Deep Learning teaching

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1 Probabilistic model to denoising

Say you have an unknown true z, but you only observe a corrupted copy $\tilde{z}$ of it. What is the best guess for a reconstruction $\hat{z}$?

More specifically: You are given a probabilistic prediction $p(z) = \mathcal{N}(\mu, \sigma_p^2)$ and an additive noise model $p(\tilde{z}|z) = \mathcal{N}(z, \sigma_n^2)$. Task is to find a reconstruction $\hat{z}$ that minimizes $\mathbb{E}_z\left[(z - \hat{z})^2\right]$.

Hints: The probability density of a normal distribution is $p(x) = \mathcal{N}(\mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]$. The first step is to compute the posterior density $p(z|\tilde{z})$ using the Bayes theorem $p(z|\tilde{z}) \propto p(\tilde{z}|z)p(z)$.