BLG 354E HW5
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HW5

(1) 4) $e^{-at}.u(t)$ $\quad x(jw) = \int_{-\infty}^{\infty} x(t)\, e^{-jwt} dt = \int_{0}^{\infty} e^{-at} . e^{-jwt} dt$

$$= \int_{0}^{\infty} e^{-t(a+jw)} dt = \frac{-1}{a+jw} e^{-(a+jw)} \Big|_{0}^{\infty}$$

$$= 0 - {}^{-1}/_{a+jw} = {}^{1}/_{a+jw}$$

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10) $\cos(w_0 t) = 1/2\,(e^{jw_0 t} + e^{-jw_0 t})$

$$x(jw) = \int_{-\infty}^{\infty} x(t) . e^{-jwt} dt = \int_{-\infty}^{\infty} 1/2 (e^{jw_0 t} + e^{-jw_0 t}) . e^{-jwt} dt$$

$$= \int_{-\infty}^{\infty} 1/2 (e^{jt(w_0 - w)} + e^{-jt(w_0 + w)}) dt$$

9th rule $\Rightarrow \frac{1}{2}\left(2\pi\delta(w-w_0) + 2\pi\delta(w+w_0)\right) = \pi\left(\delta(w-w_0) + \delta(w+w_0)\right)$

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11) $\sin(w_0 t) = 1/2j\,(e^{jw_0 t} - e^{-jw_0 t})$

$$x(jw) = \int_{-\infty}^{\infty} x(t) . e^{-jwt} dt = \int_{-\infty}^{\infty} 1/2j (e^{jw_0 t} - e^{-jw_0 t}) . e^{-jwt} dt$$

$$= \int_{-\infty}^{\infty} 1/2j (e^{jt(w_0 - w)} - e^{-jt(w_0 + w)}) dt$$

9th rule $\Rightarrow \frac{1}{2j}\left(2\pi\delta(w-w_0) + 2\pi\delta(w+w_0)\right) = \frac{\pi}{j}\left(\delta(w-w_0) + \delta(w+w_0)\right)$

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16) $rect\left(\frac{t}{\tau}\right) = \begin{cases} 1 & |t| \leq \tau/2 \\ 0 & |t| > \tau/2 \end{cases}$

$$x(jw) = \int_{-\tau/2}^{\tau/2} e^{-jwt} dt = \frac{e^{-jwt}}{-jw}\Big|_{-\tau/2}^{\tau/2} = \frac{e^{-jw\tau/2} - e^{jw\tau/2}}{-jw}$$

$$= \frac{-j\sin\tau/2 - j\sin\tau/2}{-jw} = \frac{2\sin w\tau}{w2} = \tau\,\mathrm{sinc}\left(\frac{w\tau}{2\pi}\right)$$

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(2) a) $x[n] = 4^n . u[-(n+1)]$ $\quad u[n] \xrightarrow{DTFT} e^{jw}$, $u[-n] \xleftrightarrow{DTFT} e^{jw}$

$$u[-(n+1)] \xrightarrow{DTFT} e^{jw} . X(e^{jw}) = e^{2jw}$$

$$4^n . u[-(n+1)] \xleftrightarrow{DTFT} \frac{1}{1 - 4 . e^{2jw}}$$

b) $x[n]=\begin{cases}1, & 5\geq n\geq -5\\ 0, & \text{otherwise}\end{cases}$ $\quad X(e^{j\omega})=\sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$

$$X(e^{j\omega})=\sum_{n=-5}^{5} 1.e^{-j\omega n}=\sum_{n=1}^{n=11} e^{-j\omega(n-6)}=\frac{1}{1-e^{-j\omega(n-6)}}$$

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c) $x[n]=\cos(7\pi n/16)=1/2\,(e^{j7\pi n/16}+e^{-j7\pi n/16})$

$$X(e^{j\omega})=\sum_{n=-\infty}^{\infty} 1/2\,(e^{j7\pi n/16}+e^{-j7\pi n/16})$$

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3) a) $\int_{-2}^{-1} e^{-j\omega t}dt+\int_{-1}^{0} -t.e^{-j\omega t}dt+\int_{0}^{1} t.e^{-j\omega t}dt+\int_{1}^{2} e^{-j\omega t}dt$

$$=\left.\frac{e^{-j\omega t}}{-j\omega}\right|_{-2}^{-1}+\left.\frac{(j\omega t+1)e^{-j\omega t}}{+j^2\omega^2}\right|_{-1}^{0}+\left.\frac{(j\omega t+1)e^{-j\omega t}}{-j^2\omega^2}\right|_{0}^{1}+\left.\frac{e^{-j\omega t}}{-j\omega}\right|_{1}^{2}$$

$$=\frac{e^{j\omega t}-e^{2j\omega t}}{-j\omega}+\frac{1-(1-j\omega).e^{j\omega t}}{-\omega^2}+\frac{(j\omega+1).e^{-j\omega t}-1}{-\omega^2}+\frac{e^{2j\omega t}-e^{-j\omega t}}{-j\omega}$$

$=(2j\sin(t)-2j\sin(2t)/-j\omega)+\ldots$

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b) $\int_{-2}^{-1} -e^{-j\omega t}dt+\int_{-1}^{1} t.e^{-j\omega t}dt+\int_{1}^{2} e^{-j\omega t}dt$

$$=\left.\frac{e^{-j\omega t}}{j\omega}\right|_{-2}^{-1}+\left.\frac{(j\omega t+1).e^{-j\omega t}}{-1.\omega^2}\right|_{-1}^{1}+\left.\frac{e^{-j\omega t}}{-j\omega}\right|_{1}^{2}$$

$$=\frac{e^{j\omega}-e^{2j\omega}}{+j\omega}+\frac{(1+j\omega t).e^{-j\omega}-(1-j\omega t).e^{j\omega}}{-\omega^2}+\frac{e^{2j\omega}-e^{-j\omega}}{-j\omega}$$

$=2\cos(t)-2\cos(2t)+\ldots$

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c) $\int_{0}^{1} 4.e^{-j\omega t}dt+\int_{1}^{2} 2.e^{-j\omega t}dt=4\left(\left.\frac{e^{-j\omega t}}{-j\omega}\right|_{0}^{1}\right)+2\left(\left.\frac{e^{-j\omega t}}{-j\omega}\right|_{1}^{2}\right)$

$$=4\left(\frac{e^{-j\omega}-1}{-j\omega}\right)+2\left(\frac{e^{-2j\omega}-e^{-j\omega}}{-j\omega}\right)=\frac{2e^{-2j\omega}+2e^{-j\omega}-4}{-j\omega}$$

4) a) $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} x(j\omega) \cdot e^{j\omega t} d\omega$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{5j\omega + 30}{(j\omega)^3 + 17(j\omega^2) + 80j\omega + 100} \cdot e^{j\omega t} d\omega$$

b) $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} x(j\omega) \cdot e^{j\omega t} d\omega$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{2(j\omega + 12)}{(j\omega)^2 + 4j\omega + 29} \cdot e^{j\omega t} \cdot d\omega$$

c) $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} x(j\omega) \cdot e^{j\omega t} d\omega$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{1}{(j\omega)^2 + 2500j\omega + 10^8} \cdot e^{j\omega t} d\omega$$

5) a) $y(t)=\int_{-\infty}^{\infty} x(\tau).h(t-\tau)\,d\tau = \int_{0}^{\infty} e^{-\tau}.u(\tau) . e^{-2(t-\tau)}.u(t-\tau)\,d\tau$

$= \int_{0}^{\infty} e^{-\tau}.e^{-2(t-\tau)}\,d\tau = \int_{0}^{\infty} e^{\tau}.e^{-2t}\,d\tau = e^{-2t}.\int_{0}^{\infty} e^{\tau}\,d\tau = \frac{e^{\tau-2t}}{\tau}\Big|_{0}^{\infty}$

b) $h(t)*x(t) \xrightarrow{DTFT} X(jw).H(jw)$

$X(jw)=\int_{0}^{\infty} e^{-t}.u(t).e^{-jwt}\,dt = \int_{0}^{\infty} e^{-t(1-jw)}\,dt = \frac{e^{-t(1-jw)}}{-t}\Big|_{0}^{\infty}$

$H(jw) = \frac{e^{-t(2-jw)}}{-t}\Big|_{0}^{\infty}$ $\quad X(jw).H(jw)=Y(jw) \xleftrightarrow{CTFT} Y(t)$

(convolusion property)

BONUS

5) $\int_{-\infty}^{0} e^{at}.e^{-jwt}\,dt + \int_{0}^{\infty} e^{-at}.e^{-jwt}\,dt$

$= \int_{-\infty}^{0} e^{t(a-jw)}\,dt + \int_{0}^{\infty} e^{t.(-a-jw)}\,dt = \frac{e^{t}.e^{(a-jw)}}{a-jw}\Big|_{-\infty}^{0} + \frac{e^{t}.e^{(-a-jw)}}{-a-jw}\Big|_{0}^{\infty}$

$= \frac{1-0}{a-jw} + \frac{0-1}{-a-jw} = \frac{2a}{a^2+w^2}$

BONUS

8) $\int_{-\infty}^{0} -e^{-jwt}\,dt + \int_{0}^{\infty} e^{-jwt}\,dt = \frac{e^{-jwt}}{+jw}\Big|_{-\infty}^{0} + \frac{e^{-jwt}}{-jw}\Big|_{0}^{\infty}$

$= \frac{1-0}{jw} + \frac{0-1}{-jw} = \frac{2}{jw}$