BLG 202E Homework - 2

Due 29.03.2016 23:59

- An e-report should be prepared individually. The written MATLAB codes should be included in the submitted report.
- Plagiarized assignments will be given a negative mark.
- No late submissions will be accepted.

Submissions: Please submit your report and your MATLAB codes through Ninova e-Learning System.

Questions

1. Considering fixed point iterative schemes shown in a, b, and c, write a Matlab function to find a root of $x^4 - x - 10 = 0$.

   a. $g_1(x) = \frac{10}{(x^3-1)},\ \ x_{i+1} = \frac{10}{(x_i^3-1)},\ \ i = 0,\ 1,\ 2\ldots\ \ and\, x_0 = 2.0$

   b. $g_2(x) = (x+10)^{\frac{1}{4}},\ x_{i+1} = (x_i+10)^{\frac{1}{4}},\ \ i = 0,\ 1,\ 2\ldots\ \ and\, x_0 = 2.0$

   c. $g_3(x) = \frac{(x+10)^{\frac{1}{2}}}{x},\ \ x_{i+1} = \frac{(x_i+10)^{\frac{1}{2}}}{x_i},\ \ i = 0,\ 1,\ 2\ldots\ \ and\, x_0 = 1.8$

   d. Compare the convergence steps of g_1, g_2 and g_3 functions.

2. Use LU decomposition to determine the matrix inverse for the following system. Do not use a pivoting strategy, and check your results by implementing LU decomposition in MATLAB environment.

$$10x_1 + 2x_2 - x_3 = 27$$
$$-3x_1 - 6x_2 + 2x_3 = -61.5$$
$$x_1 + x_2 + 5x_3 = -21.5$$

3. Making use of the relationship between the singular values of A and the eigenvalues of AA^T and A^TA, show the proof of the Singular Value Decomposition (SVD) of A with eigenvalue decomposition.

4. An m-by-n image can be thought as an m-by-n matrix, where entry (i, j) is interpreted as the brightness of pixel (i, j). Rather than storing all m*n matrix entries to represent the image, it is often preferred to compress the image by storing many fewer numbers, from which the original image can be still approximately reconstructed.

   Consider the image "sample.png". This 256-by-256 pixel image corresponds to a 256-by-256 matrix A. Let the $SVD\ of\ A\ as\ A = U\sum V^T$. Find the best rank-$k$ approximation of A using $A_K = \sum_{i=1}^{k} \sigma_i u_i v_i^T$ in the sense of minimizing $\| A - A_K \|_2 = \sigma_{k+1}$. Find the approximations for various values of k $(k = 3, 10, 20, 100)$ along with the relative errors $(\frac{\sigma_{k+1}}{\sigma_1})$ and compression ratios $\frac{((m+n).k)}{(m.n)}$

5. Let $\lambda_1,\ \lambda_2,\ \lambda_3 \ldots \lambda_n$ be the eigenvalues nxn of a matrix A. λ_1 is called the **dominant eigenvalue of** A if $(\lambda_1 > \lambda_2 > \lambda_3 > \lambda_n)$ The eigenvectors corresponding λ_1 to are called **dominant eigenvectors** of A. Dominant eigenvalue the λ (the eigenvalue) and a nonzero vector v (the eigenvector), such that Av = λv can be found by **Power Iteration Method (PIM)**.

   Consider 2D point cloud matrix named "A.mat". Using **PIM**, find the dominant eigenvalue and its corresponding eigenvector of ***centered*** $A^T A$, and visualize it. Also, find the eigenvalues of ***centered*** $A^T A$ with eigenvalue decomposition and compare them with eigenvalue found by PIM.