CAS 781: Data Center Design Assignment 2

Baran Kaya, 400284996

Question 1

- All 3 subsystems have to work → components in series
- Control valve subsystem has parallel components
- $R_{overall} = R_{power\_supply} \cdot R_{pumps} \cdot (1 - (1 - R_{control\_valve}) \cdot (1 - (R_{controller} \cdot R_{bypass\_valve})))$

- Reliability = 1 – Failure
- $R_{power\_supply} = 1 - (8 \times 10^{-6}) = 0.999992$
- $R_{pump} = 1 - (6 \times 10^{-4}) = 0.9994$
- $R_{pumps} = R^4 + \binom{4}{3}R^3(1-R) + \binom{4}{2}R^2(1-R)^2$
- $R_{pumps} = 0.99760215913 + 0.00239568259 + 0.0000021574 = 0.99999999912$
- $R_{control\_valve} = 1 - (2 \times 10^{-4}) = 0.9998$
- $R_{controller} = 1 - (3 \times 10^{-4}) = 0.9997$
- $R_{bypass\_valve} = 1 - (4 \times 10^{-4}) = 0.9996$
- $R_{controller}$ and $R_{bypass\_valve} = 0.99930012$

- $R_{overall} = 0.999992 \cdot 0.99999999912 \cdot (1 - 0.0002 \cdot 0.00069988)$
- $R_{overall} = 0.999992 \cdot 0.99999999912 \cdot 0.999999986002 = 0.99999198512$

- $F_{overall} = 1 - R_{overall} = 0.00000801487$

- For improving the overall system reliability, we have to improve the least reliable subsystem. In this case it is power supply with 0.999992 reliability.

Question 2

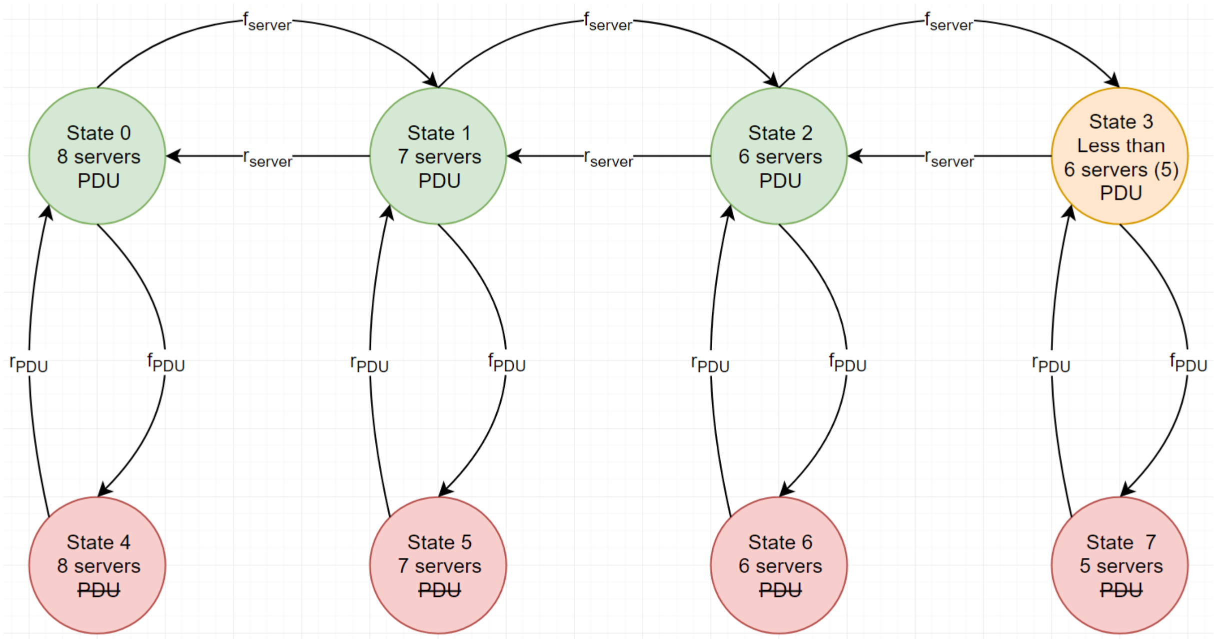
- Component availability = MTTF / (MTTF+MTTR) = (1/f) / (1/f + 1/r)
- $R_{server} = \dfrac{\frac{1}{fserver}}{\frac{1}{fserver}+\frac{1}{rserver}} = \dfrac{\frac{1}{0.004}}{\frac{1}{0.004}+1} = 0.99601593625$

- $R_{PDU} = \dfrac{\frac{1}{0.000002}}{\frac{1}{0.000002}+\frac{1}{0.02}} = 0.99990000999$

(a) With RBD

- $R_{at\_least\_6\_servers} = R^8 + \binom{8}{7}R^7(1-R) + \binom{8}{6}R^6(1-R)^2$
- $R_{at\_least\_6\_servers} = 0.9999720899 + 2.79097184 \times 10^{-5} + 3.40801178 \times 10^{-10}$
- $R_{at\_least\_6\_servers} \sim= 1$
- $R_{overall} = R_{PDU} * R_{at\_least\_6\_servers}$
- $R_{overall} = 0.99990000999 * 1$
- $R_{overall} = 0.99990000999$

(b) With CTMC



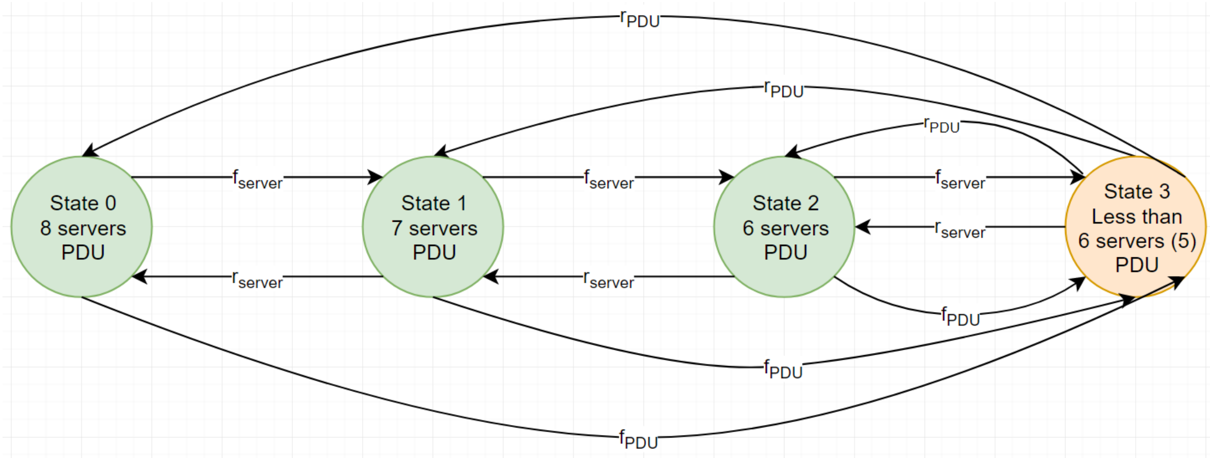
- State 0 → 8 servers and PDU working
- State 1 → 7 servers and PDU working
- State 2 → 6 servers and PDU working
- State 3 → 5 servers and PDU working
- State 4 → 8 servers working (no need to repair but offline) but PDU isn't working
- State 5 → 7 servers working but PDU isn't working
- State 6 → 6 servers working but PDU isn't working
- State 7 → 5 servers working but PDU isn't working

- State 0 → $\pi_0 \cdot f_{server} + \pi_0 \cdot f_{PDU} = \pi_1 \cdot r_{server} + \pi_4 \cdot r_{PDU}$
- State 1 → $\pi_1 \cdot f_{server} + \pi_1 \cdot f_{PDU} + \pi_1 \cdot r_{server} = \pi_2 \cdot r_{server} + \pi_5 \cdot r_{PDU} + \pi_0 \cdot f_{server}$
- State 2 → $\pi_2 \cdot f_{server} + \pi_2 \cdot f_{PDU} + \pi_2 \cdot r_{server} = \pi_4 \cdot r_{server} + \pi_6 \cdot r_{PDU} + \pi_1 \cdot f_{server}$
- State 3 → $\pi_3 \cdot r_{server} + \pi_3 \cdot f_{PDU} = \pi_1 \cdot r_{server} + \pi_5 \cdot r_{PDU}$
- State 4 → $\pi_0 \cdot f_{PDU} = \pi_4 \cdot r_{PDU}$
- State 5 → $\pi_1 \cdot f_{PDU} = \pi_5 \cdot r_{PDU}$
- State 6 → $\pi_2 \cdot f_{PDU} = \pi_6 \cdot r_{PDU}$
- State 7 → $\pi_3 \cdot f_{PDU} = \pi_7 \cdot r_{PDU}$
- $\pi_0 + \pi_1 + \pi_2 + \pi_3 + \pi_4 + \pi_5 + \pi_6 + \pi_7 = 1$

Simultaneous Linear Equations Solver for Eight Variables

	Equations	Solved Values
Eq.1:	-0.004002S + 1T + 0U + 0V + 0.02W + 0X + 0Y + 0Z = 0	S = 0
Eq.2:	0.004S + -1.004002T + 1U + 0V + 0W + 0.02X + 0Y + 0Z = 0	T = 0
Eq.3:	0S + 0.004T + -1.004002U + 0V + 1W + 0X + 0.02Y + 0Z = 0	U = 0
Eq.4:	0S + 1T + 0U + -0.004002V + 0W + 0.02X + 0Y + 0Z = 0	V = 0
Eq.5:	0.000002S + 0T + 0U + 0V + -0.02W + 0X + 0Y + 0Z = 0	W = 0
Eq.6:	0S + 0.000002T + 0U + 0V + 0W + -0.02X + 0Y + 0Z = 0	X = 0
Eq.7:	0S + 0T + 0.000002U + 0V + 0W + 0X + -0.02Y + 0Z = 0	Y = 0
Eq.8:	1S + 1T + 1U + 1V + 1W + 1X + 1Y + 1Z = 1	Z = 1

I think my state chart isn't correct. That is why I tried to build the new one but I'm not sure which one is the correct state chart.



- State 0 → 8 servers and PDU working
- State 1 → 7 servers and PDU working
- State 2 → 6 servers and PDU working
- State 3 → 5 servers or none of them working

- State 0 → $\pi_0 \cdot f_{server} + \pi_0 \cdot f_{PDU} = \pi_1 \cdot r_{server} + \pi_3 \cdot r_{PDU}$
- State 1 → $\pi_1 \cdot f_{server} + \pi_1 \cdot f_{PDU} + \pi_1 \cdot r_{server} = \pi_0 \cdot f_{server} + \pi_2 \cdot r_{server} + \pi_3 \cdot r_{PDU}$
- State 2 → $\pi_2 \cdot f_{server} + \pi_2 \cdot f_{PDU} + \pi_2 \cdot r_{server} = \pi_1 \cdot f_{server} + \pi_3 \cdot r_{server} + \pi_3 \cdot r_{PDU}$
- State 3 → $\pi_3 \cdot r_{server} + (\pi_3 + \pi_3 + \pi_3) \cdot r_{PDU} = \pi_2 \cdot f_{server} + (\pi_0 + \pi_1 + \pi_2) \cdot f_{PDU}$
- $\pi_0 + \pi_1 + \pi_2 + \pi_3 = 1$

Simultaneous Linear Equations Solver for Four Variables

	Equations	Solved Values
Eq.1:	-0.004002W + 1X + 0Y + 0.02Z = 0	W = 0.995994215200468
Eq.2:	0.004W + -1.004002X + 1Y + 0.02Z = 0	X = 0.00398592901361618
Eq.3:	0W + 0.004X + -1.004002Y + 1Z = 0	Y = 1.78640051107061e-05
Eq.4:	1W + 1X + 1Y + 1Z = 1	Z = 1.99178080469333e-06

- $\pi_0 = 0.995994215$, $\pi_1 = 0.003985929$, $\pi_2 = 1.786400511$, $\pi_3 = 1.991780804$

(c) Expected time

- In stage 0, system could fail with PDU fail or less than 6 server fail. So Stage 0 to 4 and 1 to 3 probabilities:
- $E[T_{0,4}] = E[T_{0,3}] = \frac{1}{fPDU+fserver} + E[T_{1,3}] \cdot \frac{fserver}{fserver+fPDU} + E[T_{4,4}] \cdot \frac{fPDU}{fserver+fPDU}$

- $E[T_{1,5}] = E[T_{1,3}] = \frac{1}{fPDU+fserver+rserver} + E[T_{0,4}] \cdot \frac{rserver}{fserver+fPDU+rserver} + E[T_{5,5}] \cdot \frac{fPDU}{fserver+fPDU} + E[T_{2,3}] \cdot \frac{fserver}{fserver+fPDU+rserver}$

- $E[T_{2,6}] = E[T_{2,3}] = \frac{1}{fPDU+fserver+rserver} + E[T_{1,5}] \cdot \frac{rserver}{fserver+fPDU+rserver} + E[T_{6,6}] \cdot \frac{fPDU}{fserver+fPDU} + E[T_{3,3}] \cdot \frac{fserver}{fserver+fPDU+rserver}$

- $E[T_{3,3}] = 0$
- $E[T_{4,4}] = 0$
- $E[T_{5,5}] = 0$
- $E[T_{6,6}] = 0$
- $E[T_{7,7}] = 0$
- $E[T_{3,7}] = 0$
- Expected time until less than six servers are working
- $E[T_{0,3}] + E[T_{0,4}] + E[T_{1,3}] + E[T_{1,5}] + E[T_{2,3}] + E[T_{2,6}]$

With 4 states chart

- $E[T_{0,3}] = \frac{1}{fPDU+fserver} + E[T_{1,3}] \cdot \frac{fserver}{fserver+fPDU} + E[T_{3,3}] \cdot \frac{fPDU}{fserver+fPDU}$

- $E[T_{1,3}] = \frac{1}{fPDU+fserver+rserver} + E[T_{0,3}] \cdot \frac{rserver}{fserver+fPDU+rserver} + E[T_{3,3}] \cdot \frac{fPDU}{fserver+fPDU+rserver} + E[T_{2,3}] \cdot \frac{fserver}{fserver+fPDU+rserver}$

- $E[T_{2,3}] = \frac{1}{fPDU+fserver+rserver} + E[T_{1,3}] \cdot \frac{rserver}{fserver+fPDU+rserver} + E[T_{3,3}] \cdot \frac{fPDU}{fserver+fPDU+rserver} + E[T_{3,3}] \cdot \frac{fserver}{fserver+fPDU+rserver}$

- $E[T_{3,3}] = 0$
- Expected time until less than six servers are working

Simultaneous Linear Equations Solver for Three Variables

	Equations	Solved Values
Eq.1:	-1X + 0.99950024Y + 0Z = -249.87506	X = 484606.87791
Eq.2:	0.99601395X + -1Y + 0.00398405Z = -0.9960139	Y = 484599.18614
Eq.3:	0.00000199X + 0.99601395Y + -1Z = -0.9960139	Z = 482669.50993

- $E[T_{0,3}] + E[T_{1,3}] + E[T_{2,3}] = 1451875.57398$

Question 3

None of the papers mention the reliability of the components. When cooling systems fail, most likely to both systems cannot reach the performance constraints. However, since both of the algorithms are aware of the thermal situation of the servers, they can reduce the utilization. Decreasing utilization will decrease the temperature and servers can run a little bit more without thermal protection shut down.

Zapater et al paper uses various kinds of cooling systems. This could increase the reliability however; their cooler has lots of subsystems. That much subsystems decreases the overall reliability, because they are serial modules. If one of them fails, whole cooling system fails. Also, they used water in cooling system. If the water system fails, this could affect the servers and repair time of the whole data center increases. After cooler fail, algorithm distribute the workload to all servers and decrease the utilization and overall heat.

TACOMA uses TASP and TAWD. If cooling systems fails both algorithms can work to reduce the heat of the system without cooling. TASP can assign new servers and TAWD can distribute the workload to idle servers. With that, system's temperature can be decreased and system could work for a while. However, in order to calculate the TASP and TAWD results the calculation unit must work. If this unit fails, the whole cooling algorithm fails.

If the cooling system fails during a normal time, some servers need to boot up in order to decrease utilization. However, booting up new servers can increase the data centers temperature even more.

Baran Kaya, 400284996, kayab@mcmaster.ca