BLG 311E – FORMAL LANGUAGES AND AUTOMATA
SPRING 2016
HOMEWORK 4

1) Use pumping lemma to prove that the language defined below is non-regular (i.e. cannot be accepted by any finite automaton).

$$L = cc^r \mid c \in \{0,1\}^*$$

2) For the language $L = \{a^i b^{i+j} a^j \mid i > 0, j \geq 0\}$,

- **a)** Write the grammar production rules.
- **b)** Design a PDA for this language.
- **c)** Show how the strings $aabbba$ and $aaabbb$ are accepted by the PDA you designed.

IMPORTANT: You must do this homework **by hand** and **submit it using the box in the secreteriat**.

SOLUTIONS:

1) Assumptions:

- Suppose that there exists a finite automaton M having n states and accepting L
- We choose the string $x = 0^n 1^n 1^n 0^n$ so $x \in L$ and $|x| \geq n$

By pumping lemma:

- $x = uvw$, $|v| > 0$ and $|uv| \leq n$
- For all possible splits that satisfy these rules: $v = 0^m$ where $1 \leq m \leq n$
- Lemma states that for a regular language all $uv^i w$ must also belong to the language ($i \geq 0$)
- Consider the string uv^2w ($i = 2$)
- $uv^2w = 0^{n+m}1^n1^n0^n$
- The string does not belong to L
- This is a contradiction so L is not regular

2)

a) $L = \{a^i b^{i+j} a^j \mid i > 0, j \geq 0\} \rightarrow L = \{a^i b^i b^j a^j \mid i > 0, j \geq 0\}$

<S> ::= <A><B>
<A> ::= a<A>b | ab
<B> ::= b<B>a | Λ

Chomsky Type 2.

b) $M = (S, \Sigma, \Gamma, \delta, s_0, F)$

$S = \{q_0, q_1, q_2, q_3, f\}$, $\Sigma = \{a, b\}$, $\Gamma = \{a, b, c\}$, $s_0 = q_0$, $F = f$

$\delta = \{ \underbrace{[(q_0, a, \Lambda), (q_1, ac)]}_{a},$ → push c to be able to check if the stack is empty

$\underbrace{[(q_1, a, \Lambda), (q_1, a)]}_{a^{i-1}}, \underbrace{[(q_1, b, a), (q_2, \Lambda)]}_{b}, \underbrace{[(q_2, b, a), (q_2, \Lambda)]}_{b^{i-1}}, \underbrace{[(q_2, \Lambda, c), (f, \Lambda)]}_{\text{accept } a^i b^i},$

$\underbrace{[(q_2, b, c), (q_2, bc)]}_{b}, \underbrace{[(q_2, b, b), (q_2, bb)]}_{b^{j-1}}, \underbrace{[(q_2, a, b), (q_3, \Lambda)]}_{a}, \underbrace{[(q_3, a, b), (q_3, \Lambda)]}_{a^{j-1}}, \underbrace{[(q_3, \Lambda, c), (f, \Lambda)]}_{\text{accept } a^i b^{i+j} a^j} \}$

c)

State	Tape	Stack	Transition Rule	State	Tape	Stack	Transition Rule
q_0	$aabbba$	Λ	$[(q_0, a, \Lambda), (q_1, ac)]$	q_0	$aaabbb$	Λ	$[(q_0, a, \Lambda), (q_1, ac)]$
q_1	$abbba$	ac	$[(q_1, a, \Lambda), (q_1, a)]$	q_1	$aabbb$	ac	$[(q_1, a, \Lambda), (q_1, a)]$
q_1	$bbba$	aac	$[(q_1, b, a), (q_2, \Lambda)]$	q_1	$abbb$	aac	$[(q_1, a, \Lambda), (q_1, a)]$
q_2	bba	ac	$[(q_2, b, a), (q_2, \Lambda)]$	q_1	bbb	$aaac$	$[(q_1, b, a), (q_2, \Lambda)]$
q_2	ba	c	$[(q_2, b, c), (q_2, bc)]$	q_2	bb	aac	$[(q_2, b, a), (q_2, \Lambda)]$
q_2	a	bc	$[(q_2, a, b), (q_3, \Lambda)]$	q_2	b	ac	$[(q_2, b, a), (q_2, \Lambda)]$
q_3	Λ	c	$[(q_3, \Lambda, c), (f, \Lambda)]$	q_2	Λ	c	$[(q_2, \Lambda, c), (f, \Lambda)]$
f	Λ	Λ		f	Λ	Λ	