Actividad 2 - Fase 1

viernes, 4 de noviembre de 2022 11:54

Ecuaciones Lotka-Volterra

$$\begin{cases} p'(t) = a_1 p(t) - a_2 p(t) d(t) \\ d'(t) = -\beta_1 d(t) + \beta_2 p(t) d(t) \end{cases}$$

Linealizar

- $P_1 = (p,d) = (0,0)$
- $P_2 : (p,d) = (\beta_1/\beta_2 , a_1/a_2)$

$$\begin{cases} f(p,d) = a_1 p(t) - a_2 p(t) d(t) \\ g(p,d) = -\beta_1 d(t) + \beta_2 p(t) d(t) \end{cases}$$

Construir la matriz Jacobiana

$$J(p,d) = \begin{pmatrix} \frac{\partial f}{\partial p} & \frac{\partial f}{\partial d} \\ \frac{\partial g}{\partial p} & \frac{\partial g}{\partial d} \end{pmatrix} \quad ; \quad J(0,0) \text{ y } J(\beta_1/\beta_2 , a_1/a_2)$$

- $\frac{\partial f}{\partial p} = a_1 - a_2 d(t)$
- $\frac{\partial f}{\partial d} = -a_2 p(t)$
- $\frac{\partial g}{\partial p} = \beta_2 d(t)$
- $\frac{\partial g}{\partial d} = -\beta_1 + \beta_2 p(t)$

$$J(p,d) = \begin{pmatrix} a_1 - a_2 d(t) & -a_2 p(t) \\ \beta_2 d(t) & -\beta_1 + \beta_2 p(t) \end{pmatrix}$$

1: $J(0,0) = \begin{pmatrix} a_1 & 0 \\ 0 & -\beta_1 \end{pmatrix}$ 2: $J(\beta_1/\beta_2 , a_1/a_2) = \begin{pmatrix} 0 & -a_2 \beta_1/\beta_2 \\ \beta_2 a_1/a_2 & 0 \end{pmatrix}$

Eigenvalores y eigenvectores

1: $(\lambda I - J) = 0$

$$\begin{bmatrix} \lambda - a_1 & 0 \\ 0 & \lambda + \beta_1 \end{bmatrix}$$

Valores: $(\lambda - a_1)(\lambda + \beta_1) - (0)(0)$

$\lambda_1 = a_1$; $\lambda_2 = -\beta_1$

Punto de silla

Vectores: $(\lambda I - J) = 0$

a_1 $\quad -\beta_1$

a) $((a_1) I - J)\vec{x} = 0$

$$\begin{bmatrix} a_1 - a_1 & 0 \\ 0 & a_1 + \beta_1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$= (a_1 + \beta_2) x_2 = 0$

$\to (0, x_2)$

b) $((-\beta_1) I - J)\vec{x} = 0$

$$\begin{bmatrix} -\beta_1 - a_1 & 0 \\ 0 & -\beta_1 + \beta_1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$= (-\beta_1 - a_1) x_1 = 0$

$\to (x_1, 0)$

2: $(\lambda I - J) = 0$

$$\begin{bmatrix} \lambda - 0 & -a_2 \beta_1/\beta_2 \\ \beta_2 a_1/a_2 & \lambda - 0 \end{bmatrix}$$

Valores: $(\lambda)(\lambda) - (-a_2 \beta_1/\beta_2)(\beta_2 a_1/a_2)$

$\lambda^2 - \left(\frac{-\cancel{a_2} \beta_1 \cancel{\beta_2} a_1}{\cancel{\beta_2} \cancel{a_2}}\right)$

$\lambda^2 - (-\beta_1 a_1)$

$\lambda^2 + \beta_1 a_1$

$\lambda_1 = i\sqrt{\beta_1}\sqrt{a_1}$; $\lambda_2 = -i\sqrt{\beta_1}\sqrt{a_1}$

Centro ; punto crítico estable pero no asintóticamente estable.

Vectores: $(\lambda I - J) = 0$

$i\sqrt{\beta_1}\sqrt{a_1}$ $\quad -i\sqrt{\beta_1}\sqrt{a_1}$

a) $((i\sqrt{\beta_1}\sqrt{a_1}) I - J)\vec{x} = 0$

$$\begin{bmatrix} i\sqrt{\beta_1}\sqrt{a_1} - 0 & -a_2\beta_1/\beta_2 \\ \beta_2 a_1/a_2 & i\sqrt{\beta_1}\sqrt{a_1} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$= i\sqrt{\beta_1}\sqrt{a_1}\, x_1 - \frac{a_2 \beta_1}{\beta_2} x_2 = 0$

$= \frac{\beta_2 a_1}{a_2} x_1 + i\sqrt{\beta_1}\sqrt{a_1}\, x_2 = 0$

$\to \left(\frac{-i a_2 \sqrt{\beta_1}}{\beta_2 \sqrt{a_1}} x_2 , x_2\right)$

b) $((-i\sqrt{\beta_1}\sqrt{a_1}) I - J)\vec{x} = 0$

$$\begin{bmatrix} -i\sqrt{\beta_1}\sqrt{a_1} - 0 & -a_2\beta_1/\beta_2 \\ \beta_2 a_1/a_2 & -i\sqrt{\beta_1}\sqrt{a_1} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$= -i\sqrt{\beta_1}\sqrt{a_1}\, x_1 - \frac{a_2 \beta_1}{\beta_2} x_2 = 0$

$= \frac{\beta_2 a_1}{a_2} x_1 - i\sqrt{\beta_1}\sqrt{a_1}\, x_2 = 0$

$\to \left(\frac{i a_2 \sqrt{\beta_1}}{\beta_2 \sqrt{a_1}} x_2 , x_2\right)$