$$\begin{cases} \text{Max } U = X^{\frac{2}{3}} Y^{\frac{1}{3}} \\ 300 = 10X + 20Y \end{cases}$$

$X = 20 \quad Y = 50$

$$\begin{cases} \text{Max } U = X^{\frac{2}{3}} Y^{\frac{1}{3}} \\ 300 = 20X + 20Y \end{cases}$$

$$\frac{2Y}{X} = \frac{P_X}{P_Y} = \frac{20}{20} = 1$$

$$Y = \frac{1}{2}X$$

$$\Rightarrow X = 10 \quad Y = 5$$

$$U = X^{\frac{2}{3}} Y^{\frac{1}{3}} = (20)^{\frac{2}{3}} (5)^{\frac{1}{3}} = (2000)^{\frac{1}{3}}$$

$Y = \frac{1}{2}X$ 代入 $U = (2000)^{\frac{1}{3}}$

$$U = X^{\frac{2}{3}} Y^{\frac{1}{3}} = \left(\frac{1}{2}X^3\right)^{\frac{1}{3}} = (2000)^{\frac{1}{3}}$$

$$X = (4000)^{\frac{1}{3}} \approx 15.874 \qquad Y = (500)^{\frac{1}{3}}$$

邊際：

$$(X, Y) = (20, 5) \sim \left[(4000)^{\frac{1}{3}}, (500)^{\frac{1}{3}}\right]$$

X 的邊際 $= (4000)^{\frac{1}{3}} - 20 < 0$

所得：

$$(X, Y) = \left[(4000)^{\frac{1}{3}}, (500)^{\frac{1}{3}}\right] \sim (10, 5)$$