

HOMEWORK 1

Dr. Didem Çınar
Res. Assist. Alperen Bal

Assigned date: 02 March 2017
Due Date: 16 March 2017

- You can constitute a group of **max. 3 students** to submit HWs. No cheating will be tolerated! Names and numbers of the group members should be indicated on the cover.
- The homework brought after **16 March 2017** at **15:00 will not be accepted**.
- Your submission will have two parts, one will be uploaded to ninova and the other is printout. Look at the following instruction for submission.

Instruction:

1. Submit your solutions as printouts to Dr. Didem Çınar.
2. Do not print out the GAMS models. “.gms” file should be uploaded to ninova.
3. In gms file, names and ID's of group members should be written at the top as a comment. In order to write a comment use *. For example;
*Didem Çınar 070000341
*Alperen Bal 070000330
4. The name of the gms file should be changed as “BLG_*the full name of one of the group members.gms*”. For example: BLG_didemcinar.gms
5. Gams file should be compiled without any error.

1. (50 pts.) A paper-recycling plant processes box board, tissue paper, newsprint, and book paper into pulp that can be used to produce three grades of recycled paper (grades 1, 2, and 3). The prices per ton and the pulp contents of the four inputs are shown in the following Table. Two methods, de-inking and asphalt dispersion, can be used to process the four inputs into pulp. It costs \$20 to de-ink a ton of any input. The process of de-inking keeps 90% of the input's pulp to produce de-inked pulp. It costs \$15 to apply asphalt dispersion to a ton of material. The asphalt dispersion process keeps 80% of the input's pulp. At most, 3,000 tons of input can be run through each process (the asphalt dispersion process and the de-inking process). Grade 1 paper can only be produced with newsprint or book paper pulp; grade 2 paper, only with book paper, tissue paper, or box board pulp; and grade 3 paper, only with newsprint, tissue paper, or box board pulp. To meet its current demands, the company needs 500 tons of pulp for grade 1 paper, 500 tons of pulp for grade 2 paper, and 600 tons of pulp for grade 3 paper.

| Input | Cost (\$) | Pulp Content (%) |
|--------------|-----------|------------------|
| Box board | 5 | 15 |
| Tissue paper | 6 | 20 |
| Newsprint | 8 | 30 |
| Book paper | 10 | 40 |

- a. Formulate an LP to minimize the cost of meeting the demands for pulp in general form (by using \forall and \sum notation)
- b. Find the optimal solution with GAMS.

2. (10 pts.) Use the graphical method to find the optimal solution to the following LP.

$$\begin{array}{ll}\max & z = x_1 - x_2 \\ \text{s.t.} & -x_1 - x_2 \leq 1 \\ & 2x_1 - x_2 \leq 1 \\ & -2 \leq x_1 \leq 1 \\ & x_2 \leq 2\end{array}$$

3. (15 pts.) Find all basic solutions of the following system.

$$\begin{array}{rcl} -x_1 + 2x_2 + x_3 + x_4 - 2x_5 & = & 4 \\ x_1 - 2x_2 + 2x_4 - x_5 & = & 3 \end{array}$$

4. (10 pts.) Use the simplex algorithm to find the optimal solution to the following LP.

$$\begin{array}{ll}\max & z = -x_1 + 2x_2 \\ \text{s.t.} & 3x_1 + 4x_2 \leq 12 \\ & 2x_1 - x_2 \leq 12 \\ & x_1, x_2 \geq 0\end{array}$$

5. (15 pts.) Use the 2-phase simplex algorithm to find the optimal solution to the following LP.

$$\begin{array}{ll}\max & z = 5x_1 - 2x_2 + x_3 \\ \text{s.t.} & 2x_1 + 4x_2 + x_3 \leq 6 \\ & 2x_1 + x_2 + 3x_3 \geq 2 \\ & x_1, x_2 \geq 0 \\ & x_3 \text{ urs.}\end{array}$$