ITESM

Waiting Lines Model

Functional Programming

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Abstract.

The purpose of this project is to describe the context of the problematic, explain about waiting lines model and its applications, as well as asses real life problematics. As for the objective in the evaluation, it is to analyze two empirical cases based on models M/G/1 and M/M/1 or M/M/S, evaluate each outcome and demonstrate possible solutions for each of the cases.

Methodology for evaluation: First, each of the models correspond to a different case by a certain organization. Data has been simulated in order to make a significant impact in difference of costs.

Study Limitations: Data is limited by the two empirical cases, for a more profound study, analyzing information of hundreds of companies will allow for projections and long term decisions.

Keywords.

Waiting time, Servers, Probability, Functional Programming, Programming paradigm.

Introduction.

Waiting in lines is part of everyday life, from waiting at a supermarket or convenience store, or waiting for a fun ride at a rollercoaster, to ships waiting for cargo load. And we have been experiencing them since we can recall. So it is fair to say that they are important. According to El Mundo, we spend around 4 years of our lives waiting in line.

All this time spent waiting, depends on several factors, one of the is its system. A waiting line system is defined by two elements: its population source of its costumer and its process or service system.

A service may not be a tangible object, rather it is defined as economic activities where its results are produced and consumed at the same time. Which makes it nearly impossible for data to be stored as the business is transformed through its customers. So, given this intangibility of service, loyalty of costumers has its strength through word of mouth.

Problematic.

Businesses struggle to keep track of costumers and service, which turns out to be expensive if not taken care of. Therefore, it must be measured in order to take smart and accurate decisions. Since lines can be frustrating, it is a must to keep track of.

It applies both to person and object lines, for example, time of waiting lines in a fast food restaurant will be the difference between a regular or irregular costumer. For transport businesses, time will be the differentiator between early or late shipments.

Most important, it is implicit that time is money. So, how is money related to waiting in lines? Businesses struggle in other things. Aside from customers review, they must pay for each employee in a separate box, or as it is called, Server. Each server can be a relief or a stress. For example, if a bank decides to open 30 boxes, customers will be really happy because it is more likely for them to find an empty box, therefore a quick service, But what about all of the unused boxes? They might cost a lot of money for a business to maintain such high costs. Same goes for Cargo companies, if they have multiple servers but only few shipments, it is not a smart idea to keep them, or the other way around, if they have few servers but loads of shipments, cost of delay will play a hand in the equation .

Solution.

Waiting lines systems are meant to measure all of the above, it will become a must in a company that wishes to improve and to save money. Waiting line systems are not the same for every company. They are classified by Kendall notation.

David G. Kendall, born in England, develop queue notation, which is used to determine each characteristic depending on the first letter of each section. It is denoted as A/B/X/Y/Z where:

A is the arrival model which can be

- M = Markovian (exponential arrivals)
- D = Deterministic
- G = General (Any distribution)

B is service model, which can take same values as A.

X is server number.

Y is systems capacity

Z is discipline

V is number of states.

For this project, the model A/B/X is going to be used.

Types of servers:

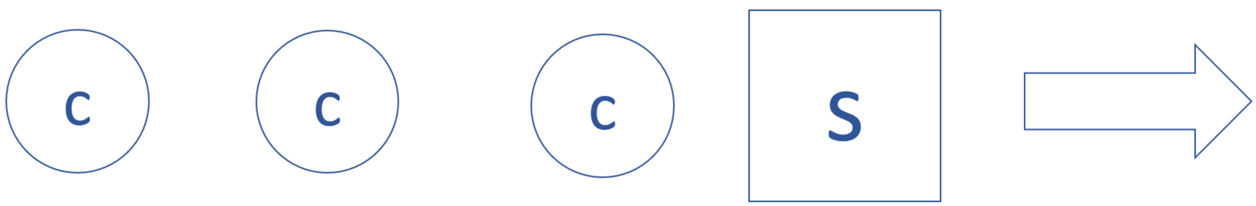
There are several combinations that can be obtain through Kendall Notation, on most businesses Markovian is going to be used as arrivals, when in a automated process it can be deterministic or general. The reason behind this is that customers or cargos are not always at the same time, they can be delayed or can arrive at a random time (on most cases Poisson distribution is considered effective).

As for Service model, M and D are going to be evaluated and for Servers will depend on system need which will be explained further.

Classification

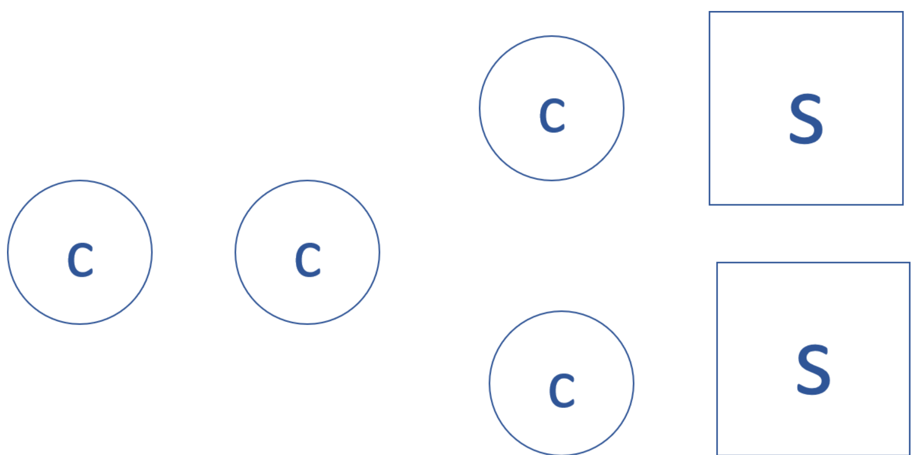
M/D/1 M/G/1 M/M/1

They have exponential arrival and a set service, the only difference is the standard deviation which in D it will be 0. They are single server therefore , it is a single line single server.



M/M/S

Number of servers is greater than one.



Methodology

As for the evaluation, there are several formulas that will help the results to be accurate, to measure probabilities and cost of services and delays.

Total cost of system

$$TC = TCS + TCD$$

Cost of service (CS)

$$TCS = Cs * S$$

* Where Cs stands for daily service costs and S the number of servers.

Cost of Delay (CD)

$$TCD = \ CD * L$$

Where Cd is unit delay costs $/(day)(unit)

And L is the average customers within the system.

Average number of clients is given by L. For a M/G/1 model this expression is the following.

$$L = p + \frac{\lambda^2\sigma^2 + \rho^2}{2(1-\rho)}$$

Being Lambda the mean of arrival in the system. Ro the factor of occupation and o the standard deviation of service time.

If The standard deviation is 0 model is converted from M/G/1 to M/D/1

$$L = p + \frac{\rho^2}{2(1-\rho)}$$

Where Ro (p) is given by.

$$\rho = \frac{\lambda}{\mu}$$

Been u the service rate which must be in the same units as lambda.

For a M/M/1 system L is given by the following

$$L = \ \frac{\rho}{(1-\rho)}$$

And for M/M/S :

$$L = pS + \ \frac{(pS)^2}{S!}\frac{1}{(1-\rho)^2}P0$$

Where Po is the probability that the system is empty. Which can be estimated by the following.

$$P0 = \frac{1}{\sum_{k=0}^{S} \frac{(\rho S)^k}{k!} + \frac{(\rho S)^S \rho}{S!\,(1-\rho)}}$$

Now, the following formula minimizes the total cost for model M/M/1

$$S = \frac{\lambda + \sqrt{\frac{\lambda \mu C D}{C S}}}{\mu}$$

Functional Programming

As definition, it is a programming paradigm where programs are constructed by applying and composing functions. Which is executed sequentially.

Functions are specific blocks of code which are used to perform an specific task in the program. They can be general or anonymous. They are helpful because of the following

1. Reduces code redundancy
2. Improves modularity
3. Helps solving complex problems
4. Increases maintainability

Programming languages

For this project Scheme will be used.

Scheme is a dialect of Lisp developed during 1970s at MIT AI Lab and was released by its developers. It was the first dialect of List to choose lexical scope and to require implementations to perform tail-call optimization.

Use in this project.

Due to the type of data that will be handled, a functional approach will be helpful to demonstrate which of the models are best suited and its limitations. Formulas are dependent to other results therefore the system must wait for this calls. Each distribution , probability or formula has to be encapsulated within a function which will be helpful for multiple calls.

Lambda functions are helpful as well because they reduce implementation details. Arguments are not evaluated when invoked, the first element contains the formal parameters and the second argument is the body, it returns an unnamed procedure and that procedure can be invoked.

Recursion

Scheme is best implemented when using tail recursion. The main difference between head and tail recursion is the following.

Tail : if nothing has to be done after the call returns. The value is immediately returned from the calling function

Head: the first statement is the recursive call, after it is done traversing then elements are returned.

Usage.

Also, scheme notation is helpful when dealing with mathematical equations such as this evaluation.

It is simpler to understand and as mentioned above it has to finish some instructions before moving on to the next one.

Case Analysis.

So, which of the following models is best suited for businesses? It depends, in the following section two empirical cases will be presented in order to demonstrate the importance of decision making and how each change may affect each model, both M/G/1 or M/D/1 and M/M/1 M/M/S will be evaluated in order to conclude for best scenarios.

First Case.

A logistics and transport company situated Southern Mexico, which is medium sized (has around 10,000m2) delivers services throughout the country. Its main product is car manufacturing which is why they have multiple trucks suited to transport all these materials needed for costumers. They all pick them up in the same station.

This station works nonstop 14 hours a day, after a rough evaluation, it has been following a poison distribution of 34 trucks per day , on which each load takes about 2 hours , which gives a standard deviation of 1.96 hours (being .14 days). This system is a M/G/1.

Loading trucks is done by using 8 forklifts, they have a cost of 2500 pesos per day and each daily delay has a cost of 4200.

Given all this information we can state the following.

Lambda is 34, service rate is 7 trucks per forklift given they are working 14 hours a day.

Standard deviation is 1.96 hours.

Evaluation.

$$\rho = \frac{34}{(5)(7)} = 0.971$$

Minimum value of S is 5, because it has to be the minimum value greater than lambda. Therefore 7 * 5 will be 35 which is greater than 34.

For L it can be calculated by

$$L = 0.971 + \frac{(34)^2(0.14)^2 + 0.971^2}{2(1-0.971)} = 413.99$$

And now we can calculate the total cost

(4200) * (413.99) *+ (2500) (5) = 1751273

Which is a lot.

After realizing several calculations with different number of servers, data is as follows.

M/G/1					
S	P	L	CS	CD	CT
5	0.971429	413.99	12500	1738774	1751273
6	0.809524	62.005	15000	260425	275425
7	0.871795	38.48769	17500	161648	179148.3
8	0.607143	29.9132	20000	125635	145635.6

In order to compare this to its values in M/D/1, the standard deviation must be set to 0

$$L = 0.971 + \frac{0.971^2}{2(1-0.971)} = 17.486$$

M/D/1					
S	P	L	CS	CD	CT
5	0.971429	17.48571	12500	73440	85940
6	0.809524	2.529762	15000	10625	25625
7	0.871795	1.480272	17500	6217.143	23717.14
8	0.607143	1.076299	20000	4520.455	24520.45

As we compare tables, when the standar deviation is 0 it will reduce the total cost.

Second Case

A medium sized business company (around 30,000m2) specializes in shipment, they have this huge cranes where all of cargo loading is done. They have an exponential service of 45 ships per day which a service rate of 12 daily ships per crane. Therefore this type of model is M/M. its operational cost (service cost) is 1100 pesos per day per crane, and costs of delay rise to 6000 pesos per day.

This company uses 6 cranes but has no clue of how efficient it is.

So , after analyzing information Lambda is 45, u is 12.

First minimum servers must be calculated which is in this case 4 (12 * 4 = 48)

In order to know the number of optimal cranes in a M/M/1 model it is obtained by the following

$$S = \frac{45 + \sqrt{\frac{(45)(12)(6{,}000)}{1{,}100}}}{12} = 8.27$$

Therefore 8 cranes have to be subsistems so for each day only 45/8 ships will arrive and then calculate ro by the following.

$$\rho = \frac{5.625}{12} = 0.469$$

L is given by:

$$L = \frac{0.469}{1 - 0.469} = 0.882$$

And now we can calculate total costs.

For optimal: (6000) * (0.882) * (1100) 8 = 14094

M/M/1					
S	P	L	CS	CD	CT
4	0.9375	15	4400	90000	94400
5	0.75	3	5500	18000	23500
6	0.625	1.666667	6600	10000	16600
7	0.535714	1.153846	7700	6923.077	14623.08
8	0.46875	0.882353	8800	5294.118	14094.12

If a similar approach is made with a M/M/S model the following has to be done.

First the probability of having no users in the system. (no ships) done for the minimun amount of cranes (4)

$$\sum_{k=0}^{4} \frac{(\rho S)^k}{k!} = \frac{3.75^0}{0!} + \frac{3.75^1}{1!} + \frac{3.75^2}{2!} + \frac{3.75^3}{3!} + \frac{3.75^4}{4!} = 28.81$$

$$P_0 = \frac{1}{28.81 + \frac{(3.75)^4 (0.9375)}{4!(1-0.9375)}} = 0.00656$$

Now L given by the following table.

M/M/S					
S	P	L	CS	CD	CT
4	0.9375	4.70904	4400	28742	33142.78
5	0.75	3.789	5500	22737.59	28237.5
6	0.625	3.7535	6600	22520	29120.12
7	0.535714	3.75031	7700	22501.88	30201
8	0.46875	0.882353	8800	22500.17	31300.18

Case Conclusions.

After evaluating both cases, it is found that there are several differences between each waiting line models, and that there is some areas to consider when implementing a model within a business company.

For the first scenario in a M/G/1 System, if there is a longer service rate with a greater standard deviation, servers must be incremented and the minimum cost as well, which could cause a devastating impact within the organization. As recommendation, if the only option is to have a M/G/1 system, full focus should be in lowering service response time due that the impact on the standard deviation is huge.

So for businesses with this type of model, if customers are incoming exponentially, measures within tracking service time should be applied in order to minimize the risk of having greater expenses. This is demonstrated by M/D/1 models where standard deviation is cero, due to its deterministic service time it is favorable.

For the second analysis, in systems using M/M/S and M/M/1, there is a notorious difference aside the number of servers which makes this recommendable. If a business has the budget and capability of implementing a M/M/S it will save a lot of money daily. The only problem will be physical space and money.

As for study limitations, monitoring and control will help businesses take advantage of this measurement tools to take smarter decisions.

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