departamento de matemática **universidade de aveiro**

1. Determine cada um dos seguintes integrais indefinidos.

(a) $\displaystyle\int x^3\,dx$

(b) $\displaystyle\int \left(x^4+3x^2-2\right)\,dx$

(c) $\displaystyle\int \frac{1}{x^7}\,dx$

(d) $\displaystyle\int \frac{1}{\sqrt[5]{x^3}}\,dx$

(e) $\displaystyle\int \frac{\sqrt[3]{x}}{x^2}\,dx$

(f) $\displaystyle\int \left(3\sqrt{x}-\frac{2}{x^2}\right)\,dx$

(g) $\displaystyle\int \left(\frac{\sqrt{x}+5\sqrt[3]{x}}{x}\right)\,dx$

(h) $\displaystyle\int \left(\sqrt[5]{x}+\frac{6}{x^3}-\frac{2}{\sqrt{x^7}}\right)\,dx$

(i) $\displaystyle\int \left(\frac{1}{x^5}+\frac{7}{\sqrt[3]{x}}+\frac{9\sqrt{x}}{x^7}\right)\,dx$

(j) $\displaystyle\int \left(8x^4-7\sqrt[3]{x}+\frac{4}{x^5}\right)\,dx$

(k) $\displaystyle\int \frac{(x+1)^2}{x^3}\,dx$

(l) $\displaystyle\int \left(x^2+1\right)^3\,dx$

(m) $\displaystyle\int \left(\frac{3}{4x}+10e^x\right)\,dx$

(n) $\displaystyle\int e^{3x+2}\,dx$

(o) $\displaystyle\int x^3e^{3+5x^4}\,dx$

(p) $\displaystyle\int x^4e^{x^5}\,dx$

(q) $\displaystyle\int \frac{x^2}{e^{x^3}}\,dx$

(r) $\displaystyle\int \frac{e^{\frac{1}{x}}}{x^2}\,dx$

(s) $\displaystyle\int x\left(x^2+2\right)^3\,dx$

(t) $\displaystyle\int x^2\left(x^3+1\right)^4\,dx$

(u) $\displaystyle\int \frac{1}{(x+1)^2}\,dx$

(v) $\displaystyle\int \frac{1}{(3-2x)^8}\,dx$

(w) $\displaystyle\int \frac{x}{\left(3x^2-5\right)^3}\,dx$

(x) $\displaystyle\int \frac{3}{\sqrt[3]{(1-x)^5}}\,dx$

(y) $\displaystyle\int \frac{3x}{\sqrt[5]{1+5x^2}}\,dx$

(z) $\displaystyle\int \frac{8}{\sqrt{(4-3x)^5}}\,dx$

(aa) $\displaystyle\int e^x\left(2+3e^x\right)^2\,dx$

(ab) $\displaystyle\int \frac{\ln(x)}{x}\,dx$

(ac) $\displaystyle\int \frac{\sqrt[3]{\ln(x)}}{x}\,dx$

(ad) $\displaystyle\int \frac{1}{x\sqrt{\ln(x)}}\,dx$

(ae) $\displaystyle\int \frac{1}{x+3}\,dx$

(af) $\displaystyle\int \frac{x}{x^2+3}\,dx$

(ag) $\displaystyle\int \frac{x+2}{x^2+4x}\,dx$

(ah) $\displaystyle\int \frac{e^{2x}}{5+e^{2x}}\,dx$

(ai) $\displaystyle\int \frac{e^{x+1}}{1+e^x}\,dx$

(aj) $\displaystyle\int \frac{2x^2}{1+16x^3}\,dx$

(ak) $\displaystyle\int \frac{1}{x\ln(x)}\,dx$

(al) $\displaystyle\int \operatorname{sen}\left(\frac{x}{5}\right)\,dx$

(am) $\displaystyle\int \operatorname{sen}(7x)\,dx$

(an) $\displaystyle\int \frac{\operatorname{sen}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx$

(ao) $\displaystyle\int x^2\cos\left(x^3\right)\,dx$

(ap) $\displaystyle\int \cos(-3x)\,dx$

(aq) $\displaystyle\int \cos\left(\frac{3x}{4}\right)\,dx$

(ar) $\displaystyle\int \cos(x)\operatorname{sen}^2(x)\,dx$

(as) $\displaystyle\int \operatorname{sen}\left(\frac{x}{7}\right)\cos^3\left(\frac{x}{7}\right)\,dx$

(at) $\displaystyle\int \frac{\operatorname{sen}(-x)}{\cos^6(-x)}\,dx$

(au) $\displaystyle\int \frac{\cos(3x)}{\operatorname{sen}(3x)}\,dx$

(av) $\displaystyle\int \frac{\operatorname{sen}(2x)}{6-\cos(2x)}\,dx$

(aw) $\displaystyle\int \cos(5x)e^{\operatorname{sen}(5x)}\,dx$

(ax) $\displaystyle\int \frac{3\operatorname{sen}(x)}{\left(1+\cos(x)\right)^2}\,dx$

(ay) $\displaystyle\int \cos^3(x)\operatorname{sen}^3(x)\,dx$

(az) $\displaystyle\int \frac{4}{\sqrt{1-x^2}}\,dx$

(ba) $\displaystyle\int \frac{3x}{\sqrt{1-25x^4}}\,dx$

(bb) $\displaystyle\int \frac{3}{\sqrt{1-7x^2}}\,dx$

(bc) $\displaystyle\int \frac{3x^3}{\sqrt{1-x^8}}\,dx$

(bd) $\displaystyle\int \frac{1}{\sqrt{x}\sqrt{1-x}}\,dx$

(be) $\displaystyle\int \frac{9}{1+x^2}\,dx$

(bf) $\displaystyle\int \frac{3}{3x^2+1}\,dx$

(bg) $\displaystyle\int \frac{5x}{1+x^4}\,dx$

(bh) $\displaystyle\int \frac{e^x}{1+e^{2x}}\,dx$

(bi) $\displaystyle\int \frac{1}{x\left(1+\ln^2(x)\right)}\,dx$

(bj) $\displaystyle\int \frac{1}{x^2+2}\,dx$

(bk) $\displaystyle\int \frac{\operatorname{arctg}^2(x)}{1+x^2}\,dx$

(bl) $\displaystyle\int \frac{1}{\sqrt{1-x^2}\,\operatorname{arcsen}^3(x)}\,dx$

(bm) $\displaystyle\int \frac{1+x}{\sqrt{1-x^2}}\,dx$

(bn) $\displaystyle\int \frac{3x-1}{x^2+9}\,dx$

(bo) $\displaystyle\int \operatorname{tg}^2(7x)\,dx$

(bp) $\displaystyle\int \cos^2(3x)\,dx$

(bq) $\displaystyle\int \frac{\cos\left(\ln\left(x^2\right)\right)}{x}\,dx$

(br) $\displaystyle\int \operatorname{tg}^3(x)\,dx$

(bs) $\displaystyle\int \frac{6}{\sqrt{6x-x^2-5}}\,dx$

(bt) $\displaystyle\int \frac{3x-7}{x^2+4x+9}\,dx$

2. Considere a função g definida por $g(x) = \dfrac{\ln^2(x)}{x}$, com $x \in \mathbb{R}^+$.
   (a) Determine a família de todas as primitivas de g.
   (b) Indique a primitiva da função g que se anula para $x = e$.

3. Sabendo que a função h satisfaz a igualdade $\displaystyle\int h(x)\,dx = \operatorname{sen}(x) - x\cos(x) - \frac{x^2}{2} + C$, com $C \in \mathbb{R}$, determine $h\left(\dfrac{\pi}{4}\right)$.

4. Determine a função $f : \mathbb{R} \to \mathbb{R}$ tal que $f'(x) = \dfrac{2e^x}{3+e^x}$ e $f(0) = \ln(4)$.

5. Determine a função g que verifica as seguintes condições:

$$g'(x) = \frac{1}{\left(1+\operatorname{arctg}^2(x)\right)\left(1+x^2\right)} \qquad \text{e} \qquad \lim_{x\to+\infty} g(x) = 0$$

1.
(a) $\frac{x^4}{4}+C,\ C\in\mathbb{R}$;
(b) $\frac{x^5}{5}+x^3-2x+C,\ C\in\mathbb{R}$;
(c) $-\frac{1}{6x^6}+C,\ C\in\mathbb{R}$;
(d) $\frac{5}{2}\sqrt[5]{x^2}+C,\ C\in\mathbb{R}$;
(e) $-\frac{3}{2\sqrt[3]{x^2}}+C,\ C\in\mathbb{R}$;
(f) $2\sqrt{x^3}+\frac{2}{x}+C,\ C\in\mathbb{R}$;
(g) $2\sqrt{x}+15\sqrt[3]{x}+C,\ C\in\mathbb{R}$;
(h) $\frac{5}{6}\sqrt[5]{x^6}-\frac{3}{x^2}+\frac{4}{5\sqrt{x^5}}+C,\ C\in\mathbb{R}$;
(i) $-\frac{1}{4x^4}+\frac{21}{2}\sqrt[3]{x^2}-\frac{18}{11\sqrt{x^{11}}}+C,\ C\in\mathbb{R}$;
(j) $\frac{8}{5}x^5-\frac{21}{4}\sqrt[3]{x^4}-\frac{1}{x^4}+C,\ C\in\mathbb{R}$;
(k) $\ln|x|-\frac{2}{x}-\frac{1}{2x^2}+C,\ C\in\mathbb{R}$;
(l) $\frac{x^7}{7}+\frac{3}{5}x^5+x^3+x+C,\ C\in\mathbb{R}$;
(m) $\frac{3}{4}\ln|x|+10e^x+C,\ C\in\mathbb{R}$;
(n) $\frac{1}{3}e^{3x+2}+C,\ C\in\mathbb{R}$;
(o) $\frac{1}{20}e^{3+5x^4}+C,\ C\in\mathbb{R}$;
(p) $\frac{1}{5}e^{x^5}+C,\ C\in\mathbb{R}$;
(q) $-\frac{1}{3e^{x^3}}+C,\ C\in\mathbb{R}$;
(r) $-e^{\frac{1}{x}}+C,\ C\in\mathbb{R}$;
(s) $\frac{1}{8}(x^2+2)^4+C,\ C\in\mathbb{R}$;
(t) $\frac{1}{15}(x^3+1)^5+C,\ C\in\mathbb{R}$;
(u) $-\frac{1}{x+1}+C,\ C\in\mathbb{R}$;
(v) $\frac{1}{14(3-2x)^7}+C,\ C\in\mathbb{R}$;
(w) $-\frac{1}{12(3x^2-5)^2}+C,\ C\in\mathbb{R}$;
(x) $\frac{9}{2\sqrt[3]{(1-x)^2}}+C,\ C\in\mathbb{R}$;
(y) $\frac{3}{8}\sqrt[5]{(1+5x^2)^4}+C,\ C\in\mathbb{R}$;
(z) $\frac{16}{9\sqrt{(4-3x)^3}}+C,\ C\in\mathbb{R}$;
(aa) $\frac{1}{9}(2+3e^x)^3+C,\ C\in\mathbb{R}$;
(ab) $\frac{\ln^2(x)}{2}+C,\ C\in\mathbb{R}$;
(ac) $\frac{3}{4}\sqrt[3]{\ln^4(x)}+C,\ C\in\mathbb{R}$;
(ad) $2\sqrt{\ln(x)}+C,\ C\in\mathbb{R}$;
(ae) $\ln|x+3|+C,\ C\in\mathbb{R}$;
(af) $\frac{1}{2}\ln|x^2+3|+C,\ C\in\mathbb{R}$;
(ag) $\frac{1}{2}\ln|x^2+4x|+C,\ C\in\mathbb{R}$;
(ah) $\frac{1}{2}\ln|5+e^{2x}|+C,\ C\in\mathbb{R}$;
(ai) $e\ln|1+e^x|+C,\ C\in\mathbb{R}$;
(aj) $\frac{1}{24}\ln|1+16x^3|+C,\ C\in\mathbb{R}$;
(ak) $\ln|\ln(x)|+C,\ C\in\mathbb{R}$;
(al) $-5\cos(\frac{x}{5})+C,\ C\in\mathbb{R}$;

(am) $-\frac{1}{7}\cos(7x)+C,\ C\in\mathbb{R}$;

(an) $-2\cos(\sqrt{x})+C,\ C\in\mathbb{R}$;

(ao) $\frac{1}{3}\operatorname{sen}(x^3)+C,\ C\in\mathbb{R}$;

(ap) $-\frac{1}{3}\operatorname{sen}(-3x)+C,\ C\in\mathbb{R}$;

(aq) $\frac{4}{3}\operatorname{sen}(\frac{3x}{4})+C,\ C\in\mathbb{R}$;

(ar) $\frac{1}{3}\operatorname{sen}^3(x)+C,\ C\in\mathbb{R}$;

(as) $-\frac{7}{4}\cos^4\left(\frac{x}{7}\right)+C,\ C\in\mathbb{R}$;

(at) $\frac{-1}{5\cos^5(-x)}+C,\ C\in\mathbb{R}$;

(au) $\frac{1}{3}\ln|\operatorname{sen}(3x)|+C,\ C\in\mathbb{R}$;

(av) $\frac{1}{2}\ln|6-\cos(2x)|+C,\ C\in\mathbb{R}$;

(aw) $\frac{1}{5}e^{\operatorname{sen}(5x)}+C,\ C\in\mathbb{R}$;

(ax) $\frac{-3}{1+\cos(x)}+C,\ C\in\mathbb{R}$;

(ay) $-\frac{1}{4}\cos^4(x)+\frac{1}{6}\cos^6(x)C,\ C\in\mathbb{R}$;

(az) $4\operatorname{arcsen}(x)+C,\ C\in\mathbb{R}$;

(ba) $\frac{3}{10}\operatorname{arcsen}(5x^2)+C,\ C\in\mathbb{R}$;

(bb) $\frac{3}{\sqrt{7}}\operatorname{arcsen}(\sqrt{7}x)+C,\ C\in\mathbb{R}$;

(bc) $\frac{3}{4}\operatorname{arcsen}(x^4)+C,\ C\in\mathbb{R}$;

(bd) $2\operatorname{arcsen}(\sqrt{x})+C,\ C\in\mathbb{R}$;

(be) $9\operatorname{arctg}(x)+C,\ C\in\mathbb{R}$;

(bf) $\frac{3}{\sqrt{3}}\operatorname{arctg}(\sqrt{3}x)+C,\ C\in\mathbb{R}$;

(bg) $\frac{5}{2}\operatorname{arctg}(x^2)+C,\ C\in\mathbb{R}$;

(bh) $\operatorname{arctg}(e^x)+C,\ C\in\mathbb{R}$;

(bi) $\operatorname{arctg}(\ln(x))+C,\ C\in\mathbb{R}$;

(bj) $\frac{\sqrt{2}}{2}\operatorname{arctg}\left(\frac{x}{\sqrt{2}}\right)+C,\ C\in\mathbb{R}$;

(bk) $\frac{1}{3}\operatorname{arctg}^3(x)+C,\ C\in\mathbb{R}$;

(bl) $\frac{-1}{2\operatorname{arcsen}^2(x)}+C,\ C\in\mathbb{R}$;

(bm) $\operatorname{arcsen}(x)-\sqrt{1-x^2}+C,\ C\in\mathbb{R}$;

(bn) $\frac{3}{2}\ln|x^2+9|-\frac{1}{3}\operatorname{arctg}\left(\frac{x}{3}\right)+C,\ C\in\mathbb{R}$;

(bo) $\frac{1}{7}\operatorname{tg}(7x)-x+C,\ C\in\mathbb{R}$;

(bp) $\frac{1}{2}x+\frac{1}{12}\operatorname{sen}(6x)+C,\ C\in\mathbb{R}$;

(bq) $\frac{1}{2}\operatorname{sen}\left(\ln\left(x^2\right)\right)+C,\ C\in\mathbb{R}$;

(br) $\ln|\cos(x)|-\sec^2(x)+C,\ C\in\mathbb{R}$.

(bs) $6\operatorname{arcsen}\left(\frac{x-3}{2}\right)+C,\ C\in\mathbb{R}$;

(bt) $\frac{3}{2}\ln|x^2+4x+9|-\frac{\sqrt{5}}{5}\operatorname{arctg}\left(\frac{x+2}{\sqrt{5}}\right)+C,\ C\in\mathbb{R}$.

2. (a) $\frac{\ln^3(x)}{3}+C,\ C\in\mathbb{R}$; (b) $G(x)=\frac{\ln^3(x)}{3}-\frac{1}{3}$.

3. $\frac{\pi}{8}\left(\sqrt{2}-2\right)$.
4. $f(x) = 2\ln(e^x+3) - \ln(4)$.
5. $g(x) = \operatorname{arctg}\left(\operatorname{arctg}(x)\right) - \operatorname{arctg}\left(\frac{\pi}{2}\right)$.