departamento de matemática **universidade de aveiro**

1. Aplicando a técnica da integração por partes, determine cada um dos seguintes integrais indefinidos.

   (a) $\int xe^{-x}\,dx$

   (b) $\int x\ln(x)\,dx$

   (c) $\int -2x\ln(x+3)\,dx$

   (d) $\int x\cos(x)\,dx$

   (e) $\int 3x\,\mathrm{sen}(5x)\,dx$

   (f) $\int 4x\cos(2x)\,dx$

   (g) $\int (x+3)e^{\frac{3}{2}x}\,dx$

   (h) $\int (x+4)\cos(3x)\,dx$

   (i) $\int \frac{x+2}{3}\cos(5x)\,dx$

   (j) $\int x^2\,\mathrm{sen}(x)\,dx$

   (k) $\int 5x^2e^{4x}\,dx$

   (l) $\int \left(x^2+6x-2\right)e^{\frac{x}{3}}\,dx$

   (m) $\int \frac{x^2-2x+5}{e^x}\,dx$

   (n) $\int x^3e^{2x}\,dx$

   (o) $\int \ln(1-x)\,dx$

   (p) $\int \ln\left(x^2+1\right)\,dx$

   (q) $\int \ln^2(x)\,dx$

   (r) $\int x^7e^{x^4}\,dx$

   (s) $\int 3e^x\cos(5x)\,dx$

   (t) $\int e^{4x}\,\mathrm{sen}(1-x)\,dx$

   (u) $\int \mathrm{sen}(x)\cos(3x)\,dx$

   (v) $\int \mathrm{sen}\left(\ln(x)\right)\,dx$

   (w) $\int x\,\mathrm{arctg}(x)\,dx$

   (x) $\int x\,\mathrm{arcsen}\left(x^2\right)\,dx$

   (y) $\int \sec^3(x)\,dx$

   (z) $\int \frac{x^2}{\sqrt{(1-x^2)^3}}\,dx$

2. Determine cada um dos seguintes integrais indefinidos de funções racionais.

(a) $\int \frac{x^2-4x+5}{x-2}\,dx$

(b) $\int \frac{x^3-2x^2+5}{x+1}\,dx$

(c) $\int \frac{x^3+6x^2+10x}{x+2}\,dx$

(d) $\int \frac{9+x^3-3x}{2x-3}\,dx$

(e) $\int \frac{2x^3+6x^2+8x}{2x^2+1}\,dx$

(f) $\int \frac{x^2+5x+7}{x^2+3x+4}\,dx$

(g) $\int \frac{2x^3+2x+1}{4x^2+1}\,dx$

(h) $\int \frac{2x^3+6x^2+8x}{5+x^2}\,dx$

(i) $\int \frac{3x-7}{x^3+x^2+4x+4}\,dx$

(j) $\int \frac{4x+1}{x^2+6x+8}\,dx$

(k) $\int \frac{1}{x(x^2+1)}\,dx$

(l) $\int \frac{4x^3+2x^2-5x-18}{(x-4)(x+1)^3}\,dx$

(m) $\int \frac{x^3+x-1}{(x^2+2)^2}\,dx$

(n) $\int \frac{x^4+x+1}{x^3-x}\,dx$

(o) $\int \frac{x^5}{x^3-1}\,dx$

(p) $\int \frac{x^6-x^3+1}{x^4+9x^2}\,dx$

(q) $\int \frac{x+2}{(x-1)^2(x^2+4)}\,dx$

(r) $\int \frac{1}{x^3+8}\,dx$

(s) $\int \frac{x^3+3x-1}{x^4-4x^2}\,dx$

(t) $\int \frac{1}{x(x^2+1)^2}dx$

(u) $\int \frac{x^4}{x^4-1}\,dx$

(v) $\int \frac{1}{x^2+4x+5}dx$

(w) $\int \frac{x^8}{1+x^2}\,dx$

(x) $\int \frac{x^5+x^4-8}{x^3-4x}dx$

3. Efetuando a mudança de variável indicada entre parênteses, determine cada um dos seguintes integrais indefinidos.

   (a) $\displaystyle\int \frac{x}{\sqrt{x+1}}\,dx$ $\qquad (x = t^2 - 1)$

   (b) $\displaystyle\int \frac{\sqrt{x}}{1+\sqrt[3]{x}}\,dx$ $\qquad (x = t^6)$

   (c) $\displaystyle\int \frac{1}{x\sqrt{1+\ln(x)}}\,dx$ $\qquad (x = e^t)$

   (d) $\displaystyle\int \frac{e^{2x}}{\sqrt[4]{e^x+1}}\,dx$ $\qquad (x = \ln(t^4 - 1))$

   (e) $\displaystyle\int \sqrt{e^x - 1}\,dx$ $\qquad (x = \ln(t^2 + 1))$

   (f) $\displaystyle\int \frac{1}{\sqrt{e^x+1}}\,dx$ $\qquad (x = \ln(t^2 - 1))$

   (g) $\displaystyle\int \frac{\sqrt{\operatorname{arctg}(\sqrt{x})}}{\sqrt{x}(1+x)}\,dx$ $\qquad (x = t^2)$

   (h) $\displaystyle\int \frac{e^{2x} - e^{-x}}{1+e^{2x}}\,dx$ $\qquad (x = \ln(t))$

4. Aplicando a técnica da integração por substituição, determine cada um dos seguintes integrais indefinidos.

   (a) $\displaystyle\int e^{\sqrt{5x-2}}\,dx$

   (b) $\displaystyle\int x\sqrt{x+1}\,dx$

   (c) $\displaystyle\int \left(\sqrt{x}+3\right)^4\,dx$

   (d) $\displaystyle\int \frac{1}{x\sqrt{2x-3}}\,dx$

   (e) $\displaystyle\int \frac{1}{x+1+\sqrt{x+1}}\,dx$

   (f) $\displaystyle\int \frac{1}{(1+x)\sqrt{x}}\,dx$

   (g) $\displaystyle\int \frac{x}{\sqrt[3]{x-1}}\,dx$

   (h) $\displaystyle\int \operatorname{sen}\left(\sqrt[3]{x}\right)\,dx$

   (i) $\displaystyle\int \frac{\sqrt{x}}{x+1}\,dx$

   (j) $\displaystyle\int \frac{1}{x\sqrt{2x+4}}\,dx$

(k) $\displaystyle\int \frac{1}{x\sqrt{4-x^2}}\,dx$

(l) $\displaystyle\int \frac{1}{x\sqrt{9+x^2}}\,dx$

(m) $\displaystyle\int \frac{1}{x^2\sqrt{x^2-25}}\,dx$

(n) $\displaystyle\int \frac{x}{\sqrt{4-x^2}}\,dx$

(o) $\displaystyle\int \frac{x^3}{\sqrt{9x^2+49}}\,dx$

(p) $\displaystyle\int \frac{1}{x^4\sqrt{x^2-3}}\,dx$

(q) $\displaystyle\int \frac{\ln(x)}{x\sqrt{1+\ln(x)}}\,dx$

(r) $\displaystyle\int \frac{1+\operatorname{tg}^2(x)}{\operatorname{tg}(x)-1}\,dx$

(s) $\displaystyle\int x(2x+5)^{10}\,dx$

(t) $\displaystyle\int (1+x)^{-2}\left(\frac{1-x}{1+x}\right)^{\frac{1}{3}}\,dx$

5. Determine os seguintes integrais indefinidos.

(a) $\displaystyle\int \operatorname{sen}(2x)e^{\cos^2(x)}\,dx$

(b) $\displaystyle\int \frac{1}{\sqrt{x}-\sqrt[4]{x}}\,dx$

(c) $\displaystyle\int \frac{1}{x^2\sqrt{x^2-9}}\,dx$

(d) $\displaystyle\int x^2 \operatorname{arctg}(x)\,dx$

(e) $\displaystyle\int \frac{x+2}{x(x^2+4)}\,dx$

(f) $\displaystyle\int \frac{x^2}{\sqrt{1+x^3}}\,dx$

(g) $\displaystyle\int \frac{1}{x^2\sqrt{1+x^2}}\,dx$

(h) $\displaystyle\int \frac{3x-1}{x^3+x}\,dx$

(i) $\displaystyle\int \frac{-\cos(x)}{(1+\operatorname{sen}(x))^2}\,dx$

(j) $\displaystyle\int x\ln\left(1+x^2\right)\,dx$

(k) $\displaystyle\int \cos(x)\ln(\operatorname{sen}(x))\,dx$

(l) $\displaystyle\int \frac{1}{x\sqrt{x^2-4}}\,dx$

(m) $\displaystyle\int x\sqrt{(1+x^2)^3}\,dx$

(n) $\displaystyle\int \frac{1}{\sqrt{2x-x^2}}\,dx$

(o) $\displaystyle\int \operatorname{sen}^5(x)\cos^2(x)\,dx$

(p) $\displaystyle\int \frac{1}{\sqrt{2+x^2}}\,dx$

6. Determine a função g tal que $g'(x) = \dfrac{5x-4}{x(x^2-2x+2)}$ e $\lim\limits_{x\to+\infty} g(x) = 0$.

7. Determine a função $h : \mathbb{R} \to \mathbb{R}$ tal que $h(0) = 1$, $h'(0) = 2$ e $h''(x) = 12x$, para todo $x \in \mathbb{R}$.

1. (a) $-e^{-x}(x+1)+C, \quad C\in\mathbb{R}$;
   (b) $\frac{1}{2}x^2\left(\ln(x)-\frac{1}{2}\right)+C, \quad C\in\mathbb{R}$;
   (c) $(9-x^2)\ln(x+3)+\frac{1}{2}x^2-3x+C, \quad C\in\mathbb{R}$;
   (d) $x\,\mathrm{sen}(x)+\cos(x)+C, \quad C\in\mathbb{R}$;
   (e) $\frac{3}{25}\,\mathrm{sen}(5x)-\frac{3}{5}x\cos(5x)+C, \quad C\in\mathbb{R}$;
   (f) $2x\,\mathrm{sen}(2x)+\cos(2x)+C,\ C\in\mathbb{R}$;
   (g) $2e^{\frac{3}{2}x}\left(\frac{1}{3}x-\frac{7}{9}\right)+C, \quad C\in\mathbb{R}$;
   (h) $\left(\frac{1}{3}x+\frac{4}{3}\right)\mathrm{sen}(3x)+\frac{1}{9}\cos(3x)+C, \quad C\in\mathbb{R}$;
   (i) $\left(\frac{1}{15}x+\frac{2}{15}\right)\mathrm{sen}(5x)+\frac{1}{75}\cos(5x)+C, \quad C\in\mathbb{R}$;
   (j) $(2-x^2)\cos(x)+2x\,\mathrm{sen}(x)+C, \quad C\in\mathbb{R}$;
   (k) $\frac{5}{4}e^{4x}\left(x^2-\frac{1}{2}x+\frac{1}{8}\right)+C, \quad C\in\mathbb{R}$;
   (l) $3e^{\frac{x}{3}}\left(x^2-2\right)+C, \quad C\in\mathbb{R}$;
   (m) $-e^{-x}\left(x^2+5\right)+C, \quad C\in\mathbb{R}$;
   (n) $\frac{1}{2}e^{2x}\left(x^3-\frac{3}{2}x^2+\frac{3}{2}x-\frac{3}{4}\right)+C, \quad C\in\mathbb{R}$;
   (o) $x\ln(1-x)-x-\ln|1-x|+C, \quad C\in\mathbb{R}$;
   (p) $x\ln\left(x^2+1\right)-2x+2\,\mathrm{arctg}(x)+C, \quad C\in\mathbb{R}$;
   (q) $x\left((\ln(x))^2-2\ln(x)+2\right)+C, \quad C\in\mathbb{R}$;
   (r) $\frac{1}{4}e^{x^4}\left(x^4-1\right)+C, \quad C\in\mathbb{R}$;
   (s) $\frac{3e^x}{26}(\cos(5x)+5\,\mathrm{sen}(5x))+C, \quad C\in\mathbb{R}$;
   (t) $\frac{e^{4x}}{17}(\cos(1-x)+4\,\mathrm{sen}(1-x))+C, \quad C\in\mathbb{R}$;
   (u) $\frac{1}{8}(\cos(x)\cos(3x)+3\,\mathrm{sen}(x)\,\mathrm{sen}(3x))+C, \quad C\in\mathbb{R}$;
   (v) $\frac{x}{2}(\mathrm{sen}(\ln(x))-\cos(\ln(x)))+C, \quad C\in\mathbb{R}$;
   (w) $\frac{x^2}{2}(\mathrm{arctg}(x)-\frac{1}{2}\left(x-\mathrm{arctg}(x)\right)+C, \quad C\in\mathbb{R}$;
   (x) $\frac{x^2}{2}(\mathrm{arcsen}(x^2)+\frac{1}{2}\sqrt{1-x^4}+C, \quad C\in\mathbb{R}$;
   (y) $\frac{1}{2}\left(\sec(x)\,\mathrm{tg}(x)-\ln|\sec(x)+\mathrm{tg}(x)|\right)+C, \quad C\in\mathbb{R}$;
   (z) $\frac{x}{\sqrt{1-x^2}-\mathrm{arcsen}(x)}+C, \quad C\in\mathbb{R}$.
2. (a) $\frac{1}{2}x^2-2x+\ln|x-2|+C, \quad C\in\mathbb{R}$;
   (b) $\frac{1}{3}x^3-\frac{3}{2}x^2+3x+2\ln|x+1|+C, \quad C\in\mathbb{R}$;

(c) $\frac{1}{3}x^3 + 2x^2 + 2x - 4\ln|x+2| + C, \quad C \in \mathbb{R}$;

(d) $\frac{1}{6}x^3 + \frac{3}{8}x^2 - \frac{3}{8}x + \frac{63}{16}\ln|2x-3| + C, \quad C \in \mathbb{R}$;

(e) $\frac{1}{2}x^2 + 3x - \frac{7}{4}\ln|2x^2+1| - \frac{3}{\sqrt{2}}\operatorname{arctg}\left(\sqrt{2}x\right) + C, \quad C \in \mathbb{R}$;

(f) $x + \ln|x^2+3x+4| + C, \quad C \in \mathbb{R}$;

(g) $\frac{3}{8}\ln|4x^2+1| + \frac{1}{2}\operatorname{arctg}(2x) + \frac{1}{4}x^2 + C, \quad C \in \mathbb{R}$;

(h) $x^2 + 6x - \ln|x^2+5| - 6\sqrt{5}\operatorname{arctg}\left(\frac{x}{\sqrt{5}}\right) + C, \quad C \in \mathbb{R}$;

(i) $\ln\left(\frac{x^2+4}{(x+1)^2}\right) + \frac{1}{2}\operatorname{arctg}\left(\frac{x}{2}\right) + C, \quad C \in \mathbb{R}$;

(j) $\frac{1}{2}\ln\left|\frac{(x+4)^{15}}{(x+2)^7}\right| + C, \quad C \in \mathbb{R}$;

(k) $\ln\left|\frac{x}{\sqrt{x^2+1}}\right| + C, \quad C \in \mathbb{R}$;

(l) $\ln\left((x-4)^2(x+1)^2\right) - \frac{3}{2(x+1)^2} + C, \quad C \in \mathbb{R}$;

(m) $\frac{2-x}{4(x^2+2)} + \ln\left(\sqrt{x^2+2}\right) - \frac{1}{4\sqrt{2}}\operatorname{arctg}\left(\frac{x}{\sqrt{2}}\right) + C, \quad C \in \mathbb{R}$;

(n) $\frac{x^2}{2} + \frac{1}{2}\ln\left|\frac{(x-1)^3(x+1)}{x^2}\right| + C, \quad C \in \mathbb{R}$;

(o) $\frac{1}{3}\left(x^3 + \ln|x3-1|\right) + C, \quad C \in \mathbb{R}$;

(p) $\frac{1}{3}x^3 - 9x - \frac{1}{9x}\frac{1}{2}\ln(x^2+9) + \frac{728}{27}\operatorname{arctg}(x) + C, \quad C \in \mathbb{R}$;

(q) $-\frac{1}{25}\ln|x-1| - \frac{3}{5(x-1)} + \frac{1}{50}\ln|x^2+4| - \frac{7}{25}\operatorname{arctg}\left(\frac{x}{2}\right) + C, \quad C \in \mathbb{R}$;

(r) $\frac{1}{12}\ln|x+2| - \frac{1}{24}\ln|x^2-2x+4| + \frac{\sqrt{3}}{12}\operatorname{arctg}\left(\frac{x-1}{\sqrt{3}}\right) + C, \quad C \in \mathbb{R}$;

(s) $-\frac{3}{4}\ln|x| - \frac{1}{4x} + \frac{13}{16}\ln|x-2| + \frac{15}{16}\ln x + 2 + C, \quad C \in \mathbb{R}$;

(t) $\ln|x| - \frac{1}{2}\ln|1+x^2| + \frac{1}{2(x^2+1)} + C, \quad C \in \mathbb{R}$;

(u) $\frac{1}{4}\left(4x + \ln|x-1| - \ln|x+1| - 2\operatorname{arctg}(x)\right) + C, \quad C \in \mathbb{R}$;

(v) $\frac{1}{2}\ln|x^2+4x+5| - \operatorname{arctg}(x+2) + C, \quad C \in \mathbb{R}$;

(w) $\frac{x^7}{7} - \frac{x^5}{5} + \frac{x^3}{3} - x + \operatorname{arctg}(x) + C, \quad C \in \mathbb{R}$;

(x) $\frac{x^3}{3} + \frac{x^2}{2} + 4x + 2\ln|x| + 5\ln|x-2| - 3\ln|x+2| + C, \quad C \in \mathbb{R}$.

3. (a) $\frac{2}{3}\left(\sqrt{x+1}\right)^3 + 2\sqrt{x+1} + C, \quad C \in \mathbb{R}$;

(b) $\frac{5}{7}\left(\sqrt[6]{x}\right)^7 - \left(\sqrt[6]{x}\right)^5 + \frac{5}{3}\left(\sqrt[6]{x}\right)^3 - 5\sqrt[6]{x} + 5\operatorname{arctg}\left(\sqrt[6]{x}\right) + C, \quad C \in \mathbb{R}$;

(c) $2\sqrt{1+\ln(x)} + C, \quad C \in \mathbb{R}$;

(d) $\frac{4}{7}\left(\sqrt[4]{e^x+1}\right)^7+\frac{4}{3}\left(\sqrt[4]{e^x+1}\right)^3+C,\quad C\in\mathbb{R}$;

(e) $2\sqrt{e^x-1}-2\operatorname{arctg}\left(\sqrt{e^x-1}\right)+C,\quad C\in\mathbb{R}$;

(f) $\ln\left|\frac{\sqrt{e^x+1}-1}{\sqrt{e^x+1}+1}\right|+C,\quad C\in\mathbb{R}$;

(g) $\frac{4}{3}\sqrt{\operatorname{arctg}^3(\sqrt{x})}+C,\quad C\in\mathbb{R}$;

(h) $e^{-x}+\frac{1}{2}\ln(1+e^{2x})+\operatorname{arctg}(e^x)+C,\quad C\in\mathbb{R}$.

4. (a) $\frac{2}{5}e^{\sqrt{5x-2}}\left(\sqrt{5x-2}-1\right)+C,\quad C\in\mathbb{R}$;

(b) $\frac{2}{5}\left(\sqrt{x+1}\right)^5-\frac{2}{3}\left(\sqrt{x+1}\right)^3+C,\quad C\in\mathbb{R}$;

(c) $\frac{1}{3}\left(\sqrt{x}+3\right)^6+\frac{6}{5}\left(\sqrt{x}+3\right)^5+C,\quad C\in\mathbb{R}$;

(d) $\frac{2\sqrt{3}}{3}\operatorname{arctg}\left(\frac{\sqrt{2x-3}}{\sqrt{3}}\right)+C,\quad C\in\mathbb{R}$;

(e) $2\ln\left(\sqrt{x+1}+1\right)+C,\quad C\in\mathbb{R}$;

(f) $2\operatorname{arctg}\left(\sqrt{x}\right)+C,\quad C\in\mathbb{R}$;

(g) $\frac{3}{5}\left(\sqrt[3]{x-1}\right)^5+\frac{3}{2}\left(\sqrt[3]{x-1}\right)^2+C,\quad C\in\mathbb{R}$;

(h) $2\left(6\sqrt[3]{x}-x\right)\cos\left(\sqrt[3]{x}\right)+6\left(\left(\sqrt[3]{x}\right)^2-2\right)\operatorname{sen}\left(\sqrt[3]{x}\right)+C,\quad C\in\mathbb{R}$;

(i) $2\sqrt{x}-2\operatorname{arctg}(\sqrt{x})+C,\quad C\in\mathbb{R}$;

(j) $\ln\sqrt{\left|\frac{\sqrt{2x+4}-2}{\sqrt{2x+4}+2}\right|}+C,\quad C\in\mathbb{R}$. (k) $\frac{1}{2}\ln\left|\frac{2-\sqrt{4-x^2}}{x}\right|+C,\quad C\in\mathbb{R}$;

(l) $\frac{1}{3}\ln\left|\frac{\sqrt{9+x^2}-3}{x}\right|+C,\quad C\in\mathbb{R}$;

(m) $\frac{x^2-25}{25x}+C,\quad C\in\mathbb{R}$;

(n) $-\sqrt{4-x^2}+C,\quad C\in\mathbb{R}$;

(o) $\frac{1}{243}\sqrt{(9x^2+49)^3}-\frac{49}{81}\sqrt{9x^2+49}+C,\quad C\in\mathbb{R}$;

(p) $\frac{(3+2x^2)\sqrt{x^2-3}}{27x^3}+C,\quad C\in\mathbb{R}$;

(q) $\frac{2}{3}\left(\sqrt{1+\ln(x)}\right)^3-2\sqrt{1-\ln(x)}+C,\quad C\in\mathbb{R}$;

(r) $2\sqrt{\operatorname{tg}(x)-1}+C,\quad C\in\mathbb{R}$;

(s) $\frac{1}{48}(2x+5)^{12}-\frac{5}{44}(2x+5)^{11}+C,\quad C\in\mathbb{R}$;

(t) $-\frac{3}{8}\left(\frac{1-x}{1+x}\right)+\frac{4}{3}\,C,\quad C\in\mathbb{R}$.

5. (a) $e^{\cos^2(x)}+C,\quad C\in\mathbb{R}$;

(b) $4\left(\frac{\sqrt{x}}{2}+\sqrt[4]{x}+\ln\left(\sqrt[4]{x}-1\right)\right)+C, \quad C\in\mathbb{R}$;

(c) $\frac{\sqrt{x^2-9}}{9x}+C, \quad C\in\mathbb{R}$;

(d) $\frac{x^3}{3}\operatorname{arctg}(x)-\frac{x^2}{6}-\frac{1}{6}\ln\left(1-x^2\right)+C, \quad C\in\mathbb{R}$;

(e) $\frac{1}{2}\ln|x|-\frac{1}{4}\ln\left(4+x^2\right)+\frac{1}{2}\operatorname{arctg}\left(\frac{x}{2}\right)+C, \quad C\in\mathbb{R}$;

(f) $\frac{2}{3}\sqrt{1+x^3}+C, \quad C\in\mathbb{R}$;

(g) $-\frac{\sqrt{1+x^2}}{x}+C, \quad C\in\mathbb{R}$;

(h) $\frac{1}{2}\left(\ln\left(x^2+1\right)-2\ln|x|+6\operatorname{arctg}(x)\right)+C, \quad C\in\mathbb{R}$;

(i) $\frac{1}{1+\operatorname{sen}(x)}+C, \quad C\in\mathbb{R}$;

(j) $\frac{x^2+1}{2}\ln\left(1+x^2\right)-\frac{x^2}{2}+C, \quad C\in\mathbb{R}$;

(k) $\operatorname{sen}(x)\ln(\operatorname{sen}(x))-\operatorname{sen}(x)+C, \quad C\in\mathbb{R}$;

(l) $\frac{1}{2}\arccos\left(\frac{2}{x}\right)+C, \quad C\in\mathbb{R}$;

(m) $\frac{1}{5}\left(1+x^2\right)^2\sqrt{1+x^2}+C, \quad C\in\mathbb{R}$;

(n) $\operatorname{arcsen}(x-1)+C, \quad C\in\mathbb{R}$;

(o) $-\frac{\cos^3(x)}{3}+\frac{2}{5}\cos^5(x)-\cos^7(x)7+C, \quad C\in\mathbb{R}$;

(p) $\ln\left|\sqrt{\frac{2+x^2}{2}}+\frac{x}{\sqrt{2}}\right|+C, \quad C\in\mathbb{R}$;

6. $g(x)=\ln\left(\frac{x^2-2x+2}{x^2}\right)+3\operatorname{arctg}(x-1)-\frac{3\pi}{2}$

7. $h(x)=2x^3+2x+1$