# Is walking better than running in the rain?

24th August 2023

I know that the best way to avoid getting wet is by having an umbrella… However, when you are a forgetful person in a place with unpredictable weather – England – it is easy to get caught off guard and left soaked.

The instinctual answer is to go as fast as you can to the closest shelter, right? By definition, that is the null hypothesis: *"The best way to minimise your exposure to the rain is by walking faster."* However, there is an alternative hypothesis: *"Going slower in the rain is a better alternative as you reduce the amount of water drops you run into."*

To determine whether the null hypothesis can be rejected, we have three options:

1. Experimental approach: to run in the rain, taking measurements. This approach might be the most accurate, but it certainly is not precise. Counting the volume of water that rains on someone is difficult to measure with small uncertainties – and besides, it is more science than maths.
2. Semi-experimental approach: to simulate a person running in the rain using software. This approach is extremely precise at counting the volume of water, but its accuracy is limited by the quality of the virtual model. Again, this is not purely mathematical as there's variability, but it should provide sufficiently reliable results so that we use this virtual model rather than the experimental results.
3. Artificial approach: instead of simulating a person in the rain, we make a mathematical model to predict how much water wets the person. We can then compare the results from the model to the semi-experimental results to compare the two models. While two models giving comparable results technically does not prove either right, there is a good probability that the models are somewhat accurate.

Before we go any further in our investigation, I will rule out the "Experimental approach." While it might be more scientific, I do not have the tools to accurately measure rainfall on a person – nor do I have the intention to. Therefore, we will only compare the other two approaches.

It is more pragmatic to start with the artificial approach rather than the semi-experimental one. Because a fully virtual approach has less limitations when it comes with how simplistic the assumptions need to be for the model to be achievable, it is more pragmatic to use the "more powerful" tool later and bind it to the limitations of the "less powerful" tool for a fairer comparison.

## The Artificial approach:

*aka, the mathematical model*

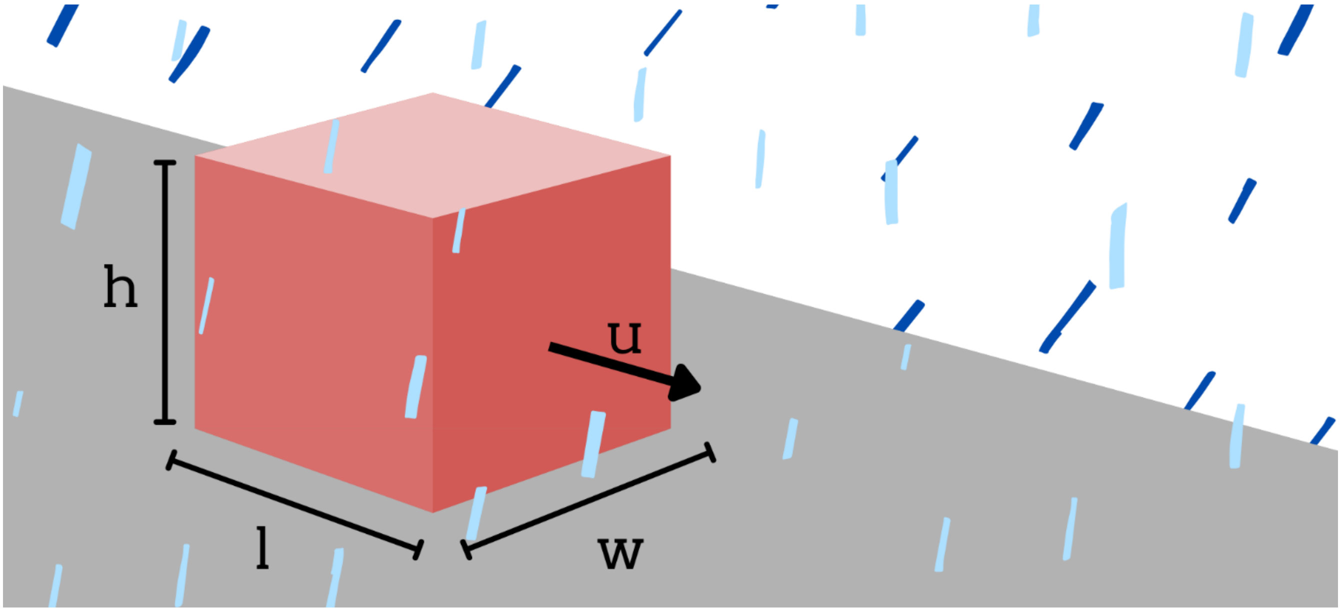



Image 1.

To make the modelling manageable, we will assume that:

1. The person is a cuboid with width w, height h, and length l.
2. Rain falls vertically. The rate of rainfall, r, is constant with space and time.

We can define the total precipitation on a person, $P_t$ as the sum of precipitation that falls from above $p_v$, and the precipitation that the person runs into $p_h$.

$$p_t\,(u) = p_v + p_h$$

The simpler precipitation to work out is the precipitation from above. The water that falls from above per second can be modelled as the surface area of the top face, multiplied by the rate of rainfall. To obtain the total precipitation from above, we multiply by the total time.

$$p_v = r(wl)t$$

Less intuitively, the water that the person runs into can be modelled as the change in volume occupied per unit time, multiplied by the coefficient of humidity. Again, to turn this into a rate we multiply by time.

$$p_h = H\frac{dV}{dt}t$$

The change of volume over time is the change of volume occupied by the person over time. Thus, all the water occupying this volume hits the person. We calculate this change in volume as the horizontal surface area, multiplied by the distance travelled in that time.

$$\frac{dV}{dt} = (wh)\frac{ds}{dt} \therefore \frac{dV}{dt} = (wh)u$$

Therefore, we can conclude that:

$$p_h = H(wh)ut \therefore p_h = H(wh)s$$

Uniting the two halves, we obtain: $p_t\,(u) = H(wh)s + r(wl)t$

We are not done yet. This equation shows that the water that hits the person depends on:

- The size of the person (w/m, h/m, l/m)
- The distance that the person needs to travel (s/m)
- The time it takes for them to cross that distance (t/s)
- The humidity coefficient of the air (H/ no unit)
- The rate of flow of rain (r/ $ms^{-1}$)

The first two constants and the variable (time) are intuitive. However, what are the humidity coefficient and the rate of flow of rain? To start determining what they mean, we should start from how rainfall is determined.

Rainfall is measured in millimetres per hour. For example, a light drizzle might be just 0.5 $mmhour^{-1}$, whereas a heavy rain might be as high as 8 $mmhour^{-1}$. To convert from rainfall, f, in millimetres per hour, to rainfall F in meters per second (SI units), simply:

$$F = \frac{f}{3600 \cdot 1000}$$

This new variable, F, is the measure of meters of rainfall per unit time in one point. Therefore, the rate of flow of rain, is simply equivalent.

$$F = r$$

On the other hand, the humidity coefficient is the ratio of volume of water for unit space.

$$H = \frac{V_w}{V_t}$$

Because the unit space is 1 $m^2$, then:

$$H = \frac{V_w}{1} = V_w$$

We therefore need to find the volume of water in one unit space.

To find it, let us consider the definition of F. It is the height of water (in meters) that falls in one point per second. This water was spread over a great height before it fell.
E.g., F = 2.0 $mms^{-1}$, and speed of rain, $u_r$ = 10 $ms^{-1}$
It is therefore clear that those 2.0mm of rain were spread over 10 meters the second before they fell. Therefore, the humidity of those meters was:

$$H = \frac{2.0 \cdot 10^{-3}}{10} = 2.0 \cdot 10^{-4}\ (no\ unit)$$

More generally, we can express the humidity as a ratio of the rain flow to the speed of rain:

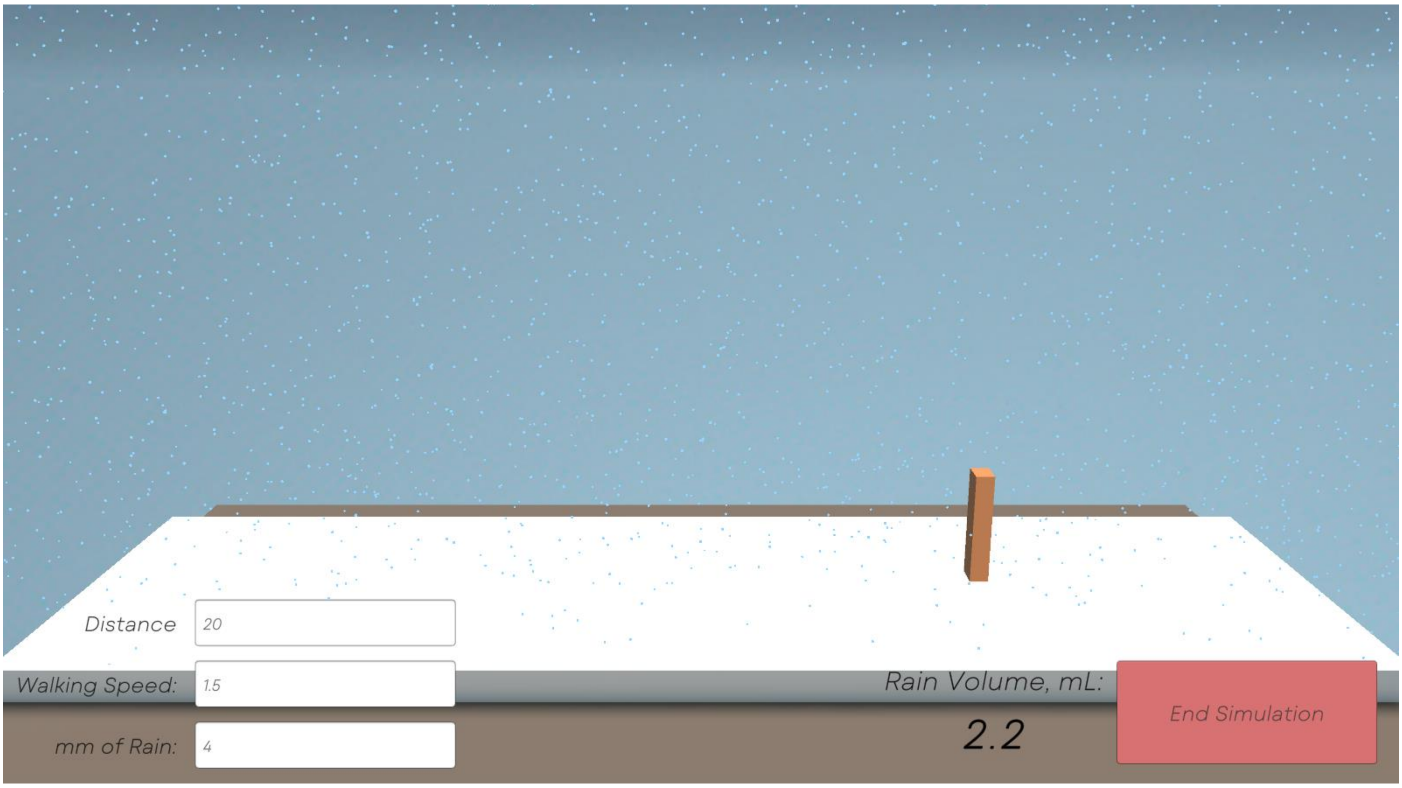
$$H = \frac{r}{u_r}$$

We can therefore conclude that: $p_t\,(u) = \frac{r}{u_r}(wh)s + r(wl)\frac{s}{u}$

This final equation shows that although rain received depends on many factors, the time in the rain is the only one that someone can influence. Because $r\mathrm{wl} > 0$ in any rainy condition, as time increases, the quantity of rain received too increases. Therefore, our artificial approach states that running is always better than walking.

# The Semi-Experimental approach:

*aka using the computer to do the work for us.*




In this process, we will use a random system rather than a deterministic one. Using the game engine Unity, we can instruct a character to move in the rain.

To ensure a good comparison, we will have the same assumptions to the mathematical model:

- The person is a cuboid.
- Rainfall is vertical. Due to how the computer simulation works, rain must be random in its distribution in space. However, this should not hinder the reliability of the results as many water drops fall per second, meaning that it will have a clear average distribution.
- The person will move in a straight line of distance s, for a time t. The cuboid faces in the direction that the person will move in.
- The rain will fall with constant speed.

The computer simulation is rather straightforward. We create a particle system to simulate the rain. The person the is instructed to move from side to side. Every time the person collides with a raindrop, the volume of the raindrop is added to the counter.

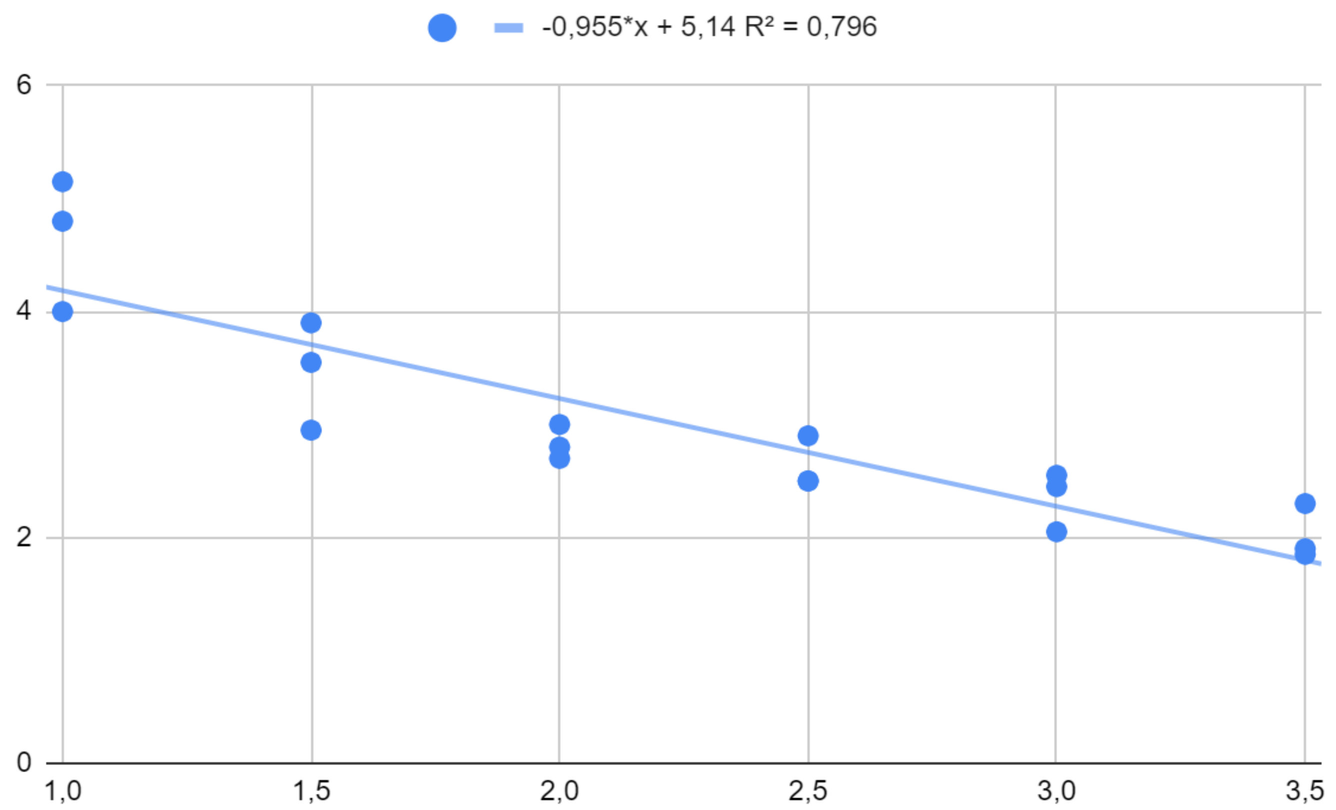
The volume of a raindrop is approximately 0.05 mL.

As said in the introduction, we will now take values from the simulation to compare with the mathematical model. In particular, the speed of the person will be the independent variable. The volume of water received is the dependent variable. Every other variable is controlled.

The following table contains the data collected from the simulation:

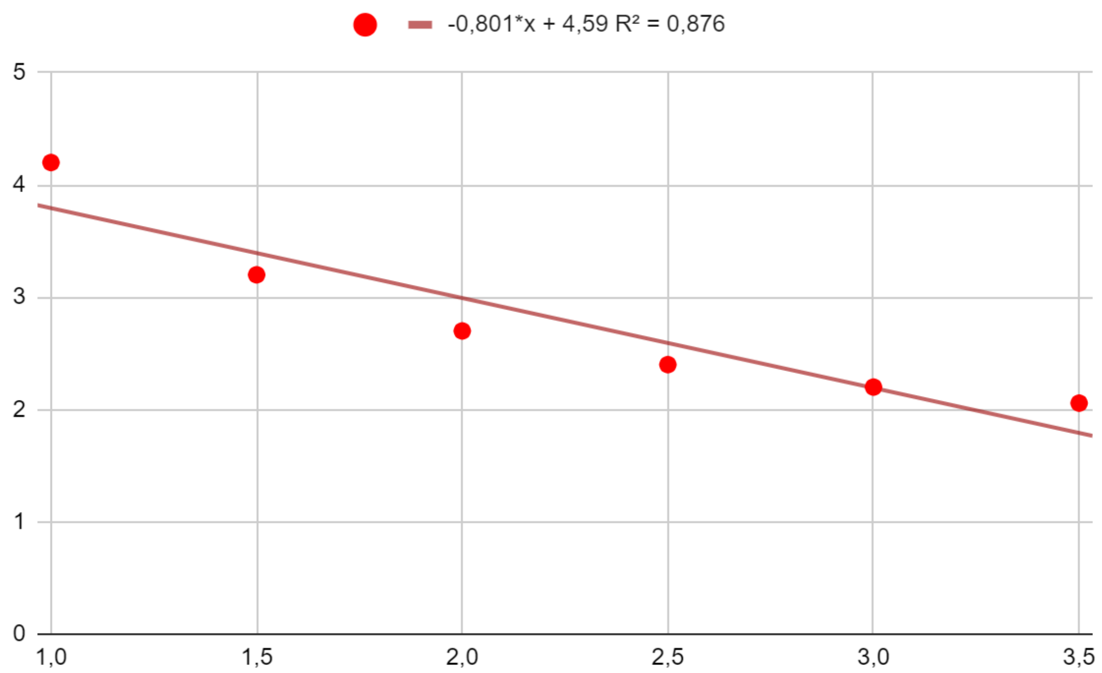
| Walking Speed / $ms^{-1}$ | Distance /m | Rain / mm $hour^{-1}$ | Rain Speed / $ms^{-1}$ | Rain volume / mL (1st repeat) | Rain volume / mL (2nd repeat) | Rain volume / mL (3rd repeat) |
|---|---|---|---|---|---|---|
| 1.00 | 20.0 | 4.00 | 15.0 | 4.80 | 4.00 | 5.15 |
| 1.50 | 20.0 | 4.00 | 15.0 | 3.55 | 3.90 | 2.95 |
| 2.00 | 20.0 | 4.00 | 15.0 | 3.00 | 2.80 | 2.70 |
| 2.50 | 20.0 | 4.00 | 15.0 | 2.90 | 2.50 | 2.50 |
| 3.00 | 20.0 | 4.00 | 15.0 | 2.05 | 2.55 | 2.45 |
| 3.50 | 20.0 | 4.00 | 15.0 | 1.90 | 2.30 | 1.85 |

Walking speed has been chosen to range from 1.00 $ms^{-1}$ to 3.50 $ms^{-1}$ because human walking speed is approximately 1.30 $ms^{-1}$ and running speed is approximately 3.30 $ms^{-1}$.




The results from the table are plotted on the graph above.

By inspecting the graph, it might seem that the data is unreliable, There is variance – especially in the lower speeds – and $R^2$ is low, of only 0.796. However, it is by considering the expected values that an interesting conclusion arises.




The expected results from the model are plotted on the graph above.

The expected value also has a low correlation coefficient. This is because the data is does not follow a linear trend. If we consider the mathematical model:

$$p_t = \frac{r}{u_r}(wh)s + r(wl)\frac{s}{u}$$

In this case, were treating $p_t$ as a function of u, the speed of the person. Therefore, we can rewrite the equation as:

$$p_t(u) = \mathrm{a} + \frac{b}{u}$$

Where a and be are constants. Thus, the model would suggest, the rain one receives is a reciprocal relationship to their speed. This explains why linear models had low correlation coefficient.

A better way to compare the expected results with the simulation, therefore, would be to conduct a chi-square test.

## Chi-squared test:

H0: the mathematical model is suitable for modelling how wet someone gets.
H1: the mathematical model is unsuitable for modelling how wet someone gets.

| | | | | | | |
|---|---|---|---|---|---|---|
| Mean Volume from Simulation / ml | 4.6500 | 3.4667 | 2.8333 | 2.6333 | 2.3500 | 2.0167 |
| Expected Volume from model / ml | 4.2000 | 3.2000 | 2.7000 | 2.4000 | 2.2000 | 2.0571 |

Goodness of fit = $\sum\left(\frac{O_i^2}{E_i}\right) - N$ = 19.25 - 17.95 = 1.3003

Because the mean is derived from a 3 by 6 table (3 repeats, for 6 values of speed), the degrees of freedom of the data are:

$$\upsilon = (3-1)(6-1) = 10$$

At a 5% significance level, the critical value is:

$$\chi^2_{10}(5\%) = 19.307$$

Because 19.307 > 1.3003, even by an order of magnitude, we have insufficient evidence to reject $H_0$. Therefore, the data from the simulation shows that the model is accurate.

## Conclusion:

By how surprisingly close the "experimental" results are to the model, it is clear that the model is accurate for the axioms we defined. Therefore, yes, the better way to avoid getting wet in the rain is by simply moving faster*. However, because the relationship is reciprocal, running much faster does not have as significant of an effect.

In the future, this project could be expanded by further generalising the axioms we defined:

- What if the person is not a cuboid?
- What if rain falls at an angle?
- What if rainfall is not constant in space and time?

Although this model does no provide sufficient answers to such questions, it might provide a starting point to such developments.

* or just having an umbrella