6.4

設 $E(X_i)=u$, $V(X_i)=\sigma^2=E(X_i^2)-u^2$

則 $E(\bar{X})=u$, $V(\bar{X})=\frac{\sigma^2}{n}=E(\bar{X}^2)-u^2$

$$E(\theta_1)=E\left(\frac{\sum_{i=1}^{n}(X_i-\bar{X})^2}{n}\right)=\frac{1}{n}E\left(\sum_{i=1}^{n}X_i^2-n\bar{X}^2\right)$$

$$=\frac{1}{n}(n\sigma^2+nu^2-\sigma^2-nu^2)=\frac{n-1}{n}\sigma^2$$

$$E(\theta_2)=E\left(\frac{\sum_{i=1}^{n}(X_i-\bar{X})^2}{n-1}\right)=\frac{1}{n-1}E\left(\sum_{i=1}^{n}X_i^2-n\bar{X}^2\right)$$

$$=\frac{1}{n-1}(n\sigma^2+nu^2-\sigma^2-nu^2)=\sigma^2$$

因此，$\theta_2=\sum_{i=1}^{n}(X_i-\bar{X})^2/(n-1)$ 為母體變異數 σ^2 之不偏估計量

而 $\theta_1=\sum_{i=1}^{n}(X_i-\bar{X})^2/n$ 為母體變異數 σ^2 之偏誤估計量。

習題1.

(1) $t_{0.025}(10)=2.228$

(2) $t_{0.95}(8)=-1.860$

(3) $\chi^2_{0.05}(12)=21.08$

(4) $\chi^2_{\alpha}(15)=7.26$ $\alpha=0.95$

(5) $\chi^2_{0.95}(10)=3.94$

(6) $F_{0.05}(5,8)=3.69$

(7) $F_{0.95}(6,7)=3.87$

(8) $F_{\alpha}(6,6)=4.28$ $\alpha=0.05$