

$$5. a. F(K, L) = K^{\frac{1}{2}} L^{\frac{1}{2}}$$

$$\Rightarrow \frac{MP_L}{MP_K} = |MRTS|$$

$$\Rightarrow \frac{0.5 L^{\frac{1}{2}} + 0.5 L^{-\frac{1}{2}} K^{\frac{1}{2}}}{0.5 K^{\frac{1}{2}} L^{-\frac{1}{2}} + 0.5 K^{-\frac{1}{2}} L^{\frac{1}{2}}} = \frac{0.5 L^{\frac{1}{2}} K^{\frac{1}{2}}}{0.5 K^{\frac{1}{2}} L^{\frac{1}{2}}} = K/L$$

$$\delta = \frac{d \ln(K/L)}{d \ln MRTS} = \frac{d \ln(K/L)}{d \ln(K/L)} = 1$$

$$b. F(K, L) = 2\sqrt{L} + L$$

$$\Rightarrow \frac{MP_L}{MP_K} = |MRTS| = \frac{1}{2}$$

$$\delta = \frac{d \ln(K/L)}{d \ln MRTS} = \frac{d \ln(K/L)}{d \ln(1/2)} = \infty$$

8. 生產函數: $Q = 3K + 2L$

(1) 函數呈現固定規模報酬

$$\Rightarrow Q(nK, nL) = nQ$$

$$\Rightarrow 3(nK) + 2(nL) = nQ$$

$$\Rightarrow n(3K + 2L) = nQ \quad (\text{合理})$$

(2) 生產函數與資本、勞動力生產力遞增

$$\Rightarrow MP_L = 2 \quad (\text{合理})$$

$$\Rightarrow MP_K = 3$$

9. 求規模報酬性質

$$(A) Q = (L^\alpha + K^\alpha)^\beta$$

$$Q(L, K) = (L^\alpha + K^\alpha)^\beta$$

$$\Rightarrow Q(\lambda L, \lambda K) = (\lambda L^\alpha + \lambda K^\alpha)^\beta$$

$$= [\lambda^\alpha (L^\alpha + K^\alpha)]^\beta$$

$$[\lambda^\alpha (L^\alpha + K^\alpha)]^\beta > \lambda Q \Rightarrow \text{為 IRTS}$$

(B)

$$(C) Q = [\min\{\alpha L, bK\}]^\alpha$$

$$f(\lambda L, \lambda K) = \min[\alpha \lambda L, b \lambda K]^\alpha$$

$$\Rightarrow \lambda \min[\alpha L, bK]^\alpha$$

$$f(\lambda L, \lambda K) = \lambda f(K, L)$$

\Rightarrow 為 CRTS

$$(B) \ln Q = 5 + 0.5 \ln L + 0.2 \ln K$$

$$\Rightarrow \ln Q(L, K) = 5 + 0.5 \ln L + 0.2 \ln K$$

$$\Rightarrow (\lambda L, \lambda K) = 5 + 0.5 \ln(\lambda L) + 0.2 \ln(\lambda K)$$

$$= 5 + 0.5[\ln \lambda + \ln L] + 0.2[\ln \lambda + \ln K]$$

$$= 5 + 0.5 \ln L + 0.2 \ln K + 0.7 \ln \lambda$$

$$= \lambda \ln Q + 0.7 \ln \lambda$$

$$f(\lambda L, \lambda K) > \lambda \ln Q \Rightarrow \text{為 IRTS}$$

α, β 不知數字大小

$$\Rightarrow f(nK, nL) > n f(K, L) \text{ 遞增}$$

$$f(nK, nL) = n f(K, L) \text{ 固定}$$

$$f(nK, nL) < n f(K, L) \text{ 遞減}$$