FACULTY OF COMPUTING
UTM Johor Bahru

MCSD1133: OPERATIONAL RESEARCH AND OPTIMIZATION

Project

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TABLE OF CONTENTS

Introduction

Sally, the manager of the North Atlantic office of the Miharja Shipping and Transport Company, is faced with the task of negotiating a new shipping contract with ChemPro, a chemical manufacturing company. The contract involves transporting hazardous chemical waste from six different plants to three waste disposal sites. Given the potential risks associated with transporting hazardous materials and the complex logistics involved, Sally needs to determine the most cost-effective shipping strategy.

Objectives

In this analysis, we will explore two transportation models to assist Sally in making an informed decision. The objectives of this analysis are divided into two sections:

a) Direct Shipping Model: We will develop a transportation model in Excel Solver and Python to find the optimal routes for shipping waste directly from the six plants to the three waste disposal sites. This model will help determine the minimum transportation cost for this approach.

b) Transshipment Model: We will develop a transshipment model using Excel Solver and Python, allowing us to use the six plants as intermediate shipping points. This model will help us find the optimal cost when using transshipment.

After solving both models, we will interpret the results in a subsequent section and provide recommendations on which approach is more cost-effective and practical for Sally to implement in her contract proposal to ChemPro. By comparing the two models, Sally can make an informed decision that balances cost efficiency and logistics complexity, ultimately benefiting both Miharja Shipping and ChemPro.

Part A

In Part A of our analysis, we explore a transportation model focused on shipping waste directly from the six plants to the three waste disposal sites. We have a set of plants (Kingsport, Danville, Macon, etc.) and three waste disposal sites (Whitewater, Los Canos, Duras). The spreadsheet is set up to show the supply of waste shipped from each plant, the demand at each disposal site, and the shipping costs associated with each route from plant to disposal site. The goal is to minimize the total cost of shipping the waste while meeting the demand at each disposal site and equal toing the supply at each plant. Introducing a dummy plant is a common technique in optimization problems, ensuring supply and demand balance without altering solution feasibility. When total supply exceeds demand, as in this case with a surplus of 27 units, a dummy destination (plant) can be added to absorb the excess without affecting real locations or artificially inflating costs. In this context, a dummy plant with a supply of 27 units is introduced to maintain a balanced transportation model, acting as a sink for excess demand. This technique ensures an efficient and optimal transportation plan while accommodating supply-demand discrepancies.

Objective Function

Minimize total cost Z across all shipping paths:

$$Z = 12x_{1A} + 15x_{1B} + 17x_{1C} + 14x_{2A} + 9x_{2B} + 10x_{2C} + 13x_{3A} + 20x_{3B} + 11x_{3C} + 17x_{4A} + 16x_{4B} + 19x_{4C} + 7x_{5A} + 14x_{5B} + 12x_{5C} + 22x_{6A} + 16x_{6B} + 18x_{6C}$$

Subject to:

Supply Constraints for each plant:

$$x_{1A} + x_{1B} + x_{1C} = 35\ (Supply\ from\ Kingsport)$$
$$x_{2A} + x_{2B} + x_{2C} = 26\ (Supply\ from\ Danville)$$
$$x_{3A} + x_{3B} + x_{3C} = 42\ (Supply\ from\ Macon)$$
$$x_{4A} + x_{4B} + x_{4C} = 53\ (Supply\ from\ Selma)$$
$$x_{5A} + x_{5B} + x_{5C} = 29\ (Supply\ from\ Columbus)$$
$$x_{6A} + x_{6B} + x_{6C} = 38\ (Supply\ from\ Allentown)$$

$x_{7A} + x_{7B} + x_{7C} = 27$ ($Supply\ from\ Dummy\ Plant$)

Demand Constraints for each waste disposal site:

$x_{1A} + x_{2A} + x_{3A} + x_{4A} + x_{5A} + x_{6A} + x_{7A} = 65$ ($Demand\ at\ Whitewater$)

$x_{1B} + x_{2B} + x_{3B} + x_{4B} + x_{5B} + x_{6B} + x_{7B} = 80$ ($Demand\ at\ Los\ Canos$)

$x_{1C} + x_{2C} + x_{3C} + x_{4C} + x_{5C} + x_{6C} + x_{7C} = 105$ ($Demand\ at\ Duras$)

And the non-negativity constraints:

$x_{mn} \geq 0$

Where x_{mn} represents the tons of waste shipped from each plant m,m =1,2,3,4,5,6,7 (including Dummy Plant) to each waste disposal site n,n=A,B,C.

Excel Solver

Objective:

The primary objective is to minimize the value in cell C15, which represents the total cost of transportation.

Variable Cells:

The variables to be optimized are located in cells C5:E11, representing the quantities of waste shipped from each plant to each disposal site.

Constraints:

1. The sum of waste shipped from each plant must equal to the plant's supply.
2. The sum of waste received at each disposal site must meet the site's demand.
3. All shipping quantities must be non-negative; negative quantities are not allowed as you can't ship a negative amount of waste.

Solver Settings:

The Solver is configured to use the Simplex LP algorithm, which is appropriate for linear programming problems. This algorithm will help find the optimal solution for minimizing the total transportation cost while satisfying the defined constraints.

1.

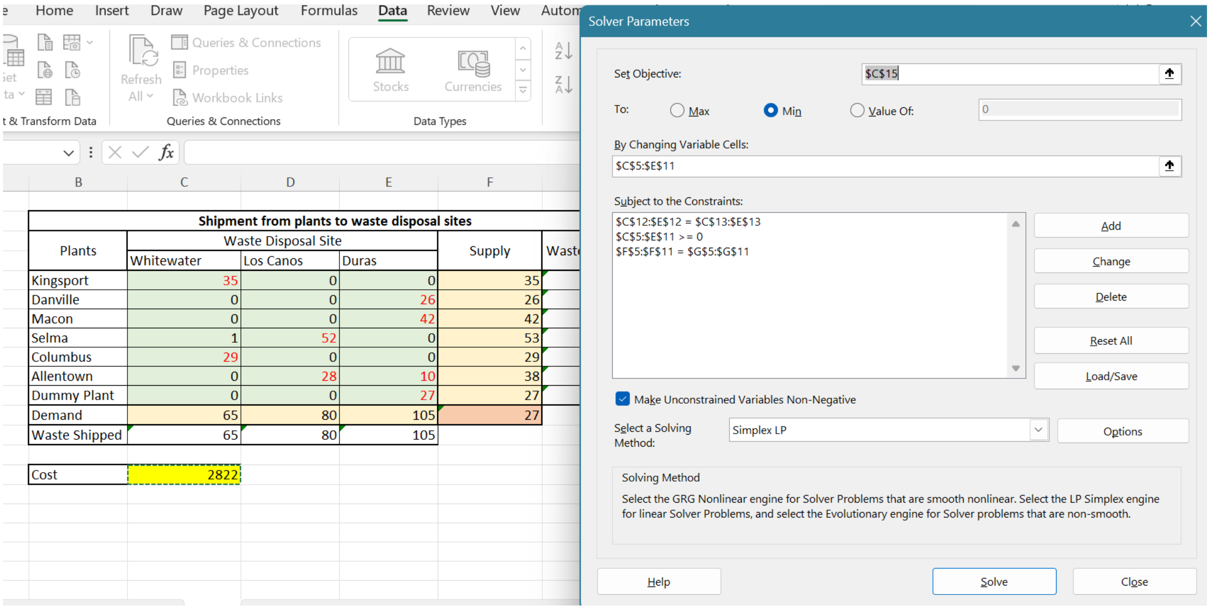
Shipment from plants to waste disposal sites					
Plants	Waste Disposal Site			Supply	Waste Shipped
	Whitewater	Los Canos	Duras		
Kingsport				35	=SUM(C5:E5)
Danville				26	=SUM(C6:E6)
Macon				42	=SUM(C7:E7)
Selma				53	=SUM(C8:E8)
Columbus				29	=SUM(C9:E9)
Allentown				38	=SUM(C10:E10)
Dummy Plant				27	=SUM(C11:E11)
Demand	65	80	105	=SUM(C12:E12)-SUM(F5:F10)	
Waste Shipped	=SUM(C5:C11)	=SUM(D5:D11)	=SUM(E5:E11)		
Cost	=SUMPRODUCT(C5:E11,J5:L11)				

2.

Shipment from plants to waste disposal sites					
Plants	Waste Disposal Site			Supply	Waste Shipped
	Whitewater	Los Canos	Duras		
Kingsport				35	0
Danville				26	0
Macon				42	0
Selma				53	0
Columbus				29	0
Allentown				38	0
Dummy Plant				27	0
Demand	65	80	105	27	
Waste Shipped	0	0	0		
Cost	0				

Cost of shipping a barrel of waste from plants to waste disposal sites			
Plants	Waste Disposal Site		
	Whitewater	Los Canos	Duras
Kingsport	12	15	17
Danville	14	9	10
Macon	13	20	11
Selma	17	16	19
Columbus	7	14	12
Allentown	22	16	18
Dummy Plant	0	0	0

3.



Shipment from plants to waste disposal sites				
Plants	Waste Disposal Site			Supply
	Whitewater	Los Canos	Duras	
Kingsport	35	0	0	35
Danville	0	0	26	26
Macon	0	0	42	42
Selma	1	52	0	53
Columbus	29	0	0	29
Allentown	0	28	10	38
Dummy Plant	0	0	27	27
Demand	65	80	105	27
Waste Shipped	65	80	105	
Cost	2822			

1. The optimal shipping quantities, represented in green cells, provide valuable insights into the solution of the optimization problem. These quantities determine the most cost-effective distribution of waste from each plant to each waste disposal site while minimizing transportation costs.

- Kingsport: This plant is shipping 35 units of waste to Whitewater, indicating that it is most cost-effective to fulfill the entire demand at Whitewater. No waste is being shipped to Los Canos or Duras, suggesting that these destinations may not offer cost-efficient transportation options or are not necessary to meet demand.

- Danville: Danville is not shipping waste to Whitewater or Los Canos. Instead, it is sending 26 units to Duras, which implies that Duras is the most cost-effective destination for Danville's waste.

- Macon: Similar to Danville, Macon is shipping all 42 units of waste to Duras, indicating that Duras is the most efficient destination for Macon.

- Selma: Selma's shipping pattern is a bit more diversified. It is sending 1 unit to Whitewater and 52 units to Los Canos. This strategy covers the minimal extra demand at Whitewater while predominantly serving Los Canos.

- Columbus: Columbus is shipping 29 units exclusively to Los Canos, suggesting that this destination offers the most cost-efficient solution for Columbus.

- Allentown: Allentown's strategy involves shipping 28 units to Whitewater and 10 units to Duras, with none going to Los Canos. This distribution optimally divides its supply to meet demands in two different locations while maintaining cost efficiency.

- Dummy Plant: The dummy plant, a theoretical construct introduced to balance the model when there is excess supply, is shipping 27 units to Duras. The inclusion of the dummy plant indicates that there might be more supply than demand, and the model utilizes this plant to ensure all constraints are met without affecting the operations of real plants.

Each plant's shipping strategy is a direct result of the cost-minimization goal. These shipping quantities ensure that the demands at each waste disposal site are exactly met, as indicated by the 'Demand' and 'Waste Shipped' rows for Whitewater, Los Canos, and Duras, which all match at 65, 80, and 105 units, respectively. No plant is shipping more waste than it supplies, and all disposal site demands are precisely fulfilled.

This optimization demonstrates the power of linear programming in operational management, especially in logistics and supply chain optimization. By using Solver in Excel, businesses can solve complex allocation problems, ensuring that resources are not wasted, and costs are minimized.

2. The Minimum Total Cost (Yellow Cell) with a value of 2,822 is the ultimate objective achieved in this optimization process. This figure signifies the lowest feasible total cost, considering the imposed constraints and the associated shipping expenses. It is calculated as the summation of the product of shipping quantities and their respective costs. In optimization problems like this one, attaining the lowest possible total cost is usually the paramount goal.

Python

The provided Python code snippet in Appendix outlines the steps taken to solve the linear programming problem using the PuLP library, which is designed for such optimization problems.

1. Importing the PuLP Library: The first step is to import PuLP, a popular optimization library in Python that allows for modeling and solving linear programming problems.

2. Defining Supply Nodes and Capacities: The plants and their respective supplies are defined, creating a 'supply' dictionary with dummy plant of 27 units. Similarly, the demand nodes (Sites) and their demands are also specified.

3. Defining Shipping Costs: The shipping costs from each plant to each site are set up in a list and then converted into a dictionary with the makeDict function. This allows for easy lookup of the cost associated with shipping between each pair of nodes.

4. Creating the Problem Variable: An LP problem named "Waste_Distribution_Transportation_Model_Question_a" is created with the goal of minimizing costs.

5. Creating Possible Routes: A list of tuples representing all potential shipping routes (from every plant to each site) is constructed.

6. Creating Decision Variables: For each route, a decision variable is created, which will be used to determine the optimal shipping quantity.

7. Objective Function: The objective function, which is the total cost of transportation, is defined as the sum of the product of the shipping quantities (decision variables) and their respective costs.

8. Supply Constraints: For each plant, a constraint is added to ensure that the sum of products shipped from it does equal to its supply.

9. Demand Constraints: For each site, a constraint is added to ensure that the sum of products received meets its demand.

10. Writing the Problem Data: The problem data is written to an LP file, which is a standard file format for representing linear programming problems.

11. Solving the Problem: The problem is solved using PuLP's default solver.

12. Output: The status of the solution is printed, followed by the optimal value for each decision variable (quantity shipped on each route) and the total cost of transportation.

13. Network Diagram Visualization: Lastly, the networkx and matplotlib libraries are used to create a visualization of the transportation network, showing the flow of goods from plants to sites.

The Python-based linear programming solution successfully found an optimal solution to the problem, matching the results from the Excel Solver:

```
Route_Allentown_Duras = 10.00
Route_Allentown_Los_Canos = 28.00
Route_Allentown_Whitewater = 0.00
Route_Columbus_Duras = 0.00
Route_Columbus_Los_Canos = 0.00
Route_Columbus_Whitewater = 29.00
Route_Danville_Duras = 26.00
Route_Danville_Los_Canos = 0.00
Route_Danville_Whitewater = 0.00
Route_Dummy_Plant_Duras = 27.00
Route_Dummy_Plant_Los_Canos = 0.00
Route_Dummy_Plant_Whitewater = 0.00
Route_Kingsport_Duras = 0.00
Route_Kingsport_Los_Canos = 0.00
Route_Kingsport_Whitewater = 35.00
Route_Macon_Duras = 42.00
Route_Macon_Los_Canos = 0.00
Route_Macon_Whitewater = 0.00
Route_Selma_Duras = 0.00
Route_Selma_Los_Canos = 52.00
Route_Selma_Whitewater = 1.00
Total Cost of Transportation = RM 2822.00
```

1. Optimal Shipping Quantities: The quantities in the matrix represent the amount of waste shipped from each plant to each disposal site, in the same structure as the Excel spreadsheet:

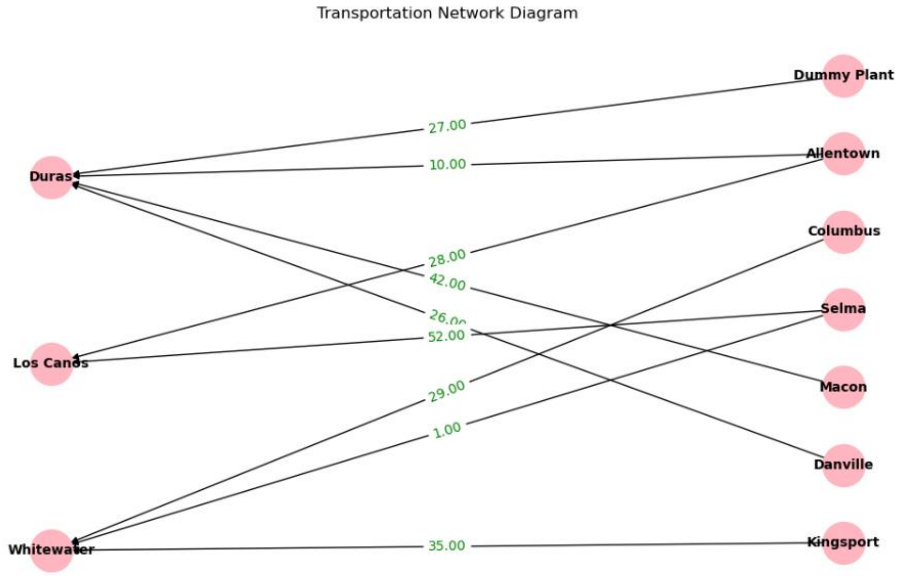
- Kingsport ships 35 units to Whitewater.
- Danville ships 26 units to Duras.
- Macon ships 42 units to Duras.
- Selma ships 1 unit to Whitewater and 52 units to Los Canos.
- Columbus ships 29 units to Whitewater.
- Allentown ships 28 units to Los Canos and 10 units to Duras.
- The Dummy Plant ships 27 units to Duras.

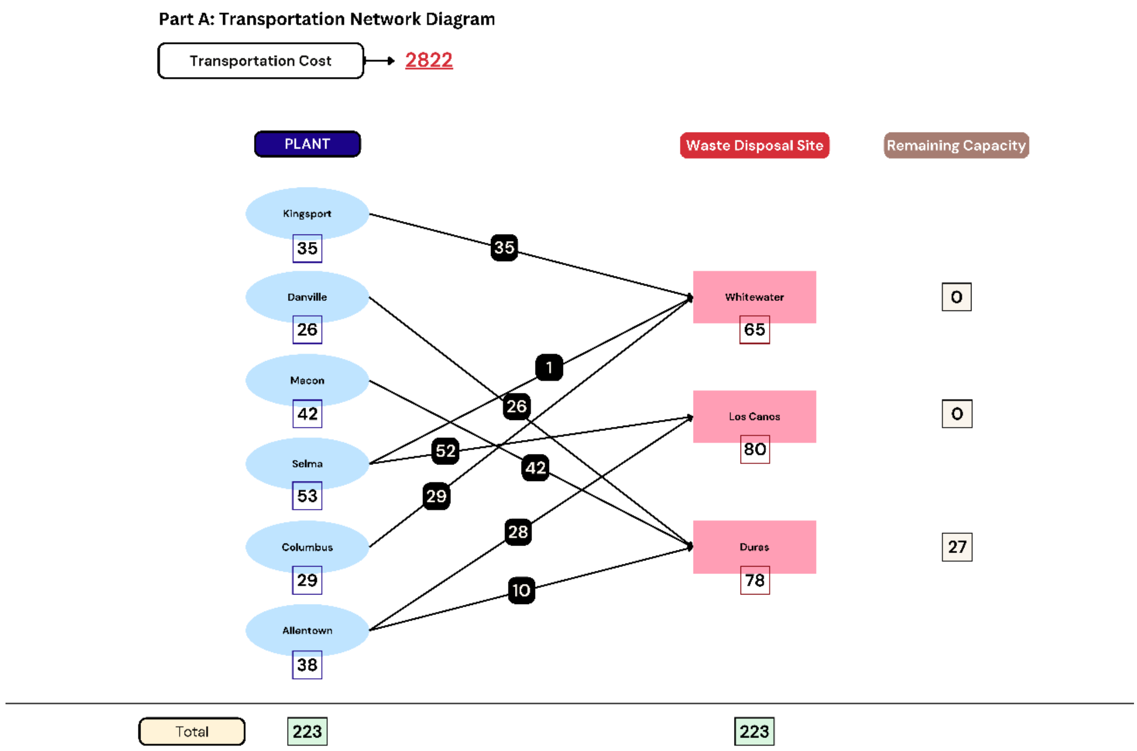
2. Minimum Total Cost: The minimum total cost achieved through this shipping plan is RM 2,822, which is the same as the cost found by the Excel Solver.

This confirms that both the Excel Solver and the Python optimization approach yield the same optimal solution for the given supply chain problem. The optimal quantities for shipping are consistent across both platforms, and the total cost is identical, indicating that the linear programming model was correctly specified in both cases and the Solver used in Excel and the optimization algorithm used in Python are equivalent in terms of their results for this problem.

Network Diagram

Network diagrams are important in transportation problems because they offer a visual representation, aiding route planning, constraint identification, and mathematical modelling. They are crucial for optimization and facilitate effective communication and scenario analysis, simplifying complex logistics management.





Part B

In Part B, we are dealing with a transshipment model where each plant not only supplies waste to disposal sites but can also act as an intermediate transfer point. This allows for the waste to be shipped indirectly through other plants before reaching its final disposal site.

Objective Function

Minimize total cost Z across all shipping paths:

$$Z = 12x_{1A} + 15x_{1B} + 17x_{1C} + 14x_{2A} + 9x_{2B} + 10x_{2C} + 13x_{3A} + 20x_{3B} + 11x_{3C} + 17x_{4A} + 16x_{4B} + 19x_{4C} + (similar\ terms\ for\ the\ remaining\ direct\ plant\ to\ disposal\ site\ costs) + 6x_{12} + 4x_{13} + 9x_{14} + 7x_{15} + 8x_{17} + 6x_{21} + 11x_{23} + 10x_{24} + 12x_{25} + 7x_{26} + (similar\ terms\ for\ the\ remaining\ inter - plant\ shipping\ costs)$$

Subject to:

Supply Constraints for each plant:

$x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} = 35$ $(Waste\ Production\ at\ Kingsport)$

$x_{21} + x_{22} + x_{23} + x_{24} + x_{25} + x_{26} = 26$ $(Waste\ Production\ at\ Danville$

$x_{31} + x_{32} + x_{33} + x_{34} + x_{35} + x_{36} = 42$ $(Waste\ Production\ at\ Macon)$

$x_{41} + x_{42} + x_{43} + x_{44} + x_{45} + x_{46} = 53$ $(Waste\ Production\ at\ Selma)$

$x_{51} + x_{52} + x_{53} + x_{54} + x_{55} + x_{56} = 29$ $(Waste\ Production\ at\ Columbus)$

$x_{61} + x_{62} + x_{63} + x_{64} + x_{65} + x_{66} = 38$ $(Waste\ Production\ at\ Allentown)$

$x_{71} + x_{72} + x_{73} + x_{74} + x_{75} + x_{76} = 27$ $(Waste\ Production\ at\ Dummy\ Plant)$

Demand Constraints for each waste disposal site:

$x_{1A} + x_{2A} + x_{3A} + x_{4A} + x_{5A} + x_{6A} + x_{7A} = 65$ $(Demand\ at\ Whitewater)$

$x_{1B} + x_{2B} + x_{3B} + x_{4B} + x_{5B} + x_{6B} + x_{7B} = 80$ $(Demand\ at\ Los\ Canos)$

$x_{1C} + x_{2C} + x_{3C} + x_{4C} + x_{5C} + x_{6C} + x_{7C} = 105$ $(Demand\ at\ Duras)$

$x_{11} + x_{21} + x_{31} + x_{41} + x_{51} + x_{61} - x_{1A} - x_{1B} - x_{1C} = 0$ $(Shipped\ into = out\ Kingsport)$

$x_{12} + x_{22} + x_{32} + x_{42} + x_{52} + x_{62} - x_{2A} - x_{2B} - x_{2C} = 0$ $(Shipped\ into = out\ Danville)$

$x_{13} + x_{23} + x_{33} + x_{43} + x_{53} + x_{63} - x_{3A} - x_{3B} - x_{3C} = 0$ $(Shipped\ into = out\ Macon) \ldots and\ so\ on\ for\ the\ remaining\ plants$

And the non-negativity constraints:

$x_{mn} \geq 0$

Where x_{mn} represents the tons of waste shipped from each plant m,m =1,2,3,4,5,6,7 (including Dummy Plant) to each waste disposal site n,n=A,B,C.s

Excel Solver

Objective:

Minimize the total cost of transportation, including the costs associated with transshipment through intermediate plants.

Variable Cells:

The variables to be optimized are in two sets of cells:

1. C5:H10, representing the quantities of waste shipped from each plant to another plant or directly to the disposal site.
2. C16:E22, representing the quantities of waste shipped from the intermediate plants to the disposal sites.

Constraints:

1. The amount of waste shipped from each plant must equal to the supply at that plant.
2. The amount of waste shipped to each disposal site must satisfy the site's demand.
3. The amount of waste shipped from a plant to another plant plus the waste shipped directly to disposal sites must equal to the supply at the originating plant.

4. The amount of waste received by a plant from other plants plus its own production should not be more than the sum of what it sends to other plants and disposal sites.
5. All shipping quantities must be non-negative.

Solver Settings:

The Solver should still use the Simplex LP algorithm suitable for linear problems with transshipment considerations.

1.

Shipment from plants to intermediate plants								
Plants	Plants						Supply	Waste Shipped
	Kingsport	Danville	Macon	Selma	Columbus	Allentown		
Kingsport							35	=SUM(C5:H5)
Danville							26	=SUM(C6:H6)
Macon							42	=SUM(C7:H7)
Selma							53	=SUM(C8:H8)
Columbus							29	=SUM(C9:H9)
Allentown							38	=SUM(C10:H10)
Dummy							27	=SUM(C11:H11)
Waste Shipped	=SUM(C5:C11)	=SUM(D5:D11)	=SUM(E5:E11)	=SUM(F5:F11)	=SUM(G5:G11)	=SUM(H5:H11)		

Shipment from intermediate plants to waste disposal sites				
Plants	Waste Disposal Site			Waste Shipped
	Whitewater	Los Canos	Duras	
Kingsport				=SUM(C17:E17)
Danville				=SUM(C18:E18)
Macon				=SUM(C19:E19)
Selma				=SUM(C20:E20)
Columbus				=SUM(C21:E21)
Allentown				=SUM(C22:E22)
Demand	65	80	105	
Waste Shipped	=SUM(C17:C22)	=SUM(D17:D22)	=SUM(E17:E22)	

Transhipment flow	
Kingsport	=C12-F17
Danville	=D12-F18
Macon	=E12-F19
Selma	=F12-F20
Columbus	=G12-F21
Allentown	=H12-F22

Cost	=SUMPRODUCT(C5:H11,M5:R11)+SUMPRODUCT(C17:E22,M17:O22)

2.

Shipment from plants to intermediate plants								
Plants	Plants						Supply	Waste Shipped
	Kingsport	Danville	Macon	Selma	Columbus	Allentown		
Kingsport							35	0
Danville							26	0
Macon							42	0
Selma							53	0
Columbus							29	0
Allentown							38	0
Dummy							27	0
Waste Shipped	0	0	0	0	0	0		

Cost of shipping a barrel of waste from plants to intermediate plants						
Plants	Plants					
	Kingsport	Danville	Macon	Selma	Columbus	Allentown
Kingsport	0	6	4	9	7	8
Danville	6	0	11	10	12	7
Macon	5	11	0	3	7	15
Selma	9	10	3	0	3	16
Columbus	7	12	7	3	0	14
Allentown	8	7	15	16	14	0
Dummy	0	0	0	0	0	0

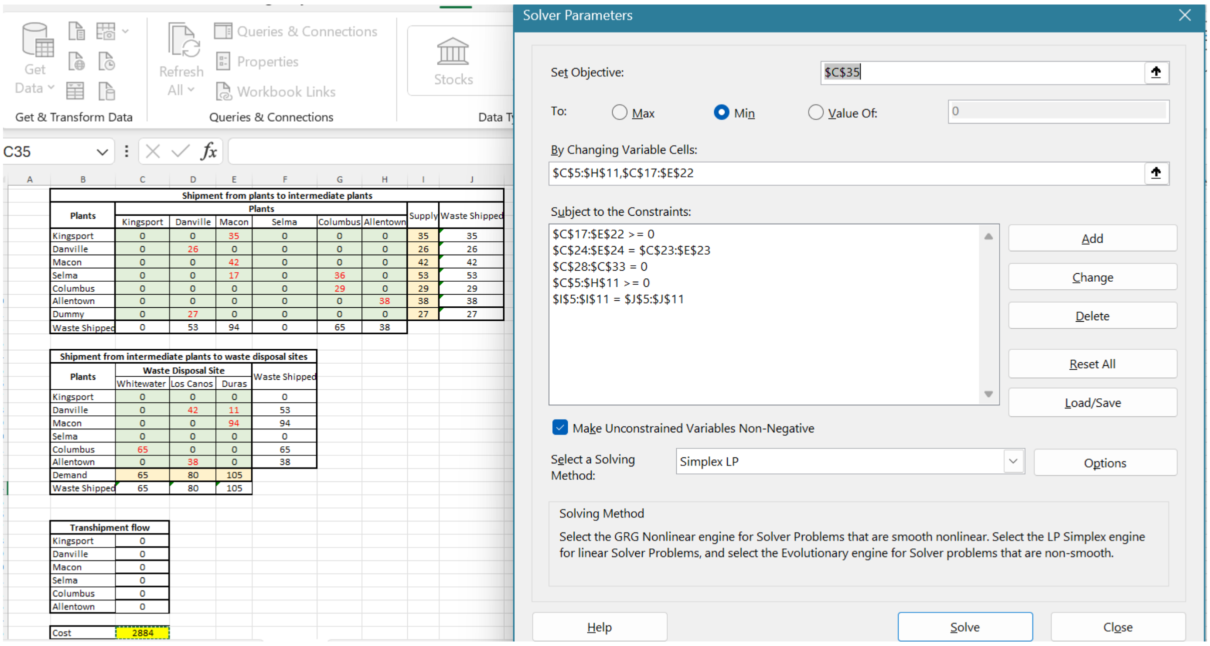
Shipment from intermediate plants to waste disposal sites				
Plants	Waste Disposal Site			Waste Shipped
	Whitewater	Los Canos	Duras	
Kingsport				0
Danville				0
Macon				0
Selma				0
Columbus				0
Allentown				0
Demand	65	80	105	
Waste Shipped	0	0	0	

Cost of shipping a barrel of waste from intermediate plants to waste disposal sites			
Plants	Waste Disposal Site		
	Whitewater	Los Canos	Duras
Kingsport	12	15	17
Danville	14	9	10
Macon	13	20	11
Selma	17	16	19
Columbus	7	14	12
Allentown	22	16	18

Transhipment flow	
Kingsport	0
Danville	0
Macon	0
Selma	0
Columbus	0
Allentown	0

Cost	0

3.



Furthermore, within the framework of Problem B, the inclusion of a dummy plant serves a specific purpose. It is introduced to ensure that supply and demand within the system remain balanced. To achieve this equilibrium, a constraint is imposed, indicating that the total amount 'shipped' to or from the dummy plant must equal 27. This constraint acts as a safeguard, preventing any supply-demand discrepancies and reinforcing the model's ability to provide efficient and feasible solutions.

1. The green cells in the Excel Solver output represent the optimal quantities to be shipped along each route between plants and from plants to waste disposal sites, given the objective of minimizing the total cost of transportation.

Inter-plant Shipments:

- Kingsport ships 35 units to Macon.
- Danville ships/stores 26 units to itself.
- Macon ships/Stores 42 units to itself.
- Selma ships 17 units to Macon and 36 units to Columbus.
- Columbus ships/stores 29 units to itself.
- Allentown ships/stores 38 units to itself.
- Dummy plant ships 27 units to Danville.

Shipments from Plants to Waste Disposal Sites:

- Kingsport does not send any waste directly to disposal sites.
- Danville sends 42 units directly to Los Canos and 11 units to Duras.
- Macon sends 94 units to Duras.
- Selma does not send any waste directly to disposal sites.
- Columbus sends 65 units directly to Whitewater.
- Allentown sends 38 units to Los Canos.

Supply and Waste Shipped from Plants:

All plants have their supply fully shipped out, whether directly to disposal sites, to other plants, or a combination of both.

2. In Part B of the optimization problem, the total transportation cost is calculated to be RM 2884. This cost represents the cumulative expense associated with transporting waste from the various plants to the waste disposal sites, considering the use of intermediate plants to minimize costs while meeting supply and demand constraints.

Python

1. Importing the PuLP Library:
   Begin by importing PuLP, a powerful library for linear optimization in Python.

2. Defining Nodes and Capacities:
   Specify dictionaries for the supplies at each plant and the demands at each disposal site. Additionally, define each plant as a potential transshipment point with the ability to send and receive shipments. When setting up your supply dictionary, include the dummy plant with its "supply" equal to the excess amount or set it up with a large supply number that will not constrain the model.

3. Defining Shipping Costs:
   Set up a nested dictionary or a 2D list that includes the shipping costs between all nodes, including the cost from each plant to every other plant and to each disposal site.

4. Creating the Problem Variable:
   Create an LP problem in PuLP named "Waste_Distribution_Transshipment_Model_Question_b" with the goal of minimizing the total transshipment cost.

5. Creating Possible Routes:
   Generate a list of tuples representing all potential shipping routes, including inter-plant shipments, and plant to disposal site shipments.

6. Creating Decision Variables:
   Establish a decision variable for every route defined in the possible routes. These variables will be used to find the optimal shipping and transshipment quantities.

7. Objective Function:
   Define the objective function as the sum of the products of the decision variables (shipping quantities) and their corresponding costs across all routes.

8. Transshipment Constraints:
   Add constraints for each plant to regulate the inflow and outflow, ensuring that the total amount shipped to and from a plant (including to itself for transshipment) does not exceed its supply and meets its demand if it is also a disposal site.

9. Supply and Demand Constraints:

- For supply constraints, make sure the sum of shipments from each plant is equal to its available supply.
- For demand constraints, ensure that the total shipments received at each disposal site fulfill its demand.

10. Writing the Problem Data:
    Write the problem data to an LP file for record-keeping and potential reuse.

11. Solving the Problem:

    Solve the problem using PuLP's chosen solver.

12. Output:

    Print the status of the solution to ensure it has been solved successfully. Then, for each route, print the decision variable's name and its optimal value to understand the transshipment strategy. Finally, display the minimized total cost of transportation and transshipment.

The Python-based linear programming solution successfully found an optimal solution to the problem, matching the results from the Excel Solver:

```
Route_1_Allentown_Allentown = 0.00
Route_1_Allentown_Columbus = 0.00
Route_1_Allentown_Danville = 38.00
Route_1_Allentown_Dummy_Plant = 0.00
Route_1_Allentown_Kingsport = 0.00
Route_1_Allentown_Macon = 0.00
Route_1_Allentown_Selma = 0.00
Route_1_Columbus_Allentown = 0.00
Route_1_Columbus_Columbus = 29.00
Route_1_Columbus_Danville = 0.00
Route_1_Columbus_Dummy_Plant = 0.00
Route_1_Columbus_Kingsport = 0.00
Route_1_Columbus_Macon = 0.00
Route_1_Columbus_Selma = 0.00
Route_1_Danville_Allentown = 0.00
Route_1_Danville_Columbus = 0.00
Route_1_Danville_Danville = 26.00
Route_1_Danville_Dummy_Plant = 0.00
Route_1_Danville_Kingsport = 0.00
Route_1_Danville_Macon = 0.00
Route_1_Danville_Selma = 0.00
Route_1_Dummy_Plant_Allentown = 0.00
Route_1_Dummy_Plant_Columbus = 0.00
Route_1_Dummy_Plant_Danville = 27.00
Route_1_Dummy_Plant_Dummy_Plant = 0.00
Route_1_Dummy_Plant_Kingsport = 0.00
Route_1_Dummy_Plant_Macon = 0.00
Route_1_Dummy_Plant_Selma = 0.00
Route_1_Kingsport_Allentown = 0.00
Route_1_Kingsport_Columbus = 0.00
Route_1_Kingsport_Danville = 0.00
Route_1_Kingsport_Dummy_Plant = 0.00
Route_1_Kingsport_Kingsport = 0.00
Route_1_Kingsport_Macon = 35.00
Route_1_Kingsport_Selma = 0.00
Route_1_Macon_Allentown = 0.00
Route_1_Macon_Columbus = 0.00
Route_1_Macon_Danville = 0.00
Route_1_Macon_Dummy_Plant = 0.00
Route_1_Macon_Kingsport = 0.00
Route_1_Macon_Macon = 42.00
Route_1_Macon_Selma = 0.00
Route_1_Selma_Allentown = 0.00
Route_1_Selma_Columbus = 36.00
Route_1_Selma_Danville = 0.00
Route_1_Selma_Dummy_Plant = 0.00
Route_1_Selma_Kingsport = 0.00
Route_1_Selma_Macon = 17.00
Route_1_Selma_Selma = 0.00
Route_2_Allentown_Duras = 0.00
Route_2_Allentown_Los_Canos = 0.00
Route_2_Allentown_Whitewater = 0.00
Route_2_Columbus_Duras = 0.00
Route_2_Columbus_Los_Canos = 0.00
Route_2_Columbus_Whitewater = 65.00
Route_2_Danville_Duras = 11.00
Route_2_Danville_Los_Canos = 80.00
Route_2_Danville_Whitewater = 0.00
Route_2_Kingsport_Duras = 0.00
Route_2_Kingsport_Los_Canos = 0.00
Route_2_Kingsport_Whitewater = 0.00
Route_2_Macon_Duras = 94.00
Route_2_Macon_Los_Canos = 0.00
Route_2_Macon_Whitewater = 0.00
Route_2_Selma_Duras = 0.00
Route_2_Selma_Los_Canos = 0.00
Route_2_Selma_Whitewater = 0.00
Total Cost of Transportation = RM 2884.00
```

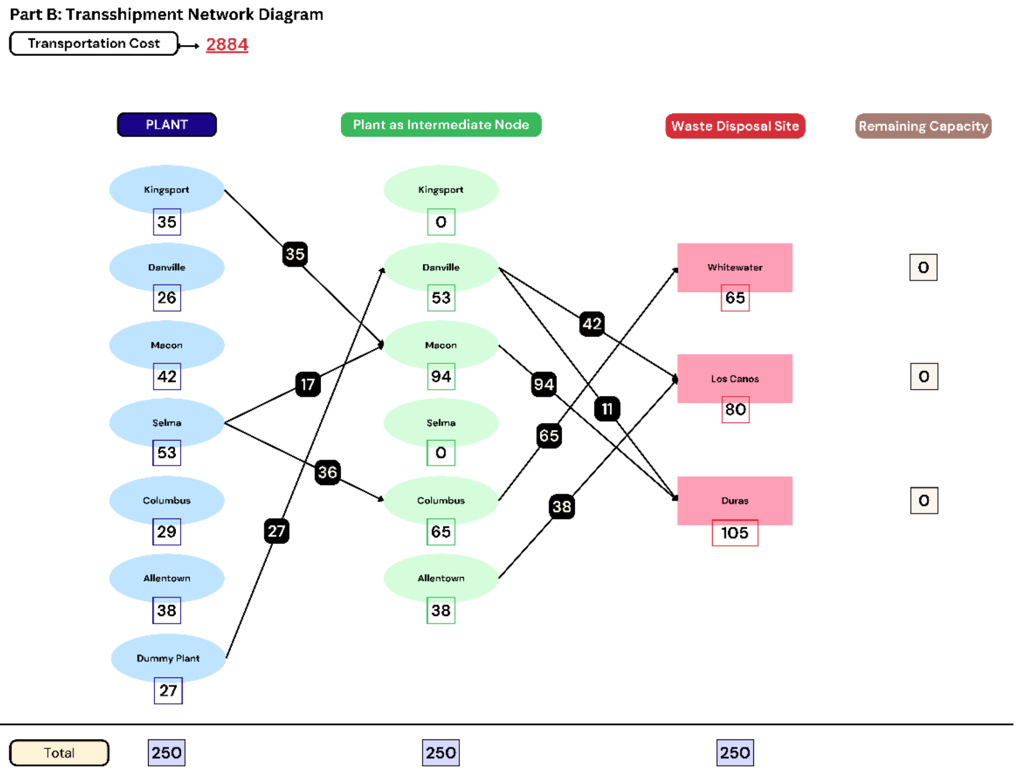
1. Optimal Shipping Quantities: The quantities in the matrix represent the amount of waste shipped from each plant to each disposal site, in the same structure as the Excel spreadsheet outputs.

2. Minimum Total Cost: The minimum total cost achieved through this shipping plan is RM 2,884, which is the same as the cost found by the Excel Solver.

This confirms that both the Excel Solver and the Python optimization approach yield the same optimal solution for the given supply chain problem. The optimal quantities for shipping are consistent across both platforms, and the total cost is identical, indicating that the linear programming model was correctly specified in both cases and the Solver used in Excel and the optimization algorithm used in Python are equivalent in terms of their results for this problem.

Network Diagram

Network diagrams are essential in transshipment models as they visually map the flow of goods through the supply chain, including intermediate nodes. They help in identifying efficient routes, bottlenecks, and cost-saving opportunities, thereby simplifying complex logistics decisions and aiding in strategic optimization of the supply network.



Part C

In analysing the results of both transportation models, it is evident that each model offers unique advantages and considerations.

a) Direct Shipping Model (Total Cost: $2,822): This model involves shipping waste directly from the six plants to the three waste disposal sites. It results in a total cost of $2,822. The key advantage of this model is its simplicity. By bypassing the need for intermediate plants, it streamlines the transportation process. In this case, Kingsport, Selma, and Allentown find cost-effective direct routes to meet demand, while other plants also contribute directly. This model ensures minimal complexity in logistics and may be preferred when efficiency in execution is paramount.

b) Transshipment Model (Total Cost: $2,884): The transshipment model introduces the concept of intermediate plants and results in a total cost of $2,884. While this model offers advantages in terms of optimizing routes and leveraging intermediate points, it leads to a slightly higher total cost. The complexity arises from the need to coordinate and manage shipments through intermediate plants, which can add operational intricacies and potentially result in increased costs.

Considering the total costs:

The direct shipping model (Part A) has a total cost of $2,822.
The transshipment model (Part B) has a slightly higher total cost of $2,884.

Given these factors, it is recommended that Sally implements the direct transportation model (Part A). It not only achieves cost savings but also provides a more adaptable and nuanced approach to waste transportation. This approach aligns with the goal of minimizing transportation costs while meeting supply and demand constraints effectively.

Conclusion

In conclusion, it is advisable for Sally to implement the direct shipping transportation model (Part A). This model offers cost-efficiency, resource optimization, and flexibility in transportation planning, aligning well with Miharja Shipping's goals and constraints in its negotiations with ChemPro.

Five key insights gained from the case study related to waste transportation optimization:

Cost Efficiency: The optimization models showcased the ability to significantly reduce transportation costs. This cost efficiency is of paramount importance, particularly in hazardous waste transportation, where minimizing expenses is crucial.

Resource Allocation: The case study demonstrates the effective allocation of waste resources from different plants to disposal sites, ensuring that the demands at each site are met while considering supply constraints. This efficient resource allocation enhances operational management.

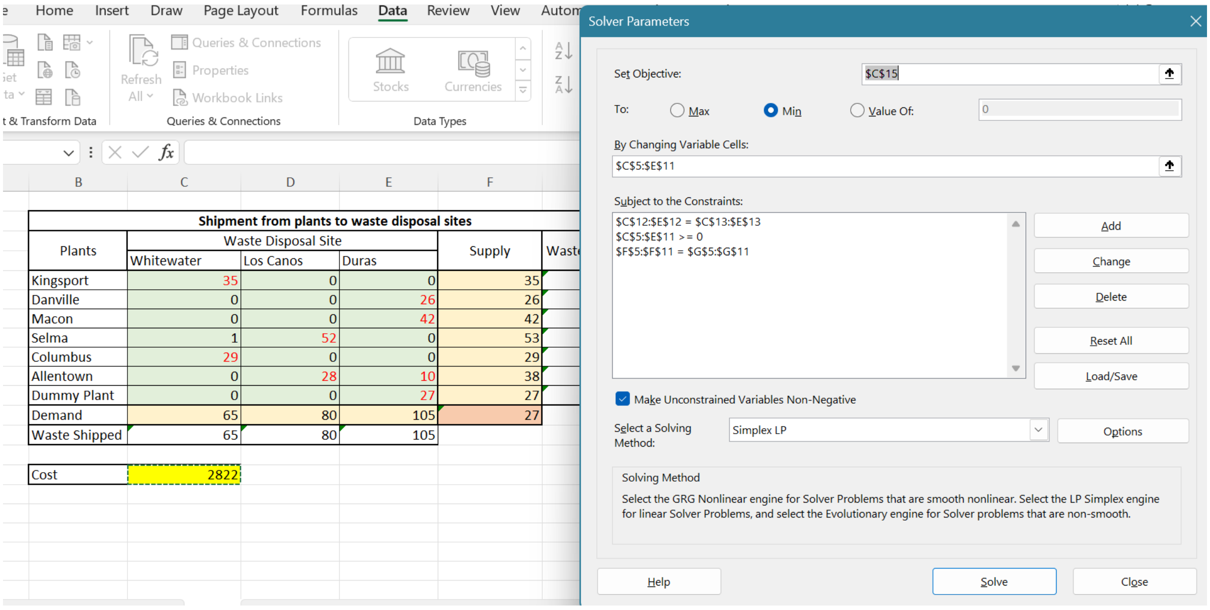
Balanced Supply and Demand: The models successfully maintain a balance between waste supply and disposal site demand. This equilibrium is vital for compliance with regulatory and safety requirements, especially when dealing with hazardous materials.

Data-Driven Decision-Making: The use of optimization techniques, such as linear programming and Solver, promotes data-driven decision-making. It provides a clear and quantitative foundation for making informed choices in logistics and supply chain strategies.

These insights collectively highlight the practical application of optimization in waste transportation, emphasizing cost reduction, resource allocation efficiency, supply-demand balance, data-driven decision-making, and flexible transportation planning as critical benefits for Miharja Shipping in its negotiations with ChemPro.

Appendixes

Part A – Excel

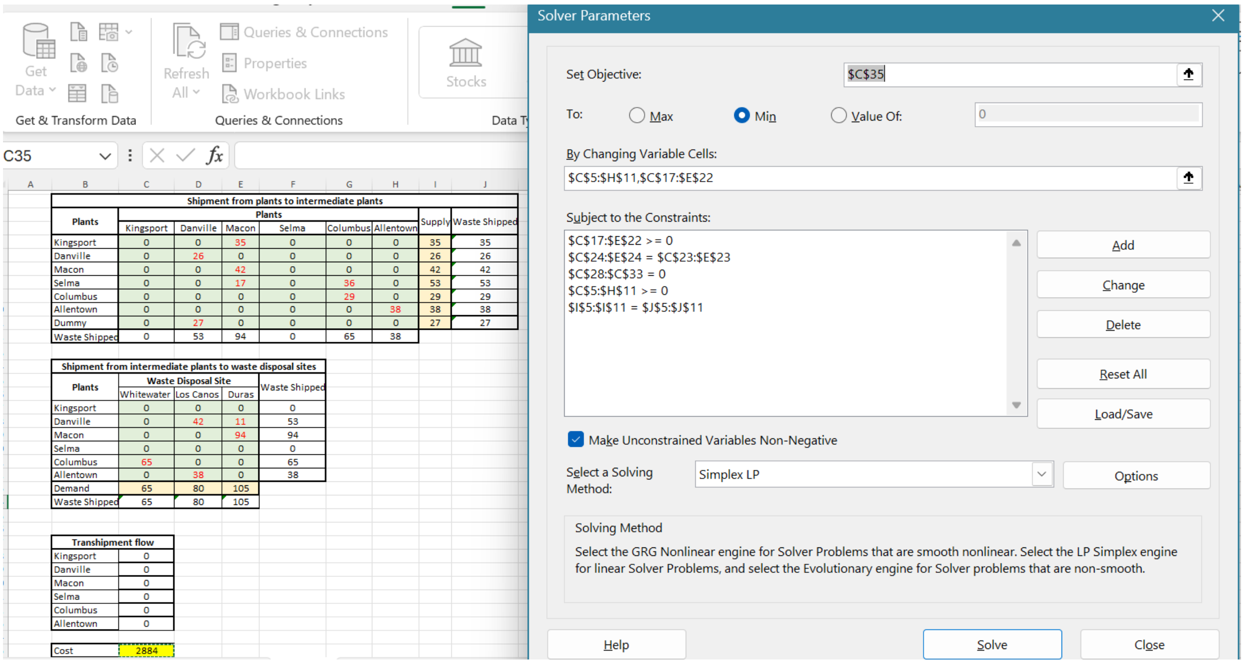


Shipment from plants to waste disposal sites					
Plants	Waste Disposal Site			Supply	Wast
	Whitewater	Los Canos	Duras		
Kingsport	35	0	0	35	
Danville	0	0	26	26	
Macon	0	0	42	42	
Selma	1	52	0	53	
Columbus	29	0	0	29	
Allentown	0	28	10	38	
Dummy Plant	0	0	27	27	
Demand	65	80	105	27	
Waste Shipped	65	80	105		
Cost	2822				

Part A – Python

```
Route_Allentown_Duras = 10.00
Route_Allentown_Los_Canos = 28.00
Route_Allentown_Whitewater = 0.00
Route_Columbus_Duras = 0.00
Route_Columbus_Los_Canos = 0.00
Route_Columbus_Whitewater = 29.00
Route_Danville_Duras = 26.00
Route_Danville_Los_Canos = 0.00
Route_Danville_Whitewater = 0.00
Route_Dummy_Plant_Duras = 27.00
Route_Dummy_Plant_Los_Canos = 0.00
Route_Dummy_Plant_Whitewater = 0.00
Route_Kingsport_Duras = 0.00
Route_Kingsport_Los_Canos = 0.00
Route_Kingsport_Whitewater = 35.00
Route_Macon_Duras = 42.00
Route_Macon_Los_Canos = 0.00
Route_Macon_Whitewater = 0.00
Route_Selma_Duras = 0.00
Route_Selma_Los_Canos = 52.00
Route_Selma_Whitewater = 1.00
Total Cost of Transportation = RM 2822.00
```

Part B – Excel



Part B – Python

```
Route_1_Allentown_Allentown = 0.00
Route_1_Allentown_Columbus = 0.00
Route_1_Allentown_Danville = 38.00
Route_1_Allentown_Dummy_Plant = 0.00
Route_1_Allentown_Kingsport = 0.00
Route_1_Allentown_Macon = 0.00
Route_1_Allentown_Selma = 0.00
Route_1_Columbus_Allentown = 0.00
Route_1_Columbus_Columbus = 29.00
Route_1_Columbus_Danville = 0.00
Route_1_Columbus_Dummy_Plant = 0.00
Route_1_Columbus_Kingsport = 0.00
Route_1_Columbus_Macon = 0.00
Route_1_Columbus_Selma = 0.00
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Route_1_Danville_Columbus = 0.00
Route_1_Danville_Danville = 26.00
Route_1_Danville_Dummy_Plant = 0.00
Route_1_Danville_Kingsport = 0.00
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Route_1_Danville_Selma = 0.00
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Route_1_Dummy_Plant_Columbus = 0.00
Route_1_Dummy_Plant_Danville = 27.00
Route_1_Dummy_Plant_Dummy_Plant = 0.00
Route_1_Dummy_Plant_Kingsport = 0.00
Route_1_Dummy_Plant_Macon = 0.00
Route_1_Dummy_Plant_Selma = 0.00
Route_1_Kingsport_Allentown = 0.00
Route_1_Kingsport_Columbus = 0.00
Route_1_Kingsport_Danville = 0.00
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Route_1_Kingsport_Kingsport = 0.00
Route_1_Kingsport_Macon = 35.00
Route_1_Kingsport_Selma = 0.00
Route_1_Macon_Allentown = 0.00
Route_1_Macon_Columbus = 0.00
Route_1_Macon_Danville = 0.00
Route_1_Macon_Dummy_Plant = 0.00
Route_1_Macon_Kingsport = 0.00
Route_1_Macon_Macon = 42.00
Route_1_Macon_Selma = 0.00
Route_1_Selma_Allentown = 0.00
Route_1_Selma_Columbus = 36.00
Route_1_Selma_Danville = 0.00
Route_1_Selma_Dummy_Plant = 0.00
Route_1_Selma_Kingsport = 0.00
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Route_2_Allentown_Los_Canos = 0.00
Route_2_Allentown_Whitewater = 0.00
Route_2_Columbus_Duras = 0.00
Route_2_Columbus_Los_Canos = 0.00
Route_2_Columbus_Whitewater = 65.00
Route_2_Danville_Duras = 11.00
Route_2_Danville_Los_Canos = 80.00
Route_2_Danville_Whitewater = 0.00
Route_2_Kingsport_Duras = 0.00
Route_2_Kingsport_Los_Canos = 0.00
Route_2_Kingsport_Whitewater = 0.00
Route_2_Macon_Duras = 94.00
Route_2_Macon_Los_Canos = 0.00
Route_2_Macon_Whitewater = 0.00
Route_2_Selma_Duras = 0.00
Route_2_Selma_Los_Canos = 0.00
Route_2_Selma_Whitewater = 0.00
Total Cost of Transportation = RM 2884.00
```