

一、長期成本 \rightarrow 生產函數為 $Q = 10L^{0.5}K^{0.5}$, 且 $w=r=10$

(A) 等成本函數方程式

$$\begin{aligned} \Rightarrow LTC &= wL + rK \\ &= 10L + 10K \end{aligned}$$

(B) 求 MRTS 的數值

$$\begin{aligned} |MRTS| &= \frac{MP_L}{MP_K} \\ &= \frac{10 \times 0.5 L^{-0.5} K^{0.5}}{10 \times 0.5 L^{0.5} K^{-0.5}} \\ &= K/L \end{aligned}$$

(C) 等產量曲線會凸向原點嗎?

會, 生產中, 設備能替代勞力, 但無法替代所有勞力, 故凸向原點, 等產量曲線的斜率一般隨著投入要素的量的增加而遞減。

(D) 求條件要素總需求函數

$$P \cdot Y = wL + rK = \pi$$

$$\Rightarrow 10(10L^{0.5}K^{0.5}) - wL - rK = \pi$$

$$\frac{\partial \pi}{\partial L} = 0: 5L^{0.5}K^{0.5} = w$$

$$\frac{\partial \pi}{\partial K} = 0: 5L^{0.5}K^{-0.5} = r$$

$$\frac{K}{L} = \frac{w}{r}$$

$$\frac{K}{L} = \frac{10}{10} = 1$$

$$K = \frac{w}{r}L \text{ 代入 } (1)$$

$$L = \frac{r}{w}K \text{ 代入 } (2)$$

$$\begin{aligned} L^* &= 5 \times 10 \times \left(\frac{w}{r}\right)^{\frac{1}{2}} \\ &= 5 \times 10 \times \left(\frac{10}{10}\right)^{\frac{1}{2}} \\ &= 5 \times 10 = 50 \end{aligned}$$

$$\begin{aligned} K^* &= 5 \times \left(\frac{r}{w}\right)^{\frac{1}{2}} \times K^{\frac{1}{2}} \\ &= 5 \times \left(\frac{10}{10}\right)^{\frac{1}{2}} \times K^{\frac{1}{2}} \\ &= 5 \times 10 = 50 \end{aligned}$$

(E) 生產一單位的最低成本為多少?

$$\frac{K}{L} = \frac{10}{10}$$

$$\Rightarrow 10K = 10L$$

$$\Rightarrow K = L$$

$$10 = L^{0.5}K^{0.5}$$

$$\Rightarrow 10 = L^{0.5}L^{0.5}$$

$$\Rightarrow L = 10, K = 10$$

$$\begin{aligned} \text{Min cost} &= LTC = 10 \times 10 + 10 \times 10 \\ &= 200 \end{aligned}$$

(F) 求總成本、平均成本、邊際成本函數

$$LTC = w \cdot L(w, r, Q) + r \cdot K(w, r, Q) = wL^* + rK^* = w \cdot 5 \left(\frac{w}{r}\right)^{\frac{1}{2}} + r \cdot 5 \left(\frac{r}{w}\right)^{\frac{1}{2}} = 5 \left[w^{\frac{1}{2}} \cdot w^{\frac{1}{2}} \cdot r^{-\frac{1}{2}} + r^{\frac{1}{2}} \cdot r^{\frac{1}{2}} \cdot w^{-\frac{1}{2}} \right]$$

$$ATC = \frac{LTC}{Q} = \frac{LTC}{10L^{0.5}K^{0.5}} = \frac{5[w^{\frac{1}{2}} \cdot w^{\frac{1}{2}} \cdot r^{-\frac{1}{2}} + r^{\frac{1}{2}} \cdot r^{\frac{1}{2}} \cdot w^{-\frac{1}{2}}]}{10L^{0.5}K^{0.5}} = ?$$

$$MC = \frac{LTC}{dQ} = \frac{LTC}{dQ} = ?$$

week 4

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