BLG 311E – FORMAL LANGUAGES AND AUTOMATA
SPRING 2016
HOMEWORK 2

1. Let α be a relation defined over the set A = {a, b, c, d}, expressed with the following matrix.

$$\alpha = \begin{vmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{vmatrix}$$

   a. Express α as a relation graph.

   b. Find rs(α) and sr(α) and show that they are equal.

   c. Find st(α) and ts(α) and show that st(α) ⊆ ts(α).

2. Design context-free grammars for the following languages:

   a. The set $\{0^n1^n \mid n \geq 1\}$, that is, the set of all strings of one or more 0's followed by an equal number of 1's.

   b. The set $\{a^ib^jc^k \mid i \neq j \text{ or } j \neq k\}$, that is, the set of strings of a's followed by b's followed by c's, such that there are either a different number of a's and b's or a different number of b's and c's, or both.

3. Consider the following grammar.

$$S \rightarrow AbB$$
$$A \rightarrow aA \mid \Lambda$$
$$B \rightarrow aB \mid bB \mid \Lambda$$

   a. Find the language generated by the grammar.

   b. What is the type of the grammar according to Chomsky hierarchy? Why?

   c. Design another grammar with a more restrictive type that generates the same expression you found in (a). (e.g. if the given grammar is Type-1 design a Type-2 or Type-3 grammar.)

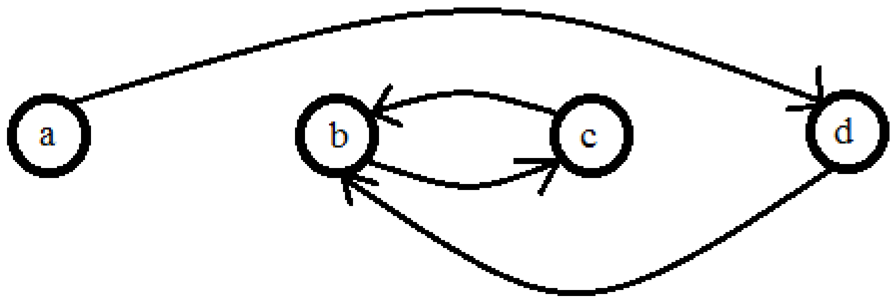
   d. What is the type of the grammar you designed in (c)? Why?

IMPORTANT: You must do this homework **by hand** and **submit it using the box in the secreteriat**.

SOLUTIONS:

1.

a. $\alpha = \{(a,d),(b,c),(c,b),(d,b)\}$

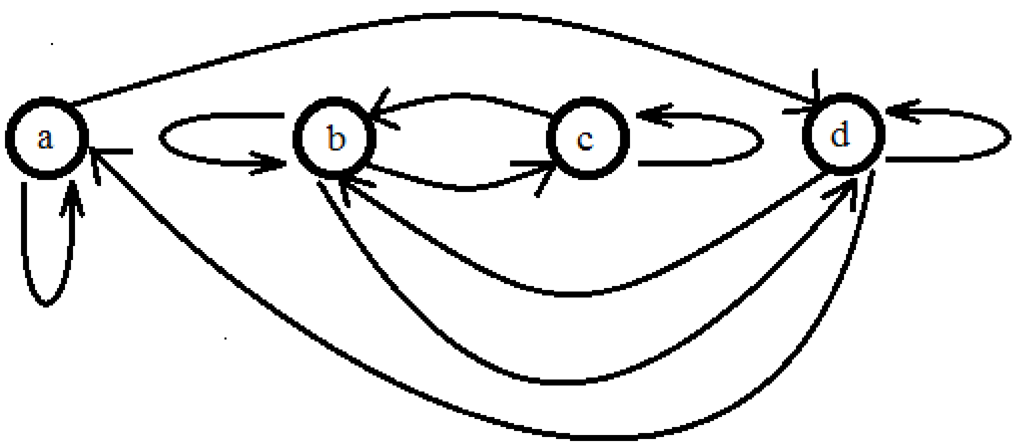


b.

$\alpha = \{(a,d),(b,c),(c,b),(d,b)\}$

$s(\alpha) = \{(a,d),(d,a),(b,c),(c,b),(d,b),(b,d)\}$
$rs(\alpha) = \{(a,d),(d,a),(b,c),(c,b),(d,b),(b,d),(a,a),(b,b),(c,c),(d,d)\}$

$r(\alpha) = \{(a,d),(b,c),(c,b),(d,b),(a,a),(b,b),(c,c),(d,d)\}$
$sr(\alpha) = \{(a,d),(d,a),(b,c),(c,b),(d,b),(b,d),(a,a),(b,b),(c,c),(d,d)\}$



Both closures are the same.

c.

$\alpha^1 = \{(a,d),(b,c),(c,b),(d,b)\}$
$\alpha^2 = \{(a,b),(b,b),(c,c),(d,c)\}$
$\alpha^3 = \{(a,c),(b,c),(c,b),(d,b)\}$
$\alpha^4 = \{(a,b),(b,b),(c,c),(d,c)\} = \alpha^2$
$\forall n \geq 2 \;\; \alpha^{n+2} = \alpha^n$

$$t(\alpha) = \bigcup_{i=1}^{\infty} \alpha^i = \alpha^1 \cup \alpha^2 \cup \alpha^3$$

$t(\alpha) = \{(a,b),(a,c),(a,d),(b,b),(b,c),(c,b),(c,c),(d,b),(d,c)\}$
$st(\alpha) = \{(a,b),(a,c),(a,d),(b,a),(b,b),(b,c),(b,d),(c,a),(c,b),(c,c),(c,d),(d,a),(d,b),(d,c)\}$

$s(\alpha)^1 = \{(a,d),(b,c),(b,d),(c,b),(d,a),(d,b)\}$
$s(\alpha)^2 = \{(a,a),(a,b),(b,a),(b,b),(c,c),(c,d),(d,c),(d,d)\}$
$s(\alpha)^3 = \{(a,c),(a,d),(b,c),(b,d),(c,a),(c,b),(d,a),(d,b)\}$
$s(\alpha)^4 = \{(a,a),(a,b),(b,a),(b,b),(c,c),(c,d),(d,c),(d,d)\} = s(\alpha)^2$
$\forall n \geq 2 \;\; s(\alpha)^{n+2} = s(\alpha)^n$

$$ts(\alpha) = \bigcup_{i=1}^{\infty} s(\alpha)^i = s(\alpha)^1 \cup s(\alpha)^2 \cup s(\alpha)^3$$

$ts(\alpha) = \{(a,a),(a,b),(a,c),(a,d),(b,a),(b,b),(b,c),(b,d),(c,a),(c,b),(c,c),(c,d),(d,a),(d,b),(d,c),(d,d)\}$

$st(\alpha) \subseteq ts(\alpha)$

2. a.

S → 0S1 | 01

b.

S → AB | BC | AC | DC | AE
A → aA | a
B → bB | b
C → cC | c
D → aDb | A | B
E → bEc | B | C

To understand how this grammar works, observe the following:

- *A* generates one or more *a*'s.
- *B* generates one or more *b*'s.
- *C* generates one or more *c*'s.
- *D* first generates an equal number of *a*'s and *b*'s, then produces either one or more *a*'s (via *A*) or one or more *b*'s (via *B*).
- Similarly, *E* generates unequal numbers of *b*'s then *c*'s.

3. a. a*b{a,b}*

b. It is type-0 since length of the left side is not always smaller than or equal to the length of the right side.

c. You can design any Type-1, 2 or 3 grammar. A type 2 example is given below.

$S \rightarrow b \mid Ab \mid AbB \mid bB$
$A \rightarrow aA \mid a$
$B \rightarrow aB \mid bB \mid a \mid b$

d. It is type-2 since (1) length of the left side is always smaller than or equal to the length of the right side, and (2) left side always consists of a single nonterminal. It is not type-3 because there are multiple nonterminals in the right side (AbB).