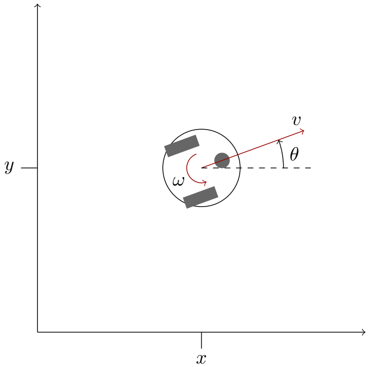
The unicycle model

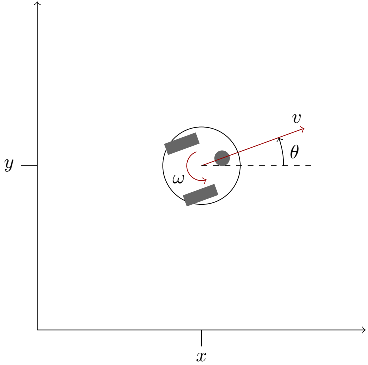
Kjartan Halvorsen

February 12, 2022

The unicycle model



The unicycle model

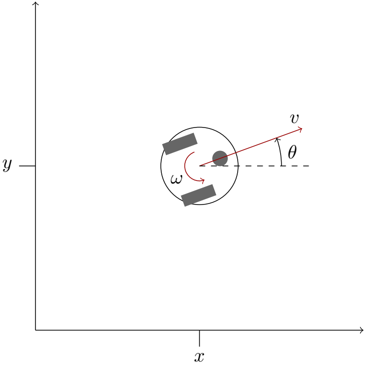


$$\xi = \begin{bmatrix} \theta \\ x \\ y \end{bmatrix}, \quad u = \begin{bmatrix} \omega \\ v \end{bmatrix}$$

$$\frac{d}{dt}\xi = \begin{bmatrix} \dot{\theta} \\ \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} \omega \\ v\cos\theta \\ v\sin\theta \end{bmatrix}$$

Called unicycle model.

The unicycle model



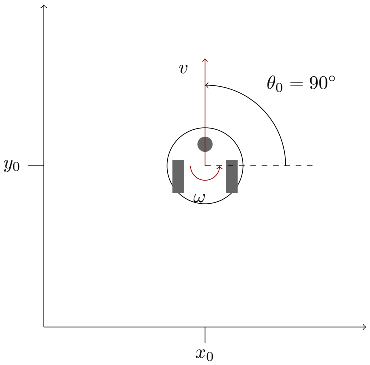
$$\xi = \begin{bmatrix} \theta \\ x \\ y \end{bmatrix}, \quad u = \begin{bmatrix} \omega \\ v \end{bmatrix}$$

$$\frac{d}{dt}\xi = \begin{bmatrix} \dot{\theta} \\ \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} \omega \\ v\cos\theta \\ v\sin\theta \end{bmatrix}$$

Called unicycle model.

Activity Can we reach any point in state space $\begin{bmatrix} x & y & \theta \end{bmatrix}^T$ by a suitably designed input signal sequence $u(t)$?

The unicycle model



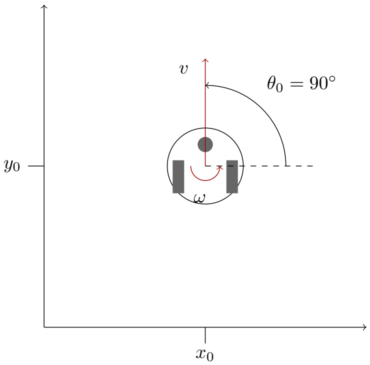
$$\frac{d}{dt}\xi = \begin{bmatrix} \dot{\theta} \\ \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} \omega \\ v\cos\theta \\ v\sin\theta \end{bmatrix}$$

Linearized model using deviation variables

$$\xi(t) = \xi_0 + z(t), \quad \frac{d}{dt}\xi = \frac{d}{dt}z$$

$$\frac{d}{dt}z = \begin{bmatrix} \omega \\ v\cos\theta_0 \\ v\sin\theta_0 \end{bmatrix} = \underbrace{0}_{A} z + \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}}_{B} \begin{bmatrix} \omega \\ v \end{bmatrix}$$

The unicycle model



$$\frac{d}{dt}\xi = \begin{bmatrix} \dot\theta \\ \dot x \\ \dot y \end{bmatrix} = \begin{bmatrix} \omega \\ v\cos\theta \\ v\sin\theta \end{bmatrix}$$

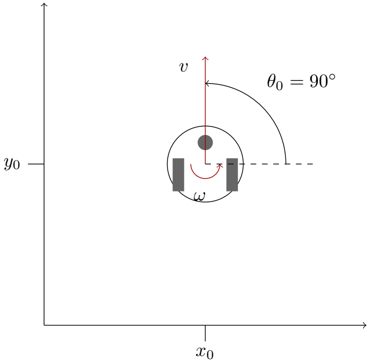
Linearized model using deviation variables

$$\xi(t) = \xi_0 + z(t), \quad \frac{d}{dt}\xi = \frac{d}{dt}z$$

$$\frac{d}{dt}z = \begin{bmatrix} \omega \\ v\cos\theta_0 \\ v\sin\theta_0 \end{bmatrix} = \underbrace{0}_{A} z + \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}}_{B} \begin{bmatrix} \omega \\ v \end{bmatrix}$$

$$\mathcal{C} = \begin{bmatrix} B & AB & A^2B \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The unicycle model



$$\frac{d}{dt}\xi = \begin{bmatrix} \dot{\theta} \\ \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} \omega \\ v\cos\theta \\ v\sin\theta \end{bmatrix}$$

Linearized model using deviation variables

$$\xi(t) = \xi_0 + z(t), \quad \frac{d}{dt}\xi = \frac{d}{dt}z$$

$$\frac{d}{dt}z = \begin{bmatrix} \omega \\ v\cos\theta_0 \\ v\sin\theta_0 \end{bmatrix} = \underbrace{0}_{A} z + \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}}_{B} \begin{bmatrix} \omega \\ v \end{bmatrix}$$

$$\mathcal{C} = \begin{bmatrix} B & AB & A^2B \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Activity Is the linearized model controllable? (Hint: What is rank $\mathcal{C}$?)