ITU
Computer Engineering Department
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Due:
16.05.2017
23:30

BLG 354E – HOMEWORK 5

Table 1: Fourier Transforms of some fundamental functions.

CT signals	Time domain $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$	Frequency domain $X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$	Comments
(1) Constant	1	$2\pi\delta(\omega)$	
(2) Impulse function	$\delta(t)$	1	
(3) Unit step function	$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$	
(4) Causal decaying exponential function	$e^{-at}u(t)$	$\frac{1}{a + j\omega}$	$a > 0$
(5) Two-sided decaying exponential function	$e^{-a\|t\|}$	$\frac{2a}{a^2 + \omega^2}$	$a > 0$
(6) First-order time-rising causal decaying exponential function	$te^{-at}u(t)$	$\frac{1}{(a + j\omega)^2}$	$a > 0$
(7) Nth-order time-rising causal decaying exponential function	$t^n e^{-at}u(t)$	$\frac{n!}{(a + j\omega)^{n+1}}$	$a > 0$
(8) Sign function	$\mathrm{sgn}(t) = \begin{cases} 1 & t > 0 \\ -1 & t < 0 \end{cases}$	$\frac{2}{j\omega}$	
(9) Complex exponential	$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$	
(10) Periodic cosine function	$\cos(\omega_0 t)$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$	
(11) Periodic sine function	$\sin(\omega_0 t)$	$\frac{\pi}{j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$	
(12) Causal cosine function	$\cos(\omega_0 t)u(t)$	$\frac{\pi}{2}[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)] + \frac{j\omega}{\omega_0^2 - \omega^2}$	
(13) Causal sine function	$\sin(\omega_0 t)u(t)$	$\frac{\pi}{2j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)] + \frac{\omega_0}{\omega_0^2 - \omega^2}$	
(14) Causal decaying exponential cosine function	$e^{-at}\cos(\omega_0 t)u(t)$	$\frac{a + j\omega}{(a + j\omega)^2 + \omega_0^2}$	$a > 0$
(15) Causal decaying exponential sine function	$e^{-at}\sin(\omega_0 t)u(t)$	$\frac{\omega_0}{(a + j\omega)^2 + \omega_0^2}$	$a > 0$
(16) Rectangular function	$\mathrm{rect}\left(\frac{t}{\tau}\right) = \begin{cases} 1 & \|t\| \le \tau/2 \\ 0 & \|t\| > \tau/2 \end{cases}$	$\tau\,\mathrm{sinc}\left(\frac{\omega\tau}{2\pi}\right)$	$\tau \neq 0$
(17) Sinc function	$\frac{W}{\pi}\mathrm{sinc}\left(\frac{Wt}{\pi}\right)$	$\mathrm{rect}\left(\frac{\omega}{2W}\right) = \begin{cases} 1 & \|\omega\| \le W \\ 0 & \|\omega\| > W \end{cases}$	
(18) Triangular function	$\Delta\left(\frac{t}{\tau}\right) = \begin{cases} 1 - \frac{\|t\|}{\tau} & \|t\| \le \tau \\ 0 & \text{otherwise} \end{cases}$	$\tau\,\mathrm{sinc}^2\left(\frac{\omega\tau}{2\pi}\right)$	$\tau > 0$
(19) Impulse train	$\sum_{k=-\infty}^{\infty}\delta(t - kT_0)$	$\omega_0\sum_{m=-\infty}^{\infty}\delta(\omega - m\omega_0)$	angular frequency $\omega_0 = 2\pi/T_0$
(20) Gaussian function	$e^{-t^2/2\sigma^2}$	$\sigma\sqrt{2\pi}e^{-\sigma^2\omega^2/2}$	

1-)(20 pts) Prove the 4th, 10th, 11th and 16th transformations given in the table. You can benefit from the table for other equalities. Use limit of the function where necessary. For the 16th transformation:

$$sinc(x) = \frac{\sin(\pi x)}{\pi x}$$

2-)(15 pts) Find the discrete-time Fourier transform of the following signals:

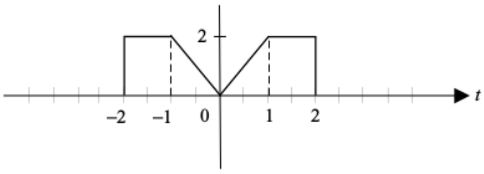
a) $x[n] = Y^n u[-(n+1)]$, where $|Y| > 1$

b) $x[n] = \begin{cases} 1, & 5 \ge n \ge -5 \\ 0, & otherwise \end{cases}$

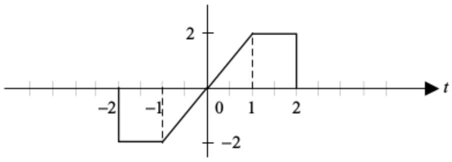
c) $x[n] = \cos(\frac{7\pi n}{16})$

3-)(15 pts) Find the continuous-time Fourier transform of the following signals:

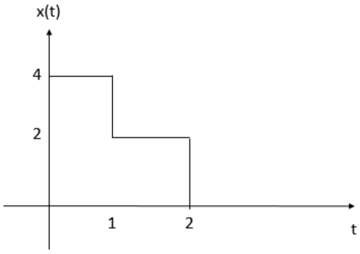
a)



b)



c)



4-)(30 pts) Calculate the inverse Fourier transform of the following functions:

a) $X(\omega) = \frac{5j\omega+30}{(j\omega)^3+17(j\omega)^2+80j\omega+100}$

b) $X(\omega) = \frac{2j\omega+24}{(j\omega)^2+4j\omega+29}$

c) $X(\omega) = \frac{1}{(j\omega)^2+25000j\omega+10^8}$

5-)(20 pts) A LTI system has an impulse response $h(t) = e^{-2t}u(t)$

a) What will be the system's output for $x(t) = e^{-t}u(t)$

b) Show that, using convolution property of Fourier transform we can obtain the same output.

BONUS) (15 pts) Prove the 5th and 8th transformations. (**Hint:** What is the similarity between u(t) and sgn(t)?)

Notes:

- Please write your answers briefly and add explanations at necessary points to make your calculations more understandable.
- If you have any questions, feel free to contact Res. Asst. Yusuf Hüseyin Şahin (sahinyu@itu.edu.tr).