

Resolución del sistema de ecuaciones

$$\begin{cases} p'(t) = \alpha_1 p(t) - \alpha_2 p(t) d(t) \\ d'(t) = -\beta_1 d(t) + \beta_2 p(t) d(t) \end{cases}$$

$$2) d'(t) = -\beta_1 d(t) + \beta_2 p(t) d(t)$$

1) Igualamos a 0 (condiciones)

$$p'(t) = \alpha_1 p(t) - \alpha_2 p(t) d(t) = 0$$

$$d'(t) = -\beta_1 d(t) + \beta_2 p(t) d(t)$$

2) Despejar

$$\alpha_1 p(t) = \alpha_2 p(t) d(t)$$

$$d(t) = \frac{\alpha_1 p(t)}{\alpha_2 p(t)}$$

$$\boxed{d(t) = \frac{\alpha_1}{\alpha_2}}$$

$$-\beta_1 d(t) = -\beta_2 p(t) d(t)$$

$$p(t) = \frac{-\beta_1 d(t)}{-\beta_2 d(t)}$$

$$\boxed{p(t) = \frac{-\beta_1}{-\beta_2}}$$

- Realizar Promedios

$$\bar{p} = \frac{\beta_1}{\beta_2}$$

$$\bar{d} = \frac{\alpha_1}{\alpha_2}$$

$$\bar{p} = \frac{715.7}{21} = 34.08095$$

$$\bar{d} = \frac{423.5}{21} = 20.16667$$

2) Despeje y sustitución

$$\frac{715.7}{21} = \frac{\beta_1}{\beta_2}$$

$$\frac{423.5}{21} = \frac{\alpha_1}{\alpha_2}$$

$$\beta_1 = \frac{715.7}{21} \beta_2$$

$$\alpha_1 = \frac{423.5}{21} \alpha_2$$

- Sustitución de datos en ecuaciones para encontrar constantes

$$p(t) = p(0) \cdot e^{\alpha_1 t} \quad (\text{población depredador baja})$$

$$d(t) = p(0) \cdot e^{-\beta_1 t} \quad (\text{población de presas bajas})$$

- Años 1909 = 25.4 1908 = 22 (conejos)

1909 = 9.1 1908 = 8.3 (lince)

2) sustitución

$$p(t) \Rightarrow 25.4 = 22 e^{\frac{423.5}{21} \alpha_2} \quad \cdot \text{aplico ln}$$

$$\ln \left| \frac{25.4}{22} \right| = \frac{423.5}{21} \alpha_2$$

$$\alpha_2 = \frac{6}{121} \ln \left| \frac{127}{110} \right| = \underline{\underline{7.12595 \times 10^{-3}}} = \alpha_2$$

- Sustitución

$$d(t) \Rightarrow 9.1 = 8.3 e^{\frac{-715.7}{21} \beta_2}$$

$$\frac{9.1}{8.3} = e^{\frac{-715.7}{21} \beta_2} \quad \cdot \text{aplico ln}$$

$$\ln \left| \frac{9.1}{8.3} \right| = \frac{-715.7}{21} \beta_2 \Rightarrow \beta_2 = \frac{-21}{715.7} \ln \left| \frac{9.1}{8.3} \right|$$

$$\beta_2 = \underline{\underline{-2.70001 \times 10^{-3}}}$$

Sustituir en EC

$$\alpha_1 = \frac{423.5}{21} \quad \alpha_2 \Rightarrow \alpha_1 = \frac{423.5}{21} (7.12595 \times 10^{-3})$$

$$\beta_1 = \frac{715.7}{21} \quad \beta_2 \Rightarrow \beta_1 = \frac{715.7}{21} (-2.70001 \times 10^{-3})$$

Resultados

$$\alpha_1 = 0.14371$$

$$\beta_1 = -0.09201$$

$$\alpha_2 = 7.12595 \times 10^{-3}$$

$$\beta_2 = -2.70001 \times 10^{-3}$$

• Ecuaciones (constantes sustituidas)

$$p'(t) = 0.14371 p(t) - 7.12595 \times 10^{-3} p(t) d(t)$$

$$d(t) = \frac{\alpha_1}{\alpha_2} = \frac{0.14371}{7.12595} = 20.16714$$

$$d'(t) = -\beta_1 d(t) + \beta_2 p(t) d(t)$$

$$d'(t) = 0.09201 d(t) - 2.70001 \times 10^{-3} p(t) d(t)$$

$$p(t) = \frac{\beta_1}{\beta_2} = \frac{-0.09201}{-2.70001 \times 10^{-3}} = 34.0776 \text{ s}$$