BLG 202E Homework - 3

Due 29.04.2016 23:59

- An e-report should be prepared individually. The written MATLAB codes should be included in the submitted report.
- Plagiarized assignments will be given a negative mark.
- No late submissions will be accepted.

Submissions: Please submit your report and your MATLAB codes through Ninova e-Learning System.

1. (25 pt.) Joe had decided to buy stocks of a particularly promising Internet company. The price per share was $100, and Joe subsequently recorded the stock price at the end of each week. With the abscissae measured in days, the following data were acquired: (0,100), (7,98), (14,101), (21,50), (28,51), (35,50).

   In attempting to analyse what happened, it was desired to approximately evaluate the stock price a few days before the crash.

   (a) Pass a linear interpolant through the points with abscissae 7 and 14. Then add to this data set the value at 0 and (separately) the value at 21 to obtain two quadratic interpolants. Evaluate all three interpolants at $x = 12$. Which one do you think is the most accurate? Explain.

   (b) Plot the two quadratic interpolants above, together with the data (without a broken line passing through the data) over the interval [0,21]. What are your observations?

2. (25 pt.) Suppose we want to approximate the function e^x on the interval [0,1] by using polynomial interpolation with $x_0 = 0$, $x_1 = 1/2$ and $x_2 = 1$. Let $p_2(x)$ denote the interpolating polynomial.

   (a) Find an upper bound for the error magnitude

$$\max_{0 \le x \le 1} |e^x - p_2(x)|.$$

(b) Find the interpolating polynomial using your favorite technique.

(c) Plot the function e^x and the interpolant you found, both on the same figure, using the commands `plot`.

(d) Plot the error magnitude $|e^x - p_2(x)|$ on the interval using logarithmic scale (the command `semilogy`) and verify by inspection that it is below the bound you found in part (a).

3. (25 pt.) Let $f(x)$ be a given function that can be evaluated at points $x_0 \pm jh$, $j = 0, 1, 2, ...$, for any fixed value of h, $0 < h \ll 1$.

   (a) Find a second order formula (i.e., truncation error $\mathcal{O}(h^2)$) approximating the third derivative $f'''(x_0)$. Give the formula, as well as an expression for the truncation error, i.e., not just its order.

   (b) Use your formula to find approximations to $f'''(0)$ for the function $f(x) = e^x$ employing values $h = 10^{-1}, 10^{-2}, ...10^{-9}$, with the default MATLAB arithmetic. Verify that for the larger values of h your formula is indeed second order accurate. Which value of h gives the closest approximation to $e^0 = 1$?

   (c) For the formula that you derived in (a), how does the roundoff error behave as a function of h, as $h \to 0$?

   (d) How would you go about obtaining a fourth order formula for $f'''(x_0)$ in general? (You do not have to actually derive it: just describe in one or two sentences.) How many points would this formula require?

4. (25 pt.) Let us denote $x_{\pm 1} = x_0 \pm h$ and $f(x_i) = f_i$. It is known that the difference formula

$$fpp_0 = (f_1 - 2f_0 + f_{-1})/h^2$$

provides a second order method for approximating the second derivative of f at x_0, and also that roundoff error increases like h^{-2}.

Write a MATLAB script using default floating point arithmetic to calculate and plot the actual total error for approximating $f''(1.2)$, with $f(x) = sin(x)$. Plot the error on a log-log scale for $h = 10^{-k}$, $k = 0 : .5 : 8$. Observe the roughly V shape of the plot and explain it. What is (approximately) the observed optimal h?

5. (This question will not be graded. Just for practice.) Based on the points $x_i = x_0 + ih, i = -l, ..., u$, where $l + u = n$, an nth order formula approximating $f'(x_0)$ is given by

$$f'(x_0) \approx \frac{1}{h} \sum_{j=-l}^{u} a_j f(x_j),$$

where

$$a_j = \begin{cases} -\sum\limits_{\substack{k=-l \\ k \neq 0}}^{u} (\frac{1}{k}), & j = 0 \\ \frac{1}{j} \prod\limits_{\substack{k=-l \\ k \neq 0 \\ k \neq j}}^{u} (\frac{k}{k-j}), & j \neq 0 \end{cases}$$

Take $l = -1, u = 1$. Write MATLAB function to calculate a_j. Assume that $f(x) = e^x$, use your function to plot $f(x)$ and $f'(x)$ with $h = 0.1, 0.01, 0.001, 0.0001, 0.00001$ for $0 \leq x \leq 5$.