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5. 算下列生產函數的替代彈性

(a)

$$F(k, L) = k^{\frac{1}{2}} L^{\frac{1}{2}}$$

$$\text{替代彈性 } \sigma = \frac{\frac{d(\frac{L}{k}) / (\frac{L}{k})}{\frac{d(MRTS)}{d(\frac{L}{k})}} = \frac{\frac{d \ln(\frac{L}{k})}{\frac{d \ln(MRTS)}{d \ln(\frac{L}{k})}} = 1$$

(b)

$$F(k, L) = 2k + L$$

$$\text{替代彈性 } \sigma = \frac{\frac{d \ln(\frac{L}{k})}{\frac{d \ln(MRTS)}{d \ln(\frac{L}{k})}} = \frac{d \ln(\frac{L}{k})}{d \ln(\frac{1}{2})} = \infty$$

因為分母為常數，微分 = 0

8.

假設生產函數的型式 $Q = \alpha k + \beta L$ 其中 k 為資本， L 為勞動， Q 為產出。考慮生產函數三個敘述：

(1) 函數呈現固定規模報酬

(2) 函數呈現資本與勞動的邊際生產力遞減

(3) 函數呈現固定的技術替代率

選出正確敘述：(1), (3)

9. 判斷生產函數規模報酬的屬性

(A)

$$Q = (L^{\alpha} + K^{\alpha})^{\beta}$$

(B)

$$\ln Q = 5 + 0.5 \ln L + 0.7 \ln K$$

(C)

$$Q = [\text{Min}\{aL, bK\}]^{\alpha}$$

$$\Rightarrow (\lambda L^{\alpha} + \lambda K^{\alpha})^{\beta}$$

$$\Rightarrow \lambda (L^{\alpha} + K^{\alpha})^{\beta}$$

$$\Rightarrow \lambda Q$$

規模報酬固定

$$b. Q = f(L, K) = L^{\alpha} K^{\beta}, \alpha, \beta > 0$$

$$(1) \text{ 產出彈性: } AP_L = \frac{Q}{L} = \frac{L^{\alpha} K^{\beta}}{L} = L^{\alpha-1} K^{\beta}$$

$$MP_L = \frac{\partial Q}{\partial L} = \alpha L^{\alpha-1} K^{\beta}$$

$$AP_K = \frac{Q}{K} = \frac{L^{\alpha} K^{\beta}}{K} = L^{\alpha} K^{\beta-1}$$

$$MP_K = \frac{\partial Q}{\partial K} = \beta L^{\alpha} K^{\beta-1}$$

$$\text{勞動產出彈性: } \frac{MP_L}{AP_L} = \frac{\alpha L^{\alpha-1} K^{\beta}}{L^{\alpha-1} K^{\beta}} = \alpha$$

$$\text{資本產出彈性: } \frac{MP_K}{AP_K} = \frac{\beta L^{\alpha} K^{\beta-1}}{L^{\alpha} K^{\beta-1}} = \beta$$

(2) 生產力彈性:

$$Q = f(\lambda L, \lambda K) = \lambda^{\alpha+\beta} L^{\alpha} K^{\beta}$$

$$\downarrow$$

$$\alpha + \beta$$

(3) 替代彈性:

$$MRTS = \frac{MP_L}{MP_K} = \frac{\alpha L^{\alpha-1} K^{\beta}}{\beta L^{\alpha} K^{\beta-1}} = \frac{\alpha}{\beta} \frac{K}{L}$$

$$\frac{d \ln(\frac{K}{L})}{d \ln(MRTS)} = \frac{d \ln(\frac{K}{L})}{d \ln(\frac{\alpha}{\beta} \frac{K}{L})} = 1$$