$E(X_i) = u \quad , \quad V(X_i) = \sigma^2 = E(X_i^2) - u$

則 $E(\overline{X}) = u \quad , \quad V(\overline{X}) = \frac{\sigma^2}{n} = E(\overline{X}^2) - u^2$

$$E(\hat{\theta}_1) = E\left(\frac{\sum_{i=1}^{n}(X_i - \overline{X})^2}{n}\right) = \frac{1}{n} E\left(\sum_{i=1}^{n} X_i^2 - n\overline{X}^2\right)$$

$$= \frac{1}{n}\left(n\sigma^2 + nu^2 - \sigma^2 - nu^2\right) = \frac{n-1}{n}\sigma^2$$

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$$E(\hat{\theta}_2) = E\left(\frac{\sum_{i=1}^{n}(X_i - \overline{X})^2}{n-1}\right) = \frac{1}{n-1} E\left(\sum_{i=1}^{n} X_i^2 - n\overline{X}^2\right)$$

$$= \frac{1}{n-1}\left(n\sigma^2 + nu^2 - \sigma^2 - nu^2\right) = \sigma^2$$

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因此 $\hat{\theta}_1 = \sum_{i=1}^{n}(X_i - \overline{X})^2 / n$ 為母体變異數 σ^2 之偏誤估計量

$\hat{\theta}_2 = \sum_{i=1}^{n}(X_i - \overline{X})^2 / (n-1)$ 為母体變異數 σ^2 之不偏估計量