≫ Unbiased Estimator

یک تخمین گر آماری ← مقدار پارامتر واقعی جمعیت را برمیگرداند

$\theta \rightarrow$ Parameter to be estimated

$\hat{\theta} \rightarrow$ estimator of θ

estimator $\hat{\theta}$ is unbiased $\xrightarrow{\text{if}}$ $E[\hat{\theta}] = \theta$

($E[\hat{\theta}]$: expected value of $\hat{\theta}$)

* مثال ها

1. Sample mean

if $X_1, X_2, \dots, X_n$ are i.i.d. random variables from a population with mean μ & variance σ^2

$$\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \xrightarrow{\text{is}} \text{an unbiased estimator of the } \mu$$

اثبات:

$$E[\overline{X}] = E\left[\frac{1}{n}\sum_{i=1}^{n} X_i\right] = \frac{1}{n}\sum_{i=1}^{n} E[X_i] = \frac{1}{n}\sum_{i=1}^{n} \mu = \mu$$

2. Sample Variance

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2 \xrightarrow{\text{is}} \text{an unbiased estimator of the population variance } \sigma^2$$

Bias in Estimators

$$E(\hat{\theta}) \neq \theta \rightarrow \text{Bias}(\hat{\theta}) = E[\hat{\theta}] - \theta$$

کاهش bias:

- Using correct Formulas → using $\frac{1}{n-1}$ instead $\frac{1}{n}$ in the sample variance formula
- Larger Sample sizes
- Advanced Techniques
  - bootstraping
  - cross-validation