432 Class 9 Slides

github.com/THOMASELOVE/2019-432

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Setup

```
library(skimr); library(broom); library(janitor)
library(Hmisc); library(rms)
library(gridExtra); library(GGally)
library(tidyverse)
```

Today's Materials (Chapters 9/10 in the Notes)

- The `maleptsd` data
- Using `ols` to fit a linear model
    - ANOVA in `ols`
    - Plot Effects with `summary` and `Predict`
    - Validating summary statistics like R^2
    - Nomogram
    - Evaluating Calibration
- Spending Degrees of Freedom on Non-Linearity
    - The Spearman ρ^2 (rho-squared) plot
- Building Non-Linear Predictors with
    - Polynomial Functions
    - Splines, including Restricted Cubic Splines

The `maleptsd` data: Background and Exploration

The `maleptsd` data

The `maleptsd` file on our web site contains information on PTSD (post traumatic stress disorder) symptoms following childbirth for 64 fathers[1]. There are ten predictors and the response is a measure of PTSD symptoms. The raw, untransformed values (`ptsd_raw`) are right skewed and contain zeros, so we will work with a transformation, specifically, `ptsd = log(ptsd_raw + 1)` as our outcome, which also contains a lot of zeros.

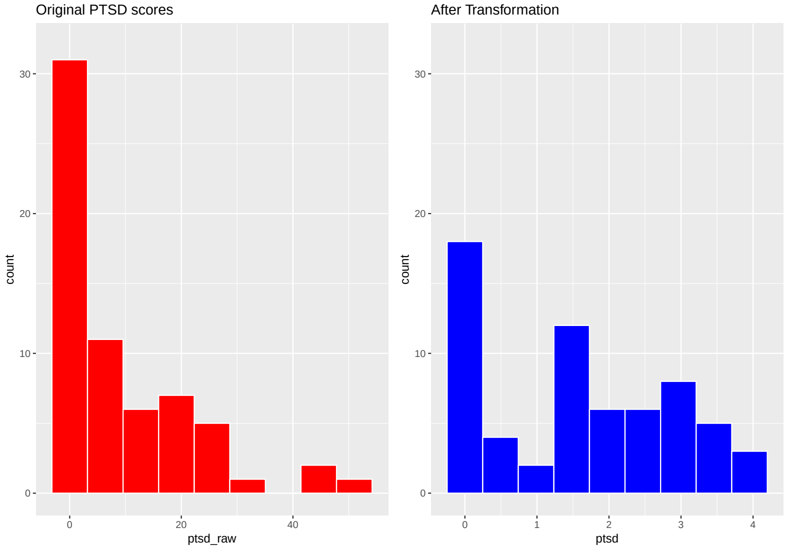
```
maleptsd <- read_csv("data/maleptsd.csv") %>%
    clean_names() %>%
    mutate(ptsd = log(ptsd_raw + 1))
```

[1] Source: Ayers et al. 2007 *J Reproductive and Infant Psychology*. The data are described in more detail in Wright DB and London K (2009) *Modern Regression Techniques Using R* Sage Publications.

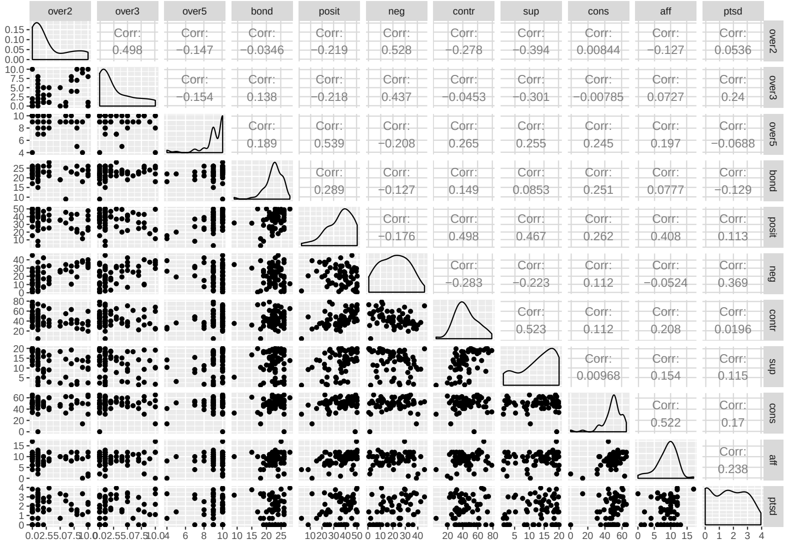
Skimming the `maleptsd` data

```
> maleptsd %>% select(-id, -ptsd_raw) %>% skim()
Skim summary statistics
 n obs: 64
 n variables: 11

-- Variable type:numeric ------------------------------------------------------------
 variable missing complete  n  mean    sd  p0   p25   p50   p75  p100     hist
      aff       0       64 64  8.84  3.08 0     7     9.5  11    17
     bond       0       64 64 22.52  3.07 9    21    23    24.25 28
     cons       0       64 64 48.98 11.15 0    45.75 51    55    65
    contr       0       64 64 44.2  14.84 4.5  35.1  41.75 53.98 78.5
      neg       0       64 64 21.09 11.6  0.7  11.17 20.65 30.42 45.4
    over2       0       64 64  2.8   3.34 0     0     1     5    10
    over3       0       64 64  2.72  3.13 0     0     1     5    10
    over5       0       64 64  9.12  1.34 4     9     9.5  10    10
    posit       0       64 64 35.44 11.02 2.5  27.05 37    43.35 50.1
     ptsd       0       64 64  1.59  1.27 0     0     1.61  2.74  3.95
      sup       0       64 64 13     5.87 1.2   9.28 14.25 18.3  20
```

Transformation of Outcome



Scatterplot Matrix



Using `ols` to fit a Two-Predictor Model

- `ols` is part of the `rms` package, by Frank Harrell and colleagues.
- A detailed discussion of `ols` is found in Chapter 10 of the Course Notes. We'll sketch out some key ideas now in a simple example, and add some more details later.

```
dd <- datadist(maleptsd)
options(datadist = "dd")

mod_first <- ols(ptsd ~ over2 + over3, data = maleptsd,
                 x = TRUE, y = TRUE)
```

Contents of `mod_first`?

```
> mod_first
Linear Regression Model

 ols(formula = ptsd ~ over2 + over3, data = maleptsd)

                Model Likelihood     Discrimination
                   Ratio Test           Indexes
 Obs        64    LR chi2      4.18    R2         0.063
 sigma1.2500      d.f.            2    R2 adj     0.033
 d.f.       61    Pr(> chi2) 0.1235    g          0.345

 Residuals

       Min        1Q     Median        3Q       Max
 -2.259244 -1.337198  0.008866  1.140664  2.333183


           Coef    S.E.   t     Pr(>|t|)
 Intercept  1.3733 0.2202  6.24 <0.0001
 over2     -0.0333 0.0544 -0.61 0.5425
 over3      0.1149 0.0579  1.98 0.0518
```

ANOVA for `mod_first` fit by `ols`

```r
anova(mod_first)
```

```
                Analysis of Variance          Response: ptsd

 Factor     d.f. Partial SS MS        F    P
 over2       1    0.5862441 0.5862441 0.38 0.5425
 over3       1    6.1458656 6.1458656 3.93 0.0518
 REGRESSION  2    6.4382526 3.2191263 2.06 0.1362
 ERROR      61   95.3127887 1.5625047
```

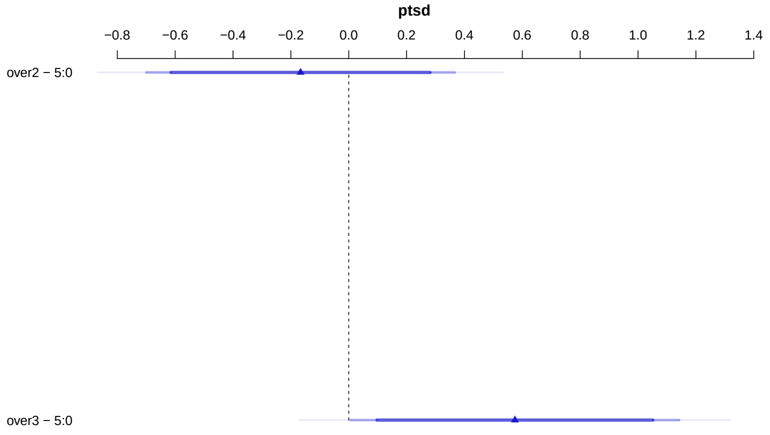
summary for `mod_first` fit by `ols`

```
summary(mod_first)
```

```
             Effects              Response : ptsd

 Factor Low High Diff. Effect   S.E.    Lower 0.95
 over2  0   5    5     -0.16658 0.27195 -0.7103800
 over3  0   5    5      0.57455 0.28970 -0.0047389
 Upper 0.95
 0.37722
 1.15380
```

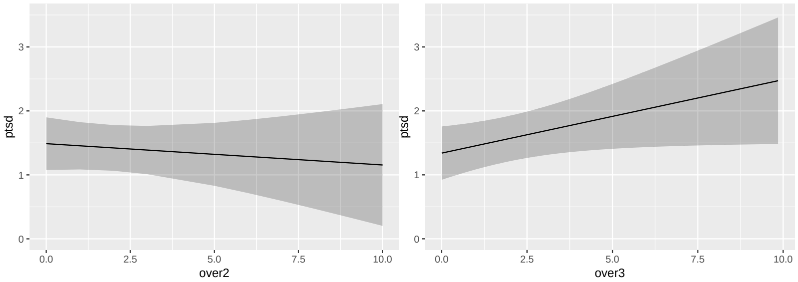
Plot the summary to see effect sizes

```
plot(summary(mod_first))
```



What do the individual effects look like?

```
ggplot(Predict(mod_first))
```



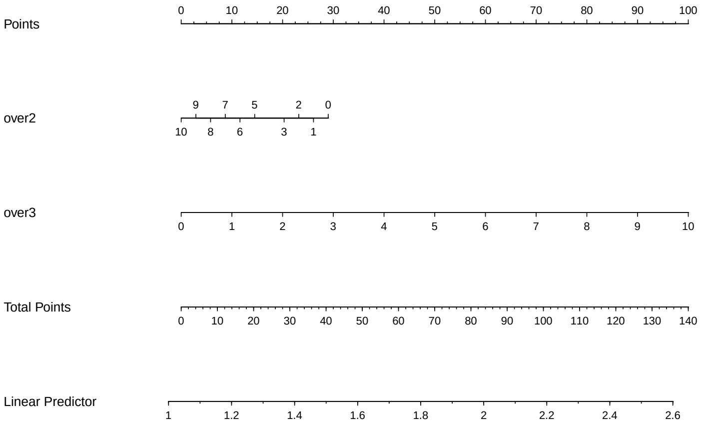
Validate the summary statistics of an `ols` fit

```
set.seed(432010); validate(mod_first)
```

```
          index.orig training   test optimism
R-square      0.0633   0.0940 0.0178   0.0762
MSE           1.4893   1.4185 1.5616  -0.1431
g             0.3450   0.3792 0.2878   0.0914
Intercept     0.0000   0.0000 0.1656  -0.1656
Slope         1.0000   1.0000 0.8854   0.1146
          index.corrected  n
R-square          -0.0129 40
MSE                1.6324 40
g                  0.2536 40
Intercept          0.1656 40
Slope              0.8854 40
```

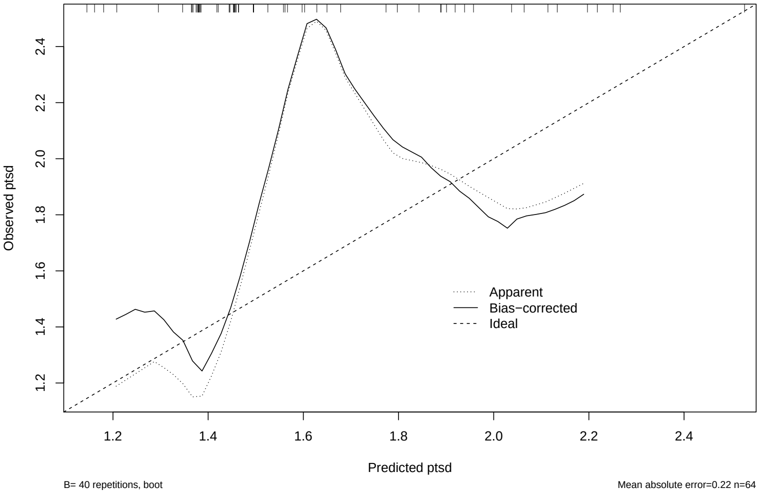
Build a nomogram for the `ols` fit

```
plot(nomogram(mod_first))
```



Is this model well-calibrated?

```
set.seed(432); plot(calibrate(mod_first))
```



Spending degrees of freedom wisely

- Suppose we have a data set with many possible predictors, and minimal theory or subject matter knowledge to guide us.
- We might want our final inferences to be as unbiased as possible. To accomplish this, we have to pay a penalty (in terms of degrees of freedom) for any "peeks" we make at the data in advance of fitting a model.
- So that rules out a lot of decision-making about non-linearity based on looking at the data, if our sample size isn't much larger than 15 times the number of predictors we're considering including in our model.
- In our case, we have $n = 64$ observations on 10 predictors.
- In addition, adding non-linearity to our model costs additional degrees of freedom.
- What can we do?

Spearman's ρ^2 plot: A smart first step?

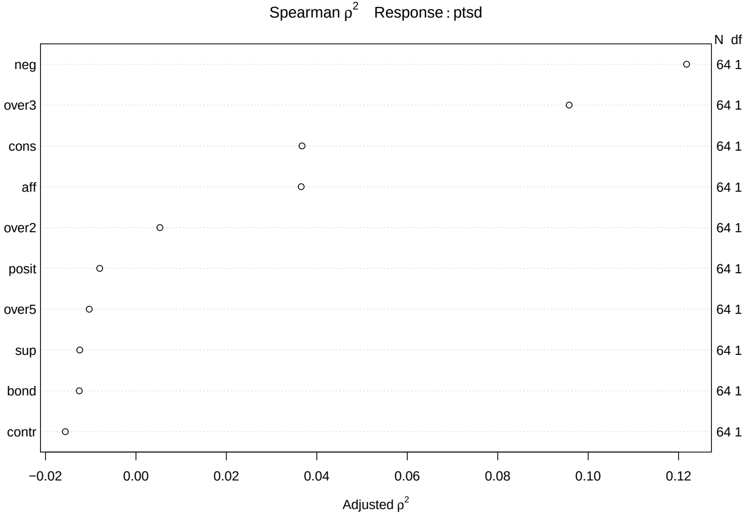
Spearman's ρ^2 is an indicator (not a perfect one) of potential predictive punch, but doesn't give away the game.

- Idea: Perhaps we should focus our efforts re: non-linearity on predictors that score better on this measure.

```
spear_ptsd <- spearman2(ptsd ~ over2 + over3 + over5 + bond +
                   posit + neg + contr + sup + cons + aff,
                 data = maleptsd)
```

Spearman's ρ^2 Plot

```
plot(spear_ptsd)
```



Conclusions from Spearman ρ^2 Plot

- `neg` is the most attractive candidate for a non-linear term, as it packs the most potential predictive punch, so if it does turn out to need non-linear terms, our degrees of freedom will be well spent.
  - By no means is this suggesting that `neg` actually needs a non-linear term, or will show significant non-linearity. We'd have to fit a model with and without non-linearity in `neg` to know that.
  - Non-linearity will often take the form of a product term, a polynomial term, or a restricted cubic spline.
  - Since all of these predictors are quantitative, we'll think about polynomial or spline terms, soon.
- `over3`, also quantitative, has the next most potential predictive punch
- these are followed by `cons` and `aff`

Grim Reality

With 64 observations (63 df) we should be thinking about models with at most 63/15 regression inputs, including the intercept, even if all were linear.

- Non-linear terms (polynomials, splines) just add to the problem, as they need additional df to be estimated.

In this case, we might choose between

- including non-linearity in one (or maybe 2) variables (and that's it),
- or a linear model including maybe 3-4 predictors, tops

in light of the small sample size.

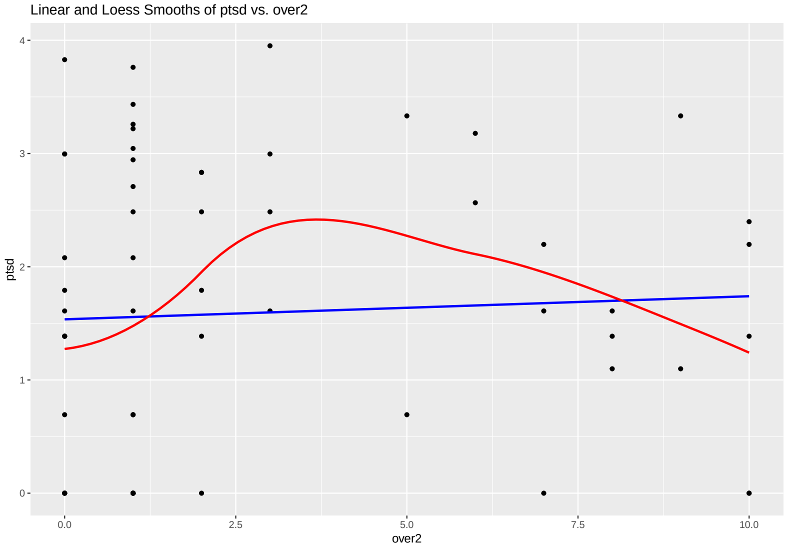
Contents of `spear_ptsd`

```
spear_ptsd
```

```
Spearman rho^2    Response variable:ptsd

       rho2    F df1 df2      P Adjusted rho2  n
over2 0.021 1.34   1  62 0.2522         0.005 64
over3 0.110 7.67   1  62 0.0074         0.096 64
over5 0.006 0.36   1  62 0.5527        -0.010 64
bond  0.004 0.22   1  62 0.6405        -0.013 64
posit 0.008 0.50   1  62 0.4825        -0.008 64
neg   0.136 9.73   1  62 0.0027         0.122 64
contr 0.001 0.03   1  62 0.8602        -0.016 64
sup   0.004 0.23   1  62 0.6357        -0.012 64
cons  0.052 3.40   1  62 0.0699         0.037 64
aff   0.052 3.39   1  62 0.0704         0.037 64
```

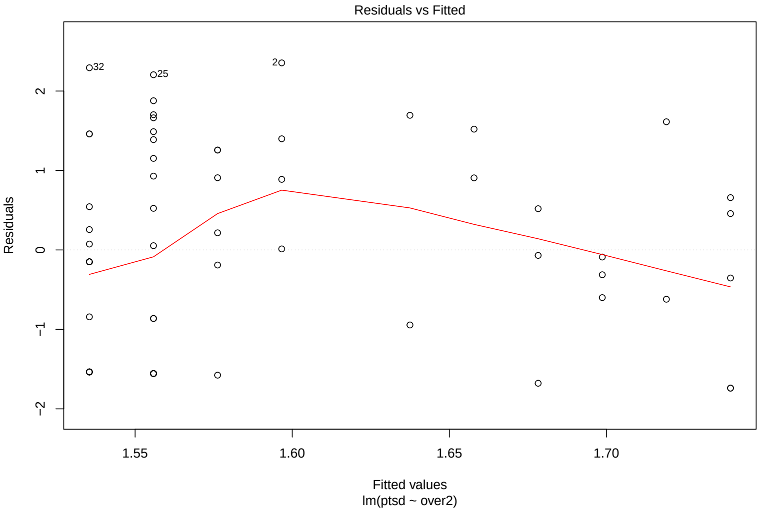
Actually Building Non-Linear Models

Predicting `ptsd` from `over2`



Linear Fit - does this work well?

```
plot(lm(ptsd ~ over2, data = maleptsd), which = 1)
```



Polynomial Regression

A polynomial in the variable `x` of degree D is a linear combination of the powers of `x` up to D.

For example:

- Linear: $y = \beta_0 + \beta_1 x$
- Quadratic: $y = \beta_0 + \beta_1 x + \beta_2 x^2$
- Cubic: $y = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3$
- Quartic: $y = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \beta_4 x^4$
- Quintic: $y = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \beta_4 x^4 + \beta_5 x^5$

Fitting such a model creates a **polynomial regression**.

Raw Quadratic Model for `ptsd` using `over2`

```r
modA <- lm(ptsd ~ over2 + I(over2^2), data = maleptsd)
modA
```

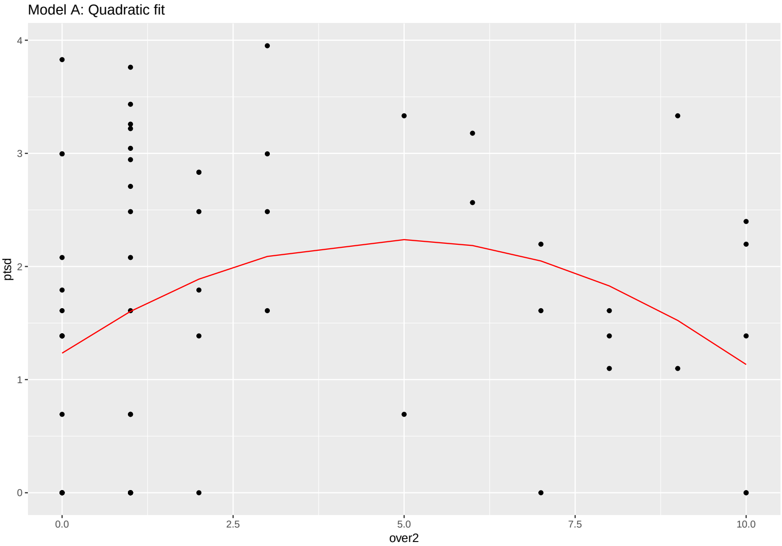
```

Call:
lm(formula = ptsd ~ over2 + I(over2^2), data = maleptsd)

Coefficients:
(Intercept)        over2   I(over2^2)  
    1.23412      0.41128     -0.04213  
```

$$ptsd = 1.234 + 0.411(over2) - 0.042(over2)^2$$

Summary of Quadratic Fit

```
> summary(modA)

Call:
lm(formula = ptsd ~ over2 + I(over2^2), data = maleptsd)

Residuals:
    Min      1Q  Median      3Q     Max 
-2.0487 -1.2341  0.1503  0.9570  2.5945 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  1.23412    0.24958   4.945 6.29e-06 ***
over2        0.41128    0.19271   2.134   0.0369 *  
I(over2^2)  -0.04213    0.02014  -2.092   0.0407 *  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.246 on 61 degrees of freedom
Multiple R-squared:  0.0696,	Adjusted R-squared:  0.03909 
F-statistic: 2.281 on 2 and 61 DF,  p-value: 0.1108
```

Plot Fitted Values of Quadratic Fit



Code for Previous Slide

```
modA_aug <- augment(modA, maleptsd)

ggplot(modA_aug, aes(x = over2, y = ptsd)) +
    geom_point() +
    geom_line(aes(x = over2, y = .fitted),
              col = "red") +
    labs(title = "Model A: Quadratic fit")
```

Another Way to fit the Identical Model

```r
modA2 <- lm(ptsd ~ pol(over2, degree = 2, raw = TRUE),
            data = maleptsd)
```

```
Coefficients:
                          (Intercept)    pol(over2, degree = 2, raw = TRUE)over2
                              1.23412                                    0.41128
pol(over2, degree = 2, raw = TRUE)over2^2
                             -0.04213
```

Do models give same fitted values?

```r
temp <- fitted(modA2) - fitted(modA)
sum(temp != 0)
```

```
[1] 0
```

Orthogonal Polynomials

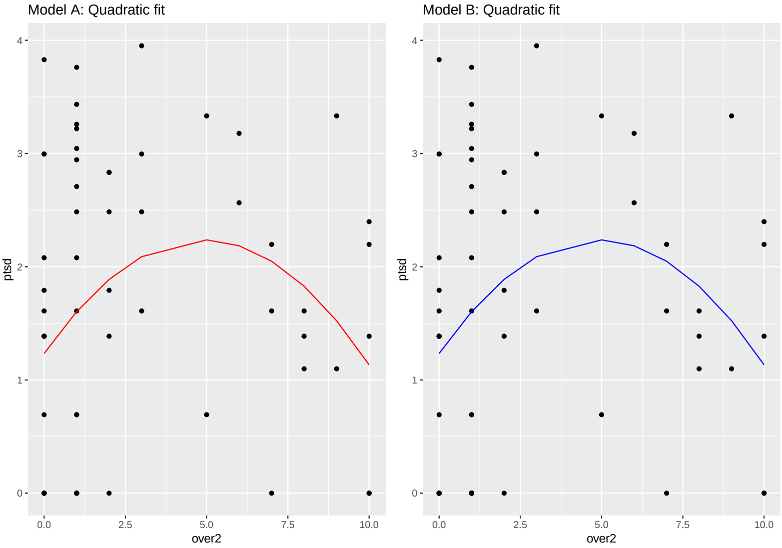
Now, let's fit an orthogonal polynomial of degree 2 to predict `ptsd` using `over2`.

```r
modB <- lm(ptsd ~ poly(over2, 2), data = maleptsd)
```

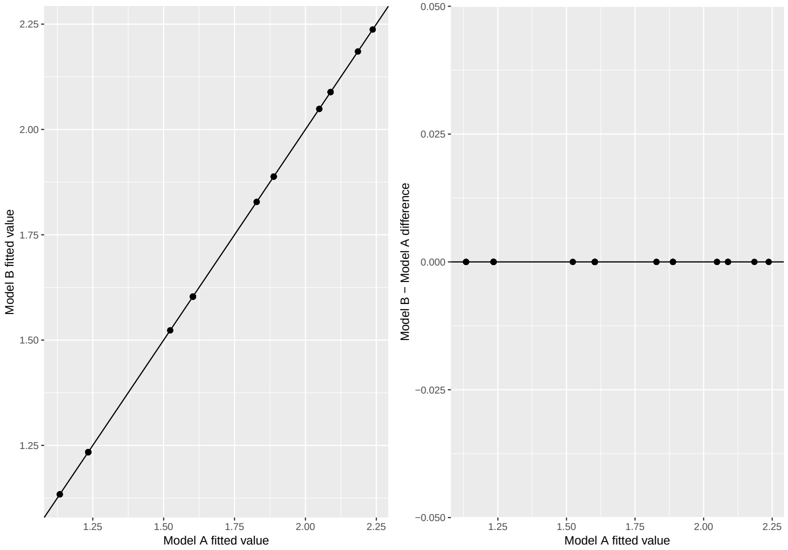
Looks very different . . .

```
> modB

Call:
lm(formula = ptsd ~ poly(over2, 2), data = maleptsd)

Coefficients:
    (Intercept)  poly(over2, 2)1  poly(over2, 2)2
         1.5925           0.5407          -2.6056
```

But it fits the same model, exactly!



Or, if you don't believe me. . .



Orthogonal Polynomial

An orthogonal polynomial sets up a model design matrix using the coding we've seen previously: `over2` and `over2^2` in our case, and then scales those columns so that each column is **orthogonal** to the previous ones.

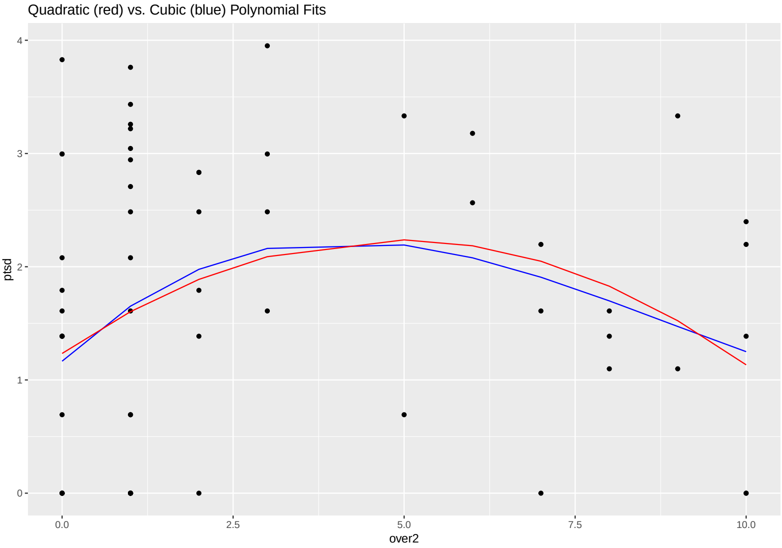
- Two columns are orthogonal if their correlation is zero.
- This eliminates the collinearity (correlation between predictors) and lets our t tests tell us whether the addition of any particular polynomial term improves the fit of the model over the lower orders.

Would adding a cubic term help predict ptsd?

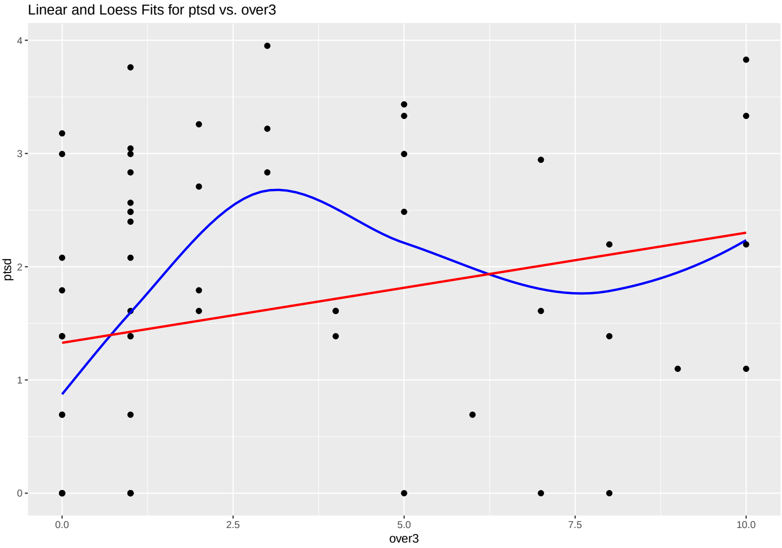
```r
modC <- lm(ptsd ~ poly(over2, 3), data = maleptsd)
```

```
> summary(modC)

Call:
lm(formula = ptsd ~ poly(over2, 3), data = maleptsd)

Residuals:
    Min      1Q  Median      3Q     Max 
-1.9770 -1.1658  0.1784  0.9220  2.6628 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept)       1.5925     0.1567  10.164 1.15e-14 ***
poly(over2, 3)1   0.5407     1.2534   0.431   0.6677    
poly(over2, 3)2  -2.6056     1.2534  -2.079   0.0419 *  
poly(over2, 3)3   0.6363     1.2534   0.508   0.6135    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.253 on 60 degrees of freedom
Multiple R-squared:  0.07357,	Adjusted R-squared:  0.02725 
F-statistic: 1.588 on 3 and 60 DF,  p-value: 0.2016
```

Comparing Quadratic (red) and Cubic (blue) Models



What if we look instead at `over3` as a predictor?

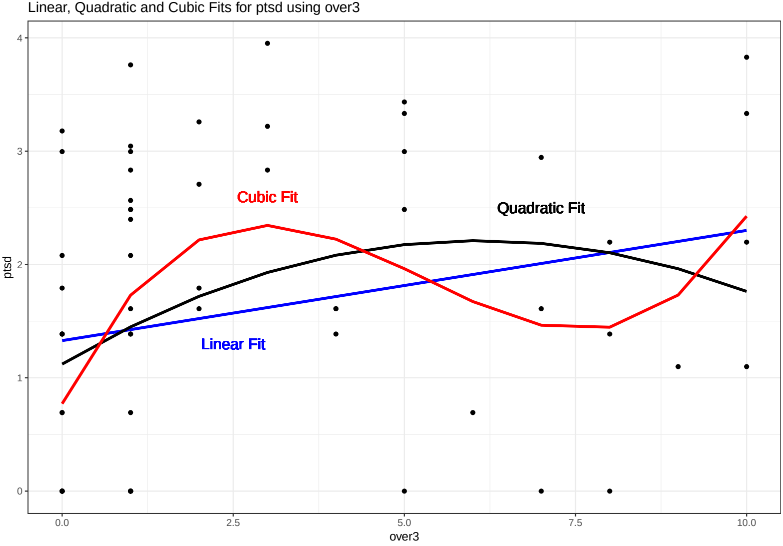


What if we predict using over3?

```r
modD1 <- lm(ptsd ~ over3, data = maleptsd)
modD2 <- lm(ptsd ~ poly(over3, degree = 2), data = maleptsd)
modD3 <- lm(ptsd ~ poly(over3, degree = 3), data = maleptsd)
```

```
> summary(modD1)$coef
              Estimate Std. Error  t value     Pr(>|t|)
(Intercept) 1.32814813 0.20649484 6.431871 2.047614e-08
over3       0.09723645 0.04999054 1.945097 5.630181e-02
> summary(modD3)$coef
                          Estimate Std. Error   t value     Pr(>|t|)
(Intercept)               1.592510  0.1460114 10.906746 7.216662e-16
poly(over3, degree = 3)1  2.419092  1.1680915  2.070979 4.267030e-02
poly(over3, degree = 3)2 -1.905737  1.1680915 -1.631496 1.080240e-01
poly(over3, degree = 3)3  3.225048  1.1680915  2.760955 7.635010e-03
```

Plotting the Fitted Models



Using Restricted Cubic Splines to Capture Non-Linearity

Splines

- A **linear spline** is a continuous function formed by connecting points (called **knots** of the spline) by line segments.
- A **restricted cubic spline** is a way to build highly complicated curves into a regression equation in a fairly easily structured way.
- A restricted cubic spline is a series of polynomial functions joined together at the knots.
    - Such a spline gives us a way to flexibly account for non-linearity without over-fitting the model.
    - Restricted cubic splines can fit many different types of non-linearities.
    - Specifying the number of knots is all you need to do in R to get a reasonable result from a restricted cubic spline.

The most common choices are 3, 4, or 5 knots.

- 3 Knots, 2 degrees of freedom, allows the curve to “bend” once.
- 4 Knots, 3 degrees of freedom, lets the curve “bend” twice.
- 5 Knots, 4 degrees of freedom, lets the curve “bend” three times.

Fitting Restricted Cubic Splines with `lm` and `rcs`

For most applications, three to five knots strike a nice balance between complicating the model needlessly and fitting data pleasingly. Let's consider a restricted cubic spline model for `ptsd` based on `over3` again, but now with:

- in `modE3`, 3 knots, and
- in `modE4`, 4 knots,

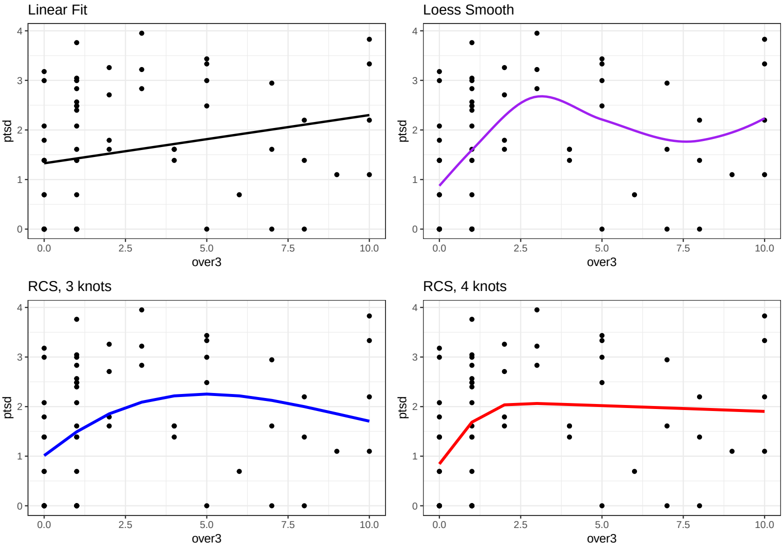
```
modE3 <- lm(ptsd ~ rcs(over3, 3), data = maleptsd)
modE4 <- lm(ptsd ~ rcs(over3, 4), data = maleptsd)
```

Summarizing the 4-knot model coefficients

Values of the estimates, and where are the knots located?

```
> round(summary(modE4)$coef,3)
                       Estimate Std. Error t value Pr(>|t|)
(Intercept)               0.843      0.290   2.908    0.005
rcs(over3, 4)over3        0.953      0.432   2.209    0.031
rcs(over3, 4)over3'      -8.914      5.982  -1.490    0.141
rcs(over3, 4)over3''     13.480      9.635   1.399    0.167
>
> attributes(rcs(maleptsd$over3, 4))$parms
[1] 0.00 1.00 2.95 9.00
```

Plotting the spline models



Does the fit improve markedly from 3 to 4 knots?

In-sample comparison via ANOVA

```
anova(modE3, modE4)
```

```
Analysis of Variance Table

Model 1: ptsd ~ rcs(over3, 3)
Model 2: ptsd ~ rcs(over3, 4)
  Res.Df    RSS Df Sum of Sq      F Pr(>F)
1     61 89.598
2     60 87.573  1    2.0246 1.3871 0.2435
```

Does the fit improve markedly from 3 to 4 knots?

In-Sample comparisons of information criteria, etc.

```
glance(modE3) %>% select(r.squared, adj.r.squared, AIC, BIC)
```

```
# A tibble: 1 x 4
  r.squared adj.r.squared   AIC   BIC
      <dbl>         <dbl> <dbl> <dbl>
1     0.119        0.0906  211.  220.
```

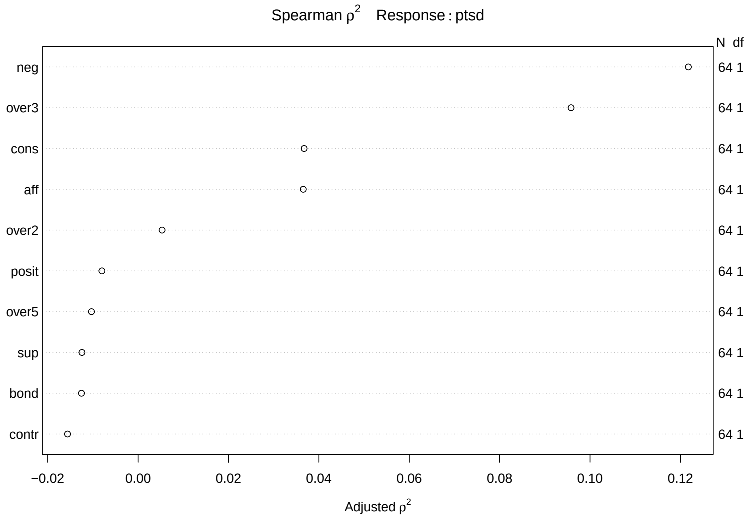
```
glance(modE4) %>% select(r.squared, adj.r.squared, AIC, BIC)
```

```
# A tibble: 1 x 4
  r.squared adj.r.squared   AIC   BIC
      <dbl>         <dbl> <dbl> <dbl>
1     0.139        0.0963  212.  222.
```

Back to the Spearman's ρ^2 Plot

Spearman's ρ^2 Plot

```r
plot(spear_ptsd)
```



Proposed New Model F

Fit a model to predict `ptsd` using:

- a 4-knot spline on `neg`
- a 3-knot spline on `over3`
- a linear term on `cons`
- a linear term on `aff`

Still more than we can reasonably do with 64 observations, but let's see how it looks.

Fit model F

```r
modelF <- lm(ptsd ~ rcs(neg, 4) + rcs(over3, 3) +
                 cons + aff, data = maleptsd)
```

```
> round(summary(modelF)$coef,3)
                   Estimate Std. Error t value Pr(>|t|)
(Intercept)          -0.425      0.749  -0.568    0.572
rcs(neg, 4)neg        0.066      0.060   1.095    0.278
rcs(neg, 4)neg'      -0.126      0.164  -0.768    0.446
rcs(neg, 4)neg''      0.492      0.537   0.916    0.363
rcs(over3, 3)over3    0.458      0.201   2.283    0.026
rcs(over3, 3)over3'  -2.125      0.943  -2.252    0.028
cons                 -0.012      0.016  -0.724    0.472
aff                   0.145      0.060   2.424    0.019
```

ANOVA for Model F

```
anova(modelF)
```

```
Analysis of Variance Table

Response: ptsd
               Df Sum Sq Mean Sq F value  Pr(>F)
rcs(neg, 4)     3 14.597  4.8657  3.7342 0.01617 *
rcs(over3, 3)  2  5.892  2.9460  2.2609 0.11369
cons            1  0.636  0.6365  0.4885 0.48751
aff             1  7.657  7.6566  5.8760 0.01860 *
Residuals      56 72.969  1.3030
---
Signif. codes:
0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Remember that this ANOVA testing is sequential.

Is Model F better than Model E3?

```
anova(modelF, modE3)
```

```
Analysis of Variance Table

Model 1: ptsd ~ rcs(neg, 4) + rcs(over3, 3) + cons + aff
Model 2: ptsd ~ rcs(over3, 3)
  Res.Df    RSS Df Sum of Sq      F  Pr(>F)
1     56 72.969
2     61 89.598 -5   -16.629 2.5524 0.03769 *
---
Signif. codes:
0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Limitations of `lm` for fitting complex linear models

We can certainly assess this big, complex model using `lm` in comparison to other models:

- with in-sample summary statistics like adjusted R^2, AIC and BIC,
- we can assess its assumptions with residual plots, and
- we can also compare out-of-sample predictive quality through cross-validation,

But to really delve into the details of how well this complex model works, and to help plot what is actually being fit, we'll probably want to fit the model using `ols`, from the `rms` package.

- Again, a detailed discussion of `ols` is found in Chapter 10 of the Course Notes. We'll look at some key ideas now.

Using `ols` to fit a complex linear model

Model F, fitted using `ols`

```
dd <- datadist(maleptsd)
options(datadist = "dd")

modF_ols <- ols(ptsd ~ rcs(neg, 4) + rcs(over3, 3) +
                 cons + aff, data = maleptsd,
                x = TRUE, y = TRUE)
```

modF_ols results (slide 1 of 2)

```
> modF_ols
Linear Regression Model

 ols(formula = ptsd ~ rcs(neg, 4) + rcs(over3, 3) + cons + aff,
     data = maleptsd, x = TRUE, y = TRUE)

                Model Likelihood     Discrimination
                   Ratio Test           Indexes
 Obs       64    LR chi2     21.28    R2       0.283
 sigma1.1415     d.f.            7    R2 adj   0.193
 d.f.      56    Pr(> chi2) 0.0034    g        0.763

 Residuals

      Min       1Q   Median       3Q      Max
 -2.06529 -0.81434  0.06745  0.81760  2.17200
```

modF_ols results (slide 2 of 2)

	Coef	S.E.	t	Pr(>|t|)
Intercept	-0.4255	0.7490	-0.57	0.5723
neg	0.0660	0.0603	1.10	0.2780
neg'	-0.1261	0.1641	-0.77	0.4456
neg''	0.4924	0.5373	0.92	0.3634
over3	0.4582	0.2007	2.28	0.0263
over3'	-2.1247	0.9433	-2.25	0.0282
cons	-0.0119	0.0164	-0.72	0.4722
aff	0.1450	0.0598	2.42	0.0186

Validation of Summary Statistics

```
set.seed(4322019); validate(modF_ols)
```

```
          index.orig training   test optimism
R-square      0.2829   0.3747 0.1457   0.2290
MSE           1.1401   0.9753 1.3582  -0.3830
g             0.7630   0.8600 0.6520   0.2080
Intercept     0.0000   0.0000 0.4452  -0.4452
Slope         1.0000   1.0000 0.7327   0.2673
          index.corrected  n
R-square           0.0538 40
MSE                1.5231 40
g                  0.5550 40
Intercept          0.4452 40
Slope              0.7327 40
```

anova results for modF_ols

```
anova(modF_ols)
```

```
                Analysis of Variance          Response: ptsd

 Factor          d.f. Partial SS MS        F    P
 neg              3   11.4062336 3.8020779 2.92 0.0420
  Nonlinear       2    1.6536591 0.8268295 0.63 0.5339
 over3            2    6.8378486 3.4189243 2.62 0.0814
  Nonlinear       1    6.6106843 6.6106843 5.07 0.0282
 cons             1    0.6826901 0.6826901 0.52 0.4722
 aff              1    7.6565797 7.6565797 5.88 0.0186
 TOTAL NONLINEAR  3    7.8079300 2.6026433 2.00 0.1248
 REGRESSION       7   28.7821644 4.1117378 3.16 0.0070
 ERROR           56   72.9688769 1.3030157
```

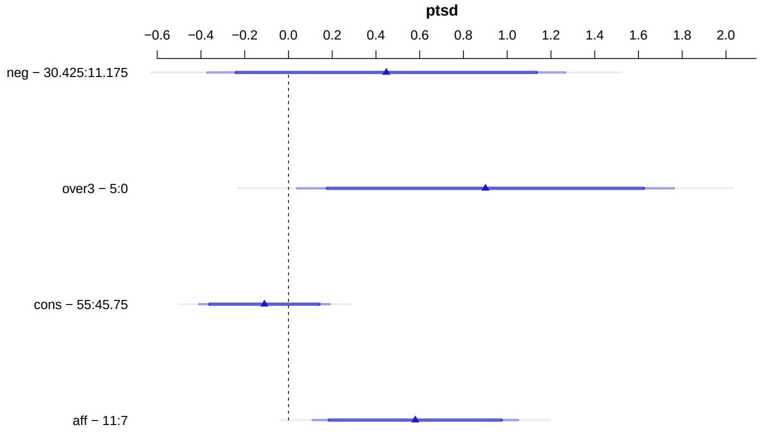
summary results for modF_ols

```r
summary(modF_ols)
```

```
             Effects              Response : ptsd

 Factor Low    High   Diff. Effect   S.E.    Lower 0.95
 neg    11.175 30.425 19.25  0.44727 0.41704 -0.388160
 over3   0.000  5.000  5.00  0.90059 0.43913  0.020902
 cons   45.750 55.000  9.25 -0.10997 0.15192 -0.414310
 aff     7.000 11.000  4.00  0.57998 0.23926  0.100680
 Upper 0.95
 1.28270
 1.78030
 0.19437
 1.05930
```

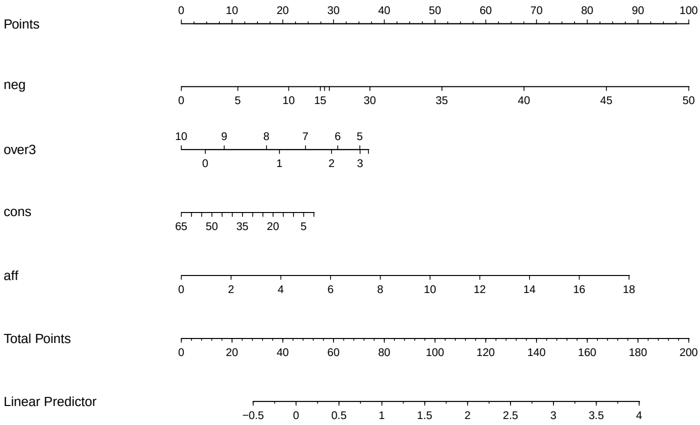
Plot of summary results for modF_ols

```
plot(summary(modF_ols))
```



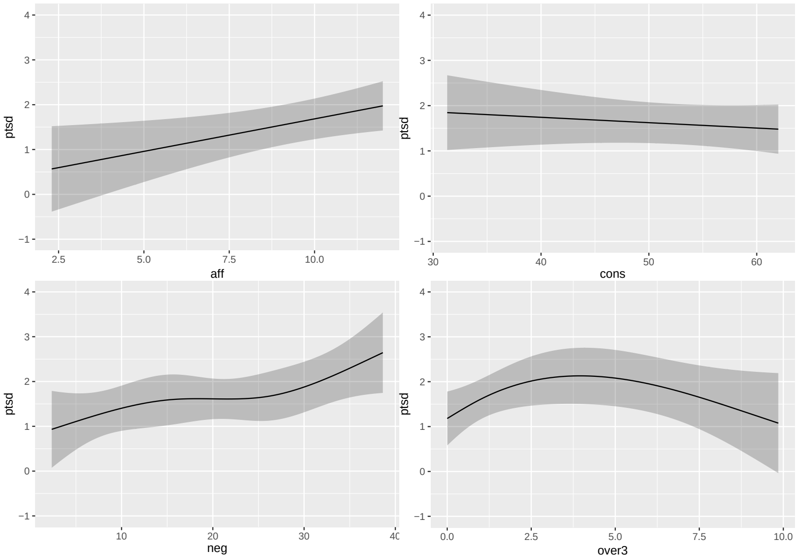
Nomogram for `modF_ols`

```
plot(nomogram(modF_ols))
```



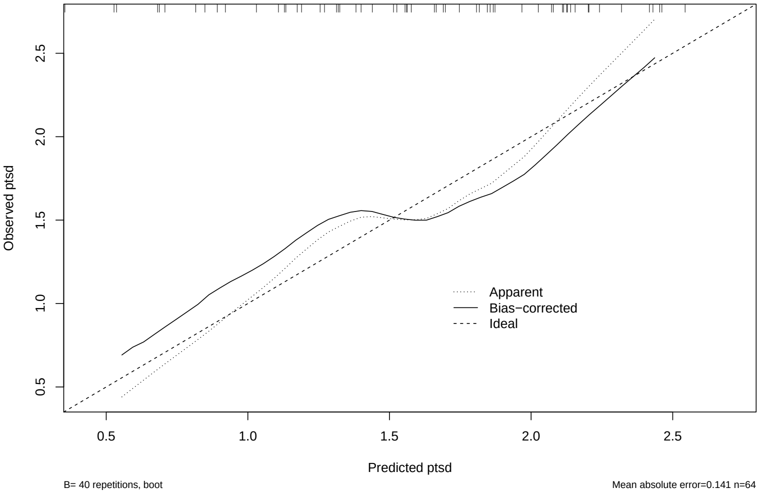
Seeing the impact of the modeling another way

```
ggplot(Predict(modF_ols))
```



Checking the model's calibration

```
set.seed(43220191); plot(calibrate(modF_ols))
```



Next Time

- The HERS data
- Fitting a more complex linear regression model
    - Dealing with categorical predictors
    - Dealing with interactions (another form of non-linearity)
    - Adding missing data into all of this, and running multiple imputation