

$$S = \sqrt{\frac{\sum (X_i - \bar{X})^2}{n-1}} = \sqrt{\frac{1284 - 6 \times 14.33^2}{6-1}} = \sqrt{10.38} = 3.22 \#$$

(2)

$$1 - \alpha = 0.9, \quad \frac{\alpha}{2} = 0.05, \quad n-1 = 5$$

$$\chi^2_{\frac{\alpha}{2}}(n-1) = \chi^2_{0.05}(5) = 11.07$$

$$\chi^2_{1-\frac{\alpha}{2}}(n-1) = \chi^2_{0.95}(5) = 1.15$$

$$\left(\sqrt{\frac{(n-1)S^2}{\chi^2_{\frac{\alpha}{2}}(n-1)}}, \sqrt{\frac{(n-1)S^2}{\chi^2_{1-\frac{\alpha}{2}}(n-1)}} \right) = \left(\sqrt{\frac{5 \times 10.38}{11.07}}, \sqrt{\frac{5 \times 10.38}{1.15}} \right) = (2.17, 6.72) \#$$

20.

(2)

$$1 - \alpha = 0.9, \quad \frac{\alpha}{2} = 0.05, \quad n-1 = 8$$

$$\chi^2_{\frac{\alpha}{2}}(n-1) = \chi^2_{0.05}(8) = 15.51$$

$$\chi^2_{1-\frac{\alpha}{2}}(n-1) = \chi^2_{0.95}(8) = 2.73$$

$$\left(\sqrt{\frac{8 \times 9.27^2}{15.51}}, \sqrt{\frac{8 \times 9.27^2}{2.73}} \right) = \left(\sqrt{\frac{687.46}{15.51}}, \sqrt{\frac{687.46}{2.73}} \right) = (6.66, 15.87) \#$$

(3)

$$F_{\frac{\alpha}{2}}(n_1-1, n_2-1) = F_{0.05}(8, 8) = 3.44$$

$$F_{1-\frac{\alpha}{2}}(n_1-1, n_2-1) = F_{0.95}(8, 8) = \frac{1}{F_{0.05}(8, 8)} = 0.29$$

$$\left(\frac{S_1^2}{S_2^2} \times \frac{1}{F_{\frac{\alpha}{2}}(n_1-1, n_2-1)}, \frac{S_1^2}{S_2^2} \times \frac{1}{F_{1-\frac{\alpha}{2}}(n_1-1, n_2-1)} \right) = \left(\frac{9.27^2}{21.15^2} \times \frac{1}{3.44}, \frac{9.27^2}{21.15^2} \times \frac{1}{0.29} \right) = (0.06, 0.66) \#$$

(1) $\sigma_1^2 \neq \sigma_2^2$

$$v = \frac{\left(\frac{9.27^2}{9} + \frac{21.15^2}{9} \right)}{\left(\frac{9.27^2}{8} + \frac{21.15^2}{8} \right)} = 10.96 \approx 11$$

$$\begin{aligned} (\bar{x} - \bar{y}) \pm t_{\frac{\alpha}{2}}(v) \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} &= (7.67 - 6.78) \pm t_{0.025}(11) \sqrt{\frac{9.27^2}{9} + \frac{21.15^2}{9}} \\ &= 0.89 \pm 2.201 \times 7.7 \\ &= 0.89 \pm 16.95 \\ &\Rightarrow (-16.06, 17.84) \# \end{aligned}$$