BLG 454E Learning From Data

Homework 1 Report

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1) Saturday: 25%

If Saturday rains, Sunday 50%

If Saturday does not rain, Sunday 25%

P(Saturday|Sunday) = p(Saturday,Sunday) / p(Sunday)

P(Sunday,Saturday) = p(Saturday) * p(Sunday|Saturday) = $\frac{1}{4}$ * $\frac{1}{2}$ = 1/8

P(Sunday) = $\frac{1}{4}$ * $\frac{1}{2}$ + $\frac{3}{4}$ * $\frac{1}{4}$ = 5/16

P(Saturday|Sunday) = p(Saturday,Sunday) / p(Sunday) = 1/8 / 5/16 = 2/5

2) 0 moves -> (A) = 1/7

1 move: 5/42

B -> A = 1/7 * 1/3

G -> A = 1/7 * 1/6

F -> A = 1/7 * 1/3

2 moves: 11/126

B -> G -> A = 1/7 * 1/3 * 1/6

C -> B -> A = 1/7 * 1/3 * 1/3

C -> G -> A = 1/7 * 1/3 * 1/6

D -> G -> A = 1/7 * 1/3 * 1/6

E -> F -> A = 1/7 * 1/3 * 1/3

E -> G -> A = 1/7 * 1/3 * 1/6

F -> G -> A = 1/7 * 1/3 * 1/6

G -> B -> A = 1/7 * 1/6 * 1/3

G -> F -> A = 1/7 * 1/6 * 1/3

Total = 1/7 + 5/42 + 11/126 = 44/126

3.a) Gaussian distribution $P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)}$

- Likelihood function

$L(\mu, \sigma^2; x1,x2,\ldots,xn) = \prod_{i=1}^{n} fx(xi; \mu\ \sigma^2)$

$= \prod_{i=1}^{n} (2\pi\sigma^2)^{-1/2}\ e^{(-\frac{1}{2}*(xi-\mu)^2/\sigma^2)}$

- Take the log

$l(\mu, \sigma^2; x1,x2,\ldots,xn) = \ln(L(\mu, \sigma^2; x1,x2,\ldots,xn))$

$= -n/2 \ln(2\,\pi) - n/2 \ln(\sigma^2) - 1/2\sigma^2 \sum_{i=1}^{n}(xi - \mu\,)\text{^}2$

- Take the derivative for μ and $\sigma^2$and equal them to zero

$d/d\,\mu\ (\ l(\mu, \sigma^2; x1,x2,\ldots,xn)\ ) = 0$

$d/d\,\sigma^2\ (\ l(\mu, \sigma^2; x1,x2,\ldots,xn)\ ) = 0$

- Results

$\mu' = 1/n \sum_{i=1}^{n}(xi\ )$

$\sigma'^2 = 1/n \sum_{i=1}^{n}(xi - \mu'\,)\text{^}2$

4.a) $P(y|x1,x2,x3) = P(x1,x2,x3|y) * P(y) / P(x1,x2,x3)$

$P(y|x1,x2,x3) = P(x1|y) * P(y) / P(x1,x2,x3) *$

$P(x2|y) * P(y) / P(x1,x2,x3) *$

$P(x3|y) * P(y) / P(x1,x2,x3)$

$P(y | x1,x2,x3) = P(y=+ | x1,x2,x3) + P(y=- | x1,x2,x3)$

$P(y | x1,x2,x3) = [P(x1|y=+) * P(y=+) / P(x1,x2,x3) * P(x2| y=+) * P(y=+) / P(x1,x2,x3) * P(x3| y=+) * P(y=+) / P(x1,x2,x3)] + [P(x1|y=-) * P(y=-) / P(x1,x2,x3) * P(x2| y=-) * P(y=-) / P(x1,x2,x3) * P(x3| y=-) * P(y=-) / P(x1,x2,x3)]$

4.b) $P(y=-) = ½$ $P(y=+) = ½$

$P(y | x1=1, x2=1, x3=1) = P(y=- | x1=1, x2=1, x3=1) + P(y=+ | x1=1, x2=1, x3=1)$

$P(y | x1=1, x2=1, x3=1) \alpha P(x1=1|y)*P(y) * P(x2=1|y)*P(y) * P(x3=1|y)*P(y)$ proportional

$P(y=- | x1=1, x2=1, x3=1) = P(x1=1, x2=1, x3=1 | y=-) * P(y=-) / P(x1=1, x2=1, x3=1)$

$P(x1 = 1 | y=-) = 2/5$ $P(x1 = 1 | y=+) = 3/5$

$P(x2 = 1 | y=-) = 2/5$ $P(x2 = 1 | y=+) = 2/5$

$P(x3 = 1 | y=-) = 1/5$ $P(x3 = 1 | y=+) = 4/5$

$P(x1=1, x2=1, x3=1 | y=-) = 2/5 * 2/5 * 1/5 = 4/125$

$P(x1=1, x2=1, x3=1 | y=+) = 3/5 * 2/5 * 4/5 = 24/125$

$P(x1=1, x2=1, x3=1) = P(y=-) * P(x1=1, x2=1, x3=1 | y=-) +$

$P(y=+) * P(x1=1, x2=1, x3=1 | y=+)$

$= [(1/2 * 2/5 *2/5 * 1/5) + (1/2 * 3/5 * 2/5 * 4/5)] = 28/250$

$P(y=- | x1=1, x2=1, x3=1) \alpha P(x1=1, x2=1, x3=1 | y=-) * P(y=-) = 4/125 * ½ = 2/125$

$P(y=+ | x1=1, x2=1, x3=1) \alpha P(x1=1, x2=1, x3=1 | y=+) * P(y=+) = 24/125 * ½=12/125$

$12/125 > 2/125 \rightarrow x1=1, x2=1, x3=1 \rightarrow y=+$

4.c) $P(x1 = 1) = 5/10$ $P(x2 = 1) = 3/10$

Independent $\rightarrow P(x1=1, x2=1) = P(x1=1 | x2=1) * P(x2=1)$

$P(x1=1, x2=1) = 5/10 + 3/10 – 1/10 = 7/10$

P(x1=1 | x2=1) * P(x2=1) = 1/3 * 3/10 = 1/10

7/10 != 1/10 -> x1 and x2 are not independent