1. Let α be a relation defined over the set $A = \{a, b, c, d\}$, expressed with the following matrix.

$$\alpha = \begin{vmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{vmatrix}$$

   a. Express α as a relation graph.

   b. Find $rs(\alpha)$ and $sr(\alpha)$ and show that they are equal.

   c. Find $st(\alpha)$ and $ts(\alpha)$ and show that $st(\alpha) \subseteq ts(\alpha)$.

2. Design context-free grammars for the following languages:

   a. The set $\{0^n1^n \mid n \geq 1\}$, that is, the set of all strings of one or more 0's followed by an equal number of 1's.

   b. The set $\{a^ib^jc^k \mid i \neq j \text{ or } j \neq k\}$, that is, the set of strings of a's followed by b's followed by c's, such that there are either a different number of a's and b's or a different number of b's and c's, or both.

3. Consider the following grammar.

$$S \rightarrow AbB$$
$$A \rightarrow aA \mid \Lambda$$
$$B \rightarrow aB \mid bB \mid \Lambda$$

   a. Find the language generated by the grammar.

   b. What is the type of the grammar according to Chomsky hierarchy? Why?

   c. Design another grammar with a more restrictive type that generates the same expression you found in (a). (e.g. if the given grammar is Type-1 design a Type-2 or Type-3 grammar.)

   d. What is the type of the grammar you designed in (c)? Why?

IMPORTANT: You must do this homework **by hand** and **submit it using the box in the secreteriat**.