BLG368E Operations Research

HOMEWORK 2 ANSWERS

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1. (30 pts.) Televco produces TV picture tubes at three plants. Plant 1 can produce 50 tubes per week; plant 2, 100 tubes per week; and plant 3, 50 tubes per week. Tubes are shipped to three customers. The profit earned per tube depends on the site where the tube was produced and on the customer who purchases the tube (see the following Table). Customer 1 is willing to purchase as many as 80 tubes per week; customer 2, as many as 90; and customer 3, as many as 100. Televco wants to find a shipping and production plan that will maximize profits.

	To ($)		
From	Customer 1	Customer 2	Customer 3
Plant 1	75	60	69
Plant 2	79	73	68
Plant 3	85	76	70

a. (5 pts.) Formulate a balanced transportation problem that can be used to maximize Televco's profits.

	Customer 1	Customer 2	Customer 3	
Plant 1	75	60	69	50
Plant 2	79	73	68	100
Plant 3	85	76	70	50
Dummy	-M	-M	-M	70
	80	90	100	

b. (15 pts.) Use Vogel's method to find a basic feasible solution to the problem.

	Customer 1	Customer 2	Customer 3		Row penalty
Plant 1	75	60	69	50	75-69=6
Plant 2	79	73	68	100	6
Plant 3	85 50	76 -	70 -	50	9*
Dummy	-M	-M	-M	70	0
	80	90	100		
Column penalty	6	3	1		

	Customer 1	Customer 2	Customer 3		Row penalty
Plant 1	75	60 -	69	50	75-69=6
Plant 2	79	73 90	68	100	6
Plant 3	85 50	76 -	70 -	X	
Dummy	-M	-M -	-M	70	0
	30	90	100		
Column penalty	4	13*	1		

	Customer 1	Customer 2	Customer 3		Row penalty
Plant 1	75	60 -	69	50	75-69=6
Plant 2	79 10	73 90	68 -	10	11*
Plant 3	85 50	76 -	70 -	X	
Dummy	-M	-M -	-M	70	0
	30	X	100		
Column penalty	4		1		

	Customer 1	Customer 2	Customer 3		Row penalty
Plant 1	75 20	60 -	69	50	75-69=6
Plant 2	79 10	73 90	68 -	X	
Plant 3	85 50	76 -	70 -	X	
Dummy	-M -	-M -	-M	70	0
	20	X	100		
Column penalty	75-(-M)=M*		M		

	Customer 1	Customer 2	Customer 3	
Plant 1	75 / 20	60 / -	69 / 30	30
Plant 2	79 / 10	73 / 90	68 / -	X
Plant 3	85 / 50	76 / -	70 / -	X
Dummy	-M / -	-M / -	-M / 70	70
	X	X	100	

c. (10 pts.) Use the transportation simplex to find an optimal solution to the problem.

ui\vj	75	69	69	
0	75 / 20	60	69 / 30	50
4	79 / 10	73 / 90	68	100
10	85 / 50	76	70	50
M-69	-M	-M	-M / 70	70
	80	90	100	

$$\bar{c}_{12} = 0 + 69 - 60 = 9$$
$$\bar{c}_{23} = 4 + 69 - 68 = 5$$
$$\bar{c}_{32} = 10 + 69 - 76 = 3$$
$$\bar{c}_{33} = 10 + 69 - 70 = 9$$
$$\bar{c}_{41} = M - 69 + 75 - M = 6$$
$$\bar{c}_{42} = M - 69 + 69 - M = 0$$

Since $\bar{c}_{ij} \geq 0$ for all nonbasic variables, optmal solution is obtained. There are alternative solution because $\bar{c}_{42} = 0$.

Optimal solution: Plant 1 sends 20 TVs to Customer 1 and 30 TVs to Customer 3. Plant 2 sends 10 TVs to Customer 1 and 90 TVs to Customer 2. Plant 3 sends 50 TVs to Customer 1. Customer 3 will not receive 70 TVs. Total profit will be 15,180$.

2. (15 pts.) General Ford has two plants, two warehouses, and three customers. The locations of these are as follows:

Plants: Detroit and Atlanta

Warehouses: Denver and New York

Customers: Los Angeles, Chicago, and Philadelphia

Cars are produced at plants, then shipped to warehouses, and finally shipped to customers. Detroit can produce 150 cars per week, and Atlanta can produce 100 cars per week. Los Angeles requires 80 cars per week; Chicago, 70; and Philadelphia, 60. The cost of shipping a car between two cities is given in the following Tables. Formulate a balanced transportation problem to determine how to meet General Ford's weekly demands at minimum cost. (Do not solve!)

	To ($)	
From	Denver	New York
Detroit	1,200	600
Atlanta	1,400	800

	To ($)		
From	Los Angeles	Chicago	Philadelphia
Denver	1,000	1,000	1,700
New York	2,800	800	100

	Denver	New York	Los Ang.	Chicago	Phil.	Dummy	
Detroit	12	6	M	M	M	0	150
Atlanta	14	8	M	M	M	0	100
Denver	M	M	10	10	17	0	250
New York	M	M	28	8	1	0	250
	250	250	80	70	60	40	

3. (20 pts.) A company is taking bids on four construction jobs. Three people have placed bids on the jobs. Their bids (in thousands of dollars) are given in the following Table (a * indicates that the person did not bid on the given job). Person 1 can do only one job, but persons 2 and 3 can each do as many as two jobs. Determine the minimum cost assignment of persons to jobs.

Person	Job 1	Job 2	Job 3	Job 4
1	50	46	42	40
2	52	48	44	*
3	*	47	45	45

	Job 1	Job 2	Job 3	Job 4	Dummy	Row min
Person 1	50	46	42	40	0	0
Person 2	52	48	44	M	0	0
Person 2	52	48	44	M	0	0
Person 3	M	47	45	45	0	0
Person 3	M	47	45	45	0	0
Column min	50	46	42	40	0	

	Job 1	Job 2	Job 3	Job 4	Dummy
Person 1	0	0	0	0	0
Person 2	2	2	2	M	0
Person 2	2	2	2	M	0
Person 3	M	1	3	5	0
Person 3	M	1	3	5	0

	Job 1	Job 2	Job 3	Job 4	Dummy
Person 1	0	0	0	0	1
Person 2	1	1	1	M	0
Person 2	1	1	1	M	0
Person 3	M	0	2	4	0
Person 3	M	0	2	4	0

	Job 1	Job 2	Job 3	Job 4	Dummy
Person 1	0	1	0	**0**	2
Person 2	0	1	**0**	M	0
Person 2	**0**	1	0	M	0
Person 3	M	**0**	1	3	0
Person 3	M	0	1	3	**0**

Optimal assignment: Person 1 does Job 4, Person 2 does Job 1 and Job 3, Person 3 does Job 2. Total cost will be 183 thousand dollars.

4. (15 pts.) Consider the following puzzle. You are to pick out 4 three-letter "words" from the following list:

DBA DEG ADI FFD GHI BCD FDF BAI

For each word, you earn a score equal to the position that the word's third letter appears in the alphabet. For example, DBA earns a score of 1, DEG earns a score of 7, and so on. Your goal is to choose the four words that maximize your total score, subject to the following constraint: The sum of the positions in the alphabet for the first letter of each word chosen must be at least as large as the sum of the positions in the alphabet for the second letter of each word chosen. Formulate an integer programming to solve this puzzle.

Decision variables:
x_i = 1 if word i is chosen, 0 otherwise $(i = 1, \dots, 8)$

Parameters:
P_{ij} = Point of the jth position of word i $(j = 1, \dots, 3)$

LP model:

max $\sum_{i=1}^{8} P_{i3} x_i$ (Total score)

s.t. $\sum_{i=1}^{8} x_i = 4$ (4 word should be chosen)

$\sum_{i=1}^{8} P_{i1} x_i \geq \sum_{i=1}^{8} P_{i2} x_i$ (The sum of the first letter must be at least the sum of the second letter)

$x_i \in \{0,1\} \quad \forall i$

5. (20 pts.) You have been assigned to arrange the songs on the cassette version of Madonna's latest album. A cassette tape has two sides (1 and 2). The songs on each side of the cassette must total between 14 and 16 minutes in length. The length and type of each song are given in the following Table. The assignment of songs to the tape must satisfy the following conditions:

1. Each side must have exactly two ballads.

2. Side 1 must have at least three hit songs.
3. Either song 5 or song 6 must be on side 1.
4. If songs 2 and 4 are on side 1, then song 5 must be on side 2.

Determine the decision variables and write down the constraints of the problem.

Song	Type	Length (in minutes)
1	Ballad	4
2	Hit	5
3	Ballad	3
4	Hit	2
5	Ballad	4
6	Hit	3
7		5
8	Ballad and hit	4

Decision variables:

$x_{ij} = 1$ if song i is placed in side j, 0 otherwise $(i = 1, \ldots, 8; j = 1,2)$

Constraints:

$14 \leq \sum_{i=1}^{8} x_{ij} \leq 16 \quad \forall j$ (each side of the cassette must total between 14 and 16 minutes in length)

$\sum_{j=1}^{2} x_{ij} = 1 \quad \forall i$ (Each song can be placed in one side)

$x_{1j} + x_{3j} + x_{5j} + x_{8j} = 2 \quad \forall j$ (Each side must have exactly two ballads)

$x_{21} + x_{41} + x_{61} + x_{81} \geq 3$ (Side 1 must have at least three hit songs)

$x_{51} + x_{61} \geq 1$ (Either song 5 or song 6 must be on side 1)

$x_{52} \geq x_{21} + x_{41} - 1$ (If songs 2 and 4 are on side 1, then song 5 must be on side 2.)