Exercise 8

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8.2 b)

Derviative of L2 regularizer from Equation 3, which is simply:

$$|| \nabla x ||_2^2 \tag{1}$$

Definition of $|| X ||_2$ (from `https://en.wikipedia.org/wiki/Norm_(mathematics)#Euclidean_norm`):

$$|| X ||_2 = \sqrt{x_1^2 + ... + x_n^2} \tag{2}$$

Therefore following equation comes up:

$$f(x) = || x ||_2^2 = ((\sum_{k=1}^{n} x_k^2)^{\frac{1}{2}})^2 = \sum_{k=1}^{n} x_k^2 \tag{3}$$

Now you can take a look at the gradient

$$\frac{\partial}{\partial x_j} f(x) = \frac{\partial}{\partial x_j} \sum_{k=1}^{n} x_k^2 = \sum_{k=1}^{2} \frac{\partial}{\partial x_j} x_k^2 \tag{4}$$

The term $\frac{\partial}{\partial x_j} x_k^2$ evaluates to 0 if $j \neq k$ and to $2x_j$ else, which you can use in Eq(4):

$$\frac{\partial}{\partial x_j} f(x) = 2x_j \tag{5}$$

It follows that:

$$|| \nabla x ||_2^2 = 2x \tag{6}$$