西安电子科技大学 XIDIAN UNIVERSITY | 空间科学与技术学院 School of Aerospace Science and Technology

# Optimal target location and tracking for multiple UAVs

Ranjun Shi

School of Aerospace Science and Technology

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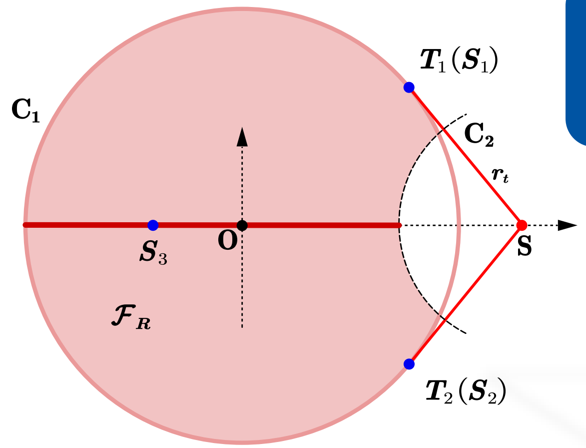
# Schedule

# 01 Introduction


**Figure: Type of Multi-UAVs cooperative control algorithm**

# 02 Research on optimal geometric deployment of multiple UAVs




## Traditional constrained optimization problem

$$\max\ f(x_t, y_t, x_i, y_i)$$

$$\begin{cases} x_t^2 + y_t^2 > \lambda^2 \\ x_i^2 + y_i^2 \le \lambda^2 \\ (x_i - x_t)^2 + (x_i - x_t)^2 \ge r^2 \end{cases} \qquad i = 1, 2, \ldots, n$$

Based on distance measurements

The restricted area is circular

## Constraint optimization problem is established

$$\max\ \det(F)$$

$$\begin{cases} x_t^2 + y_t^2 > \lambda^2 \\ x_i^2 + y_i^2 \le \lambda^2 \\ (x_i - x_t)^2 + (x_i - x_t)^2 \ge r^2 \end{cases} \qquad i = 1, 2, \ldots, n$$

$F$ is Fischer information matrix(FIM)

## Problem

- The traditional constrained optimization problem is difficult to solve.

## Solution

- The concept of **maximum feasible Angle** is proposed.
- The equivalent constraint optimization problem is established to **reduce the solving complexity**.

## Equivalent constraint optimization problem

$$\max\ \sigma^{-4} \sum_{i<j}^{n} \sin^2 \theta_{ij}$$

$$\begin{cases} \theta_{ij} \in [0, 2\varphi] \\ \pi / 4 \le \varphi < \pi / 3 \end{cases}$$

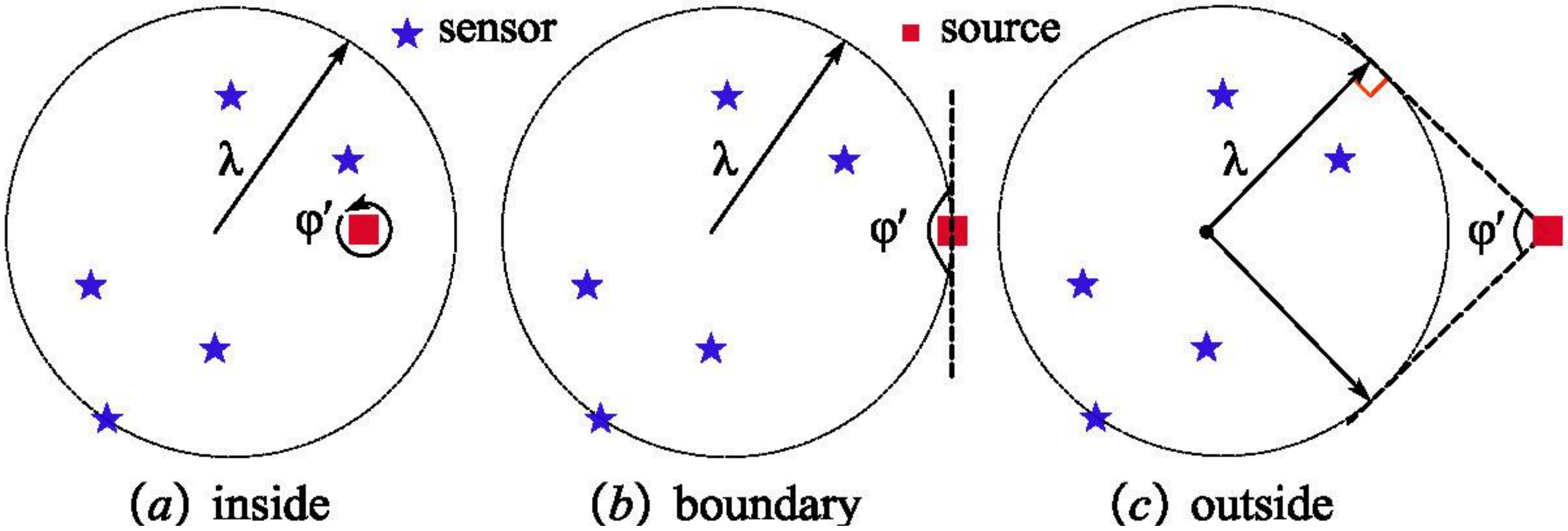



**Figure: Three different circular constraints**

## 1) Target lies outside the circular region(e.g. $n=2$, $n$ is the number of UAVs )

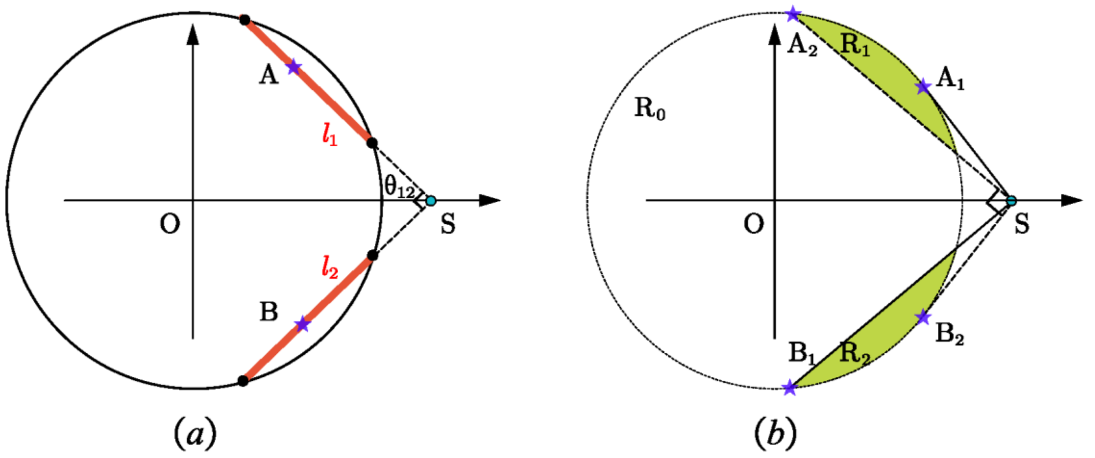



**Figure: Source lies outside the region with $n=2, \varphi > \pi/4$**

Given the position of one of UAV $(x_t, y_t)$, we can get the slope $k_1 = -y_t/(\rho - x_t)$. Use $k_1 \in [-\lambda/r_t, -r_t/\lambda]$ to determine whether there is an optimal geometric deployment, i.e., whether the separation Angle can be reached $\pi/2$, and (2-2) can then be used to determine a viable location for another drone.

**$e.g. \varphi > \pi/4$**

$\theta_{12}^* = \pi/2$

$$\max \ \sigma^{-4}\sin^2\theta_{12}$$
$$s.t.\begin{cases}\theta_{12}\in[0,2\varphi]\\ \pi/4<\varphi\le\pi/2 \\ (x_i-x_t)^2+(y_i-y_t)^2\ge\lambda^2 \quad, i=1,2\end{cases} \tag{2-1}$$

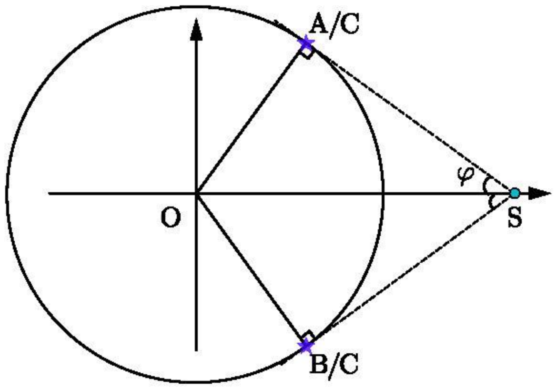
**Optimal deployment scheme**

$$S_1(x_1, k_1x_1 - k_1\rho)$$

$$S_2(x_2, -x_2/k_1 + \rho/k_1)$$

$$\begin{cases} x_1 \in \left[\frac{k_1^2\rho - \sqrt{\Theta - k_1^2\rho^2}}{1+k_1^2}, \frac{k_1^2\rho + \sqrt{\Theta - k_1^2\rho^2}}{1+k_1^2}\right] \\ x_2 \in \left[\frac{\rho + k_1\sqrt{\Theta - \rho^2}}{1+k_1^2}, \frac{\rho - k_1\sqrt{\Theta - \rho^2}}{1+k_1^2}\right] \\ \theta = k_1^2\lambda^2 + \lambda^2 \end{cases} \tag{2-2}$$

## 2) Target lies outside the circular region(e.g. $n = 3$)




(a) $0<\varphi\leq\pi/4$

**$0<\varphi\leq\pi/4$**

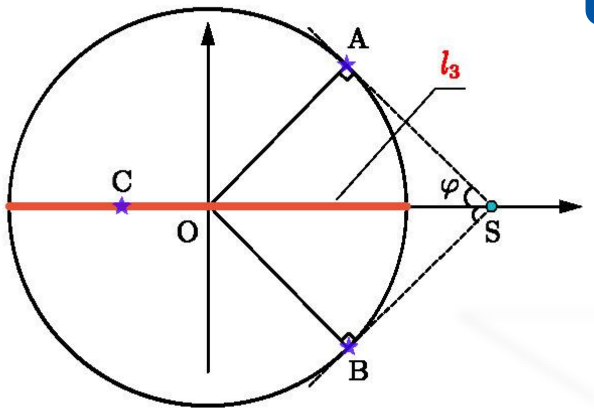
$$\max\ \sigma^{-4}(\sin^2\theta_{12}+\sin^2\theta_{13}+\sin^2\theta_{23})$$
$$s.t.\begin{cases}\theta_{13}\in[0,2\varphi]\\ \theta_{13}=\theta_{12}+\theta_{23} \\ 0<\varphi\leq\pi/3\end{cases} \qquad (2\text{-}3)$$

The objective function is analyzed and derived mathematically

**Optimal deployment scheme**

$$\begin{cases}S_1(\lambda/\rho,\lambda r_t/\rho)\\ S_2(\lambda^2/\rho,\pm\lambda r_t/\rho)\\ S_3(\lambda^2/\rho,\lambda r_t/\rho)\end{cases}$$

$$\{\theta_{12}^*,\theta_{23}^*,\theta_{13}^*\}=\{2\pi/3,2\pi/3,2\pi/3\}$$
$$\text{or }\{\pi/3,\pi/3,2\pi/3\}$$




(b) $\pi/4\leq\varphi\leq\pi/3$

**$\pi/4\leq\varphi\leq\pi/3$**

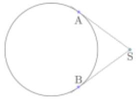
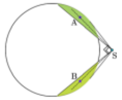
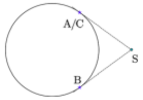
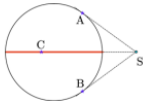
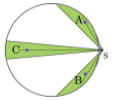
$$\max\ \sigma^{-4}(\sin^2\theta_{12}+\sin^2\theta_{13}+\sin^2\theta_{23})$$
$$s.t.\begin{cases}\theta_{12},\theta_{13},\theta_{13}-\theta_{12}\in[0,2\varphi]\\ \varphi<\pi/3\end{cases} \qquad (2\text{-}4)$$

The objective function is analyzed and derived mathematically

**Optimal deployment scheme**

$$\begin{cases}S_1(\lambda^2/\rho,\lambda r_t/\rho)\\ S_2(x_2,0)\\ S_3(\lambda^2/\rho,-\lambda r_t/\rho)\\ 2\sqrt{3}\lambda/3\leq\rho\leq\sqrt{2}\lambda\\ x_2\in[-\lambda,\lambda]\end{cases}$$

## 3) Target lies outside the circular region with $n \geq 4$

| Localization scenario | $\varphi'$ | Optimal geometry | Number of solutions | Maximum |
|---|---|---|---|---|
| $C_1: n=2,\ \rho \geq \sqrt{2}\lambda$ | $\varphi \leq \pi/4$ |  | One | $\sigma^{-4}\sin^2 2\varphi$ |
| $C_2: n=2,\ \lambda < \rho < \sqrt{2}\lambda$ | $\varphi > \pi/4$ |  | Infinity | $\sigma^{-4}$ |
| $C_3: n=3,\ \rho \geq \sqrt{2}\lambda$ | $\varphi \leq \pi/4$ |  | Two | $2\sigma^{-4}\sin^2 2\varphi$ |
| $C_4: n=3,\ 2\sqrt{3}\lambda/3 \leq \rho < \sqrt{2}\lambda$ | $\pi/4 \leq \varphi \leq \pi/3$ |  | Infinity | $\sigma^{-4}(2\sin^2\varphi + \sin^2 2\varphi)$ |
| $C_5: n=3,\ \lambda < \rho < 2\sqrt{3}\lambda/3$ | $\varphi > \pi/3$ |  | Infinity | $9\sigma^{-4}/4$ |

**Table： Optimal geometric deployment of UAVs located outside the region**

- $\boldsymbol{\varphi > \pi/3}$

UAVs can be divided into subgroups of two or three UAVs, and each Subgroups are deployed as shown in $C2$ or $C5$ of Table .

## 3) Target lies outside the circular region with $n \geq 4$

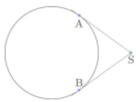
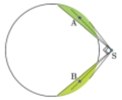
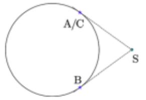
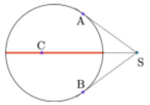
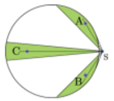
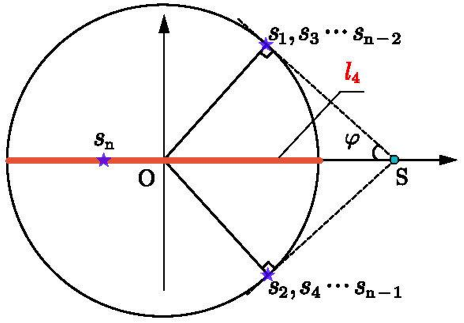
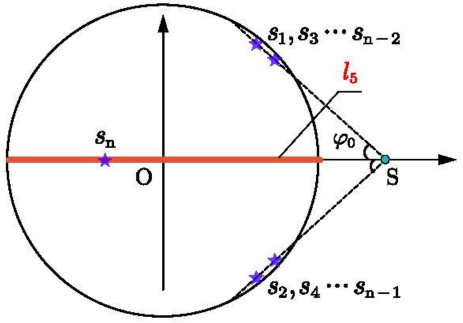
| Localization scenario | $\varphi'$ | Optimal geometry | Number of solutions | Maximum |
|---|---|---|---|---|
| $C_1: n=2,\ \rho \geq \sqrt{2}\lambda$ | $\varphi \leq \pi/4$ |  | One | $\sigma^{-4}\sin^2 2\varphi$ |
| $C_2: n=2,\ \lambda < \rho < \sqrt{2}\lambda$ | $\varphi > \pi/4$ |  | Infinity | $\sigma^{-4}$ |
| $C_3: n=3,\ \rho \geq \sqrt{2}\lambda$ | $\varphi \leq \pi/4$ |  | Two | $2\sigma^{-4}\sin^2 2\varphi$ |
| $C_4: n=3,\ 2\sqrt{3}\lambda/3 \leq \rho < \sqrt{2}\lambda$ | $\pi/4 \leq \varphi \leq \pi/3$ |  | Infinity | $\sigma^{-4}(2\sin^2\varphi + \sin^2 2\varphi)$ |
| $C_5: n=3,\ \lambda < \rho < 2\sqrt{3}\lambda/3$ | $\varphi > \pi/3$ |  | Infinity | $9\sigma^{-4}/4$ |

**Table: Optimal geometric deployment of UAVs located outside the region**




(a) $\pi/4 < \varphi \leq \frac{1}{2}\arccos\left(-\frac{1}{n-1}\right)$




(b) $\frac{1}{2}\arccos\left(-\frac{1}{n-1}\right) < \varphi < \pi/3$

- $\boldsymbol{\pi/4 < \varphi \leq \pi/3}$

If there is an even number of UAVs, the optimal geometric deployment scheme is to deploy all subgroups according to the configuration of $C_2$ in Table to ensure that the separation Angle between each pair of UAVs is the optimal value $\pi/2$.

If $\boldsymbol{\pi/4 < \varphi \leq 1/2arccos(-1/(n-1))}$, the optimal geometric deployment is the last drone deployed in $l_4$, while the rest of the UAVs according to the $C_1$ in table for deployment, as shown in Figure (a).

If $\boldsymbol{1/2arccos(-1/(n-1)) < \varphi < \pi/3}$, we only provide a sufficient but not necessary solution. In this case, it is recommended that the last UAV be deployed on $l_5$, while each pair of the remaining UAVs is deployed symmetrically in accordance with the axis about $x$ and satisfies $\varphi_0 = \pm 1/2arccos(-1/(n-1))$, as shown in Figure (b).

## 3) Target lies outside the circular region with $n \geq 4$

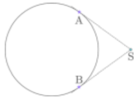
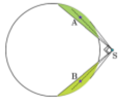
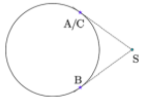
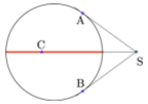
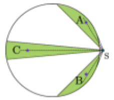
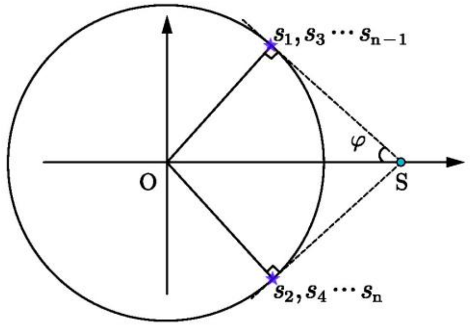
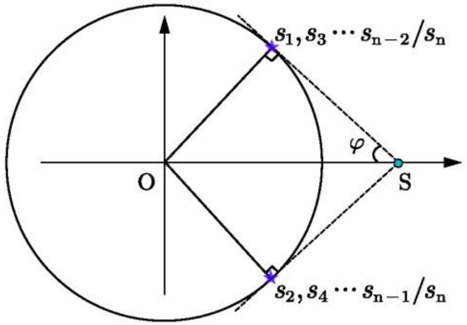
| Localization scenario | $\varphi'$ | Optimal geometry | Number of solutions | Maximum |
|---|---|---|---|---|
| $C_1: n=2,\ \rho \geq \sqrt{2}\lambda$ | $\varphi \leq \pi/4$ |  | One | $\sigma^{-4}\sin^2 2\varphi$ |
| $C_2: n=2,\ \lambda < \rho < \sqrt{2}\lambda$ | $\varphi > \pi/4$ |  | Infinity | $\sigma^{-4}$ |
| $C_3: n=3,\ \rho \geq \sqrt{2}\lambda$ | $\varphi \leq \pi/4$ |  | Two | $2\sigma^{-4}\sin^2 2\varphi$ |
| $C_4: n=3,\ 2\sqrt{3}\lambda/3 \leq \rho < \sqrt{2}\lambda$ | $\pi/4 \leq \varphi \leq \pi/3$ |  | Infinity | $\sigma^{-4}(2\sin^2\varphi + \sin^2 2\varphi)$ |
| $C_5: n=3,\ \lambda < \rho < 2\sqrt{3}\lambda/3$ | $\varphi > \pi/3$ |  | Infinity | $9\sigma^{-4}/4$ |

**Table: Optimal geometric deployment of UAVs located outside the region**




$(a)$ $n$ is even




$(b)$ $n$ is odd

- $\boldsymbol{\varphi \leq \pi/4}$

If $n$ is even, The optimal geometry is that the two UAVs in subgroups are placed as $C_1$ in Table, this indicates that half of the $n$ UAVs are placed at each tangent point, as shown in Figure (a).

The optimal geometry is that the last UAV is placed at any tangent point, and all remaining subgroups are deployed as $C_1$ in Table, which means at each tangent point $(n-1)/2$ drones, as shown in Figure (b).

# 4) Simulation

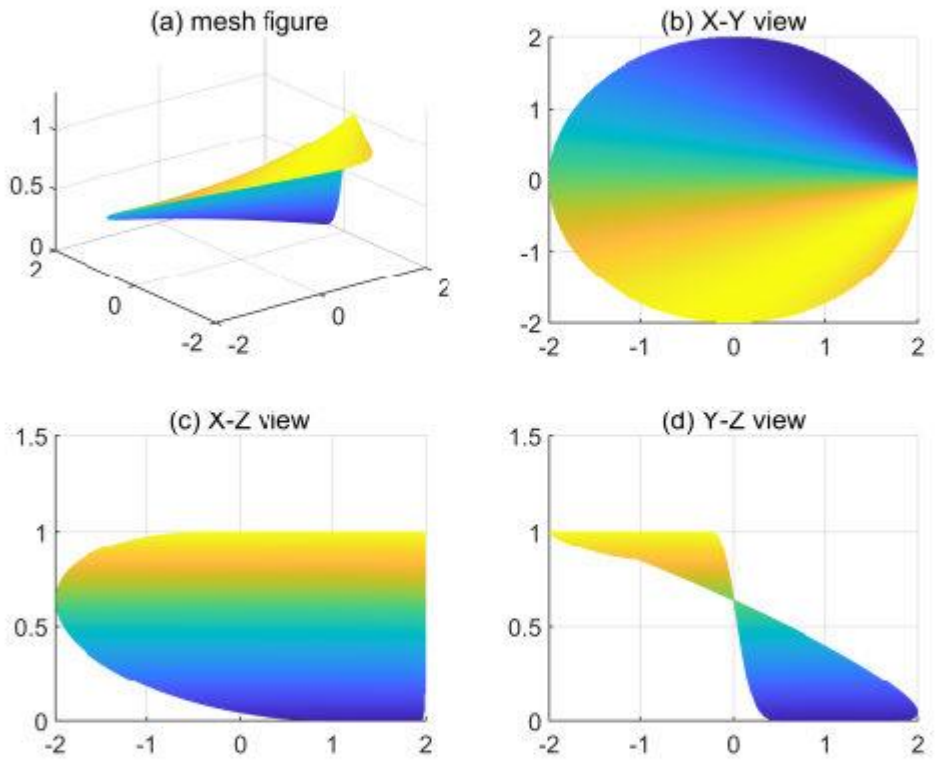



**Figure: Objective function changes with x and y coordinates of $S_2$**

The optimal solutions in Figure are consistent with those obtained by theoretical analysis. There are infinitely many optimal solutions, all of which result in an optimal separation angle of $\pi/2$ between the two sensors and the source.

## 4) Simulation

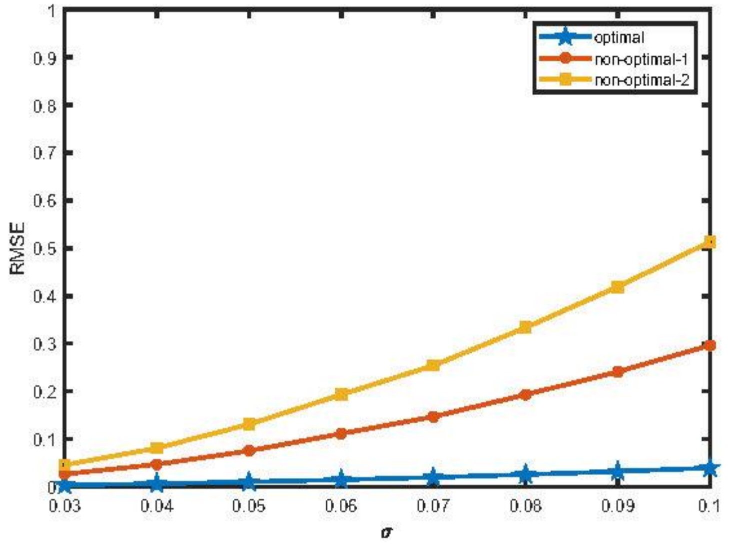



**Figure: The RMSE comparisons for different geometries**

Target: $S(2,0)$

Geometry−01: $S_{11}\ (1,\sqrt{3})$, $S_{12}\ (0,0)$, $S_{13}\ (1,-\sqrt{3})$

Geometry−02: $S_{21}\ (-0.5,1)$, $S_{22}\ (-1.5,0)$, $S_{23}\ (-0.5,-1)$

Geometry−03: $S_{31}\ (0,1)$, $S_{32}\ (-1.5,1)$, $S_{33}\ (-1,0)$

The MLE algorithm is implemented using 10,000 Monte Carlo runs for these three different localization geometries, and root mean square error (RMSE) is utilized for performance evaluation. Figure illustrates the RMSE performance for these geometries. From the analysis, We can conclude that Geometry-01 is an optimal geometry, while Geometry-02 and Geometry-03 are non-optimal geometries. The RMSE performance in the optimal geometry is smaller than those in the non-optimal geometries, and the estimation performance decreases as $\sigma$ increases .

## 5) Conclusion

- In this chapter, **the optimal geometric deployment of UAVs** in target location system based on range information is studied, especially how the relative geometric position between UAVs and targets significantly affects the accuracy and performance of the positioning system. Compared with the existing research, this study is unique in that it considers the actual scenario in which the UAV and the target must be deployed in a **specific restricted area**, which requires us to solve a constrained problem when optimizing the UAV deployment.
- In order to facilitate the theoretical analysis, the concepts of **separation angle and maximum feasible angle** are innovatively introduced in this paper, so that the complex constraint optimization problem can be transformed into a form that is easy to understand and handle.

# 03 Collaborative positioning and tracking for multiple UAVs

# 1) Target Localization and Pursuit System

**Target Model**: $\boldsymbol{X}(k+1)=\boldsymbol{f}(\boldsymbol{X}(k),\boldsymbol{U}(k))+\boldsymbol{W}(k)$, $\boldsymbol{X}(k)$ is the target state, $\boldsymbol{U}(k)$ is a known deterministic input, $\boldsymbol{W}(k)$ is independent Gaussian random processes that describe the state noises.

**Measure Model**: $\boldsymbol{Z}(k)=\boldsymbol{H}(k)\boldsymbol{X}(k)+\boldsymbol{V}(k)$, $\boldsymbol{V}(k)$ is independent Gaussian random processes that describe the state noises. $\boldsymbol{H}(k)$ is a linearized measurement matrix, which includes the distances from the trackers to the targets. $\boldsymbol{Z}(k)$ is the measurement output at discrete time $k$.

**Trackers Model**: $\boldsymbol{q}_i\,(k+1)=\boldsymbol{q}_i(k)+\boldsymbol{f}(\boldsymbol{q}_i(k),\boldsymbol{u}_i(k))*T$, $\boldsymbol{q}_i$ is the tracker state, $T$ is sampling interval.

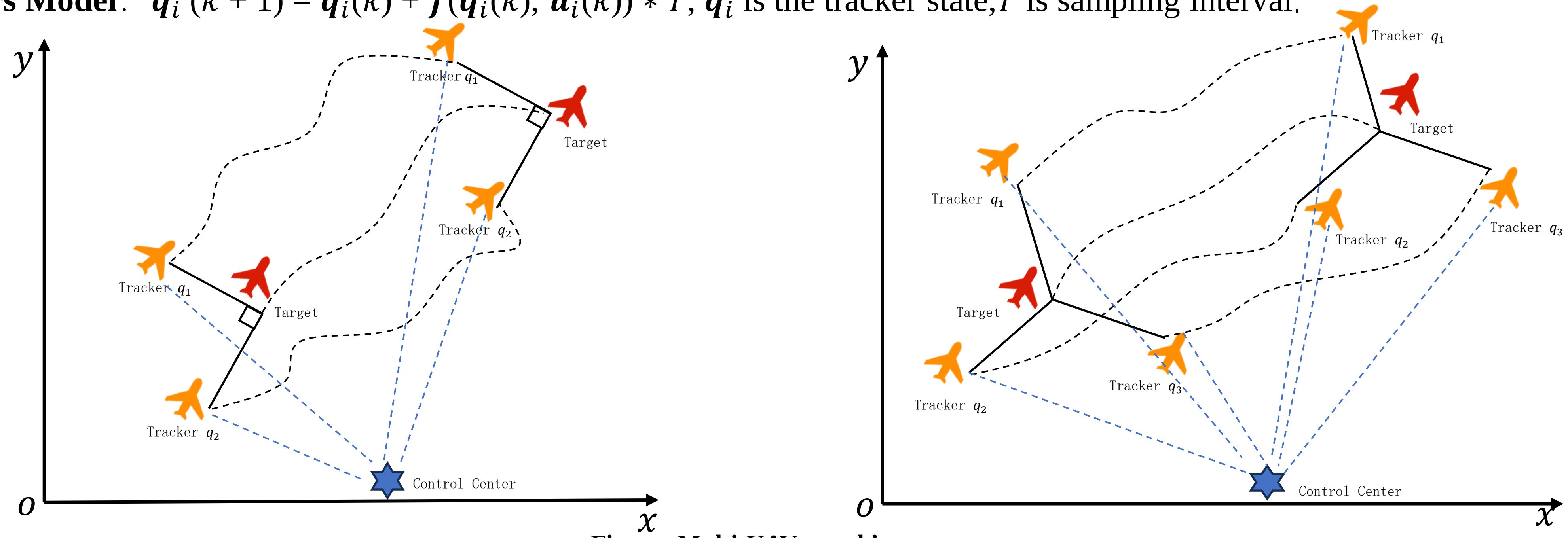



**Figure: Multi-UAVs tracking target.**

## 2) Rolling Horizon Optimization(RHO) Model

$$u^*[k+1\!:k+Tp] = argmin_u \left\{ \sum_{l=1}^{Tp} \left( \lambda_{k+l} \left( -log\big(\hat{J}_{k+l}\big) + \mu \sum_{i=1}^{n} g_i\big(d, r_i(k)\big) \right) \right) \right\}$$

$$= argmin_u \sum_{l=1}^{Tp} \left( \lambda_{k+l} \left( -log \left( det\Big( \sum_{i=1}^{n} \widehat{H}_{k+l}^{T} R_{k+l}^{-1} \widehat{H}_{k+l} \Big) \right) + \mu \sum_{i=1}^{n} \big( g_{min,i,k+l} + g_{max,i,k+l} \big) \right) \right)$$

$\hat{J}_{k+l}$ is Fisher information matrix(FIM), $u$ is the input sequence in the scrolling window, $\lambda_{k+l}$ is the weight factor in the scrolling time domain, $g(d, r_i\,(k))$ is the penalty function about the distance constraint, and $\mu$ is the penalty factor. Instead of optimizing $u$ through particle swarm optimization (PSO) algorithm, we used the first item of $u$ to update the status of the UAV. $Tp$ is the scrolled time domain window length.

## 3) Simulation(Two UAVs)

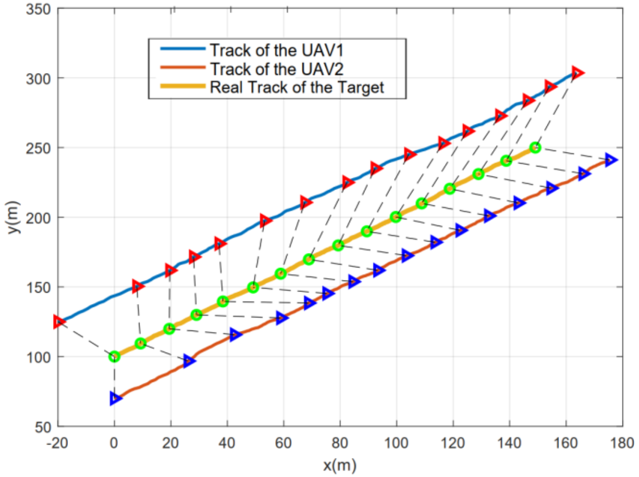



**Figure 1: Optimal tracking trajectories of two UAVs. The red and blue triangles represent UAVs. The green circle represents the target.**

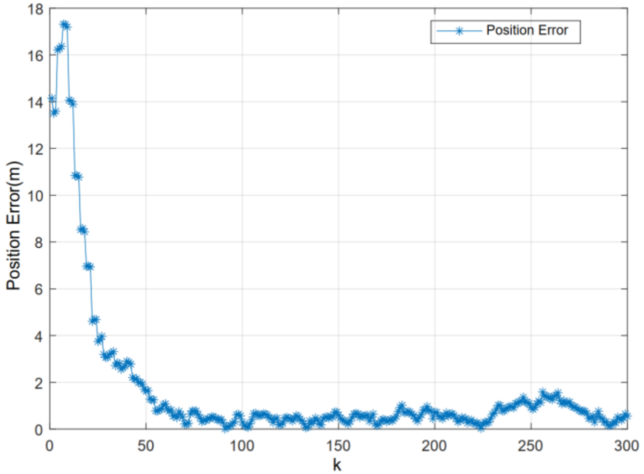



**Figure 2: The position is estimated by the Extended Information Filtering(EIF) and compared with the real position of the target.**

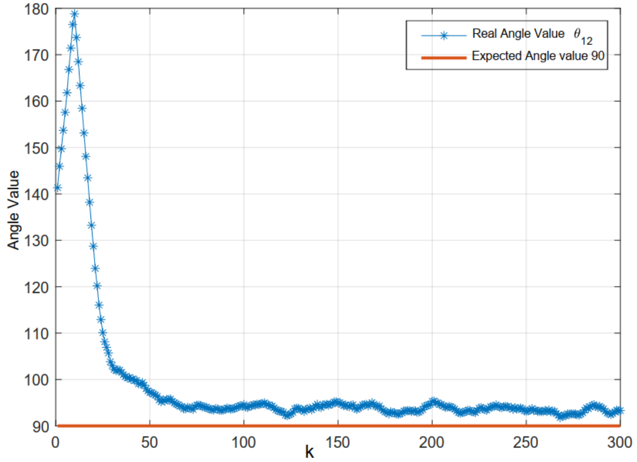



**Figure 3: The separation angle of the two UAVs. The figure above shows that the separation angle converges to the expected angle.**

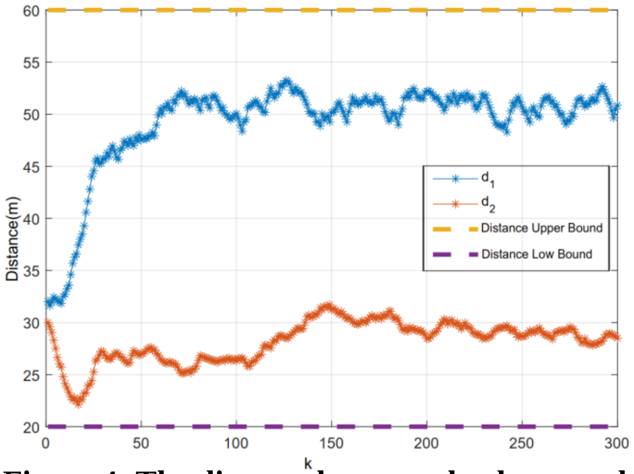



**Figure 4: The distance between the drone and the target. The results show that the UAV is always tracking within the constraint.**

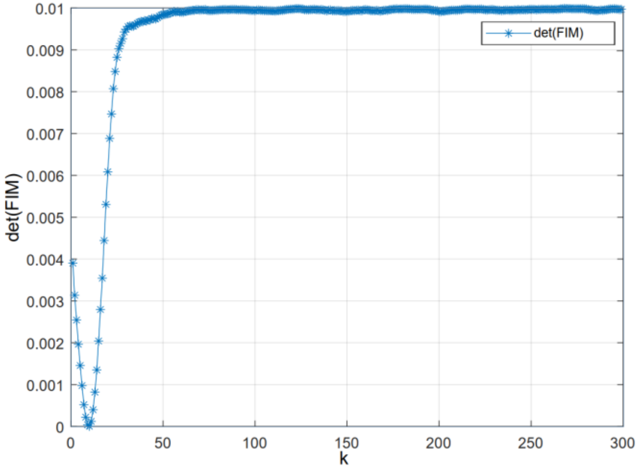



**Figure 5: The amount of target information acquired by UAV gradually increases and becomes saturated during the tracking process.**

## 3) Simulation(Three UAVs)

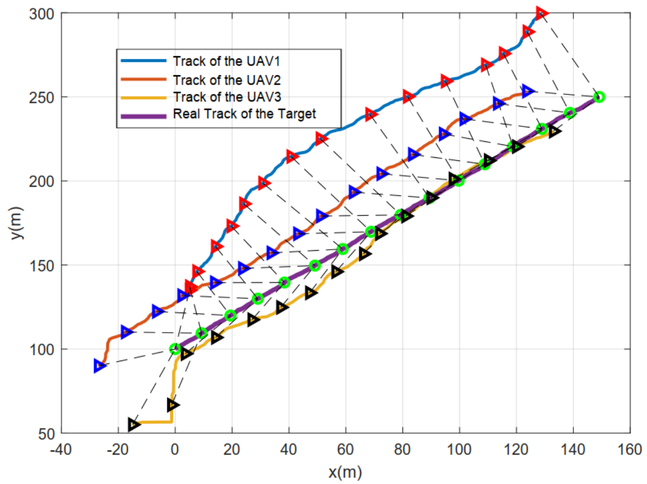
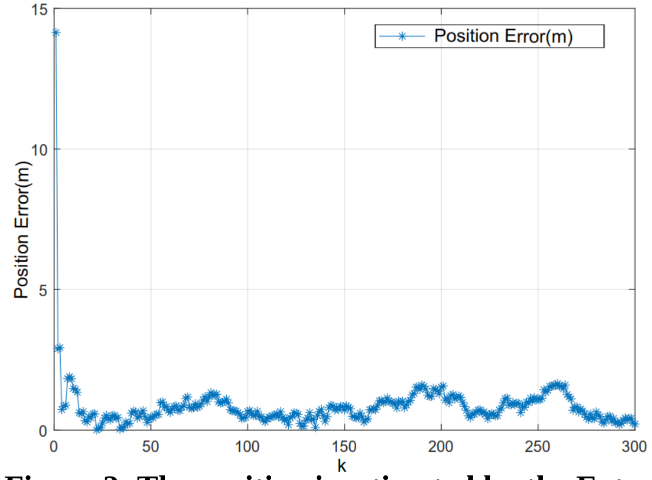



**Figure 1: Optimal tracking trajectories of three UAVs. Triangles represent UAVs. The green circle represents the target.**

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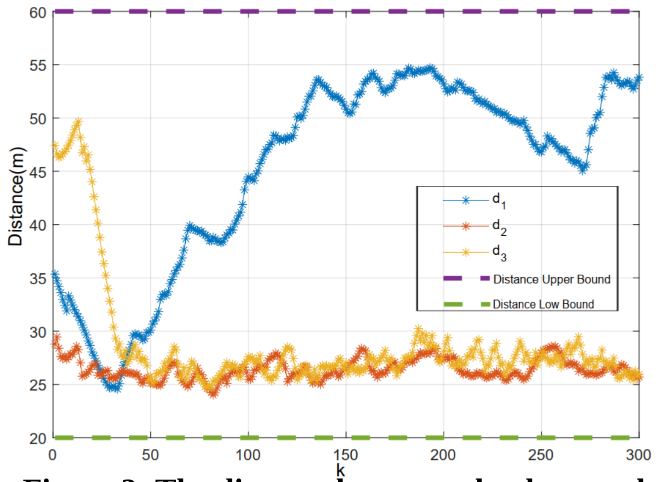


**Figure 3: The distance between the three and the target. The results show that the UAV is always tracking within the constraint.**

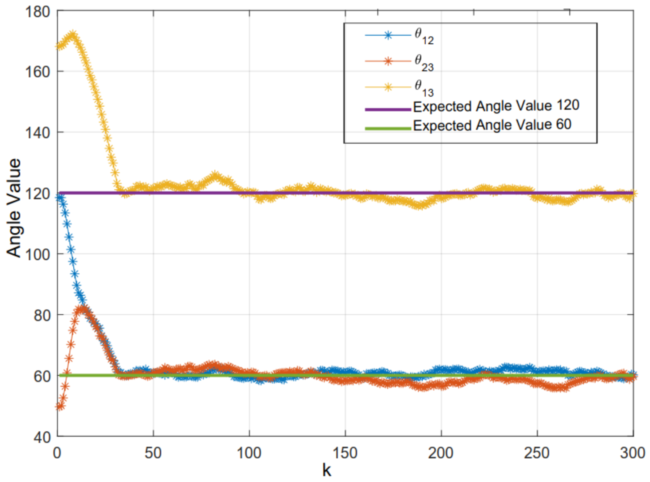


**Figure 4: The optimal geometric deployment between the three is $\{\theta_{13} = 2\pi/3, \theta_{12} = \theta_{23} = \pi/3\}$.**

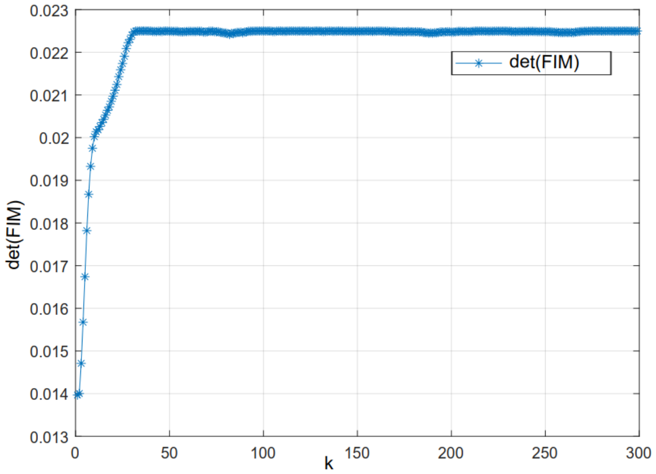


**Figure 5: The amount of target information acquired by UAV gradually increases and becomes saturated during the tracking process.**

## 4) Conclusion

- In this chapter, an optimization model based on **the Rolling Horizon Optimization** is proposed, and the target **state is estimated by EIF**, and the target information obtained by the UAV swarm is used as the key index to evaluate the system performance. This indicator comprehensively reflects the efficiency and accuracy of UAVs in positioning tasks.
- In order to constrain the distance in the Optimization process, the **penalty function is designed** and the model is solved by PSO.