## 432 Class 7 Slides

github.com/THOMASELOVE/2019-432

2019-02-14

## Setup

```
library(skimr); library(broom); library(janitor)
library(pROC); library(ROCR) # these are new to us
library(tidyverse)

lbw <- read_csv("data/lbw.csv") %>% clean_names()
```

# The Low Birth Weight data, again

# The lbw1 data (n = 189 infants)

Variable	Description		
subject	id code		
low	indicator of low birth weight (< 2500 g)		
momwt	mom's weight at last menstrual period (lbs.)		
age	age of mother in years		
ftv	count of physician visits in first trimester (0 to 6)		
ht	history of hypertension: $1 = yes$ , $0 = no$		
race_f	race of mom: white, black, other		
preterm	prior premature labor: $1 = \text{yes}$ , $0 = \text{no}$		
smoke	1=smoked during pregnancy, $0=did$ not		
ui	presence of uterine irritability: $1 = yes$ , $0 = no$		

Source: Hosmer, Lemeshow and Sturdivant, *Applied Logistic Regression* 3rd edition. Data from Baystate Medical Center, Springfield MA in 1986.

#### Our current model

```
model.1 <- glm(low ~ momwt, data = lbw1, family = binomial)
model.1</pre>
```

```
Call: glm(formula = low ~ momwt, family = binomial, data = 1)
Coefficients:
(Intercept) momwt
   0.99831 -0.01406
Degrees of Freedom: 188 Total (i.e. Null); 187 Residual
Null Deviance: 234.7
Residual Deviance: 228.7 AIC: 232.7
```

# Our logistic regression model

The logistic regression equation is:

$$logit(Pr(low = 1)) = log\left(\frac{Pr(low = 1)}{1 - Pr(low = 1)}\right) = 0.99831 - 0.01406 \times momwt$$

Suppose, for instance, that we are interested in making a prediction when Mom's weight at her last period, momwt = 130 lbs.

So we have:

$$logit(Pr(low = 1)) = 0.99831 - 0.01406 \times 130 = -0.82949$$

# Obtaining a Prediction from R for Prob(low = 1)

```
model.1 <- glm(low ~ momwt, data = lbw1, family = binomial)</pre>
```

To predict on the probability scale, we can use

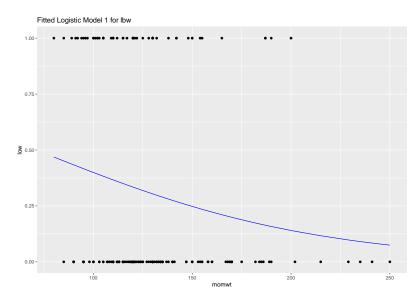
0.3038016

# Plotting the Logistic Regression Model

We can use the augment function from the broom package to get our fitted probabilities included in the data.

Results on next slide

# Plotting the Logistic Regression Model

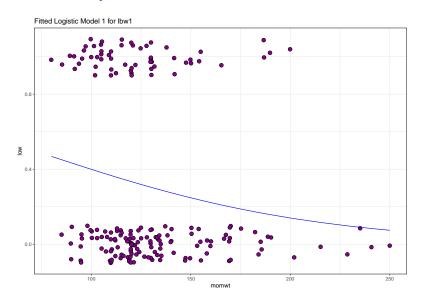


# Cleaning up the plot

I'll add a little jitter on the vertical scale to the points, so we can avoid overlap, and also make the points a little bigger.

Results on next slide

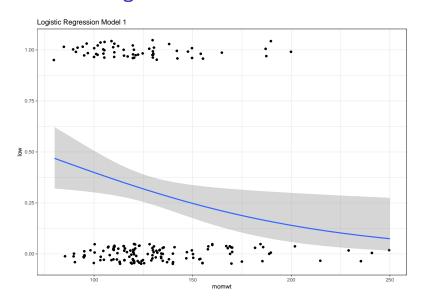
# Cleaned up Plot of Model 1



# Plotting a Simple Logistic Model using binomial smooth

```
binomial smooth <- function(...) {</pre>
  geom smooth(method = "glm",
              method.args = list(family = "binomial"), ...)
}
ggplot(lbw1, aes(x = momwt, y = low)) +
  geom_jitter(height = 0.05) +
  binomial smooth() +
    ## ...smooth(se=FALSE) to leave out interval
  labs(title = "Logistic Regression Model 1") +
  theme bw()
```

# The Resulting Plot



# Studying the Model, Again

model.1

```
Call: glm(formula = low ~ momwt, family = binomial, data = li
```

Coefficients:

(Intercept) momwt

0.99831 -0.01406

Degrees of Freedom: 188 Total (i.e. Null); 187 Residual

Null Deviance: 234.7

Residual Deviance: 228.7 AIC: 232.7

- logit(Pr(low = 1)) = 0.998 0.014 momwt
  - so ... as momwt increases, what happens to Pr(low = 1)?
  - if Harry's mom weighed 130 lbs and Sally's weighed 150 lbs, how can we compare the predicted Pr(low = 1) for Harry and Sally?

# Harry (momwt = 130) vs. Sally (momwt = 150)

0.3038016 0.2477917

- Harry's mom weighed 130 lbs, and his predicted probability of low birth weight is 0.304
- ullet Sally's mom weighed 150 lbs, and her predicted Pr(low=1)=0.248

# Interpreting the Coefficients of the Model

```
(Intercept) momwt 0.99831432 -0.01405826
```

coef(model.1)

To understand the effect of momwt on low, try odds ratios.

```
exp(coef(model.1))
```

```
(Intercept) momwt
2.7137035 0.9860401
```

Suppose Charlie's Mom weighed one pound more than Harry's.

- The **odds** of low birth weight are 0.986 times as large for Charlie as Harry.
- In general, odds ratio comparing two subjects whose momwt differ by 1 pound is 0.986

# **Comparing Harry to Charlie**

Charlie's mom weighed 1 pound more than Harry's. The estimated odds ratio for low birth weight from the model associated with a one pound increase in momet is 0.986.

- If the odds ratio was 1, that would mean that Charlie and Harry had the same estimated odds of low birth weight, and thus the same estimated probability of low birth weight, despite having Moms with different weights.
- Since the odds ratio is less than 1, it means that Charlie has a lower estimated odds of low birth weight than Harry, and thus that Charlie has a lower estimated probability of low birth weight than Harry.
- If the odds ratio was greater than 1, it would mean that Charlie had a
  higher estimated odds of low birth weight than Harry, and thus that
  Charlie had a higher estimated probability of low birth weight than
  Harry.

The smallest possible odds ratio is . . . ?

## The rest of the model's output

Degrees of Freedom: 188 Total (i.e. Null); 187 Residual

Null Deviance: 234.7

Residual Deviance: 228.7 AIC: 232.7

Model	Null	Residual	$\Delta \; (\texttt{model.1})$
Deviance (lack of fit)	234.7	228.7	6.0
Degrees of Freedom	188	187	1

- Deviance accounted for by model.1 is 6 points on 1 df
- Can compare to a  $\chi^2$  distribution for a p value via anova

AIC = 232.7, still useful for comparing models for the same outcome

## anova on a glm model

```
anova(model.1)
```

Analysis of Deviance Table

Model: binomial, link: logit

Response: low

Terms added sequentially (first to last)

Df Deviance Resid. Df Resid. Dev NULL 188 234.67 momwt 1 5.9813 187 228.69

```
pchisq(5.9813, 1, lower.tail = FALSE)
```

[1] 0.01445834

## glance on model.1

#### glance(model.1)

- Deviance =  $-2 \times \log$  (likelihood)
- AIC and BIC are based on the deviance, but with differing penalties for complicating the model
- AIC and BIC remain useful for comparing multiple models for the same outcome

## summary of model.1

```
> summary(model.1)
Call:
glm(formula = low ~ momwt, family = binomial, data = lbw1)
Deviance Residuals:
            10 Median 30 Max
   Min
-1.0951 -0.9022 -0.8018 1.3609 1.9821
Coefficients:
           Estimate Std. Error z value Pr(>|z|)
(Intercept) 0.99831 0.78529 1.271 0.2036
          -0.01406 0.00617 -2.279 0.0227 *
momwt
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
(Dispersion parameter for binomial family taken to be 1)
   Null deviance: 234.67 on 188 degrees of freedom
Residual deviance: 228.69 on 187 degrees of freedom
AIC: 232.69
Number of Fisher Scoring iterations: 4
```

# **Coefficients output**

#### Coefficients:

```
Estimate Std. Error z value Pr(>|z|)
(Intercept) 0.99831 0.78529 1.271 0.2036
momwt -0.01406 0.00617 -2.279 0.0227 *
```

- We have a table of coefficients with standard errors, and hypothesis tests, although these are Wald z-tests, rather than the t tests we saw in linear modeling.
- momwt has a Wald Z of -2.279, yielding p = 0.0227
  - $\bullet$   $H_0$ : momwt does not have an effect on the log odds of low
  - ullet  $H_A$ : momwt does have such an effect
- If the coefficient (on the logit scale) for momwt was truly 0, this would mean that:
  - the log odds of low birth weight did not change based on momwt,
  - ullet the odds of low birth weight were unchanged based on momwt (OR = 1), and
  - the probability of low birth weight was unchanged based on the momwt.

## **Confidence Intervals for Coefficients**

```
(Intercept) momwt
0.99831432 -0.01405826

confint(model.1, level = 0.95)
```

```
Waiting for profiling to be done...
```

```
2.5 % 97.5 % (Intercept) -0.48116701 2.611748138 momwt -0.02696198 -0.002650036
```

- The coefficient of momwt has a point estimate of -0.014 and a 95% confidence interval of (-0.027, -0.003).
- On the logit scale, this isn't that interpretable, but we will often exponentiate to describe odds ratios.

coef (model.1)

# Odds Ratio Interpretation of exp(Coefficient)

```
exp(coef(model.1))
(Intercept)
                  momwt
 2.7137035 0.9860401
exp(confint(model.1, level = 0.95))
               2.5 % 97.5 %
(Intercept) 0.6180617 13.6228447
momwt
      0.9733982 0.9973535
```

- Odds Ratio for low based on a one pound increase in momwt is 0.986 (95% CI: 0.973, 0,997).
  - ullet Estimated odds of low birth weight will be smaller (odds < 1) for those with larger momet values.
  - Smaller odds(low birth weight) = smaller Prob(low birth weight).

#### **Deviance Residuals**

#### Deviance Residuals:

```
Min 1Q Median 3Q Max
-1.0951 -0.9022 -0.8018 1.3609 1.9821
```

- The deviance residuals for each individual subject sum up to the deviance statistic for the model, and describe the contribution of each point to the model likelihood function. The formula is in the Course Notes.
- Logistic Regression is a non-linear model, and it doesn't come with either an assumption that the residuals will follow a Normal distribution, or an assumption that the residuals will have constant variance, so when we build diagnostics for the logistic regression model, we'll use different plots and strategies than we used in linear models.

# **Other New Things**

(Dispersion parameter for binomial family taken to be 1)

Number of Fisher Scoring iterations: 4

- Dispersion parameters matter for some generalized linear models. For binomial family models like the logistic, it's always 1.
- The solution of a logistic regression model involves maximizing a likelihood function. Fisher's scoring algorithm needed just four iterations to perform this fit. The model converged, quickly.

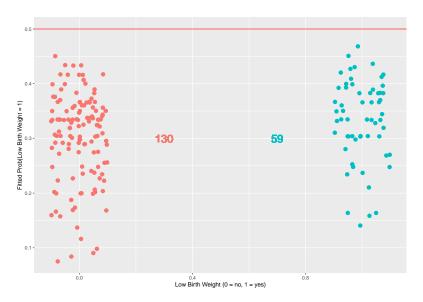
# How Well Does Our model.1 Classify Subjects?

One possible rule: if predicted  $Pr(low = 1) \ge 0.5$ , then we predict "low birth weight"

0 1
Predict Not Low 130 59

This rule might be a problem for us. What % are correct?

# A plot of classifications with the 0.5 rule

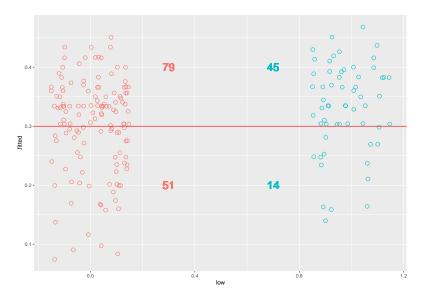


# How Well Does Our model.1 Classify Subjects?

A new rule: if predicted  $Pr(low = 1) \ge 0.3$ , then we predict "high risk of low birth weight" and otherwise, we predict "low risk of low birth weight"

Low Birth Weight OK Birth Weight
High Risk of LBW 45 79
Low Risk of LBW 14 51

# A plot of classifications with the 0.3 rule



The C Statistic (Area under the ROC Curve)

# Our Model as Diagnostic Test

We want to assess predictive accuracy of our model.

- One approach: Receiver Operating Characteristic (ROC) curve analysis.
- A common choice for assessing diagnostic tests in medicine.

Consider two types of errors made by our model, in combination with a classification rule.

- Our model uses Mom's weight at last period to predict Pr(low birth weight).
- Lighter moms had higher model probabilities, so our rule would be: Predict low birth weight if Mom's last weight is no more than R pounds.

But the choice of R is available to us. Any value we select can lead to good outcomes (of our prediction) or to errors.

#### **Test Results**

- One good outcome of our "model/test" would be if the Mom's weight is less than R and her baby is born at a low birth weight.
- The other good outcome is if Mom's weight is greater than R and her baby is born at a non-low weight.

But we can make errors, too.

- A false positive occurs when we predict Pr(low = 1) to be small, but the baby is born at a low birth weight.
- ullet A false negative occurs when we predict  $\Pr(\text{low}=1)$  to be large, but the baby is born at a non-low weight.

#### We identify two key summaries:

- The true positive fraction (TPF) for a specific weight cutoff R is  $Pr(Mom\ weight < R \mid baby\ actually\ has\ low = 1)$ .
- The false positive fraction (FPF) for a specific weight cutoff R is  $Pr(Mom\ weight < R \mid baby\ has\ low = 0)$ .

#### The ROC Curve

Since the cutoff R is not fixed in advanced, we can plot the value of TPF (on the y axis) against FPF (on the x axis) for all possible values of R, and this is what the ROC curve is.

- We calculate AUC = the area under the ROC curve (a value between 0 and 1) and use it to help summarize the effectiveness of the predictions made by the model on the following scale:
  - AUC above 0.9 = excellent discrimination of low = 1 from low = 0
  - AUC between 0.8 and 0.9 = good discrimination
  - AUC between 0.6 and 0.8 = mediocre/fair discrimination
  - AUC of 0.5 = random guessing
  - AUC below 0.5 = worse than guessing

Others refer to the Sensitivity on the Y axis, and 1-Specificity on the X axis, and this is the same idea. The TPF is called the sensitivity. 1 - FPF is the true negative rate, called the specificity.

#### **A Simulation**

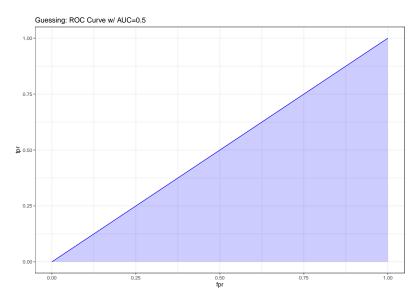
Warning: `data\_frame()` is deprecated, use `tibble()`. This warning is displayed once per session.

# What if we are guessing?

If we're guessing completely at random, then the model should correctly classify a subject (as died or not died) about 50% of the time, so the TPR and FPR will be equal. This yields a diagonal line in the ROC curve, and an area under the curve (C statistic) of 0.5.

Plot is on the next slide...

## What if we are guessing?



#### Building that ROC curve, Code part 1

This approach requires the loading of the ROCR package. . .

## Building that ROC curve, Code part 2

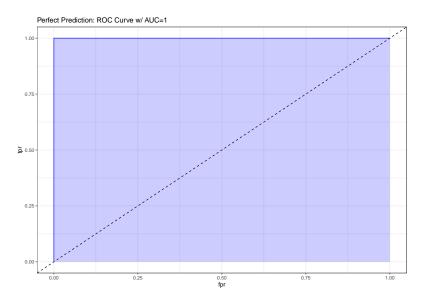
#### What if our model classifies things perfectly?

If we're classifying subjects perfectly, then we have a TPR of  $\bf 1$  and an FPR of  $\bf 0$ .

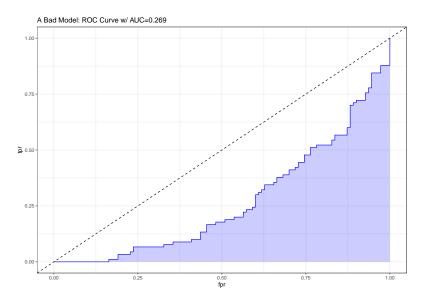
- That yields an ROC curve that looks like the upper and left edges of a box.
- If our model correctly classifies a subject (as died or not died) 100% of the time, the area under the curve (c statistic) will be 1.0.

I added in a diagonal dashed black line to show how this model compares to random guessing.

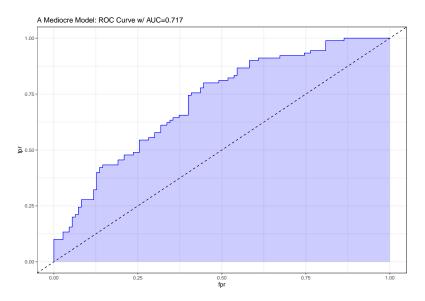
#### What if our model classifies things perfectly?



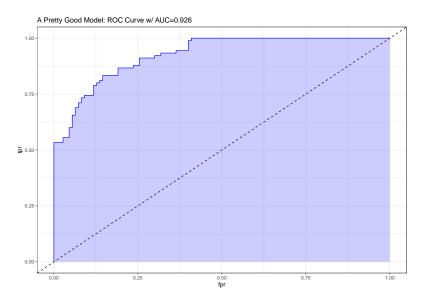
# What does "worse than guessing" look like?



## What does "better than guessing" look like?



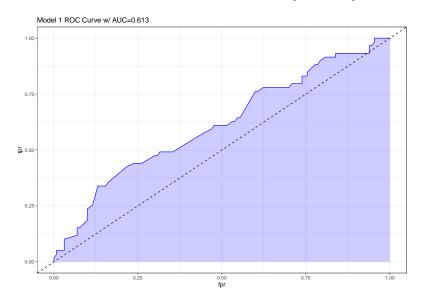
# What does "pretty good" look like?



# The ROC plot for our Model 1 (code)

```
## requires ROCR package
prob <- predict(model.1, lbw1, type="response")</pre>
pred <- prediction(prob, lbw1$low)</pre>
perf <- performance(pred, measure = "tpr", x.measure = "fpr")</pre>
auc <- performance(pred, measure="auc")</pre>
auc <- round(auc@y.values[[1]],3)
roc.data <- data.frame(fpr=unlist(perf@x.values),
                        tpr=unlist(perf@y.values),
                        model="GLM")
ggplot(roc.data, aes(x=fpr, ymin=0, ymax=tpr)) +
    geom ribbon(alpha=0.2, fill = "blue") +
    geom_line(aes(y=tpr), col = "blue") +
    geom_abline(intercept = 0, slope = 1, lty = "dashed") +
    labs(title = paste0("Model 1 ROC Curve w/ AUC=", auc)) +
    theme bw()
```

## The ROC plot for our Model 1 (Result)



# Interpreting the C statistic (0.613) for Model 1

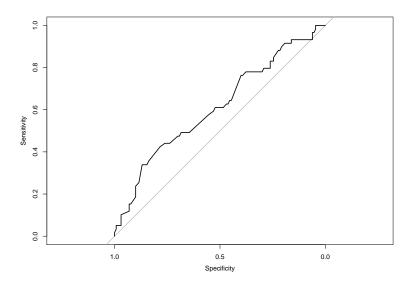
C statistic	Interpretation
0.90 to 1.00	model does an excellent job at discriminating "yes" from "no" (A)
0.80 to 0.90	model does a good job (B)
0.70 to 0.80	model does a fair job (C)
0.60 to 0.70	model does a poor job (D)
0.50 to 0.60	model fails (F)
below 0.50	model is worse than random guessing

#### Another way to plot the ROC Curve

If we've loaded the pROC package, we can also use the following (admittedly simpler) approach to plot the ROC curve, without ggplot2, and to obtain the C statistic, and a 95% confidence interval around that C statistic.

```
## requires pROC package
roc.mod1 <-
    roc(lbw1$low ~ predict(model.1, type="response"),
        ci = TRUE)</pre>
```

## Result of plot(roc.mod1)



Plotting Residuals of a Logistic Regression

#### Residual Plots for model.1?

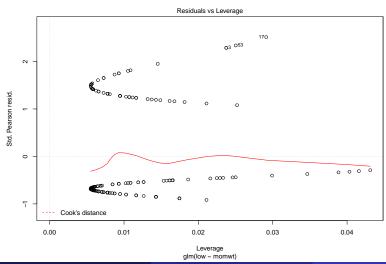
- Yes/No outcomes contain less information than quantitative outcomes
- Residuals cannot be observed predicted
  - There are several different types of residuals defined
- Assumptions of logistic regression are different
  - Model is deliberately non-linear
  - Error variance is a function of the mean, so it isn't constant
  - Errors aren't assumed to follow a Normal distribution
  - Only thing that's the same: leverage and influence

So, plot 5 (residuals/leverage/influence) can be a little useful, but that's it.

We'll need better diagnostic tools down the line.

#### Semi-Useful Residual Plot

plot(model.1, which = 5)



**Building a Bigger Model** 

## Model 2: A "Kitchen Sink" Logistic Regression

Variable	Description
low	indicator of low birth weight (< 2500 g)
momwt	mom's weight at last menstrual period (lbs.)
age	age of mother in years
ftv	physician visits in first trimester (0 to 6)
ht	history of hypertension: $1 = yes$ , $0 = no$
race_f	race of mom: white, black, other
preterm	prior premature labor: $1 = \text{yes}$ , $0 = \text{no}$
smoke	$1=smoked\ during\ pregnancy,\ 0=did\ not$
ui	uterine irritability: $1 = yes$ , $0 = no$

#### model.2

```
Call: glm(formula = low ~ momwt + age + ftv + ht + race_f + p
    smoke + ui, family = binomial, data = lbw1)
```

#### Coefficients:

```
(Intercept) momwt age ftv
0.64448 -0.01508 -0.03955 0.05090
ht race_fblack race_fother pretermyes
1.86043 1.21879 0.81944 1.21851
smoke ui
0.85946 0.71930
```

Degrees of Freedom: 188 Total (i.e. Null); 179 Residual

Null Deviance: 234.7

Residual Deviance: 196.8 AIC: 216.8

#### Comparing model.2 to model.1

```
anova(model.1, model.2)
Analysis of Deviance Table
Model 1: low ~ momwt
Model 2: low ~ momwt + age + ftv + ht + race f + preterm + smo
  Resid. Df Resid. Dev Df Deviance
       187 228.69
      179 196.75 8 31.941
pchisq(31.94, 8, lower.tail = FALSE)
```

```
[1] 9.547465e-05
```

#### Comparing model.2 to model.1

```
glance(model.2)
```

#### glance(model.1)

#### **Interpreting** model.2

> round(summary(model.2)\$coef, 3)							
	Estimate Std.	Error	z value	Pr(> z )			
(Intercept)	0.644	1.224	0.527	0.598			
momwt	-0.015	0.007	-2.143	0.032			
age	-0.040	0.038	-1.032	0.302			
ftv	0.051	0.175	0.290	0.772			
ht	1.860	0.708	2.627	0.009			
race_fblack	1.219	0.533	2.286	0.022			
race_fother	0.819	0.450	1.819	0.069			
pretermyes	1.219	0.463	2.632	0.008			
smoke	0.859	0.410	2.097	0.036			
ui	0.719	0.463	1.552	0.121			

 Larger Mom momwt is associated with a smaller log odds of LBW holding all other predictors constant.

#### Impact of these predictors via odds ratios

exp(coef(model.2)); exp(confint(model.2))

Variable	OR est.	2.5%	97.5%
momwt	0.985	0.971	0.998
age	0.961	0.890	1.035
ftv	1.052	0.739	1.478
ht	6.426	1.662	28.187
race_fblack	3.383	1.192	9.808
race_fother	2.269	0.947	5.597
pretermyes	3.382	1.378	8.575
smoke	2.362	1.067	5.375
ui	2.053	0.818	5.101

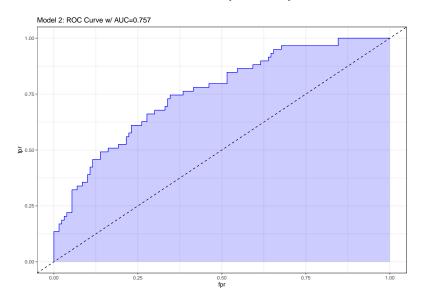
 Larger Mom momwt is associated with a smaller odds of LBW (est OR 0.985, 95% Cl 0.971, 0.998) holding all other predictors constant.

• What appears to be associated with larger odds of LBW?

# **ROC** curve for Model 2 (Code)

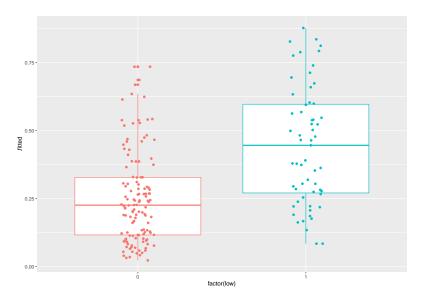
```
prob <- predict(model.2, lbw1, type="response")</pre>
pred <- prediction(prob, lbw1$low)</pre>
perf <- performance(pred, measure = "tpr", x.measure = "fpr")</pre>
auc <- performance(pred, measure="auc")</pre>
auc <- round(auc@y.values[[1]],3)
roc.data <- data.frame(fpr=unlist(perf@x.values),
                        tpr=unlist(perf@y.values),
                        model="GLM")
ggplot(roc.data, aes(x=fpr, ymin=0, ymax=tpr)) +
    geom_ribbon(alpha=0.2, fill = "blue") +
    geom line(aes(y=tpr), col = "blue") +
    geom_abline(intercept = 0, slope = 1, lty = "dashed") +
    labs(title = paste0("Model 2: ROC Curve w/ AUC=", auc)) +
    theme bw()
```

# **ROC** curve for Model 2 (Result)



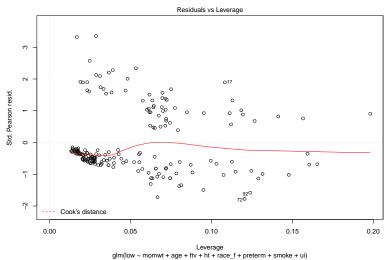
#### Using augment to capture the fitted probabilities

## Plotting Model 2 Fits by Observed LBW status



#### Residuals, Leverage and Influence

plot(model.2, which = 5)



#### **Next Time**

• Fitting Logistic Regression models with 1rm