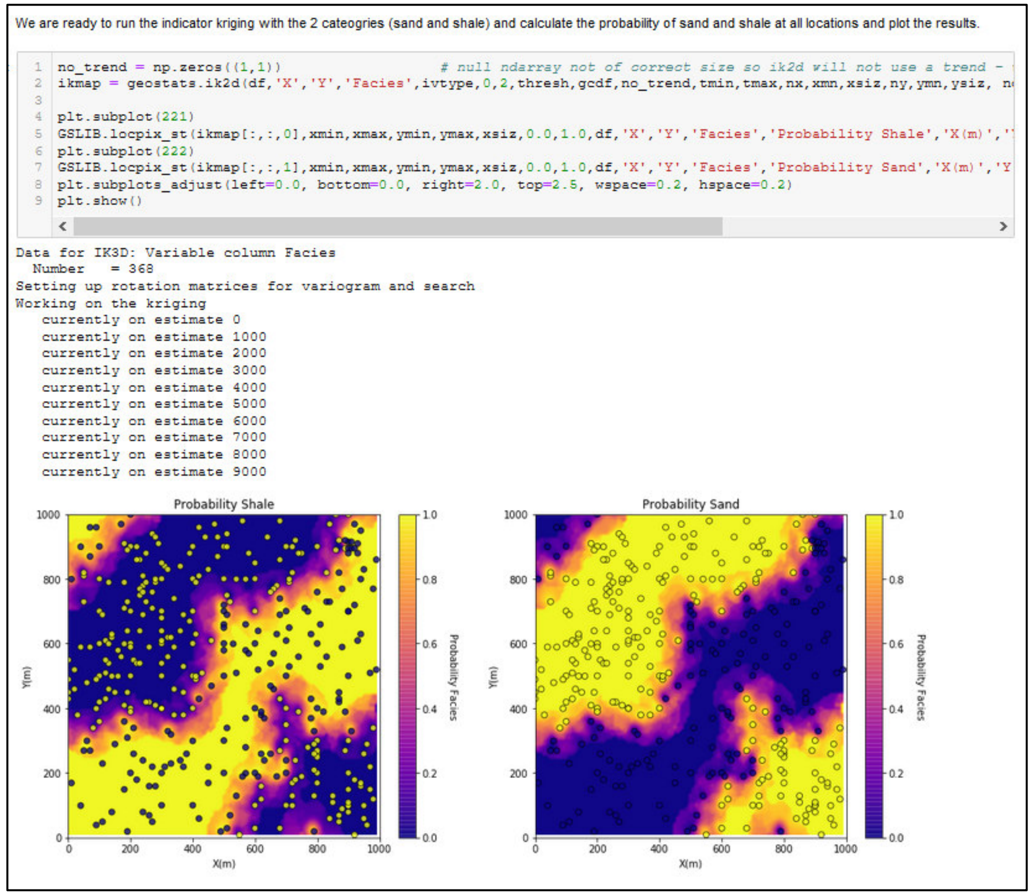
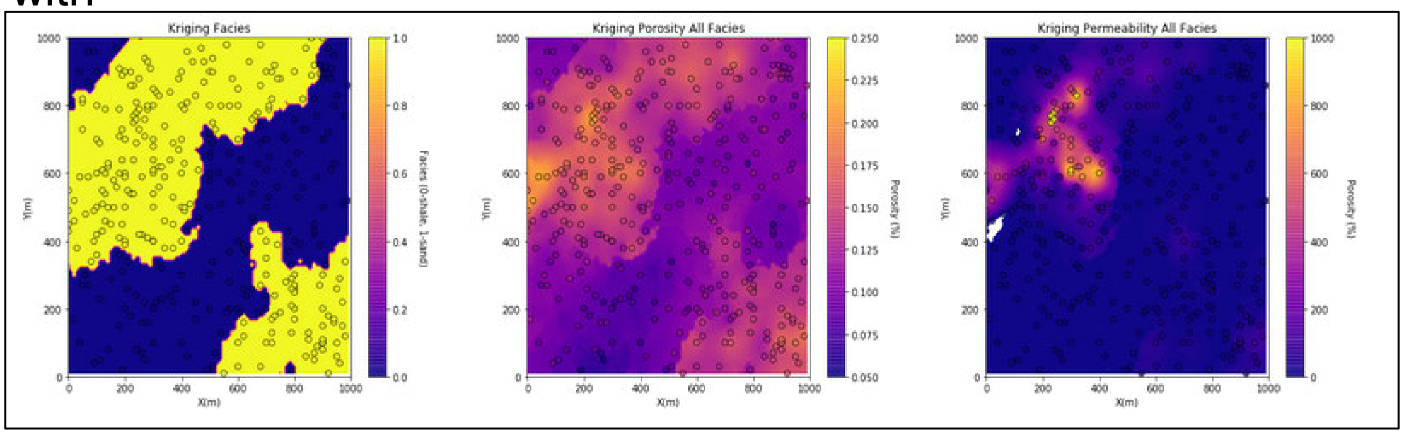
- **Kriging Workflow with GeostatsPy**

Kriging Spatial Estimation Workflow with GeostatsPy



Let's walkthrough a more thorough a spatial estimation workflow:

- facies by indicator kriging
- porosity by-facies with simple kriging



Python Jupyter kriging (GeostatsPy_kriging.ipynb).

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