

Approach:

For my approach, I chose to use a binary search that utilizes recursion to search for the requested bounds in a sorted list, then return the values of the requested bounds as a sub-list. I chose this method as I found the for loop implementation difficult to implement while keeping the algorithm at a time complexity, in the worst-case, at $O(k + \log(n))$ or less.

Subroutines:

In my implementation, I created two subroutines, `get_lo()` and `get_hi()` which are the heart of the algorithm. Each subroutine takes five parameters, `list_s`, `lo_i`, `mid_i`, `hi_i`, and the respective boundary they are tasked to find (lo or hi). The parameters `lo_i`, `mid_i`, and `hi_i` act as index pointers for `list_s` during the binary search and conditional evaluations. These subroutines take five parameters due to their recursive nature.

One of my previous implementations only took three parameters, but I found issues in keeping track of, or the overwriting of, the index pointers as the recursive process continued, creating bugs that caused unexpected sub-lists to be returned during normal functionality cases. Having these functions as separate subroutines made the algorithm easier to debug since I could pinpoint if the issues were happening when the algorithm was finding the lower bound, or when finding the upper bound. Additionally, having them as separate entities ensured I did not add bugs to one while fixing the other.

The subroutine `get_lo()` first checks if `lo_i` is greater-than or equal-to `hi_i`, which serves as the base case for the recursive process. Else, if the value of `list_s` at index `mid_i` is greater than or equal to `lo`, `hi_i` is assigned the value of `mid_i`, and `mid_i` is recalculated using the equation $(lo_i + hi_i) \text{ divided by } 2$, using integer division. Then, `lo_i` is assigned to the recursive call to `get_lo()`. The Else condition covers the case where the value of `list_s`, at index `mid_i`, is less than `lo`. In this case, `lo_i` is set to `mid_i + 1`, `mid_i` is recalculated in the same manner as the prior step, then `lo_i` is assigned to the recursive call to `get_lo()`. The subroutine then returns the index of the lowest value, within the requested bounds, of `list_s` as `lo_i`.

The subroutine `get_hi()`, works in a similar fashion as `get_lo()`, and utilizes the same base case. The only difference lies in the recursive method being called and the else-if conditional. The else-if checks if `hi` is less-than the value of `list_s` at index `mid_i`, if so, the same actions are taken as the else-if in `get_lo()`, but `lo_i` is assigned to the recursive call of `get_hi`. The else would then cover the condition of `hi` being greater-than or equal-to `list_s[mid_i]`. Again taking the same actions as the else in `get_lo()`, with the exception of the recursive call to `get_hi()`. This small

change in the conditionals ensured that the index after the highest value, within the bound, is returned as `lo_i`; ensuring the highest value is inclusive in the sub-list.

Extract routine:

The `extract(list_s, lo, hi)` functionality starts by checking the simple edge cases such as checking if the list is empty or `None`. After this check, the list length and lower, middle, and higher indices are gathered for `list_s`; and higher-level edge cases are checked. These special cases check if `lo` and/or `hi` contain the value `None`. If both `hi` and `lo` are `None`, `list_s` is returned. If `lo` is `None`, the algorithm searches for the upper bound and assigns that value to `hi_pointer` and returns `list_s[0: hi_pointer]`. If `hi` is `None`, the algorithm searches for the lower bound and assigns the value to `lo_pointer` and returns `list_s[lo_pointer:]`. The final special case ensures that the value of `lo` is not greater than `hi`; if it is, `extract()` returns `None`.

If the algorithm passes all of the edge cases, we first search for the upper bound of the requested range and assign it to `hi_pointer` with subroutine `get_hi()`. We then search for the lower bound with the `get_lo()` subroutine. The `get_lo()` return value is assigned to the variable `lo_pointer`. Finally, `list_s[lo_pointer: hi_pointer]`, which contains all values within the requested range, is returned from `extract()`

Complexity:

My algorithm implementation has a worst-case time complexity of $O(k + \log(n))$, where n is the number of values in `list_s` and k is the number of values in the requested range. This is due to the binary search cutting `list_s` in half through each recursive call to find a bound of the range within the list. To find both bounds, making this section $O(2\log(n))$. After the recursive calls, we then return the range of values within `list_s` using slicing, which is $O(k)$. Making the worst- case time complexity $O(k + \log(n))$ after the removal of constants.

Major Implementation Issue:

One of the main issues that plagued me through all versions of my code was finding the upper bound. My returned sub-list would vary in the inclusion of the highest value within the requested range; sometimes it was there and sometimes not. I found that this issue stemmed from me being too focused on the `hi_i` (high index) variable as the value being returned from the subroutine, `get_hi()`. This issue caused me to try many different types of index incrementations for `lo_i`, `mid_i`, and `hi_i` out of desperation. I later solved my issue after many hand traces of varied types of sorted lists and found that `lo_i` should be returned rather than `hi_i`.