因此，估計量 $\overline{X}$ 及 $\sum_{i=1}^{n}\varpi_i X_i / \sum_{i=1}^{n}\varpi_i$ 皆為母體參數 μ 之不偏估計量

例 6.4

假設統計量 $\hat{\theta}_1=\sum_{i=1}^{n}(X_i-\overline{X})^2/n$，$\hat{\theta}_2=\sum_{i=1}^{n}(X_i-\overline{X})^2/(n-1)$ 分別為母體變異數 σ^2 之估計量，請問何者滿足不偏性？

$V(X_i)=\sigma^2=E(X_i^2)-\mu^2$

$E(X_i^2)=\sigma^2+\mu^2$

解 依 $E(X_i)=\mu$，$V(X_i)=\sigma^2=E\left(X_i^2\right)-\mu^2$，

$E(\hat{\theta}_1)=E\left(\dfrac{\sum_{i=1}^{n}(X_i-\bar{X})^2}{n}\right)=\dfrac{1}{n}E\left(\sum_{i=1}^{n}X_i^2-n\bar{X}^2\right)$

$E(\bar{X})=\mu$

$V(\bar{X})=\dfrac{\sigma^2}{n}=E(\bar{X}^2)$

$=\dfrac{1}{n}(n\sigma^2+n\mu^2-\sigma^2-n\mu^2)=\dfrac{n-1}{n}\sigma^2$

$\Rightarrow E(\bar{X}^2)=\dfrac{\sigma^2}{n}+\mu^2$

$E(\hat{\theta}_2)=E\left(\dfrac{\sum_{i=1}^{n}(X_i-\bar{X}^2)}{n-1}\right)=\dfrac{1}{n-1}E\left(\sum_{i=1}^{n}X_i^2-n\bar{X}^2\right)=\dfrac{1}{n-1}(n\sigma^2+n\mu^2-\sigma^2-n\mu^2)=\sigma^2$