Real-Time Performance-Focused Localization Techniques for Autonomous Vehicle: A Review

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***Abstract*— Real-time, accurate, and robust localisation is critical for autonomous vehicles (AVs) to achieve safe, efficient driving, whilst real-time performance is essential for AVs to achieve their current position in time for decision making. To date, no review paper has quantitatively compared the real-time performance between different localisation techniques based on various hardware platforms and programming languages and analysed the relations among localisation methodologies, real-time performance and accuracy. Therefore, this paper discusses the state-of-the-art localisation techniques and analyses their overall performance in AV application. For further analysis, this paper firstly proposes a localisation algorithm operations capability (LAOC)-based equivalent comparison method to compare the relative computational complexity of different localisation techniques; then, it comprehensively discusses the relations among methodologies, computational complexity, and accuracy. Analysis results show that the computational complexity of localisation approaches differs by a maximum of about 10^7 times, whilst accuracy varies by about 100 times. Vision- and data fusion-based localisation techniques have about 2–5 times potential for improving accuracy compared with lidar-based localisation. Lidar- and vision-based localisation can reduce computational complexity by improving image registration method efficiency. Data fusion-based localisation can achieve better real-time performance compared with lidar- and vision-based localisation because each standalone sensor does not need to develop a complex algorithm to achieve its best localisation potential. Vehicle-to-everything (V2X) technology can improve positioning robustness. Finally, the potential solutions and future orientations of AVs' localisation based on the quantitative comparison results are discussed.**

***Index Terms*— Autonomous vehicle, localisation, sensor fusion, real-time performance, computational complexity, vehicle-to-everything.**

I. Introduction

AUTONOMOUS vehicles (AVs) are expected to play a key role in future intelligent transportation systems due to their potential in ensuring safe driving, relieving traffic pressure and reducing energy consumption. Current research on AVs has entered the road test phase. For example, Baidu has tested its Apollo 5.0 system in complex road scenarios, such as curves or intersections without special markings [1]. The Google Waymo project has also completed over 10 million miles on U.S. public roads and 7 billion miles in simulation [2]. Nevertheless, the industry still needs to address several critical challenges before the commercialisation of AVs. These challenges include a) coming up with real-time, accurate, and low-cost self-localisation solution, b) achieving real-time and accurate environmental perception model, and c) achieving smart, safe, and efficient decision making in complex scenarios. Meanwhile, the environmental perception and decision-making module significantly rely on the real-time and accurate self-localisation for AVs to achieve safe driving. Thus, Self-localisation is one of the core elements of AV. Moreover, safe driving, such as collision avoidance, can only be guaranteed when self-localisation achieves millisecond-level real-time performance and centimetre-level accuracy [3].

As a typical approach, map-matching algorithm is widely used in many localisation solutions equipped with lidar [4], radar [5], camera [6], or vehicle-to-everything (V2X) [7]. One of the map matching methods is to use an existing map to match the detected environment feature (e.g., corner and road marking) and thereby obtain vehicle location information. Another technique is simultaneous localization and mapping (SLAM) used in the application without a priori map. It achieves the vehicle position by simultaneously building an environment model (the map) for sequential mapping. The mapping algorithms mainly focus on abstract data extracted from various sensors, such as lidar, radar, camera, or their combination. In terms of sensor-based localisation techniques, it relies on on-board vehicle sensors to estimate the absolute or relative position of AV. It was discussed in detail by previous survey [8]. In many sensor-based localisation research, "sensor" was regarded as the main localisation sensors that authors tried to explore an innovative method mainly based on its measurements and aim to solve the positioning challenges in some special scenarios. It does not mean the localisation system only uses a single sensor to achieve vehicle location. As an example explaining this concept, for IMU-based localisation, reference [9] proposed an interacting multiple model (IMM) method by using IMU and odometer sensor data to eliminate the system drift caused by global positioning system (GPS) outage or GPS signal block, thereby improved the localisation robustness and integrity performance in such driving scenarios. The sensor-based localisation techniques can guide the deployment of AV localisation system, which includes how to select sensors, localisation algorithms, fusion algorithms, and

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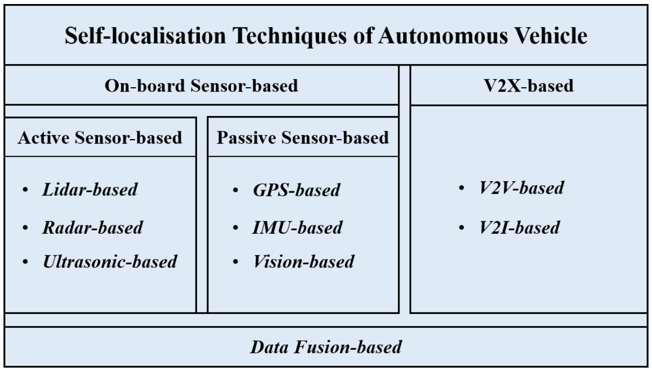

Fig. 1. Overview of different self-localisation techniques for AVs.

computational resources that can meet real-time performance. Moreover, focusing on the localisation input (sensor hardware) can allow the readers to better understand the advantages and disadvantages of different system deployments in terms of accuracy, real-time, robustness, and cost. Therefore, this survey will be organised starting from on-board sensors, to discuss different sensor-based localisation techniques, then discuss the V2X localisation techniques, and finally discuss the data fusion-based localisation.

Fig. 1 shows the different self-localisation techniques for AVs including on-board sensor-, V2X-, and data fusion-based techniques. The on-board sensor-based localisation system, which includes active and passive sensor-based technologies, relies on on-board sensors to perceive the surrounding environment and then estimate the vehicle position. The V2X-based localisation approach communicates with surrounding environment nodes (e.g., neighbour vehicles or infrastructures) to receive their pose information, including vehicle-to-vehicle (V2V)- and vehicle-to-infrastructure (V2I)-based technologies, which can provide several reference coordinates for the localisation algorithm. Data fusion is not a method of directly sensing position but of post-processing positioning sensing technologies. Its goal is to fuse the measurement results of various sensors to obtain better results than individual sensors.

Active sensor-based localisation actively perceives the surrounding environment to estimate vehicle positions through on-board sensors, including lidar, radar, and ultrasonic sensors. Their ranging principle is the same, which is based on the time-of-arrival (TOA) method. Their difference lies in the signal carriers, namely, laser, radio, and ultrasonic for lidar, radar, and ultrasonic sensors, respectively. The difference in signal carrier wavelengths results in considerable variations in the cost and accuracy of these sensors. For example, lidar usually has the highest cost but also the best precision amongst them; the converse is true for ultrasound [10]–[13].

Passive sensor-based localisation passively receives environmental information, from which it calculates the vehicle position. Sensors include GPS, IMU, and vision (e.g., monocular or binocular cameras). GPS needs three or more satellites in the open sky area to obtain vehicle position (2–10m) according to the spatial triangulation method. The advantage of GPS is its low cost, but it often suffers from multipath and non-line-of-sight (NLOS) errors in a city environment as well as the slow location update rate. IMU uses a high-frequency sampling rate (>100Hz) to measure the vehicle acceleration and rotation rate. Hence, the position and direction of the vehicle can be derived through dead reckoning by the given initial pose [14]. Although IMU has a fast location refresh rate and high reliability, it is also prone to substantial cumulative errors. Vision-based localisation estimates the vehicle position by using images from the monocular or binocular camera as inputs. This is similar to the vision system of humans that determines the position of obstacles according to plane triangulation. Rich environmental information in the image can provide satisfactory localisation performance under adequate illumination conditions but consumes substantial memory and computing resources.

V2V-based localisation refers to AVs under a vehicular ad hoc network (VANET), using dedicated short-range communication (DSRC) or long-term evolution technique to determine the pose of other vehicles and thus improve position accuracy of AVs. V2I-based localisation refers to the communication between the target vehicle and static infrastructures using their exact known location to determine the target vehicle position. The types of infrastructure include magnetic markers, radio-frequency identification (RFID) tags, roadside units (RSUs), and GPS base stations. V2X-based localisation has a broad global sensing range (∼300m [15]) but may suffer from network latency and congestion in the city.

Many surveys have been published summarising existing self-localisation techniques and have comprehensively discussed their advantages and disadvantages as well as the potential applications of each sensor-based method. However, most recent review papers only focus on the following aspects in evaluating various localisation methods:

a) economic and reliable localisation techniques [8], where economic corresponding to the cost of localisation system and reliable corresponding to how localisation performance (includes accuracy and reliability) those techniques can achieve in various driving scenarios (e.g., snow weather);

b) accuracy, reliability and availability [16], where availability corresponding to the localisation system should be available in different environments, such as a GPS-based positioning system in tunnels, a V2V approaches in situation with communication latency; and

c) robustness and scalability [17], [18], where robustness corresponding to the localisation system operates with low failure rate for long time in different seasons and traffic conditions, scalability corresponding to the capacity of the vehicle to handle large-scale autonomous driving.

The real-time performance of self-localisation is one of the key indicators to evaluate AV's safe driving. The above surveys also mentioned that researcher should carefully consider the computing load and real-time performance of different techniques when they design a localisation system. However, up to now, no survey has compared and discussed in depth the real-time performance of different self-localisation techniques. By comparing the reaction time of driver's behaviour on decision-making process [28]. We did literature review for displaying the reaction time of AV's behaviour from the

TABLE I
REACTION TIME OF AVS' BEHAVIOUR FROM OBSTACLE PERCEPTION TO ACTION EXECUTION

AVs' behaviour		Reaction time (second)	Total expected reaction time for AVs' behaviour (second)
Perception	Detection and recognition	~0.1-0.2 [19]-[21]	~0.5 [22], [23]
	Localisation	Expected millisecond-level [3]	
Judgement	Planning and decision making	~0.1-0.2 [24], [25]	
Reaction	Execution	~0.1 [26], [27]	

TABLE II
THE RELATION AND DIFFERENCE BETWEEN OUR SURVEY AND THE RECENT EXISTING SURVEYS

Year	Reference	Relation and difference
2016	[17]	SLAM techniques in industrial robots and AVs
2017	[18]	The SLAM approaches of AVs
2017	[16]	The relative positioning in vehicles
2018	[8]	The advantages and disadvantages of AV localisation techniques
-	Our survey	The innovative algorithms or approaches of AV localisation solutions and the real-time performance comparison between different solutions

moment of perceiving obstacles to the moment of executing control action, as shown in TABLE I. According to computer simulation and practical test, the reaction time of whole decision-making process for AV is usually cut to 0.5s to meet safe driving. However, in extreme cases, detection and recognition module, planning and decision-making module and execution module will take up nearly 0.5s, which results in the execution time reserved for localisation module being very limited. Hence, a fast real-time localisation solution can save the computing resources for other modules of the AV system, such as decision-making, to implement complex strategies to ensure safe driving. At present, the real-time performances of different localisation solutions are presented on various hardware platforms and programming languages. Comparing the real-time performance directly using the data provided by each self-localisation research paper makes no sense and cannot reflect the relative consumption of memory and computing resources in an AV. There is also no survey quantifying computational complexity of various positioning solutions under the same benchmark, which is related to the real-time performance and deployment cost of the localisation system. This paper aims to investigate existing the state-of-the-art localisation techniques and focuses on the proposed innovative algorithms or approaches of each solution and overall localisation performance in terms of real-time performance, accuracy, and robustness; propose an equivalent method to quantitatively compare the relative real-time performance between different localisation solutions based on various hardware platforms and programming languages; and finally conclude existing localisation techniques and discuss the potential solutions and future orientations based on the quantitative comparison results in an AV. The relation and difference between our survey and the recent existing surveys are summarised in TABLE II.

The rest of the paper is organised as follows: active sensor-, passive sensor-, V2X-, and data fusion-based localisation are discussed in Sections II, III, IV, and V, respectively. The accuracy and real-time performance of different solutions are discussed in Section VI. The conclusions are presented in Section VII.

II. ACTIVE SENSOR-BASED LOCALISATION

A. Lidar-Based Localisation

Lidar-based localisation generally needs to pre-build a reference map for matching with point cloud data or lidar reflection intensity data. But in the situation of no prior map, it will use SLAM technique to construct a real-time map to match with the previous generated map. In AVs' application, a high-dimensional map contains rich feature information, which improves position estimation accuracy but reduces memory efficiency and increases processing time [29], [30].

Im *et al.* [31] established a 1D corner map based on the vertical corner of buildings on both sides of the city road for matching and positioning. They used the iterative-end-point-fit to extract the features of the vertical corner and built the corner feature map by the length and direction of the vertical lines. Then, they applied feature matching with point cloud data to calculate vehicle position. This method reduces matching time and the map data file size ($\sim$14KB/km) due to less extracted feature information. However, the maximum horizontal position error reaches 0.46m; moreover, this method is not suitable for a region without buildings. Reference [32] constructed a 2D occupancy grid map composed of a road marking-based road reflective dense map and a vertical structure-based probabilistic occupancy grid map. First of all, they built a 1D extended line map (ELM) by extracting line features of the road marking and corner. The features contained only the latitude and longitude information of two endpoints of the line. Then, they converted the ELM into a 2D grid map for matching during positioning. Compared with [31], [32] added the road marking feature to improve the accuracy performance, but this method increases ELM data size to 134KB/km.

The 2D planar map matching for lidar-based localisation is very popular in current research. For example, Levinson *et al.* [4] obtained vehicle positioning by using a SLAM-style relaxation algorithm to build a flat ground reflective map without any potential moving object and then using the partial filter (PF) for correlating lidar measurements. In order to further improve the robustness, reference [33] used a probabilistic map represented as Gaussian distribution over remittance values instead of the previous ground map represented as fixed infrared remittance values. It enables the stationary objects and consistent angular reflectivity in the map to be quickly identified by Bayesian inference. Then they used offline SLAM to align the overlapping trajectories in previous sequential map, which makes the localisation system keep learning and improving maps. Compared with the method in reference [4], reference [33] improved the localisation accuracy and robustness of AV in dynamic urban environments. However, the map data size of these two methods has increased to about 10MB per mile. Reference [34] built a road surface reflective intensity map by extracting lane marking. This technique uses a curve approximation algorithm to represent roads as a set of piecewise parametric polynomials; thus, the pre-built map only contains line features. Both methods can reduce memory usage and computational complexity. The challenge is that they need apparent features of the ground, snow-covered roads, for example, can lead to positioning failure. In response to that problem, [35] and [36] reconstructed the incomplete lidar image of the snow-covered road surface with principal component analysis (PCA) to enhance the texture and structure of the road representation and obtain the vehicle position. These methods can obtain 0.2m lateral position accuracy. References [29] and [37] proposed a SLAM technique with matching the Gaussian hybrid maps to calculate the location, thus improving the robustness of lidar-based localisation. The highlight of the hybrid map is adding a z-height distribution to each cell in a 2D grid map. This method can achieve lateral position errors within 10 cm. Although the map data volume is quite large (44.3MB/km), it can achieve 0.1–0.2s positioning refresh rate with the help of graphics processing unit (GPU) acceleration. Other studies were dedicated to reducing the data size of the plane map and addressing the trade-off between real-time performance and positioning accuracy. For example, Hu *et al.* [38] calculated vehicle positioning in the loose constraint area, such as intersections, by building a topological-metric hybrid map, whilst a simple topological map was built for the lane-following driving scene. The data volume of the hybrid map was reduced by 50% compared with that of traditional metric maps. The pre-built maps mentioned only consider extracting the typical static features. However, positioning would fail if lidar cannot detect these typical static features due to obstacles or scattering surface. In addition to static features, considerable research has been proposed to improve position performance through tracking dynamic objects with machine learning methods of the random forest classifier [39], [40] e.g., [40] achieved successful vehicle positions in about 93% of confidence values, or SLAM approach based on a deep learning approach of multiple hypothesis trackers with an adaptive IMM [41]. By contrast, [42] proposed a method, namely, multi-layer random sample consensus (RANSAC), which did not need to detect static or dynamic features during registration to iterate and update registration results. Experiments for this method achieved horizontal and vertical root mean square errors (RMSE) of less than 0.076m and 0.15m, respectively.

The 3D map-based matching can achieve a more accurate position because it contains height information of environmental objects. Reference [43] built a 3D map by extracting road marker features. Then, the system used normal distribution transform (NDT) to process uncertain information, after which robustness and accurate positioning were derived based on PF. The 3D NDT method, however, may require a large amount of memory for keeping ND voxel (total 3D ND voxels for matching is up to 100MB [30]), which results in a positioning time that is as high as that of the second level [44]. Li *et al.* [45] proposed building a 3D occupancy grid map and then used a hybrid filtering framework (i.e., a combination of cubature Kalman filter and PF) to calculate large-scale outdoor localisation and reduce the map data size. Although the amount of data was reduced, experiments showed that this approach could maintain stable, reliable positioning performance, which means the positioning error is less than 0.097m.

B. Radar-Based Localisation

Compared with lidar- and vision-based localisation, radar-based localisation can meet real-time performance requirements because of its memory-efficient and low computing load [46], [47]. However, radar-based simultaneous localisation and mapping (SLAM) faces the risk of data registration errors in map matching due to insubstantial features that are sometimes extracted, thus leading to the risk of low positioning accuracy [5]. The trajectory-oriented extended Kalman filter (EKF)-SLAM technique uses the Fourier–Mellin transform to register radar images sequentially and calculates vehicle position without matching the features to avoid the risk brought about by such features. The disadvantage is that the positioning error reaches 13m (Mean) [46]. Reference [48] aimed to extend a semi-Markov chain by a Levy process to improve robustness in a long-term change environment, and 83% of the estimated position error was less than 0.2m. For rain and snow situations, [49] built a reference map by modelling the uncertainties of error propagation and then matched radar images to achieve reliable positioning. Reference [50] proposed a cluster-SLAM technique that used a density-based stream clustering algorithm to cluster radar signals in a dynamic environment. An environment scan without measurement noise was presented for map matching. PF was applied to use this match result for calculating the vehicle position. The size of the map used in this technique was only 200KB.

Moreover, references [51] and [52] proposed a joint spatial and Doppler-based optimisation framework to further improve the speed of positioning. This framework expresses the reference scan by building a sparse Gaussian mixture model, which is a sparse probability density function that can reduce computational complexity. The positioning refresh rate of this method can reach 17Hz. Reference [47] used the radar scanning data of the same roads to construct the reference map. Then, iterative closest point (ICP) was used to match the radar image to estimate the vehicle location. Finally, EKF was applied to smooth the estimates. This technology reduces the calculation load of map matching due to the small size of the mapping data required. However, the challenge is that it requires the latest data out from the same model of the sensor and sample from the same roads as the reference to perform the matching.

Furthermore, reference [53] designed an under-vehicle-mounted localising ground-penetrating radar (LGPR) system to build the road subsurface map. This system can resist signal interference by complicated weather because its radar is mounted under the chassis to scan the ground. Moreover, it can achieve high accuracy (RMSE of 12.7cm) and excellent real-time performance (∼126Hz refresh rate). However, the authors also mentioned that the height of LGPR array needed to be reduced further for fitting under more passenger vehicles.

C. Ultrasonic-Based Localisation

Ultrasonic-based localisation is widely used for the application of indoor robot localisation due to the low-cost ultrasonic sensor. However, short detection distance and sensitivity to environmental temperature, humidity and dust all limit the wide application of ultrasonic sensor in AV positioning [54], [55]. Moussa *et al.* [56] used the EKF algorithm to achieve an ultrasonic-based assisted navigation solution. This solution uses the ultrasonic sensor as the primary sensor for positioning when GPS fails to limit the drift of the vehicle position and enhances the robustness of the system. It can achieve excellent real-time performance (∼92Hz refresh rate), but the position error is up to 7.11m. Jung *et al.* [13] used ultrasonic sensor, encoder, gyro, and digital magnetic compass, and together with a SLAM method to estimate vehicle absolute position. The average position update time of this method is up to 10.65s. Also, the long-time SLAM calculation process can lead to some accumulated errors for the localisation system caused by IMU before the position update Therefore the average driving distance that can meet the position accuracy requirement is only about 5.2m. In summary, ultrasonic-based localisation technique can achieve a low-cost and low-power positioning system. However, its localisation accuracy and robustness performance still cannot meet the requirements of autonomous driving.

D. Discussion

Accurate and robust feature detection approach in lidar-based map matching techniques can improve the accuracy and robustness performances of AV localisation [57]. In summary, in terms of lidar-based 1D map matching techniques, the computational load and memory usage in feature registration is low since this method only takes a few special-shaped lines as features, such as vertical corners shown in reference [31] and [32]. However, this method needs to address the challenges in the scenarios where there are no vertical buildings on roadsides. Compared with 1D map, 2D map contains rich types of features but increases the map storage. The intensity-based 2D map method can enhance the road representation in snow-covered road surface scenarios. And the hybrid map-based algorithm can reduce the memory usage and address the trade-off between real-time performance and positioning accuracy, such as a topological-metric map shown in reference [38]. The 3D map-based matching algorithms can achieve an accurate and robust position which benefit from the 3D features. However, it takes the largest computational resource compared with 1D map- and 2D map-based methods, which will increase the deployment cost of AV localisation system. Compared with high-cost lidar-based localisation, radar is a cost-efficient solution, but the low resolution of the environment model obtained by millimetre-wave radar and the lack of object's height information make the localisation system difficult to achieve robustness and accuracy performance. At present, radar is widely used as an auxiliary localisation sensor to detect the distance from the vehicle to obstacles. The ultrasonic sensor detection range (∼3m) decides the ultrasonic-based localisation is mainly used in short-distancing localisation applications, such as automatic parking, in which several reference objects are at a close range.

III. Passive Sensor-Based Localisation

A. GPS -Based Localisation

GPS can provide a low-cost, efficient positioning solution for an AV. However, GPS is often affected by NLOS, multipath or signal block in an urban canyon, all of which pose challenges to the goal of providing a reliable vehicle localisation [58], [59].

Current mainstream GPS-based localisation improves accuracy and reliability by position correction technologies, including fusing measurements from different sources [60], filtering abnormal signal [61], and map aid [62]. Reference [63] improved GPS-based localisation by fusing measurements from other sources, including GPS, RFID, and V2V. The authors analysed the accuracy of different data sources and filtered out redundant connections. They only retained connections with the desired accuracy to achieve the robustness requirements in a GPS-degraded environment. The position accuracy of the proposed method is about 2.9m, and computational complexity is about 0.8% of [64]. Reference [61] proposed a GPS abnormal signal discrimination processing framework to improve the robustness for GPS-based localisation. This framework can decide to output original GPS, estimated GPS or GPS with removed the abnormal signal according to the quality of original GPS. Unlike the two previous technologies, Lu *et al.* [65] improved GPS accuracy by matching a low-precision open-source map. However, the limitation of this method is that extracting the lane marking features in road junctions is difficult. Meanwhile, [66] proposed a global navigation satellite system (GNSS)-based localisation method by removing abnormal GPS signals and combining with terrain height aiding of a digital map. Reference [67] improved GNSS accuracy by matching the NLOS signal delays. Despite the effort, the position RMS error for [66] and [67] remain as high as about 10m in an urban canyon scenario. In conclusion, achieving a reliable, accurate vehicle location using a standalone GPS receiver is difficult.

B. IMU-Based Localisation

IMU, a component of the inertial navigation system (INS), can measure the acceleration and pitch rate and has a strong anti-interference capability [68]. However, an autonomous system cannot use IMU to calculate the position for a long distance due to the disadvantage of cumulative errors. In this case, IMU is widely used as a backup sensor or one of the fusion sources for guaranteeing continuous localisation when the primary positioning sensor is short-time interrupted [69].

Reference [70] proposed using a dead reckoning (DR)-based tightly coupled (TC) scheme to improve accuracy performance in urban canyons. Reference [71] used modified TC with abnormal GPS measurement rejection to achieve continuous positioning under GPS denial environment. Wang *et al.* [72] proposed a scheme based on a set of autoregressive, moving average predictive models and occupancy grid constraints to improve positioning accuracy further; the scheme can also reduce cumulative error of the DR system and multipath interference on GPS. Xiao *et al.* [73] eliminated the effects of abnormal signals by using PF smoothing algorithm, which considered non-Gaussian noise and process noise from sensor measurements. They achieved a cost-effective positioning for AVs, although real-time performance requires further improvement. Similarly, Belhajem *et al.* [74] proposed using machine learning to determine and compensate the deviations of IMU-based localisation during GPS failure. However, they only achieved metre-level positioning accuracy. For intersection regions, DR error and distance from the stop line to the AV were used as the input states of EKF to achieve the high-precision position in [75]. The DR error in this research was calculated by matching curved lanes with waypoints. To further improve the localisation robustness, Ndjeng *et al.* [9] proposed a DR-based IMM by using constrained probabilistic models, which improved the robustness and integrity of AV localization in the driving scenarios of GPS outage or GPS signal block. Gruyer *et al.* [76] proposed a DR-based credibility IMM approach based on Transferable Belief Model (TBM). Compared with the reference [9], this method further improves the positioning accuracy and robustness.

In addition to the DR method, pattern recognition for pitch rate signal output by IMU can also be used to calculate vehicle position. The principle of this method is that the patterns of vibration and motion of a vehicle are extracted by analysing pitch rate signals. Then, pattern matching is performed with the pre-built indexed map for position estimation. This technology has no cumulative errors and thus has achieve reasonable accuracy (∼5m). The disadvantage, however, is that it can be easily affected by measurement noises [68], [77], [78].

C. Vision-Based Localisation

Vision-based localisation can typically achieve reasonable accuracy. The popularity of multi-core CPU and GPU and the improvement in their powerful parallel image processing capabilities alleviate the pressure from high computational complexity for this type of localisation methods [79], [80].

Reference [81] used four fisheye cameras, a pre-built map and the current vehicle pose to detect symmetric park markings within a given range in autonomous parking scene. Then, detections were taken as orientation marks to match with the pre-built map. This method can achieve the vehicle position with a parallel position error of 0.3m and positioning time of 0.04s. Du *et al.* [82] developed an improved sequential RANSAC algorithm to extract lane lines from images efficiently for feature matching; they achieved about 0.06m position error and a 0.12s positioning refresh rate in a scenario with lane lines. Reference [83] built a road landmark-based lightweight 3D semantic map for feature matching and then minimised the residual registration error to estimate the vehicle position. This map can reduce the memory usage, which results in just four iterations for image matching. The weakness of this method, however, is that it still needs further testing when used in a curved road scenario.

Reference [6] proposed building a planar feature map to reduce positioning time. Each planar feature includes its position, orientation, texture and observation and is projected to a camera view for matching with captured images during positioning. The position refresh rate of this method is about 0.09s. To improve the accuracy and real-time performance of vision-based localisation further, [84] and [85] matched the captured image with the predefined orthogonal map to obtain a correlation distribution, and the correlation distribution is used to update a probability distribution of the vehicle pose. Then, they estimated the vehicle position using this probability distribution. The position accuracy of this approach is less than 0.14m (RMS), and its real-time performance is about 0.057s. However, their proposed approach needs apparent surface information of the road. Reference [86] designed a coarse-grained semantics (e.g., flat terrain, shrubs, or tree) to match with a ground image and a satellite map and achieved reasonable vision-based localisation performance across seasons. Reference [87] combined a semi-dense image description based on a histogram of oriented gradient features and global descriptors from deep convolutional neural networks trained on ImageNet for image matching. These two techniques can achieve robust localisation, but their positioning accuracy needs to be improved further.

Meanwhile, reference [88] developed a topological model to obtain a set of possible nodes from a reference map that were close to a captured image. Then, they matched the extracted holistic features with the possible nodes for a closest node. Finally, they achieved reliable vehicle positioning with position accuracy of 0.45m by associating features from this node with local features from an image. This method, however, is subject to illumination sensitivity, which may result in positioning failure. Reference [89] proposed an extended hull census transform method for semantic description and feature extraction from an omni-directional image dataset to build a topological map. By combining content- and feature-based image retrieval methods for scene recognition, that work achieved a robust positioning in about 85.5% confidence value in changing lightness and dynamic obstacles scenarios by matching recognition results with a topological map. The challenge of this technique, however, is that its position refresh period is up to 2s.

D. Discussion

To sum up, the analysis in passive sensor-based localisation techniques has shown the significant advantages for obtaining low-cost AV localisation. However, it is noted that a standalone passive sensor cannot meet the accuracy and robustness requirements. GPS is often affected by NLOS, multipath or signal block in urban canyons, which pose challenges to localisation consistency and integrity. GPS-based localisation can be improved through fusing GPS measurements from different sources, imperfection signal bounding and map aid. The DR system can provide a real-time consistent vehicle position when GPS signal is unavailable. For example, the DR-based IMM method reduced the system drift and improved localisation robustness and integrity in the environments of GPS outage or GPS signal block, as shown in [9]. However, both of GPS-based and IMU-based localisation still need to further improve the accuracy, consistency, and integrity performance in the situations of long-time abnormal GPS-IMU signals occur. Vision-based localisation can achieve a positional RMSE of 0.14m. but a reasonable localisation time usually requires the system to be equipped with GPU for acceleration. Moreover, the reliability of the camera in the conditions of inadequate illumination or bad weather (e.g., fog and rain) still need to further study. The foregoing discussions show that data fusion techniques will be the trend to achieve a cost-efficient localisation solution by fusing multiple low-cost sensors. Meanwhile, the recent works about sensor fault detection and identification approaches in reference [90]–[93] have shown significant advantages for improving localisation robustness performance, such as IMM-based fault identification method, multi-model and fuzzy logic-based fault detection approach etc. Future research is required to focus on these techniques and the imperfection data modelling methods.

IV. V2X-Based Localisation

A. V2V-Based Localisation

V2V-based localisation does not require vehicles to be equipped with high-precision sensors for achieving an accurate position under a VANET. Instead, it can achieve reasonable position accuracy by fusing the coarse pose information from other connected vehicles [94]. However, its disadvantage is that the insufficient or non-uniform distribution of participating vehicles on a road may result in inadequate positioning accuracy [95], [96].

Liu *et al.* [15] proposed a weighted least square–double differences method to calculate inter-vehicle distances based on sharing GPS pseudo-range measurements with other vehicles. They used a distributed location estimate algorithm to fuse the sharing data and achieved a positioning accuracy of about 4m. This solution reduces the effects of random noise and improves the accuracy of calculating inter-vehicle distances. Reference [97] proposed using the Bayesian method to fuse the information of GPS position from target vehicle GPS positions from other vehicles and inter-vehicle distances for vehicle localisation. This method can considerably degrade positioning uncertainty. To eliminate the challenge of participating vehicles that need a predefined dynamic motion model to implement data fusion, reference [98] calculated a belief about vehicle current position, which is a probability that can infer vehicle position and broadcast it in VANET. Then, they used the angle of arrival and TOA techniques to measure inter-vehicle distances, which presented the relative position of neighbour vehicles. Finally, the vehicle position was estimated by calculating a weight sum over locations by its neighbour; the location includes a relative position and a belief. The position accuracy of this method is about 1.95m, but the refresh rate is up to 1.4s (7 vehicles access the network). Reference [99] proposed to use fuzzy logic method to achieve an accurate vehicle localisation in VANETs. The authors first obtained a weight to each nearby inter-vehicle using fuzzy logic. Then, a weighted centroid localisation method was applied to assign weights to neighbour vehicles, such as vehicles that were closer had higher weights, thereby achieve estimated position using all vehicles' weighted coordinates. The simulations results show that this method can obtain a sufficient localisation accuracy (MSE $<$ 30cm) when the network sizes increase to about 100 cars. Both reference [98] and [99] allocated confidence values to each received position estimation by assigning the beliefs or weights to neighbour vehicles and then improved localisation accuracy.

Reference [7] proposed using a cooperative map matching (CMM) method and a dynamic base station differential GPS (DDGPS) to improve vehicular positioning. The authors of that work implemented a road constraint of nearby vehicles to reduce positioning uncertainty and a DDGPS for pseudo-range correction. Then, Rivoirard *et al.* [100] proposed a Chain-Branch Leaf (CBL) clustering scheme to guarantee that the vehicle in reference [7] can exchange its state and pose and error correction. Meanwhile, this scheme can provide an accurate V2V communication service to the vehicles under a VANET. Soatti *et al.* [101] proposed an ICP approach that uses passive physical features (e.g., pedestrian and traffic light) as common noisy reference points to improve positioning accuracy. This method achieved sub-meter accuracy for 75% of confidence level. Both techniques do not need the calculation of inter-vehicle distances. Reference [102] proposed a detection–rejection approach to eliminate GNSS multipath bias and used Rao–Blackwellised particle filter-based map matching method to improve position accuracy. This solution can achieve the positioning accuracy of about 0.9m in GNSS multipath scene, but it is subject to low-frequency noise errors from the GNSS receiver. Reference [103] achieved a fast-positioning solution by deploying multi-type sensors to identify nearby vehicles and cooperating with a local map to quickly achieve accurate relative positions. The challenge of this solution is that it increases the deployment cost, although its position refresh rate is less than 0.1s.

B. V2I-Based Localisation

V2I-based localisation infers the vehicle position based on the locations of nearby infrastructure. It can achieve an accurate, real-time and robust localisation performance. The advantages of V2I technique include high-accuracy locations of infrastructure, stable data sources independent of time and low computational complexity.

References [104] and [105] proposed a magnetic marker-based V2I localisation. First of all, magnetic markers with unique Gaussian distribution of polar array are arranged on a road at a certain interval, and the position and distribution of each marker are stored in the database. Then, each marker is detected, and its Gaussian distribution is calculated during vehicle driving. Finally, vehicle position is determined by searching this distribution in the database. This method minimises the effects of distortion and achieves a centimetre level (<10cm) positioning accuracy. RFID techniques, which include low-cost RFID reader and RFID tags, are also used for localisation. RFID tags are deployed on the road surface, and a vehicle equipped with an RFID reader can determine positioning from the tags [106], [107]. As for the disadvantages, these techniques require high-density infrastructure and easily suffer from blocked infrastructure.

Wang *et al.* [108] proposed using two cooperation multiple-input multiple-output (MIMO) radar with direction-of-arrival (DOA) method for vehicle localisation. They applied the unitary sparse Bayesian learning method to degrade mutual coupling and noise of MIMO radar. The advantage of this technique is that it improves the robustness of DOA estimation. Reference [109] combined an RSU and an INS system to assist in vehicle localisation in GPS denial environments. Reference [110] used a Bayesian approach to fuse RSS data from two RSUs and improve GPS position accuracy. This method considers the effects of two-type RSU deployment (e.g., same or opposite sides) on vehicle positioning. Reference [111] estimated the carrier frequency offset of received DSRC signals from two RSUs for vehicle positioning and achieved a positioning error of less than 2m. Reference [112] used a pair of RSUs on either side of the road and implemented a two-way reciprocal TOA to achieve a position accuracy of about 3.3 m. However, this technique requires strict deployment of RSU, such as height, signal broadcast angle and transmit power [113], and it has challenges related to signal latency. Real-time kinematic (RTK)-GPS can achieve centimetre-level position accuracy. It uses a dual-frequency receiver to obtain GPS base station position and then calculates the position by using the carrier measurement technique. Differential GPS (DGPS) can improve GPS positioning accuracy to a centimetre level by using the fixed, known position of the GPS base station [8]. The challenges of using the RTK-GPS or DGPS techniques, however, are that they require deployment of expensive GPS base stations and they are subject to multipath or NLOS errors in an urban area.

C. Discussion

From the review of V2X localisation techniques, both V2V and V2I solutions do not require expensive dedicated hardware. For V2V-based solution, the sufficient and uniform distribution of participating vehicles on a road can enhance positioning accuracy and robustness. However, the continuous increasing vehicles may result in high system computational overhead but the accuracy has not improved much. An efficient clustering architecture for creating a hierarchy between the nodes can provide accurate V2V communication service under a VANET with long distancing. The challenges of accurate information exchange between inter-vehicles can be overcome by further studies on such architectures. CMM method can provide a potential way to discard the multipath errors between antennas, but the issues of propagation signal latency still need to be further addressed. The signal latency to V2X systems was suggested within 10ms [3]. The signal degradation and packet loss can be solved by optimising the network parameters (e.g., data baud rate, broadcast frequency, and antenna power etc.), which have been discussed in detail by previous survey [8]. RFID-based V2I systems can achieve a cost-efficient AV localisation. However, these methods require high-density infrastructure and easily suffer from blocked infrastructure. The RFID-based techniques are very suitable for the applications which AV driving on the fix routes, such as sightseeing bus in zoo or container handling vehicle in port. Optimising the relations between RSU height, propagation angle, and transmission power can ensure a wide range of signal strength and network coverage for achieving accurate and robust RSU-based V2I localisation. Though the signal latency still needs to be further addressed to improve localisation accuracy.

V. Data Fusion-Based Localisation

A. Multi-Sensor-Based Data Fusion Localisation

The previous discussion presented that no standalone sensor could meet the accuracy, real-time and reliability requirements for AV localisation. Data fusion of multiple sensors shows substantial potential for achieving an accurate, real-time and reliable self-localisation.

Reference [114] developed an interacting multiple model (IMM) filter, which consists of a vehicle dynamic model and a vehicle kinematics model, to achieve cost-efficient AV localisation by using low-cost sensors. GPS data and in-vehicle sensor (i.e., wheel speed sensor and steering angle sensor) data are used in this filter. The IMM filter could weigh an appropriate model for data fusion implement based on various driving scenes. This method can achieve a reasonable positioning performance in a 32-bit embedded processor. Reference [115] proposed building a model with three IMM-based UKFs to fuse low-cost sensor data, such as GPS and inertial sensors. This model reduces most of the uncertain noise from inertial sensors, predicts and compensates the positioning errors and could achieve a position accuracy of 1.18m during GPS outages. To dynamic manoeuvres situations, such as strong acceleration, high-speed turning, and START and STOP, Ndjeng *et al.* [116] showed that the IMM-based localisation system using low-cost sensors (e.g., IMU, odometer, and GPS) outperformed EKF-based. They concluded that IMM-based positioning robustness performance is better than EKF-based in high variability in the vehicle dynamics manoeuvres through practical experiments. In terms of combining with DR system, Vivacqua *et al.* [117], [118] built a back lane marking registry (BLMR) model for localisation. This model recorded the lane marking detection of the last 240m driven and continuously updated their relative position by a DR system. For the first step, pose measurement was implemented by matching BLMR lane markings with reference map lane markings. Then, robust positioning was calculated through the data fusion of pose measurements and DR. This method achieved position accuracy of about 0.3m and real-time performance of about 0.008s. Bak *et al.* [119] proposed a low-cost multi-sensor localisation solution for the scenarios with disturbance. They used a cheap visual odometry (VO) to bound the cumulative error caused by low-cost DR system and achieved a similar performance than an expensive INS sensor. Moreover, the authors also defined a median retro-projection error calculated for all inliers in image pairs data as the confidence indicator. This confidence value can offer real-time reliability assessment to system. The result showed that the robustness of this VO system is similar to that of the high-end INS system. The authors also mentioned this indicator can be used as either abnormal data filter or adapted noise model. Moreover, the bio-inspired methods are currently applied for AV localization. For example, particle swarm optimization (PSO) is an optimization method inspired by the behaviour of biological social group such as bee colonies or bird flocks [120], [121]. Godoy *et al.* [120] proposed to use particle swarm localization (PSL) algorithm to obtain AV position by fusing low-cost sensors data – e.g., GPS, IMU, odometer and a digital map. This method can achieve the localisation update time in less than 0.05s and the position accuracy is better compared with the EKF method when the swarm size is set to around 250 particles. Furthermore, Bacha *et al.* [122]–[124] proposed an optimized Kalman particle swarm (OKPS) fusion approach to achieve vehicle localisation by fusing the data from low-cost sensors (e.g., GPS, INS, and Odometer). This approach combined the advantages of different filters (such as EKF, PF, and PSO) to solve the swarm particle filter's premature convergence challenge of and traded-off the localisation reactivity and robustness issues of AV in dynamic environments. Compared to other PSO aided algorithms, this approach can meet the real-time localisation and improve the localisation accuracy and robustness performance in various scenarios (e.g., GPS failure scenarios). Furthermore, the bounded-error state estimation methods were applied to vehicle localisation in outdoor environments, such as set inversion via interval analysis (SIVIA) algorithm [125] and constraints propagation approach [126]. Kueviakoe *et al.* [127], [128] introduced a real-time interval constraint propagation algorithm for on-road vehicle orientation correction using GPS, gyro, and odometer sensor data. Wang *et al.* [129], [130] proposed to use interval constraint propagation technique to achieve localisation consistency and accuracy of a car-like robot by fusing DR, camera and map data. Both solutions considered the localisation problem as an interval constraint satisfaction problem. And then used the constraint propagation approach to solve this problem. Moreover, Xu *et al.* [131] combined an empirical mode decomposition interval threshold filter (EITF) with a least-squares support vector machine-based nonlinear autoregressive with an exogenous input model (LSSVM-NARX)/KF hybrid strategy for cost-efficient localisation. They adopted EITF to filter the noises from INS and applied LSSVM-NARX/KF to predict and compensate the INS position error. This approach achieved reasonable positioning accuracy in GPS outages and provided navigation at 20Hz.

B. Map-Based Data Fusion Localisation

The map-based data fusion technologies are based on multi-sensor measurement and improve the localisation performance by adding map information. For example, Suhr *et al.* [132] proposed fusing low-cost sensors with a digital map to improve real-time performance. They expressed the lanes and road marking features as a set of key points and used a front-end camera module to process captured images. This solution can reduce memory usage and computational overhead; moreover, its position refresh rate is about 100Hz, and its position accuracy is about 0.5m. Cai *et al.* [133] proposed a data-driven motion model without using inertial sensors to eliminate the challenge of integration errors. They corrected GPS positions and lateral distances from the camera by using an HD map, after which they used these two types of information as fusion data. The position error of this method is reduced by 1/3 compared with that of pure GPS-based localisation. Gruyer *et al.* [134], [135] proposed a map-aid data fusion method based on an accurate digital map, GPS, IMU, and two cameras to obtain a sub-decimetre accuracy AV lateral position. They first estimated the distance from the vehicle to road markings of the vehicle's right and left sides through two lateral cameras. Then, they used EKF to estimate vehicle position by using GPS and IMU sensor measurements. Finally, they combined the previously estimated vehicle position and matching segment position obtained by the point-to-segment-based map-matching algorithm to further improve the localisation accuracy and reliability. Bresson *et al.* [136] proposed a cooperative fusion architecture based on a laser-based SLAM algorithm and a lane marking detection and tracking algorithm to achieve robust localisation. This framework can select the best position output according to the state of two fusion methods and driving environment, thereby improve the accuracy and robustness performance of the localisation system. For the localisation solutions that require a map, reference [137] proposed using high-precision lidar to build a Gaussian mixture map (GMM) in order to improve positioning accuracy and robustness. Each grid-cell in GMM consisted of laser intensity and altitude, which made the matching results against challenging scenes. Error-state Kalman filter was adopted to fuse data of matching results, GNSS localisation and INS measurements. This method achieved a positioning accuracy of about 5–10cm. In terms of SLAM-based fusion techniques, Zhang *et al.* [138] have compared some SLAM-based fusion methods through theoretical analysis and have shown that the right invariant error EKF (RI-EKF) -SLAM can achieve better accuracy and consistency performance compared with special orthogonal group SO (3)-EKF-SLAM. Bounini *et al.* [139] proposed a SLAM-based real-time collaborative AV localization approach. In this method, the extended information filter (EIF)-SLAM was used to fuse vehicle sensor measurements to obtain the local fusion node (LFN) (includes vehicle status, information vectors, covariance and other information, etc.). Then, the LFN was broadcasted to its neighbours. Finally, a track-to-track fusion was applied to integrate LFN information to achieve the final vehicle position. This solution can reduce the computational complexity of cooperative localisation and improve the accuracy and consistency performance.

C. Discussion

The analysis has shown that the low-cost multi-sensor (e.g., GPS, IMU, camera, and odometer etc.) data fusion-based techniques can provide a cost-efficient commercial localisation solution for AV. Multi-sensor data fusion techniques fusing with GPS measurements still need to address the issues of GPS integrity. The IMM-based fusion methods can reduce most of the uncertain noise from inertial sensors and improve the localisation accuracy and robustness during GPS outage or GPS signal block. Nevertheless, the positioning error of the IMM is still up to the meters level. The interval method can achieve the vehicle localisation with high level of integrity and consistency through modelling the imperfection data as intervals. The position RSSE and update time of this approach can be about 15cm, about 170ms, respectively. The interval technique can offer a potential fusion-based localisation solution to the market. However, the overall localisation performance in different complex environments still need to further validation for fully AVs. The cooperative approach fusing with a map can also obtain an accurate and robust localisation solution. For example, reference [136] shown a cooperative approach that can enhance the localisation accuracy and robustness by fusing with multi-sensors (e.g., GPS, camera etc.), SLAM and a map. Furthermore, the different sensors' fault detection and identification techniques can also be focused to guarantee a more robust AV localization. To sum up, the foregoing discussions show that data fusion-based technique has significant potential to trade-off commercial AVs' localisation performances between economy, real-time, accuracy, and robustness.

VI. Accuracy and Real-Time Performance Discussion

A. Related Work of Localisation Performance Evaluation

A real-time, accurate and robust AV localisation is one of the key elements to guarantee safe driving. The performances of different localisation techniques comparison can guide the sensor selection of AV system and research purpose. Many works related to the accuracy and robustness performance comparison of different localisation algorithms have been published. Zhang *et al.* [138] theoretically analysed the convergence and consistency properties of RI-EKF-SLAM and compared its localisation performances with SO (3)-EKF-SLAM. The accuracy and consistency performance of RI-EKF-based SLAM and optimization-based SLAM were compared through 1-D, 2-D and 3-D simulations by Zhang *et al.* [140]. Moreover, Mourllion *et al.* [141] showed the performances of Kalman filter variants-e.g., EKF, UKF, and the Divided Differences of first and second order (DD1 and DD2) in predicted steps in vehicle localisation. Gruyer *et al.* [142] compared the overall localisation process (predictive and corrective step) of these KF variants using criteria based on accuracy and filters' uncertainty and consistency and multi-sensor experimental measurements. Ndjeng *et al.* [116] evaluated the accuracy and robustness performances of IMM-based and EKF-based low-cost localisation systems under dynamics manoeuvre scenarios. Up to now, few works have compared the localisation real-time performance. Reference [6] and [149] compared the same solution's localisation time based on CPU and GPU platforms. Reference [143] run a filtering algorithm on CPU and GPU to compare their execution time. However, the foregoing real-time performance comparisons are only running the same algorithm on various platforms. The real-time performances of different localisation solutions are presented on various hardware platforms and programming languages. Moreover, the localisation time of the overall solution is affected by the step of data extraction and primitive search, core localisation algorithm execution, map storage and update (if the map was used). To apply a fast real-time performance comparison between different solutions without a real test, firstly, we assume that the localisation time shown in different research papers was relative to a complete localisation solution rather than just the algorithm. Secondly, we assume that the code running in each solution has made full use of all computational sources. Therefore, the localisation time of different solutions can be converted to the same benchmark based on different hardware computational power and programming language execution efficiency. Then, the real-time performance of different solutions can be approximately and quantitatively compared.

B. Equivalent Comparison Method

The discussion of different localisation techniques shows that AV localisation mainly relies on CPU and GPU as the hardware platforms and MATLAB and C/C++ as the programming languages. As is well known, different hardware has different computing capabilities. For example, GPU is 52 times faster than CPU when the filtering algorithm is used to process lidar 3D point cloud data [143]. For the programming language, C/C++ is a compiled language that is translated into machine language before execution. MATLAB is an interpreted language wherein each line of code must be read and interpreted by the interpreter during execution, which makes it much slower than a compiled language [144], [145]. Therefore, the factor of which hardware and programming language is used must be considered when comparing the real-time performances of different localisation techniques.

As the first step, the localisation algorithm operations capability (LAOC) equivalent conversion factors between CPU/GPU families and between CPU and GPU must be determined. All CPUs/GPUs in CPU/GPU families are derived from the hardware platform of different localisation techniques. In this paper, the single-precision floating-point (SPFP) peak performance was used to determine the LAOC equivalent conversion relations of GPU/CPU families, because a localisation algorithm often involves SPFP operations. In CPU families, the SPEC CPU® 2006 benchmark [146] is designed for comparing the compute-intensive performance of different CPUs at hardware level. This depends on the factors of processor, memory architecture, and bus. This benchmark can comprehensively evaluate and compare the hardware performance of different CPUs [147]. Therefore, the LAOC equivalent conversion relations between CPU families are based on the SPECfp2006 [148], where CPU relative peak floating-point operations per second (FLOPS) performances are presented. For normalisation, the minimum value of the relative peak FLOPS performance shown in this paper was

TABLE III

SUMMARY OF CPU RELATIVE PEAK FLOPS PERFORMANCE AND ITS LAOC EQUIVALENT CONVERSION FACTORS BETWEEN DIFFERENT LOCALISATION TECHNIQUES

Technique (Reference paper)	CPU relative peak FLOPS performance between different localisation techniques	CPU LAOC equivalent conversion factor between different localisation techniques ε_{h_c}
Lidar-based		
[35], [36]	93.8	11.7
[40]	64.5	8.1
[42]	16	2
Radar-based		
[46]	42.5	5.3
[51], [52]	111	13.9
[53]	19.2	2.4
Ultrasonic-based		
[56]	11.6	1.5
IMU-based		
[72]	11.6	1.5
Vision-based		
[81]	58.1	7.3
[82]	83.2	10.4
[6]	8	1.0
[84], [85]	93.8	11.7
V2X-based		
[98]	30.5	3.8
[103]	110.4	13.8
Data fusion-based		
[117], [118]	11.6	1.5
[132]	56.9	7.1

taken as the baseline, and its LAOC equivalent conversion factor was determined as $\varepsilon_{h_c} = 1$. The LAOC equivalent conversion factors between CPU families were obtained by using $\varepsilon_{h_c} = FLOPS/FLOPS_{baseline}$, as shown in TABLE III.

For GPU families, the factors affecting the FLOPS capabilities include frequency f, number of cores N and single-precision fused multiply-add operation (FMA) per cycle of each core FMA. FMA can be found in the official website of the selected GPU. Theoretical single-precision peak performance can be estimated by using the following equation.

$$FLOPS_{peak} \approx f \times N \times FMA \tag{1}$$

For the same data transfer and copy, $FLOPS_{peak}$ can represent the actual SPFP computing capabilities of a GPU, and the transformation relations amongst GPU families are based on $FLOPS_{peak}$. For normalisation, we defined the minimum$FLOPS_{peak}$ performance presented in this paper as the baseline and its LAOC equivalent conversion factor as $\varepsilon_{h_g} = 1$. The LAOC equivalent conversion factors between GPU families were calculated by using $\varepsilon_{\mathrm{h_g}} = \mathrm{FLOPS/FLOPS_{baseline}}$, as shown in TABLE IV.

For the LAOC equivalent relations between CPU and GPU, Charmette *et al.* [6], [149] conducted many representative works in comparing CPU and GPU computing performance in localisation application. In our paper, the conversion factor between CPU and GPU was based on their latest research

TABLE IV

SUMMARY OF GPU PEAK FLOPS PERFORMANCE AND ITS LAOC EQUIVALENT CONVERSION FACTORS BETWEEN DIFFERENT LOCALISATION TECHNIQUES

Technique (Reference paper)	GPU peak FLOPS performance between different localisation techniques (Unit: Giga FLOPS)	GPU LAOC equivalent conversion factor between different localisation techniques ε_{h_g}
Lidar-based		
[29], [37]	10974.21	12.1
[45]	4372.5	4.8
Vision-based		
[6]	907.2	1.0
[87]	10974.21	12.1

conclusion [6]. The conclusion shows that the localisation time of a same methodology, GPU, is about 45 times faster than that of CPU. The authors mentioned that only one core was used for localisation in a dual-core CPU. Therefore, we considered that the peak FLOPS performance of CPU in [6] was half of the same dual-core CPU, as shown in TABLE III. The LAOC equivalent conversion factor between CPU and GPU in [6] is determined as $\varepsilon_{h_t} = 45$.

Next, the LAOC equivalent conversion factors between C/C++ and MATLAB must be determined. MATLAB is 9–11 times slower than C/C++ when it can be translated to the best C/C++ executable programme [150]. This perfect translation is unaffected by several factors, such as programming structure, style and language proficiency. It is only related to the execution efficiency of the programming language itself. Therefore, the LAOC equivalent conversion relations between C/C++ and MATLAB were based on the conclusion of a past work [150]. We considered taking C/C++ as the programming language benchmark, and its LAOC equivalent conversion factor was set as $\varepsilon_s = 1$. Thus, the LAOC equivalent conversion factor of MATLAB was determined as $\varepsilon_s = 0.1$ based on a previous work [150].

Finally, we selected the baseline peak FLOPS performance as the hardware benchmark and C/C++ as the programming language benchmark. The positioning time based on different hardware and programming languages must be transferred to this benchmark for comparison. The transformation method is given by the following equation.

$$\begin{aligned} T_C &= T_R \times \varepsilon_h \times \varepsilon_s \\ \varepsilon_h &= \begin{cases} \varepsilon_{h_c}, & CPU \\ \varepsilon_{h_g} \times \varepsilon_{h_t}, & GPU \end{cases} \end{aligned} \tag{2}$$

where T_R is the actual positioning time, and T_C is the positioning time assumed to be run on this benchmark. T_C reflects the relative computational complexity of each positioning technique.

TABLE V
COMPUTATIONAL COMPLEXITIES OF THE VALIDATION EXAMPLE

Solution	Hardware platform	T_R (s)	ε_h	ε_s	T_C (s)
A	CPU	4.562	13.5	1	61.587
B	GPU	0.114	544.5	1	62.073

C. Method Validation

In this paper, reference [29] is used to assess the proposed LAOC-based equivalent comparison method. Reference [29] has compared localisation time of the same solution based on CPU and GPU platform. T_R and T_C of CPU and GPU, the LAOC equivalent conversion factor ε_h and ε_s of hardware and software, respectively, are presented in TABLE V. TABLE V shows that, the difference in positioning times before conversion is due to the different hardware platform (CPU and GPU). The positioning time after conversion substantially increased because the peak FLOPS performance of the hardware benchmark is the lowest, and the programming language is the same. Moreover, the converted results show that $\frac{T_{C\_A}}{T_{C\_B}} = \frac{61.587}{62.073} = 0.99$, which means the positioning time based on CPU and GPU are similar after conversion. This is because both solution A and solution B are the same solution but implemented in different hardware platform. Thus, the LAOC-based equivalent comparison method is reasonable and can be used to approximately and quantitatively compared different localisation solutions. The relative computational complexities of different localisation techniques calculating by using Equation (2) are summarised in TABLE VI.

TABLE VI
SUMMARY OF THE COMPUTATIONAL COMPLEXITY OF DIFFERENT LOCALISATION TECHNIQUES

Technique (Reference paper)	Hardware platform	T_R (s)	ε_h	ε_s	T_C (s)
Lidar-based					
[29], [37]	GPU	0.2	544.5	1	108.9
[35], [36]	CPU	0.1	11.7	1	1.17
[40]	CPU	1	8.1	0.1	0.81
[42]	CPU	0.15	2	0.1	0.03
[45]	GPU	0.05	216	1	10.8
Radar-based					
[46]	CPU	1	5.3	1	5.3
[51], [52]	CPU	0.06	13.9	0.1	0.0834
[53]	CPU	0.008	2.4	1	0.0192
Ultrasonic-based					
[56]	CPU	0.01	1.5	1	0.015
Pure GPS					
GPS	-	1	-	-	1
IMU-based					
[72]	CPU	5×10^{-6}	1.5	1	7.5×10^{-6}
Vision-based					
[81]	CPU	0.045	7.3	1	0.3285
[82]	CPU	0.12	10.4	0.1	0.1248
[6]	GPU	0.09	45	1	4.05
[84], [85]	CPU	0.057	11.7	0.1	0.0667
[87]	GPU	0.133	544.5	1	72.419
V2X-based					
[98]	CPU	1.388	3.8	0.1	0.5274
[103]	CPU	0.1	13.8	1	1.38
Data fusion-based					
[117], [118]	CPU	0.008	1.5	0.1	0.0012
[132]	CPU	0.01	7.1	1	0.071

D. Discussion

1) Accuracy and Real-Time Performance: This section quantitatively compares the computational complexities and position errors of all localisation techniques mentioned above.

Fig. 2 shows that, in lidar-based localisation, the 3D map-based approach is better than the 2D map-based approach in terms of accuracy because it contains rich feature information. However, the 3D map-based technique increases the memory usage and computing load, resulting in high-computational complexity of the algorithm. Moreover, despite the less difference in accuracy between 2D map-based techniques, computational complexity varies greatly due to different methodologies. For example, computational complexity of the 2D GMM matching technique in [29] is about 2,000 times that of the combination of multi-layer RANSAC registration and 2D map matching method in [42]. The radar- and ultrasonic-based localisation techniques have lower computational complexity compared with lidar-based ones because they emit low-density electromagnetic waves. Computational complexities and position errors of radar-based localisation are between lidar- and ultrasonic-based localisation; despite the combination of particle swarm optimisation and grid map matching method achieves reasonable positioning performance, this method requires strict sensor deployment. The position accuracy of ultrasonic-based technique position accuracy is about 10m as a result of the low precision of the ultrasonic sensor.

Fig. 3 shows that, for pure GPS localisation in the open sky, the GPS receiver can output position information with frequency of 1Hz and accuracy of 2–10m without being restricted by the vehicle operating system. Compared with other sensor-based localisations, the IMU-based technique can achieve the lowest computational complexity due to its fast position refresh rate, but its cumulative error results in a positioning error of about 1m in only 10-minute of driving. In terms of vision-based localisation, rich environmental information contained in an image makes its computational complexity similar to that of the lidar-based approach. However, vision cannot accurately measure surrounding object ranges due to challenges of the image quality and lens distortion. Thus, its localisation accuracy is lower than that of the lidar-based technique. Moreover, its computational complexity decreases with the dimension of reference map, but its position accuracy does not change much.

As shown in Fig. 4, compared with lidar- and vision-based localisation, the real-time performance of V2X-based is better,

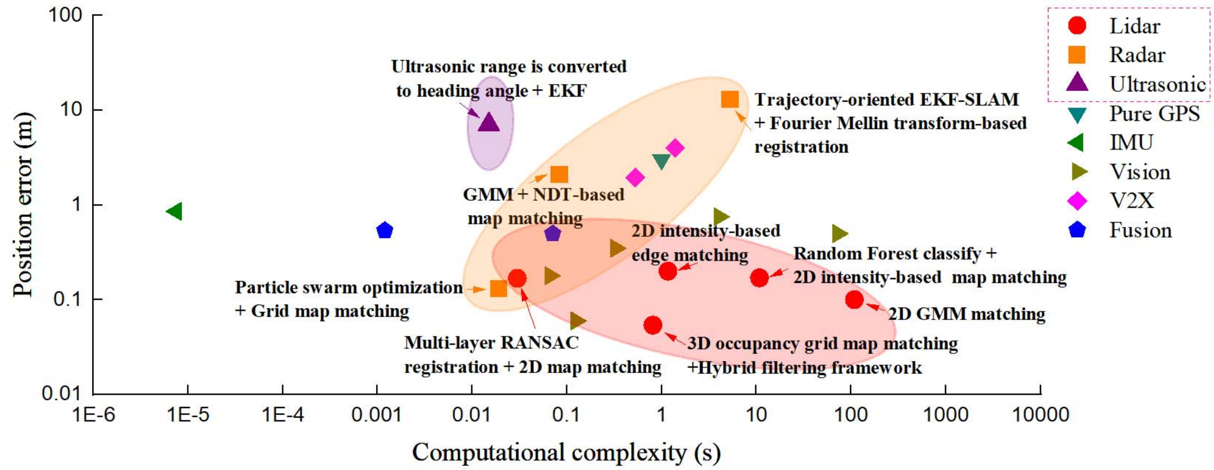

Fig. 2. Computational complexity and position error of active sensor-based localisation.

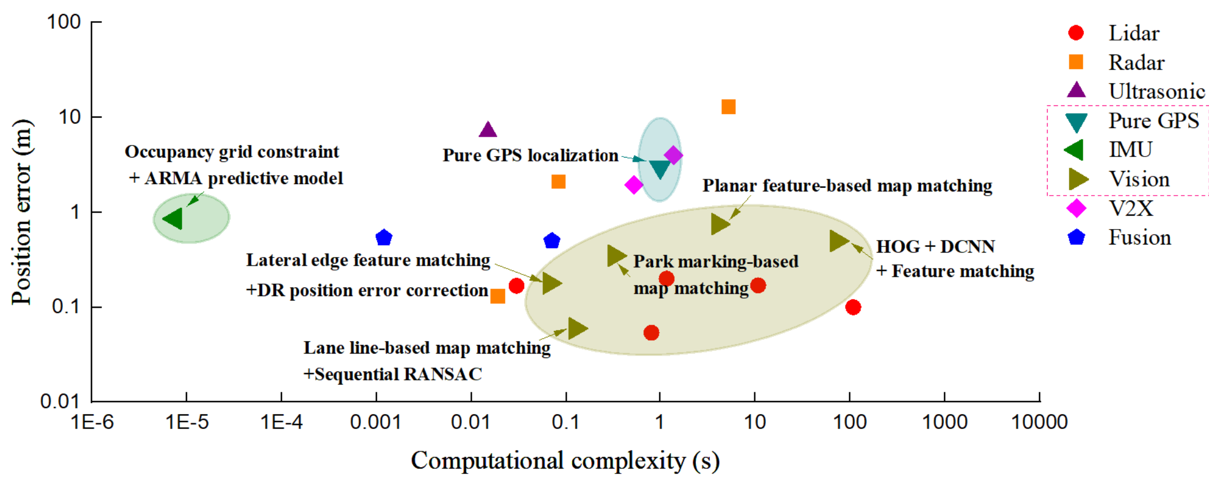

Fig. 3. Computational complexity and position error of passive sensor-based localisation.

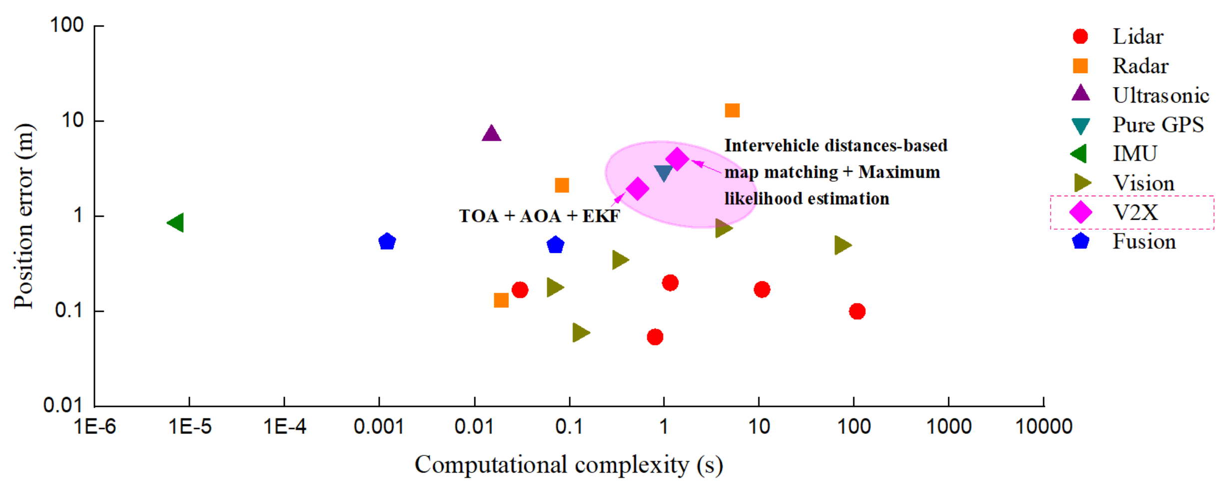

Fig. 4. Computational complexity and position error of V2X-based localisation.

but its accuracy is not satisfactory due to challenges of signal latency or inadequate participating nodes.

Fig. 5 shows that, compared with other sensor-based localisations, the data fusion-based technique can achieve a balance in terms of accuracy and real-time performance. This is because it uses the advantages of each sensor to reduce the effects of the disadvantages of other sensors, and each standalone sensor does not need to

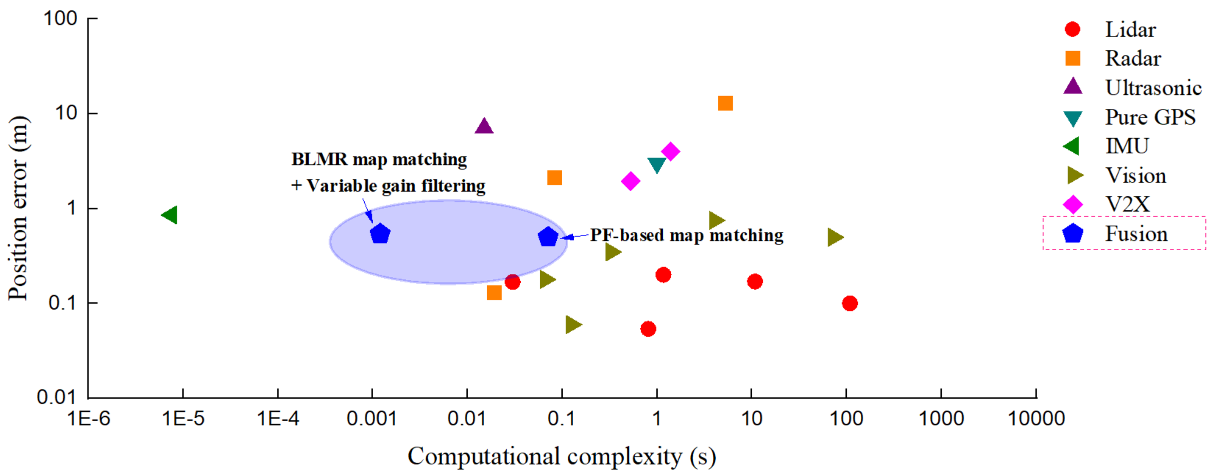

Fig. 5. Computational complexity and position error of data fusion-based localisation.

TABLE VII
ACCURACY AND REAL-TIME PERFORMANCE OF DIFFERENT SENSOR-BASED TECHNIQUES

Requirements	**High**			→		→		**Low**
Accuracy	Lidar-based	Vision-based	Data fusion-based	Radar-based	IMU-based	V2X-based	Pure GPS	Ultrasonic-based
Computational complexity	Lidar-based	Vision-based	V2X-based	Pure GPS	Radar-based	Multi-sensor fusion-base	Ultrasonic-based	IMU-based

develop a complex algorithm to achieve its best localisation potential.

In summary, the computational complexities of different sensor-based localisation techniques differ by a maximum of about 10^7 times, whereas the position errors vary by about 100 times. TABLE VII summarises the performance of different sensor-based techniques in terms of accuracy and real-time performance.

2) Application Scenarios: The accuracy and real-time performance that satisfy the requirements of safe driving for AV applications are the position errors and the position output frequency require to be less than 30cm [3] and 100ms [151], respectively. The analysis shows that the lidar-, vision-, and data fusion-based localisation have the potential to meet the accuracy performance. The lidar- and vision-based techniques that use a powerful processor, such as high-performance GPU and multi-core CPU, can meet the real-time performance requirement. The computational complexity of data fusion-based techniques fusing multiple low-cost sensors (e.g., camera, GPS, IMU, and in-vehicles) is lower than that of the lidar- and vision-based techniques. In summary, the fusion technique has considerable potential for achieving cost-efficient autonomous self-localisation.

Furthermore, TABLE VII could also guide the localisation solution selection in different scenarios. For the urban environments where pedestrians and vehicles are highly involved in traffic, the localisation accuracy and real-time requirements are the highest compared with the other common driving environments. Although the lidar-, vision-based and lidar- or vision-based data fusion techniques may increase the cost of hardware deployment to achieve real-time performance, those techniques can obtain precise positioning accuracy. The highway and suburban scenarios have fewer pedestrians and vehicles around the AV. The accuracy requirement in these scenarios can be lower than that in the urban environments. However, AVs require long-distance detection sensors or means to perceive the surrounding obstacles and high-frequency position output to meet high-speed driving. Thus, the localisation techniques with long-distance sensor perception and real-time performance could be a potential choice, such as data fusion-, radar- and V2V-based techniques. Since fewer obstacles in the dedicated-lane and low driving speed for AVs used as city bus or sightseeing bus, the accuracy and real-time requirements are lower than the situations mentioned above. In this case, the low-cost data fusion-, V2I- and radar-based localisation techniques could be the preferred options. In the automatic parking scenarios, the detection distance and positioning real-time performance do not need to be as high as in the above applications. Therefore, the low-cost ultrasonic- and radar-based techniques could be the most promising options.

VII. CONCLUSION

This paper has reviewed the state-of-the-art self-localisation techniques of active sensor-, passive sensor-, V2X-, and data fusion-based and quantitatively compared their accuracy and computational complexity performance. Lidar-based 2D map matching methods showed the most significant promise for balancing the localisation performance for commercial AVs between cost, accuracy, real-time and robustness compared with 1D map and 3D map matching approaches.

However, lidar-based localisation is more expensive solution than the other sensor-based localisation, such as radar-based, vision-based and V2X-based. Furthermore, the real-time performance of lidar-based (2D) solutions may suffer from the limitations of system computational ability and needs a powerful CPU/GPU acceleration, which can increase the deployment cost of AVs. Further improvement of the lidar-based (2D) solution is required to reduce the localisation update time with low-cost processors. Passive sensor-based localisation solution has shown the significant advantages in the low-cost of deployment. The challenges are that for the typical passive sensors, such as GPS-based and IMU, the localisation integrity and consistency make this technique still difficult to apply to AV. Vision-based localisation can achieve high-precision vehicle positions but may require GPU acceleration for proccing substantial image data. The reliability of the camera in inadequate illumination or lousy weather also needs to be further addressed. V2X techniques can offer a cost-efficient AV localisation solution under a wide range of signal strength and network coverage of VANET. The RFID-based techniques are very suitable for AV applications in the fix routes, such as a sightseeing bus in a zoo, container handling vehicle in port. However, the challenges of signal latency and packet loss in V2X systems need to be further optimised to improve localisation accuracy and consistency. Compared with other sensor-based localisation solutions, the data fusion-based technique has the most significant potential to trade-off commercial AVs' localisation performances of economy, real-time, accuracy and robustness. For example, interval theory-based technique can achieve vehicle localisation with a high level of integrity and consistency by fusing low-cost sensors data (e.g., GPS, IMU, and odometer). Further research and validation to this technique under different change environments and a variety of driving conditions such as long-distancing driving will be essential before commercialisation.

Furthermore, the comparative analysis between real-time and accuracy performance shows that the position errors of different sensor-based localisation techniques differ by a maximum of about 100 times. The lidar-, vision-, and data fusion-based localisation techniques have potential to meet the accuracy requirement (<30cm) for AV safe driving. The lidar-based techniques have achieved the best positioning accuracy compared with other sensor-based techniques, and the position accuracies achieved by different lidar-based methodologies are similar. Moreover, high-dimensional map matching or intensity-based matching approaches can reduce position error by about 2–3 times but can increase computational complexity by about 20–2,000 times. Vision- and data fusion-based localisation has about 2–5 times potential for improvement in position accuracy compared with lidar-based localisation.

In terms of real-time performance, the computational complexity between different sensor-based techniques varies by a maximum of about 10^7 times. It has great space for improvement compared with accuracy. IMU-, ultrasonic-, multi-sensor fusion-, and radar-based self-localisation can meet the real-time performance requirement (<100ms) for safe driving with low-cost processors, whereas lidar- and vision-based localisation can achieve real-time positioning by using powerful processors. However, the IMU-, ultrasonic- and radar-based techniques have inadequate positioning accuracy and are often used as assisted positioning solutions in AVs. The lidar-based techniques have the highest computational complexity and about 2,000 times maximum difference compared with different methodologies. The focus on improving the lidar image registration methods can enhance the real-time positioning performance of lidar-based techniques. The computing complexity of vision-based localisation is similar with that of lidar-based approach, which has about 1,000 times maximum difference compared with those of different methodologies. Improving the efficiency and accuracy of the captured image association can improve accuracy and real-time performance. Moreover, matching low-dimensional features can reduce computational complexity but has no substantial effects on accuracy. Compared with lidar- and vision-based localisations, data fusion-based localisation achieves a better real-time performance because each standalone sensor does not need to develop a complex algorithm to achieve its best localisation potential. In addition, it achieves the best balance between accuracy and real-time performance. In summary, lidar-, vision-, and data fusion-based techniques can still be greatly improved in terms of real-time performance.

The discussion has shown that no single sensor can meet all localisation requirements for autonomous driving. Data fusion-based techniques will be the research focus for achieving a cost-efficient self-localisation for AV compared with other single sensor-based techniques. In addition to traditional fusion information sources, such as GPS and IMU, V2X will be a promising solution mainly due to the excellent robustness against illumination and weather. It has a wide detection range (∼300m), which can increase the data sources and improve their stability. However, the trade-off among accuracy, real-time performance, and robustness still needs to be researched further. Moreover, future research is required to focus on the sensors fault detection and identification techniques as well as the imperfection data modelling approaches to ensure robust and consistent AV localisation. With the rise of new emerging methods, such as machine learning and deep learning. The map-based localisation performance can be enhanced because the artificial intelligence algorithms have great potential to learn features automatically. And we refer the reader to the recent survey by Fayyad *et al.* [152], which provides a comprehensive review of deep learning-based localisation.

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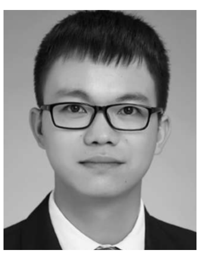

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