

Week 3 3/5

5.

a. $F(K, L) = K^{\frac{1}{2}} L^{\frac{1}{2}}$

$$\Rightarrow \frac{MP_L}{MP_K} = |MRTS|$$

$$\Rightarrow \frac{0.5 L^{\frac{1}{2}} + \frac{1}{2} L^{-\frac{1}{2}} K^{\frac{1}{2}}}{\frac{1}{2} K^{\frac{1}{2}} L^{\frac{1}{2}} + K^{\frac{1}{2}} \times 0} = \frac{K}{L}$$

$$\sigma = \frac{d \ln(\frac{K}{L})}{d \ln MRTS} = \frac{d \ln(\frac{K}{L})}{d \ln(\frac{K}{L})} = 1$$

b.

$$F(K, L) = 2K + L$$

$$\Rightarrow \frac{MP_L}{MP_K} = |MRTS| = \frac{1}{2}$$

$$\sigma = \frac{d \ln(\frac{K}{L})}{d \ln MRTS} = \frac{d \ln(\frac{K}{L})}{d \ln(\frac{1}{2})} = \infty$$

8. 生產函數 $Q = 3K + 2L$

(1) 函數呈現固定規模報酬

$$\Rightarrow Q(nK, nL) = nQ$$

$$\Rightarrow 3(nK) + 2(nL) = nQ$$

$$\Rightarrow n(3K + 2L) = nQ \text{ (合理)}$$

(2) 生產邊際資本，勞動生產力遞減

$$\Rightarrow MP_L = 2 \text{ (不合)}$$

$$\Rightarrow MP_K = 3$$

(3) 函數呈現固定的技術替代率

$$\Rightarrow \frac{MP_L}{MP_K} = \frac{2}{3} \text{ (合理)}$$

(1)(3) 正確敘述

9. 求規模報酬屬性

(A) $q = (L^\alpha + K^\alpha)^\beta$

$$\Rightarrow (nL, nK) = (nL)^\alpha + (nK)^\alpha)^\beta$$

$$= [n^\alpha (L^\alpha + K^\alpha)]^\beta$$

$$[n^\alpha (L^\alpha + K^\alpha)]^\beta > nq \Rightarrow \text{為 IRTS}$$

(B) $\ln q = 5 + 0.5 \ln L + 0.2 \ln K$

$$\Rightarrow \ln q = (L, K) = 5 + 0.5 \ln L + 0.2 \ln K$$

$$= \lambda \ln q + 0.7 \ln \lambda$$

$$f(\lambda L, \lambda K) > \lambda \ln q \Rightarrow \text{為 IRTS}$$

(C)

$$q = [\min\{aL, bK\}]^\alpha$$

$$f(\lambda L, \lambda K) = \min[\alpha \lambda L, b \lambda K]^\alpha$$

$$\Rightarrow \lambda \min[aL, bK]^\alpha$$

$$f(\lambda L, \lambda K) = \lambda f(K, L)$$

$$\Rightarrow \text{為 CRTS}$$

$$Q = (K, L)$$

$$\Rightarrow f(nK, nL) > n f(K, L) \text{ 遞增}$$

$$f(nK, nL) = n f(K, L) \text{ 固定}$$

$$f(nK, nL) < n f(K, L) \text{ 遞減}$$