departamento de matemática **universidade de aveiro**

1. Identifique a espécie dos seguintes integrais impróprios e determine a sua natureza. Em caso de convergência, indique o seu valor.

(a) $\displaystyle\int_e^{+\infty} \frac{1}{x\ln(x)}\,dx$ (b) $\displaystyle\int_{-\infty}^{+\infty} \frac{1}{1+x^2}\,dx$ (c) $\displaystyle\int_0^{+\infty} \frac{1}{\sqrt[3]{x+1}}\,dx$

(d) $\displaystyle\int_4^{+\infty} \frac{1}{e^x}\,dx$ (e) $\displaystyle\int_\pi^{+\infty} \cos(3x)\,dx$ (f) $\displaystyle\int_3^{+\infty} \frac{1}{9+x^2}\,dx$

(g) $\displaystyle\int_0^{+\infty} xe^{-3x}\,dx$ (h) $\displaystyle\int_{-\infty}^{2} \frac{1}{(4-x)^2}\,dx$ (i) $\displaystyle\int_{-\infty}^{+\infty} x\,dx$

(j) $\displaystyle\int_3^{+\infty} \frac{4}{x^2-4}\,dx$ (k) $\displaystyle\int_{-\infty}^{0} xe^{-x^2}\,dx$ (l) $\displaystyle\int_{-1}^{0} \frac{x}{\sqrt{1-x^2}}\,dx$

(m) $\displaystyle\int_{\frac{\pi}{2}}^{\pi} \operatorname{cotg}(x)\,dx$ (n) $\displaystyle\int_{-1}^{2} \frac{1}{4-x^2}\,dx$ (o) $\displaystyle\int_0^{2} \frac{1}{\sqrt{1-\frac{x^2}{2}}}\,dx$

(p) $\displaystyle\int_{-\infty}^{+\infty} \frac{1}{x^3}\,dx$ (q) $\displaystyle\int_0^{1} \frac{1}{x\ln^2(x)}\,dx$ (r) $\displaystyle\int_0^{1} \ln(x)\,dx$

(s) $\displaystyle\int_1^{+\infty} \frac{1}{x\sqrt{x^2-1}}\,dx$ (t) $\displaystyle\int_0^{+\infty} \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx$ (u) $\displaystyle\int_{-\infty}^{0} \frac{1}{|1-x|}\,dx$

(v) $\displaystyle\int_{-2}^{1} \frac{1}{|x|}\,dx$ (w) $\displaystyle\int_{-1}^{1} \frac{1}{\sqrt{1-x^2}}\,dx$ (x) $\displaystyle\int_5^{+\infty} \frac{1}{x\ln^3(x)}\,dx$

(y) $\displaystyle\int_0^{+\infty} \frac{1}{\sqrt{e^x-1}}\,dx$

2. Calcule, caso exista, $\displaystyle\int_{-\infty}^{+\infty} f(x)\,dx$ com $f(x) = \begin{cases} \dfrac{1}{x-1} & \text{se } x \leq 0 \\ \operatorname{arctg}(x) & \text{se } x > 0 \end{cases}$

3. Estude a natureza do integral impróprio $\displaystyle\int_0^{+\infty} e^{-st}e^{\alpha t}\,dt$, com $s > \alpha$, e em caso de convergência indique o seu valor.

4. Mostre que $\displaystyle\int_0^{+\infty} e^{-st}\cos(\alpha t)\,dt = \frac{s}{s^2+\alpha^2}$, para $\alpha \in \mathbb{R}$ e $s \in \mathbb{R}^+$.

5. Estude, em função de $\alpha \in \mathbb{R}$, a natureza dos integrais impróprios:

   (a) $\displaystyle\int_0^1 \frac{1}{x^\alpha}\,dx$ (b) $\displaystyle\int_0^{+\infty} \frac{1}{x^\alpha}\,dx$

6. Seja $f(x) = \begin{cases} m & \text{se } |x| \le 2 \\ 0 & \text{se } |x| > 2 \end{cases}$. Determine m de modo a que $\displaystyle\int_{-\infty}^{+\infty} f(x)\,dx = 1$.

7. Determine se os integrais impróprios seguintes são convergentes ou divergentes, aplicando os critérios de comparação ou comparação por passagem ao limite.

   (a) $\displaystyle\int_1^{+\infty} \frac{|\cos(3x)|}{x^3}\,dx$ (b) $\displaystyle\int_0^1 \frac{1}{\ln\left(1+\sqrt{x}\right)}\,dx$

   (c) $\displaystyle\int_0^1 \frac{1}{\sqrt{1-x^4}}\,dx$ (d) $\displaystyle\int_0^{+\infty} e^{-x}\left|\cos\left(\sqrt{x}\right)\right|\,dx$

   (e) $\displaystyle\int_0^{+\infty} \frac{1}{\sqrt[3]{x^4+3x+2}}\,dx$ (f) $\displaystyle\int_1^{+\infty} \frac{\ln(x)}{x\ln(x+1)}\,dx$

   (g) $\displaystyle\int_1^{+\infty} \operatorname{arcsen}\left(\frac{1}{x}\right)dx$ (h) $\displaystyle\int_1^{+\infty} \frac{x^2+1}{1+2x+x^3}\,dx$

   (i) $\displaystyle\int_0^1 \frac{\operatorname{sen}(x)}{\sqrt{1-x}}\,dx$ (j) $\displaystyle\int_0^1 \frac{e^{-x}}{x}\,dx$

   (k) $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\operatorname{sen}(x)}{x^3}\,dx$ (l) $\displaystyle\int_0^1 \frac{1}{\sqrt[5]{x}}\operatorname{sen}\left(\frac{1}{\sqrt{x}}\right)dx$

   (m) $\displaystyle\int_1^{+\infty} \frac{-1}{x^3+1}\,dx$ (n) $\displaystyle\int_0^{+\infty} \frac{x+1}{\sqrt{x^3}}\,dx$

   (o) $\displaystyle\int_2^{+\infty} \frac{x^3}{x^2(x^2+1)}\,dx$ (p) $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\operatorname{sen}\left(\sqrt{x}\right)}{\sqrt[4]{x}}\,dx$

   (q) $\displaystyle\int_2^{+\infty} \frac{1}{\sqrt[3]{x^4+2x+1}}\,dx$ (r) $\displaystyle\int_1^{+\infty} \frac{|\cos(3x)|}{x^3}\,dx$

(s) $\displaystyle\int_0^{+\infty} e^{-x}\left|\cos\left(\sqrt{x}\right)\right|\,dx$

(t) $\displaystyle\int_1^{+\infty} \frac{\sqrt{x+1}}{\sqrt{x^2+x+1}}\,dx$

(u) $\displaystyle\int_1^{+\infty} \frac{1}{x\sqrt{x}+x+\sqrt{x}}\,dx$

(v) $\displaystyle\int_1^{+\infty} \frac{1}{x+\ln(x)}\,dx$

(w) $\displaystyle\int_{-\infty}^{+\infty} \frac{\arctan(x)}{\sqrt{1+x^4}}\,dx$

(x) $\displaystyle\int_2^{+\infty} \frac{1}{\ln(x)}\,dx$

(y) $\displaystyle\int_1^{+\infty} \frac{|\operatorname{sen}(x)|}{x\sqrt{x}}\,dx$

(z) $\displaystyle\int_0^{1} \frac{1}{x\sqrt{x}+x+\sqrt{x}}\,dx$

(aa) $\displaystyle\int_0^{1} \frac{\ln(1+x^2)}{x^3}\,dx$

(ab) $\displaystyle\int_0^{1} \frac{\arctan\sqrt[3]{x}}{\sqrt[3]{x}}\,dx$

(ac) $\displaystyle\int_0^{1} \frac{1}{x-\ln(x)}\,dx$

(ad) $\displaystyle\int_1^{+\infty} \frac{\operatorname{sen}\left(\frac{1}{x}\right)}{x}\,dx$

(ae) $\displaystyle\int_0^{1} \frac{1}{\ln(x)}\,dx$

(af) $\displaystyle\int_0^{\frac{\pi}{2}} \frac{1}{x\cos(x)}\,dx$

(ag) $\displaystyle\int_0^{1} \frac{x}{e^{2x}-1}\,dx$

(ah) $\displaystyle\int_1^{+\infty} \frac{\cos\left(\frac{1}{x}\right)}{x}\,dx$

8. (a) Mostre que o integral impróprio $\displaystyle\int_1^{+\infty} xe^{-x^2}\,dx$ é convergente e indique o seu valor.

   (b) Estude a natureza do integral impróprio $\displaystyle\int_1^{+\infty} xe^{-x^2}\cos^2(x)\,dx$ sem recorrer à definição.

9. Seja $f :]1,+\infty[\to \mathbb{R}$ a função definida por $f(x) = \dfrac{1}{x\sqrt{\ln(x)}}$.

   (a) Determine a primitiva de f que se anula no ponto $x = e$.

   (b) Estude a natureza do integral impróprio $\displaystyle\int_2^{+\infty} f(x)\,dx$.

1. (a) divergente; (b) π; (c) divergente; (d) $\frac{1}{e^4}$; (e) divergente; (f) $\frac{\pi}{12}$; (g) $\frac{1}{9}$; (h) $\frac{1}{2}$; (i) divergente; (j) $\ln 5$; (k) $-\frac{1}{2}$; (l) -1; (m) divergente; (n) divergente; (o) divergente; (p) divergente; (q) divergente; (r) -1; (s) $\frac{\pi}{2}$; (t) 2; (u) divergente; (v) divergente; (w) π; (x) $\frac{1}{2\ln^2(5)}$; (y) π.
2. divergente
3. $\frac{1}{s-\alpha}$
5. (a) se $\alpha \geq 1$, é divergente; se $\alpha < 1$, é convergente e o seu valor é igual a $\frac{1}{1-\alpha}$; (b) é divergente para qualquer $\alpha \in \mathbb{R}$.
6. $m = \frac{1}{4}$.
7. (a) convergente; (b) convergente; (c) convergente; (d) convergente; (e) convergente; (f) divergente; (g) divergente; (h) divergente; (i) convergente; (j) divergente; (k) divergente; (l) convergente; (m) convergente; (n) divergente; (o) divergente; (p) divergente; (q) convergente; (r) convergente; (s) convergente; (t) divergente; (u) convergente; (v) divergente; (w) convergente; (x) divergente; (y) convergente; (z) convergente; (aa) divergente; (ab) convergente; (ac) convergente; (ad) convergente; (ae) divergente; (af) divergente; (ag) convergente; (ah) divergente.
8. (a) $\frac{1}{2e}$; (b) convergente.
9. (a) $F(x) = 2\sqrt{\ln(x)} - 2$; (b) divergente.