CAS 781: Data Center Design Assignment 3

Baran Kaya, 400284996

Question 1

A) 100 CPUs, 10000GB Memory

- A = 2 CPUs, 300GB Memory
- B = 6 CPUs, 100GB Memory
- A is memory dominant and B is CPU dominant
- 100 CPU = 2*a + 6*b
- 10000 GB = 300*a + 100*b
- A dominant → 300GB / 10000GB
- B dominant → 6CPU / 100CPU
- Solve 300a/10000 = 6b/100 ➔ a = 2b
- 100 CPU = 2*(2b) + 6*b ➔ b = 10 ➔ a = 20
- CPU A ➔ 2*a = 2*20 = 40 CPU for A
- CPU B ➔ 6*b = 6*10 = 60 CPU for B
- Memory A ➔ 300*a = 300*20 = 6000 GB memory for A
- Memory B ➔ 100*b = 100*10 = 1000 GB memory for B
- Total CPU ➔ 60 + 40 = 100 ➔ 100% usage
- Total Memory ➔ 6000+1000 = 7000GB ➔ 70% usage

B) 150 CPUs, 10000GB Memory

- A = 2 CPUs, 300GB Memory
- B = 6 CPUs, 100GB Memory
- A is memory dominant and B is CPU dominant
- 150 CPU = 2*a + 6*b
- 10000 GB = 300*a + 100*b
- A dominant → 300GB / 10000GB
- B dominant → 6CPU / 150CPU
- Solve 300a/10000 = 6b/150 ➔ 3a = 4b ➔ a = 4/3b
- 10000 GB = 300*(4/3*b) + 100*b ➔ b = 20 ➔ a = 80/3 = 26,67 ~= 26
- CPU A ➔ 2*a = 2*26 = 52 CPU for A
- CPU B ➔ 6*b = 6*20 = 120 CPU for B
- Memory A ➔ 300*a = 300*26 = 7800 GB memory for A
- Memory B ➔ 100*b = 100*20 = 2000 GB memory for B
- Total CPU ➔ 52 + 120 = 172 ➔ **115% usage**
- 150 CPU = 2*a + 6*(3/4*a) ➔ a ~= 23.08 ➔ b ~= 17.31
- CPU A ➔ 2*a = 2*23 = 46 CPU for A
- CPU B ➔ 6*b = 6*17 = 102 CPU for B
- Memory A ➔ 300*a = 300*23 = 6900 GB memory for A
- Memory B ➔ 100*b = 100*17 = 1700 GB memory for B
- Total CPU ➔ 46 + 102 = 148➔ 99% usage
- Total Memory ➔ 6900+1700 = 8600GB ➔ 86% usage

- With increased CPU resource, system's overall usage increased. System has 1% decrease in CPU usage but 19% increase in memory usage (70% to 86%). Also, A and B task numbers are closer to each other (20,10) to (23,17).

Question 2

A) Server A has 40 cores

- A requires 2 cores, B requires 8 cores and C requires 16 cores
- (1, 1, 1) → 1*2 + 1*8 + 1*16 = 26 cores
- (1, 1, 1) → 1*2 + 1*8 + 1*16 = 26 cores
- (3, 1, 1) → 3*2 + 1*8 + 1*16 = 30 cores
- (0, 2, 1) → 0*2 + 2*8 + 1*16 = 32 cores
- (0, 2, 1) → 0*2 + 2*8 + 1*16 = 32 cores
- (0, 2, 1) → 0*2 + 2*8 + 1*16 = 32 cores
- (0, 0, 2) → 0*2 + 0*8 + 2*16 = 32 cores
- (0, 0, 2) → 0*2 + 0*8 + 2*16 = 32 cores
- **Total:** 242 cores but server has 320 cores
- 5 A tasks, 9 B tasks and 10 C tasks
- This problem is a multi knapsack problem and can be solve with dynamic programming.
- **Best:** (0, 1, 2), (0, 1, 2), (0, 1, 2), (0, 1, 2), (0, 1, 2), (4, 4, 0), (1, 0, 0) with 7 servers
- **Best:** 40, 40, 40, 40, 40, 40, 2 cores

B) Minimize movement

- Min VM replacement numbers
- Constraint: Use minimum number of servers
- (1,1,1), (1,1,1), (3,1,1), (0,2,1), (0,2,1), (0,2,1), (0,0,2), (0,0,2)
- B 6 to 8
- (1,1,1), (1,1,1), (3,1,1), (0,2,1), (0,2,1), (0,1,1), (0,0,2), (0,1,2)
- B 5 to 7
- (1,1,1), (1,1,1), (3,1,1), (0,2,1), (0,1,1), (0,1,1), (0,1,2), (0,1,2)
- C 4 to 6
- (1,1,1), (1,1,1), (3,1,1), (0,2,0), (0,1,1), (0,1,2), (0,1,2), (0,1,2)
- C 3 to 5
- (1,1,1), (1,1,1), (3,1,0), (0,2,0), (0,1,2), (0,1,2), (0,1,2), (0,1,2)
- A 2 to 3
- (1,1,1), (0,1,1), (4,1,0), (0,2,0), (0,1,2), (0,1,2), (0,1,2), (0,1,2)
- C 1 to 2
- (1,1,0), (0,1,2), (4,1,0), (0,2,0), (0,1,2), (0,1,2), (0,1,2), (0,1,2)
- B 1 to 3
- (1,0,0), (0,1,2), (4,2,0), (0,2,0), (0,1,2), (0,1,2), (0,1,2), (0,1,2)
- B 4 to 3 (x2)
- (1,0,0), (0,1,2), (4,4,0), (0,0,0), (0,1,2), (0,1,2), (0,1,2), (0,1,2)
- Total movement 9

Question 3

A) S1 = $\begin{bmatrix} b0 \\ b1 \end{bmatrix}$, S2 = $\begin{bmatrix} b1 \\ b2 \end{bmatrix}$, S3 = $\begin{bmatrix} b0\ XOR\ b2 \\ b1\ XOR\ b3 \end{bmatrix}$,

- $Bx\begin{bmatrix} b0 \\ b1 \\ b2 \\ b3 \end{bmatrix} = \begin{bmatrix} b0 \\ b1 \\ b2 \\ b3 \\ b0 \oplus b2 \\ b1 \oplus b3 \end{bmatrix}$
- $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ b11 & b12 & b13 & b14 \\ b21 & b22 & b23 & b24 \end{bmatrix} x \begin{bmatrix} b0 \\ b1 \\ b2 \\ b3 \end{bmatrix} = \begin{bmatrix} b0 \\ b1 \\ b2 \\ b3 \\ b0 \oplus b2 \\ b1 \oplus b3 \end{bmatrix}$
- $B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ b11 & b12 & b13 & b14 \\ b21 & b22 & b23 & b24 \end{bmatrix} = \begin{bmatrix} b0 \\ b1 \\ b2 \\ b3 \\ b0 \oplus b2 \\ b1 \oplus b3 \end{bmatrix} x \begin{bmatrix} b0 \\ b1 \\ b2 \\ b3 \end{bmatrix}^{-1} --> B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ b2^{-1} & 0 & b0^{-1} & 0 \\ 0 & b3^{-1} & 0 & b1^{-1} \end{bmatrix}$

B) Server 2 fails:

- $\begin{bmatrix} b0 \\ b1 \\ x \\ y \\ b0 \oplus b2 \\ b1 \oplus b3 \end{bmatrix} --> B^{|} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ b2^{-1} & 0 & b0^{-1} & 0 \\ 0 & b3^{-1} & 0 & b1^{-1} \end{bmatrix} x \begin{bmatrix} b0 \\ b1 \\ b2 \\ b3 \end{bmatrix} = \begin{bmatrix} b1 \\ b2 \\ b0 \oplus b2 \\ b1 \oplus b3 \end{bmatrix}$
- For finding/recovering server 2 data (b2, b3):
- $(B^{|})^{-1} x \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ b2^{-1} & 0 & b0^{-1} & 0 \\ 0 & b3^{-1} & 0 & b1^{-1} \end{bmatrix} x \begin{bmatrix} b0 \\ b1 \\ x \\ y \end{bmatrix} = \begin{bmatrix} b1 \\ b2 \\ b0 \oplus b2 \\ b1 \oplus b3 \end{bmatrix} x (B^{|})^{-1}$
- Failed/erased data:
- $\begin{bmatrix} b0 \\ b1 \\ x \\ y \end{bmatrix} = \begin{bmatrix} b1 \\ b2 \\ b0 \oplus b2 \\ b1 \oplus b3 \end{bmatrix} x (B^{|})^{-1}$ → Matrix is not invertible because determinant is 0.

Baran Kaya, 400284996, kayab@mcmaster.ca