BLG 354E HW3

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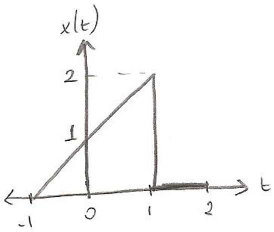
B.K.

(1) $\sin(0,1n\pi)$ $\quad T_0 = \frac{2\pi}{\Omega_0} = \frac{2\pi}{0,1\pi} = 20 \quad a_k = \frac{1}{T_0}\sum_{n=0}^{20} x[n] \cdot e^{-J.k.\Omega_0.n}$

$$a_k = \frac{1}{20}\sum_{n=0}^{20} \sin(0,1\pi n) \cdot e^{-J.k.0,1.\pi.n} = \frac{1}{20}\sum_{n=0}^{20} \frac{1}{2J}\left(e^{J.0,1.\pi.n} - e^{-J.0,1.\pi.n}\right) \cdot e^{-J.k.0,1.\pi.n}$$

$$a_k = \frac{1}{40J}\sum_{n=0}^{20} \underbrace{e^{J.0,1.\pi.n}}_{(-1)^{0,1.n}} \underbrace{\left(e^{1-k} - e^{-1-k}\right)}_{-1,1}$$

(2) a)



b) $T_0 = 3 \quad w_0 = \frac{2\pi}{T_0} = \frac{2\pi}{3}$

$$a_k = \frac{1}{3}\left[\int_{-1}^{1} (t+1) \cdot e^{-J.k.2\pi/3.t}\, dt + \int_{1}^{2} 0 \cdot dt\right]$$

$$a_k = \frac{1}{3} \cdot \left.\frac{3(2J.\pi.k(t+1)+3) \cdot e^{-2\pi J k/3}}{4\pi^2 k^2}\right|_{-1}^{1}$$

(3) a) $T_0 = K$

$\Omega_0 = \frac{2\pi}{K}$

$$a_x = \frac{1}{K}\left[\sum_{k=0}^{N} e^{-J.x.\Omega_0.k} + \sum_{k=K-N-1}^{K} e^{-J.x.\Omega_0.k}\right]$$

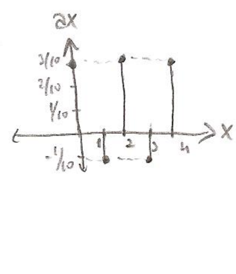
b) $T_0 = 10$

$\Omega_0 = \pi/5$

$$a_x = \frac{1}{10}\left[\sum_{k=-1}^{1} e^{-J.x.\pi/5.k}\right] = \frac{1}{10}\left[\underbrace{e^{J.x.\pi/5}}_{(-1)^{x/5}} + 1 + \underbrace{e^{-J.x.\pi/5}}_{(-1)^{-x/5}}\right]$$

$$a_x = \frac{1}{10}\left[(-1)^{x/5} + 1 + (-1)^{-x/5}\right]$$

$a_0 = 3/10$
$a_1 = -1/10$
$a_2 = 3/10$
$a_3 = -1/10$



(4) $x[k) = 0{,}5^k . u[k]$ $\quad 0 \leq k \leq 14 \quad \Rightarrow T_0 = 14 \quad \Omega_0 = 2\pi/14$

$$a_x = \frac{1}{14} \sum_{k=0}^{14} (1/2)^k . e^{-j.x.2\pi/14.k} = \frac{1}{14}\left[\sum_{k=0}^{14} (1/2)^k . e^k . e^{-j.2\pi/14.x}\right]$$

$$\boxed{\sum_{k=0}^{n-1} ar^k = \frac{1-r^n}{1-r}.a}$$

$$a_x = \frac{1}{14}\left[\frac{1-(e/2)^{15}}{1-e/2} . e^{-j.2\pi/14.x}\right]$$

(5) $T_0 = 4$
$W_0 = \frac{2\pi}{T_0} = \frac{\pi}{2}$

$$a_k = \frac{1}{4}\left[\int_1^2 e^{-j.k.\pi/2.t}dt + \int_2^3 2.e^{-j.k.\pi/2.t}dt + \int_3^4 e^{-j.k.\pi/2.t}dt\right]$$

$$a_k = \frac{1}{4}\left[\left(\frac{e^{-j.k.\pi}}{-j.k.\pi/2} - \frac{e^{-j.k.\pi/2}}{-j.k.\pi/2}\right) + 2.\left(\frac{e^{-j.k.3\pi/2}}{-j.k.\pi/2} - \frac{e^{-j.k.\pi}}{-j.k.\pi/2}\right) + \left(\frac{e^{-j.k.2\pi}}{-j.k.\pi/2} - \frac{e^{-j.k.3\pi/2}}{-j.k.\pi/2}\right)\right.$$

$$a_k = \frac{1}{4}\left(\frac{-e^{-j.k.\pi/2} - e^{-j.k.\pi} + e^{-j.k.3\pi/2} + e^{-j.k.2\pi}}{-j.k.\pi/2}\right) \quad \boxed{k = 1/2, 1, 3/2, 2}$$

BONUS $T_0 = 2\pi$
$W_0 = 1$

$$a_k = \frac{1}{2\pi}\int_0^{2\pi} 3.e^{-0{,}2.t} . e^{-j.k.1.t}dt = \frac{1}{2\pi}\int_0^{2\pi} 3.e^{-t(0{,}2+j.k)}dt$$

$$a_k = \frac{3}{2\pi} . e^{(0{,}2+jk)} . \int_0^{2\pi} e^t dt = \frac{3}{2\pi} . e^{(0{,}2+jk)} . -e^t\Big|_0^{2\pi} = \frac{3}{2\pi} . e^{(0{,}2+jk)} . (-e^{2\pi}+1)$$