Asmita's

Principles of PHYSICS- II

Grade XII

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Preface

There are a number of textbooks of +2 levels in the market. So, obviously, a natural question arises, why another one again? The major reason is that most of the textbooks tend to be dry and formal and hence often difficult or complex for the students. It was thus essential to develop a textbook that could touch the pulse of students hence, this textbook is developed with a new approach. Our approach is to recognize that physics is a description of reality starting each topic with concrete observations and experiences to enable students directly related to it. Not only does this book make the material more interesting and easier to understand but also it is closer to the way that physics is actually practiced worldwide.

This book entitled "**Principles of Physics- II**" covers the latest syllabus of class XII. The main objectives of this book are two folds: **to provide the student with a clear and logical presentation of the basic concepts and principles of physics**, and **to strengthen an understanding of the concepts and principles through a broad range of interesting applications to the real world**.

This book is an end product of our uninterrupted two decade long teaching experience. We have tried to solve all the difficulties of the students through this book. The basic parts presented in this book are explanation of theory, mathematical formulae, related figures, answers to the short questions, worked out examples and adequate self practice questions. In some observational facts, reliable reference books are mentioned to avoid the confusion for the reader. In the numerical portions, 'ALP' refers to Advanced Level Physics and 'UP' refers to University Physics. SI system of unit is used throughout the book.

We wish to acknowledge our indebtedness to the many international books which have been consummated. We would like to express our profound and sincere gratitude to our family, colleagues, students, readers, etc. from different part of the country who have adopted this book and sent us their compliments and valuable suggestions through available means. In this regard, special mention goes to Mr. Prakash Pantha, Mr. Akash Pokhrel, Mr. Shesh Nath Chaudhary, Sanjaya K. Sharma, Diwash Dahal, Bipin Bhattarai, Roshan Shrestha, Laxman Aryal and all of our students.

Last but not least, Mr. Manoj Kumar Sharma, managing director of Asmita Books Publishers & Distributors (P) Ltd. deserves our acclaim for meticulous efforts and suggestions to present this collective effort to carry out these matters in this form. We are also very much thankful to Mr. Bipin Kumar Acharya for his valuable advice and suggestions in preparing the book. Mr. Niraj Bhattarai deserves thanks and appreciation for his outstanding type settings and layout for this book.

Humbly, we would like to request our esteemed readers to kindly send us the valuable suggestions for the improvement of the book and to notify of any errors they might come across while going through it. By which both will be thankfully acknowledged and incorporated in the next edition.

Finally, we would like to thank almighty for this endless blessings and kindness.

June 2018

Authors

Syllabus

Teaching hours: 150T +50P
Nature of course: Theory +Practical

Full marks: 100 (75T + 25 P)
Pass Marks: 27T + 8P

Course Contents

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2. **Solids and Semiconductor Devices-** Structure of solids; Energy bands in solids (qualitative ideas only); Difference between metals, insulators and semi-conductors using band theory; Intrinsic and extrinsic semi-conductors; P-N Junction; Semiconductor diode: Characteristics in forward and reverse bias; Full wave rectification; Filter circuit; Zener diode; Transistor: Common emitter characteristics, Logic gates; NOT, OR, AND, NAND and NOR., Nanotechnology (introductory idea) LH 11
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4. **Nuclear Physics-** Nucleus: Discovery of nucleus; Nuclear density; Mass number; Atomic number; Atomic mass; Isotopes; Einstein's mass-energy relation, Mass Defect; Binding energy; Fission and fusion. LH6
5. **Radioactivity-** Alpha-particles; Beta-particles, Gamma rays; Laws of radioactive disintegration; Half-life and decay constant; Geiger-Muller Tube; Radio carbon dating; Medical use of nuclear radiation; Health hazards and safety precautions. LH7
6. **Nuclear Energy and Other Sources of Energy-** Sources of energy; Conservation and degradation of energy; Transformation of energy. Nuclear energy: Energy released from fission and fusion; Thermal and Hydroelectric power; Wind energy; Biofuels; Solar energy; Solar constant; Solar devices; Global energy consumption pattern and demands; Energy use in Nepal. Fuels and pollution: Global Warming; Acid rain. LH9
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WAVE MOTION

1
CHAPTER

1.1 Introduction

When we throw a stone in a quiet pond, nice circular ripples emerge on the surface of water that move in a concentric pattern outward from the point of disturbance as shown in Fig. 1.1. These ripples are the waves or more precisely surface waves. Though the spreading pattern of these surface waves seem nice and simple, the physics behind it is quite complex.

Fig. 1.1: Ripples in a pond

Actually, the stone displaces the water molecules at the point of impact from their equilibrium positions. These molecules execute a back and forth vibration and in doing so, all other neighbouring molecules through out the surface are forced to do the same about their mean positions. So, a kind of disturbance seems to propagate from the point of impact in radially outward direction. This disturbance travelling from one point to another point is called a wave in motion. During wave motion, the particles though displaced from their mean position, do not actually travel from one point to another. Rather, they transfer their energies to neighbouring molecules during the vibration and it is the energy that is being transported from one point to another. Thus, we can say that wave motion is a mode of energy transfer from one point to another point.

Fig. 1.2: Queue in front of temple

As an analogy, following example is relevant to understand the wave motion. When a person standing at last of a very long queue pushes another person in front of him, he loses his balance and all other person ahead in the queue receive a gentle push and hence lose balance to some extent as shown in Fig. 1.2. However, all of them in the line manage to return back to their initial position. Therefore, the disturbance in the form of push created at the end of a queue travelled throughout the queue to the front. But, there is no actual displacement of the person in the queue from end to front. This is the real way of wave motion in a material medium. On contrary, a running stream of water carries energy with itself as it moves along. This is not the way of energy transfer in discussion for our present situation.

1.2 Wave Motion

We can define a wave as an activity that transmits energy from one point to another point without actual transfer of matter. The most common waves that we come through in our daily life are sound waves, water ripples, light waves, etc. It might appear in water ripples that, the water has moved along the wave from its initial position; however it is not the case. In fact, at the onset of water ripples the water molecules vibrate up and down and transfer its energy to the neighbouring water molecules and thus, a chain of energy transfer is created without transfer of molecules from its mean position.

Characteristics of Wave Motion

i. Wave motion is a disturbance propagating in a medium.
ii. It transfers energy as well as momentum from one point to another.
iii. It has finite and fixed speed depending on the nature of the medium and is given by $v = f \lambda$.
iv. When it travels in a medium, there is a continuous phase difference among the successive medium particles.
v. The vibrating particles of the medium posses a kinetic as well as potential energy.
vi. The phenomena such as reflection, refraction, interference and diffraction are shown by all types of waves but polarization is shown only by transverse wave.

Types of Wave Motion

There are three ways of energy transfer by waves and hence there are three types of wave motion.

i. Electromagnetic wave
ii. Mechanical wave
iii. Matter wave

i. Electromagnetic wave: *The wave which does not require medium for its propagation is called electromagnetic wave.* For example, light, heat, radio waves. The magnitude of electromagnetic field varies during propagation of electromagnetic wave. All electromagnetic waves such as γ-rays, X-rays, microwaves etc. are non-mechanical waves.

ii. Mechanical wave: *The wave which requires medium for its propagation is called mechanical wave*. For example, waves on springs and strings, water waves, sound waves, seismic waves etc. are mechanical waves. For the propagation of a mechanical wave, the medium should have two properties: elasticity, and inertia. Due to elastic property of a medium, the mechanical wave is also called an elastic wave. The medium must be continuous to propagate such wave.

iii. Matter wave: *The waves associated with the microscopic particles such as electrons, protons, neutrons, atoms and molecules, when they are in motion are called matter waves.* The concept of matter wave was first introduced by de Broglie, so it is also called de Broglie waves. Although, these waves can be generalized to the large mass objects, they are not detectable. Matter waves are very important for the quantum mechanical description of matters. Electronic waves (i.e. matter waves) are used to visualize the very small particles in electron microscope.

Types of Mechanical Waves

There are two types of a mechanical wave based on the direction of vibration of medium particles or the fields.

i. Transverse wave
ii. Longitudinal wave

i. Transverse wave

If the particles of a medium vibrate perpendicularly to the propagation of the wave, then the wave is called transverse wave. These waves travel in the form of crests and troughs as shown in Fig. 1.3. For example, waves on strings, water ripples etc.

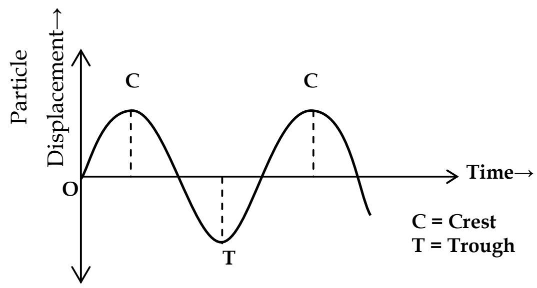


Fig. 1.3: Transverse wave

Propagation of Transverse Wave

Wave transports energy from one point to another. In transverse wave, the direction of oscillation of particles in a medium and direction of propagation of wave are perpendicular to each other. To study the propagation of transverse wave, consider nine points on a medium in which every point lies in phase difference of $\frac{T}{8}$, where T is the time period of oscillation of particles. It means, the disturbance of every preceding point transfers it to succeeding point after the time period $\frac{T}{8}$. The process of wave propagation is described below.

i. When t = 0, the particle 1 remains at rest as shown in Fig 1.4 (i), the displacement of the particle is determined by,

$$y(t = 0) = a \sin \omega t = a \sin 0 = 0$$

ii. When $t = \frac{T}{8}$, the particle 1 executes simple harmonic motion (SHM) with displacement,

$$y\left(t = \frac{T}{8}\right) = a \sin \omega\frac{T}{8} = a \sin \frac{2\pi}{T} \cdot \frac{T}{8} = \frac{a}{\sqrt{2}}$$

At the same instant, the particle 2 just starts SHM as shown in Fig 1.4 (ii)

iii. When $t = \frac{2T}{8}$, the particle 1 executes SHM with displacement

$$y\left(t = \frac{2T}{8}\right) = a \sin \omega\frac{2T}{8} = a \sin \frac{2\pi}{T} \cdot \frac{2T}{8} = a$$

In this condition, the particle displaces with maximum amplitude in positive direction.

At that instant, particle 2 is displaced by $\frac{a}{\sqrt{2}}$ and particle 3 just starts SHM as shown in Fig 1.4 (iii).

iv. When $t = \frac{3T}{8}$, the particle 1 executes SHM with displacement

$$y\left(t = \frac{3T}{8}\right) = a \sin \omega \cdot \frac{3T}{8} = a \sin \frac{2\pi}{T} \cdot \frac{3T}{8} = \frac{a}{\sqrt{2}}$$

At the same instant, particle 2 has maximum displacement, particle 3 is displaced by $\frac{a}{\sqrt{2}}$ and particle 4 starts executing SHM as shown in Fig 1.4 (iv).

v. When $t = \frac{4T}{8}$, the particle 1 executes SHM with displacement

$$y\left(t = \frac{4T}{8}\right) = a \sin \omega \cdot \frac{4T}{8} = a \sin \frac{2\pi}{T} \cdot \frac{4T}{8} = 0$$

Particle 1 returns to the mean position, particles 2, 3, and 4 have displacements $\frac{a}{\sqrt{2}}$, a, $\frac{a}{\sqrt{2}}$ respectively. Particle 5 starts executing SHM as shown in Fig 1.4 (v)

Similarly, the displacement of particle 1 executes SHM in next half cycle making the displacement as shown in Fig. 1.4 (vi), (vii), (viii), (ix).

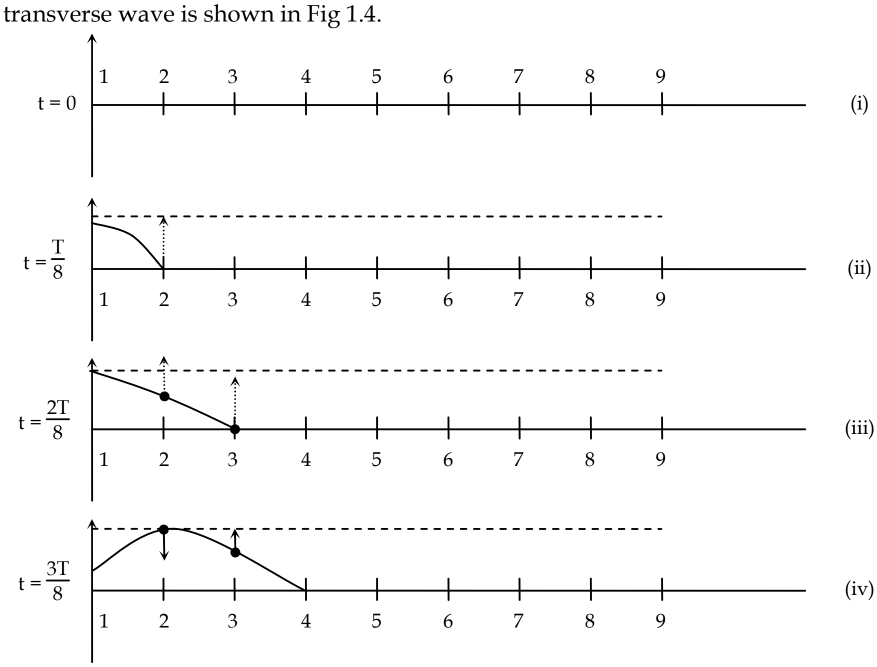
vi. When $t = \frac{5T}{8}$, $y\left(t = \frac{5T}{8}\right) = a \sin \frac{2\pi}{T} \cdot \frac{5T}{8} = -\frac{a}{\sqrt{2}}$

vii. When $t = \frac{6T}{8}$, $y\left(t = \frac{6T}{8}\right) = a \sin \frac{2\pi}{T} \cdot \frac{6T}{8} = -a$

viii. When $t = \frac{7T}{8}$, $y\left(t = \frac{7T}{8}\right) = a \sin \frac{2\pi}{T} \cdot \frac{7T}{8} = -\frac{a}{\sqrt{2}}$

ix. When $t = \frac{8T}{8} = T$, $y\,(t = T) = a \sin \frac{2\pi}{T} \cdot T = 0$

Thus, the transverse wave propagates in a medium. The process of formation of a complete transverse wave is shown in Fig 1.4.



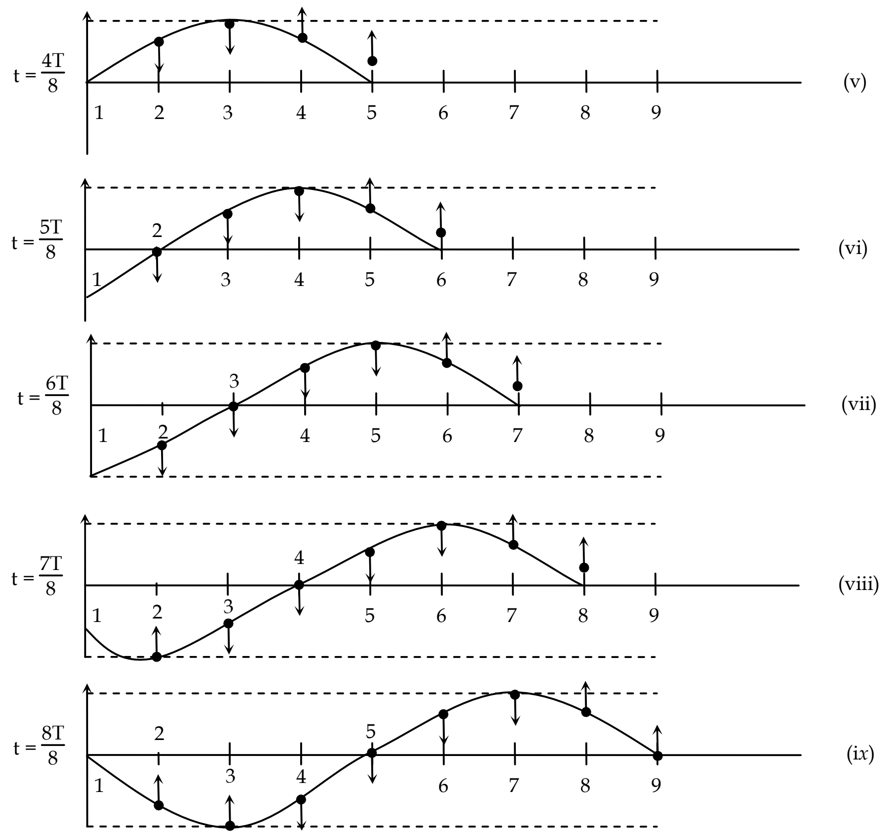


Fig. 1.4: Propagation of transverse wave

ii. Longitudinal wave

If the particles of a medium vibrate along the direction of propagation of the wave, the wave is called longitudinal wave. These waves travel in the form of compressions and rarefactions as shown in Fig. 1.5. During compressions and rarefactions, the pressure of the medium changes. That is why, they are also called pressure or compression waves. For example, waves on springs along length, sound waves in air etc. Whenever, a wave propagates in a medium, there is transfer of energy from one point to the another but, the net displacement of the particle is zero. So, when a particle in an elastic medium is disturbed from its mean position, a restoring force (property of elastic medium) acts in it; as a result it executes SHM. But, the disturbance in the form of energy is transferred to the surrounding particles and this disturbance forms a pattern of propagation known as wave propagation. The graphical representation of longitudinal wave is shown in Fig. 1.6.

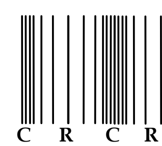


Fig 1.5: Propagation of longitudinal wave

Propagation of Longitudinal Wave

Consider an elastic medium in which the particles of the medium have to and fro motion (i.e. simple harmonic motion). Consider nine particles 1, 2, 3, 4, 5, 6, 7, 8 and 9 arranged linearly. At t = 0, all the particles occupy their mean position. When the particle at 1 is disturbed, then the disturbance is transferred to all other particle continuously. The transfer of disturbance can be explained below.

i. When t = 0, particle 1 is set into vibration.

ii. When $t = \frac{T}{8}$, particle 1 sends the disturbance to particle 2. So, particle 2 is set into vibration.

iii. When $t = \frac{2T}{8}$, particle 1 reaches to extreme position, particle 2 sends the disturbance to particle 3 and hence, particle 3 is set into vibration.

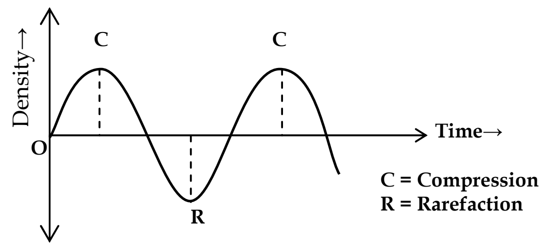


Fig. 1.6: Wave form of longitudinal wave

iv. When $t = \frac{3T}{8}$, particle 1 starts returning back to the left. Particle 2 reaches to extreme position. Particle 3 sends the disturbance to the particle 4 and is set into vibration.

v. When $t = \frac{4T}{8}$, particle 1 returns backs to its mean position. Particle 2 starts returning back to the left. Particle 3 reaches to extreme position. Particle 4 sends disturbance to particle 5.

Similarly, the disturbance travels to particles 6, 7, 8 and 9. Then, particle 1 starts oscillating in the opposite direction. Hence, the one cycle of oscillation is completed at time $t = T = \frac{8T}{8}$. Thus, the longitudinal wave transfers energy (disturbance) in an elastic medium. The formation of complete longitudinal wave is shown in Fig. 1.7.

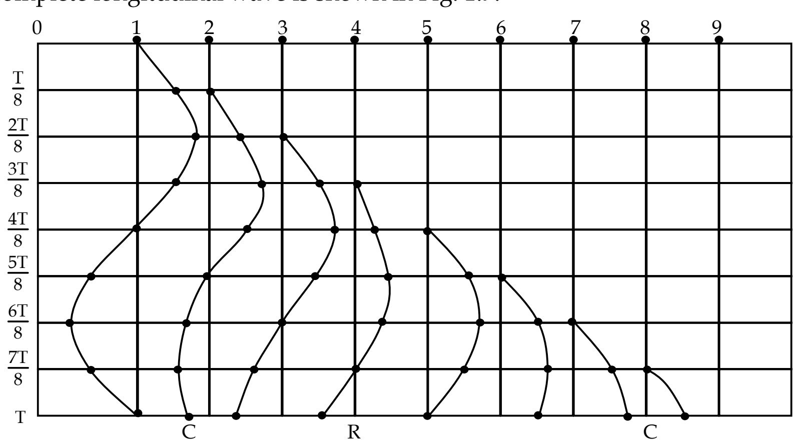


Fig. 1.7: Propagation of longitudinal wave

Difference between Longitudinal Wave and Transverse Wave

Longitudinal Wave	Transverse Wave
1. The particles of the medium vibrate along the direction of propagation of waves.	1. The particles of the medium vibrate at right angle to the direction of propagation of wave.
2. In this type of wave motion, a series of compressions and rarefactions are formed. One compression and one rarefaction constitute one wave.	2. In this type of wave motion, compressions and rarefactions are not formed. One crest and one trough constitute one wave.

3. Sound waves in air and water medium travel as longitudinal wave.	3. Light waves are transverse in nature.
4. It can travel in all types of media, solid, liquid and gas.	4. It can travel in solid and in liquid at lower depth from the surface but not in gases.
5. The pressure and density vary and are maximum at the compression region and minimum at rarefaction region.	5. The pressure and density remain the same through out any region.
6. If a wave is longitudinal it is mechanical, but if a wave is mechanical it may or may not be longitudinal.	6. If a wave is non-mechanical, it is transverse, but if a wave is transverse it may or may not be non-mechanical.

1.3 Graphical Representation of Waves

When a disturbance is created at a point of a medium, the particles in the medium get displaced from their mean position. This displacement of the particles imparts disturbances to the neighbouring particles. Thus, the disturbance travels to the surroundings forming a regular pattern of vibration of particles in the medium, which is called wave. The direction of displacement of the particles may be parallel or perpendicular to the direction of propagation of wave. The net displacement of particle is zero, although the disturbance travels long distance away. In this process, the particles execute simple harmonic motion (SHM). Therefore, the displacement of particles in a medium can be written in terms of equation of SHM, when wave travels,

$$y = a \sin\left(\omega t - \frac{2\pi}{\lambda} x\right) \qquad \ldots (1.1)$$

Physically, this equation implies that, the particle displacement (y) depends on two variables; the distance of wave propagation (x) from the mean position, and instantaneous time of oscillation (t) of a particular particle. They are explained as follows:

i. **Displacement versus distance graph:** If the displacement of a particle is taken along y-axis and distance of wave propagation along x-axis, the graph so drawn is called displacement versus distance graph as shown in Fig. 1.8. In transverse wave, the displacement of particles and distance of wave propagation are perpendicular to each other. So, it is easier to visualize the graph. But, in case of longitudinal wave, the direction of particle displacement (y) is parallel to the direction of wave propagation. Nevertheless, the graphical representation can be visualized by taking the displacement of particles (y) perpendicular to the direction of propagation of wave. Therefore, the nature of graph is shown in the Fig. 1.8.

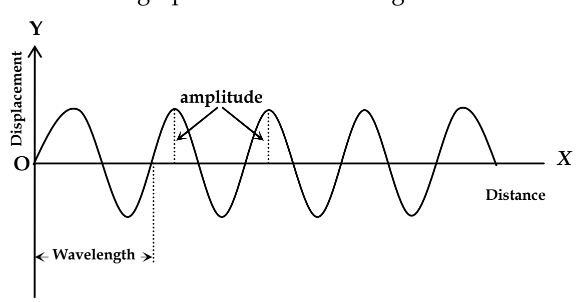


Fig. 1.8: Displacement-distance graph for a wave

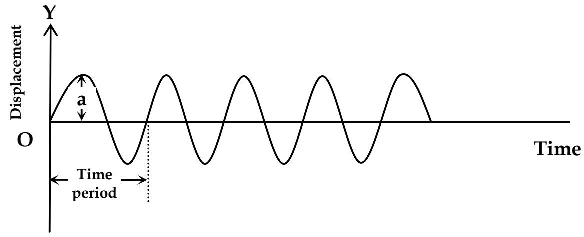


Fig. 1.9: Displacement-time graph for a wave

ii. **Displacement versus time graph:** If the displacement of particle is taken along y-axis and time of oscillation of a particle is taken along *x*-axis, the graph so drawn is known as displacement versus time graph. The nature of the graph is shown in Fig. 1.9.

1.4 Basic Terminologies of Wave

Compression (C): *The region at which the particles in the medium come closer is known as compression region*. In this region, the particles in a medium come closer and hence density and pressure increase as shown in Fig. 1.10.

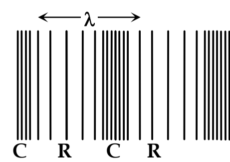


Fig 1.10: Longitudinal wave

Rarefaction (R): The region at which the particles in the medium move away from each other is called rarefaction region. In this region, the particles in a medium move away from each other and hence density and pressure decrease as shown in Fig. 1.10.

Crest: The position of maximum positive displacement i.e. the upper-most point of the transverse wave is called crest. In Fig. 1.11, C symbolizes the crest.

Trough: The position of maximum negative displacement i.e. the lowest point of the transverse wave is called trough. In Fig. 1.11, T symbolizes the trough.

Wavelength: The distance travelled by a wave in one complete cycle is called wavelength. It is denoted by λ. It is the distance between, either any two nearest crests or troughs in case of transverse waves (or any two rarefactions or compressions in case of longitudinal waves).

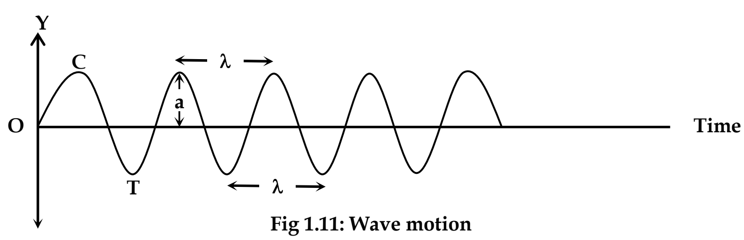


Fig 1.11: Wave motion

The wavelength can also be defined as the separation between any two nearest points which are in the same phase.

Amplitude: The maximum displacement of the particles in a medium about their mean position is known as amplitude. It is denoted by 'a' or 'A'.

Time period: The time in which a particle of medium completes one vibration about its mean position is known as time period of wave. It is denoted by 'T'.

Frequency: The number of oscillations per second is called frequency. It is denoted by 'f'. It can also be defined as the number of waves passing through a point per unit time. For N number of complete waves, the frequency is,

$$\therefore \quad f = \frac{N}{t} = \frac{N}{NT} = \frac{1}{T}$$

It is measured in cycle/s which is also called hertz.

$$\therefore \quad 1 \text{ Hz} = 1 \text{ cycle/s.}$$

Wave speed: The linear distance covered by a wave per unit time in its direction of propagation is called its wave speed.

$$\text{Wave speed (v)} = \frac{\text{Distance along propagation of wave}}{\text{Time taken}}$$

As we know, the wave travels distance 'λ' in time period T. So,

$$v = \frac{\text{Wave length } (\lambda)}{\text{Time period (T)}}$$

$$v = \frac{1}{T} \cdot \lambda$$

$$\mathbf{v = f\lambda} \qquad \text{... (1.2)}$$

i.e., wave speed = frequency × wave length

Equation (1.2) is an important relation between the speed of a wave, its frequency and wave length. This relation is valid for all kinds of waves including mechanical and electromagnetic waves.

Particle Speed

The longitudinal wave propagates due to the oscillation of molecules of an elastic medium. *The speed of particle when it oscillates to transfer the energy from one particle to another is known as particle speed.* The displacement of a particle from its mean position is written as,

$$y = a \sin (\omega t - \phi)$$

$$\frac{dy}{dt} = a\omega \cos (\omega t - \phi)$$

$$\therefore \text{ Speed of particles, } v = \frac{dy}{dt} = a\omega \cos (\omega t - \phi)$$

The velocity of oscillating particles depends on its phase, varying from zero to maximum. The maximum value of speed of particle is,

$$\mathbf{v_{max} = a\omega} \qquad \text{... (1.3)}$$

Phase of a Wave

The position of an oscillating particle during time 't' can be described in terms of angular displacement from its mean position. This angular displacement of the oscillating particle in a medium which describes its location is known as phase or phase angle of a wave. The wave equation for simple harmonic motion is,

$$y = a \sin \omega t$$

The angular term 'ωt' gives the phase of a wave.

When one wave is ahead of another by some angle, the difference of angle between them is represented by phase difference (ϕ). Then, the phase of oscillation is represented by (ωt – ϕ) or (ωt + ϕ).

Therefore, the wave equation of SHM is,

$$y = a \sin (\omega t - \phi)$$

For a wave, moving opposite to the above condition,

$$y = a \sin (\omega t + \phi)$$

Relation between Phase Difference and Path Difference

The path refers the linear displacement and phase refers the angular displacement of two points in wave propagating medium. Therefore, the linear displacement of two points in a medium is called path difference of these points of the wave. It is denoted by x. Similarly, the angular displacement of two points in a wave is called their phase difference. It is denoted by ϕ.

Consider a OA wave travelling along x-direction as shown in Fig. 1.12. Let us take a point P in the x-axis at distance x from the origin O. The relation between phase difference (ϕ) and path difference (x) of two points O and P are as follows: As we know, for path difference λ, the phase difference is 2π.

For path difference λ, the corresponding phase difference is 2π.

For path difference 1, the corresponding phase difference is $\frac{2\pi}{\lambda}$.

For, path difference x, the corresponding phase difference is $\frac{2\pi}{\lambda} x$.

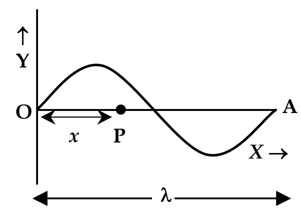


Fig. 1.12: A complete cycle of wave

Therefore,

$$\textbf{Phase difference } (\phi) = \frac{2\pi}{\lambda} x \qquad \text{. . . (1.4)}$$

i.e. $$\text{phase difference } (\phi) = \frac{2\pi}{\text{wavelength } (\lambda)} \times \text{path difference } (x)$$

This is the relation between path difference and phase difference.

The waves are said to be in the same phase if they have a phase difference of even integral multiples of π. Similarly, the waves are said to be out of phase (opposite phase), if they have a phase difference of odd integral multiples of π.

1.5 Progressive Wave

A wave which travels forward in a medium with maximum transfer of energy from one particle to another particle is called progressive wave. Progressive wave is also called travelling wave. For example, water wave, light wave, sound wave, etc. are progressive waves.

Progressive Wave Equation

Consider a progressive wave travelling along x-direction from a reference origin O with speed v. The displacement time graph for the progressive wave is shown in Fig. 1.13.

The particles in the medium execute simple harmonic motion, while the progressive wave travels from one point to another. The wave equation for such condition is written as,

$$y = a \sin (\omega t - \phi) \qquad \text{. . . (1.5)}$$

Where, y = displacement of a particle in a medium
a = amplitude
ω = angular velocity
ϕ = phase difference

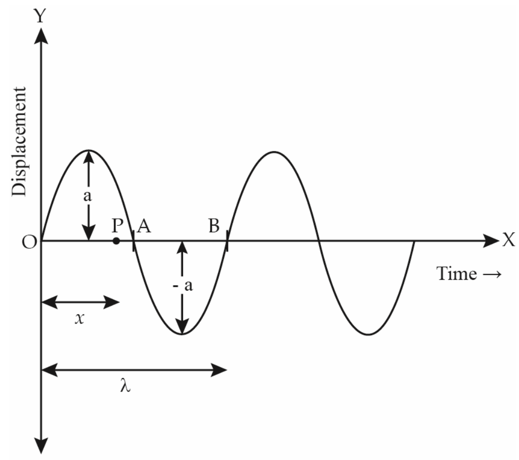


Fig. 1.13 : Progressive wave

Also, the phase difference is related to the path difference. So,

$$\phi = \frac{2\pi}{\lambda} x$$

Putting this value of ϕ in equation (1.5), we get,

$$\mathbf{y = a \sin\left(\omega t - \frac{2\pi}{\lambda} x\right)} \qquad \text{. . . (1.6)}$$

Now, taking $\omega = \frac{2\pi}{T}$ in equation (1.6), we get,

$$y = a \sin\left(\frac{2\pi}{T} t - \frac{2\pi}{\lambda} x\right)$$

$$\mathbf{y = a \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right)} \qquad \text{. . . (1.7)}$$

Also, $\omega = 2\pi f = 2\pi \frac{v}{\lambda}$, v is the wave velocity. So, equation (1.6) can also be written as,

$$y = a \sin\left(\frac{2\pi vt}{\lambda} - \frac{2\pi}{\lambda} x\right)$$

$$\therefore \quad \mathbf{y = a \sin \frac{2\pi}{\lambda} (vt - x)} \qquad \text{. . . (1.8)}$$

The term $\frac{2\pi}{\lambda}$ is called propagation constant or wave vector denoted by k i.e. $k = \frac{2\pi}{\lambda}$. So, equation(1.6) can be written as,

$$\mathbf{y = a \sin (\omega t - kx)} \qquad \text{. . . (1.9)}$$

Equations (1.6), (1.7), (1.8) and (1.9) are the progressive wave equations written in several alternative forms.

If the wave travels in opposite direction i.e. along negative X-axis, the equation (1.9) becomes,

$$\mathbf{y = a \sin (\omega t + kx)} \qquad \text{. . . (1.10)}$$

Hence, the general progressive wave equation is given by,

$$\mathbf{y = a \sin (\omega t \pm kx)} \qquad \text{. . . (1.11)}$$

Characteristics of a Progressive Wave

i. Every particle of a medium executes periodic motion.

ii. The amplitude of each particle of the medium is same, but there exists phase difference between them.

iii. The distance between two successive crests of a transverse wave and distance between a compression and rarefaction is wavelength.

iv. The change in pressure and density of a medium is similar in case of progressive waves.

v. A progressive wave travels forward, undamped and unobstructed.

vi. No particle remains permanently at rest.

vii. Energy is transferred across every plane along the direction of propagation.

viii. The progressive wave may be longitudinal or transverse.

1.6 Differential Form of Wave Equation

The general wave equation is

$$\mathbf{y = a \sin (\omega t - k\mathit{x})} \quad \text{. . . (1.12)}$$

Differentiating equation (1.12) with respect to 't',

$$\frac{dy}{dt} = a\omega \cos (\omega t - kx)$$

Again, differentiating,

$$\frac{d^2y}{dt^2} = -a\omega^2 \sin (\omega t - kx)$$

$$= -\omega^2 y$$

$$\therefore \mathbf{y = -\frac{1}{\omega^2}\frac{d^2y}{dt^2}} \quad \text{. . . (1.13)}$$

Now, differentiating (1.12) with respect to 'x',

$$\frac{dy}{dx} = -ak \cos (\omega t - kx)$$

Again, differentiating,

$$\frac{d^2y}{dx^2} = -ak^2 \sin (\omega t - kx)$$

$$= -k^2 y$$

$$\therefore \mathbf{y = -\frac{1}{k^2}\frac{d^2y}{dx^2}} \quad \text{. . . (1.14)}$$

Equating (1.13) with (1.14), we get,

$$-\frac{1}{\omega^2}\frac{d^2y}{dt^2} = -\frac{1}{k^2}\frac{d^2y}{dx^2}$$

$$\frac{k^2}{\omega^2}\frac{d^2y}{dt^2} = \frac{d^2y}{dx^2}$$

$$\frac{1}{v^2}\frac{d^2y}{dt^2} = \frac{d^2y}{dx^2} \qquad \left[\because \frac{k^2}{\omega^2} = \frac{4\pi^2/\lambda^2}{4\pi^2 f^2} = \frac{1}{f^2\lambda^2} = \frac{1}{v^2}\right]$$

$$\therefore \mathbf{\frac{d^2y}{dx^2} - \frac{1}{v^2}\frac{d^2y}{dt^2} = 0} \quad \text{. . . (1.15)}$$

Equation (1.15) is the differential form of wave equation.

Principle of Superposition of Waves

When two or more waves meet simultaneously at a point of a medium, the particles in the medium oscillate with new displacement so that a new wave pattern is formed. This phenomenon of formation of new wave by mixing of two or more waves is known as superposition of wave.

The principle of superposition of waves states that the resultant displacement of the particle is equal to the vector sum of individual displacements due to different waves. If y be the resultant displacement of a particle and $y_1, y_2, \ldots$ are displacements due to individual waves, then according to the principle of superposition of waves, we have

$$y = y_1 \pm y_2 \pm \ldots \qquad \ldots (1.16)$$

1.7 Interference of Sound

The phenomenon of redistribution of energy in the resultant sound wave formed by the superposition of two sound waves having same frequency (or wavelength) and constant phase difference when travelling in same direction is called interference of sound wave. There are two types of interference of a wave.

i. Constructive interference

ii. Destructive interference

The amplitude becomes maximum in the constructive interference and hence intensity of sound becomes maximum. In the destructive interference, amplitude and intensity becomes minimum.

In the process of interference, there is only transference of energy from one part to another. The energy missing at one point re-appears at another point. There is only redistribution of energy without any destruction or creation of energy, and so the law of conservation of energy is fully obeyed. Interference occurs in both transverse and longitudinal waves.

Expression of Interference of Two Waves

Let y_1 and y_2 be the displacements of particles in a medium due to waves of same angular frequency ω. Let a_1 and a_2 be the arbitrary amplitudes of these waves when travelling in the same direction with phase difference ϕ. The wave equations for these waves are,

$$\mathbf{y_1 = a_1 \sin(\omega t - kx)} \qquad \ldots (1.17)$$

$$\mathbf{y_2 = a_2 \sin(\omega t - kx + \phi)} \qquad \ldots (1.18)$$

Where, $k = \frac{2\pi}{\lambda}$, called wave vector

Applying superposition principle,

$$\mathbf{y = y_1 + y_2} \qquad \ldots (1.19)$$

Using (1.17) and (1.18) in (1.19), we get,

$$\begin{aligned} y &= a_1 \sin(\omega t - kx) + a_2 \sin(\omega t - kx + \phi) \\ &= a_1 \sin(\omega t - kx) + a_2 \sin(\omega t - kx) \cos\phi + a_2 \cos(\omega t - kx) \sin\phi \\ &= (a_1 + a_2 \cos\phi) \sin(\omega t - kx) + (a_2 \sin\phi) \cos(\omega t - kx) \end{aligned}$$

Putting,

$$\mathbf{a_1 + a_2 \cos\phi = A \cos\theta} \qquad \ldots (1.20)$$

$$\mathbf{a_2 \sin\phi = A \sin\theta} \qquad \ldots (1.21)$$

Where, A is the amplitude of resultant wave and θ is the phase angle.

Therefore,

$$y = A\cos\theta\sin(\omega t - kx) + A\sin\theta\cos(\omega t - kx)$$

$$\mathbf{y = A\sin(\omega t - kx + \theta)} \qquad \ldots (1.22)$$

The equation (1.22) gives the wave equation of a harmonic wave.

To find the amplitude of resultant wave, the equations (1.20) and (1.21) can be squared and added,

$$A^2\cos^2\theta + A^2\sin^2\theta = (a_1 + a_2\cos\phi)^2 + (a_2\sin\phi)^2$$

$$A^2(\cos^2\theta + \sin^2\theta) = a_1^2 + 2a_1a_2\cos\phi + a_2^2\cos^2\phi + a_2^2\sin^2\phi$$

$$A^2 = a_1^2 + 2a_1a_2\cos\phi + a_2^2$$

$$\mathbf{A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\phi}} \qquad \ldots (1.23)$$

To find the phase angle, dividing (1.21) by (1.20), we get,

$$\frac{A\sin\theta}{A\cos\theta} = \frac{a_2\sin\phi}{a_1 + a_2\cos\phi}$$

$$\tan\theta = \frac{a_2\sin\phi}{a_1 + a_2\cos\phi}$$

$$\therefore \quad \boldsymbol{\theta = \tan^{-1}\left(\frac{a_2\sin\phi}{a_1 + a_2\cos\phi}\right)} \qquad \ldots (1.24)$$

Cases:

i. When original waves overlap in phase, i.e. $\phi = 0$.

$A = a_1 + a_2$

For, $a_1 = a_2 = a$ $\quad$ $A = 2a$, (maximum amplitude)

This interference is called constructive interference. In Fig.1.14, two waves (represented by dotted lines) moving along the same positive x direction with same frequency and phase superimpose to form a single wave (represented by a solid line) which has same frequency but, has maximum amplitude (constructive interference).

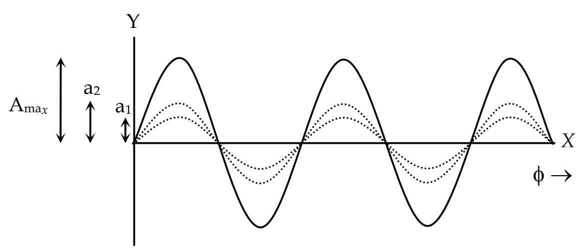


Fig.1.14: Constructive interference of two waves

ii. When original waves overlap out of phase, $\phi = 180°$

$A = a_1 - a_2$

For $a_1 = a_2$ $\quad$ $A = 0$ (minimum amplitude)

This interference is called destructive interference. In Fig. 1.15, two waves (represented by dotted lines) moving along the same +ve x-direction with same frequency but opposite phase superimpose to form a single wave (represented by solid line) which has same frequency but has minimum amplitude (destructive interference).

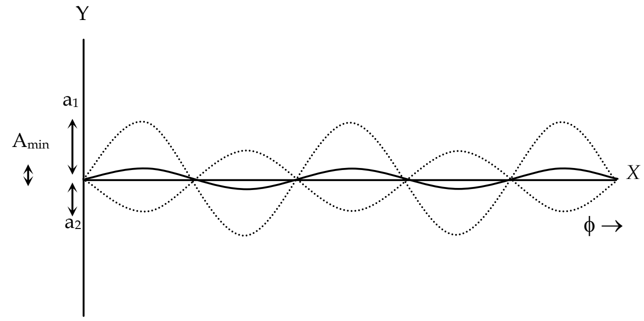


Fig 1.15: Destructive interference of two waves ($a_1 > a_2$)

1.8 Stationary Wave

When two progressive waves of same amplitude and frequency travel in a medium in exactly the opposite direction, a resultant wave is formed. This resultant wave is called stationary wave. It is also called standing wave. No energy is transferred from a particle to surrounding particles while stationary wave is formed in a medium. Each particle has its own characteristics of vibration. Hence, the amplitude of vibration of the different particles are different, ranging from zero to some maximum value. The position of particle at the zero displacement is called node (N) and the position of particle at which the maximum displacement takes place is called antinode (AN). The formation of stationary wave in a string is explained below:

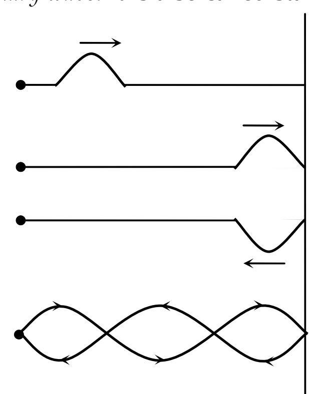


Fig. 1.16: Formation of stationary wave

Consider a rope tied to a rigid support of pole at one end, the next end being held by our hand. Now let us give a gentle up and down jerk to the free end so that a pulse travels along the length of string which reaches the next end until it is reflected back by the rigid pole. After reflection, the pulse again travels along the length of string but changes its direction.

Thus, the reflected pulse (wave) travel back to our hand and hence we have now two waves which are traveling in the opposite directions and they combine to produce a resultant wave which appears to be stationary wave Fig. 1.16. We cannot see the original wave and reflected wave separately, only a stationary wave is visible.

The frequency of the progressive wave and the stationary wave is the same. When stationary waves are formed, the amplitude becomes maximum and strain becomes minimum at certain points. At other certain points, the amplitude becomes minimum and strain becomes maximum. The points, where the amplitude is maximum and strain is minimum, are called antinodes (A.N.) and the points, where the amplitude is minimum and strain is maximum are called nodes (N). Antinodes and nodes are formed alternately in the standing wave. Thus, the wave in which antinodes and nodes are formed alternately is called a stationary wave.

Stationary Wave Equation

Let y_1 and y_2 be the displacements of two progressive waves of same amplitude a and wave length λ travelling in opposite direction simultaneously with the same velocity v. The equations of these waves may be expressed as follows,

$$\mathbf{y_1 = a \sin (\omega t - kx)} \quad \ldots (1.25)$$

$$\mathbf{y_2 = a \sin (\omega t + kx)} \quad \ldots (1.26)$$

Thus, the resultant displacement of the particle of medium due to both the waves will be determined from the principle of superposition,

$$= y_1 + y_2$$

$$= a \sin (\omega t - kx) + a \sin (\omega t + kx)$$

$$= a \left[\sin (\omega t - kx) + a \sin (\omega t + kx)\right]$$

$$\therefore \quad y = 2a \cos kx . \sin \omega t$$

$$\mathbf{y = A \sin \omega t} \quad \ldots (1.27)$$

Equation (1.27) represents a simple harmonic wave whose amplitude is A = 2a cos kx. It is evident that, for different values of x, the amplitude will have different values. Obviously, the frequency of stationary wave is equal to the interfering waves i.e. there is no change in frequency.

Condition for maximum amplitude

The amplitude A = 2 a cos kx will be maximum if, A = ± 2a

So, $2a \cos kx = \pm 2a$

$$\cos kx = \pm 1$$

$$\cos \frac{2\pi x}{\lambda} = \pm 1$$

$$\cos \frac{2\pi x}{\lambda} = \cos n\pi, \text{ where } n = 0, 1, 2, 3, \ldots$$

or, $\dfrac{2\pi x}{\lambda} = n\pi$

or, $$\mathbf{x = \frac{n\lambda}{2}} \quad \ldots (1.28)$$

The equation (1.28) is the condition for antinode formation,

For $n = 0$, $x_0 = 0$, For $n = 1$, $x_1 = \dfrac{\lambda}{2}$

For $n = 2$, $x_2 = \dfrac{2\lambda}{2}$ For $n = 3$, $x_3 = \dfrac{3\lambda}{2}$

Hence, the antinodes occurs at the positions where,

Phase difference (ϕ) = 0, π, 2π, 3π, . . . nπ, and

Path difference (x) = $0, \dfrac{\lambda}{2}, \dfrac{2\lambda}{2}, \dfrac{3\lambda}{2} \ldots, \dfrac{n\lambda}{2}$

Therefore, the condition of antinode is,

$$x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \ldots, \frac{n\lambda}{2}$$

The distance between two consecutive antinodes = $\dfrac{n\lambda}{2} - \dfrac{(n-1)\lambda}{2} = \dfrac{\lambda}{2}$.

Condition for minimum amplitude

The amplitude A = 2a cos kx will be minimum if, A = 0

So, $2a \cos kx = 0$

$$\cos kx = 0$$

$$\cos \frac{2\pi x}{\lambda} = 0$$

$$\cos \frac{2\pi x}{\lambda} = \cos (2n + 1)\frac{\pi}{2}, \text{ where } n = 0, 1, 2, 3, \ldots$$

or, $$\frac{2\pi x}{\lambda} = (2n + 1)\frac{\pi}{2}$$

or, $$\mathbf{x = (2n + 1)\frac{\lambda}{4}} \qquad \ldots \textbf{(1.29)}$$

The equation (1.29) is the condition for node formation.

For n = 0, $x_0 = \frac{\lambda}{4}$ For n = 1, $x_1 = \frac{3\lambda}{4}$

For n = 2, $x_2 = \frac{5\lambda}{4}$ For n = 3, $x_3 = \frac{7\lambda}{4}$

For n = 4, $x_4 = \frac{9\lambda}{4}$ Similarly, $x = \frac{\lambda}{4}, \frac{3\lambda}{4} \ldots, (2n + 1)\frac{\lambda}{4}$.

Hence, the nodes occur at the positions where,

Phase difference (ϕ) = $\frac{\pi}{2}, \frac{3\pi}{2}, \ldots (2n + 1)\frac{\pi}{2}$

Path difference (x) = $\frac{\lambda}{4}, \frac{3\lambda}{4}, \ldots, (2n + 1)\frac{\lambda}{4}$

The distance between two consecutive nodes = $(2n + 1)\frac{\lambda}{4} - \{2(n - 1) + 1\}\frac{\lambda}{4} = \frac{\lambda}{2}$, which is equal to the distance between two consecutive antinodes.

The distance between any consecutive node and antinode = $(2n + 1)\frac{\lambda}{4} - \frac{n\lambda}{2} = \frac{\lambda}{4}$.

1.9 Stationary Waves in Boundary

During the propagation of sound wave in air, the nodes are formed both at rarefactions and compressions but the antinodes are formed in between these rarefactions and compressions. At compression, a cross section is found in which the neighbouring molecules exert pushing force of equal and opposite direction so that this cross section becomes stationary (rest). Similarly at rarefaction, a cross section is found in which the neighbouring molecules exert pulling force equal and opposite direction so that this cross section is also at rest. The wave pattern for stationary wave depends on the type of boundary (the position from which the wave reflects). There are two types of boundaries viz., open boundary and closed boundary.

i. **Stationary wave in open boundary:** Open boundary is that boundary from which the waves are reflected but the particles are not reflected rather they move along in forward direction. Such boundaries do not have rigid surfaces to reflect the wave. From such boundaries, compressions are reflected as rarefactions and vice-versa. The equation of stationary wave in equation (1.27) is determined considering the open boundary. The vibration pattern of particles and wave form in this boundary is as shown in Fig. 1.17.

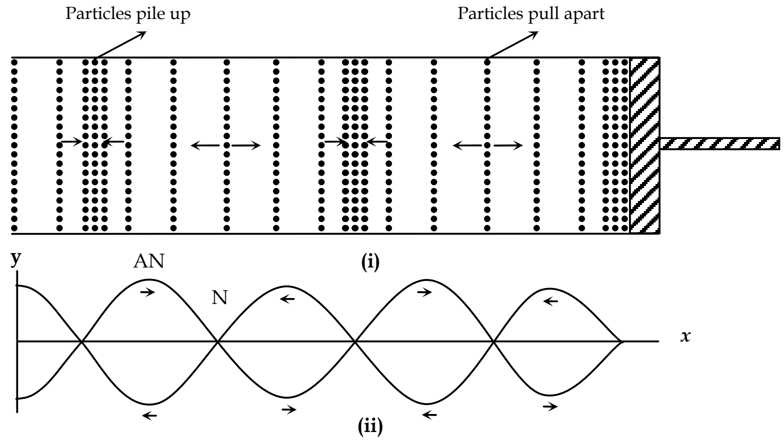


Fig 1.17: (i) Vibration pattern of particles, (ii) Stationary wave in open boundary.

At the boundary, for $x = 0$

$A = 2a \cos kx = 2a = A_{max}$ i.e. antinode is formed.

ii. **Stationary wave in closed boundary:** Closed boundary is such boundary from which both waves and particles reflect back. Such boundaries have rigid surface to reflect the wave. From these boundaries rarefactions are reflected as rarefactions and compressions are reflected as compressions. However, the incident particles reflect with phase reversal of π. So, the displacement equation of particles which are incident on the boundary and reflected from the boundary can be respectively written in the following forms.

$$\therefore \quad \mathbf{y_1 = a \sin (\omega t - kx)} \qquad \ldots (1.30)$$

$$\mathbf{y_2 = a \sin (\omega t + kx + \pi)} \qquad \ldots (1.31)$$

$\therefore$ From superposition principle,

$$\mathbf{y = y_1 + y_2} \qquad \ldots (1.32)$$

Using (1.30) and (1.31) in (1.32)

$$\begin{aligned} y &= a \sin (\omega t - kx) + a \sin (\omega t + kx + \pi) \\ &= a \sin (\omega t - kx) + a \sin (\pi + (\omega t + kx)) \\ &= a \sin (\omega t - kx) - a \sin (\omega t + kx) \\ &= a [\sin (\omega t - kx) - \sin (\omega t + kx)] \\ &= a \left[2 \sin \frac{\omega t - kx - \omega t - kx}{2} \cos \frac{\omega t - kx + \omega t + kx}{2} \right] \\ &= -2a \sin kx \cos \omega t \\ &= \mathbf{A \cos \omega t} \qquad \ldots (1.33) \end{aligned}$$

where, amplitude of resultant wave is

$$A = -2a \sin kx$$

At the boundary $x = 0$, $A = 0$

The vibration pattern of particles and wave form at the closed boundary is shown in Fig 1.18.

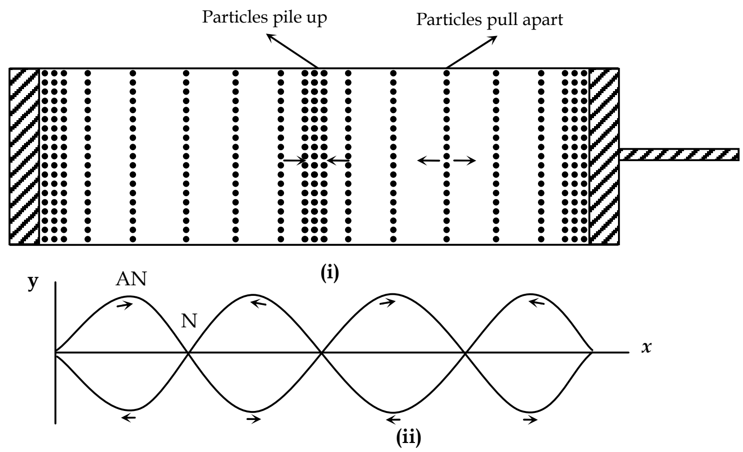


Fig 1.18: (i) Vibration pattern of particles, (ii) Stationary wave in closed boundary.

Characteristics of a Stationary Wave

i. Only the particles other than those at the nodes executes periodic motion.

ii. The phase difference between particles of the medium is same, but amplitude is different.

iii. In case of stationary wave each particle attains its stationary position twice during one complete vibration.

iv. In this wave, nodes and antinodes are formed alternately and the separation between any two consecutive nodes or antinodes is $\lambda/2$.

v. The amplitude is minimum at nodes and maximum at antinodes.

vi. In the stationary wave, the change in pressure and density of the medium is not uniform. It is maximum at the nodes and minimum at antinodes.

vii. The sound is heard more intense at nodes but less at antinodes. This is because the sound is heard due to pressure variation and pressure variation becomes maximum at nodes.

viii. There is energy variation in the standing wave within the limited area where the wave is confined.

ix. The stationary wave also occurs in light waves, radio waves etc.

Difference between Progressive Wave and Stationary Wave

Progressive Wave	Stationary Wave
1. It carries energy in the forward direction.	1. It does not carry any energy.
2. The amplitude of each particle is same but the phase changes continuously.	2. The amplitude of the different particles are different, ranging from zero at the nodes to maximum at anti-nodes.
3. There is no formation of nodes and antinodes in this wave.	3. There is formation of nodes and antinodes in this wave.
4. There is transmission of energy across every plane.	4. There is no transmission of energy across any plane.
5. No particles is permanently at rest.	5. The particles at the nodes are permanently at rest.

6. All the particles attain the same velocity when they pass through their mean position.	6. All the particles attain their own maximum velocity at the same time when they pass through their mean position.
7. In the case of a longitudinal progressive wave all the parts of the medium undergo similar variation of density one after the other. At every point there will be a density variation.	7. In the case of a longitudinal stationary wave, the variation of density is different at different points being maximum at the nodes and zero at the antinodes.

Tips for MCQs

1. Wave nature:

i. Waves in string is one dimensional

ii. Wave in water surface is two dimensional and

iii. Waves of sound wave and light waves are three dimensional

2. Relations:

i. $v = f\lambda$

ii. $\lambda = vT$

iii. $k = \frac{2\pi}{\lambda} = \frac{2\pi f}{v} = \frac{\omega}{v}$

iv. $\omega = 2\pi f = \frac{2\pi}{T}$

3. Frequency is the nature of source, so it does not depend on the medium of wave propagation.

4. About particles:

i. Speed of oscillating particles, $u = \omega\sqrt{a^2 - y^2}$

ii. Maximum speed of particle, $u_{max} = a\omega = akv$

iii. Maximum distance travelled by a particle in one complete cycle in transverse wave is four times of amplitude.

iv. Relation between wave velocity and particle velocity, $u = v\left(-\frac{dy}{dx}\right)$

5. About phase difference and path difference

i. Phase difference (ϕ) = $\frac{2\pi}{\lambda}$ × path difference (x)

ii. $\phi = \frac{2\pi x}{\lambda} = \frac{2\pi}{T}t = 2\pi ft$

iii. For two sources which emit waves of frequencies f_1 and f_2, the phase difference,

$$\phi = \phi_1 - \phi_2 = 2\pi f_1 t - 2\pi f_2 t = 2\pi(f_1 - f_2)\,t$$

iv. If an initial phase ϕ_0 is taken as for consideration, $\phi = (\omega t - kx) + \phi_0$

Wave equation is $y = a \sin\{(\omega t - kx) + \phi_0\}$

a. In open boundary, $\phi_0 = 0$, so the equation of reflected wave, $y_r = a\sin(\omega t + kx)$ ($\because x \rightarrow -x$)

b. In closed boundary, $\phi_0 = \pi$, so the equation of reflected wave, $y_r = a\sin(\omega t + kx + \pi)$

vi. Variation of phase with distance, $\Delta\phi = \frac{2\pi}{\lambda}\Delta x$

vii. Variation of phase with time, $\Delta\phi = \frac{2\pi}{T}\Delta t$

6. The forms of progressive wave equations are

$y = a\sin(\omega t - \phi)$ $\quad$ $y = a\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$

$y = a\sin(\omega t - kx)$ $\quad$ $y = a\sin\frac{2\pi}{\lambda}(vt - x)$

7. The equation of a stationary wave is y = A sin ωt, where A = 2a cos kx is the amplitude of the stationary wave.
8. A wave is a propagating disturbance that carries energy as well as momentum but not matter.
9. A wave is characterized by its amplitude, wavelength and speed.
10. Distance between two particles (i) vibrating out of phase is $\frac{\lambda}{2}$ (ii) vibrating phase is λ.
11. Sound waves can travel in the form of both longitudinal and transverse elastic wave.

Worked Out Problems

1. Radio Nepal Transmission Centre transmits the frequency of 100 MHz in FM service. Find out the wavelength of the waves transmitted from Radio Nepal.

SOLUTION

Given,

Frequency (f) = 100 MHz = 100×10^6 Hz

We know, velocity (c) = 3×10^8 ms^{-1}

Wavelength (λ) = ?

We know,

$$c = f\lambda$$

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{100 \times 10^6} = 3 \text{ m}$$

∴ Wavelength of the waves is 3 m.

2. A sound wave of frequency 400 Hz is travelling in air at a speed of 320 ms^{-1}. Calculate the difference in phase between two points on the wave 0.2 m apart in the direction of travel.

SOLUTION

Given, speed of wave (v) = 320 ms^{-1}

Path difference (Δx) = 0.2 m

Frequency of wave (f) = 400 Hz

Phase difference (Δϕ) = ?

We know,

$$\text{Wavelength } (\lambda) = \frac{v}{f} = \frac{320}{400} = 0.8 \text{ m}$$

$$\text{Now, phase difference } (\Delta\phi) = \frac{2\pi}{\lambda}\Delta x = \left(\frac{2\pi}{0.8}\right) \times 0.2 = 1.57 \text{ rad}$$

3. The equation of motion of a wave is y = 1.2 sin (3.5 t – 0.5 x), where distances and time are expressed in meter and second respectively. Determine the amplitude, frequency, wavelength and velocity of the wave. Also, find the maximum speed of particles in that medium. The equation of a wave with double the amplitude and double the frequency but travelling exactly is the opposite direction.

SOLUTION

The given equation of the wave is,

$$y = 1.2 \sin (3.5\, t - 0.5\, x) \quad \ldots \text{(i)}$$

Comparing equation (i) with general equation of progressive wave,

$$y = a \sin (\omega t - kx) \quad \ldots \text{(ii)}$$

We get,

a = 1.2 m

ω = 3.5 rad s^{-1}

k = 0.5 m^{-1}

Amplitude (a) = 1.2 m. Also,

$\omega = 3.5$

$2\pi f = 3.5$

$$f = \frac{3.5}{2\pi} = 0.56 \text{ Hz}$$

To find the wavelength,

$k = 0.5$

$$\frac{2\pi}{\lambda} = 0.5$$

$\lambda = \frac{2\pi}{0.5} = 12.57$ m

Now, velocity (v) $= f\lambda$

$= 0.56 \times 12.57 = 7.04\ ms^{-1}$

The maximum speed of particles,

$v_{max} = a\omega = 1.2 \times 3.5 = 4.2\ ms^{-1}$

Here, new amplitude (a') = 2a

$= 2 \times 1.2 = 2.4$ m

New frequency f' = 2f

$= 2 \times 0.56$

$= 1.12$ Hz

So, the new wavelength $(\lambda') = \frac{v}{f'} = \frac{7.04}{1.12} = 6.29$ m

$\therefore\ \omega' = 2\pi f' = 7.04\ rad\ s^{-1}$.

Now, the equation of wave that is travelled in opposite direction including the changed amplitude, frequency and wavelength,

$y = a' \sin\left(\omega' t + \frac{2\pi}{\lambda'}x\right)$

$= 2.4 \sin\left(7.04t + \frac{2\pi}{6.29}x\right)$

$y = 2.4 \sin(7.04t + x)$

4. **[HSEB 2053] A wave has the equation (*x* in metres and t in seconds), y = 0.02 sin (30t – 4*x*)**
Find:
   i. **Its frequency, speed and wave length.**
   ii. **The equation of wave with double amplitude but traveling in the opposite direction.**

SOLUTION

Given,

The given equation is

$y = 0.02 \sin(30t - 4x)$

Comparing this equation, with the equation of the standard wave,

$y = a \sin(\omega t - kx)$, where $k = \frac{2\pi}{\lambda}$, we have

$\omega = 30$

or, $2\pi f = 30$

or, $f = \frac{30}{2\pi} = \frac{15}{\pi}$

$\therefore$ Frequency, $f = \frac{15}{\pi} = 4.77$ Hz

$k = 4\ m^{-1}$

or, $\frac{2\pi}{\lambda} = 4$

or, $\lambda = \frac{2\pi}{4} = \frac{\pi}{2}$

$\therefore$ Wavelength, $\lambda = \frac{\pi}{2} = 1.57$ m

and speed, v = ?

We know that,

Speed, $v = \lambda f = \frac{\pi}{2} \times \frac{15}{\pi} = 7.5\ ms^{-1}$

Hence, frequency (f) = 4.77 Hz, speed (v) = 7.5 ms^{-1} and wavelength (λ) = 1. 571 m and the equation of the wave moving in opposite direction and double the amplitude is,

$y = 0.04 \sin(30t + 4x)$

5. **A stationary wave is produced due to the superposition of waves given as,**
$y_1 = 0.1 \sin(4\pi t - 10x)$ and $y_2 = 0.1 \sin(4\pi t + 10x)$
where, y is measured in cm and *x* in meter and t in second. Find the displacement of particle at 2 m away from the origin.

SOLUTION

Given, $y_1 = 0.1 \sin(4\pi t - 10x)$

$y_2 = 0.1 \sin(4\pi t + 10x)$

Applying the superposition principles, $y = 2a \cos kx \cdot \sin \omega t$, we get

$y = 2 \times 0.1 \times \cos 10x \sin 4\pi t$

(Here, a = 0.1 cm and x = 2 m)

Here, resultant amplitude (A) = $0.2 \cos(10 \times 2) = 0.2 \cos 20 = 0.19$ cm

6 **A stone is dropped into a well and a splash is heard affect 2.6 s. Calculate the depth of the well. (Velocity of sound = 334 ms^{-1})**

SOLUTION

Given,

Total time (t) = 2.6 s

Velocity of sound $(v_s) = 334\ ms^{-1}$

In the given problem, total time is the sum of (i) time of stone to reach on water surface after dropping (t_1) (ii) time for sound to come outside the well (t_2)

So,

$t = t_1 + t_2$

$2.6 = t_1 + t_2$ ... (i)

Also, the distance travelled by stone and sound is equal.

Now, distance travelled by stone (d) is

$d = ut_1 + \frac{1}{2} at_1^2$

i.e. $d = 0.t_1 + \frac{1}{2} gt_1^2$

$\therefore$ $d = \frac{1}{2} gt_1^2$ ... (ii)

Then, distance travelled by sound,

$d = v_s \times t_2$

$= 334 \times t_2$ ... (iii)

Now, equating (ii) and (iii), we get,

$334 \times t_2 = \frac{1}{2} gt_1^2$ ... (iv)

Using equation (i) in (iv)

$334 \times (2.6 - t_1) = \frac{1}{2} \times 9.8 \times t_1^2$

$868.4 - 334 t_1 = 4.9 t_1^2$

$4.9 t_1^2 + 334 t_1 - 868.4 = 0$

So, $t_1 = \frac{-334 \pm \sqrt{(334)^2 + 4 \times 4.9 \times 868.4}}{2 \times 4.9}$

$= 2.508$ s

Since t is not taken negative.

So, Using (iii),

$d = 334 (2.6 - 2.508)$

$= 30.73$ m

$\therefore$ The depth of well is 30.73 m.

Challenging Problems

1. **[UP]** A certain transverse wave is described by $y(x, t) = (6.5 \text{ mm}) \cos 2\pi \left(\frac{x}{28.0 \text{ cm}} - \frac{t}{0.0360\text{s}}\right)$.

   Determine the wave's

   a. nature and direction of propagation
   b. amplitude
   c. wavelength
   d. frequency
   e. speed of propagation

   Ans: (b) 6.5×10^{-3} m (c) 28.0×10^{-2} m (d) 27.78 Hz (e) 7.78 m/s

2. **[UP]** Ultrasound is the name given to frequencies above the human range of hearing, which is above 20,000 Hz. Waves above this frequency, can be used to penetrate the body and to produce images by reflecting from surfaces. In a typical scan, the waves travel with a speed of 1500 m/s. For a good detailed image, the wavelength should be no more than 1.0 mm. What frequency is required?

   Ans: 1.5×10^6 Hz

3. **[UP]** The speed of sound in air at 20°C is 344 m/s.

   a. What is the wavelength of sound wave with a frequency of 784 Hz, corresponding to a note on a piano?
   b. What is the frequency of a sound wave with a wavelength of 0.0655 mm?

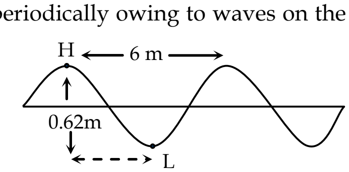
   Ans: (a) 0.439 m (b) 5.25×10^6 Hz

4. **[UP]** Transverse waves on a string have a wave speed 8.00 m/s; only amplitude 0.0700 m, wavelength 0.320 m. The waves travel in the negative x-direction, and t = 0, the x = 0 and of the string has its maximum upward displacement.

   a. Find the frequency, period and wave number of these waves
   b. Write a wave function describing the wave.
   c. Find the transverse displacement of a particle at x = 0.360 m at t = 0.150 s.

   Ans: (a) 25 Hz; 0.0400 s; 19.6 rad/ m (b) 0.0400 s; $y(x, t) = 0.0700 \text{ m} \cos 2\pi\left(\frac{x}{0.32 \text{ m}} + \frac{t}{0.0400 \text{ s}}\right)$ (c) – 0.0495 m

5. **[UP]** A fisher man notices that his boat is moving up and down periodically owing to waves on the surface of water. It takes 2.5 second for the boat to travel from its highest point to its lowest, a total distance of 0.62 m. The fisherman sees that the wave crests are spaced 6 m apart.



   a. What is the amplitude of wave?
   b. How fast are the waves travelling?

6. **[ALP]** If the velocity of sound in air is 340 m/s, calculate (i) wavelength when the frequency is 256 Hz, (ii) the frequency when the wavelength is 0.85 m.

7. Provided that the amplitude is sufficiently great, the human ear can respond to longitudinal waves over a range of frequencies from about 20.0 Hz to above 20,000.0 Hz. Compute the wavelength corresponding to these frequencies (a) for waves in air (v = 344 m/s). (b) for waves in water (v = 1480 m/s).

8. **[ALP]** A small piece of cork in a ripple tank oscillates up and down as ripples pass it. If the ripples travel at 0.20 ms^{-1}, have a wavelength of 15 mm and amplitude of 5.0 mm, what is the maximum velocity of the cork?

Ans: 0.42 ms^{-1}

9. **[ALP]** A progressive and a stationary simple harmonic wave each have the same frequency of 250 Hz and the same velocity of 30 ms^{-1}. Calculate (i) phase difference between two vibrating points on the progressive wave which are 10 cm apart, (ii) the equation of motion of the progressive wave if its amplitude is 0.03 meter, (iii) the distance between nodes in the stationary wave.

Ans: (i) $\frac{5\pi}{3}$ radian (ii) 0.03 sin2π $\left(250t - \frac{25}{3}x\right)$ (iii) 0.06 m

10. **[ALP]** A plane-progressive wave is represented by the equation $y = 0.1 \sin\left(200\pi t - \frac{20\pi x}{17}\right)$ where y is the displacement in millimeters, t is in seconds and x is the distance from a fixed origin O in meters. Find: (i) the frequency of the wave (ii) its wavelength (iii) its speed, (iv) the phase difference in radians between a point 0.25 m from O and a point 1.10 m from O (v) the equation of a wave with double the amplitude and double the frequency but travelling exactly in the opposite direction.

Ans: (i)100 Hz (ii) 1.7 m (iii) 170 ms^{-1} (iv) π radian (v) 0.2 sin $\left(400\pi t + \frac{40\pi x}{17}\right)$

11. **[ALP]** The equation y = a sin (ωt – kx) represents a plane wave travelling in a medium along X-direction, y being the displacement at the point x at time t. If a = 1.0×10^{-7} m, ω = 6.6×10^{3} s^{-1} and k = 20 m^{-1}, calculate (a) speed of the wave, (b) the maximum speed of a particle of the medium due to the wave.

Ans: (a) 330 ms^{-1} (b) 6.6×10^{-4} ms^{-1}

12. **[ALP]** It is noted that a sharp tap made in front of a flight of stone steps gives rise to a ringing sound. Explain this and, assuming that each step is 0.25 m deep, estimate the frequency of the sound. (velocity of sound may be taken to be 340 ms^{-1}). **Ans: 680 Hz**

*[**Note:** Hints to challenging problems are given at the end of this chapter.]*

Conceptual Questions with Answers

1. What is mechanical wave?

➪ The wave which transmits energy via material medium is known as mechanical wave. Sound wave is an example of mechanical wave. It transmits energy and momentum through the limited motion of the particles of the material medium, while the medium remains un-shifted.

2. What is progressive wave?

➪ If a disturbance is produced at a point in a medium continuously, the particles in that medium oscillate continuously. It means, the energy in a medium transmits regularly from a source of disturbance to its surroundings. This type of continuous energy flow in terms of wave pattern is known as progressive wave. As we disturb at a point on water surface, ripples are observed transmitting to the surroundings. This is an example of progressive wave.

3. What is stationary wave?

➪ When two identical waves travel in a bounded medium with equal speed but in opposite directions, they superimpose and form a new type of wave which appears stationary in a medium. Such type of wave is known as stationary wave. The net transfer of energy in stationary wave is zero, hence it is named so. For example, when a wave is sent along a string, it is reflected from the end of the string and the incident and reflected wave superimpose to form a stationary wave in the string.

4. How can you show experimentally that there is a transfer of energy by the wave?

↳ Suppose a stone is thrown in a pond and a leaf is placed near the surface of water at some distance. As the disturbance reaches the leaf, the leaf begins to oscillate up and down about its original position, and the disturbance travels ahead. The oscillation of leaf occurs due to the transfer of energy from the point of disturbance. Thus, we see that there is the transfer of energy by the wave.

5. In which category, transverse or longitudinal the following waves belong? Ripple on water surface, sound wave in air, light wave, waves in string.

↳ The given waves can be categorized as follows:
Longitudinal wave $\rightarrow$ sound wave in air
Transverse wave $\rightarrow$ ripple on water surface, light wave, waves in string

6. Two persons cannot talk to each other in the moon, why? [HSEB 2068]

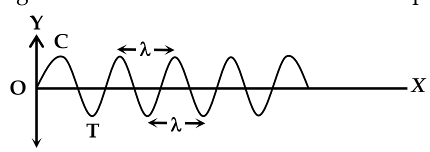
↳ Sound wave is a mechanical wave. It requires medium to travel from one place to another. There is no atmosphere on the moon's surface, i.e. there is no material medium to transfer the sound energy. Therefore, the voice cannot reach from one person to another. Hence, two persons cannot talk to each other in the moon.

7. What is a harmonic wave?

↳ The wave in a medium where the particles repeat their path is called harmonic wave. A wave function described by a sine or a cosine function is called a harmonic function.

8. What is the distance between two consecutive crests in a transverse wave?

↳ A complete wave is equivalent to the distance between two consecutive crests or two consecutive trough. Therefore, the length between two consecutive crests is equal to wavelength (λ) of the wave.



9. Can sound be a transverse wave? Explain.

↳ Yes, sound wave in solid and water surface can be of transverse nature.

In case of solid rod, when it is struck perpendicular to its length, the particles in the rod oscillate perpendicular to the direction of wave propagation along the length. Thus, the transverse wave is obtained. In liquid, the particles on the surface oscillate up and down, however the wave travels along the horizontal surface.

10. A wave transmits energy. Does it transmit linear momentum?

↳ The kinetic energy of a particle is related to the linear momentum by the relation i.e. $p^2 = 2mE$, where p is momentum and E is the kinetic energy of particle. As the energy transfer is non zero, the momentum is also nonzero. It means, the transfer of momentum is possible if a wave transmits energy.

11. Why transverse waves cannot be set up in a gas?

↳ The wave motion depends on two properties of medium: elasticity and inertia. To generate the transverse wave, modulus of rigidity is essential, but there is no rigidity in gas medium, so transverse wave is impossible in gas.

12. The distance between two consecutive nodes in a stationary wave is 20 cm. If the speed of the wave be 330 ms^{-1}, calculate the frequency.

↳ The distance between two consecutive nodes in a stationary wave is $\frac{\lambda}{2}$.

So, $\frac{\lambda}{2} = 20$ cm

$\lambda = 40$ cm $= 0.4$ m
Speed of wave (v) = 330 ms^{-1}

So, frequency (f) = $\frac{v}{\lambda} = \frac{330}{0.4}$ = 825 Hz.

13. Why a stationary wave is so named?

➯ In stationary wave, two identical oppositely moving waves superimpose to each other. So, the energy transmitted by one wave is exactly reversed by another wave so that no net transfer of energy occurs. Therefore, it is named stationary.

14. An observer at a sea – coast observes waves reaching the coast. What type of waves does he observe?

➯ He can see the elliptical wave. The wave on the surface of water is transverse, but the wave just beneath the surface is longitudinal. The resultant of these waves forms the elliptical wave.

15. Longitudinal waves are also called pressure waves. Why?

➯ Series of compressions and rarefactions are formed during the propagation of longitudinal wave. At the position of compression, particles of medium come closer and pressure increases. On the other hand, the particles move farther in rarefaction and hence the pressure decreases. Thus, the pressure varies in the medium, while longitudinal wave travels. Therefore, longitudinal wave is also called pressure waves.

16. Even huge explosions on the other planets are not heard on the earth, why?

➯ Sound wave is mechanical wave, it requires continuous material medium to travel. As we know, there is no continuous medium (i.e. vacuum) between other planets and the earth, sound wave cannot reach on the earth. Hence, we cannot hear such explosions of other planets on the earth.

17. What is the principle of superposition of waves?

➯ It states that "the displacement due to a number of waves acting simultaneously at a point in a medium is the sum of the displacement vectors due to each one of them acting separately."

For displacements y_1, y_2, y_3, . . . y_n of particles acting at a point, the resultant displacement at that point is,

$y = y_1 \pm y_2 \pm y_3 \pm \ldots \pm y_n$

18. Where does a person hear loud sound, at node or antinodes? Explain.

➯ The intensity of sound depends on the pressure amplitude in a medium. The pressure amplitude at the node is maximum, but minimum at the anti-node. So, the sound is heard louder at the node.

19. What is the phase difference between two consecutive troughs?

➯ The path difference of two consecutive trough is equal to wavelength λ. For,

$$\text{Phase difference } (\phi) = \frac{2\pi}{\lambda} \times \text{path difference } (x)$$

$$\phi = \frac{2\pi}{\lambda} \times \lambda = 2\pi$$

Therefore, the phase difference of two consecutive trough is 2π radian.

20. Which quantity is the most fundamental in wave motion?

➯ **F**requency of a wave is the most fundamental quantity. It is independent with the nature of medium. The speed of wave depends on the wavelength. When the sound travels from one medium to another, wavelength changes and so the speed changes but the frequency remains same.

Exercises

Short-Answer Type Questions

1. Distinguish between progressive wave and stationary wave.
2. Sound on the water surface has both longitudinal and transverse characteristics. Can you polarize it?
3. Write down some examples of the combination of longitudinal and transverse wave.

4. What is the phase difference between two nearest crest?
5. What is the path difference between two nearest crest?
6. If the wavelength of a sound source is reduced by a factor of 2, what happens to its frequency? Its speed?
7. Give two reasons why circular water waves decreases in amplitude as they travel away from the source.
8. Why do different objects make different sounds when dropped on a floor?
9. Is it possible for one sound wave to cancel another? Explain.
10. How are compression and rarefaction produced?
11. How does energy transmit through medium although net displacement of particles is zero?
12. Why is progressive wave called so?
13. Define matter waves.
14. How is sound propagation related to simple harmonic motion?
15. Distinguish between phase change and path change. How they are related?
16. If a stone is dropped near a leaf floating on a still water, what will happen in the leaf, move away or remains at the mean position?
17. Draw the displacement and time graph of a wave.
18. Which types of wave propagate in liquid, explain.
19. Distinguish between light waves and sound waves

Long-Answer Type Questions

1. Define longitudinal wave. Describe the mechanism of longitudinal wave motion.
2. Define transverse wave. Describe the mechanism of transverse wave motion.
3. What is progressive wave? Derive an equation for progressive wave.
4. What is stationary wave? Derive an equation for stationary wave.
5. Distinguish between progressive wave and stationary wave.
6. Derive the relation between wave velocity, wavelength and frequency.
7. Write down the different characteristics of progressive wave and stationary wave.
8. State and explain the stationary wave. Obtain the condition for nodes and antinodes of a stationary wave. [HSEB 2063]
9. Write down the different characteristics of wave motion and classify it with examples.
10. Use the principle of superposition of two waves to find the position of displacement nodes and antinodes in a standing wave. [HSEB 2061]
11. Define interference of waves. Show that interference of wave obeys the vector addition rule.

Numerical Problems

1. The speed of sound of frequency 200 Hz in air is 340 ms^{-1}. Calculate the wavelength of the wave.
   Ans: 170 m
2. The velocity of sound in air on a certain day is 365 ms^{-1}. The frequency of sound heard on a certain day is 480 Hz. What is the wave length of the sound wave?
   Ans: 76 cm
3. A note emitted has a wavelength of 1 m in still air. Calculate the frequency of the note emitted. (velocity of sound in air = 340 ms^{-1})
   Ans: 340 Hz
4. The equation of transverse wave travelling along a string is $y = 2 \sin \pi (0.5x - 200t)$ where x and y are in cm and t in sec. Find the amplitude, wavelength, frequency and velocity of propagation.
   Ans: 2×10^{-2} m, 4 $\times10^{-2}$ m,100 Hz, 4 m/s
5. If the velocity of sound in air is 340 ms^{-1}. Calculate (i) the wavelength when the frequency is 256 Hz, (ii) the frequency when the wavelength is 0.85 m. **Ans: 1.33 m, 400 Hz**

6. A wave travelling in the positive x–direction has an amplitude of 2 cm, frequency 75 Hz and velocity of propagation 45 m/s. Calculate the wave number, the displacement of the particle velocity and particle acceleration at x = 1.35 m from the origin at t = 3 sec.

Ans: wave number = $1/\lambda = 1.66 \times 10^{-4}\ m^{-1}$, 2×10^{-2} m, 0 ms^{-1}, $4.5 \times 10^{3}\ m/s^{2}$

7. A stone dropped from the top of a cliff of height 44.1 m splashes into water in a pond near the base of the cliff, after 3.126 s. Find the velocity of sound in air?

Ans: 350 m/s

8. A stone is dropped into a well and a splash is heard after 2.6 sec. Calculate the depth of the well. (Velocity of sound = 334 m/s)

Ans: 30.73 m

9. A travelling wave along positive x-axis has an amplitude of 0.2 m, wavelength of 0.5 m and frequency of 10 Hz. Find angular wave number period, angular frequency and wave speed.

Ans: 12.6 rad m^{-1}, 0.1 sec, 62.8 rad s^{-1}, 5 m/s^{-1}

Multiple Choice Questions

1. The distance between two particles in a wave motion in the same phase (λ = wavelength) is

a. $\lambda/4$ b. $\lambda/2$
c. $3\lambda/4$ d. λ

2. A transverse wave is described by the equation $y = a \sin 2\pi\left(ft - \frac{x}{\lambda}\right)$. The maximum particle velocity is equal to four times the wave velocity if

a. $\lambda = \frac{\pi a}{4}$ b. $\lambda = \frac{\pi a}{2}$
c. $\lambda = \pi a$ d. $\lambda = 2\pi a$

3. A wave of frequency 1000 Hz travels between X and Y, a distance of 600 m in 2 sec. How many wavelengths are there in distance XY?

a. 3.3 b. 300
c. 180 d. 2000

4. The equation of a traveling wave is y = 60 cos (1800 t – 6x) where y is in microns, t in secs and x in meter. The ratio of maximum particle velocity to wave velocity is

a. 3.6×10^{-11} b. 3.6×10^{-6}
c. 3.6×10^{-4} d. 3.6×10^{-2}

5. In stationary wave, the particle velocity at the nodal positions is

a. zero. b. maximum and finite.
c. minimum but non-zero. d. infinity.

6. What is the phase change in a travelling wave when reflects from open boundary.

(a) π rad (b) $\frac{\pi}{2}$ rad
(c) no change (d) $\frac{\pi}{4}$ rad

7. A travelling wave in a medium is described by the equation, y = a sin (ωt – kx). What is the maximum particle velocity?

(a) $a\omega$ (b) $a\omega^2$
(c) $a^2\omega$ (d) $a^2\omega^2$

8. What is the frequency of radio waves transmitted by a station, if the wavelength of these waves is 300 m?

(a) 4 MHz (b) 3 MHz
(c) 2 MHz (d) 1 MHz

9. Which property of material is responsible to generate transverse wave of sound?
   (a) Thermal property (b) Modulus of rigidity
   (c) Melting (d) Bulk modulus
10. Where does the energy go from the low intensity position of interference pattern?
   (a) Vanishes (b) Shift to high intensity position
   (c) Violate conservation property (d) Remains same position but inaudible
11. The relation between frequency f and velocity of propagation v of wave is:
   a. $f = v\lambda$ b. $f = \lambda/v$
   c. $f = v/\lambda$ d. $f = v\lambda^2$
12. The reciprocal of wavelength is called:
   a. Velocity b. Wave frequency
   c. Amplitude d. Wave number
13. The frequency of radio-waves is 15 MHz. What is its wavelength?
   a. 20 m b. 15 m
   c. 5 m d. 25 m
14. Node has:
   a. Maximum displacement and minimum strain
   b. Minimum displacement and maximum strain
   c. Minimum displacement and minimum strain
   d. Maximum displacement and maximum strain

Answers

1. (d) 2. (b) 3. (d) 4. (c) 5. (a) 6. (c) 7. (a) 8. (d) 9. (b) 10. (b) 11. (c) 12. (d) 13. (a) 14. (b)

Hints to Challenging Problems

Hint:1

Given equation,

$$y(x, t) = (6.5\text{ mm}) \cos 2\pi\left(\frac{x}{28.0\text{ cm}} - \frac{t}{0.0360\text{ s}}\right) \quad \ldots \text{(i)}$$

a. It is transverse wave moving in positive *X*-a*x*is.
   The general equation of transverse wave is

$$y(x, t) = a \cos 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right) \quad \ldots \text{(ii)}$$

Comparing (i) and (ii), we get

b. $a = 6.5\text{ mm} = 6.5 \times 10^{-3}\text{ m}$
c. $\lambda = 28.0\text{cm} = 28.0 \times 10^{-2}\text{ m}$
d. $T = 0.0360\text{ s}$ then $f = \frac{1}{T}$
e. $v = f\lambda$

Hint: 2

Given,

Speed of ultrasound (v) = 1500 m/s
Wavelength (λ) = 1 mm = 10^{-3} m
Frequency (f) = ?

Required formula,

$\therefore\ f = \frac{v}{\lambda}$

Hint: 3

Given,

Temperature of air (T) = 20°C
Speed of sound at 20°C (v_{20}) = 344 ms^{-1}

a. $\lambda = \frac{v_{20}}{f}$

b $f = ?$ for $\lambda = 0.0655$ mm

Required formula,

$\therefore\ f = \frac{v_{20}}{\lambda}$

Hint: 4

Given,

Speed of wave (v) = 8.00 m/s
Amplitude (a) = 0.0700 m
Wave length (λ) = 0.320 m

Required formula,

a. $f = \frac{v}{\lambda}$, $T = \frac{1}{f}$, $k = \frac{2\pi}{\lambda}$

b. The displacement of the travelling wave is the function of x and t. So, we can write the equation in X-direction as

$$y(x, t) = a \cos 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)$$

c. Use x = 0.360 m, t = 0.150 s in equation,

$$y(x, t) = 0.0700 \cos 2\pi\left(\frac{x}{0.32\text{ m}} - \frac{t}{0.0400\text{ s}}\right)$$

Hint: 5

Given, According to questions,

H is the highest position and L is its lowest position.

Time taken from H to L = t = 2.5 s

Vertical distance between H and L = 0.62 m

Distance between two consecutive crests = 6 m

a. Amplitude (a) = ?

Vertical distance between extreme positions = 0.62 m and amplitude is the half of this value so we can write

$$a = \frac{0.62}{2}$$

b. Distance between two consecutive crests = 6 m

$\therefore$ λ = 6 m

Since, time taken from the highest point to the lowest point (t) = 2.5 s

So, time period of oscillation is double of this time i.e. T = 2t

Hence, speed of wave = $f\lambda = \frac{\lambda}{T}$

Hint: 6

Given,

Velocity of sound in air (v) = 340 ms^{-1}

Wavelength (λ) = ?

when frequency (f) = 256 Hz

Required formula, $\lambda = \frac{v}{f}$

Also,

f = ? when λ = 0.85 m

$\therefore$ $f = \frac{v}{\lambda}$

Hint: 7

Given,

Low frequency (f_1) = 20 Hz

High frequency (f_2) = 20,000 Hz

a. In air,

Wave speed (v) = 344 m/s

Wave length for low frequency,

λ_1 = ?

$\therefore$ $\lambda_1 = \frac{v}{f_1}$

Wavelength for high frequency, λ_2 = ?

$\therefore$ $\lambda_2 = \frac{v}{f_2}$

b. In water,

Wave speed (v) = 1480 m/s

Wavelength for low frequency, $\lambda' = \frac{v}{f_1}$

Wavelength for high frequency, $\lambda'' = \frac{v}{f_2}$

Hint: 8

Given,

Speed of ripple, v = 0.20 ms^{-1}

Wave length, λ = 15 mm = 15 × 10^{-3} m.

Amplitude, a = 5 mm = 5 × 10^{-3} m.

Maximum velocity of cork (v_{max}) = ?

We know that

$$v = \omega\sqrt{a^2 - y^2}$$

For maximum velocity, y = 0. So,

$$v_{max} = \omega a$$

$$= 2\pi f \times a \quad (\because \omega = 2\pi f)$$

$$= 2\pi\frac{v}{\lambda} \times a \quad (\because v = f\lambda)$$

Hint: 9

Given,

Frequency (f) = 250 Hz

Speed (v) = 30 ms^{-1}

i. Phase difference (ϕ) = ?

Path difference (x) = 10 cm = 0.10 m

We know that

$$\phi = \frac{2\pi}{\lambda}.x$$

$$= \frac{2\pi}{\frac{v}{f}}.x$$

ii. Equation of progressive wave = ?

Amplitude (a) = 0.03 m.

The general progressive wave equation is given by

$$y = a \sin\frac{2\pi}{\lambda}(vt - x)$$

According to the question, we can write

$$y = 0.03 \sin 2\pi\left(\frac{v}{\lambda}t - \frac{x}{\lambda}\right)$$

$$= 0.03 \sin 2\pi\left(ft - \frac{x}{v} \times f\right)$$

iii. Distance between two consecutive nodes = ?

As we know that distance between two

consecutive nodes is $\frac{\lambda}{2}$. So, distance between two nodes $= \frac{\lambda}{2} = \frac{v}{2f}$

Hint:10

Given,

$$y = 0.1 \sin\left(200\pi t - \frac{20\pi x}{17}\right) \quad \ldots \text{(i)}$$

i. The general progressive wave equation is

$$y = A \sin\left(2\pi ft - \frac{2\pi x}{\lambda}\right) \quad \ldots \text{(ii)}$$

Comparing (i) and (ii), we get

$200\pi = 2\pi f$

ii. $\frac{20\pi}{17} = \frac{2\pi}{\lambda}$

iii. Wave velocity, $v = f\lambda$

iv. Phase difference (ϕ) = ?

Path difference (x) = 1.10 – 0.25 = 0.85 m

We have

$$\phi = \frac{2\pi x}{\lambda}$$

v. Equation of a wave = ?

Amplitude, $a_1 = 2a = 2 \times 0.1 = 0.2$ mm

Frequency, $f_1 = 2f = 2 \times 100 = 200$ Hz

Wavelength, $\lambda_1 = \frac{v}{f_1}$

The general equation of travelling wave is

$$y = a \sin \frac{2\pi}{\lambda}(vt - x)$$

When wave travels in opposite direction,

$$y = a_1 \sin\frac{2\pi}{\lambda_1}(vt - (-x))$$

$$y = a_1 \sin\frac{2\pi}{\lambda_1}(vt + x)$$

$$= a_1 \sin\left(2\pi \frac{v}{\lambda_1} t + \frac{2\pi x}{\lambda_1}\right)$$

$$= 0.2 \sin\left(2\pi f_1 t + \frac{2\pi x}{\lambda_1}\right)$$

Hint: 11

Given,

Amplitude, $a = 1 \times 10^{-7}$ m

Angular speed, $\omega = 6.6 \times 10^{3}\ s^{-1}$

$k = 20\ m^{-1}$

a. Speed of the wave, $v = f\lambda$

But $\omega = 2\pi f$ and $k = \frac{2\pi}{\lambda}$

So, $v = \frac{\omega}{2\pi} \times \lambda = \frac{\omega}{\frac{2\pi}{\lambda}} = \frac{\omega}{k}$

b. $v_{max} = \omega\sqrt{a^2 - 0} = \omega a$

Hint:12

Given, each step is 0.25 m deep for giving sound, so

Depth of each step = Distance between two cons

or $0.25 = \frac{\lambda}{2}$

or $\lambda = 0.50$ m

Frequency of sound, f = ?

We have

$\therefore\ f = \frac{v}{\lambda}$

❖❖

MECHANICAL WAVES

2

CHAPTER

2.1 Introduction

Sound wave carries energy in a material medium. It is a form of energy like light energy, electric energy, magnetic energy, etc. It gives the sensation of hearing. It is produced by the vibrations of sounding body. It requires material medium to travel. For the propagation of sound, the material medium must be continuous and should possess the elastic and inertial properties. Sound source does not emit its own particles like photons in an electromagnetic radiation. When a body is vibrated in a medium, it produces the disturbances to the near-by particles. During this process, energy is transferred to these neighbouring particles. Further, other particles nearby them also set into vibration and so on. Thus, the sound wave propagates long distance away from the source. If there is no medium, i.e. vacuum, in its path, sound wave can not travel ahead.

We do not hear all the vibrations that are produced in a medium. The audible range of sound basically depends on intensity as well as frequency. The intensity for threshold of hearing is 10^{-12} Wm^{-2} and frequency range is 20 Hz to 20 kHz. As the sound travels in the form of wave, it obeys the wave phenomena like reflection, refraction, interference and diffraction. However, it cannot be polarized which confirms that it is not the transverse wave.

Reflection

The phenomenon in which the sound returns back after falling on a surface is known as reflection of sound. The reflection of sound obeys the laws as followed by the light. We are familiar with examples of reflected sound in our daily life. If we speak loudly near a tall buildings or bridges, the reflectors (tall buildings or bridges) mimic the same voice that we produce. This happens due to the reflection of sound.

If the reflector is very near to the source, we do not hear the reflected sound. To hear the reflected sound distinctly, the reflector must be a certain distance away from the source such that the reflected sound must come back to source at least after 0.10 s. Our hearing sensitivity distinguishes two sound events only when the time difference is at least 0.10 s. The least distance for the distinct sound to be heard is determined as follows:

The speed of sound in air at 20ºC is about 340 m/s

i.e. at $T = 20^oC$, $v = 340\ ms^{-1}$

Our persistence of hearing, $t = 0.10$ s

So, the total distance travelled by sound in 0.10 s at 20ºC = 340 × 0.10 = 34 m.

In reflection, the sound travels same path two times. So, the minimum distance of reflector from the source must be $\frac{34}{2}$ = 17 m.

The laws of reflection of sound can be verified by an experiment. Suppose a long hollow pipe is bent and fitted to a rigid support such that each of the bent part is at equal inclination with the horizontal of rigid support as shown in Fig. 2.1. The end A is fixed while the end B can be rotated at our will. The sound wave is made to travel through the end A so that its reflected wave can be heard from the end B. It is found experimentally that, reflected sound has different intensity at different positions of end B. The reflected sound is heard with maximum intensity when both the ends are at equal inclination to the horizontal. But the intensity decreases as the end B moves to any other positions such as C and D. This verifies that angle of incidence is equal to angle of reflection of sound.

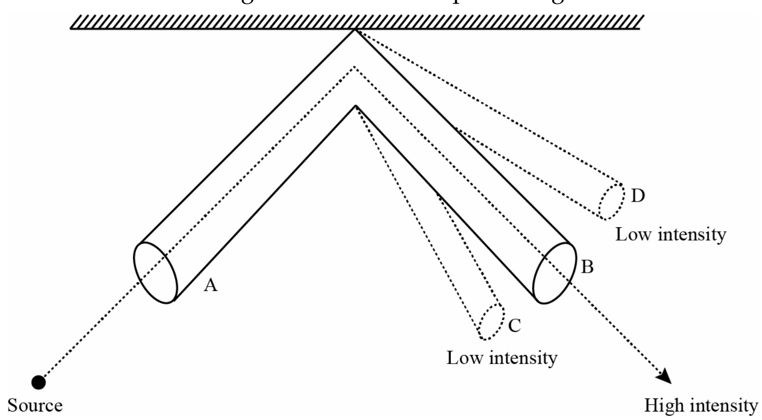


Fig. 2.1: Reflection of sound

The reflected sound can be categorized into two types: Echo and Reverberation.

Echo

It is the reflected sound that is detected only once after reflection. When the sound source and reflector are at least 17 m apart, echo can be heard. Many medical devices, like stethoscope, ultrasound machine depends on the sound echo.

Reverberation

Reverberation is the multiple reflection of sound when the reflector is situated nearer than 17 m from the sound source. If the reflector lies nearer than 17 m, the reflected sound is not distinctly heard but it prolongs the original sound. On an auditorium hall, the multiple reflection of sound from the walls produces the reverberation. To remove the reverberation, sound absorber are kept on the walls of the hall. Moreover, we hear the rolling sound of thunder due to the reverberation of sound, when reflects from different layers of cloud.

Refraction

Refraction of sound waves is the change on direction of waves as they pass from one medium to another. The refraction of wave takes place due to the change in speed and wavelength of sound, when it travels from one medium to another. If the medium is changed, the speed is also changed. Thus, waves passing from one medium to another will undergo refraction. Refraction also takes place in the same medium, if the temperature is different at different points.

At night, the lower layer of atmosphere is cold and upper layer is relatively hot. So, speed of sound travelling upward gradually increases and the sound deviates away from the normal on the layer of air formed on the atmosphere and finally returns downwards from a certain height in the atmosphere as shown in Fig. 2.2. Therefore, sound is detected louder at night than at day time.

Ultrasound produces the images of internal parts of our body exploiting the reflection and refraction phenomena of sound.

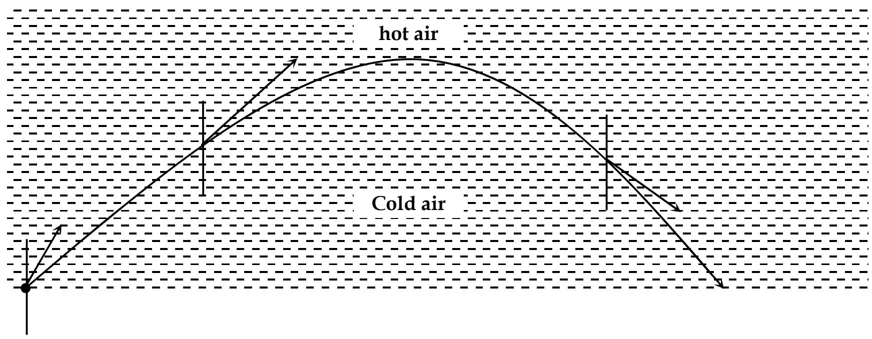


Fig. 2.2: Refraction of sound

Interference

Interference is a phenomenon in which two or more waves overlap so that a resultant wave is formed whose amplitude may be greater, smaller or same as the amplitude of original waves. It is a basic property of sound that exhibits its wave nature. Interference refers to the interaction of waves that are similar to each other and in fact correlated to each other. This phenomenon can be observed in all types of waves like light waves, sound waves and matter waves.

The interference effects can be studied using two identical sound sources (having same wavelengths and certain phase difference). When waves from these sources superimpose to each other, a resultant wave is formed whose amplitude changes but frequency remains unchanged. In this process, energy of wave is distributed differently than the original waves. *"The phenomenon of redistribution of energy in the resultant wave formed by the superposition of two waves having same frequency (or wavelength) and constant phase difference is called interference of waves."*

The term "redistribution of energy" means the shifting of energy from one place to another. When two sound waves from coherent sources superimpose in such a way that the energy is divided into definite ways: one in which energy appears at a point and disappears completely at the nearby point. The point, where the energy appears maximum, is the position in which the waves are in the same phase while the point, where the energy appears minimum, is the position in which waves are in the opposite phase. In this way, we see that the energy is only redistributed during interference but total energy still remains the same. That is why, there is no violation of law of conservation of energy.

i. **Constructive interference:** *The interference, in which similar phase of two waves overlap each other is called constructive interference.* The amplitude becomes maximum in case of constructive interference and hence intensity of sound also becomes maximum and therefore a loud sound is heard.

ii. **Destructive interference:** *The interference, in which opposite phases of two waves overlap each other is called destructive interference.* In destructive interference, the amplitude and intensity of resultant wave become minimum which results in low intensity of sound.

Diffraction

The phenomenon of bending (spreading) of light around the corner (edge) of the obstacle placed in its path is called diffraction of light. The phenomenon of diffraction is common for all waves. It occurs when sound (or any wave) is incident on the openings or obstacles but it becomes noticeable when dimension of openings or obstacles are comparable to wavelength. Diffraction of sound is easily observed because sound has long wavelength.

Sound waves are diffracted easily from the corners of doors and windows of our home. Due to this, we can hear the sound if someone calls us from outside. Sound waves can also be diffracted from the space between the large trees and rocks in the forest. So, many forest animals take advantage of this diffraction property of sound to communicate with missing members of their groups.

2.2 Speed of Mechanical Wave

Sound wave is a mechanical wave. It requires continuous medium to travel from one point to another. Two basic properties of medium are essential for the propagation of the mechanical wave; they are elasticity and inertia. This means the speed of sound depends on the elastic and inertial properties of the medium. When a point in a medium is disturbed, the energy is readily transferred to the nearby molecules so that the molecules execute simple harmonic motion. Simple harmonic motion in the medium is possible only when the medium possesses the property of elasticity. The density of medium characterises the inertial property of the medium.

1. **Elastic property:** If two particles of a body are moved apart, they should be pulled to the original position to prevent from deformation. This behaviour which prevents from deformation is called elastic property of a body. In wave motion, the supplied force tends to deform the material medium, but the elastic behavior of medium restores it to its initial position. Thus, the wave motion is possible, only when the medium possesses the elastic properly.
2. **Property to inertia:** Inertia is produced due to the mass in a particle. This property does not let the displaced particle to stop suddenly. So, the particle oscillates about a mean position. Thus, the wave is generated in a material medium.

A dimensional method is used to relate the speed of sound with modulus of elasticity (E) and density (ρ) of a medium. So,

$$v \propto E^a$$

and $$v \propto \rho^b$$

where, a and b are dimensions of E and ρ respectively.

$$\therefore \quad v \propto E^a \rho^b$$

or, $$\mathbf{v = k\,E^a \rho^b} \qquad \ldots (2.1)$$

Where k is a dimensionless constant and its experimental value is 1, i.e. k = 1.

Now,

The dimension of E, $[E] = [ML^{-1}T^{-2}]$

The dimension of ρ, $[\rho] = [ML^{-3}]$

The dimension of v, $[v] = [LT^{-1}]$

Thus, from equation (18.14) we have,

$$[LT^{-1}] = [ML^{-1}T^{-2}]^a\,[ML^{-3}]^b$$

or $$[LT^{-1}] = [M^{a+b}L^{-a-3b}T^{-2a}]$$

Equating the powers of M, L and T, we get,

$$a + b = 0,$$
$$-a - 3b = 1$$
$$\text{and } -2a = -1$$
$$\therefore \quad a = ½, b = -½$$

Hence, from equation (2.1), we get,

$$v = k\,E^{1/2}\rho^{-1/2}$$

$$\text{or} \quad v = 1 \times \sqrt{\frac{E}{\rho}}$$

$$\therefore \quad \mathbf{v} = \sqrt{\frac{\mathbf{E}}{\rho}} = \sqrt{\frac{\textbf{Modulus of elasticity}}{\textbf{Density}}} \qquad \textbf{. . . (2.2)}$$

This is the required expression for speed of sound in a medium.

So, we can write,

$$v = \sqrt{\frac{\text{Elastic property}}{\text{Inertial property}}}$$

i. Speed of sound wave in solid : When a wave travels along the rod, Young's modulus *Y* is relevant for modulus of elasticity. Thus,

$$\mathbf{v} = \sqrt{\frac{\mathbf{Y}}{\rho}} \qquad \textbf{. . . (2.3)}$$

ii. Speed of sound wave in liquid: The wave propagates in all directions, bulk modulus K, is relevant for modulus of elasticity.

$$\mathbf{v} = \sqrt{\frac{\mathbf{K}}{\rho}} \qquad \textbf{. . . (2.4)}$$

Note

The speed of sound appears greater in gas than in solid as the density of solid is greater than gas, but it is not so. In reality, the speed of sound not only depends on density (ρ) but also on modulus of elasticity (E). The ratio of modulus of elasticity to density $\left(i.e.\ \frac{E}{\rho}\right)$ is greater in solid than in gas. Hence, sound travels faster in solid than in gas. Moreover, the speed of sound is greater in liquid than in gas.

iii. Speed of transverse wave through a stretched string: Speed of transverse wave through a stretched string is determined with the formula,

$$\mathbf{v} = \sqrt{\frac{\mathbf{T}}{\mu}} \qquad \textbf{...(2.5)}$$

Where, T is the tension of the string and μ is the mass per unit length of the string.

iv. Speed of electromagnetic wave: The speed of electromagnetic (EM) wave is determined with the formula,

$$\mathbf{v} = \sqrt{\frac{\mathbf{1}}{\epsilon\mu}} \qquad \textbf{...(2.6)}$$

Where, ϵ and μ are the permittivity and permeability of a medium though which EM wave is propagated.

v. Speed of sound in extended solid: The speed of sound wave in extended solids such as the crust of the earth is written as,

$$v = \sqrt{\frac{K + 4\frac{\eta}{3}}{\rho}} \qquad \ldots(2.7)$$

Where, K = bulk modulus of elasticity

and η = modulus of rigidity

2.3 Speed of Sound in Gaseous Medium

In our daily life, we hear the sound propagating through gas medium. Although, the sound wave is generated by solid and liquid, it approaches to our ear through gas. So, determining the speed of sound in gaseous medium is very important in our life.

Newton's Formula

Sound wave is also called pressure wave, since the pressure of material medium is changed during the propagation of sound. Due to the variation of pressure, the temperature is also varied at the positions of compressions and rarefaction. Newton studied the speed of sound regarding the variation of pressure and volume in gas while sound wave propagates in it. He assumed that, the propagation of sound wave is very slow. Hence, it obeys the thermodynamic process, called the isothermal process. In such process, temperature variation is negligible (ideally zero). There is no any temperature difference between the region of compression and rarefaction. For a given mass of a gas at pressure *P* and volume V, Boyle's law is stated as,

$$PV = \text{Constant}$$

Differentiating, we get,

$$d(PV) = 0$$

or, $$PdV + VdP = 0$$

$$P = -\frac{dP}{\frac{dV}{V}} \qquad \ldots(2.8)$$

Here, dP and dV refer the change of pressure and volume in gas.

Also, the bulk modulus of elasticity (K) is the ratio of change of pressure to change of volume in a medium.

i.e. $$K = -\frac{dP}{\frac{dV}{V}} \qquad \ldots(2.9)$$

From equations (2.8) and (2.9), we get,

$$\therefore \quad K = P \qquad \ldots(2.10)$$

From the expression of speed of sound in a medium, we have,

$$v = \sqrt{\frac{K}{\rho}} \qquad \ldots(2.11)$$

From equations (2.10) and (2.11), we get,

$$v = \sqrt{\frac{P}{\rho}} \qquad \ldots(2.12)$$

This is Newton's formula for the speed of sound in a gas.

For air at NTP,

$$P = 760 \text{ mm of Hg,}$$
$$= 760 \times 10^{-3} \times 13600 \times 9.8$$
$$= 1.013 \times 10^{5} \text{ Nm}^{-2}$$

and the density, $\rho = 1.29 \text{ kgm}^{-3}$

$$\therefore \quad v = \sqrt{\left(\frac{1.013 \times 10^{5}}{1.29}\right)} \approx 280 \text{ m/s.}$$

Experimentally observed value of speed of sound in air at NTP is 332 m/s. Due to this difference between theoretical and experimental values, it was thought that Newton's formula needed to be modified with a necessary correction. Later on, Laplace corrected the above relation with necessary modification.

Laplace's Correction

Unlike Newton's assumption, Laplace assumed that the propagation of sound wave is very fast so that heat cannot be shared by compressions and refractions in such very short time. So, the temperature of gas changes, i.e. the process is adiabatic rather than isothermal. Thus, the relation between pressure (P) and volume (V) of the gas through which a sound wave is propagating, is given by the adiabatic equation for a gas, i.e.

$$PV^{\gamma} = \text{Constant}$$

Where $\gamma = \frac{c_p}{c_v}$ is the specific heat ratio.

Differentiating, we get,

$$d(PV^{\gamma}) = 0$$

or $\quad P\gamma V^{\gamma-1} dV + V^{\gamma} dP = 0$

Dividing by $V^{\gamma-1}$, we get,

or $\quad P\gamma \, dV + VdP = 0$

or $\quad \gamma P = -\dfrac{dP}{\frac{dV}{V}} = K \qquad \left(\because K = -\dfrac{\text{Change in pressure}}{\text{Volume strain}}\right) \qquad \textbf{. . . (2.13)}$

The speed of sound in a medium is given by,

$$v = \sqrt{\frac{K}{\rho}} \qquad \textbf{. . . (2.14)}$$

From equations (2.13) and (2.14), we get,

$$v = \sqrt{\frac{\gamma P}{\rho}} \qquad \textbf{. . . (2.15)}$$

Equation (2.15) is Laplace's formula for the speed of sound in a gas.

Taking $\gamma = 1.41$ for air, the speed of sound in air at NTP is calculated as,

$$v_{NTP} = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{1.41 \times 1.01 \times 10^{5}}{1.29}} = 331.6 \text{ ms}^{-1}$$

This value is closely agreed with the experimental value. Hence, Laplace's formula for the speed of sound in gaseous medium is correct formula with correct assumptions.

2.4 Factors Affecting the Speed of Sound in a Gas

i. **Effect of Temperature:** For a gas of mass m in a volume V,

$$\text{the speed of sound (v)} = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma PV}{\rho V}}$$

$$= \sqrt{\frac{\gamma nRT}{m}} \quad (\because PV = nRT)$$

$$= \sqrt{\frac{\gamma nRT}{nM}} \quad (\because m = nM, \text{ and } M = \text{molar mass of a gas})$$

$$= \sqrt{\frac{\gamma RT}{M}}$$

Since γ and M are constant for a gas,

$$v \propto \sqrt{T}$$

Let v_1 and v_2 be the speed of sound at temperatures T_1 and T_2 respectively in a gas. Then, the relation of speed and temperature is written as,

$$\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}} \qquad \ldots (2.16)$$

Thus, the speed of sound is directly proportional to the square-root of the absolute temperature of the gas.

ii. **Effect of molar mass:** From Laplace correction,

$$\text{the speed of sound (v)} = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma PV}{\rho V}}$$

$$= \sqrt{\frac{\gamma nRT}{m}}$$

$$= \sqrt{\frac{\gamma nRT}{nM}} \quad (\because M = \text{molar mass of a gas})$$

$$= \sqrt{\frac{\gamma RT}{M}}$$

For γ and T are taken constant,

$$v \propto \sqrt{M}$$

Let v_1 and v_2 be the speed of sound in gas with molar masses M_1 and M_2 respectively. Then, the relation of speed and molar mass is written as,

$$\frac{v_1}{v_2} = \sqrt{\frac{M_2}{M_1}} \qquad \ldots (2.17)$$

Thus, the speed of sound is inversely proportional to the square-root of the molar mass of the gas at constant temperature.

iii. **Effect of pressure:** From the Laplace correction, the speed of sound in gas is,

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

At constant temperature, the product of volume and pressure remains constant for a given mass of gas, i.e., PV = constant. So,

$$\frac{Pm}{\rho} = \text{constant}$$

$$\frac{P}{\rho} = \frac{\text{constant}}{m}$$

$\therefore \quad \frac{P}{\rho}$ = constant $\qquad (\because$ m is constant)

Also, $\quad v = \sqrt{\gamma}\sqrt{\frac{P}{\rho}}$

Since both γ and $\frac{P}{\rho}$ are constant for constant temperature,

$$v = \text{constant}$$

This concludes that, the speed of sound in gas is independent to the variation of pressure at constant temperature.

iv. **Effect of change in density:** Consider two different gases at same temperature and pressure with different densities. The speeds of sound in these gases media are,

$$v_1 = \sqrt{\frac{\gamma_1 P}{\rho_1}} \text{ and } v_2 = \sqrt{\frac{\gamma_2 P}{\rho_2}}$$

If we take same atomicity of gas, like diatomic gases, $\gamma_1 = \gamma_2 = \gamma$

$$v_1 = \sqrt{\frac{\gamma P}{\rho_1}} \text{ and } v_2 = \sqrt{\frac{\gamma P}{\rho_2}}$$

so, $\quad \frac{v_1}{v_2} = \sqrt{\frac{\rho_2}{\rho_1}}$

Thus, the speed of sound in a gas is inversely proportional to the square root of density of the gas.

v. **Effect of humidity:** Humid air contains a large proportion of water vapour i.e. greater proportion of hydrogen (i.e. H_2). However, the dry air contains the greater proportion of nitrogen. Hence, the density of moist air is less than the density of dry air at constant pressure.

Let ρ_m and ρ_d are the density of moist air and density of dry air respectively. Then, speed of sound in these media are,

$$v_m = \sqrt{\frac{\gamma_m P}{\rho_m}} \text{ and } v_d = \sqrt{\frac{\gamma_d P}{\rho_d}}$$

Where, v_m = speed of sound in moist air

v_d = speed of sound in dry air.

Now,

$$\frac{v_m}{v_d} = \sqrt{\frac{\gamma_m P}{\rho_m}} \times \sqrt{\frac{\rho_d}{\gamma_d P}}$$

$$= \sqrt{\frac{\gamma_m}{\gamma_d}} \times \sqrt{\frac{\rho_d}{\rho_m}}$$

Experimental result in humid and dry air shows that

$$\frac{\gamma_m}{\gamma_d} = 0.9 \text{ and } \frac{\rho_d}{\rho_m} = 1.6$$

so, $\quad \frac{v_m}{v_d} = \sqrt{0.9 \times 1.6} = 1.2 > 0$

This shows that $v_m > v_d$.

Therefore, the speed of sound is greater in humid air than the dry air. Thus, the sound travels faster in rainy days than on a dry day.

vi. Direction of wind: Consider u and v be the speed of wind and speed of sound wave respectively. Let the wind blows at an angle θ with the line joining the sound source (S) and sound observer (O) as shown in Fig. 2.3.

The resultant speed of sound wave

$V_R = v + u \cos\theta$

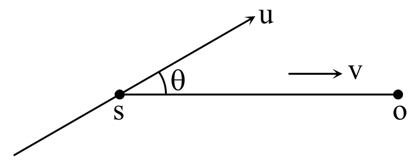


Fig. 2.3: Effect of direction wind on speed of sound

i. If the wind blow towards the observer (i.e. $\theta = 0°$)

$V_R = v + u \cos 0° = v + u$ (maximum velocity)

ii. If the wind blow in perpendicular direction of source – observer position, ($\theta = 90°$)

$V_R = v + u \cos 90°$

$= v$ (no effect)

iii. If the wind blows in opposite direction of observer ($\theta = 180°$)

$V_R = v + u \cos 180°$

$= v - u$ (minimum velocity)

This shows that the speed of sound increases along the direction of wind flow.

vii. Effect of frequency and wavelength: The frequency and wavelength of sound wave are not involved in the formula regarding the speed of sound. So, the speed of sound is independent of its frequency and wavelength.

viii. Effect of amplitude: Amplitude of sound is merely independent to the speed of sound, however, very large amplitude possesses the significantly large oscillation of particles in the material medium. This ultimately increases the temperature of the medium and then the speed of sound slightly increases.

ix. Effect of atomicity of gas: Gas molecules remain in different atomic forms: Monoatomic, diatomic, and triatomic gases. Helium, Fluorine, etc. are monoatomic gases, hydrogen, nitrogen, oxygen etc. are diatomic gases and carbondioxide, ammonia, etc. are triatomic gases. Thus, they have different atomicity, which have different values of specific heat ratio $\gamma\left(=\frac{c_p}{c_v}\right)$. Hence, the speed of sound is different in different atomicity of gas.

Tips for MCQs

1. Speed of sound in any medium depends on elasticity and density of medium,

$$v = \sqrt{\frac{E}{\rho}} = \frac{\text{Elasticity}}{\text{Density}}$$

2. Speed of longitudinal wave,

i. in solid, $v = \sqrt{\frac{E}{\rho}}$

ii. in extended solid such as earth crust, $v = \sqrt{\dfrac{K + \dfrac{4\eta}{3}}{\rho}}$

where, K = Bulk modulus of elasticity, η = modulus of rigidity

iv. in liquid, $v = \sqrt{\dfrac{K}{\rho}}$

v. in gas, $v = \sqrt{\dfrac{\gamma P}{\rho}}$, $\gamma = \dfrac{C_p}{C_v}$

3. Factors affecting the speed of sound in gas,

   i. Temperature, $v \propto \sqrt{T}$, so $\dfrac{v_1}{v_2} = \sqrt{\dfrac{T_1}{T_2}}$

   ii. Pressure, no effect at constant temperature

   iii. density, $v \propto \dfrac{1}{\sqrt{\rho}}$, so $\dfrac{v_1}{v_2} = \sqrt{\dfrac{\rho_1}{\rho_2}}$ (for different gases)

   iv. Molar mass, $v \propto \dfrac{1}{\sqrt{M}}$, so $\dfrac{v_1}{v_2} = \sqrt{\dfrac{M_2}{M_1}}$

   v. Humidity, $v_{humid} > v_{dry}$

3. Temperature coefficient of speed of sound,

$$\frac{v_t}{v_0} = \sqrt{\frac{273 + \theta}{273}} = \sqrt{1 + \frac{\theta}{273}} = \left(1 + \frac{\theta}{273}\right)^{1/2}$$

$$v_t = v_0\left(1 + \frac{1}{2}\frac{\theta}{273} + \ldots\right)$$

$$v_t = v_0\left(1 + \frac{1}{2}\left(\frac{1}{273}\right)\theta\right)$$

$$v_t = v_0\left(1 + \frac{1}{2}\alpha\,\theta\right)$$

Where, α temperature coefficient of speed of sound

4. Speed of electromagnetic wave,

$$v = \frac{1}{\sqrt{\epsilon\mu}}$$

Where, ϵ = permeability of a medium
μ = permittivity of the medium

In vacuum, $v = \dfrac{1}{\sqrt{\epsilon_0\mu_0}} = 3 \times 10^8$ ms^{-1}

5. Speed of transverse wave in a stretched string,

$v = \sqrt{\dfrac{T}{\mu}}$, where T = tension on string and μ = mass per unit length

Worked Out Problems

1. Velocity of sound in air at 0°C is 330 ms^{-1}. Calculate the velocity of sound in air at 27°C.

Solution

Given,

Initial temperature (T_1) = 0°C = 273 K
Initial velocity (v_1) = 330 ms^{-1}
Final temperature (T_2) = 27°C
= 27+273=300 K
Final velocity (v_2) = ?

We know,

$$\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}}$$

$$v_2 = \sqrt{\frac{T_2}{T_1}} \times v_1$$

$$= \sqrt{\frac{300}{273}} \times 330 = 345.93\ ms^{-1}.$$

Therefore, the velocity of sound at 27°C is 345.93 ms^{-1}.

2. At what temperature the velocity of sound is double than at the temperature 27°C?

SOLUTION

Given,

Initial temperature (T_i) = 27°C = 27 + 273 = 300 K

Initial velocity (v_1) = v

Final velocity (v_2) = 2v

Final temperature (T_2) = ?

We know,

$$\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}}$$

$$\frac{v_2^2}{v_1^2} = \frac{T_2}{T_1}$$

$$\frac{(2v)^2}{v^2} = \frac{T_2}{T_1}$$

$4T_1 = T_2$

$T_2 = 4 \times 300$

$= 1200$ K

$\therefore$ $T_2 = (1200 - 273)$°C

= 927°C

$\therefore$ The velocity of sound is double at 927°C than that of 27°C.

3. The velocity of sound in air being 332 ms^{-1} at 0°C. Find the change in velocity per degree rise in temperature.

SOLUTION

Given,

Initial velocity (v_1) = 332 ms^{-1}

suppose, the temperature is raised to 1°C,

i.e. T_2 = 274 K

Now,

$$\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}}$$

Initial temperature (T_1) = 0°C = 273 K

$$v_2 = \sqrt{\frac{274}{273}} \times 332$$

= 332.61 ms^{-1}

$\therefore$ Change in velocity per degree rise in temperature

(Δv) = $v_2 - v_1$

= 332.61 – 332

= 0.61 ms^{-1}

4. [NEB 2075] At what temperature the speed of sound is increased by 50% than that travels in 27°C?

SOLUTION

Given,

Initial temperature (T_1) = 27°C = 27 + 273 = 300 K

Let initial velocity (v_1) = v

Final velocity = v + 50% of v

$$= v + \frac{50}{100} \times v$$

$$= v + \frac{1}{2}v = \frac{3}{2}v$$

Final temperature (T_2) = ?

We know,

$$\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}}$$

or, $\dfrac{\frac{3}{2}v}{v} = \sqrt{\dfrac{T_2}{300}}$

or, $\dfrac{3}{2} = \sqrt{\dfrac{T_2}{300}}$

or, $\dfrac{9}{4} = \dfrac{T_2}{300}$

or, $T_2 = \dfrac{9}{4} \times 300 = 675$ K

$\therefore$ $T_2 = (675 - 273)$°C = 402°C

Therefore, at 402°C, the speed of sound is increased by 50% than that of 27°C.

5. [HSEB 2073] Calculate the bulk modulus of a liquid in which longitudinal waves the frequency of 250 Hz have the wavelength of 8 m if the density of liquid is 900 kgm^{-3}.

SOLUTION

Given, Frequency of wave (f) = 250 Hz

Wavelength (λ) = 8 m

and density of liquid (ρ) = 900 kgm^{-3}

Bulk modulus (K) = ?

The speed of sound (v) = fλ

$= 250 \times 8$

$= 2000\ ms^{-1}$

Now, the velocity, $v = \sqrt{\frac{K}{\rho}}$

$v^2 = \frac{K}{\rho}$

$K = v^2 \rho$

$= (2000)^2 \times 900 = 3.6 \times 10^9\ Nm^{-2}$

∴ The bulk modulus of given liquid is $3.6\times 10^9\ Nm^{-2}$.

6. Densities of oxygen and nitrogen are in the ratio of 16:14. At what temperature, the speed of sound in oxygen will be the same as at 15°C in nitrogen?

SOLUTION

From Laplace formula, we know that

$v = \sqrt{\frac{\gamma P}{\rho}}$

From ideal gas equation, we have

$PV = RT$

or $P\frac{V}{M} = \frac{RT}{M}$

or $\frac{P}{\rho} = \frac{RT}{M}$, where M is the molar mass of a gas

Therefore,

$v = \sqrt{\frac{\gamma RT}{M}}$

Let speed of sound in oxygen at θ°C be the same as in nitrogen at 15°C.

Speed of sound in oxygen at θ°C

$= \sqrt{\frac{\gamma R(273 + \theta)}{M_o}}$

Speed of sound in nitrogen at 15°C

$= \sqrt{\frac{\gamma R(273 + 15)}{M_N}}$

Since these two speeds are the same so we can write

$\sqrt{\frac{\gamma R(273 + \theta)}{M_0}} = \sqrt{\frac{\gamma R(273 + 15)}{M_N}}$

or $\frac{273 + \theta}{M_0} = \frac{288}{M_N}$

or $273 + \theta = \frac{M_0}{M_N} \times 288$

or $\theta = \frac{16}{14} \times 288 - 273$

∴ $\theta = 56.14°C$

7. A source of sound of frequency 550 Hz emits waves of wavelength 600 nm in air at 20°C. What is the speed of sound in air at this temperature? What would be the wavelength of the sound from the source in air at 0° C?

SOLUTION

Given,

Frequency (f) = 550 Hz

Wavelength at 20°C (λ_{20}) = 600 mm = 0.60 m

Speed of sound at 20° C (v_{20}) = ?

Also, we know that

$v_{20} = f \times \lambda_{20} = 550 \times 0.60 = 330\ ms^{-1}$

Now, Wavelength of sound at 0° C (λ_0) = ?

We know that

$v \propto \sqrt{T}$

or $\frac{v_0}{v_{20}} = \sqrt{\frac{T_0}{T_{20}}}$

or $v_0 = \sqrt{\frac{T_0}{T_{20}}} \times v_{20}$

or $v_0 = \sqrt{\frac{273}{273 + 20}} \times 330$

or $v_0 = \left(\sqrt{\frac{273}{293}} \times 330\right)$

or, $f\lambda_0 = 318.50$

or, $\lambda_0 = \frac{318.50}{f} = 318.54 \times \frac{1}{550}$

∴ $\lambda_0 = 0.579$ m

8. A source of sound produces a note of 512 Hz in air at 17 °C with wavelength 64.5 cm. Find the ratio of molar heat capacities at constant pressure to constant volume at NTP. Densities of air and mercury at NTP are 1.293 kgm^{-3} and 13600 kgm^{-3} respectively.

SOLUTION:

Given,

Frequency (f) = 512 Hz

Wavelength (λ) = 64.5 cm

Initial temperature (T_1) = 17 °C = 17 + 273 = 290 K

Density (ρ_0) = 1.293 kgm^{-3}

Density of mercury (ρ_{Hg}) = 13600 kgm^{-3}

The speed of sound at 17 °C (v_1) = fλ

= 512 × 0.665

= 340.48 ms^{-1}

Now, the speed of sound at 0°C (v_2) = ?

We have, $\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}}$

$$\frac{v_2}{340.48} = \sqrt{\frac{273}{290}}$$

$$V_2 = \sqrt{\frac{273}{290}} \times 340.48$$

$$= 330.45 \ ms^{-1}$$

Also, the atmospheric pressure at NTP,

P = hρg

h = 760 mm of Hg = 760 × 10^{-3} m of Hg

ρ_{Hg} = 13600 kgm^{-3}

g = 9.8 ms^{-2}

$\therefore$ P = 760 × 10^{-3} × 13600 × 9.8

= 1.013 × 10^5 Nm^{-2}

Also, speed of sound of NTP

$$v_2 = \sqrt{\frac{\gamma P}{\rho_a}}$$

$$v_2^2 \rho = \gamma P$$

$$\gamma = \frac{v_2^2 \rho}{P} = \frac{(330.45)^2 \times 1.293}{1.013 \times 10^5}$$

$$\gamma = 1.36$$

$\therefore$ The ratio of molar heat capacities, γ = 1.36

9. The interval between the flash of lighting and the sound of thunder is 2 seconds, when temperature is 10 °C. How far is the storm if the velocity of sound in air at 0°C is 330 ms^{-1}?

SOLUTION

Given,

Let time for lighting to reach a point = t_1

Time for sound to reach that point = t_2

Time interval (Δt) = $t_2 - t_1$ = 2 s

Temperature (T) = 10 °C = 10 + 273 = 283 K

Velocity of sound at 0°C (v_a) = 330 °C

Distance of source (d) = ?

We know, speed of light (c) = 3 × 10^8 ms^{-1}

To find velocity of sound at 10 °C,

$$\frac{v_{10}}{v_0} = \sqrt{\frac{T_{10}}{T_0}}$$

$$\frac{v_{10}}{330} = \sqrt{\frac{283}{273}}$$

$$v_{10} = \sqrt{\frac{283}{273}} \times 330 = 336 \ ms^{-1}$$

Since the distance travelled by sound and light is equal,

$(vt)_{sound} = (vt)_{light}$

$336 \times t_2 = 3 \times 10^8 \times t_1$

$336 \ t_2 = 3 \times 10^8 (t_2 - 2)$

$336 \ t_2 = 3 \times 10^8 \ t_2 - 6 \times 10^8$

$3 \times 10^8 \ t_2 - 336 \ t_2 = 6 \times 10^8$

$2.999 \times 10^8 \ t_2 = 6 \times 10^8$

$$t_2 = \frac{6 \times 10^8}{2.999 \times 10^8}$$

t_2 = 2.00067 s

So, the distance (d) = $v_{sound} \times t_2$

= 336 × 2.00067 = 672.2 m

Challenging Problems

1. **[UP]** At a temperature of 27.0°C, what is the speed of longitudinal waves in (a) Hydrogen (molar mass 2.02 g/mol)? (b) Helium (molar mass 4.00 g/mol)? (c) argon (molar mass 39.9 g/mol)? Compare your answers for parts (a), (b) and (c) with the speed in air at the same temperature. [γ for H_2 = 1.41, γ for He = 1.67, γ for Ar = 1.67]

   Ans: (a) 1320 ms^{-1} (b) 1020 ms^{-1}(c) 323 ms^{-1}

2. **[UP]** A jet airliner is cruising at high altitude at space of 850 km/h. This is equal to 0.85 times the speed of sound at that altitude. What is the air temperature at this altitude? (γ = 1.4, R = 8.31 J/mol.K, M_{air} = 28.8 × 10^{-3} kg/mol)

   Ans:– 82° C

3. **[UP]** The speed of sound in air at 20°C was found to be 344 m/s. What is the change in speed for a 1.0 °C change in air temperature?

Ans: 0.59 ms^{-1}

4. **[UP]** An 80.0 m long brass rod is struck at one end. A person at the other end hears two sounds as a result of two longitudinal waves, one traveling in the metal rod and the other traveling in the air. What is the time interval between the two sounds? (The speed of sound in air is 344 m/s. Y for brass = 9×10^{10} Pa, ρ for brass = 8.6×10^3 kgm^{-3})

Ans: 0.208 s

5. **[UP]** What is the difference between the speed of longitudinal waves in air at 27.0°C and their speed at – 13.0°C?

Ans: 24 ms^{-1}

6. **[UP]** a. In a liquid with density 1300 kg/m^3, longitudinal waves with frequency 400 Hz are found to have wavelength 8.00 m. Calculate the bulk modulus of the liquid.
   b. A metal bar with a length of 1.50 m has density 6400 kg/m^3. Longitudinal sound waves take 3.90×10^{-4} s to travel from one end of the bar to the other. What is Young's modulus for this metal?

Ans: (a) 1.33×10^{10} Nm^{-2}; (b) 6400 = 9.47×10^{10} Nm^{-2}

7. **[UP]** What must be the stress (F/A) in a stretched wire of a material whose Young's modulus is Y for the speed of longitudinal waves to equal 30 times the speed of transverse waves?

Ans: $\frac{Y}{900}$

8. **[ALP]** If a detonator is exploded on a railway line, an observer standing on the rail 2.0 km away hears two reports. What is the time interval between these reports? (Young modulus for steel $=2.0 \times 10^{11} Nm^{-2}$, density of steel = 8.0×10^3 kgm^{-3}, density of air =1.4 kgm^{-3}, ratio of the molar heat capacities of air = 1.40, atmospheric pressure = 10^5 Nm^{-2}). [HSEB 2069]

Ans: 5.92 sec.

9. **[ALP]** Calculate the speed of sound in air at 27°C.
(density of air at s.t.p.= 1.29 kgm^{-3}; ratio of molar heat capacities = 1.4)

Ans: 348 ms^{-1}

10. **[ALP]** A man standing at one end of a closed corridor 57 m long blew a short blast on a whistle. He found that the time from the blast to the sixth echo was two seconds. If the temperature was 17°C, what was the speed of sound at 0°C?

Ans: 331.94 ms^{-1}

11. A sound source of frequency 220 Hz produces sound of wavelength 1.5 m in air at STP. Calculate the increase in wavelength at 27°C.

Ans: 0.075 m

*[**Note:** Hints to challenging problems are given at the end of this chapter.]*

Conceptual Questions with Answers

1. Why echo cannot be heard in small room? [NEB 2075]

➪ Eco is heard after the original sound and it has been reflected from a reflecting surface. To hear the clear echo, the reflecting surface must be situated 17 m away from source. If the reflecting surface lies nearer than 17 m, the reflected sound is mixed to original sound and the echo cannot be heard. In a small room, the walls lie nearer than 17 m, so the echo cannot be heard.

2. We knock the door to inform the person inside the room, why?

➪ Sound wave is easily reflected from the surface of matter, but very small fraction of sound intensity gets refracted. If we call a person from outside of door, the voice may not reach inside the easily. But the vibration produced while knocking the door can reach to the person inside the room. So it is better to knock the door rather than to call from outside.

3. Velocity of sound in solids is more than that in liquids, Why? [NEB 2074]

➯ The velocity of sound depends on two factors: modulus of elasticity and density of medium through which it propagates. The relation is formulated as,

$$v = \sqrt{\frac{E}{\rho}}$$

The ratio $\frac{E}{\rho}$ for solid is greater than for liquid. Therefore, sound travels faster in solid than that of liquid.

4. When sound waves travel through a medium, does the temperature at various points remain constant? Explain. [HSEB 2072]

➯ When sound waves travel through a medium, it forms compression and rarefaction at different points. In compression, particles get crowded and collide to each other. So, the temperature increases. In rarefaction, particles move far away from each other, so the internal energy decreases. As a result, temperature decreases. That is why, the temperature varies at different points of a medium when sound travels through it.

5. Why does sound travel faster in metals than in air? [HSEB 2071]

➯ The speed of sound in a medium is determined

$$v = \sqrt{\frac{\text{Modulus of elasticity}}{\text{density}}} = \sqrt{\frac{E}{\rho}}$$

In case of metal, suppose steel,

$E = 2 \times 10^{11}\ Nm^{-2}$ $\rho = 7800\ kgm^{-3}$

So, speed of sound (v) $= \sqrt{\frac{E}{\rho}} = \sqrt{\frac{2 \times 10^{11}}{7800}} = 5063.7\ ms^{-1}$

in air, at STP, $E = 1.42 \times 10^5\ Nm^{-2}$

$\rho = 1.29\ kg\ m^{-3}$

$$v = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{1.42 \times 10^5}{1.29}} = 331.7\ ms^{-1}$$

This shows that the ratio $\frac{E}{\rho}$ for metal is much greater than that of gas,

i.e. $\left(\frac{E}{\rho}\right)_{Metal} > \left(\frac{E}{\rho}\right)_{gas}$

Hence, the sound travels faster in metal than in gas.

6. The speed of sound in humid air is greater than that in dry air, why?

➯ Humid air contains a large proportion of water vapour i.e. greater proportion of hydrogen (i.e. H_2). However, the dry air contains the greater proportion of nitrogen. Hence, the density of moist air is less than the density of dry air.

Let ρ_m and ρ_d are the density of moist air and density of dry air respectively. Then, speed of sound in these media are,

$$v_m = \sqrt{\frac{\gamma_m P}{\rho_m}} \text{ and } v_d = \sqrt{\frac{\gamma_d P}{\rho_d}}$$

Where, v_m = speed of sound in moist air

v_d = speed of sound in dry air.

Now,

$$\frac{v_m}{v_d} = \sqrt{\frac{\gamma_m P}{\rho_m}} \times \sqrt{\frac{\rho_d}{\gamma_d P}} = \sqrt{\frac{\gamma_m}{\gamma_d}} \times \sqrt{\frac{\rho_d}{\rho_m}}$$

Experimental result in humid and dry air shows that

$\frac{\gamma_m}{\gamma_d} = 0.9$ and $\frac{\rho_d}{\rho_m} = 1.6$

so, $\frac{v_m}{v_d} = \sqrt{0.9 \times 1.6} = 1.2 > 0$

This shows that $v_m > v_d$.

Therefore, the speed of sound is greater in humid air than the dry air. For the same reason, the sound travels faster on a rainy day than on a dry day.

---

7. Although the density of solid is high, the velocity of solid is greater in solid. Why?

➯ The speed of sound apparently seems greater in gas than in solid, as the density of solid is greater than gas, but it is not so. In reality, the speed of sound not only depends on density (ρ) but also on modulus of elasticity (E). The ratio of modulus of elasticity to density $\left(\text{i.e. } \frac{E}{\rho}\right)$ is greater in solid than in gas. Hence, sound travels faster in solid than in gas. Moreover, the speed of sound is greater in liquid than in gas.

---

8. Velocity of sound increases on a cloudy day. Why? [HSEB 2066]

➯ The air contains more moisture in cloudy day than the dry day. As we know, the sound travels faster in moist air than dry air, velocity of sound increases on a cloudy day.

---

9. Do sound waves need a medium travel from one point to other point in space? What properties of the medium are relevant?

➯ Sound requires continuous medium to travel from one point to another point in space. To travel the sound, the medium must possess two basic properties: elastic properties and inertial property. The elastic property is associated with modulus of elasticity and inertial property is associated with mass (or density) of that medium.

---

10. Why sound produced at a distance can be heard distinctly at night than in day time?

➯ The temperature of air nearer the earth's surface is lower than the upper part of atmosphere at night. So, the sound propagating upward gradually deviates away from the original direction and finally reflects back to earth surface: i.e. total internal reflection occurs in sound waves. This process increases the sound intensity nearer the earth's surface at night. So, the sound produced at a distance can be heard distinctly at night than in day time.

---

11. The audible frequency range of a human ear is 20 Hz – 20 kHz. Convert this into corresponding wavelength range. Take the speed of sound in air at ordinary temperature to be 340 ms^{-1}.

➯ Here, the audible range of human ear is 20 Hz to 20 kHz and the speed of sound in air is 340 ms^{-1}.

i. the longest wavelength $\lambda_{longest} = \frac{v}{f_{smallest}}$

$$= \frac{340}{20} = 17 \text{ m}$$

ii. The shortest wavelength, $\lambda_{shortest} = \frac{v}{f_{largest}}$

$$= \frac{340}{20 \times 1000} = 17 \text{ mm}$$

Therefore, the wavelength range for audible sound wave is 17 mm to 17 m.

---

12. Deduce the velocity of longitudinal waves in a metal rod. Given: modulus of elasticity = 7.5×10^{10} Nm^{-2} and density = 2.7×10^3 kgm^{-3}.

➯ Here, the given quantities, $Y = 7.5 \times 10^{10}$ Nm^{-2}

$\rho = 2.7 \times 10^3$ kgm^{-3}

The velocity of the wave $(v) = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{7.5 \times 10^{10}}{2.7 \times 10^3}} = 5270$ ms^{-1}.

Hence, the velocity of longitudinal waves in given metal rod is 5270 ms^{-1}.

13. Do sound waves undergo reflection, refraction, and polarization phenomena? Explain.

➯ The reflection and refraction phenomena occur whether the wave is longitudinal or transverse. Sound wave propagates in longitudinal form in air. So, reflection and refraction occur in sound wave. But for the polarization of wave, it must be the transverse, which is not possible in sound wave. Hence, sound wave cannot be polarized.

14. Why is Laplace correction required to determine the velocity of sound?

➯ Newton assumed the isothermal process in gas medium when sound propagates in it. But in reality adiabatic process occurs while sound propagates in gas. Laplace assumed the adiabatic process in gas medium and derived the correct formula.

15. What is the effect of temperature of gas medium on velocity of sound?

➯ The velocity of sound is directly proportional to the square root of absolute temperature of gas, i.e. $v \propto \sqrt{T}$. Therefore, the velocity increases when temperature of gas increases.

16. What is the effect of pressure on the velocity of sound?

➯ At constant temperature, the variation of pressure and density of gas medium changes simultaneously. So, the ratio of pressure to density always remains constant for a gas $\left(\text{i.e. } \frac{P}{\rho} = \text{constant at constant temperature}\right)$. Hence, pressure of gas is independent on the velocity of sound.

17. Why do we hear rolling sound of thunder?

➯ When the cloud of different electric potentials come closer, they collide and sound is produced. This sound gets reflected many times in the different layers of cloud and produces the echo. These repeated echos are heard as the rolling sound of thunder.

Exercises

Short-Answer Type Questions

1. Why do sound refract?
2. Eco has lower intensity than original sound at the point of source, why?
3. We do not hear the echo, if the reflected is situated nearer than 17 m. Give reason.
4. Compare the speed of sound in solid, liquid and gas.
5. How is the speed of sound affected by the direction of wind flow?
6. Write down the Newton's formula for the speed of sound in a gas. Why did his formula give wrong results?
7. How did Laplace's correct Newton's formula?
8. What is the effect of frequency and amplitude on the speed of sound?
9. Suppose you set your watch by the sound of a distance siren: will it go fast or slow?
10. Why does speed of sound increase with increase in temperature?
11. What is the drawback of Newton's formula for the speed of sound in a gas?
12. How does speed of sound in air change when temperature rises by 10°C?
13. Is it ever, always, or never true to say that the speed of sound at sea level will be greater when the atmospheric pressure is higher? Explain your answer.
14. What is the effect of humidity of air on the speed of sound?
15. Why does the flash of light reach the earth before than the sound coming from the same thunder?
16. How does speed of sound in air change when temperature rises by 1°C?
17. Speed of sound of oxygen is less than that in hydrogen, why?
18. Why is speed of sound independent of pressure in a gas?
19. Why is a given sound louder in a hall than in the open?

Long-Answer Type Questions

1. What are the two fundamental properties of a medium on which speed of sound depends? Derive an expression of speed of sound in a medium.
2. Discuss the effect of pressure, temperature and density of elastic medium on the speed of sound. [HSEB 2052]
3. What is Newton's formula for the speed of sound? What correction was made by Laplace? [HSEB 2053, 2061, 2067]
4. Describe Newton's expression for the speed of sound in a gas with Laplace correction.
5. Discuss Laplace's correction and derive the formula for the speed of sound in a gas. [HSEB 2056]
6. Derive an expression for the speed of sound in a medium by dimensional method. Discuss the effect of change in pressure and temperature on the speed of sound in air. [HSEB 2060]
7. Explain the significance of Laplace's correction of Newton's formula for the speed of sound and derive the corrected formula. [HSEB 2065]
8. Write down the factors on which the speed of sound in air depends with necessary explanation. [HSEB 2066]
9. Discuss the effect of pressure, temperature and humidity on the speed of sound? Does wind affect the speed of sound in air?

Numerical Problems

1. At what temperature, the velocity of sound is $\frac{2}{3}$ of the velocity of sound at 127°C?

   Ans: – 95.22°C
2. The velocity of sound in air at 0°C is 280 ms^{-1}, calculate the velocity of sound at 819°C temperature.

   Ans: 560 ms^{-1}
3. A compressional wave of frequency 256 Hz is set up in an iron rod and this later passes from the rod into air. The speed of the wave in iron is 5120 ms^{-1} and that in air is 352 ms^{-1}. Calculate the wavelength of the wave in each medium.

   Ans: 20 m, 1.375 m
4. At a pressure of 10^5 Nm^{-2}, the volume strain of water is 5×10^{-5}, calculate the speed of sound in water.

   Ans: 1.414×10^3 ms^{-1}
5. Calculate the velocity of sound in oxygen if the velocity of sound in hydrogen is 1248 ms^{-1}.

   Ans: 312 ms^{-1}
6. For air at standard temperature and pressure, the density is 0.001293 gcm^{-3}. Deduce the velocity of longitudinal wave using (i) Newton's formula (ii) Laplace Formula. Given $\gamma = 1.4$

   Ans: 2.8×10^2 ms^{-1}, 3.3×10^2 ms^{-1}
7. What is the percentage increase in the speed of sound when temperature increases from –5°C to 32°C?

   Ans: 6.7%
8. The speed of sound in hydrogen is 1320 m/s. What will be the speed of sound in a mixture of 2 parts by volume of hydrogen and one part by volume of oxygen?

   Ans: 538.58 ms^{-1}
9. The frequency of a tuning fork is 240 Hz. If it is made to vibrate at 27°C, what is the wavelength of the sound emitted? Speed of sound at 0°C is 330 m/s.

   Ans: 1.44 m
10. A body vibrating with a certain frequency sends waves 15 cm long through a medium A and 20 cm long through a medium B. the velocity of waves in A is 1200 cm/s. Find the velocity in B.

    Ans: 16 m/sec

11. Find at what temperature the speed of sound in air is double the speed of sound in air at the temperature of freezing point of water.

Ans: 819°C

12. Show that the speed of sound in air is given by $v_\theta = v_0 + \frac{v_0}{546}\theta$, where v_0 is the speed of sound in air at 0°C and v_θ at θ°C.

13. Calculate the speed of sound in air if temperature is increased by 20°C and pressure is doubled. The speed of sound in air at 20°C is 340 ms^{-1}.

Ans: 351.4 ms^{-1}

14. Show that the speed of sound in a gas for which $\gamma = 1.41$, is 0.68 c, where c is the root mean square velocity of the gas molecules.

Multiple Choice Questions

1. At what temperature will the speed of sound be double of its value at 0°C?
   a. 819 °C b. 919 °C
   c. 1092 °C d. 1192 °C
2. At what temperature the velocity of sound in oxygen is same as that in hydrogen at 27° c.
   a. 4050 °C b. 27 °C
   c. 4527 °C d. 4848 °C
3. Which thermodynamic process is assumed in Laplace correction?
   a. Isothermal b. Adiabatic
   c. Isochoric d. Isobaric
4. The ratio of velocity of sound in hydrogen gas ($\gamma = 7/5$) to that in helium gas ($\gamma = 5/3$) at the same temperature is:
   a. $\sqrt{21}:5$ b. 1 : 1
   c. $\sqrt{42}:5$ d. 2 : 1
5. Speed of sound is maximum in
   a. monoatomic gas. b. diatomic gas.
   c. polyatomic gas. d. equal at all.
6. Speed of sound does not depend on
   a. temperature. b. humidity.
   c. molar mass. d. pressure.
7. Velocity of sound in air isn't dependent on:
   a. Pressure b. Temperature
   c. Moisture d. Composition of air
8. A man heard the thunder 6 seconds layer he saw a lightning. The temperature of air is 27°C. How far was the flash of light from the man? (Velocity of sound in air at 0°C is 332 m/s)
   a. 1822 m b. 2332 m
   c. 2088 m d. 2445 m
9. The velocity of sound in air at NTP is 300 m/s. If the pressure is increased to 4 times the atmosphere pressure, then the velocity of sound will be:
   a. 150 ms^{-1} b. 300 ms^{-1}
   c. 600 ms^{-1} d. 1200 ms^{-1}
10. Laplace's formula for the velocity of sound is:
   a. $v = \sqrt{\frac{\gamma P}{\rho}}$ b. $v = \sqrt{\frac{P}{\rho}}$
   c. $v = \sqrt{\frac{\gamma\rho}{P}}$ d. $v = \sqrt{\frac{\gamma R}{M}}$

11. The intensity of sound at night increases because of:
    a. Low temperature
    b. Increase in density
    c. Decrease in density
    d. Calmness
12. Velocity of sound at 300 K is V. At what temperature velocity of sound becomes doubled:
    a. 300 K
    b. 600 K
    c. 800 K
    d. 1200 K

Answers

1. (a) 2. (c) 3. (b) 4. (c) 5. (a) 6. (d) 7. (a) 8. (c) 9. (b) 10. (a) 11. (c) 12. (d)

Hints to Challenging Problems

HINT: 1

Given,

Temperature of the gas (T) = 27°C = 300 K

Molar mass of (H_2) = 2.02 g/mol = 2.02×10^{-3} kg/mol

Molar mass of (He) = 4 g/mol = 4×10^{-3} kg/mol

Molar mass of argon (Ar) = 39.9 g/mol = 39.9×10^{-3} kg/mol

For hydrogen, $\gamma = 1.41$

For helium, $\gamma = 1.67$

For argon, $\gamma = 1.67$

R = 8.31 J/mol K

For a, b, c, use formula

$$v = \sqrt{\frac{\gamma RT}{M}}$$

For air,

Molar mass of air = 28.8×10^{-3} kg/mol

$\gamma_{air} = 1.4$

Find speed of sound in air by, $v = \sqrt{\frac{\gamma RT}{M}}$

then find, $\frac{v_{H2}}{v_{air}}, \frac{v_{He}}{v_{air}}, \frac{v_{Ar}}{v_{air}}$

HINT: 2

Given,

Molar heat ratio (γ) = 1.4

Universal gas constant (R) = 8.31 J/mol K

Molar mass of air (M_{air}) = 28.8 g

Velocity of jet,

$$v_{Jet} = 850 \text{ km/h} = \frac{850 \times 1000 \text{ m}}{3600 \text{ s}} = 236.11 \text{ ms}^{-1}$$

Also, given condition, $v_{jet} = 0.85 \times v$

$$\therefore \quad v = \frac{v_{jet}}{0.85}$$

Let T be the required temperature. Now velocity,

$$v = \sqrt{\frac{\gamma RT}{M}}$$

$$\text{or} \quad v^2 = \frac{\gamma RT}{M}$$

$$\therefore \quad T = \frac{v^2 M}{\gamma R}$$

HINT: 3

Given, $T_1 = 20°C$

$v_{20} = 344 \text{ ms}^{-1}$

Change in speed for a 1°C in air temperature,

$\Delta v = ?$

$\Delta T = 1°C = 1 K$

$\therefore$ New temperature (T_2) = (20 + 1) °C = 21°C

We know that

$$v_{21} = \sqrt{\frac{\gamma RT_2}{M}}$$

Also,

$$v_{20} = \sqrt{\frac{\gamma RT_1}{M}}$$

$$\therefore \quad \frac{v_{21}}{v_{20}} = \sqrt{\frac{T_2}{T_1}}$$

$$\text{or} \quad v_{21} = \sqrt{\frac{T_2}{T_1}} \times v_{20}$$

Then use,

$\Delta v = v_{21} - v_{20}$

HINT: 4

Given,

Length of brass (l) = 80 m

Speed of sound in air (v_a) = 344 m/s^{-1}

Young's modulus of brass (Y) = 9×10^{10} Pa

Density of brass (ρ) = 8.6×10^3 kgm^{-3}

Time Interval ($t_a - t_m$) = ?

Now, Time interval $= (t_a - t_m)$

$$= \frac{l}{v_a} - \frac{l}{v_m}$$

$$= \frac{l}{v_a} - \frac{l}{\sqrt{\frac{Y}{\rho}}}$$

HINT: 5

Given,

$T_{27} = (27 + 273) = 300$ K

Molecular mass of air = 28.8×10^{-3} kg/mol

For air $\gamma = 1.4$

$T_{-13} = (273 - 13) = 260$ K

R = 8.31 $Jmol^{-1}K^{-1}$

By questions;

$v_{27} - v_{-13} = ?$

We know that

$$v = \sqrt{\frac{\gamma RT}{M}}$$

Therefore,

$$v_{27} - v_{-13} = \sqrt{\frac{\gamma RT_{27}}{M}} - \sqrt{\frac{\gamma RT_{-13}}{M}}$$

$$= \sqrt{\frac{\gamma R}{M}}\left(\sqrt{T_{27}} - \sqrt{T_{-13}}\right)$$

HINT: 6

Given,

Density of liquid (ρ_l) = 1300 kg/m^3

Frequency (f) = 400 Hz

Wavelength (λ) = 8 m

a. Bulk modulus (K) = ?

We know that

$$v = \sqrt{\frac{K}{\rho_l}}$$

or $(f\lambda)^2 = \dfrac{K}{\rho_l}$ ($\because v = f\lambda$)

or K $= (f\lambda)^2 \times \rho_l$

b. Length of bar (l) = 1.50 m

Density of bar (ρ) = 6400 kg/m^3

Time taken (t) = 3.90×10^{-4} s.

Young's modulus (Y) = ?

We know,

$$v = \sqrt{\frac{Y}{\rho}}$$

or $\left(\dfrac{l}{t}\right)^2 = \dfrac{Y}{\rho}$ [$\because v = \dfrac{\text{distance}}{\text{time}} = \dfrac{l}{t}$)

or $Y = \left(\dfrac{l}{t}\right)^2 \times \rho$

HINT: 7

Given,

Stress (F/A) = ?

Speed of longitudinal wave = $30 \times v_t$ (speed of transverse wave)

or $\sqrt{\dfrac{Y}{\rho}} = 30 \times \sqrt{\dfrac{F}{\mu}}$, where μ is the mass per unit length.

or $\dfrac{Y}{\rho} = 900 \times \dfrac{F}{\mu}$

or $\dfrac{Y}{\rho} = 900 \times \dfrac{F}{A \times \rho}$ ($\because \mu = \dfrac{\text{mass}}{\text{lengh}} = \dfrac{v \times \rho}{l} = A\times\rho$)

or $\dfrac{F}{A} = \dfrac{Y}{900}$

HINT: 8

Given,

Distance, s = 2 km = 2×10^3 m

Let, t_a be the time taken in air and t_r in rail.

Time interval between two reports = $t_a - t_r$ = ?

Young modulus of steel, Y = 2×10^{11} Nm^{-2}

Density of steel, $\rho = 8 \times 10^3$ kgm^{-3}

Density of air, σ = 1.4 kgm^{-3}

Ratio of molar heat capacities of air (γ) = 1.40

Atmospherics pressure, P = 10^5 Nm^{-2}

Now,

Time interval = $t_a - t_r$

$= \dfrac{s}{v_a} - \dfrac{s}{v_r}$ ($\because$ speed (v) = $\dfrac{\text{distance (s)}}{\text{time (t)}}$)

$= \dfrac{s}{\sqrt{\dfrac{\gamma P}{\sigma}}} - \dfrac{s}{\sqrt{\dfrac{Y}{\rho}}}$ ($\because v_a = \sqrt{\dfrac{\gamma P}{\sigma}}$ and $v_r = \sqrt{\dfrac{Y}{\rho}}$)

HINT: 9

Speed of sound in air at 27°C, v_{27} = ?

Density of air at S.T.P., ρ_0 = 1.29 kgm^{-3}

Ratio of molar heat capacities, γ = 1.4

Normal pressure, P = 1.013×10^5 Nm^{-2}

Now,

Speed of sound in air at STP,

$$v_0 = \sqrt{\frac{\gamma P}{\rho_0}} = \sqrt{\frac{1.4 \times 1.013 \times 10^5}{1.29}}$$

$\therefore$ v_0 = 331.57 ms^{-1}

We know that $v \propto \sqrt{T}$

$$\therefore \frac{v_0}{v_{27}} = \sqrt{\frac{T_0}{T_{27}}}$$

HINT: 10

Given,

Time for six echoes (t) = 2 s

Temperature (T_{17}) = 17 + 273 = 290 K

Speed of sound at 0° C (v_0) = ?

$\because$ Distance for one echo = 57 × 2 m

$\therefore$ Distance for six echoes = (57 × 2) × 6 m

$$v_{17} = \frac{\text{total distance for six echoes}}{\text{total time taken}}$$

$$= \frac{(2 \times 57) \times 6}{2}$$

$\therefore$ $v_{17} = 342 \text{ ms}^{-1}$

We know that

$$v \propto \sqrt{T}$$

$$\therefore \quad \frac{v_0}{v_{17}} = \sqrt{\frac{T_0}{T_{17}}}$$

HINT: 11

Given,

Frequency of source, $f = 220$ Hz

Wavelength at STP, $\lambda_0 = 1.5$ m

$\therefore$ Speed of sound at STP,

$v_0 = f\,\lambda_0 = 220 \times 1.5 = 330 \text{ ms}^{-1}$

Let λ_{27} and v_{27} be the wavelength and speed of sound at 27°C respectively. Then,

$$\frac{v_{27}}{v_0} = \sqrt{\frac{273 + 27}{273}}$$

Then find v_{27} and use, $\lambda_{27} = \dfrac{v_{27}}{f}$

Finally, increase in wavelength $= \lambda_{27} - \lambda_0$

❖ ❖ ❖

WAVES IN PIPES AND STRINGS

3
CHAPTER

3.1 Tone, Note, Harmonics and Overtones

Tone

The sound of definite frequency is called a tone. A tone is produced when a body of certain length and mass is vibrated. The sound produced by a tuning fork is an example of a tone. The sound produced by a string of guitar is also another example of tone. If the length of string is changed, the frequency of tone is also changed.

Note

Note of sound is the combination of many tones. If the strings of guitar are vibrated pressing different lengths, tones of different frequencies are produced. A person listening the music of guitar can not detect single tone, rather he/she listens the combination of tones, which is termed as note of sound.

Harmonics

A harmonics is a signal or wave whose frequency is an integral multiple of the some reference signal. For a wave whose fundamental frequency is f, it is called the first harmonic. Then, second harmonic, third harmonic, fourth harmonic, …, etc are represented by 2f, 3f, 4f, …, etc. respectively. The signal occurring at frequencies of 2f, 4f, 6f..., etc. are called even harmonics and the signals at frequencies 3f, 5f, 7f, …, etc. are called odd harmonics. Theoretically, a signal can infinitely have many harmonics.

Overtone

An overtone is a musical tone which is a part of the harmonic series above the fundamental tone. If a sound possesses all possible harmonics f, 2f, 3f, 4f, 5f, etc…, 2f is called first overtone, 3f is called second overtone and so on. If a sound have odd harmonics, f, 3f, 5f, 7f, … etc, 3f is called first overtone, 5f is called second overtone, and so on.

Note

The frequency pattern in harmonics and overtones can be studied making an analogy with the energy level of hydrogen atom in which the harmonics corresponds to principle quantum number (n), the fundamental note corresponds to ground state energy level and overtone corresponds to excited state. For example, when n = 2, the hydrogen atom is in the first excited state. Correspondingly, f_2 in the second harmonics is the first overtone. Similarly, f_3, f_4 are the 2nd and 3rd overtone and so on.

3.2 Organ Pipes

Organ pipes are the musical devices that produce the sound of certain frequency by pressurizing the air into them. Each pipe is tuned to a specific note of musical scale. Organ pipes are generally made up of wood or metal and have mainly three different shapes: cylindrical, conical or rectangular. Flute, horn, whistle, etc. are some examples of organ pipes. The longitudinal waves traveling along the length of pipe when interfere with the reflected waves from the another end, stationary or standing waves are produced. Resonance condition occurs into it when the disturbance has the equal frequency with the natural frequency of air molecules.

Organ pipes are basically categorized into two types: closed end organ pipe (closed organ pipe) and open end organ pipe (open organ pipe).

Closed Organ Pipe

The organ pipe whose one end is open to air and another end is closed off is closed organ pipe. The air at the closed end of the pipe does not move, however the air moves freely at the open end. Since the air is not moving at the closed end, a node (N) is formed at this position. The movement of air is maximum at the opening, so an anti-node (A) is formed at this end as shown in Fig. 3.1 (ii).

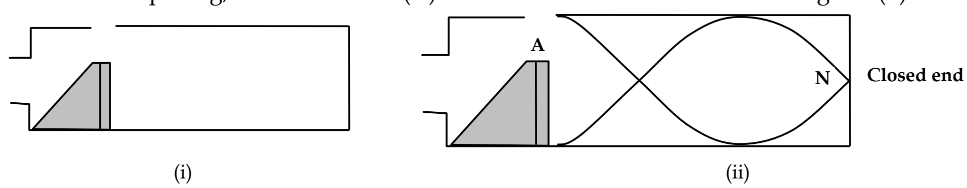


Fig. 3.1: Closed organ pipe

When the air is set into vibration at the open end, the longitudinal waves (compressions and rarefactions) travel into the pipe towards the closed boundary. These waves, then reflect back towards the open end after colliding the air molecules at the closed end. In reflection, compression reflects back as the rarefaction as the phase reversal. The wave travelling from the open end when superimposed with the reflected wave from the closed end, stationary (or standing) wave is formed into the pipe.

Modes of Vibration in Closed Organ Pipe

Different values of frequency of sound can be obtained into the closed end pipe, although the length is constant. The vibrations that possess the frequency in specific pattern are known as modes of vibration. There are various modes of vibration into the closed end pipe. The modes of vibration depends on the number of nodes or anti-nodes formed into the pipe. The mechanism and nature of some modes of vibration are described below.

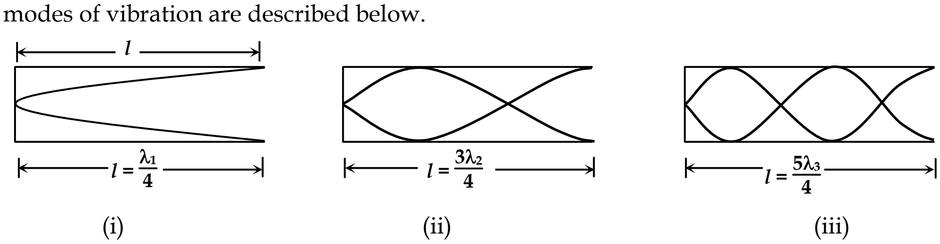


Fig. 3.2: Modes of vibration of closed organ pipe

i. **First mode of vibration:** If a node at the closed end and an anti-node at open end are formed in a pipe, the corresponding mode of vibration is called first mode of vibration. In this mode, minimum possible frequency of sound is produced into the pipe, so the mode is called fundamental mode and corresponding frequency is called fundamental frequency.

Let l be the length of a closed end pipe and λ_1 be the wavelength of fundamental tone as shown in Fig. 3.2 (i). Also, consider f_1 be the fundamental frequency of vibration in the pipe and v be the speed of sound in air. Then,

$$\mathbf{f_1 = \frac{v}{\lambda_1}} \text{ (v is constant in air at constant temperature)} \quad \text{... (3.1)}$$

One quarter part of a complete wave is formed into the pipe, so,

$$l = \frac{\lambda_1}{4}$$

$$\therefore \quad \boldsymbol{\lambda_1 = 4l} \quad \text{... (3.2)}$$

Therefore, the fundamental frequency of sound in this pipe is,

$$\mathbf{f_1 = \frac{v}{4l}} \quad \text{... (3.3)}$$

It is the lowest frequency of sound produced by the pipe, which is called the first harmonic in closed organ pipe.

$$\text{In air, } v = \sqrt{\frac{\gamma P}{\rho}}$$

$$\text{so,} \quad f_1 = \frac{1}{4l}\sqrt{\frac{\gamma P}{\rho}}$$

ii. Second mode of vibration: In this mode of vibration, two nodes and two anti-nodes are formed in the pipe as shown in Fig. 3.2 (ii). Let l be the length of pipe and λ_2 be the wavelength of note produced in this mode of vibration. Then, the corresponding frequency, f_2 of the vibration is,

$$\mathbf{f_2 = \frac{v}{\lambda_2}} \quad \text{...(3.4)}$$

Three quarter part of a complete wave is formed into the pipe, so,

$$l = 3\frac{\lambda_2}{4}$$

$$\therefore \quad \boldsymbol{\lambda_2 = \frac{4l}{3}} \quad \text{... (3.5)}$$

Therefore, the frequency of sound in this mode is,

$$f_2 = \frac{v}{\left(\frac{4l}{3}\right)}$$

$$f_2 = 3\frac{v}{4l}$$

$$\mathbf{f_2 = 3\,f_1} \quad \text{... (3.6)}$$

Equation (3.6) gives the frequency of vibration in second mode of vibration. It is called third harmonic or first overtone. As the frequency of vibration in this mode is three times greater than the fundamental mode, the harmonic is called third harmonic.

iii. Third mode of vibration: In this mode of vibration, three nodes and three anti-nodes are formed in the pipe as shown in Fig. 3.2 (iii). Let λ_3 be the wavelength of note produced in this mode at velocity v in the pipe of length l, the corresponding frequency, f_3 of vibration is written as,

$$\mathbf{f_3 = \frac{v}{\lambda_3}} \qquad \text{... (3.7)}$$

One complete and one quarter part of a wave is formed in this mode of vibration, so,

$$l = \lambda_3 + \frac{1}{4}\lambda_3$$

$$l = \frac{5\lambda_3}{4}$$

$$\therefore \quad \mathbf{\lambda_3 = \frac{4l}{5}} \qquad \text{... (3.8)}$$

Therefore, the frequency of sound in this mode is,

$$f_3 = \frac{v}{\left(\frac{4l}{5}\right)}$$

$$f_3 = 5\frac{v}{4l}$$

$$\mathbf{f_2 = 5\ f_1} \qquad \text{... (3.9)}$$

Equation (3.9) gives the frequency of sound in third mode of vibration. It is called fifth harmonic or second overtone. As the frequency of vibration is five times greater than the fundamental frequency, this is called fifth harmonic. In this way, the pattern of frequency in the succeeding modes of vibration must be $7f_1$, $9f_1$, $11f_1$, … etc. This concludes that, only odd multiple of fundamental frequency or odd harmonics are possible in closed organ pipe. In this pipe, even harmonics are absent, so the sound is imperfect and is not sweet.

Conclusions

i. Only odd harmonics are possible in closed end pipe i.e., frequency ratio is 1 : 3 : 5 and so on.

ii. The even harmonics (2^{nd}, 4^{th}, 6^{th} etc.) are missing and hence sound is not pleasant to ear. That is why this organ pipe is not generally used as a musical instrument.

iii. The fundamental frequency in case of closed organ pipe is half as compared to that of an open organ pipe of the same length.

iv. In general, frequency of n^{th} overtone = $(2n + 1)$ times the fundamental frequency, n = 1, 2, 3, …

v. The number of nodes is equal to the number of antinode in each mode.

vi. For $(2n - 1)^{th}$ harmonic, number of nodes or number of antinodes = n
(where n = 1, 2, 3, . . .) but only odd harmonics are present.

vii. Fundamental frequency (f) $\propto \frac{1}{l}$ i.e., frequency increases as the length of pipe is decreased.

viii. Also f $\propto$ v where v is the speed of sound in air. Also, $v \propto \sqrt{T}$ where T is absolute temperature.

$\therefore$ $f \propto \sqrt{T}$, i.e., the fundamental frequency is directly proportional to the square root of absolute temperature.

3.3 Open Organ Pipe

An organ pipe whose both ends are open to air is called open organ pipe. At both ends of this pipe, air is free to move, so antinodes are formed at these positions as shown in Fig. 3.3. When a vibration is set up at one end of the pipe, it travels towards the another end in the form of compressions and rarefactions. The nature of reflection is quite complicated and depends on how wide or narrow the pipe is, in comparison with wavelength of sound. In most of the musical instruments, the tube is comparatively narrow relative to wavelength. In such situation, when compression reaches at open end, the density of air readily decreases. Hence, the rarefaction travels back to the pipe. Also, when a rarefaction reaches to the open end, compression reflects back. Finally, the stationary wave is set up into the pipe due to the superposition of incident wave traveling towards the open end and reflected wave from that end.

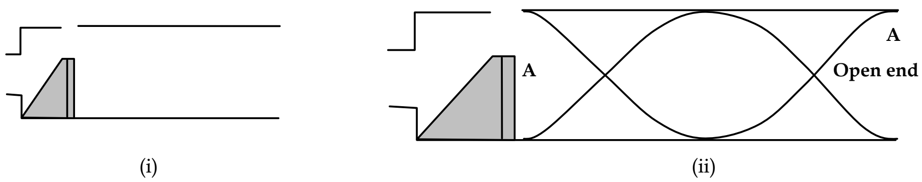


Fig. 3.3: Open organ pipe

Modes of Vibration in Open Organ Pipe

There are various modes of vibration in open organ pipe. The mechanism and nature of some modes of vibration in open organ pipe are described below:

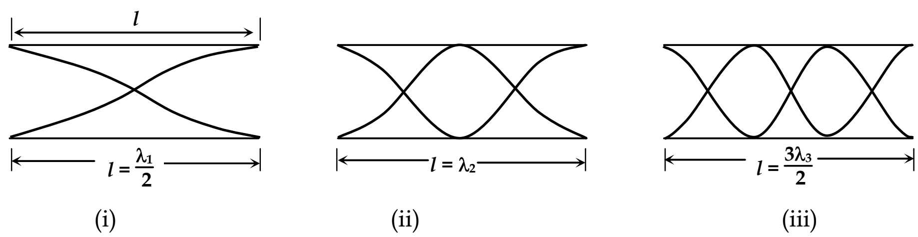


Fig. 3.4: Modes of vibration of open organ pipe

i. **First mode of vibration:** In this mode of vibration, two anti-nodes and one node are formed into the pipe as shown in Fig. 3.4 (i). In this mode, the minimum possible frequency of sound is produced in the pipe, so it is called fundamental mode and the corresponding frequency is called fundamental frequency. Let l be the length of open organ pipe, in which a longest possible wavelength λ_1 is formed. The frequency f_1 of vibration at velocity v in the pipe is,

$$\mathbf{f_1 = \frac{v}{\lambda_1}} \qquad \text{... (3.10)}$$

One half part of a complete wave is formed into the pipe in fundamental mode, so,

$$l = \frac{\lambda_1}{2}$$

$$\therefore \quad \mathbf{\lambda_1 = 2l} \qquad \text{... (3.11)}$$

Therefore,

$$\mathbf{f_1 = \frac{v}{2l}} \qquad \text{... (3.12)}$$

It is the lowest frequency produced by the open pipe, which is called the first harmonic of sound.

$$\text{In air, } v = \sqrt{\frac{\gamma P}{\rho}}$$

$$\text{So,} \quad f_1 = \frac{1}{2l}\sqrt{\frac{\gamma P}{\rho}}$$

ii. **Second mode of vibration:** Three anti-nodes and two nodes are formed in this mode of vibration as shown in Fig. 3.4 (ii). Let λ_2 be the wavelength of vibration in the pipe of length *l*. Then, the corresponding frequency f_2, of vibration at velocity v is,

$$\mathbf{f_2 = \frac{v}{\lambda_2}} \qquad \ldots (3.13)$$

One complete wave is formed into the pipe so,

$$\mathbf{l = \lambda_2} \qquad \ldots (3.14)$$

Therefore, the frequency of sound in second mode of vibration is,

$$f_2 = \frac{v}{l}$$

$$f_2 = \frac{2}{2}\frac{v}{l}$$

$$f_2 = 2\left(\frac{v}{2l}\right)$$

$$\therefore \quad \mathbf{f_2 = 2f_1} \qquad \ldots (3.15)$$

Equation (3.15) gives the frequency of sound in second mode of vibration. It is called the second harmonic or first overtone. As the frequency of sound in this mode is double than fundamental frequency, it is called second harmonic.

ii. **Third mode of vibration:** Four anti-nodes and three nodes are formed in this mode of vibration as shown in Fig. 3.4 (iii). Let λ_3 be the wavelength of vibration in the pipe of length *l*. Then, the corresponding frequency f_3 of vibration of wave at velocity v is,

$$\mathbf{f_3 = \frac{v}{\lambda_3}} \qquad \ldots (3.16)$$

One complete and half of one complete wave is formed into the pipe in the third mode of vibration, so,

$$l = \lambda_3 + \frac{1}{2}\lambda_3$$

$$l = \frac{3\lambda_3}{2}$$

$$\therefore \quad \mathbf{\lambda_3 = \frac{2l}{3}} \qquad \ldots (3.17)$$

Therefore,

$$f_3 = \frac{v}{\left(\frac{2l}{3}\right)}$$

$$f_2 = 3\left(\frac{v}{3l}\right)$$

$$\therefore \quad \mathbf{f_3 = 3f_1} \qquad \ldots (3.18)$$

Equation (3.18) gives the frequency of sound in third mode of vibration. It is called the third harmonic or second overtone. As the frequency of sound in third mode of vibration is three times greater than the fundamental frequency, it is called third harmonic.

In this way, the pattern of frequency in the succeeding modes of vibration must be $4f_1$, $5f_1$, $6f_1$, … etc. This shows that open end pipe can produce the sound of frequency in the integer multiple of fundamental frequency (i.e. f_1, $2f_1$, $3f_1$, $4f_1$, … etc.) Thus, it produces the sound of both even and odd harmonics. Therefore, the sound produced by open end pipe is sweet.

Conclusions

i. All harmonics are present and hence, the sound is richer in quality.

ii. In general, the frequency of n^{th} overtone = $(n + 1)$ times the fundamental frequency, n = 1,2, 3, …

iii. The frequencies of the various harmonics of open organ pipe are integral multiple of fundamental frequency. i.e. $f_n = nf_1$.

iv. The frequency of fundamental note of an open organ pipe is double than that of closed organ pipe of same length. Since, in open organ pipe two antinodes are formed at open ends and one node at the middle of the pipe, its length, $l = \frac{\lambda}{2}$, but $f_o = \frac{v}{\lambda} = \frac{v}{2l}$. But in case of closed organ pipe, there is one antinode at open end and one node at closed end. So $l = \frac{\lambda}{4}$, but $f_c = \frac{v}{\lambda} = \frac{v}{4l}$.

So, $f_c = 2f_o$ i.e. frequency of fundamental tone of an open organ pipe is twice of that of closed organ pipe.

v. The frequency ratio is 1 : 2 : 3 and so on. i.e., $f_1 : f_2 : f_3 \ldots = 1 : 2 : 3 : \ldots$

vi. For n^{th} harmonic, number of nodes equals n and number of antinodes is (n + 1).

3.4 End Correction of Organ Pipe

In the previous study of nature of sound waves in closed and open organ pipes, antinode of wave is considered exactly at the open end of the pipe. But, the experimental result shows that the antinode is formed slightly outside the end where the air molecules become completely free for the vibration. It means, the acoustic length of the pipe is slightly greater than the physical length due to the free vibration of wave particles outside the end. Therefore, the frequency of sound in different modes of vibration calculated in the previous discussion is erroneous. Hence, a correction factor has to be incorporated to the length of pipe. This factor is called end correction of organ pipe. Therefore, *the end correction is defined as the distance of real position of antinode outside the end from end of pipe*. It is denoted by e or c.

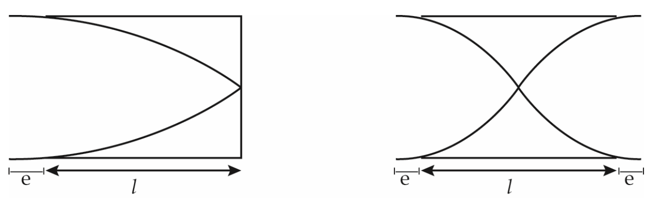


Fig. 3.5: End correction in (i) closed end pipe; (ii) open end pipe

Lord Rayleigh determined the end correction experimentally, taking the tubes of different diameters and formulated the result empirically. His formula, regarding the end correction is known as Rayleigh correction. The Rayleigh correction formula is, e = 0.6R, where R is the radius of the tube.

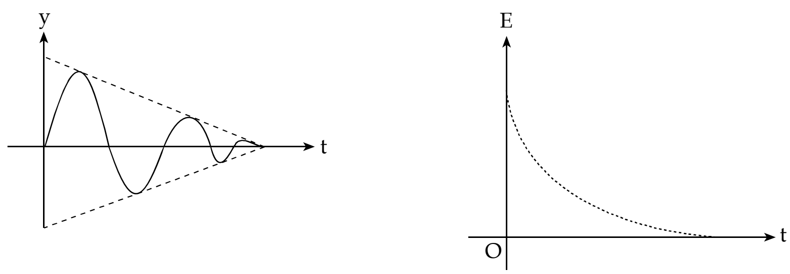
End correction is measured at an open end, but not at the closed end. So, the end should be corrected at one end of closed end pipe and at both ends in open end pipe. The end correction in closed end pipe and open end pipe are shown in Fig. 3.5.

i. For a closed end pipe of physical length l, as shown in Fig. 3.5 (i), the corrected length is given by $l + 0.6R$ $(= l + e)$

ii. For an open end pipe of physical length *l*, as shown in Fig. 3.5 (ii), the corrected length is given by $l + 2 \times 0.6R$ $(= l + 2e)$

3.5 Forced and Damped Oscillation

The energy of an ideal oscillating system always remains constant. But an oscillator almost always lies in a resistive medium. Some part of energy is dissipated in overcoming such resistive forces so that energy continuously decreases and the oscillation dies out. Such oscillations are called damped oscillations. The displacement time graph for such oscillation is shown in Fig. 3.6 (i).



(i) Displacement-time graph (ii) Energy-time graph

Fig. 3.6 : Damped oscillation

Every body in this universe has a characteristic tendency to vibrate when external force is applied. The amplitude and frequency of vibration of the body depends on its shape, size and the elastic properties. This frequency of vibration is called natural frequency.

When a body is displaced from its mean position, it vibrates under the action of restoring force. If there are no resistive forces such as friction, viscous force etc, the body keeps on vibrating with its natural frequency at a constant amplitude. Such vibration is called free oscillation or vibration. The total mechanical energy of such vibration is always conserved. The displacement-time graph and energy time graph for such oscillations are as shown below.

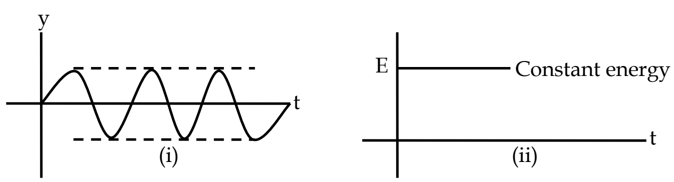


Fig 3.7: (i) Displacement-time graph for free oscillation (ii) Energy-time graph

When external periodic force is applied continuously on a damped oscillator, it can have sustained oscillations. If the energy provided by the external force is equal to the energy lost by dissipative forces, then the oscillation continues with constant amplitude. Such oscillations are called forced oscillations.

3.6 Resonance

Resonance is a special case of forced vibration. If an external force is applied to vibrate a system, the amplitude of vibration of the system is maximum at a specific frequency. To set the system in maximum amplitude, the frequency of vibration of vibrating system must be equal to its natural frequency. *This phenomenon in which the amplitude of vibration is maximum when the frequency of* *vibrating system is equal to its natural frequency is known as resonance.* The corresponding frequency of vibration is called resonant frequency.

The natural frequency of a body can be understood considering the very familiar example of simple pendulum. The time period of oscillation of simple pendulum is,

$$\text{Time period (T)} = \frac{1}{f} = 2\pi\sqrt{\frac{l}{g}}$$

$$\therefore \quad \frac{1}{f} = 2\pi\sqrt{\frac{l}{g}}$$

For effective length of 50 cm (0.5 m),

$$\frac{1}{f} = 2\pi\sqrt{\frac{0.5}{9.8}} = 1.42$$

$$\therefore \quad f = \frac{1}{1.42} = 0.70 \text{ Hz}$$

It shows that the frequency of oscillation depends on the effective length (l) of pendulum at a place. For a constant length, the frequency of oscillation is constant, whatever distance the pendulum is displaced from the mean position.

This tells us that if the pendulum is forced with vibration of frequency of 0.7 Hz, the amplitude of vibration of the pendulum is maximum. The variation of amplitude of vibration for a body when it is given a gradually changing frequency, is shown in Fig. 3.8.

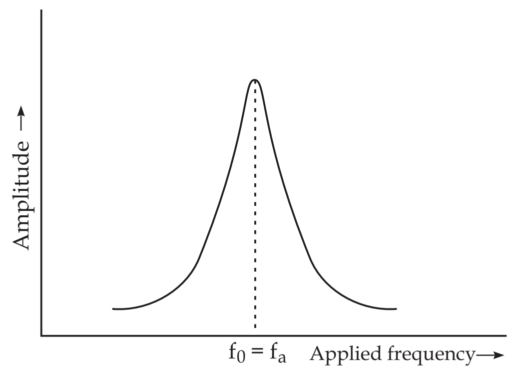


Fig. 3.8: Resonant frequency

Where, f_0 = natural frequency

f_a = frequency of forced vibration

Consequences of Resonance

1. The soldiers are asked to break their marching steps while crossing the bridge. If the frequency of walking steps of soldiers is equal to the natural frequency of vibration of bridge, the amplitude of its variation is very large. Thus, the resonance occurs. In this situation, the bridge suffers large extension and may cross the elastic limit and collapse.
2. When we set the frequency in our radio that matches the frequency broadcasted from station, the radio produces the sound. For example, if we set the FM radio at 100 MHz in Nepal, we receive the broadcasting from Radio Nepal.
3. A music expert can produce the musical note that may be matched to the natural frequency of oscillation of glass tumbler. In this condition, resonance occurs in the vibration of the glass and it may break. Therefore, it is believed that one of the Nine 'Jewels' of Emperor Akbar, widely known as Tansen, the king of music was able to break a glass by singing the appropriate note.
4. Resonance can cause great damage in an earthquake. If the natural frequency of a building matches the frequency of periodic oscillations present within the earth, then resonance occurs and building vibrates with large amplitude. So, the building may be collapsed. Probably, the great damage in Sindupalchowk, Gorkha earthquake 2072 occurred due to the resonance of

seismic wave with natural frequency of houses at this location, although it is far from epicenter at Barpark, Gorkha.

Tuning Fork

A tuning fork is a U-shaped acoustic resonator and is connected to a common base extended as a metal bar, called the stem. The U-shaped forks of the tuning forks are called prongs. The prongs are struck on a rubber pad to produce pure tone. When the prongs are struck on the pad by holding on the stem, these prongs move alternately towards and away from each other. The tone produced from the vibration of tuning fork is specific, whatever the force you applied to produce the sound. The frequency of tone depends on mass and length of prongs as shown in Fig. 3.9.

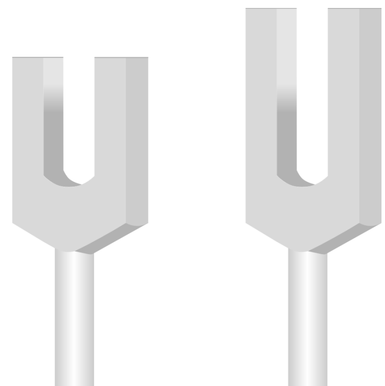


Fig. 3.9: Tuning forks (i) greater frequency; (ii) smaller frequency

Tuning fork was invented in 1711 by John Shore, a renowned musician, instrument maker and trumpeter to the English Royal Court. Usually a tuning fork of C512, is used to detect the hearing ability of the patient in ENT department of hospitals. It is also used in physics laboratory to perform many experiments regarding resonance phenomena.

The prongs of tuning fork oscillate in transverse pattern when striked on a rubber pad. Transverse vibrations superimpose at upper part of the stem and the vibration longitudinally propagates to the lower end of stem. Hence, the wave does not damp readily although we hold on it.

3.7 Resonance Tube Apparatus

Resonance tube is a metallic tube that works on the principle of resonance of the air column inside it. Resonance occurs into the tube when natural frequency of air column inside the tube is equal to the frequency of vibration supplied by tuning forks. In this condition, the air inside the tube vibrates with maximum amplitude and hence sound is produced. It is an example of closed end pipe.

Resonance tube apparatus consists of a resonance tube, a transparent pipe and a water reservoir with common base as shown in Fig. 3.10 (i). The resonance tube is opaque metallic tube and the experiment for the resonance is performed in it. The level of water in the tube can not be visualized directly, so a transparent pipe is fitted at the common base. The water level in the tube is measured with the help of this pipe. A meter scale is fixed in the pipe such that variation of water level can easily be noticed. A water reservoir is used to adjust the water level into the tube.

Resonance tube apparatus is basically used to determine the speed of sound in air and find the end correction of the tube used for the experiment.

Measurement of Speed of Sound and End Correction of Tube

To start the experiment, a tuning fork is struck on a rubber pad and held over the upper end of resonance tube. In the beginning, the water level is filled upto the rim of resonance tube. Then, the level is lowered gradually, until the loud sound is heard into the tube. This is the condition of resonance and is called first resonance. The length of air column above the water level is called first resonating length as shown in Fig. 3.10 (ii). In this case, one quarter of a complete wave is formed in the tube.

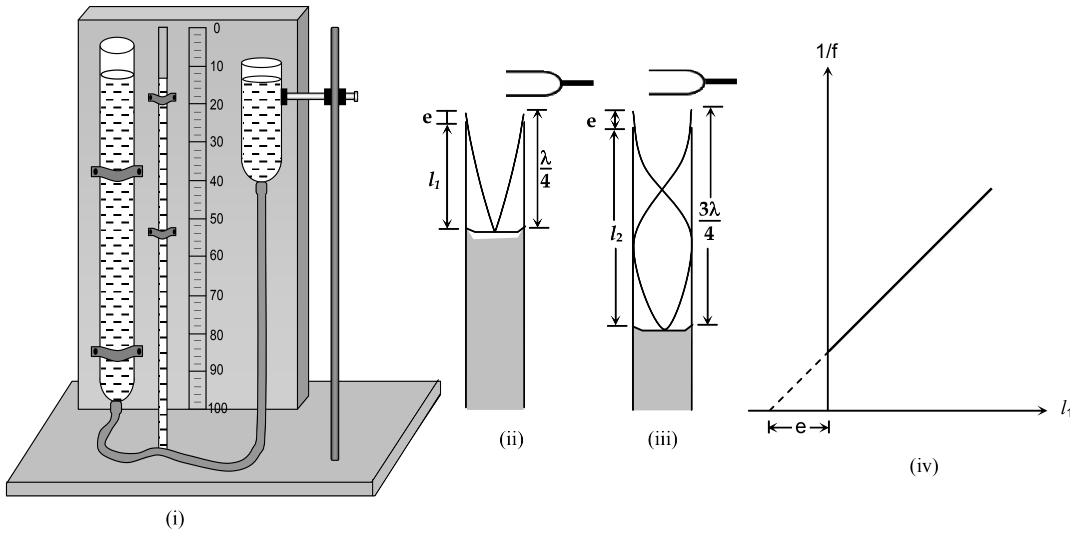


Fig. 3.10: Resonance tube apparatus

Let l_1 be the first resonating length and e be the end correction of the tube. Then, for the condition of first resonance, we write,

$$l_1 + e = \frac{\lambda}{4} \qquad \dots(3.19)$$

Where, λ is the wavelength of sound produced into the tube

Again, water level is gradually lowered from the position of first resonance. After lowering the water level more than three times the first resonating length, the tube will resound with high intensity. This condition is called second resonance and the corresponding length of air column above the water level is called second resonating length. Let l_2 be the second resonating length of air column as shown in Fig. 3.10 (iii). Then,

$$l_2 + e = 3\frac{\lambda}{4} \qquad \dots(3.20)$$

Determination of Speed of Sound

The wavelength of sound into the tube is determined in terms of first and second resonating length. For this, subtracting equation (3.19) from equation (3.20) i.e.

$$l_2 - l_1 = \frac{3\lambda}{4} - \frac{\lambda}{4}$$

$$l_2 - l_1 = \frac{\lambda}{2}$$

$$\therefore \quad \lambda = 2(l_2 - l_1) \qquad \dots (3.21)$$

The speed of sound wave,

$$v = f\lambda \qquad \dots (3.22)$$

where, f is the frequency of tuning fork used to set the air column at resonance.

Substituting the value of λ from equation (3.21) to equation (3.22), we get,

$$v = 2f\,(l_2 - l_1) \qquad \dots (3.23)$$

This is the required expression to determine the speed of sound in air.

Determination of End Correction

Now, multiplying the equation (3.19) by 3 and equating equations (3.19) and (3.20), we get,

$$l_2 + e = 3l_1 + 3e$$

$$\therefore \quad 3e - e = l_2 - 3l_1$$

$$2e = l_2 - 3l_1$$

$$\therefore \quad \mathbf{e = \frac{l_2 - 3l_1}{2}} \qquad \text{... (3.24)}$$

By measuring the values of l_1 and l_2, the speed of sound and end correction can be measured. The value of end correction that is determined from the experiment can be compared to the end correction determined from Rayleigh formula,

i.e., $e = 0.6R$

Where, R = radius of resonance tube

Furthermore, the speed of sound can be corrected including the effect of humidity, which is called the humidity correction. The required formula for the velocity of sound taking account of the humidity correction is,

$$\mathbf{v_{STP} = v_\theta \sqrt{\frac{P - 0.375f}{P}} \left(1 - \frac{1}{2}\alpha\theta\right)} \qquad \text{... (3.25)}$$

Where, P = atmospheric pressure

f = saturated vapour pressure at temperature θ^oC

α = volume coefficient

θ = room temperature

It is to be noted that, any type of liquid can be used for the resonance tube experiment. The role of liquid is merely to produce the closed end pipe with variable length of smooth boundary. Water is non-toxic, universally available and economically cheap liquid. So, it is usually used in resonance tube.

3.8 Waves in String

A string is a solid material which is physically very thin, perfectly flexible and has perfectly uniform diameter throughout the length. It can be vibrated when a jerk is given at a point in its length. In the musical devices, the string is fixed at two ends and sound is produced giving the disturbance at different points. Guitar, sitar, harmonium, violin are some examples of musical devices which exploit to play the music.

Suppose a string is set into vibration from a free end. Then, the wave travels towards the another end. If the another end is fixed at a rigid support, the propagated wave reflects back towards the free end making the node at the fixed joint. The transverse wave propagating towards the fixed end and reflected wave from that end when superimposed along the length of string, standing wave (or stationary wave) is formed as shown in Fig. 3.11.

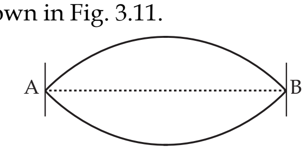


Fig. 3.11: Stationary wave in a string

Velocity of Transverse Wave along a Stretched Wire

The formula for velocity of transverse wave along stretched string can be deduced by two methods:

i. Dimensional method
ii. Employing centripetal force technique

i. Dimensional method: Let the velocity v depends on the values of tension (T), length (l) and mass (m) of the string along which wave travels. So, we can write,

$$v \propto T^a,\ v \propto l^b \text{ and } v \propto m^c$$

Combining these relations, we get,

$$\mathbf{v \propto T^a\, l^b\, m^c} \qquad \textbf{... (3.26)}$$

$$\textbf{or,} \quad \mathbf{v = k\, T^a\, l^b\, m^c} \qquad \textbf{... (3.27)}$$

where a, b and c are numbers and k is a dimensionless constant whose experimental value is one. Writing dimension of each term, we get,

$$[L\,T^{-1}] = [M\,L\,T^{-2}]^a\,[L]^b\,[M]^c$$

$$\therefore \quad [L\,T^{-1}] = [M^{a+c}\,L^{a+b}\,T^{-2a}]$$

Equating the indices of *M, L, T* on both sides, we get,

For M, a + c = 0,
For L, a + b = 1,
For T, –2a = –1

$$\therefore \quad a = \tfrac{1}{2},\ b = \tfrac{1}{2} \text{ and } c = -\frac{1}{2}$$

Putting these values in (3.27), we get,

$$v = k\,T^{1/2}\,l^{1/2}\,m^{-1/2}$$

$$\therefore \quad v = k\sqrt{\frac{Tl}{m}} = k\sqrt{\frac{T}{m/l}}$$

$$\therefore \quad \mathbf{v = k\sqrt{\frac{T}{\mu}}} \qquad \textbf{... (3.28)}$$

where, $\mu = \frac{m}{l}$ is mass per unit length of the string or linear density of the string, and k = 1.

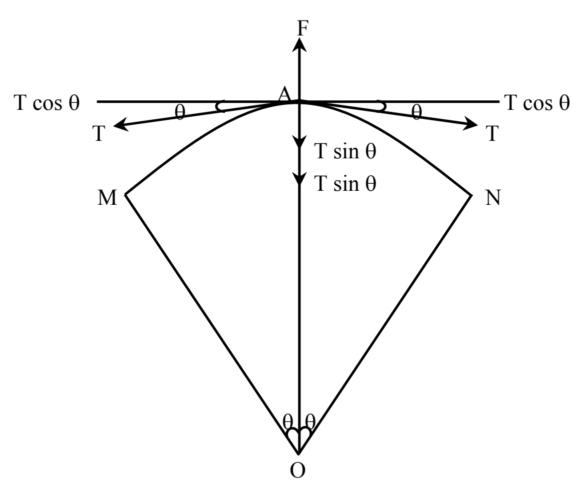


Fig. 3.12: Transverse vibration of string

ii. Employing centripetal force technique: Consider a string, fixed at two ends M and N. The mid-point A of string is plucked up and left to oscillate. The oscillation of string produces the transverse wave. Let T be the tension at mid point of string due to the plucking. Tension T is directed tangentially out in both sides of the point as shown in Fig. 3.12. Also θ be the angle made by tangent with its horizontal component. The point 'O' is the center of curvature of the arc 'MAN' of the string.

Here, horizontal components of tension are equal and opposite in a line so they cancel to each other. However, vertical components are directed linearly towards the centre O of curvature. Let R be the radius of curvature of the string.

Here, the tension along the centre of curvature is

$$F = 2\,T \sin\theta \qquad \text{... (3.29)}$$

For very small angle of θ, sin θ ≈ θ. So,

$$F = 2T.\,\theta$$

$$\mathbf{F = T\,.\,2\theta} \qquad \text{... (3.30)}$$

From the arc MAN,

$$2\,\theta = \frac{\overset{\frown}{MAN}}{R}$$

Therefore,

$$\mathbf{F = T\,.\,\frac{\overset{\frown}{MAN}}{R}} \qquad \text{... (3.31)}$$

This force provides centripetal force to pull the string towards the centre of curvature, hence,

$$F' = \frac{mv^2}{R}$$

Where, 'm' is the total mass of vibrating string.

Also,

$$m = \mu\,.\,\overset{\frown}{MAN}$$

Here, μ = mass per unit length of string.

$$\mathbf{F' = \mu\,\overset{\frown}{MAN}\,\frac{v^2}{R}} \qquad \text{... (3.32)}$$

Equation (3.31) and (3.32),

$$F = F'$$

$$T\,.\,\frac{\overset{\frown}{MAN}}{R} = \mu\,\overset{\frown}{MAN}\,\frac{v^2}{R}$$

$$\therefore \quad T = \mu v^2$$

$$v^2 = \frac{T}{\mu}$$

$$\mathbf{v = \sqrt{\frac{T}{\mu}}} \qquad \text{... (3.33)}$$

3.9 Modes of Vibration of a Stretched String

Consider a string, fixed at two end points A and B. If the string is plucked at a point, transverse waves are set up and travel towards the fixed ends. These waves reflect back from the fixed ends. As a result, the incident wave from the point of disturbance superimposes with reflected wave and hence, produce the stationary wave as shown in Fig. 3.13.

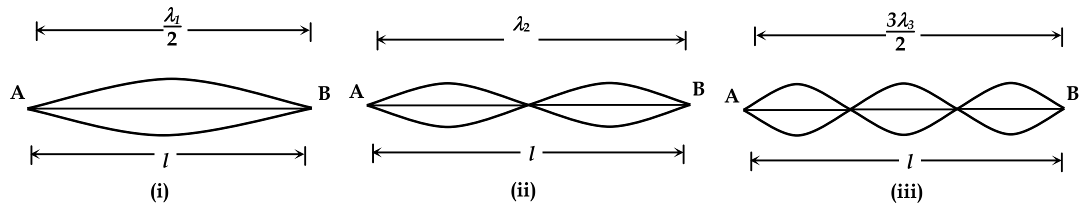


Fig. 3.13: Modes of vibration of stretched string

In the vibration of string, both ends are fixed, so no particle oscillates at these points. Therefore, nodes are formed at the fixed ends. The formation of waves in the string can be studied in different modes of vibration.

i. **First mode of vibration:** If the two ends are fixed and the string is plucked from the centre and left to oscillate, a half part of one complete wave is produced along the total length of string as shown in Fig. 3.13 (i). This oscillation produces the lowest possible frequency produced by the string. So, it is called fundamental frequency. Let l be the length of string and λ_1 be the longest possible wavelength that is produced in its vibration.

Then, the frequency of transverse vibration, f_1, produced in the vibration of string at speed v,

$$f_1 = \frac{v}{\lambda_1}$$

As the half part of a complete wave is formed, we have,

$$l = \frac{\lambda_1}{2}$$

$$\therefore \quad \lambda_1 = 2l$$

Then, the fundamental frequency of vibration of the string is,

$$\mathbf{f_1 = \frac{v}{2\mathit{l}}} \qquad \text{... (3.34)}$$

It is the lowest possible frequency produced in the transverse vibration of string, which is called the first harmonic of vibration of string.

In string $v = \sqrt{\frac{T}{\mu}}$

So, $\quad f_1 = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$

ii. **Second mode of vibration:** If the string is supported tightly at the middle and plucked from one forth part of its length, a complete wave is formed as shown in Fig. 3.13 (ii). This is the next consecutive frequency of vibration in the string. Hence, it is called second mode of vibration.

Let λ_2 be the wavelength of standing wave produced by the string in second mode of vibration. Then, the frequency of vibration, f_2 produced in this mode is,

$$f_2 = \frac{v}{\lambda_2}$$

Since a complete wave is formed in the total length of string,

$$l = \lambda_2$$

Therefore, the frequency of vibration in the string is,

$$\therefore \quad f_2 = \frac{v}{l}$$

$$f_2 = \frac{2}{2}\frac{v}{l}$$

$$\mathbf{f_2 = 2 \cdot \frac{v}{2l}} \qquad \ldots(3.35)$$

$$f_2 = 2f_1$$

Equation (3.35) gives the frequency of sound produced by string in second mode of vibration. It is called the second harmonic or first overtone. As the frequency of sound in this mode is double than the frequency of fundamental mode, it is called second harmonic.

iii. Third mode of vibration: If the string is supported rigidly at the point one third of its length and plucked from one sixth of its length, one complete and one-half part of standing wave is formed as shown in Fig. 3.13 (iii). This mode of vibration is called the third mode of vibration of string.

Let λ_3 be the wavelength of sound produced by the transverse vibration of string of length l. Then, the frequency of vibration is,

$$f_3 = \frac{v}{\lambda_3}$$

Since a complete and one half wave is formed in the total length,

$$l = \lambda_3 + \frac{1}{2}\lambda_3$$

$$l = \frac{3\lambda_3}{2}$$

$$\therefore \quad \lambda_3 = \frac{2l}{3}$$

Therefore, the frequency of vibration produced in the string is,

$$\therefore \quad f_3 = \frac{v}{\left(\frac{2l}{3}\right)}$$

$$f_3 = 3\left(\frac{v}{2l}\right)$$

$$\mathbf{f_3 = 3f_1} \qquad \ldots(3.36)$$

Equation (3.36) gives the frequency of vibration in the string in third mode of vibration. It is called the third harmonic or second overtone. As the frequency of sound in this mode is three times greater than the frequency of fundamental mode, it is called third harmonic.

Laws of Transverse Vibration of String

The fundamental frequency of transverse vibration of string is given by,

$$\mathbf{f = \frac{v}{2l}}$$

and the speed of transverse vibration,

$$\mathbf{v = \sqrt{\frac{T}{\mu}}}$$

Therefore, the frequency of fundamental tone is,

$$\mathbf{f = \frac{1}{2l}\sqrt{\frac{T}{\mu}}} \qquad \ldots(3.37)$$

Equation (3.37) shows that, the frequency of vibration of stretched string depends on three factors (i) resonating length (l), (ii) tension in it (T) and (iii) mass per unit length (μ). On the basis of these factors depending on frequency of sound, the laws of transverse vibration of a stretched string are derived. There are three laws of transverse vibration of a stretched string which are (i) Law of length, (ii) Law of tension and (iii) Law of mass.

i. **Law of length:** If tension 'T' and linear density 'μ' of a stretched string are taken constant, frequency 'f' of fundamental note is inversely proportional to the resonating length 'l' of string,

i.e. $f \propto \frac{1}{l}$, when T and μ are constant.

ii. **Law of tension:** If length and linear density of a stretched string are taken constant, frequency of fundamental note is directly proportional to the square root of the tension of string,

i.e. $f \propto \sqrt{T}$, when l and μ are constant.

iii. **Law of mass:** If length and tension of stretched string are taken constant, frequency of fundamental note is inversely proportional to the square root of mass per unit length of string,

i.e. $f \propto \frac{1}{\sqrt{\mu}}$, when l and T are constant.

Here, $\mu = \frac{\text{mass}}{\text{length}}$ (mass per unit length)

or, $\mu = \frac{\text{volume} \times \text{density}}{\text{length}} \quad (\because \rho = \frac{m}{V})$

$= \frac{V \cdot \rho}{l}$

If d be diameter of string, we have

Volume $= \pi r^2 l = \pi \left(\frac{d}{2}\right)^2 \cdot l$

$\therefore$ Volume per unit length $\left(\frac{V}{l}\right) = \frac{\pi d^2}{4}$

$\therefore$ Mass per unit length, $\mu = \frac{\pi d^2}{4} \times \rho$

Thus, we have from (3.37)

$$f = \frac{1}{2l}\sqrt{\frac{T}{\frac{\pi d^2 \rho}{4}}} = \frac{1}{2l}\sqrt{\frac{4T}{\pi d^2 \rho}} = \frac{2}{2ld}\sqrt{\frac{T}{\pi\rho}}$$

$$= \mathbf{\frac{1}{ld}\sqrt{\frac{T}{\pi\rho}}} \qquad \ldots \textbf{(3.38)}$$

Hence, from the law of mass, on the basis of equation (3.38), two more laws are derived which are (a) Law of diameter and (b) Law of density.

a. **Law of diameter:** If length, tension and density of the string are taken constant, frequency of fundamental tone is inversely proportional to the diameter of string i.e. $f \propto \frac{1}{d}$, when l, T and ρ are constant.

b. **Law of density:** If length, tension and diameter are taken constant, frequency of the fundamental tone is inversely proportional to the square root of density of the string is, $f \propto \frac{1}{\sqrt{\rho}}$, when l, T and d are constant.

Sonometer

A sonometer is a device (apparatus) used to study the transverse vibration of stretched strings. It consists of a hollow wooden rectangular box containing two bridges and a pulley at one end. A wire is attached to one end of the wooden box, which runs over the bridges and pulley, and carries a weight hanging at the free end as shown in Fig. 3.14. There are movable bridges C and D placed beneath the string. They can be moved to adjust the suitable length to achieve the resonance condition.

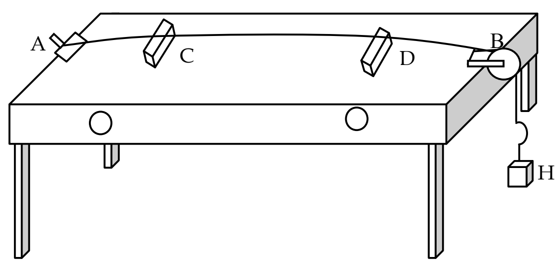


Fig. 3.14: A sonometer

The usual sonometer is horizontal and the tension is supplied by the weight of masses hung on the end of the wires after they pass over the pulley. Two or more wires can be attached in the sonometer box. It is used for many purposes: to determine the frequency of tuning fork, to find the density of a wire, to determine the frequency of a.c. mains, etc.

3.10 Verification of Laws of Vibrating Strings

Law of Length

A sonometer with single string is taken. One end of string is fixed at a rigid support and another end is free and is passed over a pulley as shown in Fig. 3.14. A set of tuning forks of different frequencies is taken. A small piece of paper rider is put on the string. Two wedge shaped bridges are kept beneath the stretched string. The segment of string between the bridges is vibrated using the tuning forks. In this process, the prongs of the tuning forks are vibrated by hitting them on a rubber pad and the bottom of stem is placed on the wooden surface of the sonometer. This process continues to obtain a particular length of the wire between the wooden bridge such that the paper rider kept over it flies off the string. This happens when the vibration of the string is maximum. Then, the lengths between the bridges are noted. This length of string is called resonating length for the specific frequency. Above process is repeated for the remaining tuning forks for the same string under constant tension.

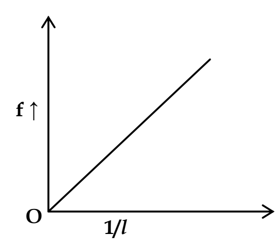


Fig. 3.15: Law of length

After determining the resonating length of string for corresponding frequencies of tuning forks, these parameters are plotted taking reciprocal of length versus frequency (f). The graph between $\frac{1}{l}$ and f is found to be a straight line passing through the origin as shown in Fig. 3.15. This verifies the law of length experimentally.

Law of Tension

To begin with, two identical wires, (having equal diameter and same material), X and Y are stretched parallel over the length of the sonometer by hanging loads of different magnitudes at their free ends. Another end of each wire is fixed at the rigid support. A small tension T_1 is given to the wire X and oscillating length is segmented by bridges PQ. A fixed tension T is given to the another wire Y and bridges RS are placed beneath the wire Y as shown in Fig. 3.16 (i). It is to be noted that, the vibrating length for wire X and tension on wire Y are taken constant for whole experiment.

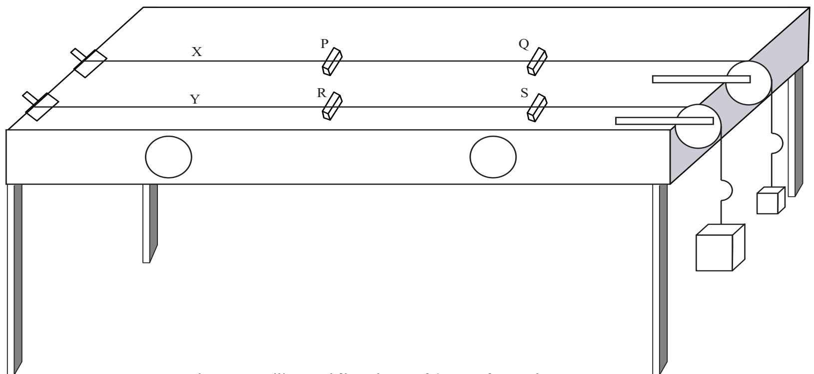


Fig. 3.16: (i) Verification of law of tension

The vibrating length of Y is adjusted by sliding the bridges RS. In the experiment, the bridge RS are moved until the vibration in X and Y became unison (i.e. match exactly). Then, the vibrating length is determined by sliding the bridges RS in wire Y, achieving the unison in X and Y. Previous steps are repeated for different values of tension in wire X. Let, l_1, l_2, l_3 .... be the vibrating length of wire Y at tensions T_1, T_2, T_3, ... respectively. In the experiment it is found that

$$\frac{1}{l_1}:\frac{1}{l_2}:\frac{1}{l_3}:\ldots\ldots = \sqrt{T_1}:\sqrt{T_2}:\sqrt{T_3}\ldots\ldots$$

Now, from the verification of law of length,

$$f_1 : f_2 : f_3 : \ldots\ldots = \frac{1}{l_1}:\frac{1}{l_2}:\frac{1}{l_3}:\ldots\ldots$$

So, the relation of frequency of note and tension of the vibrating wire is,

$$f_1 : f_2 : f_3 : \ldots\ldots = \sqrt{T_1}:\sqrt{T_2}:\sqrt{T_3}$$

$$\text{i.e. } f \propto \sqrt{T}$$

If the graph is plotted for f versus $\sqrt{T}$, a straight line is found passing through the origin as shown in Fig. 3.16 (ii). This verifies the law of tension.

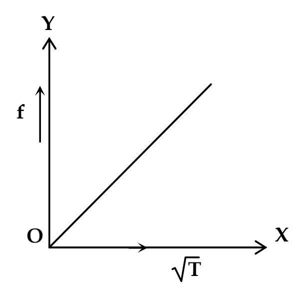


Fig. 3.16: (ii) Law of tension

Law of mass

Many experimental wires of different diameters are mounted on a sonometer (the wires may be of similar or different materials). One end of each wire is fixed at the rigid support and another end is passed through a pulley as shown in Fig. 3.17 (i). Free end of these wires are given equal tension and the vibrating lengths are also adjusted equal by using bridges. Another wire, called standard wire, is taken in the Sonometer which has the same tension as other experimental wires, but the length can be varied to produce the required pitch of sound. In the experiment, pitch of sound produced by the experimental wires should be compared with pitch of sound produced by the standard wire.

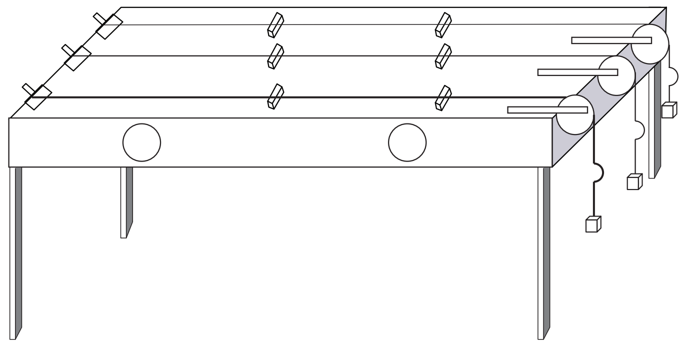


Fig. 3.17 (i) : Verification of law of mass

To perform the experiment, the length of standard wire is so adjusted that it is unison (same pitch) with the experimental wires turn by turn. Since the diameters of the wires are different, the sound produced on oscillating them has different pitch although the length is same. So, the vibrating length of standard wire should be changed to produce same pitch as that produced by experimental wires. Let, l_1, l_2, l_3, ...be the vibrating lengths of standard wires which are in unison with the experimental wires of mass per unit length μ_1, μ_2, μ_3, ... respectively. From the experiment, it is found that,

$$l_1 : l_2 : l_3 : .... \sqrt{\mu} : \sqrt{\mu_2} : \sqrt{\mu_3}$$

Also, the frequency of sound produced by experimental wires are related with vibrating length as,

$$f_1 : f_2 : f_3 : .... = \frac{1}{l_1} : \frac{1}{l_2} : \frac{1}{l_3} : ....$$

Obviously,

$$f_1 : f_2 : f_3 : .... = \frac{1}{\sqrt{\mu_1}} : \frac{1}{\sqrt{\mu_2}} : \frac{1}{\sqrt{\mu_3}} : ....$$

i.e. $f \propto \frac{1}{\sqrt{\mu}}$

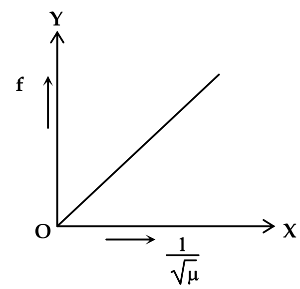


Fig. 3.17 (ii) Law of mass

If a graph is drawn for values of frequencies produced by vibrating wires versus reciprocal of square root of corresponding mass per unit length, a straight line is found passing through the origin as shown in Fig. 3.17 (ii). This verifies law of mass.

Tips for MCQs

1. **Organ Pipes:**
   i. Resonance occurs and standing waves are set up into the air of pipe.
   ii. Sound wave of definite patterns of frequency are produced, called harmonics.
2. **Closed Organ Pipe:**
   i. Closed end contains a node and opened contains an antinode.
   ii. Only odd harmonics are present and even harmonics are missing. So, the sound is relatively harsh.

iii. The fundamental frequency is,

$$f_1 = \frac{v}{4l} = \frac{1}{4l}\sqrt{\frac{\gamma P}{\rho}} = \frac{1}{4l}\sqrt{\frac{\gamma RT}{M}}$$

iv. Facts for closed organ pipe

Mode of vibration	Harmonic	Tone	Number of anti nodes	Number of Nodes	Frequency	Wave length
First or fundamental	First	Fundamental	1	1	$f_1 = \frac{v}{4l}$	$4l$
Second	Third	First overtone	2	2	$3f_1$	$\frac{4l}{3}$
Third	Fifth	Second overtone	3	3	$5f_1$	$\frac{4l}{5}$
....	....	....	....	....	....	....
n^{th}	$(2n-1)^{th}$	$(n-1)^{th}$	n	n	$(2n-1)f_1$	$\frac{4l}{2n-1}$

3. Open organ Pipe:

i. Both ends contain antinodes.

ii All harmonics, even and odd, are present. So, the sound produced by this pipe is sweet and pleasing.

iii. The fundamental frequency is,

$$f_1 = \frac{v}{2l} = \frac{1}{2l}\sqrt{\frac{\gamma P}{\rho}} = \frac{1}{2l}\sqrt{\frac{\gamma RT}{M}}$$

iv. Facts for open organ pipe

Mode of vibration	Harmonic	Tone	Number of anti nodes	Number of Nodes	Frequency	Wave length
First or fundamental	First	Fundamental	2	1	$f_1 = \frac{v}{2l}$	$2l$
Second	Second	First overtone	3	2	$2f_1$	$\frac{2l}{2}$
Third	Third	Second overtone	4	3	$3f_1$	$\frac{2l}{3}$
....	....	....	....	....	....	....
n^{th}	n^{th}	$(n-1)^{th}$	$(n+1)$	n	nf_1	$\frac{2l}{n}$

4. End correction:

i. $e = \frac{l_2 - 3l_1}{2}$ and (ii) $e = 0.3d$, d = diameter of resonance tube

ii. Fundamental frequency in closed organ pipe,

$$f_1 = \frac{v}{4(l + e)}$$

iii. End correction is not observed in string fixed at two ends.

4. Waves in string:

i. Transverse wave is produced in a stretched wire between two rigid support.

ii. Both ends contain nodes.

iii. All harmonics, even and odd, are present.

iv. The fundamental frequency, $f_1 = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$

The speed of transverse vibration , $v = \sqrt{\frac{T}{\mu}}$

v. For a cylindrical wire, $\mu = A.\rho = \frac{\pi d^2}{4}.\rho$

vi. Facts for vibrating strings:

Mode of vibration	Harmonic	Tone	Number of loops	Number of anti nodes	Number of Nodes	Frequency	Wave length
First or fundamental	First	Fundamental	1	1	2	$f_1 = \frac{v}{2l}$	$\frac{2l}{1}$
Second	Second	First overtone	2	2	3	$2f_1$	$\frac{2l}{2}$
Third	Third	Second overtone	3	3	4	$3f_1$	$\frac{2l}{3}$
....	....	....	....	…	....	....	....
n^{th}	n^{th}	$(n-1)^{th}$	n	n	n+1	nf_1	$\frac{2l}{n}$

Worked Out Problems

1. [HSEB 2067] An open pipe 30 cm long and a closed pipe 23 cm long both of the same diameter are each sounding its first overtone and they are in unison. What is the end-correction of these pipes?

SOLUTION

Given,

End correction for both pipes of same diameter (e) = ?

For the open pipe in first overtone, we can write

$\lambda_0 = l_0 + 2e$ . . . (i)

For the closed pipe in first overtone, we can write

$\frac{3\lambda_c}{4} = l_c + e$

or $\lambda_c = \frac{4}{3}(l_c + e)$ . . . (ii)

Since the vibrations in both the pipes are unison so they should have the same frequency and wavelength. So, from (i) and (ii), we get,

$\lambda_0 = \lambda_c$

or $l_0 + 2e = \frac{4}{3}(l_c + e)$

or $3l_0 + 6e = 4l_c + 4e$

or $6e - 4e = 4l_c - 3l_0$

or $2e = 4 \times 23 - 3 \times 30$

or $e = \frac{92 - 90}{2} = \frac{2}{2}$

$\therefore$ e = 1 cm

2. A 1.5 m long rope is stretched between two supports with a tension that makes the speed of transverse waves 48 m/s (a) what are the wavelength and frequency of the fundamental? (b) What are the wavelength and frequency of the second overtone?

SOLUTION

Given,

Length of rope, $l = 1.5$ m

Speed of wave, v = 48 m/s

a. Fundamental frequency (f_0) and wavelength (λ_0) = ?

We know that,

$f_0 = \frac{v}{2l}$

or $f_0 = \frac{48}{2 \times 1.5} = 16$ Hz

$\because$ $v = f_0 \lambda_0$

$\therefore$ $\lambda_0 = \frac{v}{f_0} = \frac{48}{16} = 3$ m

b. Frequency (f_3) and wavelength (λ_3) in second overtone = ?

We know that,

$f_n = nf_0$

or $f_3 = 3 \times 16 = 48$ Hz

Also,

$v = f_3\lambda_3$

$\therefore$ $\lambda_3 = \frac{v}{f_3} = \frac{48}{48} = 1$ m

3. [HSEB 2056] An organ pipe is turned to a frequency of 440 Hz when the temperature is 27°C. Find its frequency when the temperature drops to 0°C. Assume both ends of the pipe open.

SOLUTION

Given,

Frequency at 27°C (f_1) = 440 Hz

Temperature T_1) = 27°C = 27 + 273 = 300 K

Frequency at 0°C (f_2) = ?

Temperature (T_2) = 0°C = 0 + 273 = 273 K

For open organ pipe, we have

$$f_1 = \frac{v_1}{2l} \quad \ldots \text{(i)}$$

$$\text{and } f_2 = \frac{v_2}{2l} \quad \ldots \text{(ii)}$$

Dividing (ii) and (i) we get,

$$\frac{f_2}{f_1} = \frac{v_2}{2l} \times \frac{2l}{v_1} = \frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} \qquad \left[\begin{array}{l} \therefore v \propto \sqrt{T} \\ \Rightarrow \frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} \end{array}\right]$$

$$f_2 = \sqrt{\frac{273}{300}} \times 440 \qquad \therefore f_2 = 419.7 \text{ Hz}$$

Hence, the frequency at 0°C is 419.7 Hz.

4. [HSEB 2070] In a resonance tube experiment, the first and the second resonance positions were observed respectively at 17 cm and 52.6 cm with a tuning fork of frequency 512 Hz at 27°C. Calculate the velocity of sound in air at 0°C and the end correction of the tube.

SOLUTION

Given,

First resonance length (l_1) = 17 cm = 0.17 m

Second resonance length (l_2) = 52.6 cm = 0.526 m

Velocity of sound at 0°C (v_o) = ?

Frequency of tunning fork (f) = 512 Hz

We have,

At 27°C

$$v_{27} = 2f(l_2 - l_1)$$

$$= 2 \times 512 (0.526 - 0.17) = 364.5 \text{ m/s.}$$

Now,

$$\frac{v_o}{v_{27}} = \sqrt{\frac{T_o}{T_{27}}} = \sqrt{\frac{273}{273 + 27}}$$

$$v_o = \sqrt{\frac{273}{300}} \times 364.5 = 347.75 \text{ m/s}$$

Again, end correction (e) $= \frac{l_2 - 3l_1}{2} = \frac{0.526 - 3 \times 0.17}{2} = 0.008$ m

5. [HSEB 2053] A wire of diameter 0.040 cm and made of steel of density 8000 kg/m³ is under constant tension of 80 N. What length of this wire should be plucked to cause it to vibrate with a frequency of 840 Hz?

SOLUTION

Given,

Diameter of wire (d) = 0.04 cm = 0.04×10^{-2} m

Density of wire (ρ) = 8000 kg/m³

Tension (T) = 80 N

Frequency (f) = 840 Hz

Resonating length (l) = ?

We know that

$$f = \frac{1}{2l}\sqrt{\frac{T}{\mu}} = \frac{1}{2l}\sqrt{\frac{T}{\rho A}} = \frac{1}{2l}\sqrt{\frac{T}{\rho \times \frac{\pi d^2}{4}}}$$

$$\text{or } l = \frac{1}{fd}\sqrt{\frac{T}{\pi\rho}}$$

$$= \frac{1}{840 \times 0.04 \times 10^{-2}}\sqrt{\frac{80}{3.14 \times 8000}} = 0.168 \text{ m}$$

6. A steel wire of length 40 cm and diameter 0.0250 cm vibrates transversely in unison with a tube, open at each end and effective length 60 cm, when each is sounding its fundamental note. The air temperature is 27°C. Find the tension in the wire. (Assume that the velocity of sound in air at 0° C is 331 ms⁻¹ and the density of steel is 7800 kgm⁻³).

SOLUTION

Given,

Length of wire = 40 cm = 0.40 m

Diameter = 0.0250 cm = 0.00025 m

Effective length of tube = 60 cm = 0.60 m

Temp (θ) = 27°C

Tension in the wire (T) = ?

Since,

$$f = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$$

$$\text{or } f^2 = \frac{1}{4l^2}\frac{T}{\mu} \qquad \left(\because \mu = \frac{\text{mass}}{\text{length}} = \frac{v \times \rho}{l} = \frac{A \times l \times \rho}{l} = A \times \rho = \frac{\pi d^2 \rho}{4}\right)$$

$$\text{or } T = 4l^2f^2\mu$$

$$= 4l^2f^2\frac{\pi d^2\rho}{4}$$

$$= \pi d^2 l^2 f^2 \rho \quad \dots \text{(i)}$$

For the tube open at ends, one loop is formed in which antinodes are at the ends. So,

$$\text{Effective length} = \frac{\lambda}{2}$$

$$\text{or } 0.6 = \frac{\lambda}{2}$$

$$\text{or } \lambda = 1.2 \text{ m}$$

Again,

$$\therefore v = f\lambda$$

$$\text{or } f = \frac{v}{\lambda}$$

$$\text{or } f = \frac{v_{27}}{\lambda}$$

$$= \frac{\sqrt{\frac{273 + 27}{273}} \times v_0}{1.2}$$

$$\left(\because \frac{v_\theta}{v_o} = \sqrt{\frac{273 + \theta}{273}}\right.$$

$$\text{as } v \propto \sqrt{T} \quad \therefore v_{27} = \sqrt{\frac{273 + 27}{273}} \times v_0$$

$$= \frac{1}{1.2}\sqrt{\frac{300}{273}} \times 331 = 289 \text{ Hz}$$

Thus from (i) we have

$$T = \pi \times (2.5 \times 10^{-4})^2 \times (0.40)^2 \times (289)^2 \times 7800$$

$$= 20.5 \text{ N}$$

7. [HSEB 2055] A piano string has length of 2.0 m and a density of 800 kgm^{-3}. When the tension in the string produces a string of 1%, the fundamental note obtained from the string in the transverse vibrations in 170 Hz. Calculate Young's modulus for the material of the string.

SOLUTION

Given,

Length of string (l) = 2 m

Density (ρ) = 800 kg/m^3

Strain = 1% = $\frac{1}{100}$

Frequency (f) = 170 Hz

Young's Modulus (Y) = ?

Now, we have,

$$Y = \frac{\text{stress}}{\text{strain}}$$

or, stress = Y × strain

$$\text{or, } \frac{\text{Tension}}{\text{Area}} = Y \times \frac{1}{100}$$

$$\text{or, } \frac{T}{A} = \frac{Y}{100} \quad \ldots \text{(i)}$$

Then,

For fundamental mode of vibration of string; we have,

$$f = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$$

$$\text{or, } f = \frac{1}{2l}\sqrt{\frac{T}{A \times \rho}} \quad \ldots \text{(ii)}$$

Putting the value of $\frac{T}{A}$ from equation (i) in equation (ii), we get,

$$f = \frac{1}{2l}\sqrt{\frac{Y}{100} \times \frac{1}{\rho}}$$

$$\text{or, } 170 = \frac{1}{2 \times 2}\sqrt{\frac{Y}{100} \times \frac{1}{800}}$$

$$\text{or, } 680 = \sqrt{\frac{Y}{80000}}$$

$$\text{or, } 462400 = \frac{Y}{80000}$$

$\therefore$ Y = 3.7 × 10^{10} Nm^{-2}

Here, the required value of Young's Modulus is 3.7 × 10^{10} Nm^{-2}.

Challenging Problems

1. **[UP]** With what tension must a rope with length 2.5 m and mass 0.120 kg be stretched for transverse waves of frequency 40 Hz to have a wavelength of 0.75 m?

Ans: 43.2 N

2. **[UP]** One end of a horizontal rope is attached to a prong of an electrically driven tuning fork that vibrates at 120 Hz. The other end passes over a pulley and supports a mass of 1.5 kg. The linear mass density of the rope is 0.055 kg/m.
   a. What is the speed of a transverse wave on the rope?
   b. What is the wavelength?
   c. How would your answers to part (a) and (b) change if mass were increased to 3 kg.

Ans: (a) 16.35 m/s (b) 0.136 m (c) 23.12 m/s; 0.193 m

3. **[UP]** One end of 14 m long rubber tube, with total mass 0.800 kg is fastened to fixed support. A cord attached to the other end passes over a pulley and supports an object with a mass of 7.50 kg. The tube is struck a transverse blow at one end. Find the time required for the pulse to reach the other end.

Ans: 0.390 s

4. **[UP]** A simple harmonic oscillator at the point x = 0 generates a wave on a rope. The oscillator operates at a frequency of 40 Hz and with an amplitude of 3 cm. The rope has a linear mass density 50 g/m and is stretched with a tension of 5 N.
   a. Determine the speed of the wave.
   b. Find the wavelength.
   c. Write the wave function y(x, t) for the wave.

Ans: (a) 10 m/s (b) 0.25 m (c) y (x, t) = (0.03 m) cos (8 πx – 80 πt)

5. **[UP]** Adjacent antinodes of a standing wave on a string are 15 cm apart. A particle at an antinode oscillates in simple harmonic motion with amplitude 0.85 cm and period 0.075 s. The string lies along the positive X-axis and is fixed at x = 0.

a. Find the displacement of a point on the string as a function of position and time.
b. Find the speed of propagation of a transverse wave in the string.

Ans: (a) $y(x, t) = (0.85)\text{ cm} \cdot \sin\left(\frac{2\pi x}{0.3\text{ m}}\right) \cdot \sin\left(\frac{2\pi t}{0.075\text{ s}}\right)$ (b) 4 m/s

6. **[UP]** A wire with a mass of 40.0 g is stretched so that its ends are tied down at points 80.0 cm apart. The wire vibrates in its fundamental mode with frequency 60.0 Hz and with an amplitude at the antinodes of 0.3 cm. [HSEB 2072]
   a. What is the speed of propagation of transverse wave in the wire?
   b. Calculate the tension in the wire.
   c. Determine the maximum transverse velocity and acceleration of particles in the wire.

Ans: (a) 96 m/s (b) 416 N (c) 1013 m/s, – 426 m/s^2

7. **[UP]** A thin, taut string tied at both ends and oscillating in its third harmonic has its shape described by the equation y (x, t) = (5.6 cm) sin [(0.0340 rad/cm) x]. sin [(50. 0 rad/s)t, where the origin is at the left end of the string, the X-axis is along the string and the Y-axis is perpendicular to the string.
   a. Find the amplitude of the two travelling waves that make up thin standing wave.
   b. What is the length of string?
   c. Determine the speed of the wave.

Ans: (a) 2.8 cm (b) 227 cm (c) 1471 cms^{-1}

8. **[ALP]** A tube is closed at one end and closed at the other by a vibrating diaphragm may be assumed to be a displacement node. It is found that when the frequency of the diaphragm is 2000 Hz a stationary wave pattern is set up in the tube and the distance between adjacent nodes is then 8.0 cm. When the frequency is gradually reduced the stationary wave pattern disappears but another stationary wave pattern reappears at a frequency of 1600 Hz. Calculate (i) the speed of sound in air (ii) the distance between adjacent modes of a frequency of 1600 Hz, (iii) the length of the tube between the diaphragm and the closed end, (iv) the next lower frequency at which a stationary wave pattern will be obtained.

Ans: i. 320 ms^{-1}; ii. 10 cm; iii. 0.4 m ; iv. 1200 Hz

9. **[ALP]** A string fixed at both ends is vibrating in the lowest mode of vibration for which a point a quarter of its length from one end is a point of maximum vibration. The note emitted has a frequency of 100 Hz. What will be the frequency emitted when it vibrates in the next mode such that this point is again a point of maximum vibration?

Ans: 300 Hz

10. **[ALP]** Write down in terms of wavelength λ, the distance between (i) Consecutive nodes, (ii) a node and an adjacent antinode (iii) consecutive antinodes. Find the frequency of the fundamental of a closed pipe 15 cm long if the velocity of sound in air is 340 ms^{-1}.

Ans: 567 HZ

11. **[ALP]** Explain the increase in loudness (or resonance) which occurs when a sounding tuning fork is held near the open end of organ pipe when the length of the pipe has certain values, the other end of the pipe being closed. Find the shortest length of such a pipe which resonates with a 440 Hz tuning fork, neglecting end corrections. (velocity of sound in air =350 ms^{-1})

Ans: 0.199 m

12. **[ALP]** Neglecting edge effects, find the lengths of (a) closed organ pipe and (b) and open organ pipe, each of which emits a fundamental note of frequency 256 Hz. [Speed of sound in air = 330 ms^{-1}]

Ans: (a) 0.322 m (b) 0.645 m

13. **[ALP]** A uniform tube, 60 cm long stands vertically with its lower end dipping into water. When the length above water is 14.8 cm, and again when it is 48 cm, the tube resounds to a vibrating tuning-fork of frequency 512 Hz. Find the lowest frequency to which the tube will resound when it is open at both ends.

Ans: 267 Hz

14. **[ALP]** A piano string 1.5 m long is made of steel of density 7.7×10^3 kgm^{-3} and Young's modulus 2×10^{11} Nm^{-2}. It is maintained at a tension which produces an elastic strain of 1% in the string. What is the fundamental frequency of transverse vibration of the string?

Ans: 170 Hz

15. **[ALP]** A sonometer wire is stretched by hanging a metal cylinder of density 8,000 kgm^{-3} at the end of the wire. A fundamental note of frequency 256 Hz is sounded when the wire is plucked. Calculate the frequency of vibration of the same length of wire when a vessel of water is placed so that the cylinder is totally immersed.

Ans: 239.5 Hz

16. **[ALP]** A wire whose mass per unit length is 10^{-3} kgm^{-1} is stretched by a load of 4 kg over the two bridges of a sonometer 1 m apart. If it is struck at its middle point, what will be (a) the wavelength if its subsequent fundamental vibrations, (b) the fundamental frequency of the note emitted?

Ans: (a) 2 m (b) 100 Hz

*[**Note:** Hints to challenging problems are given at the end of this chapter.]*

Conceptual Questions with Answers

1. The frequency of a fundamental note of a closed organ pipe and that of an open organ pipe are the same. What is the ratio of their lengths? [HSEB 2074]

➪ Let l_1 and l_2 be the length of closed organ pipe and open organ pipe respectively in which both pipes produce the same frequency f.

Consider the fundamental frequency for both the pipes.

i. In closed organ pipe,

$$f = \frac{v}{4l_1} \quad \ldots \text{(i)}$$

ii. In open organ pipe

$$f = \frac{v}{2l_2} \quad \ldots \text{(ii)}$$

Equating frequency in above equations (i) and (ii), we get,

$$\frac{v}{4l_1} = \frac{v}{2l_2}$$

$$\frac{l_1}{l_2} = \frac{1}{2}$$

$$l_2 = 2l_1$$

Therefore, the ratio of length in closed organ pipe to open organ pipe is 1:2. (open pipe is two times longer than closed pipe)

---

2. By what factor does the velocity of transverse wave in the string change when the tension in the stretched string is increased by four times? [HSEB 2073]

➪ The velocity of transverse wave in a string is,

$$v = \sqrt{\frac{T}{\mu}}$$

If the tension is increased by four time,

$$v' = \sqrt{\frac{4T}{\mu}} = 2\sqrt{\frac{T}{\mu}}$$

$$\therefore \quad v' = 2v$$

Therefore, the velocity is doubled than the initial velocity of transverse wave

$$\% \text{ change} = \frac{2v - v}{v} \times 100\% = 100\%$$

---

3. The six strings of a guitar are of the same length and are under nearly the same tension, but have different thickness. On which string do waves travel the fastest?

➪ The speed of transverse wave in the string is, for constant length string

$v' = \sqrt{\frac{T}{\mu}}$

If T is also taken constant, $v \propto \frac{1}{\sqrt{\mu}}$.

$$v \propto \frac{1}{\sqrt{\frac{\pi d^2}{4}}}$$

i.e $v \propto \frac{1}{d}$

This shows that, transverse wave travels fastest in the thinnest string.

---

4. How does the pitch of an organ pipe change with temperature? [HSEB 2072]

➯ The frequency of sound produced in organ pipes is directly proportional to the speed of sound, if the length is taken constant,

i.e. $f \propto v$ . . . (i)

We know, fundamental frequency of open organ pipe, $f = \frac{v}{2l}$ and that of closed organ pipe is, $f = \frac{v}{4l}$

Also, in gas, $v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}}$

$\therefore \quad v \propto \sqrt{T}$ . . . (ii)

So, from (i) and (ii), we get,

$f \propto v \propto \sqrt{T}$

Therefore, the pitch of an organ pipe increases with increase in temperature.

---

5. What happens to the frequency of transverse vibration of a stretched string, if its tension is halved and area of cross – section of the string is doubled? [HSEB 2071]

➯ The fundamental frequency of transverse vibration of a stretched string is,

$f = \frac{1}{2l}\sqrt{\frac{T}{\mu}} = \frac{1}{2l}\sqrt{\frac{T}{A\rho}}$

Where T = Tension on the string
A = Cross-sectional area of string
ρ = density of material of string

When tension is halved and cross-section is doubled, the new fundamental frequency is,

$$f' = \frac{1}{2l}\sqrt{\frac{T/2}{2A.\rho}} = \frac{1}{2}\left(\frac{1}{2l}\sqrt{\frac{T}{A\rho}}\right) = \frac{f}{2}$$

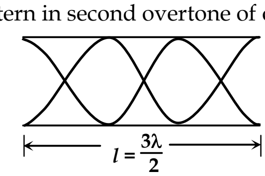
i.e. $f' = \frac{f}{2}$

It means, the new frequency is half of the original frequency.

---

6. Explain, with figure, the formation of second overtone of waves in an open organ pipe.

➯ One and half of a complete wave is formed into the open organ in second overtone. It is also called the third harmonic. The wave pattern in second overtone of open organ pipe is shown in figure.



In this mode of vibration,

$\frac{3\lambda}{2} = l$

$\lambda = \frac{2l}{3}$

So, the corresponding frequency of sound,

$f = \frac{v}{\lambda} = 3\frac{v}{2l} = 3f,$

$f = 3f_1$

The frequency of vibration into the pipe in second overtone is three times greater than the fundamental frequency.

---

7. A loud sound is heard in resonance. Why?

➯ Resonance occurs when supplied frequency to a vibrating object is equal to its natural frequency. In this condition, the particles in the vibrating medium oscillate with very high amplitudes. As we know, the intensity of sound is directly proportional to the square of amplitude (i.e. $I \propto a^2$), greater amplitude of particle produces the greater intensity. So, a loud sound is heard in resonance.

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8. Why are bells made of metal not of wood?

➯ Metals have high modulus of elasticity. So, the oscillation in metal damps gradually producing the sound for relatively long time. But the wave produced by wood damps suddenly. Therefore, the bells are made of metal, not of wood.

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9. Why are all string instruments provided with hollow boxes?

➯ Hollow boxes contain air molecules. When a vibration is set up in the air molecules, they oscillate with greater amplitudes as the same force produces in solid and liquid. Therefore, to produce the high intensity of sound from the string instruments, wooden boxes are made up hollow. Actually, the intensity of sound is directly proportional to the square of amplitude of vibrating particles.

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10. One of the 'Nine Jewels' of Emperor Alkbar, widely known as Tansen, the king of music was able to break a glass by singing the appropriate note. What physical phenomenon could account for this?

➯ The physical phenomenon, resonance, is relevant for the given condition. When the frequency of tone produced by the music is equal to the natural frequency of glass, the glass oscillates and may break the elastic limit. If the amplitude of vibration exceeds the elastic limit of the glass, the glass can be broken by music.

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11. Compare close and open organ pipes.

➯ Following few points show the comparison between closed and open organ pipes;

i. Fundamental note in closed organ pipe has half the frequency than that produced by open organ pipe of same length.

ii. Odd harmonics are present in closed organ pipe, whereas all the harmonics are present in open organ pipe.

iii. The musical sound produced by an open pipe is richer than the musical sound produced by a closed organ pipe.

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12. When are tones called harmonics?

➯ The tones are called harmonics if the frequencies of the fundamental tone and other overtones produced by a source of sound are in harmonic series.

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13. What is end correction? What are the factors on which it depend?

➯ The antinode of stationary wave in organ pipe lies outside the open end. It means, the acoustic length of wave is slightly greater than the physical length of pipe. So, the end for wave should be corrected to determine the different physical parameters if sound waves produced by the waves. Actually, the end correction is a distance of antinode from the end of stationary waves produced into the pipe. End correction is empirically related to diameter of organ pipe i.e. $e = 0.3d$, where d is the diameter of the pipe.

The end correction depends on, the diameter of wave producing pipe and the wavelength of the note used.

14. A flute has several holes in it. Why?

➪ The frequency of sound produced in pipes depend on their length. For example, the fundamental frequency of sound that is produced by open pipe is

$$f = \frac{v}{2l}$$

In a medium at constant temperature, v is constant.

$$\therefore \quad f \propto \frac{1}{l}$$

The length of flute can be varied by closing various holes in it, which ultimately changes the pitch of sound produced. To produce the sound of different frequencies, a flute is made with many holes.

15. What will happen on the frequency of the sonometer wire if the load stretching the sonometer wire is immersed in water?

➪ Due to the upthrust experienced by the load, the effective weight shall decrease. Then, the tension shall decrease consequently, the frequency of sound shall decrease.

16. What are the differences between forced vibration and resonance?

➪ The difference between forced vibration and resonance are as follows:

Forced vibration	Resonance
a. If the energy provided by external force is equal to the energy lost by dissipative force in an oscillating object, the oscillation continues with constant amplitude. Such oscillation or vibration is called forced vibration.	a. The phenomenon in which the amplitude of vibration is maximum when the frequency of vibrating system is equal to its natural frequency is known as resonance.
b. All the forced vibration is not resonance.	b. Resonance is a case of forced vibration.
c. The amplitude due to forced vibration can be small or large.	c. The amplitude due to resonance is always maximum for a particular condition.

17. Why are soldiers ordered to break their steps while crossing a bridge?

➪ During march pass, soldiers step in similar pattern. If the frequency of marching steps of soldier is equal to the natural frequency of vibration of bridge, the amplitude of its vibration is very large. In this condition, the bridge suffers large extension, that may cross the elastic limit and finally may have the possibility of collapses.

18. When water is used in a resonance tube is replaced by oil, how does the frequency change?

➪ Resonance tube is a type of closed end pipe in which the resonance occurs in space containing air. Below the air column, the tube is filled with liquid, usually water. The liquid surface acts as closed boundary. The sound of various frequencies is produced varying the level of liquid, however the frequency of sound does not depend on what liquid is used to make the boundary. Hence, the frequency of sound does not change, although you replace water by oil.

19. In mechanics, massless strings are often assumed. Why is this not a good assumption when discussing waves on strings?

➪ The speed of wave on strings is extremely sensitive with linear mass density. i.e.,

$v = \sqrt{\frac{T}{\mu}}$, where, T = tension on the sting

μ = linear mass density

As we know, $\mu = \frac{m}{l}$

If the mass of string is considered zero, i.e.

m = o, then $\mu = 0$

This shows that, $v = \sqrt{\frac{T}{0}}$

$v = \infty$

Experimentally, it is impossible. Therefore, the mass of string should not be assumed zero for meaningful result.

20. How is the wave speed affected if radius of a stretched wire is reduced to half?

The wave speed in a stretch wire is

$$v = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$$

We know, $\mu = \frac{m}{l} = \frac{\pi d^2}{4}.\rho = \pi r^2.\rho$

If the radius is halved, $\mu' = \pi\left(\frac{r}{2}\right)^2 \rho = \frac{\pi r^2 \rho}{4} = \frac{\mu}{4}$

∴ the new speed

$$v' = \frac{1}{2l}\sqrt{\frac{T}{\left(\frac{\mu}{4}\right)}} = 2\frac{1}{2l}\sqrt{\frac{T}{\mu}}$$

$v' = 2v$

The speed of wave in wire becomes double if its radius is halved.

21. A resonance tube resonates with a tuning fork of 256 Hz. If the length of the resonated air column are 32 cm and 100 cm, what is the value of end correction? What does it mean?

The end correction of a tube is,

$$e = \frac{l_2 - 3l_1}{2} = \frac{100 - 3 \times 32}{2} = 2 \text{ cm.}$$

Therefore, the end correction is 2 cm. It means, the anti-node of standing wave is formed 2 cm above the geometrical end of the tube.

Exercises

Short-Answer Type Questions

1. Define tone and note of sound.
2. Differentiate between tone and note.
3. What do you mean by harmonics and overtones?
4. What happens when tuning fork is made with wood?
5. The sound produced by open organ pipe is more sonorous than that by closed organ pipe, why?
6. If one end of an open organ pipe is closed, how will the fundamental frequency change?
7. What are the differences between forced vibration and resonance?
8. Why are soldiers ordered to break their steps while crossing a bridge?
9. Why is end correction of a pipe needed?
10. The vibrations of a tuning fork stops when its prongs are touched but they do not stop if stem of a fork is touched. Why?
11. Why is playing of a musical instrument not allowed on a bridge?
12. Why is it possible to tune a radio to a station?
13. Why is the sound produced by telegraphic wires heard more distinctly when we put our ear close to the telegraph post?

14. If oil is used in place of water in a resonance tube, how does the frequency change?
15. What is the effect of density of materials of string on the frequency of sound produced by sonometer?
16. The sound produced by open pipes sweeter than that from a closed pipe. Why?
17. How are the standing wave formed in a string?

Long-Answer Type Questions

1. Describe the various modes of vibrations of the air column in an organ pipe.
2. Describe the various modes of vibration of the air column in a closed organ pipe.
3. What do you understand by harmonics and overtones in the case of organ pipes? Prove that only odd harmonics are produced in closed organ pipe. [NEB 2075]
4. Prove that both types of harmonics, odd and even, can be produced in an organ pipe open at both ends.
5. What are harmonics? Explain the formation of overtones in an open and a closed organ pipe. [NEB 2075]
6. How are standing waves produced in open pipe?
7. How are standing waves produced in closed pipe?
8. Explain briefly the phase reversal from the closed end and opened end of organ pipe.
9. What is end correction? How can you determine end correction of a closed pipe by resonance method?
10. Describe a method of resonance to determine the speed of sound in air without end correction.
11. Describe an experiment to verify the laws of vibrations of a stretched string.
12. Discuss transverse vibrations in a stretched string. Derive the formula for the frequency of various mode of vibration.
13. State the laws of transverse vibrations of string. Using only dimension, show that the speed of propagation of a transverse wave depends only on tension and mass per unit length. [HSEB 2059]
14. State the laws of transverse vibration of string. Describe an experiment to verify the law of mass, and law of length. [HSEB 2062]
15. State and explain principle of superposition and formation of stationary waves. Show that the frequency of the fundamental note of a closed organ pipe is half as compared to that of an open pipe of the same length. [HSEB 2064]
16. What do you understand by "harmonics" and "overtones" in the case of organ pipe? Also prove that only odd harmonics are produced in closed ended organ pipe. [HSEB 2065, 2072]
17. Describe an experiment giving the necessary theory by which the speed of sound in air may be determined using resonance air column method. [HSEB 2067]
18. What is resonance? Explain it with an example.

Numerical Problems

1. Find the fundamental, first overtone and second overtone frequencies of an organ pipe of length 20 cm speed of sound in air is 340 ms^{-1}. **Ans: 850 Hz, 1700 Hz, 2550 Hz**
2. If the velocity of sound in air at 0^oC be 332 ms^{-1}, find the shortest wavelength in an open pipe that will be thrown into resonant vibrations by a tuning fork of frequency 256 Hz when the temperature of air is 50^oC.

   Ans: 0.71 m
3. A pipe 30.0 long is open at both ends. Which harmonic mode of the pipe resonates a 1.1 KHz source? Will resonance with the same source be observed if one end of the pipe is closed? Take the speed of sound in air is 330 ms^{-1}.

   Ans: 2
4. Two successive resonance frequencies in an open organ pipe are 1944 Hz and 2592 Hz. Find the length of the tube. The speed of sound in air is 324 ms^{-1}.

   Ans: 25 cm

5. The fist overtone frequency of a closed organ pipe P_1 is equal to the fundamental frequency of an open organ pipe P_2. If the length of pipe P_1 is 30 cm, what will be the length of P_2.

Ans: 20 cm

6. A string of a certain sonometer vibrates 100 times a second. Its length is doubled and its tension altered until it makes 150 vibrations in a second. Find the ratio of the new tension to the original.

Ans: 9 : 1

7. A wire of length 50 cm is stretched by a load of 10 kgwt. Find the change in (i) the length of the wire (ii) the stretching force which will increase the frequency of its fundamental tone by 1%.

Ans: (i) decrease by 0.5 cm (ii) increase by 0.2 kgwt

8. What is the velocity of transverse wave in a wire 30 m long weighing 0.09 kg, when it is under a tension of 270 N?

Ans: 300 ms^{-1}

9. Two strings A and B of equal thickness are made of the same material. The length of B is twice that of A while tension in A is twice that in B. Find ratio of the velocities of the transverse wave in the two strings.

Ans: $\sqrt{2}$: 1

10. A string 10 m long and mass 0.2 kg is stretched with a force of 5 kg wt. How long will the transverse wave take to travel the length of the string?

Ans: 0.25 sec

11. The mass of wire of length 5 m is 2 kg. What should be the tension in the wire so that the speed of transverse wave on the wire is 340 m/s?

Ans: 4.62 × 104 N

12. A string is under a tension of 6 N through which a transverse wave travels with a speed of 20 m/s. If the tension is changed to 13.5 N, find the speed of the wave in the string?

Ans: 30 ms^{-1}

13. A copper wire of length 20 m and a steel wire of length 30 m are connected end to end and stretched to a tension of 150 N. If the radius of both wires is 0.5 mm, how long does it take a transverse wave to travel the entire length of the two wires?

Ans: 0.329 sec

14. A cylindrical pipe of length 28 cm closed at one end is found to be at resonance when a tuning fork of frequency 864 Hz is sounded near the open end. Calculate the end correction when the speed of sound in air is 340 m/s.

Ans: 1.5 cm

15. A string when attached by a weight of 4 kg gives a note of frequency 256 Hz. What weight will produce an octave of this note?

Ans. 16 kg

16. A stretched wire under a tension of 1 kg wt is in unison with a fork of frequency 520 Hz. What change in tension would make the wire vibrate in unison with a fork of frequency 260 Hz?

Ans. 0.7 kg

17. A wire of length 50 cm when stretched by a load of 8 kg vibrates with a frequency of 280 vibrations per second. Find its mass.

Ans. 0.5 gm

18. A wire of linear mass density of 5.0×10^{-1} kgm^{-1} is stretched between two rigid supports with a tension of 450 N. The wire resonates at a frequency of 420 Hz. The next higher frequency at which the same wire resonates is 490 Hz. Find the length of the wire.

Ans. 2.14 m

19. Find the speed of transverse wave in wire if tension of wire is increased by 4%. Calculate the percentage change in the frequency of wire.

Ans. (i) 600 ms^{-1}, (ii) 2%

20. A piano string has a length of 2 m and a density of 8×10^3 kgm^{-3} when the tension in the string produces a strain of 1%, the fundamental note obtained from the string in transverse vibration is 170 Hz. Calculate the young modulus value for the material of the string.

Ans: 3.7×10^{11} Nm^{-2}

21. Sound waves of frequency 100 Hz fall normally on a smooth wall. At what distances have (a) maximum (b) minimum amplitude of vibration? (Speed of sound = 340 m/s)

Ans: (a) 0.85, 2.55, 4.25 m . . . from the wall, (b) 1.7, 3.4, 5.1 m . . . from the wall

22. A resonating tube resonated with a tuning fork of 256 Hz. If lengths of resonating columns are 32.5 cm and 99 cm. Find the value of the end correction and velocity of sound.

Ans: 0.75 cm, 340.5 m/s

23. A wire under the tension vibrates with a fundamental frequency of 240 Hz. What would be the fundamental frequency if the wire were half as long twice as thick and under one fourth of the tension?

Ans: 120 Hz

24. The tension in a wire of length 1 m is 50 N. Calculate the change in tension required to raise the pitch over one octave if the length is reduced to 0.8 m.

Ans: 78 N

25. The fundamental frequency of a sonometer wire increases by 5 Hz if its tension is increased by 21%. How will the frequency be affected if its length is increased by 10%?

Ans: 45. 45 Hz

Multiple Choice Questions

1. In a transverse vibration of string with fundamental frequency 256 Hz, the length is made half, tension increased to 4 times and width is doubled. Then the fundamental frequency of vibration becomes:

   a. 256 Hz b. 512 Hz
   c. 1024 Hz d. 626 Hz

2. One open organ pipe of 27 cm and closed organ pipe of length 21 cm sound in unison in their 1st overtone. Calculate the end correction for both pipes.

   a. 1.5 cm b. 0.6 cm
   c. 0.9 cm d. 1.2 cm

3. An open organ pipe and a close organ pipe resonate with same tuning fork. The ratio of the lengths of open pipe to close pipe remains in the ratio.

   a. 2 : 1 b. 1 : 2
   c. 1 : 4 d. 4 : 1

4. An organ pipe P_1 closed at one end and vibrating in its first overtone and another pipe P_2 open at both ends are vibrating in its third overtone are in resonance with a given tuning fork. The ratio of the length of P_1 and P_2 is:

   a. $\frac{8}{3}$ b. $\frac{3}{8}$
   c. $\frac{1}{2}$ d. $\frac{1}{3}$

5. The end correction of a resonance tube is 1.0 cm. Then the diameter of tube is nearly

   a. 2 cm b. 3.3. cm
   c. 1.65 cm d. 6.6. cm

6. The frequency of a vibrating wire is f. When the area of the cross section of a wire is halved and the tension doubled, the frequency becomes

   a. f b. 2f
   c. 3f d. 5f

7. A sonometer wire vibrates with a frequency f. If it is replaced by another wire of three times the diameter; while the tension and other parameters remain constant, the frequency of vibration of the wire will be

a. 9 f   b. 3 f

c. $f/\sqrt{3}$ d. $f/\sqrt{5}$

8. When the prongs of the tuning fork are cut, its frequency

a. decreases.   b. increases.

c. remains unchanged.   d. may increase or decrease.

Answers

1. (b) 2. (a) 3. (a) 4. (c) 5. (b) 6. (b) 7. (c) 8. (b)

Hints to Challenging Problems

HINT: 1

Given,

Length of rope, l = 2.5 m

Mass of rope, m = 0.120 kg

f = 40 Hz

λ = 0.75 m

∴ Tension produced, T = ?

We know that

$$v = \sqrt{\frac{T}{\mu}}$$

$$\text{or } f\lambda = \sqrt{\frac{T}{\frac{m}{l}}}$$

$$\text{or } f^2\lambda^2 = \frac{T \times l}{m}$$

$$\text{or } T = \frac{f^2\ \lambda^2\ m}{l}$$

HINT: 2

Given,

Frequency of fork, f = 120 Hz

Mass (m) = 1.5 kg

Linear density (μ) = 0.0550 kg/m

a. Speed of transverse wave, v = ?

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{mg}{\mu}}$$

b. Wavelength, λ = ?

$$\therefore \lambda = \frac{v}{f}$$

c. Given,

m = 3 kg   v = ? λ = ?

$$v = \sqrt{T/\mu} = \sqrt{\frac{mg}{\mu}}$$

Then, use,

$$\therefore \lambda = \frac{v}{f}$$

HINT: 3

Given,

Length of rubber, l = 14 m

Mass of tube, m = 0.800 kg

∴ Linear density (μ) = $\left(\frac{m}{l}\right) = \frac{0.80}{14} = 5.71 \times 10^{-2}$ kgm^{-1}

Mass of load (m') = 7.50 kg

∴ Speed of transverse wave $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{m'g}{\mu}}$

∴ Time taken to travel the length (t) = $\frac{l}{v}$

HINT: 4

Given,

Frequency of oscillator, f = 40 Hz

Amplitude, a = 3 cm = 0.03 m

Mass per unit, μ = 50 g/m = 50×10^{-3} kg/m

Tension, T = 5 N

a. $v = \sqrt{\frac{T}{\mu}}$

b. $\lambda = \frac{v}{f}$

c. Wave function, y = ?

$$y(x, t) = a \cos 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)$$

$$\text{or } y(x, t) = a \cos 2\pi\left(\frac{x}{\lambda} - tf\right) \qquad \left(\because f = \frac{1}{T}\right)$$

HINT: 5

Given,

Distance between two consecutive antinodes

$$= \frac{\lambda}{2} = 15 \text{ cm}$$

∴ λ = 30 cm = 0.3 m

a = 0.85 cm = 0.0085 m

T = 0.075 s

a. Displacement, y = ?

For standing wave, we have ,

$y(x, t) = a \sin kx. \sin \omega t$

or $y(x, t) = a \sin \frac{2\pi}{\lambda} x. \sin \frac{2\pi}{T} t$

or $y(x, t) = (0.85)\text{ cm} \cdot \sin\left(\frac{2\pi x}{0.3\text{ m}}\right) \cdot \sin\left(\frac{2\pi t}{0.075\text{ s}}\right)$

b. Speed of transverse wave, $v = f\lambda = \frac{\lambda}{T}$

HINT: 6

Given,

Mass of wire, $m = 40.0\text{ g} = 40 \times 10^{-3}\text{ kg}$

Resonating length, $l = 80\text{ cm} = 80 \times 10^{-2}\text{ m}$

Frequency, $f_1 = 60.0\text{ Hz}$

Amplitude, $a = 0.3\text{ cm} = 0.3 \times 10^{-2}\text{ m}$

a. Speed of transverse wave, $v = f_1 \times 2l$

b. Tension in the wire (T) = ?

We have, $v = \sqrt{\frac{T}{\mu}}$

or $v^2 = \frac{T}{\mu}$

or $T = \mu v^2 = \frac{m}{l} \times v^2$

c. Given, $a = 0.3\text{ cm} = 0.3 \times 10^{-2}\text{ m}$

$\therefore$ Maximum particle velocity of vibration,

$v_{max} = \omega a = 2\pi f a$

Acceleration $= -\omega^2 a = -(2\pi f)^2 a$

HINT: 7

Given,

Number of harmonics, n = 3

Amplitude of the two travelling waves, a = ?

Length of string, l = ?

Speed of wave, v = ?

The given equation of the standing wave is

$y(x,t) = (5.6\text{ cm}) \sin [0.034\, x] \sin [50\, t]$ . . . (i)

But the standard equation of standing wave is

$y(x, t) = 2A \sin (kx) \cdot \sin \omega t$ . . . (ii)

Comparing (i) and (ii), we have

$\therefore$ $2A = 5.6\text{ cm}$, $k = 0.034\text{ rad/cm}$

$\omega = 50\text{ rad/s}$

a. Since, A = 2a

$\therefore$ $a = \frac{A}{2} = \frac{5.6}{2} = 2.8\text{ cm}$

b. Since, $k = \frac{2\pi}{\lambda}$

$\therefore$ $\lambda = \frac{2\pi}{k}$

Now, we know that

$l = n \times \frac{\lambda}{2}$

c. Given,

$\because$ $\omega = 2\pi f$

or $f = \frac{\omega}{2\pi}$

Also,

Speed of wave, $v = f\lambda$

HINT: 8

Given,

frequency of the diaphragm (f) = 2000 Hz

Distance between two adjacent nodes $\left(\frac{\lambda}{2}\right)$

$= 8.0\text{ cm} = 8 \times 10^{-2}\text{ m}$

or $\lambda = 16 \times 10^{-2}\text{ m}$

i. The speed of sound in air,

$v = f\lambda = 2000 \times 16 \times 10^{-2}$

$\therefore$ $v = 320\text{ ms}^{-1}$

ii. For the second stationary wave,

f = 1600 Hz

$\therefore$ $\lambda = \frac{v}{f} = \frac{320}{1600} = 0.2\text{ m}$

Distance between adjacent nodes $= \frac{\lambda}{2}$

$= \frac{0.2}{2} = 0.1\text{ m} = 10\text{ cm}$

iii. Let l be the length of the tube between diaphragm and the closed end and n be the number of loops or (segments) formed when frequency of the diaphragm is 2000 Hz. So

$l = n \times 8$ . . . (i)

(Since distance between adjacent nodes is 8 cm so total length of n loops will be 8 × n which is equal to l)

When the frequency of the diaphragm is 1600 Hz, then

$l = (n - 1) \times 10$ . . . (ii)

(Since in this case the distance between two adjacent nodes is 10 cm)

From (i) and (ii) , we get

$8n = (n - 1)\, 10 = 10n - 10$

or $n = 5$

$\therefore$ from (i), we get

$l = n \times 8 = 5 \times 8 = 40\text{ cm} = 0.4\text{ m}$

iv. For the next lower frequency,

Number of loops formed = n – 2 = 5 – 2 = 3

So, we can write

$(n - 2)\frac{\lambda}{2} = l$

or $(5 - 2)\frac{v}{f \times 2} = l$

or $3 \times \frac{320}{f \times 2} = 0.4$

or $f = \frac{3 \times 320}{2 \times 0.4}$

$\therefore$ $f = 1200\text{ Hz}$

HINT: 9

Given,

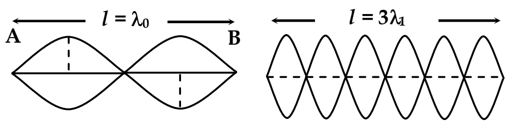


Fig. 1 Fig. 2

Fundamental frequency in the first case (f_0) = 100 Hz

Frequency in the second case (f_1) = ?

If l be the length of string and v be the speed of vibration then from Fig. 1, we have

$l = \lambda_0$

But, $v = f_0\lambda_0 = f_0 \times l$

In the second case as shown in Fig. 2, we have

$l = 3\lambda_1$

or $\lambda_0 = 3\lambda_1$

or $\frac{v}{f_0} = 3 \times \frac{v}{f_1}$

HINT:10

i. Distance between consecutive nodes = $\frac{\lambda}{2}$

ii. Distance between a node and an adjacent antinode = $\frac{\lambda}{4}$

iii. Distance between consecutive antinodes = $\frac{\lambda}{2}$

Fundamental frequency of a closed pipe, f = ?

Length of pipe, l = 15 cm = 0.15 m

Speed of sound, v = 340 ms^{-1}

For a closed pipe, we have

$f = \frac{v}{4l}$

HINT: 11

Given,

v = 350 ms^{-1} f = 440 Hz

Shortest length of a closed pipe, l = ?

For the shortest length of the closed pipe, there is first mode of vibration so the frequency of first mode is given by;

$f = \frac{v}{4l} \Rightarrow l = \frac{v}{4f}$

HINT:12

Given,

Speed of sound in air, v = 330 ms^{-1}

Fundamental frequency, f = 256 Hz

a. For a closed organ pipe,

$f = \frac{v}{4\,l}$

$\therefore\ l = \frac{v}{4\,f}$

b. For an open organ pipe,

$f = \frac{v}{2l}$

$\therefore\ l = \frac{v}{2f}$

HINT: 13

Length of tube, l = 60 cm = 0.6 m

Frequency of fork, f = 512 Hz

First resonating length (l_1) = 14.8 cm = 0.148 m

Second resonating length (l_2) = 48 cm = 0.48 m

Lowest frequency (f_0) = ?

We know that

$v = 2f(l_2 - l_1)$

End correction of the pipe, $e = \frac{l_2 - 3l_1}{2}$

Now, the tube behaves of an open pipe so its lowest frequency (f_0) is given

$f_0 = \frac{v}{2(l + 2e)}$

HINT: 14

Given,

Length of string, l = 1.5 m

Density, $\rho = 7.7 \times 10^3$ kgm^{-3}

Young's modulus, $Y = 2\times 10^{11}$ Nm^{-2}

Strain = 1 % in string = $\frac{1}{100}$

f = ?

The fundamental frequency of transverse vibration in the stretched string is given by

$$f = \frac{1}{2l}\sqrt{\frac{T}{\mu}} \quad \text{. . . (i)}$$

Now,

$Y = \frac{\text{stress}}{\text{strain}}$

or stress = Y × strain

or $\frac{T}{A}$ = Y × strain

or T = Y × A × strain

From (i), we get

$$f = \frac{1}{2l}\sqrt{\frac{YA \times \text{strain}}{\mu}}$$

$$= \frac{1}{2l}\sqrt{\frac{YA \times \text{strain}}{A\rho}} \left[\therefore\ \mu = \frac{\text{mass (m)}}{\text{length}(l)} = A\rho\right]$$

$$= \frac{1}{2l}\sqrt{\frac{Y \times \text{strain}}{\rho}}$$

HINT: 15

Given,

Density of water, $(\sigma) = 10^3$ kg m^{-3}

Density of metal cylinder, $(\rho) = 8000$ kgm^{-3}

Fundamental frequency of vibration, $(f_1) = 256$ Hz

If V be the volume of cylinder, its weight in air will be mg = Vρg. Due to this weight, tension T_1 is developed in the wire.

$\therefore$ $T_1 = V\rho g$.

If f_1 be the frequency in this case, we have

$$f_1 = \frac{1}{2l}\sqrt{T_1/\mu} \qquad \ldots(i)$$

When cylinder is immersed in the water, its weight (W) decreases due to upthrust (U) so tension (T_2) in this case is

$T_2 = W - U$

$= W - V\sigma g$

($\therefore$ U = weight of water displaced by cylinder = $V\sigma g$)

$= V\rho g - V\sigma g$

$= Vg(\rho - \sigma)$

If f_2 be the frequency of vibration in this case,

$$f_2 = \frac{1}{2l}\sqrt{\frac{T_2}{\mu}} \qquad \ldots \textbf{(ii)}$$

From (i) and (ii), we get

$$\frac{f_1}{f_2} = \sqrt{\frac{T_1}{T_2}}$$

or $f_2 = \sqrt{\dfrac{T_2}{T_1}} \times f_1$

$$= \sqrt{\frac{Vg(\rho - \sigma)}{Vg\rho}} \times f_1$$

HINT: 16

Given,

Resonating length, $l = 1$ m

Mass, m = 4 kg

Mass per unit length, $\mu = 10^{-3}$ kgm^{-1}

a. When wire is strucked at the middle, it will vibrate in one loop. The nodes are formed at bridges while antinode is at the middle. Clearly

$$l = \frac{\lambda}{2}$$

$\therefore$ $\lambda = 2$ m

b. Now,

$$f = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$$

$$= \frac{1}{2l}\sqrt{\frac{mg}{\mu}}$$

❖ ❖ ❖

ACOUSTIC PHENOMENA

4

CHAPTER

4.1 Introduction

Acoustics is a branch of physics which deals about the production, control, transmission, reception and effect of sound wave. The term is derived from Greek word "akoustos" which gives the meaning "hearing". This branch studies all properties of mechanical waves in solids, liquids and gases. The science of acoustics incorporates many aspects of human society music, medicine, architecture, industrial production, welfare and many more. In this chapter, we primarily focus on the intensity, quality, beats and Doppler's effect of sound wave.

4.2 Pressure Amplitude

In chapter one, we have discussed about longitudinal wave in which the displacement of the particle is parallel to the direction of propagation of wave. *The maximum linear displacement of particles from their mean position is called displacement amplitude of a wave.* The displacement equation for a plane wave travelling in the positive x-axis with angular velocity ω can be written as,

$$\mathbf{y = a \sin (\omega t - kx)} \quad \ldots(4.1)$$

The simplest form of longitudinal wave in a medium is the sound wave. When sound wave travels in a medium there is pressure variation at different points of the medium.

The pressure variation in a medium relies on the displacement of the particles. This change of pressure at different points of the medium occurs due to the parallel oscillation of particles with the direction of propagation of wave. Therefore, the characteristics of sound propagation can be studied in terms of pressure variation. *The maximum value through which the pressure increases (or decreases) in the medium as the sound wave travels through it is called the pressure amplitude.* It is measured in units of Pascals (Pa) or Mega Pascals (MPa).

Consider a cylindrical volume of air with cross section A and undisturbed column length Δx, so that the original volume is, $V = A\Delta x$. If sound wave propagates into the cylinder, the particles displace from their mean position. Let y_1 and y_2 be the displacement of particles at the left and right ends of the cylinder, respectively as shown in Fig. 4.1.

The change in volume $(\Delta V) = A(y_2 - y_1) = A\Delta y$

if $y_2 > y_1$, the length of air column expands, and

if $y_2 < y_1$, the length of air column is compressed.

Now, the volume strain = $\frac{\Delta V}{V} = \frac{A\Delta y}{A\Delta x} = \frac{\Delta y}{\Delta x}$ x

For very small length of cylinder, Δx i.e. $\Delta x \to 0$.

$$\frac{\Delta V}{V} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$$

$\therefore$ Volume strain = $\frac{dy}{dx}$

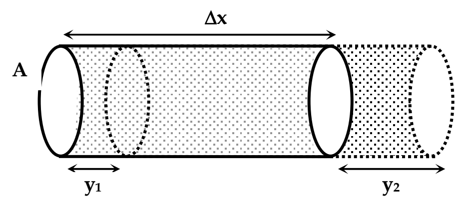


Fig 4.1: Pressure variation in air

$\therefore$ Bulk modulus of elasticity of an elastic medium is

$$B = \frac{\text{Normal stress (change in pressure)}}{\text{Volume strain}}$$

$$= -\frac{P}{\frac{dy}{dx}}$$

$$\mathbf{P = -B\frac{dy}{dx}} \qquad \ldots\mathbf{(4.2)}$$

Where, negative sign indicates that as pressure increases, volume decreases and vice versa.
Now, differentiating equation (4.1) with respect to x, we get,

$$\frac{dy}{dx} = -ak\cos(\omega t - kx)$$

Putting the value of $\frac{dy}{dx}$ in equation (4.2), we get,

$$P = Bak\cos(\omega t - kx)$$

Also, the longitudinal wave velocity in a medium of bulk elasticity B and density ρ is, $v = \sqrt{\frac{B}{\rho}}$, so, $B = v^2\rho$.

$$\therefore P = v^2\rho ak\cos(\omega t - kx)$$

Obviously, $v^2\rho ak$ is the maximum value of P (i.e. maximum change in pressure) and is called pressure amplitude P_0. Thus,

$$\mathbf{P = P_0\cos(\omega t - kx)} \qquad \ldots\mathbf{(4.3)}$$

Equation (4.3) is the pressure equation of a longitudinal wave. Hence a longitudinal wave, such as sound wave, may be represented either by a displacement wave equation (4.1) or of a pressure wave equation (4.3). A comparison of equations (4.1) and (4.3) shows that the displacement wave is 90^o out of phase with the pressure wave. It means that, when the displacement at a point is zero, the change in pressure is maximum, and vice-versa.

The variation of pressure amplitude and displacement is explained in Fig.4.2, considering the molecular vibrations of gas in a closed tube. At the compression, particles are piled up and a cross section is found in which the particles displacement is almost zero, so the node is formed at such positions. Similarly, at rarefaction, a cross section is found at which the particles are pulled apart in opposite direction so that net displacement of the particles is again zero and hence, another node is formed at that position. Hence, in both compressions and rarefaction, nodes are formed at positions where particles are at rest. The antinodes are formed at the centre between compression and rarefactions. The comparison of displacement amplitude and pressure amplitude are shown in Fig.4.2 (i) and (iii). Sound pressure is measured by microphone in air and hydrophone in water.

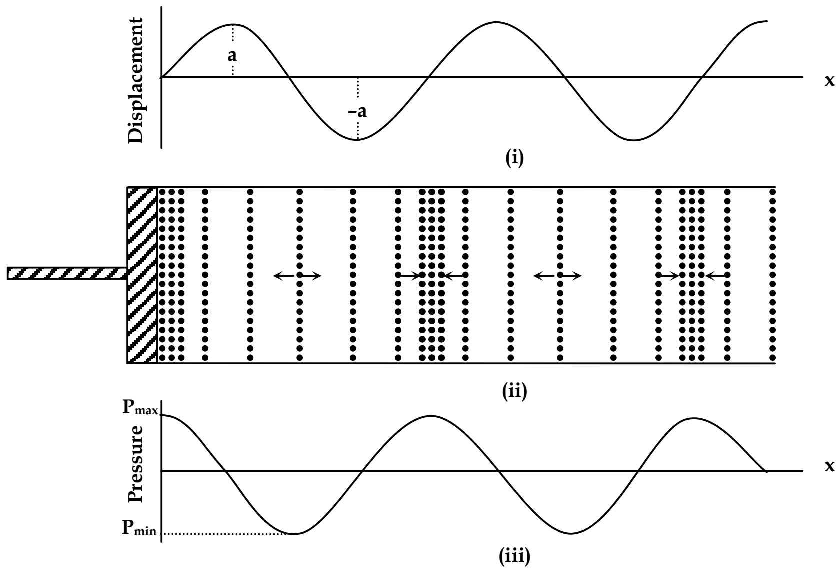


Fig.4.2: Displacement and pressure amplitude (i) Displacement versus position at t = 0. (ii) Oscillation pattern of particle in a cylinder. (iii) Pressure fluctuation versus position at t = 0.

4.3 Characteristics of Sound

We hear many kinds of sound in our daily life. The horn of vehicles, the sound of engines, music in radio and television, warbling of birds, etc. are some examples of sound with which we are familiar in our day to day life. If we start listening to our favourite music, then we don't want to turn the radio off. But, if we stay at the traffic chowks of crowded city, we do not like to stay there any more because of irritating sound. There are various types of sound we hear, but in physics, the sound is broadly categorized into two types (i) musical sound and (ii) Noise.

i. **Musical sound:** The sound which is regular, periodic and continuous is called musical sound. Music is pleasant to hear. Someone may say that s/he does not like the melody of a song, so it is noise for her/him. But, in physics, the musical sound does not solely depend on personal sense, rather it is a pattern of sound generation and propagation. If a sound contains regular and periodic vibration, this sound is certainly a music, though someone likes or dislikes it. In musical sound, the frequency is generally high.

ii. **Noise:** The sound wave which is irregular, non-periodic, and very short in duration is called noise. Noise is unpleasant to us. Sound of engine, horn of vehicles, hammer striking on anvil are some examples of noise.

Differences between Music and Noise

Music	Noise
1. Music has pleasing effect on ears and mind.	1. Noise appears to be irritating and a nuisance.
2. Music has a combination of frequencies and their harmonics.	2. Noise has no such properties.

3. Music has high frequency and there are recognizable patterns of changes in wavelength and amplitude.	3. Noise has low frequency and has irregular wavelengths and produces sudden changes in amplitude and wavelength.
4. It is periodic and regular.	4. It is non-periodic and irregular.
5. It lasts for long time.	5. It damps in very short duration.

Characteristics of Musical Sound

The musical sound is basically distinguished from its characteristics depending on its frequency, energy flow and pattern of propagation. On such basis, the characteristics of musical sound are divided into three types: pitch, intensity and quality.

Pitch

The characteristics of musical sound which is pertaining to frequency is known as pitch. Pitch is somehow perception, though it depends on frequency. Pitch of sound is a subjective quantity. High frequency sound is called the high pitch sound. High pitch sound is shriller than the low pitch sound. Although the frequency of sound produced by a source is constant, its observed frequency may be different due to the relative motion between the sound source and the observer.

If we compare the sound of mosquito with the roaring of lion, we certainly find that the energy propagation in the roaring of lion is greater than the sound of mosquito. But, if we compare the pitch of these sounds, the pitch of sound of mosquito is greater than the roaring of lion (i.e. the sound of mosquito has greater frequency than the frequency of sound produced in roaring of a lion). Other many examples that we experience in our daily life are:

1. Voice of female has greater pitch than the voice of male.
2. Voice of child has greater pitch than the voice of adult.
3. The sound produced by thin wire has greater pitch than the thick one.
4. The sound produced by different keys of harmonium has different pitch.
5. The flute produces sound of different pitch when different holes are closed.

Differences between Frequency and Pitch

Frequency	Pitch
1. Frequency is observable quantity which can be measured accurately.	1. Pitch is sensation pertaining to frequency but this is not measured accurately.
2. Frequency is discussed in all forms of waves including electromagnetic waves and mechanical waves.	2. Pitch is observed only in sound wave.
3. Frequency is a very well defined quantity.	3. Pitch is not well defined.
4. It can not be zero for a deaf person.	4. It is zero for a deaf person.
5. Frequency is the characteristics of all type of oscillations and vibrations.	5. Pitch is the characteristics of sound waves only.

Intensity

Waves transport energy from one place to another. The amount of energy transported by a wave is explained in terms of intensity. The intensity of sound is defined as the amount of sound energy propagated per second per unit surface area. It is denoted by I. Its unit is Wm^{-2} (watt per square metre).

Let E be the total energy transported through the surface area A in time t. The intensity of sound is,

$$\text{Intensity (I)} = \frac{\text{Energy transported (E)}}{\text{Area (A)} \times \text{Time (t)}}$$

$$I = \frac{E}{At}$$

$$I = \left(\frac{E}{t}\right)\frac{1}{A}$$

$$\mathbf{I = \frac{P}{A}} \qquad \text{...(4.4)}$$

Where, $P = \frac{E}{t}$ = power transported

To determine the total energy flow through a surface, a sphere of radius r is sketched around a point source, the total intensity is written as,

$$I = \frac{P}{4\pi r^2} \qquad [\because \text{Surface area of sphere} = 4\pi r^2]$$

Quality or Timber

Eventhough the pitch and loudness of two sounds are same, it can be distinguished from our hearing perception. If one of your friends comes to your home and calls you from outside, and still without seeing you can recognize from his/her voice. This subjective part of wave which possess its individual character is called the quality or timber of a sound. It is also called the tone colour. The sound quality is distinguished from the overtones of sound.

4.4 Relations of Intensity and Amplitude of Wave

Waves carry energy from one point to another. In case of mechanical wave, the disturbance at a point is spread out in a medium via the oscillation of molecules in that medium. This disturbance is transported by the molecules in the medium in the form of wave pattern. The wave pattern is in the form of sine wave. So, the displacement of particle's oscillation is written as,

$$\mathbf{y = a \sin(\omega t - kx)} \qquad \text{...(4.5)}$$

Where, a = amplitude of particles

ω = angular velocity of particles

The total energy transfer is the sum of energy carried by oscillating particles in the medium, which contains both kinetic energy due to the motion of particles and potential energy due to displacement of particles from their mean position. So, the total energy transfer is,

$$E = \text{Kinetic energy } (E_K) + \text{Potential Energy } (E_P)$$

$$\text{i.e., } \mathbf{E = E_K + E_p} \qquad \text{...(4.6)}$$

Applying conservation of energy, when $E_P = 0$, E_K is maximum. So, total energy,

$$E = (E_K)_{max}$$

$$\mathbf{E = \frac{1}{2} mv_{max}^2} \quad \text{...(4.7)}$$

To find the maximum velocity of particles, we differentiate equation (4.5) with respect to time,

$$\therefore \mathbf{v = \frac{dy}{dt} = a\omega \cos(\omega t - kx)} \quad \text{...(4.8)}$$

The maximum velocity of particles, in a medium,

$$\mathbf{v_{max} = a\omega} \quad \text{...(4.9)}$$

Using equation (4.9) in equation (4.7), we get,

$$E = \frac{1}{2} m (a\omega)^2$$

$$\mathbf{E = \frac{1}{2} ma^2\omega^2} \quad \text{...(4.10)}$$

Let V be the volume of the medium of density ρ, influenced by the disturbance so,

$$m = V\rho$$

$$\mathbf{m = Al\rho \qquad (\because V = Al)} \quad \text{...(4.11)}$$

Now, using equation (4.11) in equation (4.10), we get,

$$\mathbf{E = \frac{1}{2} Al\rho\, a^2\omega^2} \quad \text{...(4.12)}$$

Now, the intensity of sound wave,

$$I = \frac{E}{At}$$

$$= \frac{1}{2} \frac{Al\rho a^2\omega^2}{At} = \frac{1}{2}\left(\frac{l}{t}\right) \rho a^2\omega^2$$

$$= \frac{1}{2} v\rho a^2\omega^2$$

Where, $v = \frac{l}{t}$ = velocity of sound in that medium.

In a medium, ρ, a, and ω remains constant so the intensity of sound is directly proportional to the square of amplitude (i.e. $I \propto a^2$).

The intensity of sound in terms of frequency of particle oscillation is,

$$I = \frac{1}{2} v\rho a^2 (2\pi f)^2 \qquad (\omega = 2\pi f)$$

$$= \frac{1}{2} v\rho a^2\, 4\pi^2 f^2$$

$$\therefore \mathbf{I = 2\pi^2 v\rho a^2 f^2} \quad \text{...(4.13)}$$

Relation between pressure amplitude and intensity of sound

The pressure amplitude of sound is,

$$P = Bak \cos(\omega t - kx)$$

$$\therefore P_{max} = Bak$$

Squaring both sides, we get,

$$P_{max}^2 = B^2a^2k^2 = v^4\rho^2 a^2k^2$$

$$= v^4\rho^2a^2\left(\frac{\omega}{v}\right)^2 \quad [\because \omega = vk]1$$

$$= v^2\rho^2\, a^2\omega^2$$

$$= 2v\rho\left(\frac{1}{2}v\rho\, a^2\omega^2\right) = 2v\rho\,(2\pi^2\, v\rho a^2f^2)$$

From equation (4.13), $I = 2\pi^2 v\rho a^2 f^2$

Therefore, $P_{max}^2 = 2v\rho I$

$$\therefore \quad I = \frac{P_{max}^2}{2v\rho}$$

This expression gives the relation between intensity of sound and the pressure amplitude.

The term $v\rho$ is also called the acoustic impedance of a medium, and is denoted by Z.

(i.e. $Z = v\rho$)

So, the relation of intensity of sound, pressure amplitude and acoustic impedance of a medium is written as,

$$I = \frac{P_{max}^2}{2Z}$$

Loudness

Loudness refers to the perception of sound wave in our ear. In everyday language, the loudness and intensity are used interchangeably. In physics, we make distinction between the two. Actually, loudness is directly proportional to the logarithmic value of intensity which, is given by Weber-fetches law. It is denoted by L. So,

$$\text{Loudness (L)} \propto \log_{10} I$$

$$\text{Loudness (L)} = k \log_{10} I$$

The experimental value of k is 1. So,

$$\text{Loudness (L)} = \log_{10} I$$

In this relation, I is taken in numerical value of intensity.

The loudness of sound is measured on an arbitrary scale that corresponds roughly to the sensation of sound wave with respect to a standard value of intensity.

The perception of loudness of sound on our ear is based on 10^n, n = order of power. For example, 100 Wm^{-2} intensity is perceived just double than the loudness provided by intensity 10 Wm^{-2}.

1. For $I = 10\ Wm^{-2}$, Loudness (L) $= \log_{10}10^1 = 1$
2. For $I = 100\ Wm^{-2}$, Loudness (L) $= \log_{10}10^2 = 2$
3. For $I = 1000\ Wm^{-2}$, Loudness (L) $= \log_{10}10^3 = 3$

Differences between Intensity and Loudness

Intensity	Loudness
1. Sound intensity is the property of the sound source.	1. Loudness depends on the sound source, the medium and the receiver.
2. Sound intensity is measured in watt per square meter (Wm^{-2}).	2. Loudness is measured in phon.

3.	Sound intensity holds a small significance in problems involving human hearing system.	3.	Loudness is very important property to consider in such problem.
4.	It is a measurable physical quantity.	4.	It is comparable quantity with a standard value.
5.	There is a fixed relation between intensity and frequency ($I \propto f^2$).	5.	There is no direct relation between loudness and frequency.

Threshold of Hearing

The minimum loudness of sound that can just be heard by normal ear is known as threshold of hearing. It is denoted by L_0.

$$\therefore \quad L_0 = \log_{10} I_0$$

The corresponding value of intensity for the threshold condition is 10^{-12} Wm^{-2}. It means, the energy transfer per unit time per unit surface area is, $I_0 = 10^{-12}$ Wm^{-2} in a medium. Such minimum value of intensity is heard by normal ear. If the intensity is smaller than 10^{-12} Wm^{-2}, the sound is not heard, and above such value, it is efficiently heard.

The threshold value for different person may be different. In the elder age, in particular, the threshold value is increased (i.e. you have to speak louder to make the elder person hear). The value 10^{-12} Wm^{-2} is the average value for healthy person.

4.5 Intensity Level

The loudness itself does not make any complete sense in physics, however the difference of loudness of a sound with respect to threshold of hearing makes a perfect sense. Therefore, the relative value of loudness is taken into practice rather than absolute value of intensity. The loudness of sound is usually expressed in terms of the ratio to a standard value that is called intensity level. *The logarithm value of ratio of intensity of sound (I) to the standard intensity (I_0) is known as intensity level*. It is denoted by β.

$$\therefore \quad \text{Intensity level of sound, } \beta = \log \frac{I}{I_0}$$

It is to be noted that in this chapter logarithm values are taken in base 10 (i.e., $\log_{10}$)

If I and I_0 be the sound intensities of two sound notes corresponding to loudness L and threshold of hearing L_0 respectively, the law regarding the loudness of sound, Weber-Fechner's law is written as,

$$L_0 = k \log I_0 \text{ and}$$
$$L = k \log I$$

Then,

$$L - L_0 = k \log I - k \log I_0$$
$$= k(\log I - \log I_0) = k \log \frac{I}{I_0}$$

This value ($L - L_0$) gives the intensity level of sound having loudness L.

$$\therefore \quad \beta = L - L_0 = k \log \frac{I}{I_0}$$

The experimental value of k is 1. So,

$$\boldsymbol{\beta = L - L_0 = \log \frac{I}{I_0}} \qquad \ldots\textbf{(4.14)}$$

Units of Intensity Level: Bel, Decibel

Since, intensity level is the logarithm value of ratio of two intensities, sound intensity level has no dimension but the unit is assigned. Its unit is bel, in the honour of Alexander Graham Bell, the inventor of telephone.

$$\therefore \beta = \log \frac{I}{I_0} \text{ (bel)}$$

For $I = 10\ I_0$

$$\beta = \log \frac{10\ I_0}{I_0} = 1 \text{ bel}$$

Thus, one bel is defined as the sound intensity level at which the intensity of sound is ten times greater than the standard intensity. i.e. $I = 10\ I_0$.

The unit 'bel' is inconveniently large for daily purposes of our surroundings, therefore the 'decibel' unit is appropriate in practice. The 'decibel' unit of sound intensity level is one tenth value of bel unit.

$$\therefore 1 \text{ decibel} = \frac{1}{10} \text{ bel}$$

$\therefore$ We write, $\beta = 10 \log \frac{I}{I_0}$ dB.

i. If the intensity of sound is equal to the threshold value i.e. $I = I_0$,

$$\beta = 10 \log \frac{I}{I_0} = 10 \log \frac{I_0}{I_0} = 0 \text{ dB}$$

This value of sound intensity level is considered as the threshold value of loudness.

ii. If the intensity is equal to 1 Wm^{-2}, the intensity level, $\beta = 10 \log \left(\frac{1}{10^{-12}}\right) = 120$ dB.

The sound intensity level corresponding to 120 dB is painful to ear.

Comparison of sound intensity level

The intensity levels for two sounds of intensities I_1 and I_2 are:

$$\beta_1 = 10 \log \frac{I_1}{I_0} \quad \text{and} \quad \beta_2 = 10 \log \frac{I_2}{I_0}$$

The difference of sound intensity level,

$$\Delta\beta = \beta_1 - \beta_2$$
$$= 10 \log \frac{I_1}{I_0} - 10 \log \frac{I_2}{I_0}$$
$$= 10 (\log I_1 - \log I_0 - \log I_2 + \log I_0)$$
$$= 10 (\log I_1 - \log I_2)$$
$$\mathbf{\Delta\beta = 10 \log \frac{I_1}{I_2}} \quad \text{...(4.15)}$$

For the intensities of sound I_1 and I_2 at distance r_1 and r_2 respectively from a standard source, we have,

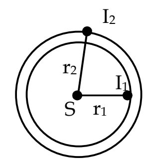
$$I_1 = \frac{P}{4\pi r_1^2} \quad \text{and} \quad I_2 = \frac{P}{4\pi r_2^2}$$

$$\mathbf{\frac{I_1}{I_2} = \left(\frac{r_2}{r_1}\right)^2} \quad \text{...(4.16)}$$

$\therefore$ Difference of intensity level, from equations (4.15) and (4.16), we have,

$$\Delta\beta = 10 \log \left(\frac{r_2}{r_1}\right)^2$$

$$\Delta\beta = 20 \log \left(\frac{r_2}{r_1}\right) \text{dB}$$



Threshold of Pain

The intensity level of sound wave which produces the irritation in our ear is called threshold of pain. If the intensity level is greater than the value of threshold of pain, it gradually damages the hearing capacity of ear. 90 decibel value is considered as the threshold of pain. If children stay longer time being exposed to such type of sound at high intensity level, they may lose their hearing capacity. In many traffic chowks of Kathmandu valley, the intensity level of noise is about 100 db (i.e. above the threshold of pain).

4.6 Infrasonics, Audible, Ultrasonics and Supersonics

Sound waves are categorized in accordance with their frequency (f). Our ear cannot hear very small and very large frequency of sound. On the basis of frequency, the sound waves are categorized as follows:

Types	Frequency Range	Property	Source
Infrasonics	$f < 20$ Hz	Inaudible	Seismic waves, waves of pendulum oscillation.
Audible	$20 \text{ Hz} \le f \le 20$ kHz	Audible	All sound waves we hear in our daily life.
Ultrasonics	$f > 20$ kHz	Inaudible	Piezolectric effect in quartz crystal.
Supersonics	velocity greater than sound wave	not actually the vibrational wave	Objects move faster than sound.

Infrasonics

The sound waves which have frequency smaller than the audible range are known as infrasonics. These waves have the frequency smaller than 20 Hz. These waves are not heard with normal human ear. Infrasonic waves are produced due to the oscillation of large objects, so the wavelengths are large enough to generate the frequency below audible range. They are produced in earthquakes, volcanic eruptions, nuclear bomb tests, waterfalls, oscillation of simple pendulum, calving of ice bergs etc.

Some animals are supposedly assumed to hear infrasonics pertaining to the fact that they behave in weird way just before earthquakes and tsunamis and are not usually the victim of the devastation because of this sensing ability. These evidences probably support the belief of infrasonic sensing ability of animals. It has been believed that the animals sense the infrasonic and get alert for the eminent natural disasters. Some animals like whales, elephants, hippos, Giraffes, and rhinoceros are known to use infrasonic waves to communicate long distance away.

Ultrasonics

The sound waves whose frequency lies above the audible range are known as ultrasonics. The frequency of ultrasonics is greater than 20 kHz. It is inaudible to normal human ear. Ultrasonics are used to visualize the internal body parts in human body. Though ultrasonic is not heard by human beings, it can be defected by many other animals. For example; bat (a mammal), produces ultrasonic and also receives its echo reflecting from obstacles which guides the bat to fly without clashing.

Ultrasound is artificially produced by pressuring the quartz crystal, called the piezo – electric effect on quartz crystal. Piezo refers the pressure and electric refers the electricity. So, piezo – electric effect refers the conversion of pressure into electricity. In radiology department of hospitals, a probe (a part of ultrasound machine that is run on the human body) produces the ultrasound and propagates into the human body. Then, it is reflected from various interfaces of organs like kidneys, stomach, liver etc. because of different acoustic impedance of these organs. The probe also receives the reflected ultrasound and passes it to the computer CPU. Finally, the internal organs are sketched into the computer monitor.

The important uses of ultrasound are listed below.

Ultrasound is a sound wave whose frequency is greater than 20 kHz and is inaudible. They can easily travel into our body. Ultrasound is greatly used in hospitals to detect things into the body. People named it video X-ray in hospitals, in reality, X-rays are not used, rather the sound wave of very large frequency is used in ultrasound diagnosis. Some important uses of ultrasound are as follows:

Applications of Ultrasonic Waves

a. Ultrasound is mainly used to get the image of our internal body parts.
b. Eco method on ultrasound is used to find the depth of sea, rock, etc.
c. A short wavelength ultrasound is used to signal the specific direction.
d. Ultrasound is used to observe the infant growth.
e. The method of echoes is used to detect the existence of tumour inside the brain.
f. Ultrasound echo is used to measure the thickness of a material.
g. A blind man can walk on the road without any fear by using ultrasonic sound.

Supersonics

Supersonics have the speed greater than the speed of audible sound. The body which has the speed greater than sound wave is known as supersonic body. For example, jet plane flies faster than sound, so it is supersonic body.

When supersonic body moves in air, it produces energetic waves, that propagates backward in the form of cone with increasing amplitude. These waves are called shock waves.

The speed of supersonic body is measured in terms of match numbers,

$$\text{Match number} = \frac{\text{speed of supersonic body}}{\text{speed of sound in air}}$$

If the match number is greater than 1, the body travels in supersonic speed.

4.7 Beats

When two sound waves of slightly different frequencies propagating along the same direction in a medium superimpose, a resultant wave is formed. The intensity of resultant wave so formed fluctuates periodically i.e. its intensity rises and falls alternately in equal interval of time. *This phenomenon of alternate rise and fall of intensity of resultant wave due to superposition of sound waves of slightly different frequencies is called beat.* One rise and one fall produce a beat. The time interval between two consecutive high or consecutive low intensity is called beat period. The number of beats per second is called beat frequency.

Mathematically, the beat frequency is equal to the frequency difference between two interfering waves. For two sound waves of slightly different frequencies f_1 and f_2, the beat frequency is written as,

$$f_b = f_1 - f_2 \text{ (for } f_1 > f_2\text{)}$$

$$\text{and } f_b = f_2 - f_1 \text{ (for } f_2 > f_1\text{)}$$

Our persistence of hearing is 0.1 sec i.e. if time interval between two sound events is shorter than 0.1 s, we can't distinguish them as two different events. Therefore, the beat sensation, i.e. $f_b = \frac{1}{T_b} = \frac{1}{0.1\text{ s}} =$ 10 Hz. This means, we cannot distinguish beat if beat frequency is more than 10 Hz.

Conditions for Formation of Beats

- The amplitude of the two interfering waves should be same.
- The difference between the frequencies of interfering waves should be small. The beats can be heard only if the frequency difference is less than 10 Hz. (i.e. f < 10 Hz)

Graphical Representation of Beats

We consider two sound waves of slightly different frequencies f_1 and f_2 propagating in same medium. When they superimpose, the amplitude of resultant wave varies with time at a point as shown in Fig. 4.3. At a point, when two waves overlap in opposite phase, fading sound is heard. If the point contains two waves of similar phase, intense sound is heard.

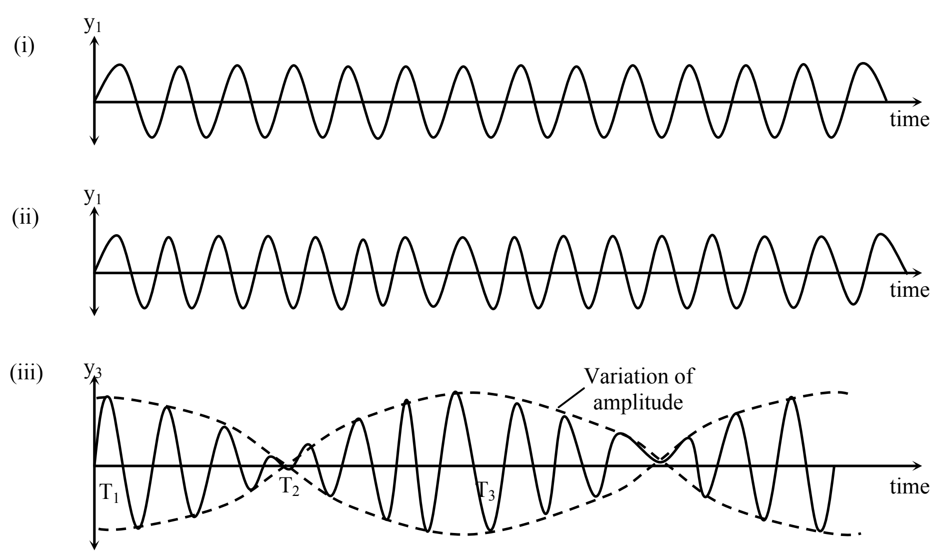


Fig. 4.3: Formation of Beats

Analytical Treatment of Beats

Consider two sound waves of slightly different angular velocity ω_1 (= $2\pi f_1$) and ω_2 (= $2\pi f_2$), propagating simultaneously in a medium with same initial phase and amplitude. The displacements of particles when the waves propagate are written as,

$$\mathbf{y_1 = a \sin \omega_1 t} \qquad \ldots (4.17)$$

$$\mathbf{y_2 = a \sin \omega_2 t} \qquad \ldots (4.18)$$

After superposition of waves, the displacement of resultant wave y is,

$$\mathbf{y = y_1 + y_2} \qquad \ldots (4.19)$$

Substituting values from equation (4.17) and (4.18) in equation (4.19), we have,

$$y = a \sin \omega_1 t + a \sin \omega_2 t$$

$$= a \left\{ 2 \sin \left(\frac{\omega_1 t + \omega_2 t}{2} \right) . \cos \left(\frac{\omega_1 t - \omega_2 t}{2} \right) \right\}$$

$$\left[\because \sin A + \sin B = 2 \sin \frac{A+B}{2} . \cos \frac{A-B}{2} \right]$$

$$= 2a \sin \left(\frac{\omega_1 + \omega_2}{2} \right) t \times \cos \left(\frac{\omega_1 - \omega_2}{2} \right) t$$

$$= \{2a \cos \left(\frac{\omega_1 - \omega_2}{2} \right) t\} \sin \left(\frac{\omega_1 + \omega_2}{2} \right) t$$

$$\mathbf{y = A \sin \left(\frac{\omega_1 + \omega_2}{2} \right) t} \qquad \ldots (4.20)$$

This is the expression for resultant wave equation. The amplitude of resultant wave is,

$$\mathbf{A = 2a \cos \left(\frac{\omega_1 - \omega_2}{2} \right) t} \qquad \ldots (4.21)$$

and angular velocity, $\omega = \frac{\omega_1 + \omega_2}{2}$

Equation (4.21) shows that the amplitude of resultant wave depends on time.

Condition for maxima

For resultant amplitude A to be maximum, we have,

$$A = \pm 2a$$

or, $2a \cos \left(\frac{\omega_1 - \omega_2}{2} \right) t = \pm 2a$

or, $2a \cos 2\pi \left(\frac{f_1 - f_2}{2} \right) t = \pm 2a$

or, $\cos 2\pi \left(\frac{f_1 - f_2}{2} \right) t = \pm 1 = \cos n\pi$, where $n = 0, 1, 2, 3, \ldots$

or, $\pi (f_1 - f_2)\, t = n\pi$

or, $\mathbf{t = \frac{n}{f_1 - f_2}}$ $\qquad \ldots$ **(4.22)**

This is the condition for maxima.

For $n = 0$, $t_1 = 0$ (first maxima),

For $n = 1$, $t_2 = \frac{1}{f_1 - f_2}$ (Second maxima),

For $n = 2$, $t_3 = \frac{2}{f_1 - f_2}$ (Third maxima)

and so on.

The time interval between two successive maxima = $\frac{1}{f_1 - f_2}$ seconds. Since, time interval is the reciprocal of frequency so,

$$\text{i.e.} \quad T_b = \frac{1}{f_1 - f_2}$$

$$\text{or,} \quad \frac{1}{f_b} = \frac{1}{f_1 - f_2}$$

$$\therefore \quad \mathbf{f_b = f_1 - f_2} \qquad \ldots (4.23)$$

Condition for minima

Similarly, for A to be minimum, we have,

$$A = 0$$

$$\text{or,} \quad 2a \cos\left(\frac{\omega_1 - \omega_2}{2}\right) t = 0$$

$$\text{or,} \quad 2a \cos 2\pi\left(\frac{f_1 - f_2}{2}\right) t = 0$$

$$\text{or,} \quad \cos 2\pi\left(\frac{f_1 - f_2}{2}\right) t = 0$$

$$\text{or,} \quad 2\pi\left(\frac{f_1 - f_2}{2}\right) t = (2n - 1)\frac{\pi}{2} \quad (\because \text{if } \cos\theta = 0 \text{ then } \theta = (2n-1)\frac{\pi}{2})$$

Where n = 1, 2, 3,. . . , etc.

$$\textbf{or,} \quad \mathbf{t = \frac{(2n - 1)}{2\,(f_1 - f_2)}} \qquad \ldots (4.24)$$

This is the condition for minima.

For n = 1, $t_1 = \frac{1}{2(f_1 - f_2)}$, first minima,

For n = 2, $t_2 = \frac{3}{2(f_1 - f_2)}$, second minima,

For n = 3, $t_3 = \frac{5}{2(f_1 - f_2)}$, third minima

and so on.

The time interval between two successive minima = $\frac{1}{(f_1 - f_2)}$ seconds.

$$\text{i.e.} \quad T_b = \frac{1}{f_1 - f_2}$$

$$\text{or,} \quad \frac{1}{f_b} = \frac{1}{f_1 - f_2}$$

$$\therefore \quad \mathbf{f_b = f_1 - f_2} \qquad \ldots (4.25)$$

From equations (4.23) and (4.25), we have,

Beat frequency of maxima = $f_1 - f_2$ = beat frequency of minima.

Thus, number of maxima or minima per second is $(f_1 - f_2)$ but one maxima and one minima of sound constitutes one beat. So, number of beats per second is equal to the difference in frequencies of the two sound waves i.e. $f = f_1 - f_2$.

Applications of Beats

i. Beats are used for the detection of harmful gases in mines. For this, two identical organ pipes are taken; one filled with pure air and other filled with air from the mine are blown together. If there are no beats, then the mine air is pure, but if beats are heard the mine-air is toxic.

ii. Beats are used in tuning musical instruments like sitar, violin, etc. The musical instrument is sounded with another instrument of known frequency. If the beats are heard, it is slightly adjusted so that there are no beats. This is called tuning.

iii. In sonometer experiment, beats can be used to adjust the vibrating length between the two bridges.

iv. Beats are used to find the unknown frequency of tuning fork. To find the unknown value of frequency produced by a tuning fork, it should be oscillated simultaneously with another tuning fork of known frequency. This can be done by two methods: (a) loading the fork; (b) filing the fork.

a. Loading the fork: The frequency of sound produced by a tuning fork decreases when the prongs are loaded with some extended masses. It is usually done by pasting the wax on prongs. In the beginning, given tuning fork (let tuning fork B) whose frequency is to be determined is sounded simultaneously with another tuning fork (A) of known frequency. During sounding, the beats produced in the superposition of these waves are recorded. Then, the tuning fork B is loaded with wax and are resounded simultaneously with tuning fork A. Due to loading, the frequency of B is changed and hence the beat frequency. Then, the beat frequency for the second condition is also noted.

Analysis

Let f_b be the beat frequency before loading fork B with wax. So, the two possible frequency of tuning fork B are:

$$f_2 = f_1 \pm f_b$$

Where, f_1 = known frequency of tuning fork A

f_2 = possible frequency of tuning fork B

Also, f_b' be the beat frequency after loading with wax. So, the possible frequency of tuning fork B are,

$$f_2' = f_1 \pm f_b'$$

Then, the result is expressed in table below.

Conditions		Upper value	Lower value
Before loading	f_2	$f_1 + f_b$	$f_1 - f_b$
After loading	f_2'	$f_1 + f_b'$	$f_1 - f_b'$

As mentioned above, the frequency of tuning fork decreases on loading, so the result is confirmed as below.

i. If the beat frequency increases after loading, f_2 must be $f_1 - f_b$.

ii. If the beat frequency decreases after loading, f_2 must be $f_1 + f_b$.

b. By filing the tuning fork: The filing process reduces the mass of prongs of tuning fork so that the frequency of vibration increases. The procedure is similar to the above experiment as done in (a). The result can be explained as below:

i. If the beat frequency increases after filing f_2 must be $f_1 + f_b$.

ii. If the beat frequency decreases after filling, f_2 must be $f_1 - f_b$.

Example:

A standard tuning fork 'A' has frequency 280 Hz and it gives 4 beats/sec when sounding simultaneously with another tuning fork 'B'. If tuning fork B is loaded with wax, the beat frequency is observed 3 beats/sec when above process is repeated.

SOLUTION

Original frequency of tuning fork A, $f_A = 280$ Hz

Beat frequency before loading, $f_b = 4$ Hz

Beat frequency after loading, $f_b' = 3$ Hz

The possible frequencies of tuning fork B before loading (280 ± 4) Hz i.e. 284 Hz or 276 Hz.

The possible frequencies of tuning fork B after loading, (280 ± 3) Hz i.e. 283 Hz or 277 Hz.

To compare the frequencies:

Before Loading	284	276
After Loading	283	277

Since, the frequency of sound produced by tuning fork decreases on loading with wax, the correct value of original frequency of tuning fork B must be 284 Hz.

This will be just reverse in case of filing.

4.8 Doppler's Effect

When source of sound and observer (or listener) are at rest, real (true) frequency of the sound is heard. But, if there is relative motion between source of sound and observer, the observer will not hear the sound of the real frequency. The frequency of sound so heard is called apparent frequency. This phenomenon was first noticed by Austrian physicist Christian Johann Doppler (C.J. Doppler) in 1845 in sound waves and is known as Doppler's effect after his name. *Hence, the apparent change in the frequency of sound heard due to relative motion between source and observer is called Doppler's effect.* The apparent frequency may be higher or lower than the actual frequency depending on how the source and observer are moving.

The whistle blown by a train is heard shriller when the train approaches a stationary passenger at the railway platform whereas it is heard smaller when train moves away from it. This is due to Doppler's effect.

Let us consider a source of sound is at any point A and an observed (listener) is standing at another point B as shown in Fig. 4.4. If both the source and observer are stationary as shown in Fig 4.4 (i), the wavelength and hence frequency of sound heard by observer doesn't change. However, if there is relative motion between the source and observer as shown in Fig. 4.4 (ii), there is apparent change in frequency of the sound heard by observer. Let us analyze the following different cases.

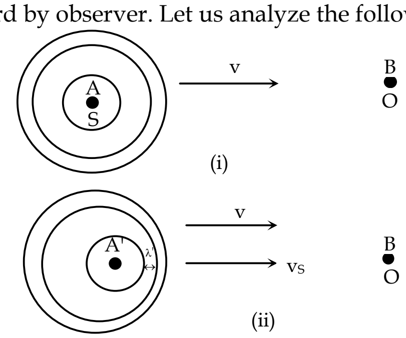


Fig.4.4: (i) Stationary source and stationary observer (ii) Moving source towards the observer

Note

We consider only that sound which is received by the observer, although sound travels in all directions. So, in this case, the sound produced by the source is taken only in the direction of observer.

a. Moving source and stationary observer

Let the waves from A take Δt time to reach at B and at the same time, the source moves from point A to A'. Therefore, the waves lying between A and B when they were stationary, now has to be confined within a smaller distance A'B, i.e. the waves are now crowded between the source and observer in this condition, as shown in Fig. 4.5 (Comparable to the distance between the consecutive turns of compressed spring). This means the wave length of sound decreases and hence, frequency increases. On the other hand, if the sound source moves away from the observer O as shown in Fig.4.6, the waves rarefies between the source and observer (comparable to the distance between the consequent turns of stretched spring). This means, the wavelength of sound increases and hence, frequency decreases.

The apparent frequency of sound is determined by

$$f' = \frac{v}{\lambda'} \quad \ldots(4.26)$$

$$\text{and } \lambda' = \frac{v \pm v_s}{f} \quad \ldots(4.27)$$

Where, v = velocity of sound wave

λ' = apparent wavelength

v_s = velocity of source

f = original frequency of sound

The positive or negative sign are chosen in accordance with the direction of motion of sound source.

Case (i): When source moves towards a stationary observer: In this case, the sound source and sound wave move in the same direction as shown in Fig. 4.5. Therefore, using the equation (4.27), the wavelength of sound is,

$$\lambda' = \frac{v - v_s}{f} \quad \ldots(4.28)$$

Therefore, the apparent frequency,

$$f' = \left(\frac{v}{v - v_s}\right) f \quad \ldots(4.29)$$

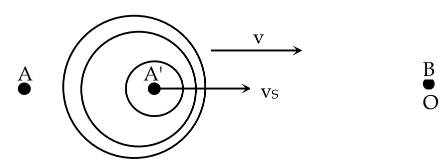


Fig 4.5: Moving source towards the observer

Clearly, f' > f. It shows that the apparent frequency of sound is greater than the real frequency.

Case (ii): When source moves away from a stationary observer: In this case, the sound source and sound wave move in the opposite direction to each other as shown in Fig.4.6. Therefore, using equation (4.27), the wavelength of sound is,

$$\lambda' = \frac{v + v_s}{f} \quad \ldots(4.30)$$

Applying equation (4.30) in equation (4.27), the apparent frequency of sound from equation (4.26) becomes,

$$f' = \left(\frac{v}{v + v_s}\right) f \quad \ldots(4.31)$$

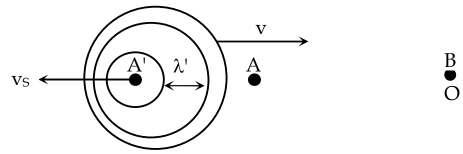


Fig 4.6: Moving source away from the observer

Clearly, f' < f. It shows that the apparent frequency of sound is smaller than real frequency.

b. Moving observer and stationary source

Let an observer initially at a position move towards/away from the stationary source. Due to the relative velocity between the sound wave (produced by the source) and the observer, there is apparently change in the frequency of sound heard by the observer.

When the observer moves towards a stationary source, the velocity of sound wave and observer are opposite to each other. So, their velocities add up to give a large resultant velocity. This means the frequency also increases. And when the observer moves away from stationary source their respective velocities are in the same direction. So, their velocities add up to give a smaller resultant which apparently decreases the frequency.

The apparent frequency of sound source, f' is determined by,

$$\mathbf{f'} = \frac{\mathbf{v'}}{\lambda} = \left(\frac{\mathbf{v} \pm \mathbf{v_0}}{\mathbf{v}}\right)\mathbf{f} \qquad \text{...(4.32)}$$

Where, v = velocity of sound wave

λ' = apparent wavelength

v_0 = velocity of observer

f = original frequency of sound

The positive or negative sign are chosen in accordance with the direction of motion of observer.

Case (i) When observer moves towards the stationary source: In this case, sound wave and observer move in opposite direction as shown in Fig.4.7. Therefore, the relative velocity of sound wave, and observer is,

$$\mathbf{v'} = \mathbf{v} + \mathbf{v_0} \qquad \text{...(4.33)}$$

Now, using appropriate condition in equation (4.32) from equation (4.33) the apparent frequency of sound is,

$$\mathbf{f'} = \left(\frac{\mathbf{v} + \mathbf{v_0}}{\mathbf{v}}\right)\mathbf{f} \qquad \text{...(4.34)}$$

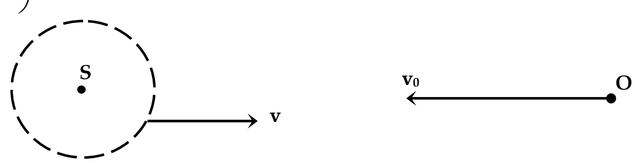


Fig 4.7: Observer is moving towards the stationary source

Clearly, f' > f. It shows that the apparent frequency of sound is greater than the real frequency.

Case (ii) When observer moves away from the stationary source: In this case, the sound wave and observer move in the same direction as shown in Fig.4.8. So, the resultant velocity of sound and observer is,

$$\mathbf{v'} = \mathbf{v} - \mathbf{v_o} \qquad \text{...(4.35)}$$

Now, using appropriate condition in equation (4.32) from equation (4.35), the apparent frequency of sound is,

$$\mathbf{f' = \left(\frac{v - v_o}{v}\right) f} \qquad \ldots\mathbf{(4.36)}$$

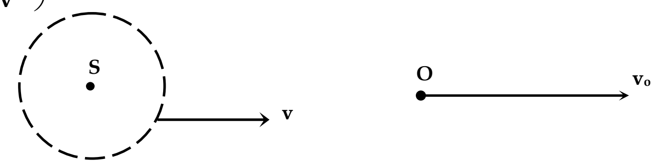


Fig 4.8: Observer is moving away from the stationary source

Clearly, f' < f. It shows that the apparent frequency of sound is smaller than the real frequency.

c. Both source and observer in motion

When both the source and observer are in motion, there is apparent change in frequency due to the relative velocities between sound wave and observer as well as the change in wavelength of the sound due to motion of source of sound.

When observer is moving relative to the sound the velocity of observer (v_o) is added up or subtracted from the velocity of sound wave (v). If the observer is travelling towards the sources, the direction of v and v_o are in opposite direction. So, the relative velocity will be $v + v_o$. If the observer is travelling away from the source, the direction of v and v_o are same. So, the relative velocity is $v - v_o$. In the combined form, the relative velocity is

$$v' = v \pm v_o$$

If the source is moving relative to the sound wave, the wavelength of sound changes in accordance with the direction of motion of source. If the source is moving towards the observer, the waves suffer crowded in between them, hence the wavelength decreases, $\lambda' < \lambda$. If the source is moving away from the observer, the waves suffer rarefied between them, hence the wave length of sound increases,

i.e., $\lambda' > \lambda$.

In combined form,

$$\lambda' = \frac{v \pm v_S}{f}$$

In this case, the apparent frequency, f' of sound wave is

$$\mathbf{f' = \frac{v'}{\lambda'} = \left(\frac{v \pm v_o}{v \pm v_s}\right) f} \qquad \ldots\mathbf{(4.37)}$$

Where, v = velocity of sound wave

λ' = apparent wavelength

v_s = velocity of source

v_o = velocity of observer

f = original frequency of sound

The positive or negative sign are chosen in accordance with the direction of motion of sound source and observer.

Case (i): When source and the observer move to each other.

In this case, sound and source move in the same direction, while sound wave and observer move in the opposite direction as shown in Fig.4.9. In such condition, the directions of v and v_o are opposite, so they add up to find the relative velocity.

So, the resultant velocity of sound and observer is,

$$v' = v + v_o \qquad \ldots(4.38)$$

As the source is travelling toward the observer the waves are crowded. In such conditon , the direction of v and v_s is same, hence, apparent wavelength,

$$\lambda' = \frac{v - v_s}{f} \qquad \ldots(4.39)$$

Using appropriate condition in equation (4.37) from (4.38) and (4.39),

$$\therefore \quad \textbf{Apparent frequency, } f' = \frac{v + v_o}{v - v_s}.f \qquad \ldots(4.40)$$

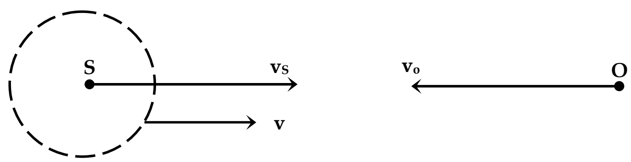


Fig. 4.9: Observer and source approach each other

Clearly, f' > f. It shows that the apparent frequency of sound is greater than the real frequency, when the observer and source approach to each other.

Case (ii): When moving source and moving observer leave each other: In this case, the sound wave and source move in the opposite direction while the sound and observer move in the same direction as shown in Fig.4.10. In such condition, the direction of v and v_o is same, so v_o is subtracted from v to find the relative velocity. So, the resultant velocity of sound and observer,

$$v' = v - v_o \qquad \ldots(4.41)$$

As the source is travelling away from the observer, the waves are rarified. In such condition, v and v_s are in opposite direction, hence apparent wavelength is,

$$\lambda' = \frac{v + v_s}{f} \qquad \ldots(4.42)$$

Using appropriate condition in equation (4.37) from (4.41) and (4.42)

$$\text{Now, the apparent frequency, } f' = \frac{v - v_o}{v + v_s}.\, f$$

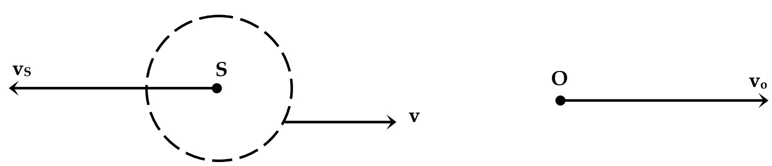


Fig. 4.10: Source and observer moving away from each other

Clearly, f' < f. It shows that the apparent frequency of sound is smaller than the real frequency.

Case (iii) When source is followed by the observer: In this case, the source moves away from the observer and the observer moves towards the source. In such case, sound wave and source move in the opposite direction and, the sound wave and observer also move in the opposite direction as shown in Fig. 4.11. So, the resultant velocity of sound is,

$$v' = v + v_o \qquad \ldots(4.43)$$

and the apparent wavelength,

$$\lambda' = \frac{v + v_s}{f} \qquad \ldots(4.44)$$

Using appropriate condition in equation (4.37) from equations (4.43) and (4.44)

∴ The apparent frequency of sound,

$$\mathbf{f'} = \left(\frac{\mathbf{v + v_o}}{\mathbf{v + v_s}}\right)\mathbf{f} \qquad \ldots(4.45)$$

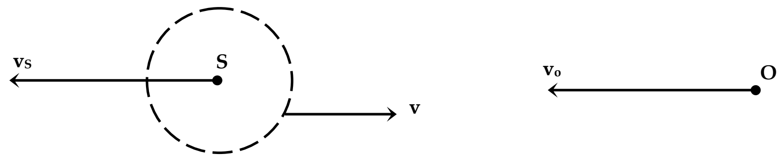


Fig. 4.11: Observer follows the moving source

For $v_o > v_s$, f' > f. It shows that the apparent frequency of sound is greater than the real frequency.

For $v_o < v_s$, f' < f. It shows that the apparent frequency of sound is smaller than the real frequency.

Case (iv): When observer is followed by source: In this case, the observer moves away from the source and the source comes behind it. In such case, the sound wave and source move in same direction and the sound wave and observer also move in same direction as shown in Fig.4.12. So, the resultant velocity of sound and observer is,

$$\mathbf{v' = v - v_o} \qquad \ldots(4.46)$$

and the apparent wave,

$$\lambda = \frac{\mathbf{v - v_s}}{\mathbf{f}} \qquad \ldots(4.47)$$

Using appropriate condition in equation (4.37) from (4.46) and (4.47), the apparent frequency of sound,

$$\mathbf{f'} = \left(\frac{\mathbf{v - v_0}}{\mathbf{v - v_s}}\right)\mathbf{f} \qquad \ldots(4.48)$$

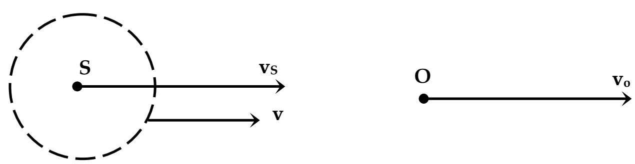


Fig. 4.12: Observer being followed by source

For $v_o < v_s$, f' > f. It shows that the apparent frequency of sound is greater than the real frequency.

For $v_o > v_s$, f' < f. It shows that the apparent frequency of sound is smaller than the real frequency.

Notes

1. *With all this understanding, the sign convention to summarize the Doppler's effect is:*

$$f' = \left(\frac{v \pm v_0}{v \pm v_s}\right)f$$

Sign conventions:

a. *The reference is always the direction of sound i.e., direction of v.*

b. *For stationary observed $v_o = 0$ and for stationary source $v_s = 0$.*

For observer.

i. *If observer moves opposite to direction of sound, the relative velocity of sound must decrease. So, we choose + sign between v and v_o.*

ii. If observer moves towards the direction of sound, the relative velocity of sound must increase. So, we choose – sign between v and v_o.

For Source

i. If source moves opposite to direction of sound, the relative velocity of sound must decrease. So, we choose + sign between v and v_s. Since, v_s affects the denominator term of above equation.

ii. If source moves towards the direction of sound, the relative velocity of sound must increase. So, we choose – sign between v and v_s. Since, v_s affects the denominator term of above equation.

With these conventions, for simplicity one may only inspect the direction of observer.

If the observer moves opposite to the direction of sound, one chooses + sign on the numerator and since the source is moving towards the direction of sound, the denominator would have – sign.

2. *The word 'apparent' means 'seeming to be real or to exist'. So, apparent means not real but it only appears due to certain circumstances. Many terms in Physics are followed by apparent such as 'apparent depth', apparent weight, apparent expansion, apparent frequency etc. Here, the real frequency of source appears different due to relative motion. The changed frequency i.e. new observed frequency is the apparent frequency. In fact actual frequency remains constant.*
3. *One thing to be clear that frequency and intensity of sound are extremely different physical terms, it is obvious that the intensity (i.e. loudness) increases as the source is company towards the observer. In addition, the pitch (i.e. frequency of sound increases when the source moves towards the observer).*
4. *Doppler's effect is a phenomenon common to all waves. This is applicable only when there is relative velocity between the source and the observer.*

Effect of Motion of the Medium

1. Effect of motion of medium on Doppler's effect

If the medium itself is moving in a direction, the speed of sound wave in that medium changes. This effect, ultimately changes the frequency of sound while propagating in this medium. Thus, the apparent frequency is heard different from the original value.

If the medium moves in the direction of propagation of sound wave, the resultant speed of sound wave is $v + v_m$ where v is speed of sound wave in still medium and v_m is the speed of medium. However, if the medium moves in opposite direction, the resultant speed of sound is $v - v_m$. So,

i. If the source, observer and the medium are moving in same direction, the apparent frequency is,

$$f' = \frac{(v + v_m) - v_0}{(v + v_m) - v_s} \times f \text{ (observer followed by source)}$$

ii. If the medium is moving in the opposite direction of source and observer, the apparent frequency is,

$$f' = \frac{(v - v_m) - v_0}{(v - v_m) - v_s} \times f \text{ (observer followed by source)}$$

Other situations can be explained using the above conditions as explained in (i) and (ii).

Comparison of Doppler's Effect in Sound Wave and Light Wave

Doppler's effect can also be observed in light wave. The Doppler's effect on light wave is a firm evidence of expansion of universe. There is a basic difference between the Doppler's effect in sound wave and light wave. Basically, the relative velocity of light wave with respect to its source and observer does not alter its net velocity, which eventually changes the situation.

The change in frequency of sound waves depends upon whether the source is moving with respect to stationary observer or observer is moving with respect to stationary source. Even if the relative velocity in the two cases is the same, the change of frequency is different. So, the Doppler's effect in sound is asymmetric. But, the Doppler's effect in light is symmetric.

Condition of no Doppler's effect

The frequency of sound heard does not change in the following conditions.

1. When both the source and the observer move in the same direction with the same speed.

   i.e. $f' = \frac{v \pm v_o}{v \pm v_s} f$

   For v_o and v_s are equal and directed in the same direction,

   $f' = f$

2. When either the source or the observer is at the center of a circle and the other is moving along it with an uniform speed.
3. When both the source and the observer are at rest and the wind alone is blowing.

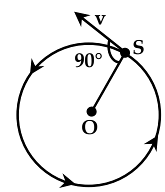


Fig. 4.13 A person moving in a circle around another

Shockwave and Sonic Boom

When the speed of a source exceeds the speed of sound ($v_s > v$), the wavefronts lag behind the source in a cone shaped region with the source at the vertex as shown in Fig. 4.14. The edge of the cone forms a supersonic wavefront with an unusually large amplitude called a shockwave. Shockwaves create a loud sound like a type of explosion which is called sonic boom. Sonic boom generates significant amount of sound energy, sounding much like an explosion to the human ear.

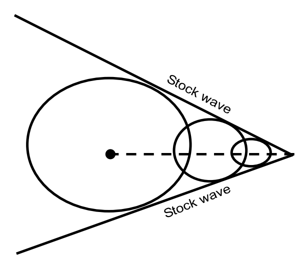


Fig. 4.14: Formation of shock wave ($v_{Source} > v_{sound}$)

When an aircraft passes through the air, it creates series of pressure waves in front of and behind it. If the speed of source (i.e. aircraft) is greater than speed of sound, the waves are forced together, or compressed, because they can not get out of the way of each other. Finally, they merge into a single shockwave.

Applications of Doppler’s Effect

i. It is used to detect a moving submarine under water and to estimate its velocity as well.
ii. It is used to detect the airplane in air and to estimate its velocity as well.
iii. It is used to study the movement of stars in the universe.
iv. It is used in thief alarm.
v. Doppler's effect is used to detect the condition of blood flow in ultrasonography.

Doppler's effect on echo

In our common sense, sound comes to us from the original source, but the situation is different in case of echo. The echo comes from reflector wherever the source is situated. In such condition, the sound source is always considered behind the reflector as shown in Fig 4.15.

i. If the sound source and observer are moving towards the reflector.

Let S be the original source of sound and O be the observer. In this condition, the source of echo is considered at equal distance away from the reflector as the mirror reflection. Let S' be the source of echo. If the original source S travels towards the reflector, S' also comes nearer to the reflector. The magnitude and direction of speed of sound (v), observer (v_o) and source (v_s) are shown in Fig. 4.15.

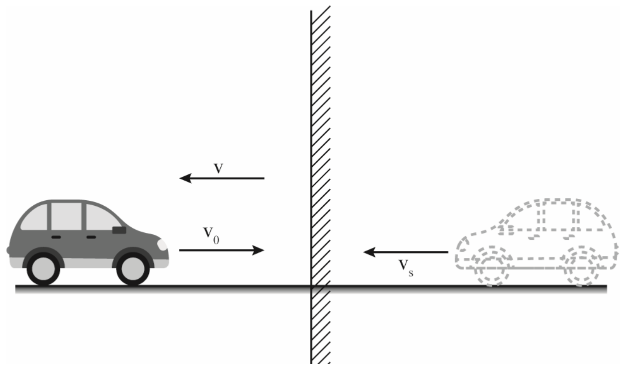


Fig. 4.15: Doppler's by reflection

Closely analyzing the figure, we get the situation that the source and observer are approaching to each other. So, the apparent frequency of sound is,

$$f' = \frac{v + v_0}{v - v_s} \times f$$

Where, f is the original frequency of sound if the original source and observer are moving together, $v_o = v_s$.

ii. If the sound source and observer are moving away from the reflector.

In this condition, the source of echo S' is observed moving away from the reflector. The magnitude and directions of speed of sound (v), source (v_s) and observer (v_o) are shown in Fig.4.16.

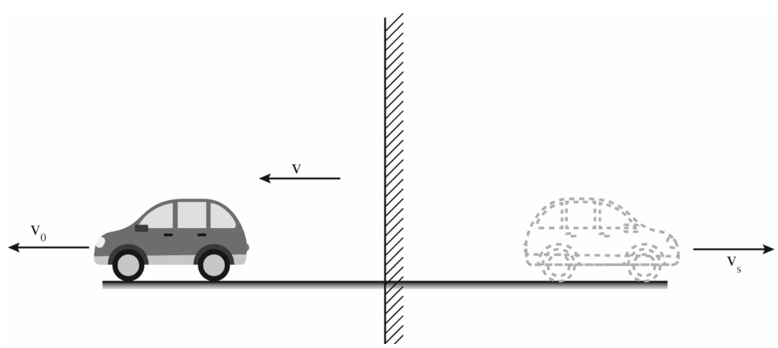


Fig. 4.16: Doppler's by reflection

It is to be noted that the speed of sound is always directed from reflector to observer. In this condition, the apparent frequency of sound is,

$$f' = \frac{v - v_0}{v + v_s} \times f$$

If observer and source are moving together, we write $v_o = v_s$.

iii. If the reflector is moving itself towards the stationary source and observer

Although both source and observer are stationary, Doppler's effect can be observed, if the reflector itself is moving towards or away from them. To tackle such problem, the condition can be simplified considering two situations: (a) stationary source and observer moving towards the source and (b) source moving towards the stationary observer taking the frequency given by (a) as a original frequency.

a. For stationary source and observer moving towards the source: The apparent frequency is,

$$f' = \frac{v + v_0}{v} \times f$$

b. Source moving towards the stationary observer taking the frequency and by (i) as original the apparent frequency is,

$$f'' = \frac{v}{v - v_s} \times f'$$

$$\text{so,} \quad f'' = \frac{v}{v - v_s} \times \frac{v + v_0}{v} \times f$$

$$f'' = \frac{v + v_0}{v - v_s} \times f$$

4.9 Noise, Noise Pollution and its Control

Noise pollution is defined as any level of nuisance caused by sound that is generally harmful to environment and creates disturbance in human life, which has an adverse effect on mental and psychological well being.

Fig. 4.17: Sources of noise pollution

Causes of Noise Pollution

Causes of noise pollution are as follows:

1. Most of the industries use big machines which are capable of producing large amount of noise. Industrialization is a major factor causing noise pollution.

2. Noise is at its peak during the social events. Pubs, parties, etc. are the places where people produce and experience such nuisance.
3. Large number of vehicles on roads, aeroplanes, trains, etc. creates noise. Basically, transportation is another major factor contributing noise pollution.
4. Different construction activities like mining, construction of bridge, dams, buildings, flyovers, etc. also contribute to noise pollution.

Remedial Measures for Noise Pollution

In order to reduce noise pollution, we need to take a few things into consideration which are explained below:

1. **Computers, televisions:** such type's electronics produce sound that may cause stress on the ears over a period of time. So, we can turn them off when we are not using them.
2. **Control at receiver's end:** meaning building the sound proof space in order to cancel the noise created in the surrounding using furniture and noise cancellation devices is a good option.
3. **Suppression of noise at source:** Sources like huge machines, noise creating devices should be monitored on daily basis. Moreover, lubricating, repairing helps reduce noise at its source.
4. **Planting of trees:** Plants and trees somehow help to absorb the noise created in surrounding environment.
5. **Legislative measures:** Effective legislation and law should be implemented in order to reduce noise level at urban area.

Tips for MCQs

1. **Intensity**
   - i. $I = \frac{\text{Power transfer}}{\text{surface area}} = \frac{P}{A} = 2a^2f^2a^2\rho v = \frac{P^2_{max}}{2\rho v}$
   - ii. $\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$ (inverse square law)
   - iii. Threshold of hearing, $L_0 = \log I_0$, $I_0 = 10^{-12}\,Wm^{-2}$
   - iv. Intensity level, $\beta = 10 \log \frac{I}{I_0}$ (decibel)
   - v. Comparison, $\Delta\beta = \beta_1 - \beta_2 = 20 \log \left(\frac{r_2}{r_1}\right)$.
   - vi. Energy density, $U = \frac{\text{Energy transfer}}{\text{volume}}$
   - viii. Intensity, point source (spherical wave) $I = \frac{P}{4\pi r^2}$, $a \propto \frac{1}{\sqrt{r}}$
2. **Types of sound:**

Types	Frequency Range	Property	Source
Infrasonics	f < 20 Hz	Inaudible	Seismic waves, waves of pendulum oscillation.
Audible	20 Hz ≤ f ≤ 20 kHz	Audible	All sound waves we hear in our daily life.

Ultrasonics	f > 20 kHz	Inaudible	Piezolectric effect in quartz crystal.
Supersonics	velocity greater than sound wave	not actually the vibrational wave	Objects move faster than sound.

3. Beats

i. The beat frequency, $f_b = f_1 - f_2$ (for $f_1 > f_2$) and $f_b = f_2 - f_1$ (for $f_2 > f_1$)

ii. Beat of sound may be possible, if more than two sound waves interfere.

4. Doppler's effect

i. The general formula for the apparent frequency heard by the observes, $f' = \frac{v \pm v_0}{v \pm v_s} f$.

ii. The speed of sound is always directed from source to observer.

iii. All the velocities along the direction of source to observer and taken as negative and all the speeds along the direction of observer to listener are taken as positive.

iv. When a source goes past a stationary observer, number of beats heard per second is given by

$$\Delta f_b = \frac{v}{v - v_s} f - \frac{v}{v + v_s} f = \frac{2vv_s}{v^2 - v_s^2} f$$

v. When a source goes past a stationary source, number of beats heard per second is given by

$$\Delta f_b = \frac{2v_0}{v} f$$

vi. Apparent frequency heard by driver moving towards a hill (i.e. reflector)

$$f' = \frac{v + v_0}{v - v_s} \times f$$

vii. Apparent frequency of echo of horn of his car heard by driver moving towards the hill (i.e. reflector)

$$f' = \frac{v + v_0}{v - v_s} \times f$$

viii. Doppler's effect in light

Apparent frequency of light received by an observer, is

$$f' = \left(1 \pm \frac{v}{c}\right) f$$

Positive sign is chosen when the source and the observer are approaching each other and negative sign is chosen when the source and the observer are receding away.

Worked Out Problems

1. The intensity level from a loud speaker is 100 dB at a distance of 10 m. What is the intensity level at distance of 200 m?

SOLUTION:

Given,

Intensity level at first point, $\beta_1 = 100$ dB

Distance from speaker, $r_1 = 10$ m

Intensity level at next point, $\beta_2 = ?$

Distance of next point from speaker (r_2) = 200 m

The difference of intensity level, $\Delta\beta = \beta_1 - \beta_2$

We know, $\Delta\beta = 20 \log\left(\frac{r_2}{r_1}\right) = 20 \log\left(\frac{200}{10}\right) = 26$ dB.

Now, to find β_2,

$\beta_1 - \beta_2 = 26$

$\beta_2 = \beta_1 - 26$

$= 100 - 26 = 74$ dB

2. The ratio of the intensities of two interfering waves is 81:1. What is the ratio of the maximum to minimum intensity?

SOLUTION

Given,

$\frac{I_1}{I_2} = \frac{81}{1}$

We know that

$I \propto a^2$

or $\frac{I_1}{I_2} = \left(\frac{a_1}{a_2}\right)^2$

or $\frac{a_1}{a_2} = \sqrt{\frac{I_1}{I_2}} = \sqrt{\frac{81}{1}} = \frac{9}{1}$

$\therefore$ $a_1 = 9 a_2$

Now,

$I_{max} = (a_1 + a_2)^2$

and

$I_{min} = (a_1 - a_2)^2$

So,

$\frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2}$

$= \frac{(9a_2 + a_2)^2}{(9a_2 - a_2)^2} = \frac{100a_2^2}{64a_2^2} = \frac{25}{16} = 1.56$

3. If two tuning forks vibrate with frequencies 440 Hz and 444 Hz respectively, then find the frequency of resultant wave and beat frequency.

SOLUTION

Given,

$f_1 = 440$ Hz

$f_2 = 444$ Hz

frequency of resultant wave, f = ?

beat frequency, f' = ?

As we know that the frequency of resultant wave is the mean frequency of the component wave frequencies. So, we can write

$f = \frac{f_1 + f_2}{2} = \frac{440 + 444}{2} = 442$ Hz

Also, from beat frequency formula,

$f' = f_2 - f_1 = 444 - 440 = 4$ Hz

4. What is the velocity of sound in a gas in which the two waves of wavelength 1.0 m and 1.01 m produce 4 beats per second?

SOLUTION

Given,

$\lambda_1 = 1.0$ m, $\lambda_2 = 1.01$ m

Beat frequency (f) = 4 beats/s

From beat frequency formula, we know that

$f = f_1 - f_2$

or $4 = \frac{v}{\lambda_1} - \frac{v}{\lambda_2}$

or $4 = v\left(\frac{1}{1.0} - \frac{1}{1.01}\right)$

$\therefore$ $v = 404$ ms^{-1}

5. Two tuning forks A and B produce 4 beats per second. The frequency of A is 10^3 Hz. When end of prong of B is loaded with wax, 5 beats per second are heard. Find the frequency of B before and after loading

SOLUTION

Given,

Frequency of fork A (f_A) = 10^3 Hz

Beat per second (f) = 4 Hz

If f_B is the frequency of fork B before loading, then we have

$f_B = f_A \pm f = 1000 \pm 4 = 996$ Hz or 1004 Hz

when the fork B is loaded, the beat frequency is increased from 4 to 5 so the frequency of B will be $(f_A - f) = 996$ Hz

After loading,
f = 5 Hz
so, $f_B = f_A - f = 1000 - 5 = 995$ Hz .

6. When two open organ pipes are sounded together at 20°C, they produce 42 beats in 5 seconds. If the temperature is raised to 70°C, how many beats would they produce in the same time?

SOLUTION

Given,
$T_1 = 20°C = 292$ K

∵ Number of beats in 5 s = 42

∴ Number of beats in 1 s = $\frac{42}{5}$

i.e., beat frequency (f) = $\frac{42}{5}$ beats/s

Let l_1 and l_2 be the lengths of the pipes. Then we can write

Let n be the number of beats in 5 seconds at 70°C.

Then beat frequency = $\frac{n}{5}$

Now,

$$f' = \frac{v_{70}}{2l_1} \text{ and } f'' = \frac{v_{70}}{2l_2}$$

From beat frequency formula, we can write

$$f = f' - f''$$

$$\text{or } \frac{n}{5} = \frac{v_{70}}{2l_1} - \frac{v_{70}}{2l_2}$$

$$\text{or } \frac{n}{5} = \frac{v_{70}}{2}\left(\frac{1}{l_1} - \frac{1}{l_2}\right) \quad \text{. . . (i)}$$

$$f_1 = \frac{v_{20}}{2l_1} \text{ and}$$

$$f_2 = \frac{v_{20}}{2l_2}$$

From beat frequency formula, we have

$$f = f_1 - f_2$$

$$\text{or } \frac{42}{5} = \frac{v_{20}}{2l_1} - \frac{v_{20}}{2l_2} = \frac{v_{20}}{2}\left(\frac{1}{l_1} - \frac{1}{l_2}\right) \quad \text{. . . (i)}$$

$$f = f_1 - f_2$$

$$\text{or, } \frac{42}{5} = \frac{v_{20}}{2l_1} - \frac{v_{20}}{2l_2} = \frac{v_{20}}{2}\left(\frac{l}{l_1} - \frac{l}{l_2}\right) \quad \text{...(ii)}$$

Dividing (ii) by (i), we get

$$\frac{n}{42} = \frac{v_{70}}{v_{20}}$$

$$\text{or } \frac{n}{42} = \sqrt{\frac{70 + 273}{20 + 273}} \quad (\because v \propto \sqrt{T})$$

$$\text{or } n = \sqrt{\frac{340}{293}} \times 42$$

$$\therefore n = 65 \text{ beats}$$

7. [HSEB 2059] A note produces 2 beat/s with a tuning fork of frequency 480 Hz and 6 beats/s with a tuning fork of 472 Hz. Find the frequency of the note.

SOLUTION

Given,

Note produced by a tuning fork of frequency 480 Hz = 2 beat/ s

Note produced by a tuning fork of frequency 472 Hz = 6 beat/ s

Since, the note produces 2 beats with the tuning fork of frequency 480 Hz, the frequency of the note = 480 ± 2 = 482 or 478 Hz.

Also, the note produces 6 beat/s with the tuning fork of 472 Hz, the frequency of the note = 472 ± 6 = 478 or 466 Hz.

As the frequency of 478 is common in both cases, the frequency of the note is 478 Hz.

8. [NEB 2074] A car is approaching towards a cliff at a speed of 20 ms^{-1}. The driver sounds a whistle of frequency 800 Hz. What will be the frequency of the echo as heard by the car driver? Velocity of sound in air = 350 ms^{-1}.

SOLUTION

Original frequency (f) = 800 Hz

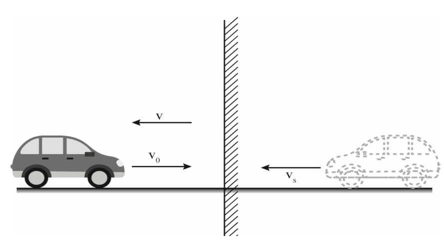
Speed of car (v_c) = 20 ms^{-1}

As the source and observer are at same car,

$v_o = v_s = 20$ms^{-1}

Speed of sound = 350 ms^{-1}

This is the case in which the observer receives echo, so we consider the source behind the cliff.



The apparent frequency (f') $= \dfrac{v + v_o}{v - v_s} \times f$

$= \dfrac{350 + 20}{350 - 20} \times 800 = 896.97$ Hz

9. A source of sound generates sound waves which travel with a speed of 340 ms^{-1}. The frequency of the source is 500 Hz. Find the frequency of the sound heard if:

i. The source is moving towards the stationary observer with a speed of 30 ms^{-1}.

ii. The observer is moving towards the stationary source with a speed of 30 ms^{-1}.

iii. Both source and observer move with a speed of 30 ms^{-1} and approach one another.

SOLUTION

Given

Speed of sound (v) = 340 ms^{-1}

Frequency of source (f) = 500 Hz

i. Speed of source (v_s) = 30 ms^{-1}

Since the source is moving towards stationary observer, the apparent frequency is,

$f' = \dfrac{v}{v - v_s} \times f$

$= \dfrac{340}{340 - 30} \times 500$

$= 548.4$ Hz

ii. Speed of observer (v_o) = 30 ms^{-1}.

Since the observer is moving towards the stationary source, the apparent frequency is,

$f' = \dfrac{v + v_o}{v} \times f$

$= \dfrac{340 + 30}{340} \times 500$

$= 544.1$ Hz

iii. For both source and observer are moving with equal speed the and approach to each other,

So, apparent frequency (f') $= \dfrac{v + v_o}{v - v_s} \times f$

$= \dfrac{340 + 30}{340 - 30} \times 500$

$= 596.8$ Hz

10. [HSEB 2067] An observer traveling with constant velocity of 20 m/s, passes close to a stationary source of sound and notices that there is a change of frequency of 50 Hz as he passes the source. What is the frequency of the source? Speed of the sound in air = 340 m/s.

SOLUTION

Given,

Velocity of observer (v_o) = 20 m/s

Velocity of source of sound (v_s) = 0

Change of frequency = 50 Hz

Speed of sound in air (v) = 340 m/s

Frequency of the source (f) = ?

When observed approaches to stationary source,

$f_1' = \dfrac{v + v_o}{v} \times f$

$= \dfrac{340 + 20}{340} \times f = \dfrac{360}{340} \times f$

or, $f_1' = \dfrac{18f}{17}$

Again,

When observer passes the source; we have

$f_2' = \dfrac{v - v_o}{v} \times f = \dfrac{340 - 20}{340} \times f = \dfrac{320}{340} \times f$

or, $f_2' = \dfrac{16f}{17}$

Then, change of frequency.

$f_1' - f_2' = \dfrac{18f}{17} - \dfrac{16f}{17}$

or, $50 = \dfrac{18f - 16f}{17}$

or, $\dfrac{2f}{17} = 50$

$\therefore$ f = 425 Hz

Hence, the required frequency is 425Hz

11. [HSEB 2073] A car travelling with a speed of 60 kmhr^{-1} sounds a horn of frequency 500 Hz. The sound is heard in another car travelling behind the first car in the same direction with a speed of 80 kmhr^{-1}. What frequencies will the driver of the second car hear before and after overtaking the first car if the velocity of sound is 340 ms^{-1}?

SOLUTION

Given,

Here, the first car is the source of sound and the driver of second car is the observer.

Speed of source (v_s) = 60 km hr^{-1} = 16.67 ms^{-1} Speed of observer (v_o) = 80 km hr^{-1} = 22.22 ms^{-1}

Original frequency (f) = 500 Hz Speed of sound (v) = 340 ms^{-1}

(i) In first case, the observer is approaching towards the moving source,

$$\therefore \quad f' = \frac{v + v_0}{v + v_s} \times f = \frac{340 + 22.22}{340 + 16.67} \times 500 = 507.8 \text{ Hz}$$

(ii) In second case, the observer is moving away from the moving source,

$$f'' = \frac{v - v_0}{v - v_s} f = \frac{340 - 22.22}{340 - 16.67} \times 500 = 491.4 \text{ Hz}$$

12. [HSEB 2072] A stationary motion detector sends sound waves of 150 kHz towards a truck approaching at a speed of 120 km/hr. What is the frequency of wave reflected back to detector? (Velocity of sound in air = 340 m/s)

SOLUTION

Given,

Frequency of sound (f) = 150 kHz = 150000 Hz

Velocity of observer (v_o) = 120 km/hr $= \frac{120 \times 1000}{3600}$ m/s = 33.33 m/s

Velocity of sound (v) = 340 m/s

Apparent frequency (f') = ?

Here, the reflector is approaching towards the detector. So, the condition is similar to 'source and observer are approaching to each other.

We have,

$$f' = \frac{v + v_0}{v - v_s} f = \frac{340 + 33.33}{340 - 33.33} \times 150000 = 182605.1 \text{ Hz} = 182.6 \text{ kHz}$$

Challenging Problems

1. **[UP]** Two sinusoidal sound waves with frequencies 108 Hz and 112 Hz arrive at your ear simultaneously. Each wave has the amplitude of 1.5×10^{-8} m. (a) How many beats are heard per sec? (b) Determine the maximum and minimum amplitude of total sound wave arriving at the ear.

   Ans: (a) 4 Hz (b) 3×10^{-8} m; 0

2. **[UP]** A railroad train is travelling at 30 m/s in still air. The frequency of the note emitted by the train whistle is 262 Hz. What frequency is heard by a passenger on a train moving in the opposite direction to the first at 18 m/s and (a) approaching the first? (b) receding from the first? (v = 344 m/s)

   Ans: a. 302 Hz; b. 228 Hz

3. **[UP]** For a person with normal hearing, the faintest sound that can be heard at a frequency of 400 Hz has a pressure amplitude of about 6.0×10^{-5} Pa. Calculate the corresponding intensity and sound intensity level at 20°C. (Take at 20°C, speed of sound in air = 344 m/s and density of air = 1.2 kg/m^3)

   Ans: 4.36×10^{-12} W/m^2; 6.4 dB

4. **[UP]** A baby's mouth is 30 cm from her father's ear and 1.50 m from her mother's ear. What is the difference between the sound intensity levels heard by the father and by the mother?

   Ans: 13.96 dB

5. **[UP]** A sound wave in air at 20°C has a frequency of 150 Hz and a displacement amplitude of 5.00×10^{-3} mm. For this sound wave calculate the (a) pressure amplitude (in Pa); (b) intensity (in W/m^2); (c) sound intensity level (in decibels).

   [Bulk modulus of air (B) = 1.42×10^5 Pa; Velocity of sound in air (v) = 344 ms^{-1},
   Density of air at 20°C = 1.2 kgm^{-3}]

Ans: (a) 1.95 Pa (b) 4.58 × 10^{-3} W/m^2 (c) 96.6 dB

6. **[UP]** On the planet Arrakis a male ornithoid is flying toward his mate at 25.0 m/s while singing at a frequency of 1200 Hz. If the stationary female hears a tone of 1240 Hz, what is the speed of sound in the atmosphere of Arrakis? **Ans: 775 m/s**

7. **[UP]**
   a. A sound source producing 1.00 k Hz waves moves toward a stationary listener at 1/2 the speed of sound. What frequency will the listener hear?
   b. Suppose instead that the source is stationary and the listener moves toward the source at 1/2 the speed of sound. What frequency does the listener hear? (speed of sound = 340 m/s)

   Ans: (a) 2,000 Hz (b) 1500 Hz

8. **[ALP]** A column of air is set into vibration and the note emitted gives 10 beats per second when a tuning fork of frequency 440 Hz is sounded, the temperature being 20^{0}C. The frequency of the beats decreases when the tuning fork is loaded with a small piece of plasticine. At what temperature will the unloaded fork and the air column be in unison? (Assume that the wavelength of the note emitted by the air column remains constant and that the frequency of the fork is independent of temperature.)

   Ans: 33.79°C

9. **[ALP]** At a point 20 m from a small source of sound the intensity is 0.5 microwatt cm^{-2}. Find a value for the rate of emission of sound energy from the source.

   Ans: 25 W

10. **[ALP]** An observer travels with a constant velocity of 30 ms^{-1} towards a distant source of sound which has a frequency of 1000 Hz. Calculate the apparent frequency of the sound heard by the observer. What frequency is heard after passing the source of sound? (Assume velocity of sound = 330 ms^{-1})

    Ans: 1090 Hz; 909 Hz

11. **[ALP]** An observer travelling with a constant velocity of 20 ms^{-1}, passes close to a stationary source of sound and notices that there is a change of frequency of 50 Hz as he passes the source. What is the frequency of the source? (Speed of sound in air = 340 ms^{-1})

    Ans: 425 Hz

12. **[ALP]** A whistle of frequency 1000 Hz is sounded on a car travelling towards a cliff with a velocity of 18 ms^{-1}, normal to the cliff. Find the apparent frequency of the echo as heard by the car driver.

    Ans: 1115.4 Hz

13. **[ALP]** A car travelling at 10 ms^{-1} sounds its horn, which has a frequency of 500 Hz, and this is heard in another car which is travelling behind the first car, in the same direction, with a velocity of 20 ms^{-1}. The sound can also be heard in the second car by reflection from a bridge head. What frequencies will the driver of the second car hear? (Speed of sound in air = 340 ms^{-1})

    Ans: 545.5 Hz

14. **[ALP]** The locomotive of a train approaching a tunnel in a cliff face at 95 kmhr^{-1} is sounding a whistle of frequency 1000 Hz. What will be the apparent frequency of the echo from the cliff face heard by the driver? What would be the apparent frequency of the echo if the train were emerging from the tunnel at the same speed? (Take the velocity of sound in air as 330 ms^{-1}).

    Ans: 852 Hz

*[**Note:** Hints to challenging problem are given at the end of this chapter.]*

Conceptual Questions with Answers

1. What is the threshold of hearing? Define one bel. [NEB 2074]

   The minimum loudness of sound that can just be heard by normal ear is known as threshold of hearing. It is denoted by L_0.

➢ The corresponding value of intensity for the threshold condition is 10^{-12} Wm^{-2}.

One bel (1 bel) is defined as the sound intensity level at which the intensity of sound is ten times greater than the standard intensity. i.e. $I = 10\ I_o$

2. Whistle of an approaching train is shriller, Why?

✎ The frequency of sound is apparently changed when source and observer are in relative motion. The apparent frequency of sound when the source (train)is approaching nearer to the listener, is

$$f' = \frac{v}{v - v_s} \times f$$

Where, v = speed of sound

v_s = speed of source

f = true frequency of sound

Obviously, $v > v - v_s$, So, $f' > f$.

This shows that apparent frequency of sound is greater than true frequency of sound. Hence, it appears shriller.

3. If the pressure amplitude of a sound wave is halved, by what factor does the intensity of the wave change? [HSEB 2072]

✎ The intensity of sound in terms of pressure amplitude (P_0) is

$I = \frac{P_0^2}{2\rho v}$, where, ρ = density of medium

v = wave velocity

If the pressure amplitude is halved

$$I' = \frac{\left(\frac{P_0}{2}\right)^2}{2\rho v} = \frac{1}{4}\left(\frac{P_0^2}{2\rho v}\right)$$

i.e. $I' = \frac{1}{4} I$. This shows that intensity is reduced 4 times.

4. A tuning fork has two prongs. Why? [HSEB 2072]

✎ A tuning fork is a acoustic resonator. When the prongs are struck on the rubber pad holding on the stem, these prongs move alternately towards and away from each other. They oscillate in transverse pattern. Transverse vibrations superimpose at upper part of the stem and the vibration longitudinally propagates to the lower end of stem. Hence, the wave does not damp readily although we hold on it. If it does not contain two prongs, the vibration does not sustain more.

5. 'An empty vessel makes much noise.' How would you justify the proverb? [HSEB 2070]

✎ The intensity of sound wave is directly proportional to the square of amplitude of vibrating particles. An empty vessel contains air molecules. These molecules oscillate with greater amplitude than the liquid molecules. Due to the greater amplitude of vibration in air filled vessel, it makes much noise.

6. Bats catch their prey in the dark even when they don't see the prey. How can this happen?

✎ Bat produces the ultrasound to recognize the things around it. The transmitted ultrasound propagates around the bat and reflects from the obstacles. It has also the echo receiver senser, which estimates the shape, size and distance of obstacle around it. Thus, the bat recognizes the prey around it and catches.

7. Is there a physical difference between intensity and intensity level of a wave? How are these quantities related? [HSEB 2069]

✎ Certainly, they have the difference in physical meaning. Intensity is defined as the energy transmitted per unit area per unit time, however the intensity level is the comparison of loudness with respect to threshold of hearing. Intensity is dimensional physical quantity, but the intensity level is dimensionless physical quantity. They are related with a formula $\alpha = 10 \log \frac{I}{I_0}$, where α is intensity level and I is the intensity.

---

8. Two notes, one produced by violin and the other by a sitar, may have the same frequency, as we can distinguish between them, why? [HSEB 2068]

✎ Although violin and sitar produce the note of same frequency, they can be distinguished by their sound quality. The quality of sound is different due to the difference of harmonics of two sounds. Quality is the individual characteristics of sound source.

---

9. Which has a more direct influence on the loudness of a sound wave: the displacement amplitude or the pressure amplitude? Explain your reasoning.

✎ The loudness of sound depends on the pressure amplitude in a medium. In the location, where the pressure amplitude is high, the molecules of the medium compress and the sound intensity increases.

---

10. If a child continuously blows a whistle while on a marry go round, Explain what you hear as the child comes by each time?

✎ A child in a marry – go – round revolves in a circular path. A person sitting at a point can see the child coming nearer to him in half circle and moving away in next half circle. When the child comes towards the observer (sitting person), the frequency of sound, $f' = \frac{v}{v - v_s} f$

i.e. $f' > f$. i.e. apparent frequency is larger than true frequency.

But in next circle, $f' = \frac{v}{v + v_s} \times f$

i.e. $f' < f$ i.e. apparent frequency is smaller than true frequency.

It shows that the sound of whistle is gradually shriller in one half circle and blurrer in next half circle of revolution.

---

11. Can a person standing at the center of a circle hear apparent change in frequency of sound produced by a whistle moving in the circle?

✎ In the given situation, the source of sound is revolving in a circle and the listener stays at the center of circle. The component of source speed along the center is $v_c = v_s \cos 90°$ and speed of listener $v_o = 0$.

So, apparent frequency of sound,

$$f' = \frac{v \pm v_0}{v \pm v_s} \times f = \frac{v \pm 0}{v \pm 0} \times f$$

$$\therefore \quad f' = f$$

This shows that there is no change in frequency that the listener hears at the center of the circle.

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12. Why the bells of colleges and temples are of large size?

✎ Larger the area of the source of sound, more is the energy transmitted into the medium. Consequently, the intensity of sound is large and loud sound is heard.

---

13. The ratio of the amplitudes of two waves is 3:4. What is the ratio of the intensities of two waves.

✎ The intensity of sound is directly proportional to the square of amplitude, i.e. $I \propto a^2$.

For the given condition.

$$\frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} = \frac{3^2}{4^2}$$

$$\therefore \quad \frac{I_1}{I_2} = \frac{9}{16}$$

Therefore, the ratio of intensities of the waves is 9:16.

---

14. Define persistence of hearing.

➪ It is the duration of the sound for which a syllable's impression remains in the ear. The time of persistence of hearing is 0.1 s.

---

15. What is supersonic?

➪ The term supersonic is used to define a speed of any object that is greater than the speed of sound. Actually, supersonic is not a type of sound, it is related to the body that has greater speed than sound. Jet plane can fly faster than sound, so it is supersonic.

---

16. What is ultrasonic?

➪ A type of sound wave which has the frequency greater than 20 kHz (i.e. $f > 20$ kHz) is called ultrasonics. The speed of ultrasonic is same as the speed of audible sound. Ultrasonics has important applications in medical field, it is used to take the image of internal parts of our body.

---

17. What are the uses of ultrasonics?

➪ There are many uses of ultrasonics. Some of them are written below:

i. In medicine, ultrasonic devices are used to examine internal organs without surgery. It is free from radiation.

ii. Ultrasonics technology is used extensively for the testing, cleaning, and soldering of electronic devices.

iii. Ultrasonic whistles, which can not be heard by human beings, are audible to dogs.

iv. Ultrasonics are used to measure the depth of sea and ocean.

---

18. What is the difference between the sound of mosquito and roaring of lion?

➪ The pitch of sound of mosquito is greater than the roaring of lion. Pitch is frequency related quantity, so the frequency of sound of mosquito is greater than the roaring of lion. However, the energy transfer in roaring of lion is much greater than the sound of mosquito.

---

19. Differentiate between music and noise.

➪ Some important differences between music and noise are as follows:

Music	Noise
1. Musical sound is continuous, regular, and long vibration.	1. Noise is discontinuous, irregular, and very short.
2. It is pleasing sound.	2. It is irritating sound.
3. For example: songs, music, etc.	3. For example: horn of engine, sound of crowded market, etc.

---

20. What is intensity level? What is its unit?

➪ The difference of loudness of sound and the threshold value of sound is called intensity level. It is denoted by β. Let L and L_0 be the loudness of a sound and threshold of hearing respectively. Then, the intensity level, β is defined as,

$$\beta = L - L_0$$

$$= \log I - \log I_0$$

$$\beta = \log \frac{I}{I_0}$$

The unit of intensity level is bel or decibel (dB).

1 bel = 10 dB

---

21. What is beat and beat frequency?

➪ When two sound waves of nearly equal frequencies propagated simultaneously in a medium, then the intensity of resultant sound produced by their superposition increases and decreases alternately with time. The phenomenon of rise and fall in intensity of sound is called beat. The number of times the intensity of sound rises and falls in one second is called beat frequency (f_b). Beat frequency can be detected only when the frequency difference between superposed waves must be smaller than 10 Hz.

22. What are the differences between interference and beats?

↳ Some important differences between interference and beats are as follows:

Interference	Beats
1. Two interfering waves have exactly same frequency.	1. Two interfering waves have slightly different frequencies.
2. The phase difference between two waves at a point remains constant.	2. The phase difference between two waves at a point varies with time.
3. The intensity of sound at every point remains constant.	3. The intensity of sound at every point changes with time and becomes maximum and minimum alternately.

23. How is it that one can recognize a friend from his voice without seeing him?

↳ Even though the pitch and loudness of sound appears same, the quality of sound is different in the voice of human. The quality of sound is subjective quantity and depends on the overtones of sound. The quality of voice of every person is different. As we are familiar with the quality of voice of our friend, we can recognize without seeing him.

24. How does the frequency of echo change when (T) observer and source are moving towards the reflector (ii) both are moving away from reflector?

i. If the source and observer and moving towards the reflector, the apparent frequency heard by observer is,

$$f' = \frac{v + v_0}{v - v_s} \times f$$

So, f' > f. The observer hears greater pitch than original.

ii. If the source and observe is moving away from the reflector, the apparent frequency heard by observer is,

$$f' = \frac{v - v_0}{v + v_s} \times f$$

so, f' < f. The observer hears the lower pitch than the original.

25. The intensity level of sound is 50 dB. What is the intensity of it?

Given,

Intensity level (β) = 50 dB

We know, threshold intensity (I_0) = 10^{-12} Wm^{-2}

Required intensity (I) = ?

We know,

$$\beta = 10 \log \frac{I}{I_0}$$

$$50 = 10 \log \frac{I}{I_0}$$

$$\log \frac{I}{I_0} = 5$$

Taking antilog,

$$\frac{I}{I_0} = 10^5$$

$$I = 10^5 I_0$$

$$= 10^5 \times 10^{-12}$$

$\therefore$ $I = 10^{-7}$ Wm^{-2}

$\therefore$ Required intensity is 10^{-7} Wm^{-2}.

Exercises

Short-Answer Type Questions

1. Differentiate between pressure amplitude and displacement amplitude.
2. What is the difference between the sound of a male and female?
3. Differentiate between musical sound and noise.
4. Why is it harmful to stay in crowded traffic region?

5. Why is not beat heard above 10 Hz?
6. What happens in the frequency of sound produced by a tuning fork when (i) it is loaded (ii) it is field?
7. What is the use of beat in laboratory works?
8. Write the relation of pressure amplitude and intensity of a sound.
9. You recognize your friend from his voice too. Which characteristic of sound is relevant?
10. Doppler's effect is not occurred in two uniformly moving source and observer, why?
11. Explain how a musical instrument such as a piano may be tuned using the phenomenon of beats.
12. An airplane mechanic notices that the sound from a twin-engine aircraft rapidly varies in loudness when both engines are running. What could be causing this variation from loud to soft?
13. Name the factors on which Doppler's effect depends.
14. Is it possible that the apparent frequency of the sound heard by a moving listener is the same as the true frequency? If so give an example.
15. Can we apply Doppler's effect to a source of sound moving faster than the velocity of sound?
16. List out typical sources of noise pollution.
17. What are the impacts of noise?
18. What are the methods to control noise pollution?
19. What are the noise exposure limits in a workspace environment?
20. What are the ambient noise limits?
21. The tone quality of an acoustic guitar is different when the strings are plucked near the bridge (the lower end of the strings) than when they are plucked near the sound hole (close the center of the strings). Why?

Long-Answer Type Questions

1. What is pressure amplitude? Derive pressure equation for a longitudinal wave with necessary figure.
2. What are the characteristics of musical sound? Describe them.
3. What do you mean by intensity and intensity level of sound? Define bel and decibel. [HSEB 2058]
4. Define the intensity of sound and prove that $I = \frac{1}{2}\rho v a^2\omega^2$ where the symbols have their usual meaning. [HSEB 2071]
5. Define threshold of hearing. Distinguish between intensity and intensity level.
6. What are beats? Prove that the number of beats per second is equal to the difference between the frequencies of two superposing waves. [HSEB 2060]
7. What beats? Obtain the expression for the beat frequency when beats are produced by superposing two waves of slightly different frequencies. [HSEB 2067]
8. Discuss the analytical treatment for the formation of beat.
9. Define intensity of sound. Show that the intensity of sound for a given frequency is directly proportional to the square of amplitude of vibration.
10. What is Doppler’s effect? Derive an expression for the apparent frequency received by a stationary observer when a source is moving away from him. [HSEB 2057]

11. What is Doppler's effect? Derive the change in frequency when an observer moves towards a stationary source.
12. Discuss the phenomenon of Doppler's effect. Find the change in frequency when a moving source of sound passes a stationery observer. [HSEB 2062]
13. What is Doppler's effect? Obtain on expression for the apparent pitch when a source moves away from stationary observer. [NEB 2075]
14. What is Doppler's effect? Obtain an expression for the apparent frequency heard by a listener due to a source when both are approaching each other. [NEB 2075]

Numerical Problems

1. Calculate the decibel increase if there is a two fold increase in the intensity of a wave?

   Ans: 3.01 dB

2. The noise level of classroom in absence of the teacher is 50 dB when 50 students are present. Assuming that on the average each student outputs same sound energy per second, what will be the noise level if the number of students is increased to 100?

   Ans: 53.01 dB

3. Two sitar strings A and B playing the note 'Dha' are slightly out of tune and produce beats of frequency 5 Hz. The tension of the string B is slightly increased and the beat frequency is found to decrease to 3 Hz. What is the original frequency of B if the frequency of A is 427 Hz?

   Ans: 422 Hz

4. Two tuning forks A and B are sounded simultaneously in air. The original frequency of A is 512 Hz and B in unknown. In sounding, they produce beat frequency 5 Hz. Now, tuning fork B is loaded with wax and resounded both. Then the beat frequency is observed 2 Hz. What is the original frequency of tuning forks?

   Ans: 517 Hz

5. When a jet plane is flying on elevation of 1000 m the sound level on the ground is 4.0 dB. What would be the intensity level on the ground when its elevation is as low as 100 m. [HSEB 2069]

   Ans: 24 dB

6. Two observes A and B are provided with source of sound of frequency 500 Hz. A remains stationary and B moves away from him at a velocity of 1.8 ms^{-1}. How many beats per second are observed by B, The velocity of sound in air being 330 ms^{-1}? [HSEB 2054]

   Ans: 27 beats/s]

7. What is the intensity level in a car when the sound intensity is 0.500 $\mu W/m^2$ [$I_o = 10^{-12} W/m^2$]

   Ans: 57 dB

8. Two tuning forks A and B give 6 beats/second. A resounds with closed column of air 15 cm long and B with an open column 30.5 cm long. Calculate their frequencies.

   Ans: 366 Hz and 360 Hz.

9. The prongs of a tuning fork A, originally in unison with a tuning fork a B, are filed. Now the two tuning forks on being sounded together produce 2 beats/s. What is the frequency of A after filling, if the frequency of B is 250 cycles/s.

   Ans: 252 cycle/sec

10. Two tuning forks A and B when sounded together give 4 beats per second. A is then loaded with a little wax and the number of beats/s is found to decrease. If the frequency of A is 256 Hz, find that of B.

    Ans: 252 Hz

11. Two sitar strings A and B playing the note Ga are slightly out of tune and produce beats of frequency 6 Hz. The tension in the string A is slightly reduced and the beat frequency is found to reduce to 3 Hz. If the original frequency of A is 324 Hz. What is the frequency of B?

    Ans: 318 Hz

12. A tuning fork of frequency 300 Hz is unison with a sonometer wire. How many beats per second will be heard if the tension of the wire is increased by two percent?

Ans: 3

13. A rain standing at the outer signal of a railway station blows a whistle of 400 Hz in still air. What is the frequency of the whistle for a platform observer when the train
    a. approaches the platform with a speed of 10 ms^{-1}.
    b. recedes from the platform with a speed of 10 ms^{-1}?
    c. what is the speed of sound in each case (speed of sound in air = 340 ms^{-1}).

Ans: (a) 412.12 Hz (b) 388.87 Hz (c) 340 m/s

14. The pressure amplitude of a sound wave in air is 0.84 N/m^2 and displacement amplitude is 5.5×10^{-6} m. Find the minimum wavelength of the sound.

Ans: 5.81 m

Multiple Choice Questions

1. A car travelling with a velocity equal to one-tenth of the velocity of sound. When it is approaching towards a siren of frequency 1000 Hz. The frequency appears to the driver 1100 HZ. The velocity of car is:
   a. 17 ms^{-1} b. 34 ms^{-1}
   c. 51 ms^{-1} d. 68 ms^{-1}
2. A radar sends a signal of frequency 78×10^9/s towards aeroplane moving with certain velocity a frequency difference of 2.7×10^3/s is reflected from aeroplane. Find the velocity of aeroplane.
   a. 1.87×10^2 km/hr b. 2.87×10^2 km/hr
   c. 0.87×10^2 km/hr d. 3.74×10^3 km/hr
3. Two tuning forks of frequency 256 and 258 vibrations per sec are sounded together, then the time interval between two consecutive maxima heard by an observer is:
   a. 2 s b. 0.5 s
   c. 250 s d. 2.52 s
4. 25 tuning forks are arranged in series with decreasing frequency 3 beats/s. If the frequency of last tuning fork is octave of the first, then the frequency of 1st tuning fork is:
   a. 142 b. 144
   c. 146 d. 140
5. A source and listener is moving in the same direction with a velocity equal to half the velocity of sound, what is the change in frequency?
   a. 0% b. 100%
   c. 25% d. 50%
6. When two tuning forks of frequencies 484 Hz and 486 Hz are sounded together. What will be beat frequency?
   a. 4 Hz b. 2 Hz
   c. 3 Hz d. 6 Hz
7. The intensity of sound in a normal conversation at home is of the order of:
   a. 10^{-2} Wm^{-2} b. 10^{-5} Wm^{-2}
   c. 10^{-2} Wm^{-2} d. 10^{5} Wm^{-2}
8. For production of beats, the radio should tune with:
   a. Same frequency, different phase b. Different frequency, same phase
   c. Different frequency constant phase d. Same amplitude and same frequency

Answers

1. (b) 2. (a) 3. (b) 4. (b) 5. (a) 6. (b) 7. (b) 8. (c)

Hints to Challenging Problems

HINT: 1

Given,

$f_1 = 108$ Hz, $f_2 = 112$ Hz

amplitude of each wave, $a = 1.5 \times 10^{-8}$ m

a. beat frequency, $f_b = f_2 - f_1$

b. Maximum amplitude of total sound wave $(A_{max}) = 2a$

Minimum amplitude of total sound wave $(A_{min}) = 0$

c. When the beat is formed, the resultant amplitude, $A = 2a\cos \pi (f_1 - f_2)t$

HINT: 2

Given,

$v_s = 30$ m/s, $f = 262$ Hz, $v_o = 18$ m/s, $v = 344$ m/s

a. Frequency heard when they are approaching,

$$f' = \left(\frac{v + v_o}{v - v_s}\right) f$$

b. Frequency heard when they are receding,

$$f'' = \left(\frac{v - v_o}{v + v_s}\right) f$$

HINT: 3

Given,

$f = 400$ Hz, threshold of hearing $(I_0) = 10^{-12}$ W/m^2

$P_{max} = 6.0 \times 10^{-5}$ Pa

$v = 344$ m/s

$\rho = 1.2$ kg/m^3

We know that

$$\text{Intensity, } I = \frac{P^2_{max}}{2\rho v}$$

$\therefore$ Intensity level in dB, $\beta = 10 \log (I/I_0)$

HINT: 4

Given,

$r_1 = 30$ cm $= 0.3$ m

$r_2 = 1.5$ m

Difference between sound intensity levels,

$\Delta\beta = (\beta_1 - \beta_2) = ?$

We have,

$$\Delta\beta = 20 \log\left(\frac{r_2}{r_1}\right)$$

HINT: 5

Given,

Speed of sound at 20°C, $v = 344$ m/s

Frequency (f) = 150 Hz,

Threshold of hearing $(I_0) = 10^{-12}$ Wm^{-2}

Amplitude (a) $= 5.00 \times 10^{-3}$ mm $= 5.00 \times 10^{-6}$ m

Bulk modulus of air (B) $= 1.42 \times 10^5$ Pa

a. $$P_{max} = Bka = \frac{B \times 2\pi}{\lambda} \times a = \frac{B \times 2\pi f}{v} \times a$$

b. $$\text{Intensity } (I) = \frac{P_{max}^2}{2\rho v}$$

c. $$\beta = 10 \log \frac{I}{I_o}$$

HINT: 6

Given,

Velocity of source $(v_s) = 25$ m/s

Velocity of listener $(v_0) = 0$

Real frequency (f) = 1200 Hz

Apparent frequency (f') = 1240 Hz

Speed of sound (v) = ?

When the source is moving towards a stationary listener, $f' = \left(\frac{v}{v - v_s}\right) f$

HINT: 7

Given

Real frequency of sound (*f*) = 1.0 kHz = 1000 Hz

a. Speed of sound (v) = 340 m/s

$$\text{Speed of source } (v_s) = \frac{1}{2} \times v = \frac{340}{2} = 170 \text{ m/s}$$

$$\text{Then, } f' = \left(\frac{v}{v - v_s}\right) f$$

b. $$\text{Speed of listener } (v_0) = \frac{1}{2} \times v = \frac{340}{2} = 170 \text{ m/s}$$

$$\text{Then, } f'' = \left(\frac{v + v_0}{v}\right) f$$

HINT: 8

Given,

Beat frequency (f) = 10 beats per sec

Frequency of tuning fork $(f_1) = 440$ Hz

Frequency of air vibration $(f_2) = f_1 \pm f = 440 \pm 10$
$= 450$ Hz or 430 Hz

Since the beat frequency decreases on loading, the true frequency of vibration of air must be 430 Hz.

Let θ be the required temperature at which unloaded fork and air column are in unison i.e.

Frequency of unloaded fork
= frequency of air column at θ°C

or $440 = f_\theta$

$\therefore$ $f_\theta = 440$ Hz

$\therefore$ $v_\theta = f_\theta \times \lambda = 440 \times \lambda$

Now,

$v \propto \sqrt{T}$ so we can write

$$\frac{v_{20}}{v_\theta} = \sqrt{\frac{273+20}{273+\theta}}$$

or $$\frac{430 \times \lambda}{440 \times \lambda} = \sqrt{\frac{293}{273+\theta}}$$

HINT: 9

Given,

$r = 20$ m

$$I = 0.5\ \mu W/cm^2 = \frac{0.5 \times 10^{-6}\ W}{(10^{-2})^2\ m^2}$$

$$= 0.5 \times 10^{-2}\ W/m^2$$

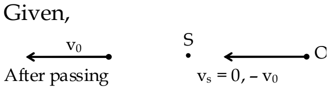
Power transmission (P) = ?

We have, $I = \frac{P}{A}$

$\therefore\ P = I \times A = I \times 4\pi r^2$

HINT: 10

Given,



i. When an observer is coming towards the stationary

$v_s = 0$

$\therefore\ f' = \frac{v + v_0}{v} \times f$

ii. After passing the stationary source,

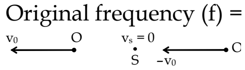
$f'' = \frac{v - v_0}{v} \times f$

HINT: 11

Given,

Speed of observer (v_0) = 30 ms^{-1}

Original frequency (f) = 100 Hz



$v_0 = 20\ ms^{-1}$

$v\ = 340\ ms^{-1}$

frequency of source (f) = ?

i. when observer is moving towards the stationary source,

$f_1 = \frac{v - (-v_0)}{v - 0} \times f = \frac{v + v_0}{v} \times f$

ii. When observer passes the source, $f_2 = \frac{v - v_0}{v} \times f$

Given, $f_1 - f_2 = 50$ Hz

or $\frac{v + v_0}{v} \times f - \frac{v - v_0}{v} \times f = 50$

Use given values and find f.

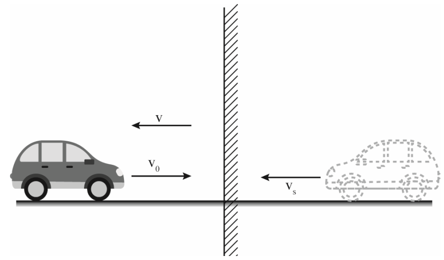
HINT: 12

Given,

f = 1000 Hz $\quad v_s = 18\ ms^{-1}$

v = 330 ms^{-1} $\quad v_0 = 18\ ms^{-1}$

Apparent frequency of echo heard by the car driver (observer), f' = ?



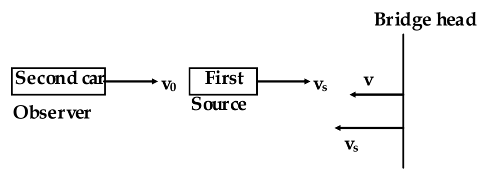
$f' = \frac{v + v_0}{v - v_s} \times f$

HINT: 13

Given,

f = 500 Hz, v = 340 ms^{-1}

$v_s = 10\ ms^{-1}$, $v_0 = 20\ ms^{-1}$



For the direct sound heard by the observer in the second car, we can write

i. $f' = \frac{v + v_0}{v + v_s} \times f$

ii. $f'' = \frac{v + v_0}{v - v_s} \times f$

HINT: 14

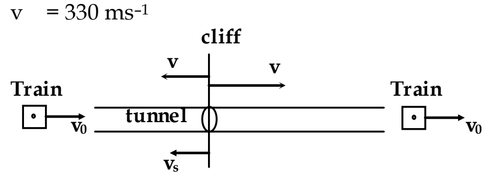
Given,

$v_o = 95$ km hr^{-1}

$= \frac{95 \times 1000}{60 \times 60}\ ms^{-1} = 26.39\ ms^{-1}$

f $\quad$ = 1000 Hz

v $\quad$ = 330 ms^{-1}



i. $f_1 = \frac{v + v_0}{v - v_s} \times f$

ii. $f_2 = \frac{v - v_0}{v + v_s} \times f$

❖ ❖ ❖

SPEED OF LIGHT

5

CHAPTER

5.1 Introduction

Until the late seventeenth century, it was not known whether light travels instantaneously or with finite speed. Many scientists believed that light is observed instantly at all distances regardless where the source is situated. Against the belief of those scientists, Galileo put his view that light could have finite speed. He tried to measure the time, a light beam takes to travel to a distant mirror and back, but he could not measure it because of the time interval being very short. Later on, Many scientists attempted to measure the speed of light by blinking the lanterns on and off between distant mountain tops. These experiments also could not give the expected result.

In 1676, Danish astronomer, Olaus Roemer first demonstrated that light travels at a finite speed by studying the apparent motion of Jupiter's innermost moon I_O. In 1865, Maxwell proposed that light was an electromagnetic wave, and therefore travelled at the speed c ($\approx 3 \times 10^8$ ms^{-1}), in his theory of electromagnetism. In 1905, Albert Einstein postulated in his theory of relativity that speed of light (c) is a universal physical constant. The experiments measuring the speed of light were performed by Armand Fizeau (in 1849), Foucault (1862) and Albert Michelson (in 1880). Among these, Foucault's experiment and Michelson's experiment are described below.

5.2 Foucault's Method

The experimental apparatus set up for the Foucault's method to measure the speed of light is shown in Fig. 5.1. An intense light from source S is allowed to fall on the partially reflecting glass plate P, which then transmits light towards a converging lens L. The incident light on lens L converges light at a point, say I. A plane mirror M_1 is placed at point C, which can rotate across an axis passing through C. As the mirror is inserted between L and I, the light rays converge at point B, rather at I. A concave mirror M_2 is placed at point B. The distance between M_1 and M_2 are so adjusted that centre of curvature of the mirror M_2 lies at point C on M_1. This arrangement resembles the light rays as if they return back through the same path as they incident on M_2. Then, the light rays return towards the original source S. The glass plate P which is inclined at an angle 45° with principal axis of lens L partially reflects the light and finally converges at Y as shown in Fig. 5.1.

In the next step, the mirror M_1 is rotated with uniform speed about the axis passing through C. During this condition, the mirror M_1 changes its position from M_1 to M_1', as the light rays traverses from M_1 to M_2 and return to M_1. Due to the shifting of position of M_1, final image does not lie at S, rather at point S'. The glass plate P reflects the light to another point Y'. The image of S' can be projected to I' beyond the mirror M_1.

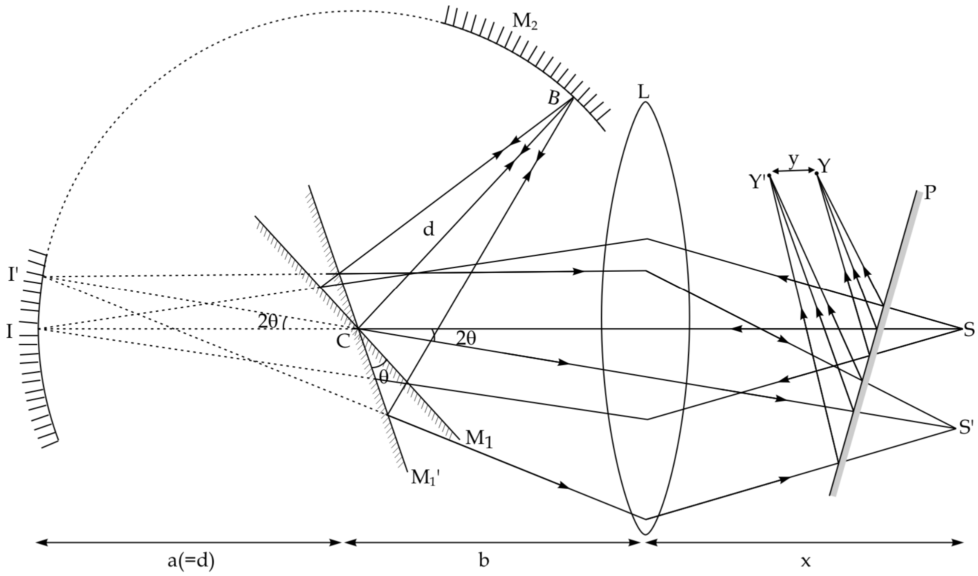


Fig. 5.1: Foucault's method

Theory

Let d be the distance between M_1 and M_2, which is ultimately equal to the radius of curvature of M_2 (i.e. d = R). Then, the time (t) for light to travel from M_1 to M_2 and reflect back to M_1 is,

$$\mathbf{t = \frac{2d}{c}} \qquad \ldots(5.1)$$

Where, c is the speed of light which is to be determined in this experiment.

Let θ be the angle of rotation of mirror M_1 at the same time t as taken by the light to ravel to M_2 from M_1 and reflect to M_1'. Suppose the angular speed of rotation of the mirror is,

$$\omega = \frac{\theta}{t}$$

$$t = \frac{\theta}{\omega}$$

$$\mathbf{t = \frac{\theta}{2\pi f}} \qquad \ldots(5.2)$$

Where, f is the number of rotation of mirror M_1 per second (i.e. frequency of rotation of mirror).

Now, equating equation (5.1) and equation (5.2), we get,

$$\frac{\theta}{2\pi f} = \frac{2d}{c}$$

$$\mathbf{c = \frac{4\pi f d}{\theta}} \qquad \ldots(5.3)$$

The direct measurement of θ is impossible, since it has very small value and is formed in very small interval of time. So, θ is determined from the following technique.

Determination of θ

As the mirror is rotated through an angle θ, the reflected ray is rotated through an angle 2θ.

So, from Fig. 5.1, we have,

The angle subtended by an arc at the centre of a circle is the ratio of arc to radius, $\angle ICI' = 2\theta$

In arc II', Angle = 2θ and radius = a

As the C is considered as the centre of curvature of concave mirror M_2, the value of 'a' is equal to the value of 'd' (equal to the radius of curvature R of M_2).

$$2\theta = \frac{II'}{a}$$

$$\therefore \quad \mathbf{II' = a \times 2\theta} \qquad \text{... (5.4)}$$

Let 'b' be the distance between plane mirror and lens, and x be the distance between the lens and source S in Fig. 5.1. Since S and S' respectively are conjugate points with respect to I and I' for the lens L, so we can write,

$$\frac{SS'}{II'} = \frac{x}{a+b} \qquad \left(\because m = \frac{h_i}{h_0} = \frac{v}{u}\right)$$

$$II' = \frac{(a+b)\,SS'}{x}$$

$$\therefore \quad \mathbf{II' = \frac{(a+b)\,y}{x}} \qquad \text{... (5.5)}$$

From (5.4) and (5.5), we have,

$$\frac{(a+b)y}{x} = a \times 2\theta \qquad (\because a = d)$$

$$\therefore \quad \mathbf{\theta = \frac{(a+b)y}{2ax}} \qquad \text{... (5.6)}$$

From equations (5.3) and (5.6), we get,

$$\frac{(a+b)y}{2ax} = \frac{4\pi fa}{c}$$

$$\therefore \quad \mathbf{c = \frac{8\pi fa^2 x}{(a+b)\,y}} \qquad \text{... (5.7)}$$

Knowing the values of 'f', 'a', 'b', 'x' and 'y', the speed of light 'c' in air can be calculated. The value of 'c' found by Foucault is 2.98×10^8 m/s.

Advantages of Foucault's Method

The main advantages are:

i. It can be performed inside a experimental lab.
ii. The speed of light in any optical medium can be measured.
iii. The adjustment of uniform speed of rotating mirror is quite easy.
iv. In addition, the refractive index of transparent medium can also be measured using this experiment.

Disadvantages of Foucault's Method

The main disadvantages are:

i. The image obtained is not very bright, due to multiple reflection and refraction of light from mirrors and lens.
ii. The apparatus arrangement is complicated, so it is difficult to locate the final image of the light source.
iii. The displacement of image y is very small, so it is difficult to measure correctly.

Michelson Method

Michelson experiment is the most precise method for measuring the speed of light. He spent more than 50 years to set the apparatus to measure the precise value of speed of light. He was awarded by Nobel Prize in 1907, for this successful experiment after his 50 years endeavour.

The experiment set up for measuring the speed of light by Michelson method is schematically shown in Fig. 5.2. It consists of a intense source of light S from which the light is allowed to fall on a face, say face 1, of polygonal mirror (specifically, octagonal mirror) M_1 after being collimated through a slit. The octagonal mirror is connected to an electric motor which provides the suitable rotation across an axis passing through centre of the mirror M_1. The reflected light from a face of mirror M_1 is sent to a concave mirror M_2 which is mounted many kilometers away from M_1. A plane mirror M_3 is kept at the focal plane of concave mirror M_2. Then, the multiple reflections take place between M_2 and M_3 and finally the light returns to M_1. The light again reflects from another face, say face 3, and is received in telescope T as shown in Fig. 5.2.

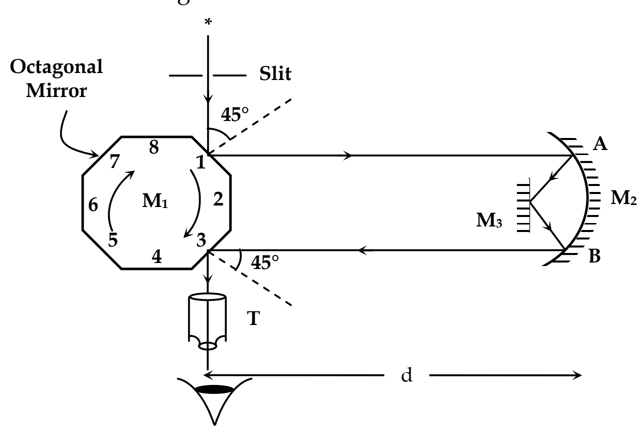


Fig. 5.2: Michelson's method

The experiment is performed in two steps. Firstly, the octagonal mirror is set stationary. The mirrors, slits and telescope are so adjusted that the light falls on face 1 of mirror M_1 at an angle 45° and is received in the telescope after it suffers many reflections from mirrors. Then, the telescope is also fixed at the same position. In the next step, the octagonal mirror is rotated by using electric motor. In the beginning of rotation of mirror M_1, the light disappears from the telescope. Then, the speed of rotation of mirror is gradually increased until light reappears in the telescope. During this process, every preceding face of M_1 is exactly occupied by every succeeding face with same inclination.

Theory

Let 'd' be the distance between mirrors M_1 and M_2. Then, the time taken by the light to travel from M_1 to M_2 and back to M_1 is 2d/c, where *c* is the speed of light, i.e.

$$t = \frac{2d}{c} \qquad \ldots (5.8)$$

Then, the polygonal mirror (here octagonal) is set into rotation through an angle θ during time 't' as expressed in equation (5.8). So, the total angle rotated by all m sides will be mθ (here, m = 8) which will be equal to 2π i.e.

$$m\theta = 2\pi$$

$$\text{or,} \quad \boldsymbol{\theta = \frac{2\pi}{m}} \qquad \ldots (5.9)$$

Let 'f' be the number of revolution per second of the mirror, we can write,

$$\omega = \frac{\theta}{t}$$

$$\text{or,} \quad 2\pi f = \frac{\theta}{t}$$

$$\text{or,} \quad t = \frac{\theta}{2\pi f} = \frac{2\pi}{m} \cdot \frac{1}{2\pi f}$$

$$\therefore \quad \boldsymbol{t = \frac{1}{mf}} \qquad \ldots (5.10)$$

From (5.8) and (5.10), we get,

$$\frac{2d}{c} = \frac{1}{mf}$$

$$\text{or,} \quad \boldsymbol{c = 2mfd} \qquad \ldots (5.11)$$

For m = 8, c = 16fd

In Michelson's original experiment, he used the values of f = 528 rev/s and d = 35 km = 35×10^3 m. Then, he found the speed of light in vacuum was 2.99910×10^8 ms^{-1}.

After the advancement of new technology, the speed of light has been measured with more sophisticated optical devices using highly coherent and unidirectional laser beam. The most precised value of speed of light in free space (vacuum) is found c = 2.99774×10^8 ms^{-1}, which is approximately equal to 3×10^8 ms^{-1}.

Advantages of Michelson's Method

The main advantages are:

i. The value is most accurate with the real value of speed of light.
ii. The apparatus arrangement is comparatively easier than Foucault's method.
iii. The image obtained is quite bright, so it is easier to locate the final beam of light emerging from the rotating mirror.
iv. The experiment is performed at a large distance, so the value possesses small error.

Disadvantages of Michelson Method

The main disadvantages are:

i. A long distance (many kilometres) is required to perform the experiment, so it is not a lab method. The experimental set up is difficult.
ii. It is very difficult to maintain the constant rotational speed of light for a long time.
iii. Rotational mirror may be damaged due to the high speed rotation in an axis.

5.3 Importance of measuring speed of light

Speed of light in vacuum is considered as universal constant. Its value is 3×10^8 ms^{-1}. Moreover, the interesting fact is that no object or radiation can travel faster than speed of light in vacuum. It is used in various fields of science as a standard value. Some of them are described below:

i. Refractive index of transparent medium can be determined with the help of c, i.e. $\eta = \frac{c}{v}$.

ii. The mass reduction, length contraction and time dilation in relativistic condition can be determined using the value of 'c'.

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}, \ \ell = \frac{\ell_0}{\sqrt{1 - \frac{v^2}{c^2}}}, \ t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

iii. The equivalent mass of radiation can be determined by using the value of c, $m = \frac{E}{c^2}$.

iv. Maxwell's electromagnetic theory can be verified by using 'c'.

Tips for MCQs

1. Danish astronomer, Olaus Roemer, in 1676, first demonstrated that speed of light is finite.
2. Speed of light in vaccum, $c = 3 \times 10^8$ ms^{-1}, is a universal constant. Nothing moves faster than this speed.
3. The symbol of speed of light in vacuum 'c' is taken from the Latin Word, 'Celeritas', which means 'Swiftness'.
4. The speed of light is absolute, not relative.
5. The formula used to find out the speed of light in Foucault's method is $c = \frac{4\pi fa}{\theta} = \frac{8\pi\ fa^2x}{(a + b)\ y}$
6. The formula used to find out the speed of light in Michelson's method is c = 2mfd, for octagonal mirror, c = 16fd.
7. The momentum of light photon, $p = \frac{hf}{c}$

Worked Out Problems

1. A plane mirror is placed at the center of a concave mirror having radius of curvature 40 m. The plane mirror rotates at the speed of 2600 revolutions per second. Calculate the angle between ray incidents on the plane mirror and then reflected from it after the light has travelled to the concave mirror and back to the plane mirror. Given speed of light is 3×10^8 ms^{-1}.

SOLUTION

Given,

Radius of curvature (R) = d = 40 m

Rotational frequency (f) = $2600 \frac{\text{rev}}{\text{s}}$

Rotation of reflected light (2θ) = ?

Speed of light (c) = 3×10^8 ms^{-1}

We have,

$$c = \frac{4\pi fd}{\theta}$$

$$\theta = \frac{4\pi fd}{c} = \frac{4\pi \times 2600 \times 40}{3 \times 10^8} = 4.36 \times 10^{-3}$$

Then, $2\theta = 8.71 \times 10^{-3}$ rad.

$$= \left(8.71 \times 10^{-3} \times \frac{180}{\pi}\right) = 0.5^\circ.$$

Therefore, required angle is 0.5°.

2. In Foucault's method for the speed of light, the distance between the moving and static mirrors was 3 km and the speed of the moving mirror was 50° rev/s. If the displacement of the returned beam be 7°12', find the speed of light.

SOLUTION

Here,

$$2\theta = 7^\circ 12' = 7 + \frac{12}{60} = 7.2^\circ$$

$\theta = 3.6^o = 3.6 \times \frac{\pi}{180}$ rad = 0.063 rad

d = 3 km = 3×10^3 m

f = 500 rev/s

c = ?

We have,

$$c = \frac{4\pi fd}{\theta} = \frac{4\,\pi \times 500 \times 3 \times 10^3}{0.063}$$

$= 3 \times 10^8$ ms^{-1}

3. [HSEB 2058] The radius of curvature of the curved mirror is 20 m and the plane mirror is rotated at 20 revs^{-1}. Calculate the angle in degree between a ray incident on the plane mirror and then reflected from it after the light has travelled to the curved mirror and back to the plane mirror. (c = 3 × 10^8 m/s)

SOLUTION

Here,

Radius of curvature of concave mirror is equal to the separation between plane mirror and the concave mirror i.e,

R = d = 20 m

c = 3×10^8 ms^{-1}

f = 20 rev s^{-1}

Angle between incident ray and reflected ray from the plane mirror (α) = 2θ = ?

The speed of light (c) = $\frac{4\pi fd}{\theta}$

$\therefore \theta = \frac{4\pi fd}{c}$

Since angle through which reflected ray turns, is twice the angle of rotation of mirror (θ) so we can write

$$\alpha = 2\theta$$

$$= 2 \times \frac{4\pi fd}{c}$$

$$= \left(\frac{8 \times \pi \times 20 \times 20}{3 \times 10^8}\right) \text{radian}$$

$$= \left(\frac{8 \times \pi \times 400}{3 \times 10^8} \times \frac{180}{\pi}\right) \text{degree}$$

$= 1.92 \times 10^{-3}$ degree

4. [HSEB 2071] In a Michelson experiment for measuring speed of light, the distance travelled by light between two reflections from the rotating mirror is 4.8 km. The rotating mirror has a shape of regular octagon. At what frequency of rotation of mirror the image is formed at the position where non-rotating mirrors forms it?

SOLUTION

Speed of light (c) = 3×10^8 m/s

No. of face of mirror (m) = 8

No. of rotation (f) = ?

Distance traveled (d) = 4.8 km = 4800 m

We have,

c = 2mfd

or, $3 \times 10^8 = 2 \times 8 \times n \times 4800$

$\therefore$ f = 3906 rev/s

Challenging Problems

1. In Michelson's method, to determine the speed of light in air, the distance traveled by light between reflections from opposite faces of the octagonal mirror is 75 km. The image appears stationary when the minimum speed of rotation of the octagonal mirror is 500 rotations per second. Calculate the speed of light in air.

ANS: 3×10^8 M/S

2. **[ALP]** A beam of light after reflection at a plane mirror rotating 2000 times per minute passes a distant reflector. It returns to a rotating mirror from which it is reflected to make an angle of 1° with

the original direction. Assuming that the speed of light is 3×10^5 kms^{-1}, calculate the distance between the mirrors.

ANS: 6250 M

3. A beam of light is reflected by a rotating mirror onto a fixed mirror which sends back to the rotating mirror from which it is again reflected and then makes an angle of 3.6° with the original direction. The distance between the two mirrors is 1km and the rotating mirror is making 750 $revs^{-1}$. Calculate the speed of light.

ANS: 3×10^8 M/S

4. **[ALP]** A horizontal beam of light is reflected by a vertical plane mirror A, travels a distance of 250 m is then reflected back along the same path and is finally reflected again by the mirror A. when A is rotated with constant angular speed about a vertical axis in its plane, the emergent beam is deviated through an angle of 18 minutes. Calculate the number of revolutions per second made by the mirror. ($c = 3 \times 10^8$ m/s)

ANS: 250 REV/SEC

5. In a Michelson's arrangement for determining the speed of light, the distance between octagonal reflector and distant stationary mirror is 32 km. Determine the frequency of revolution to turn the octagonal reflector for forming image after reflection from succeeding reflector. ($c = 3 \times 10^8$ m/s)

ANS: 586 HZ

*[**NOTE**: Hints To Challenging Problems Are Given At The End Of This Chapter.]*

Conceptual Questions with Answers

1. Who successfully found that light has finite speed?

↳ A Danish astronomer, Olans Romer, in 1676 found that light has finite speed. He observed the speed of light from the eclipses on one of the Jupiter's Satellite.

2. Who found that the speed of light in vacuum is a physical constant?

↳ Great scientist, Albert Einstein, found that the speed of light in vacuum is a physical constant. Its value is $c = 3 \times 10^8$ ms^{-1}. In his theory of relativity, he explained that no object has greater velocity than the velocity of light in vacuum. Speed of light is not relative. It is the optimum speed in universe.

3. Does the speed of light depend on the nature of its source?

↳ No. The speed of light depends on the nature of transparent medium. The transparent medium whose refractive index is high, the speed is low i.e. speed, $v = \frac{c}{\eta}$, where c = speed of light in vacuum and η = refractive index of medium. That is why, speed of light in glass is smaller than that of water.

4. What are the advantages of Foucault's method?

↳ The main advantages of Foucault's method of measurement velocity of light are:

i. It can be performed inside a experimental lab.

ii. The speed of light in any optical medium can be measured.

5. What are the disadvantages of Foucault's method of measurement of speed of light?

↳ The main disadvantages are:

i. The image obtained is not very bright, due to multiple reflection and refraction of light from mirrors and lens.

ii. The apparatus arrangement is complicated, so it is difficult to locate the final image of the light source.

6. What are the advantages of Michelson's method of measurement of speed of light?

↳ The main advantages are:

i. The value is most accurate with the real value of speed of light.

ii. The apparatus arrangement is comparatively easier than Foucault's method.
iii. The image obtained is quite bright, so it is easier to locate the final beam of light emerging from the rotating mirror.
iv. The experiment is performed in a large distance, so the value possesses small error.

7. What are the disadvantages of Michelson Method?

✎ The main disadvantages are:

i. A long distance (many kilometres) is required to perform the experiment, so it is not a lab method. The experimental set up is difficult.
ii. It is very difficult to maintain the constant rotational speed of light for a long time.
iii. Rotational mirror may get damaged due to the high speed rotation in an axis.

8. What are the importance of measuring speed of light?

✎ Speed of light is a universal constant. It has many important significances in the study of science.

i. The refractive index of transparent medium can be determined.
ii. The Einstein's mass energy relation is based on the speed of light, $E = mc^2$.
iii. It is the optimum speed of wave, so the existence of particles in a certain space can be examined comparing with c. The non-existence of electron in nucleus can be confirmed by comparing with speed of light.
iv. The Length contraction, mass reduction and time dilation phenomena in relativity can be measured on the basis of speed of light in vacuum.

9. When light travels from one medium to another, does its energy change?

✎ The energy of single photon remains same. But, the light photon may interact with the particles in a medium. So, some of the photons may be absorbed and intensity of light may decrease so that the energy is ultimately decreased.

Exercises

Short-Answer Type Questions

1. Is speed of light a measurable quantity?
2. What are the importance of measuring speed of light?
3. What is advantage of Michelson experiment over Foucault's experiment?
4. What is the advantage of Foucault's experiment over Michelson experiment?
5. When light travels from a denser to a rarer medium, its velocity increases. Will the energy carried by the light wave increase?
6. Speed of light in a denser medium is less than that in rarer medium. What is its effect on the energy of light?
7. Define refractive index in terms of speed of light.
8. Why is it important to know the accurate value of the speed of light?

Long-Answer Type Questions

1. Describe Focault's method with necessary theory to determine speed of light and write down the significance of this method.
2. Describe Michelson's method with neat labeled diagram and necessary theory to determine speed of light. Discuss its advantages and disadvantages.
3. Describe the history of measurement of speed of light. Why is the measurement of light important in physics.

Numerical Problems

1. A beam of light after reflection at a plane mirror, rotating 2000 times per minute passes to a reflecting mirror, placed 6250 m away from the rotating mirror. It returned to the rotating mirror from which it is reflected to make an angle of 1° with its original direction. Calculate the speed of light.

ANS: 3×10^{8} M/S

2. In a Foucault's experiment, the distance between the rotating mirror and stationary mirror is 120 m and the distance of the light source from the rotating mirror is 50 m, when the mirror is rotated 200 times per sec, the displacement of the image is 10 cm. Find the speed of light.

ANS: 3×10^{8} M/S

3. In a Foucault's experiments, the reflected rays made an angle of 18° with its original direction, due to the reflection from the revolving mirror. The distance of the fixed mirror from revolving mirror was 10^{4} m. If the frequency of rotation of the mirror is 375 rev per sec, calculate the speed of light.

[HSEB 2067]

ANS: 3×10^{8} M/S

4. In Foucault's method the distances of the rotating mirror from the fixed mirror and the lens were 20 m and 6 m respectively. The source of the light was placed at a distance of 210 cm from the lens. When the plane mirror was rotated at the rate of 258 times per sec. the shift of the image was recorded to be 0.7 mm. Calculate the speed of light.

ANS: 3×10^{8} M/S

5. In Michelson's method for finding the speed of light, an octagonal prism was used. It was found that first reappearance of image occurs when prism is rotated at a speed of 500 rev/s. Calculate the speed of light if the distance between the prism and the distant mirror is 37 km.

ANS: 2.96×10^{8} M/S

7. In Michelson method to measure the speed of light, a strong source of light is reflected from one face of an octagonal equiangular mirror and travels a distance of 35 km to a stationary mirror from which it returns and after second reflection from octagonal mirror forms an image of the source on a screen. What is the angular speed of rotation of the rotating mirror? ($c = 3 \times 10^{8}$ m/s)

ANS: 3364 RAD/S

Multiple Choice Questions

1. The speed of light in vacuum is 3×10^{8} ms^{-1}. Then, what is the speed of light in glass of refractive index 1.5?

   a. 4×10^{8} ms^{-1} b. 3×10^{8} ms^{-1}
   c. 1.5×10^{8} ms^{-1} d. 0.5×10^{8} ms^{-1}

2. Which of the following is not a lab experiment?

   a. Michelson experiment b. Foucault's experiment
   c. Double slit experiment d. Fizeau's experiment

3. Who firstly demonstrated that speed of light is finite?

   a. Michelson b. Foucault
   c. Young d. Romer

4. Which is the most accurate method of determination of speed of light?

   a. Foucault's method b. Michelson's method
   c. Fizeau's method d. Romer's method

5. What type of mirror for rotation was used during the Michelson experiment?

   a. Plane mirror b. Concave mirror
   c. Octagonal mirror d. convex mirror

Answers

1. (c) 2. (a) 3. (d) 4. (b) 5. (c)

Hints to Challenging Problems

HINT:1

Here,

$2d = 75$ km

$d = 37.5$ km $= 37.5 \times 10^3$ m

$f = 500$ rotations per second

Since,

$c = 16fd$

HINT: 2

The reflected ray is rotated by $1°$

$\therefore$ The plane mirror is rotated by $\frac{1°}{2}$

Now,

The time taken by the mirror to rotate 2000 times = 1 min = 60 sec.

In one complete rotation, 360° angle is covered in 60 sec.

$\therefore$ In 200 complete rotations = $(2000 \times 360)°$ in 60 sec.

$1°$ covered in $\frac{60}{2000 \times 360}$ sec.

$\therefore$ $\frac{1°}{2}$ covered in $\frac{60}{2000 \times 360} \times \frac{1}{2} = 4.17 \times 10^{-5}$ sec.

Let the distance between two mirrors be 'd'.

$\therefore$ $2d = ct$

$\therefore$ $d = \frac{ct}{2}$

HINT: 3

Here,

$2\theta = 3.6°$

$\therefore$ $\theta = 1.8° = 1.8 \times \frac{\pi}{180}$ rad $= 3.14 \times 10^{-2}$ rad

$f = 750$ rev/s

Then, $c = \frac{4\pi fd}{\theta}$

HINT: 4

Here,

$d = 250$ m

$c = 3 \times 10^8$ m/s

$f = ?$

$$2\theta = 18' = \left(\frac{18}{60}\right)^{\circ} = \left(\frac{18}{60}\right) \times \left(\frac{\pi}{180}\right) \text{ rad}$$

or, $2\theta = 0.3$ rad.

$\therefore$ $\theta = 0.15$ rad.

Then, $c = \frac{4\pi fd}{\theta}$

$\therefore$ $f = \frac{c\theta}{4\pi d}$

HINT: 5

Here,

Number of faces, $m = 8$

$d = 32$ km $= 32 \times 10^3$ m

$c = 3 \times 10^8$ m/s

$f = ?$

$c = 2mfd$

$\therefore$ $f = \frac{c}{2md}$

❖ ❖ ❖

PHYSICAL OPTICS

6

CHAPTER

6.1 Introduction

We have till now studied the geometric nature of light in which we considered that the light travels in a straight line path called rays. Many optical phenomena such as reflection, refraction were studied geometrically, and the same geometric principles were used to model many optical instruments such as mirror, lens, prisms, microscopes, telescopes, periscopes, etc.

However, there are certain phenomena such as interference, diffraction and polarization which can't be explained on the basis of geometric optics. So, we will now discuss a different nature of light known as wave nature to deal with these phenomena. The study of nature of light considering it to behave as a wave known as wave theory of light is the outcome of rigorous efforts of many scientists, some of whom are Huygen, Young, Fresnel. The different theories presented at different times in the history and their experimental confirmation has made us to believe light as a wave. The wave theory successfully describes all the phenomena such as reflection, refraction, interference, diffraction and polarization.

After the advancement of Planck's theory of quantum mechanics, it has been believed that light also has particle nature. Photoelectric effect and Compton effects are the fundamental properties of light that exhibits the particle nature of light. Eventually, the modern theory of quantum mechanics incorporates both wave nature and particle nature of light, called wave particle duality. Some eminent theories regarding the nature of light were proposed by some scientists which are briefly explained below:

Newton's Corpuscular Theory

Sir Isaac Newton proposed that light is made up of small discrete particles called "corpuscles" (little particles) which travel in a straight line with a finite speed. He claimed that the geometric nature of reflection and refraction of light could be explained if light was made of particles. Because of the Newton's great prestige, Newton's corpuscular theory was predominant for more than 100 years. The corpuscular theory was failed when it could not explain the interference, diffraction and polarization phenomena of light. Moreover, its next evidence of failure is that the speed of light is greater in denser medium than the rarer medium. To some extent, Newton's corpuscular theory is similar to the quantum nature of light.

Huygen's Wave Theory

Dutch physicist Christian Huygen in 1678, first proposed the wave nature of light. His model satisfactorily explained the reflection and refraction of light. To explain the wave nature of light, Huygen proposed that the space of universe is filled with a hypothetical liquid called 'luminiferous ether'. He supposed that light waves are mechanical waves which are propagated in the space, oscillating the particles of ether, just as sound propagates vibrating the air molecules. Huygen's theory was also similar to Newton's theory that the speed of light is greater in denser medium than the rarer medium. Later on, the experiment showed that the speed of light was greater in air than the water. Various scientists attempted to detect the ether, but none was successful. Finally in 1887, Michelson and Morley carried out their famous experiment, which provided strong evidence against the ether concept.

Maxwell's Electromagnetic Theory

In 1865, James Clerk Maxwell explained that light was an electromagnetic wave which can travel even in empty space. In his publication "A Dynamic Theory of Electromagnetic Field", he demonstrated that electric and magnetic fields can travel through space and they oscillate perpendicular to the direction of propagation of wave. He formulated four important equations regarding the electromagnetic theory. They are known as Maxwell's equations, then the electromagnetic signal propagation has become possible. The radio signals, television signals and telecommunication are being possible only after the Maxwell discovery on electromagnetism.

Particle nature of light (Planck's theory)

In 1900, a German scientist Max Planck changed the understanding of light. Before this discovery, light could be explained only by wave nature, but not the particle nature. Although the Newton's corpuscular theory proposed the particle nature of light, it could not explain even the change of speed of light in different media. Planck's theory proposed that the amount of energy radiated by luminous source is directly proportional to the frequency of electromagnetic waves ($E = hf$). Planck's quantum theory successfully explained the photoelectric effect, Compton effect and pair production. Planck's discovery of quantum nature of light is a remarkable finding for the modern science, which changed the existing concept of light at that time.

Dual nature of light

Young's double slit experiment on interference and Fresnel's experiment on diffraction of light showed that light behaves like a wave. Without wave property of light, the redistribution of energy (dark and bright patterns) in superposition is impossible. On the other hand, photoelectric effect and compton effect showed that light behaves like a particle. Thus, a debate regarding the nature (wave or particle) continued for a very long time and finally resolved when de Brogile proposed a concept of wave particle duality correlating these two conceptually different aspects of light, called de Broglie hypothesis. This concept was firmly established when electron wave was detected experimentally. As a similar consequence, gravitational wave has also been detected experimentally in September 2015 and announced at Feb 11, 2016.

6.2 Electromagnetic Waves

The wave which is composed by mutually perpendicular electric and magnetic fields is called electromagnetic wave. At any point in its propagation, the electric field is perpendicular to the magnetic field, and both are perpendicular to the direction of motion of the wave. These waves are produced from the conversion of kinetic energy of oscillating charge. Electromagnetic wave can travel in vacuum. The speed of electromagnetic wave in vacuum is 3×10^8 ms^{-1}. Its speed is constant whatever the frequency or wavelength or intensity of the radiation. Visible light is simply an electromagnetic wave. The schematic diagram of electromagnetic spectrum is shown in Fig. 6.1.

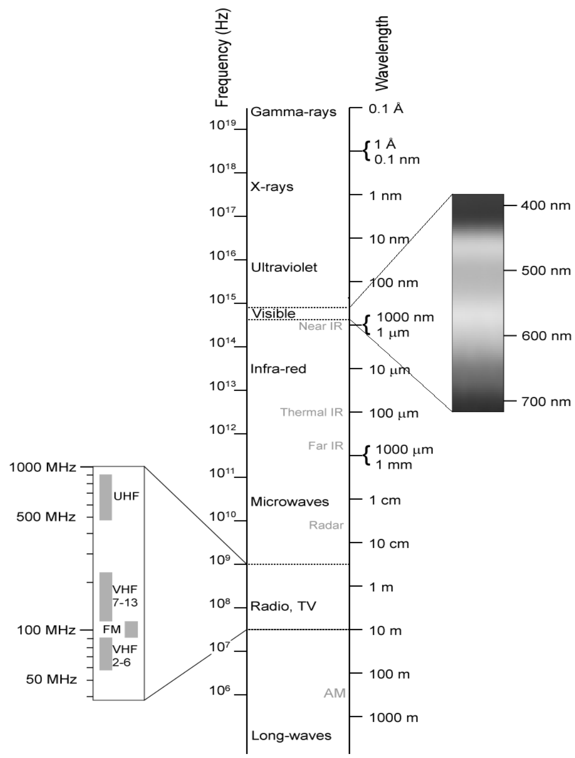


Fig. 6.1: Electromagnetic Spectrum

Electromagnetic Spectrum

In optics, spectrum refers the arrangement of waves in accordance with wavelength or frequency. Electromagnetic waves incorporate a wide range of wavelengths ranging from a few kilometers to about 10^{-14} m. There are basically seven types of waves in electromagnetic spectrum. They are radiowaves, microwaves, infrared, visible, ultraviolet, x-rays and γ-rays.

i. **Radiowaves:** Radiowaves have the frequency ranging from a few kilometers down to 0.3 m. The frequency range is from a few Hz to 10^9 Hz. Radiowaves are basically used in radio and television communication signals. The amplitude modulation (AM) band ranges from 530 kHz to 1710 kHz. Higer frequencies upto 54 MHz can be used for 'short wave' bands. Television (TV) waves range from 54 MHz to 890 MHz. The frequency modulation (FM) radio band extends from 88 MHz to ultrahigh frequency (UHF) band. The accelerated motion of charges in conducting wires produce radio waves.

ii. **Microwaves:** Microwaves have the wavelength ranging from 0.3 m down to 10^{-3} m. The frequency range is from 10^9 Hz to 3×10^{11} Hz. Microwaves can pass easily through the earth's atmosphere with less interference with longer wavelengths. Special vacuum tubes like Klystrons, magnetrons and Gunn diodes are used to produce microwaves. They can be used to transmit power over long distances. They are also used in communication satellite transmission. A microwave oven uses a magnetic microwave generator.

iii. Infrared waves: Infrared waves have the wavelength ranging from 10^{-3} m down to 7.8×10^{-7} m. The frequency range is from 3×10^{11} Hz to 4×10^{14} Hz. Infrared waves are sometimes called heat waves. They are produced by hot bodies and molecules. The infrared wave coming from the sun keeps the earth warm. In hospitals, they are used in muscular therapy in physiotherapy departments. Infrared lies nearer to the red colour of visible spectrum, so it is called infrared.

iv. Visible waves: The objects around us are visualized by using visible waves. It has narrow band compared to other waves of electromagnetic spectrum. The wavelength of visible light extends from 7.8×10^{-7} m down to 3.8×10^{-7} m. The frequency ranges from 4×10^{14} Hz to 8×10^{14} Hz. These waves are produced from the electron transition in various orbits of atoms. It is named "visible " because it provides the visibility in human eye. The visibility range of other animals can be different from human. Visible spectrum contains seven colours: red, orange, yellow, green, blue, violet.

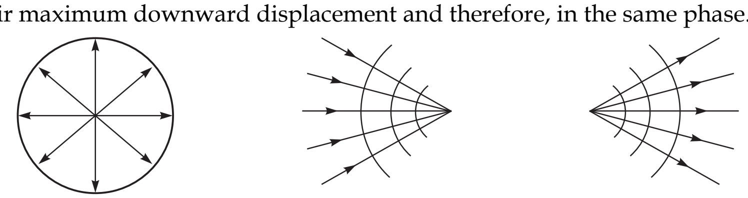
v. Ultraviolet waves: The wavelength of ultraviolet waves lie nearer to the violet light of visible spectrum. So, it is named ultraviolet. Its wavelength ranges from 4×10^{-7} m down to 6×10^{-10} m. The frequency ranges from 8×10^{14} Hz to 5×10^{17} Hz. Ultraviolet rays are a part of the solar spectrum. These waves are produced by atoms and molecules in electrical discharges. They are very harmful to the living tissues. It may cause skin cancer. These waves are used to preserve food stuffs as the rays kill germs.

vi. X-rays: X-rays are familiar from clinical approaches. X-rays are also electromagnetic waves in which the wavelength extends from nearly 10^{-9} m down to 6×10^{-12} m. The frequency ranges from 3×10^{17} Hz to 5×10^{19} Hz. X-rays are commonly produced when energetic electrons are bombarded on high atomic number metal like tungsten. Besides many uses in medical diagnosis and therapy, they are harmful for our body. Biological cells are destroyed, if x-rays are exposed on them.

vii γ-rays: γ-rays are most powerful among seven waves of electromagnetic spectrum. They can easily penetrate even a concrete wall. Their wavelength ranges from nearly 10^{-10} m below to 10^{-14} m. The frequency ranges from 3×10^{18} Hz to 3×10^{22} Hz. They are produced in nuclear changes. γ-rays are emitted in radioactivity. These rays have serious effect on human cells. Besides harmful effects, they are used to kill the cancerous cells in radiotherapy.

6.3 Wavefronts and Wavelets

If a stone is dropped on the surface of still water, circular waves produce and travel away from the point of impact. The water particles lying on a crest oscillate in the position of their maximum upward displacement and hence in the same phase. Also, all particles lying on a trough are in the position of their maximum downward displacement and therefore, in the same phase.



(i) Spherical wavefront **(ii) Converging Spherical wavefront** **(iii) Diverging Spherical wavefront**

Fig. 6.2: Different types of wavefronts

Similar phenomenon can be observed in light wave while propagating through a homogeneous medium. If the locus is drawn around the source of light of same phase, spherical shaped waves are obtained. The locus of such points oscillating in the same phase is termed as a wavefront. Therefore, *a wavefront is defined as the continuous locus of a wave which are oscillating in same phase at any instant.* A wavefront is in fact a surface of wave in which every point oscillates in constant phase.

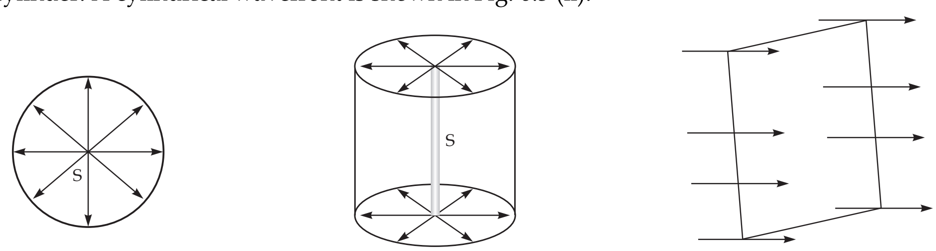
The shape of wavefronts are different in different conditions. Its shape usually depends on shape of source. For example, a point source produces the spherical wavefront, a linear source produces the cylindrical wavefront. When these wavefronts travels long distance away, they appear plane shaped.

Types of wavefronts

The shape of wavefront, of course, depends on the shape of source. Some common shapes of wavefront are explained below:

i. **Spherical wavefront:** *The waves originating from a point source are spherical in shape and the wavefronts so produced are called spherical wavefronts.* The wavefront produced from the point source are spherical because all points of that wavefront are equidistant from the point source and the disturbance starting from that point will reach all these points simultaneously. A spherical wavefront in shown in Fig. 6.3 (i).

ii. **Cylindrical wavefront:** *When a linear source, such as a linear slit, produces waves, they are cylindrical in shapes and the wavefronts so produced are called cylindrical wavefronts.* The wavefront produced by "tube light" is cylindrical in shape. A linear source produces the cylindrical wavefront because the locus of all such points which are equidistant from the linear source will be a cylinder. A cylindrical wavefront is shown in Fig. 6.3 (ii).



(i) (ii) (iii)

Fig. 6.3: (i) Spherical wavefront (ii) Cylindrical wavefront (iii) Plane wavefront

iii. **Plane wavefront:** Whether the source is point shaped or linear, the wavefronts produced from them expands. Then, the curvature decreases progressively. A small portion of such spherical or cylindrical wavefront at a large distance from the source will be a plane wavefront. The wavefronts produced by the sun appear perfectly plane when we observe on the earth. A plane wavefront is shown in Fig. 6.3 (iii).

Wavelets

According to wave theory of light, every point of a wavefront acts as the independent source of light. As the point acts as a source, new waves are produced from these points. These secondary waves produced from the points of wavefront care called wavelets. Wavelets are usually spherical in a homogeneous medium. The velocity, frequency and wavelength of each wavelet is same as that of original wavefront.

Rays and wavefronts

In geometrical optics, the direction of propagation of light is represented by a straight line with an arrowhead. But, in wave optics, the propagation of light is shown in terms of wavefront (i.e. in terms of a surface) as shown in Fig. 6.4. Actually, the energy of a wave travels in a direction perpendicular to the wavefront. Therefore, the ray shows the direction of energy flow, whereas wavefront shows the pattern of energy distribution in the space. Hence, the ray is also defined as an arrow drawn perpendicular to the wavefront in the direction of propagation of a wave.

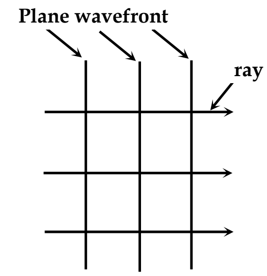


Fig. 6.4 : Ray and wavefront

6.4 Wave Theory of Light

After the downfall of Newton's corpuscular theory about nature of light, Christian Huygen proposed that light was a wave phenomenon. Huygen's theory successfully described the reflection and refraction of light. However many scientists could not believe his theory because this theory contradicted the Newton's corpuscular theory. The prestige and academic height of Newton was not comparable in the scientific community till that date. Huygen's wave theory was not comparable in the contemporary scientific community. Huygen's wave theory was firmly established when Thomas Young demonstrated the double slit experiment in 1801 to prove the interference phenomenon of light. On the basis of Huygen's assumption on wave theory, Fresnel showed the diffraction phenomenon of light.

To explain model of wave theory, Huygen proposed some assumptions, which are called Huygen's principle.

Huygen's Principle

Huygen proposed the following assumptions to explain the wave nature of light. They are,

i. Each point on a wavefront acts as secondary source of light. The newly produced waves are called wavelets or secondary waves.

ii. The secondary waves spread out in all direction with the speed of light in a medium.

iii. The new wavefront at any later time is given by the tangential surface in the forward direction of the secondary wavelets at that time.

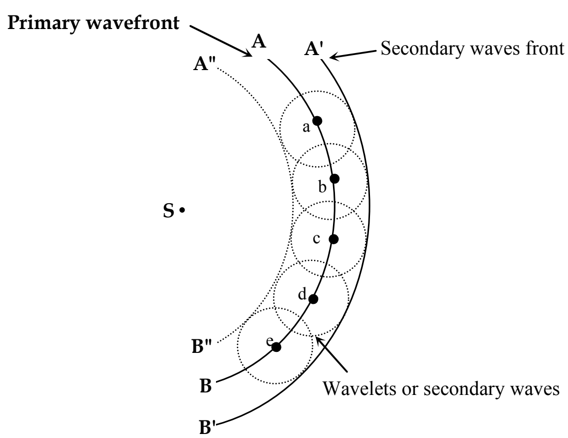


Fig. 6.5: Huygens' construction

Explanation of Huygen's Principle

Consider a point source of light S in space from which waves are produced and travelled in the surroundings. Let AB be a wavefront at any instant of time as shown in Fig. 6.5.

i. According to first principle of Huygen, every point of wavefront acts as secondary source of light. Points a, b, c, d, e are the fresh source of light and new waves are produced from these points. Equal radii spheres are formed from each point of wavefront. The radius of each sphere is r (= ct), where c is velocity of light and t is the time to form the wavelet.

ii. The wavelets spreads around the secondary source point of wavefront. Although the wavelets expand in forward and backward directions, backward transmission of wave is discarded. There is no backward transmission of energy.

iii. The wavelets spreading from each point of a wavefront are equal in radius. If the tangent's are drawn on each of wavelet taking same phase in the forward direction, a new wavefront is obtained. In Fig. 6.5. A'B' and A''B'' are two new wavefronts. The wavefront A'B' is formed in the forward direction and another wavefront A''B'' is constructed in the backward direction. But the wavefront A'B' in the forward direction is possible. No backward wavefront A''B'' is possible.

6.5 Laws of Reflection of Light from Wave Theory

Consider a plane wavefront AB be incident on a reflecting surface XY with an oblique angle. Let i be the angle of incidence at point A on the surface. Suppose the point A of the wavefront touches the surface at first, then gradually neighbouring points and finally by the point B at B' of the surface. As stated by Huygen's principle, every point of wavefront AB acts as the secondary source of light and spherical wavelets are produced from these points. So, the first wavelet generated from point A will grow into a sphere of radius AA', at the same time point B of that wavefront approaches at B' on plane XY. Thus, a series of wavelets are found travelling away with gradually decreasing radii from A' to B' as shown in Fig. 6.6. If a locus is drawn taking the points of these grown wavelets of same phase, a new wavefront A'B' will be obtained. Let r be the angle of reflection at point B'. Also, AN and B'N' be the normals at point A and B' of the reflecting surface XY.

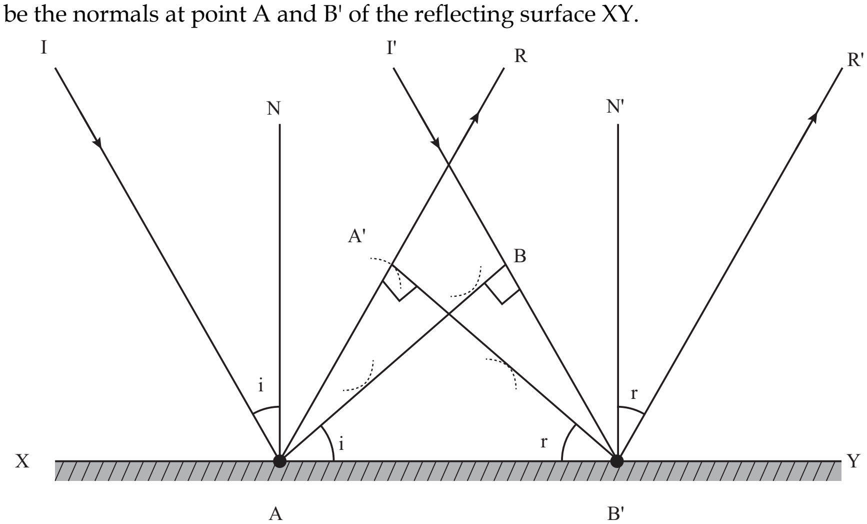


Fig. 6.6: Reflection at plane surface

Here,

$\angle IAN = i$

$\angle N'B'R' = r$

In Fig. 6.6, BA $\perp$ IA and NA $\perp$ XY

Since, the angles between two planes is equal to angle between their normals, then $\angle IAN = \angle BAB' = i$

Similarly, A'B' $\perp$ B'R' and N'B' $\perp$ XY, as explained the reason above, $\angle A'B'A = \angle N'B'R' = r$

Now, taking two right angle triangles ABB' and AA'B'

1.	BA = A'B'	Same wavefront in same medium after time 't'
2.	$\angle ABB' = \angle AA'B'$	Right angles
3.	BB' = AA'	Distance travelled by light in a same medium after an equal interval of time

Therefore, Δ^s ABB' and AA'B' are congruent triangles. It gives the result,

$$\angle BAB' = \angle A'B'A$$

i.e. $i = r$.

This verifies the first law of reflection. If we consider the incidence plane of wavefront is parallel to the plane of paper, the plane of reflected wavefront and plane of normal all lie in that plane. This proves the another law of reflection.

Thus, the laws of reflection is proved from wave theory of light.

6.6 Laws of Refraction of Light from Wave Theory

Let XY be a plane surface separating air from a denser medium (say glass) of refractive index η. Consider a plane wave AB be incident on this plane with an oblique angle. As stated by Huygen's principle, every point of wavefront acts as the secondary source of light and wavelets are produced from these points.

When the light from point B on incident wavefront approaches to B' in air, the secondary wave-lets from point A on the refracting surface XY will grow into a sphere in the denser medium of radius AA'. As the time taken by light to travel from A to A' in glass medium to that of light is equal travelling from B to B', we write as,

$$\frac{AA'}{v} = \frac{BB'}{c} \qquad \ldots (6.1)$$

Where, v = speed of light in denser medium

c = speed of light in air

In our consideration, the radius of wavelet AA' is largest into glass medium, since it is produced at first from wavefront AB and the radius of wavelet at B is the shortest, others have the gradually reducing radius from A to B' as shown in Fig. 6.7. Thus, A'B' be the refracted wavefront in the denser medium.

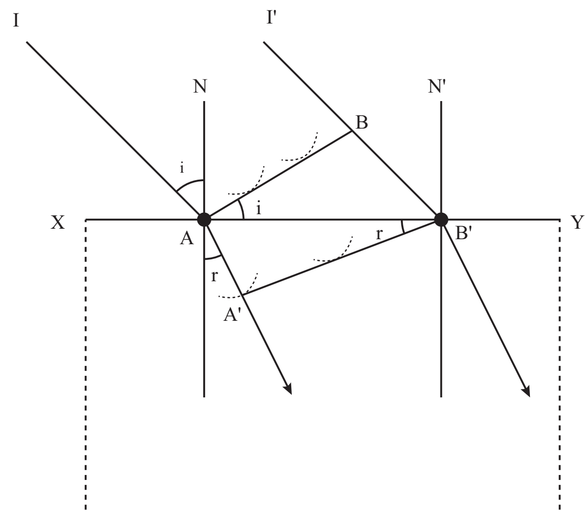


Fig. 6.7: Refraction of light

In Fig. 6.7, NA ⊥ XY and BA ⊥ IA. Also the angle between two planes is equal to the angles between their normals, we have,

$$\angle IAN = \angle BAB' = i$$

Similarly, for N'B' ⊥ XY and A'A ⊥ B'A', then, ∠AB'A' = r

$$\text{So,} \quad \sin i = \frac{BB'}{AB'} \text{ and } \sin r = \frac{AA'}{AB'}$$

Taking ratio of sin i to sin r, we get,

$$\frac{\sin i}{\sin r} = \frac{\frac{BB'}{AB'}}{\frac{AA'}{AB'}}$$

$$\therefore \frac{\mathbf{\sin i}}{\mathbf{\sin r}} = \frac{\mathbf{BB'}}{\mathbf{AA'}} \qquad \ldots \textbf{(6.2)}$$

From equation (6.1), $\frac{\mathbf{BB'}}{\mathbf{AA'}} = \frac{\mathbf{c}}{\mathbf{v}} = \eta$ **. . . (6.3)**

Comparing equations (6.2) and (6.3), we get,

$$\eta = \frac{\sin i}{\sin r}$$

This expression verifies the Snell's law of refraction of light.

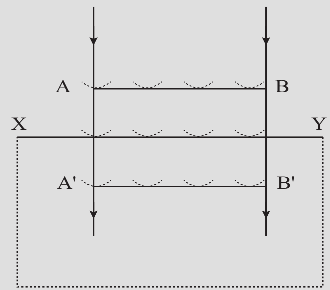
Also, if we consider the incident wavefront AB be parallel with the plane of paper, the refracted wavefront A'B' and the normals AN' and B'N' lie on the same plane.

These conditions verifies the laws of refraction of light.

Note

Why normally incident light refracted without deviation?

When a wavefront AB incident normally on an interface of denser medium, then all the points of that wavefront produce wavelets on the interface at the same time. Then, each wavelet grows equally (i.e. equal radii) into the denser medium. So that refracted wavefront is also parallel to the incident wavefront. As the *incident wavefront and refracted wavefront are parallel to each other, light appears undeviated from the interface.*



Tips for MCQs

1. Ray of light is perpendicular to the wavefront

First physicist	Theory
Sir Isaac Newton	Corpuscular theory
Christian Huygen	Wave theory of light
Maxwell	Electromagnetic theory
Max Planck	Quantum theory
de Broglie	Dual nature of light

2. Refractive index is calculated from the wavelength of light as

$$\eta = \frac{\text{wavelength of light in vacuum}}{\text{wavelength of light in medium}} = \frac{\lambda}{\lambda'}$$

3. The light wave shows the phenomena of reflection, refraction, interference, diffraction and polarization.
4. The front portion of secondary wavelets add up to give rise to a wavefront in a forward direction. The backward portion of secondary wavelets add up to zero, so no backward wavefront is possible.
5. According to dual nature of light, it shows both particle and wave nature but not at the same time
6. All electromagnetic waves propagate with the speed of light and they are non-mechanical waves.

Conceptual Questions with Answers

1. In what sense, Maxwell's electromagnetic theory is different from Huygen's wave theory?

➩ According to Maxwell's electromagnetic theory, light waves are transverse in nature and they do not require any material medium for their propagation. However, in Huygen's wave theory, light waves are longitudinal in nature and they require a material medium for their propagation.

2. Which parameter of light does not change on refraction?

➩ Frequency is the characteristics of source. It does not change on refraction. It is the number of waves produced per unit time, which remains equal in any medium. Therefore, frequency of wave is considered as the most fundamental parameter. However, speed and wavelength of light change on refraction.

3. What is Huygen's principle?

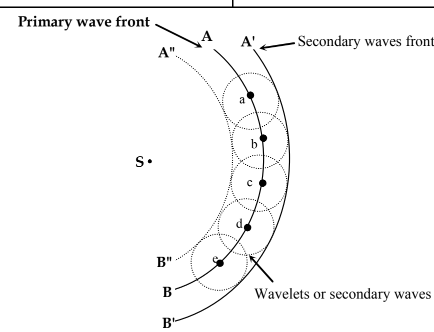
➩ Huygen proposed the following assumptions to explain the wave nature of light. They are:

i. Each point on a wavefront acts as secondary source of light. The newly produced waves are called wavelets or secondary waves.
ii. The secondary waves spread out in all direction with the speed of light in a medium.
iii. The new wavefront at any later time is given by the tangential surface in the forward direction of the secondary wavelets at that time.

---

4. Explain with proper sketch, the differences between wavefronts and wavelets.

➩ Main differences between wavefronts and wavelets are as follows:

Wavefronts	Wavelets
1. A wavefront is defined as continuous locus of all particles of the medium after same time which are vibrating in the same phase.	1. Each point on a wavefront acts as a fresh source of new disturbance, which are called wavelets.
2. Wavefronts are produced from the original source of light. They may also be sketched from wavelets.	2. Wavelets are produced from wavefronts.
	

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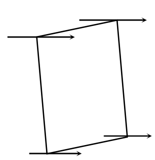
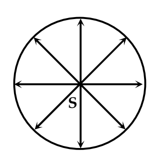
5. Differentiate between plane wavefront and a spherical wavefront.

➩ The differences between plane wavefront and spherical wavefront are as follows:

Plane wavefront	Spherical wavefront
1. A small portion of spherical or cylindrical wavefront when observed from a long distance is called plane wavefront.	1. The waves originating from a point source are spherical in shape and the wavefront so produced is called spherical wavefront.
2. It is plane shaped.	2. It is spherical in shape.
3. 	3. 

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6. A normally incident wavefront does not deviate from the boundary of two different media, why?

➩ When a wavefront AB incident normally on an interface of denser medium, then all the points of that wavefront produce wavelets on the interface at the same time. Then, each wavelet grows equally (i.e. equal radii) into the denser medium. So that refracted wavefront is also parallel to the incident wavefront. As the incident wavefront and refracted wavefront are parallel to each other, light appears undeviated from the interface.

---

7. Can two wavefronts cross one another?

➩ No, it is not possible. If they intersect each other, there must be two different directions of propagations of energy at the point of intersection which is practically impossible.

8. When a wave undergoes reflection at a denser medium, what happens to its phase and frequency?

(i) When a wave is reflected from the surface of denser medium, it undergoes a phase change of π radian. (ii) The frequency of wave does not change in reflection because it is the characteristics of source.

9. What are the reasons to believe that light is a wave motion?

Light wave undergoes interference, diffraction and polarization. These phenomena establish that light is a wave motion.

10. How is a wavefront different from a ray?

A wavefront is a surface obtained by joining all points vibrating in the same phase. A ray is line drawn perpendicular to the wavefront in the direction of propagation of light wave.

11. What determines the shape of a wavefront?

The shape of wavefront depends on two main factors: the shape of source and the distance of observed wavefront from the source. For example, a spherical wavefront is obtained near a point source, but it appears a plane wavefront at very far distance from the source.

Exercises

Short-Answer Type Questions

1. Who proposed wave theory of light first?
2. Differentiate between ray and wavefront of light.
3. Why backward flow of energy is discarded?
4. What are the use of microwaves and infrareds?
5. Normally incident light does not deviate from the boundary of transparent media, why?
6. The sun is considered a point source of light, why?
7. Name the various theories of light.
8. What were the drawbacks of the corpuscular theory?
9. What is a wavefront? Mention the different types of wavefront.
10. Mention the sources of (i) Spherical wavefront (ii) Cylindrical wavefront and (iii) plane wavefront.
11. Explain Huygen's principle. What are secondary wavelets?
12. When a light ray passes from one medium to another the ray bends at the surface of separation. Why?

Long-Answer Type Questions

1. Discuss the various theories regarding the nature of light.
2. Verify laws of reflection of light using Huygen's principle.
3. State Huygen's principle. Prove the laws of reflection of light using the wave theory. [HSEB 2072]
4. Verify laws of refraction of light using Huygen's principle. [HSEB 2064]
5. Define Huygen's principle and prove Snell's law by the help of wave theory of light. [HSEB 2063]
6. State and explain Huygen's principle. Use the principle to show that a plane wavefront incident obliquely on a plane mirror is reflected as a plane wavefront so that the angle of incidence is equal to the angle of reflection. [NEB 2074]

Multiple Choice Questions

1. Who gave concept of corpuscular theory of light?

   a. Newton b. Huygen
   c. Maxwell d. Planck

2. Which parameter does not change on reflection from the surface of denser medium?
   a. Velocity
   b. Frequency
   c. Wavelength
   d. Phase
3. Which wave has the longest wave length?
   a. Infrared
   b. Visible
   c. Ultravoilet
   d. X-rays
4. Which phenomenon shows the wave nature of light?
   a. photoelectric effect
   b. Compton effect
   c. Interference
   d. Pair production
5. Who introduced the wave property of light?
   a. Newton
   b. Archimedes
   c. Planck
   d. Huygen
6. Which experiment discarded the concept of 'ether' in the universe?
   a. Double slit experiment
   b. Michelson-Morley experiment
   c. Fresnel experiment
   d. Doppler's experiment

Answers

1. (a) 2. (b) 3. (a) 4. (c) 5. (d) 6. (b)

❖ ❖ ❖

INTERFERENCE OF LIGHT

7

CHAPTER

7.1 Introduction

Interference is a phenomenon in which two or more waves overlap so that a resultant wave is formed whose amplitude may be greater, lower or same as the amplitudes of original waves. It is a basic property of light that exhibits its wave nature. Interference refers to the interaction of waves that are similar to each other and in fact correlated to each other. This phenomenon can be observed in all types of waves like light waves, sound waves and matter waves.

"The phenomenon of redistribution of energy in the resultant light wave formed by the superposition of two light waves having same frequency (or wavelength) and constant phase difference is called interference of light." The term "redistribution of energy" means that shifting of energy from one place to another. When two light waves from coherent sources superimpose in such a way that the energy is imparted into definite ways: one in which energy appears and the other in which energy disappears completely.

The interaction effects can be studied using two identical light sources (having same wavelengths and certain phase difference). When waves from these sources superimpose to each other, a resultant wave is formed whose amplitude changes but frequency remains unchanged. When a crest of one wave overlaps to the crest of another wave or trough of one wave overlaps to the trough of another wave, the resultant wave will have greater amplitude. If the crest of one wave overlaps to trough of another wave or vice versa, the resultant wave will have low amplitude.

7.2 Coherent Sources

The interference occurs when two waves of some special characters overlap to each other. These characters of waves are their wavelengths, amplitudes and phases. The sustainable interference patterns can be obtained only when two interfering waves possess the equal wavelength, and constant phase difference. Two waves which obey the above requirements to produce the observable interference are called coherent sources. Therefore, *two sources of light are said to be coherent when they have the equal wavelength, and constant phase difference*. Same frequency can maintain constant phase difference over a time and distance, hence these sources can be coherent. Otherwise, the sources are not coherent.

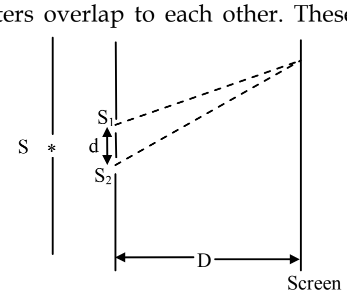


Fig. 7.1: Coherent sources

Two independent sources can never be coherent. To be coherent, both the sources must depend on same origin as when in Fig. 7.1. Since the sources are based on single source, it is named coherent.

Light sources emit the waves of very short wavelength (~ 10^{-7} m). These waves are produced from sources as a series of pulses of energy. The pulses so generated last for a very short time about a nanosecond (~ 10^{-8} s). Within the length 10^{-7} m and time interval 10^{-8} s, there is an abrupt change in the phase of the waves. Waves from two separate sources may be in phase at one instant, but out of phase in the next instant. In the fraction of nanosecond, the human eye is not so sensitive to catch such rapid changes, so the interference pattern is not observable. To produce the sustainable interference pattern, two light waves must be coherent, i.e. equal wavelength, amplitude and constant phase difference. However, to produce the interference patterns, it is not essential for the amplitudes of the waves from two sources to be the same. If two sources produce the waves of unequal amplitudes, completely dark patterns never be obtained. This reduces the contrast of interference patterns.

Note

Interference can be observed with two in dependent tuning forks but it cannot be observed two independent bulbs, why?

i. *If two tuning forks are struck simultaneously they produce sound waves almost in same phase. Their phase difference varies slowly with time. Such variations can be detected easily by the human ear. So, interference pattern is easily observable.*

ii. *The phase difference between two independent light sources changes about 10^8 times per second. Therefore the interference pattern also changes 10^8 times per second. Such rapid variations cannot be detected by our eyes. So, interference pattern is not observable.*

Constructive and Destructive Interference

When the crest of one wave overlaps the crest of another or trough of one wave overlaps the trough of another, their individual effects add together. The result is a wave of increased amplitude. This is known as constructive interference. It is also called reinforcement. The constructive interference of two waves in same phases is shown in Fig. 7.2 (i).

If crest of one wave overlaps the trough of another, their individual effects are reduced. The high part (i.e. crest) of one wave falls to the depth (trough) of another. This is called destructive interference. It is also called cancellation. The destructive interference occurs by overlapping of two waves in opposite phases which is as shown in Fig. 7.2 (ii).

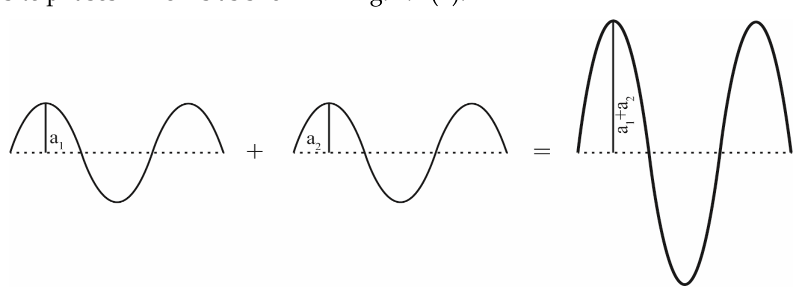


Fig. 7.2 (i): Constructive interference

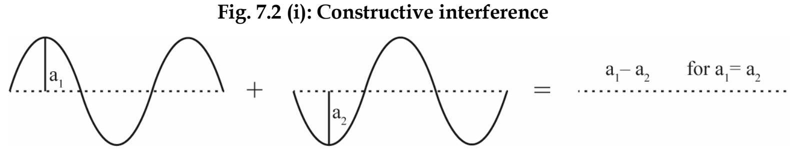


Fig. 7.2 (ii): Destructive interference

If the two incoming waves have the same frequency f and equal amplitude a, the resultant wave produced by constructive interference has an amplitude double than individual wave (i.e. 2a). The frequency of the resultant is the same as that of incoming waves. If the two waves have equal amplitudes and overlap out of phase (i.e. 180° or π radian), the resultant wave has zero amplitude.

Conditions for Sustained Interference

In order to obtain the phenomenon of continuous or sustained interference of light the following conditions should be satisfied.

i. The two sources must be coherent.
ii. The interfering waves should have a constant phase difference.
iii. The amplitude of the interfering waves must be equal or nearly equal.
iv. The two sources must lie very close to each other.
v. The distance of the screen from the coherent sources should be large as compared to the width of slit.
vi. Two interfering sources should emerge from narrow slits (dimension of the order of wavelength of incident light).

Optical Path

The speed of light is different in different media. Its speed is maximum in vacuum ($c = 3 \times 10^8\ ms^{-1}$). If light is allowed to pass through air and water at a time, it travels different distance in equal interval of time.

The distance which the light travels in vacuum during the same time for which it travels in a medium is called optical path.

Let c and v be the speed of light in vacuum and a medium respectively. The distance travelled by light in vacuum and that medium in equal interval of time is determined as,

The distance travelled by light in vacuum, $d = ct$

$$\text{i.e.}\quad \mathbf{t = \frac{d}{c}} \qquad \ldots(7.1)$$

And the distance travelled by light in that medium, $x = vt$

$$\text{i.e.}\quad \mathbf{t = \frac{x}{v}} \qquad \ldots(7.2)$$

For equal interval of time,

$$\frac{d}{c} = \frac{x}{v}$$

$$d = \frac{c}{v}x$$

$$\mathbf{d = \eta x} \qquad \ldots(7.3)$$

In equation (7.3), d is the optical path and x is the geometrical path of light. This expression also tells that optical path is the product of refractive index of a medium and geometrical path in that medium.

In air $\eta \approx 1$. So, the geometrical path in air is almost equal to the optical path. In other media, refractive index is greater than 1, so the optical path is always greater than the geometrical path.

Insertion of Denser Medium

When a thin transparent sheet of thickness 't' and refractive index 'η' is inserted in one of the interfering beams, the path difference is determined by using following technique.

Consider two points A and B at d distance apart in air (or in vacuum) as shown in Fig. 17.3. If a denser object (for example a mica sheet) of thickness 't' is inserted between A and B, then the optical path is changed. Let L be the optical path between A and B, then

L = Optical path in air + Optical path in medium
$= (d - t) + \eta t$
$= d + (\eta - 1)t$

Now, optical path difference in above cases,

$$x = L - d = (\eta - 1)t \quad \text{...(7.4)}$$

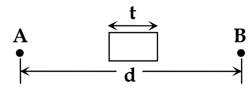


Fig. 7.3: Insertion of denser medium

Principle of superposition

The principle of superposition states that, when two or more waves meet at a point, the resultant displacement of wave at that point is equal to the sum of the displacements of the individual waves at that point.

Let $Y_1, Y_2, Y_3, \ldots, Y_n$ be the displacements of waves that meet at a point is space. Then, the resultant displacement at the point where they meet is,

$Y = Y_1 \pm Y_2 \pm Y_3 \pm \ldots \pm Y_n$

The displacement is a vector quantity, therefore the individual displacements are added taking the account of their directions. The principle applies to all types of wave like sound wave, radio wave, microwave etc.

7.3 Analytical Treatment of Interference of Light

Consider two light waves of same angular frequency 'ω' and amplitudes a_1 and a_2 emitting from two slits acting as coherent sources S_1 and S_2 as shown in Fig. 7.4. The nature of interference is obtained on the screen at distance D away from the slits S_1 and S_2.

Let Y_1 and Y_2 be the displacements of two light waves emitted from coherent sources S_1 and S_2 respectively. The wave equations for these waves are,

$$Y_1 = a_1 \sin \omega t \quad \text{... (7.5)}$$

$$Y_2 = a_2 \sin (\omega t + \phi) \quad \text{... (7.6)}$$

Where ϕ is the phase difference of the waves which are taken for the consideration. The value of ϕ depends on what difference of path the waves travel from their sources to the position of interference on the screen.

(i. e. $\phi = \frac{2\pi}{\lambda} x$)

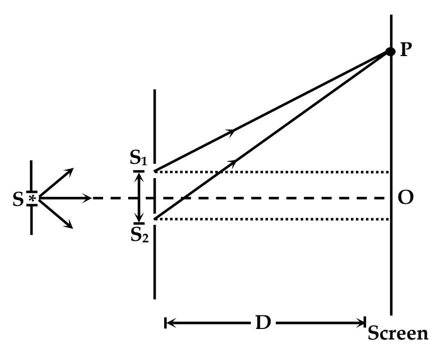


Fig. 7.4: Theory of interference fringes

If Y be the displacement of resultant wave due to the superposition of Y_1 and Y_2, the wave equation for resultant wave is,

$$Y = Y_1 + Y_2 \quad \text{. . . (7.7)}$$

Substituting equations (7.5) and (7.6) in equation (7.7), we get,

$$Y = a_1 \sin \omega t + a_2 \sin (\omega t + \phi)$$

$$= a_1 \sin \omega t + a_2 \sin \omega t \cos \phi + a_2 \cos \omega t \sin \phi \quad \text{. . . (7.8)}$$

As two waves of similar nature are superimposed, the resultant wave also behaves as the same nature with same frequency, but different amplitude and phase. So, we take the following consideration for the resultant wave.

$$a_1 + a_2 \cos \phi = A \cos \theta \quad \text{. . . (7.9)}$$

$$\text{and } a_2 \sin \phi = A \sin \theta \quad \text{. . . (7.10)}$$

Where A and θ are the amplitude and initial phase of resultant wave.

Using the equations (7.9) and (7.10) in equation (7.8), we get,

$$Y = A \sin \omega t \cos \theta + A \cos \omega t \sin \theta$$

$$= A (\sin \omega t \cos \theta + \cos \omega t \sin \theta)$$

$$Y = A \sin (\omega t + \theta) \quad \text{. . . (7.11)}$$

Equation (7.11) is the wave equation for resultant wave.

Amplitude and initial phase of resultant wave

Squaring and adding equations (7.9) and (7.10), we get,

$$\text{or,} \quad A^2 \cos^2 \theta + A^2 \sin^2 \theta = (a_1 + a_2 \cos \phi)^2 + a_2^2 \sin^2 \phi$$

$$\text{or,} \quad A^2 (\cos^2 \theta + \sin^2 \theta) = a_1^2 + 2a_1a_2 \cos \phi + a_2^2 \cos^2 \phi + a_2^2 \sin^2 \phi$$

$$\text{or,} \quad A^2 = a_1^2 + 2a_1 a_2 \cos \phi + a_2^2 (\cos^2 \phi + \sin^2 \phi)$$

$$\text{or,} \quad A^2 = a_1^2 + 2a_1 a_2 \cos \phi + a_2^2$$

$$\therefore \quad A^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos \phi$$

$\therefore$ The resultant amplitude,

$$A = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos \phi} \quad \text{. . . (7.12)}$$

Dividing equation (7.9) by equation (7.10), we get,

$$\frac{A \sin \theta}{A \cos \theta} = \frac{a_2 \sin \phi}{a_1 + a_2 \cos \phi}$$

$$\text{or,} \quad \tan \theta = \frac{a_2 \sin \phi}{a_1 + a_2 \cos \phi}$$

$\therefore$ The resultant phase,

$$\theta = \tan^{-1} \left(\frac{a_2 \sin \phi}{a_1 + a_2 \cos \phi} \right) \quad \text{. . . (7.13)}$$

It is to be noted that, the interference of two waves can be observed when the amplitudes of two waves have equal magnitude.

Suppose, $a_1 = a_2 = a$, equations (7.12) and (7.13) are written as follows,

$$\therefore \quad A^2 = a^2 + a^2 + 2a^2 \cos \phi$$

$$= a^2 (2 + 2 \cos \phi)$$

$$= 2 a^2 (1 + \cos \phi) = 2 a^2 . 2 \cos^2 \frac{\phi}{2}$$

$$\text{or,} \quad A^2 = 4a^2 \cos^2\frac{\phi}{2}$$

$$\therefore \quad \mathbf{A = \pm 2a \cos\frac{\phi}{2}} \qquad \mathbf{...(7.14)}$$

Equation (7.14) shows that the amplitude of resultant wave depends on the phase difference ϕ of original waves at the location of superposition and on the amplitude of each interfering waves.

$$\text{i.e.} \quad A \propto \cos\frac{\phi}{2}$$

The intensity of wave is directly proportional to the square of amplitude,

i.e. $I \alpha A^2$, then,

$$I \propto \cos^2\frac{\phi}{2}$$

Also, the initial phase of resultant wave is

$$\theta = \tan^{-1}\left(\frac{a \sin\phi}{a + a\cos\phi}\right)$$

$$= \tan^{-1}\left(\frac{\sin\phi}{1 + \cos\phi}\right)$$

$$= \tan^{-1}\left(\frac{2\sin\frac{\phi}{2}\cos\frac{\phi}{2}}{2\cos^2\frac{\phi}{2}}\right) = \tan^{-1}\left(\tan\frac{\phi}{2}\right)$$

$$\therefore \quad \theta = \frac{\phi}{2}$$

Conditions for constructive and destructive interference

i. Constructive interference:

The constructive interference of two waves provides the maximum intensity at a position. For such condition, the term $\cos^2\frac{\phi}{2}$ must be maximum.

$$\text{i.e.} \quad \cos^2\frac{\phi}{2} = 1$$

$$\cos\frac{\phi}{2} = \pm 1$$

$$\cos\frac{\phi}{2} = \cos n\pi, \text{ where } n = 0, 1, 2, 3, \ldots$$

$$\therefore \quad \frac{\phi}{2} = n\pi$$

$$\boldsymbol{\phi = 2n\pi} \qquad \mathbf{...(7.15)}$$

It concludes that, constructive interference (maximum intensity) is obtained when the phase difference of two waves is in the order of 0, 2π, 4π, 6π, . . . $2n\pi$. In such condition, bright patterns (bright fringe) are produced on the screen.

The relation of path difference and phase difference is.

$$\text{Phase difference, } (\phi) = \frac{2\pi}{\lambda} \times \text{Path difference (x)}$$

$$\text{i.e.} \quad \phi = \frac{2\pi}{\lambda}.\,x$$

$$\therefore \quad \mathbf{x = \frac{\lambda}{2\pi}\phi} \qquad \ldots (7.16)$$

Where, λ is the wavelength of light emitted by coherent sources.

Using equation (7.15) in equation (7.16), we get

$$x = \frac{\lambda}{2\pi}.\, 2n\pi$$

$$\mathbf{x = n\lambda} \qquad \ldots (7.17)$$

Therefore, for constructive interference, the path difference of two waves must be integral multiple of wavelength (λ) of source, i.e. 0, λ, 2λ, 3λ, 4λ, . . ., nλ. This condition can be summarized as,

Order (n)	Phase difference (ϕ)	Path difference (x)	Fringe
n = 0	0	0	Central bright
n = 1	2π	λ	First bright
n = 2	4π	2λ	Second bright
n = 3	6π	3λ	Third bright
. . .	. . .	. . .	. . .
n = n	2nπ	nπ	n^{th} bright

ii. Destructive interference

Destructive interference provides the minimum intensity of light. For such condition, the term $\cos^2\frac{\phi}{2}$ must be minimum.

$$\text{i.e.} \quad \cos^2\frac{\phi}{2} = 0$$

$$\cos\frac{\phi}{2} = 0$$

$$\cos\frac{\phi}{2} = \cos\left(\frac{2n+1}{2}\right)\pi, \quad \text{where } n = 0, 1, 2, 3, \ldots$$

$$\therefore \quad \frac{\phi}{2} = \left(\frac{2n+1}{2}\right)\pi$$

$$\therefore \quad \mathbf{\phi = (2n+1)\pi} \qquad \ldots (7.18)$$

Therefore, for destructive interference (i.e. minimum intensity), the phase difference of two waves must be odd multiple of π, i.e. π, 3π, 5π, . . ., (2n + 1)π. In such condition, dark patterns (dark fringes) are produced on the screen.

Also, the path difference, $\mathbf{x = \frac{\lambda}{2\pi}\phi}$ . . . (7.19)

Now, using equation (7.18) in equation (7.19), we get,

$$x = \frac{\lambda}{2\pi}(2n+1)\,\pi$$

$$\mathbf{x = (2n+1)\frac{\lambda}{2}} \qquad \ldots (7.20)$$

It concludes that, the dark patterns are produced on the screen when path difference of interfering waves are in the odd multiple of $\frac{\lambda}{2}$, i.e., $\frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \ldots, (2n+1)\frac{\lambda}{2}$.

This condition can be summarized as

Order of dark fringe (n)	Phase difference (ϕ)	Path difference (x)
n = 0	π	$\frac{\lambda}{2}$
n = 1	3π	$\frac{3\lambda}{2}$
n = 2	5π	$\frac{5\lambda}{2}$
n = 3	7π	$\frac{7\lambda}{2}$
. . .	. . .	. . .
n = n	$(2n + 1)\pi$	$(2n + 1)\frac{\lambda}{2}$

In conclusion, the path difference in the position and spacing of bright and dark fringes are tabulated below.

Path difference (in the multiple of λ)	0	-	1	-	2	…	-	n	Bright
	-	0.5	-	1.5	-	…	(2n + 1)	-	Dark

This shows that alternate bright and dark fringes of equal width are produced on the screen with equal spacing in Young's double slit experiment.

Note:

Students may be confused in writing the order of dark fringe, whether (2n + 1) or (2n – 1). To remove the confusion, we use the following techniques.

i. *If the order is represented by (2n – 1), the value of n should be started from 1, i.e., n = 1, 2, 3, . . .*

ii. *If the order is represented by (2n + 1), the value of n should be started from 0 i.e. n = 0, 1, 2, 3, . . .*

Students must be careful in writing the order, in writing the conditions for Young's double slit experiment we use the order (2n - 1), for the dark fringe because central fringe is bright one, no dark fringe is in zero order.

But in Newton's ring experiment, central ring is dark, so we use (2n + 1) to represent the order of dark ring.

For dark pattern, the phase difference and path difference of two waves, may be expressed in the following form.

a. *Phase difference* $(\phi) = (2n - 1)\pi$, *for* $n = 1, 2, 3, \ldots$

b. *path difference* $(x) = (2n - 1)\frac{\lambda}{2}$, *for* $n = 1, 2, 3, \ldots$

(If central fringe is not dark, this condition is more appropriate).

Interference and Principle of Conservation of Energy

It appears that the energy is destroyed at the position of dark pattern and energy is created in the bright pattern of interference. So, principle of conservation of energy appears to be violated. But in reality, it is not so. The energy is only transferred from dark pattern to bright pattern - average energy being always equal to the sum of energies of the interfering waves.

Let I be the intensity ($\propto$ energy) of a wave of amplitude a. The energy,

$$E \propto I \propto a^2$$

For two identical waves,(energy before interference)

$$\textbf{Total energy} \propto 2I \propto 2a^2 \qquad \ldots(7.21)$$

To find energy after interference:

The amplitude of constructive interference, (i.e. bright pattern) = a + a = 2a

$\therefore$ Energy $\propto I \propto (2a)^2$

$\therefore$ $I_{max} \propto 4a^2$

The amplitude of destructive interference, (i.e. dark pattern) = a – a = 0

$\therefore$ $I_{min} = 0$

$$\therefore \text{Average energy} \propto \frac{I_{max} + I_{min}}{2}$$

$$\propto \frac{4a^2 + 0}{2}$$

$$\propto \mathbf{2a^2} \qquad \ldots(7.22)$$

Comparing equations (7.21) and (7.22), we conclude that average energy of dark and bright pattern is equal to the sum of energy contributed by two waves individually.

7.4 Young's Double Slit Experiment

Young's double slit experiment is a powerful evidence of wave nature of light. Thomas Young, in 1801, demonstrated how light waves could produce the interference patterns. His experiment is famous in the name of Young's double slit experiment after his name Young and double slit apparatus.

The experimental arrangement of Young's double slit experiment is shown in Fig. 7.5. It consists of two slits S_1 and S_2 in front of a monochromatic primary light source. These slits S_1 and S_2, receive the light from primary source S, and share the light of same wave front. Therefore, they are coherent and create a sustained and observable interference patterns. Light waves diffracted from two slits interfere in front of the slits and interference patterns can be visualized on the screen (usually, the screen is a travelling microscope). Bright fringes (B) are seen where constructive interference occurs and dark fringes (D) are seen where destructive interference occurs.

In the Fig. 7.5, similar phases of two waves interfere at the center line of the screen, hence this line is seen bright. At both sides of central bright line, opposite phases of waves overlap. So, the dark lines are obtained. Likewise, beyond side of dark lines, constructive interference occurs, hence the bright lines are obtained. This process is repeated for many other alternate dark and bright lines of equal width as shown in Fig 7.5.

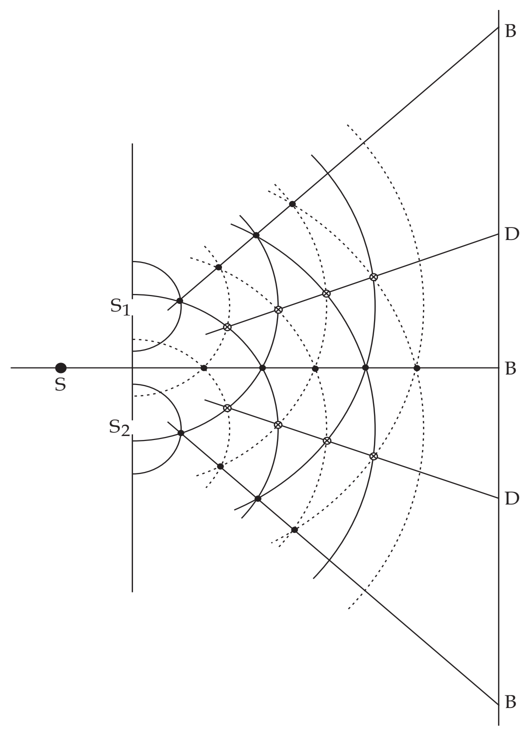


Fig. 7.5: Young's double slit experiment

Although Young's original double slit experiment was carried out with light, the conditions for constructive and destructive interference apply for all types of wave like sound waves, water waves microwaves etc.

Note

White light fringes:

The interference phenomenon of light can be studied by using white light. If the double slits are illuminated with a white light, each of the colour produces its own fringe pattern. At the center of the pattern, a white maximum is seen because of the zero path difference for all colours. Thereafter, the maxima and minima of the different colours overlap in such way as to produce a pattern of coloured fringes. In the careful observation, only a few coloured are visible due to the overlapping of different colours.

7.5 Theory of interference

Consider two monochromatic coherent light sources S_1 and S_2 depending on a single monochromatic light source S. Let the slit separation (i.e. distance between S_1 and S_2) is d and a screen is taken at D distance away from the slits as shown in Fig. 7.6. The waves propagating from slits S_1 and S_2 should be ensured that they must have the same phase. Then, the phenomenon of superposition of light is observed on the screen.

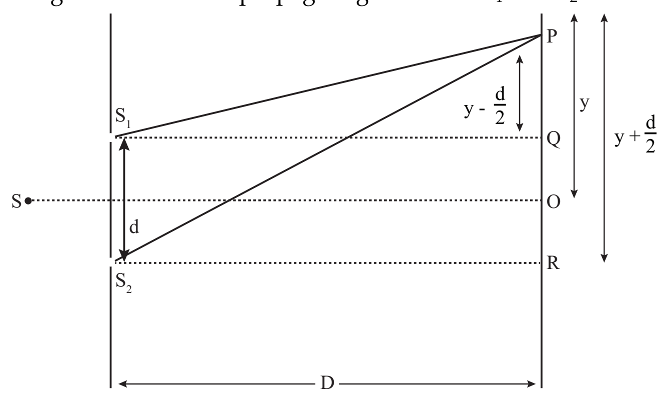


Fig. 7.6: Interference of Light

Let us take a point P on the screen at distance y from the centre of screen O. The light waves emerging from slits S_1 and S_2 continuously fall upon the point P with certain phase difference so that the interference patterns can be observed on it.

Let x be the path difference of two waves to reach at P after emerging from S_1 and S_2 with same phase. Path difference is the difference of distance traveled by light waves. So,

i. To form the constructive interference at P, the path difference (x) = $n\lambda$, where n = 0, 1, 2, 3, . . . and phase difference, $\phi = 2n\pi$, where, n = 0, 1, 2, 3, . . .

ii. Also, to form the destructive interference, at P, the path difference, n = $(2n - 1)\frac{\lambda}{2}$, where, n = 1, 2, 3, …

The actual path difference in the Fig 7.6 is,

$$x = S_2P - S_1P$$

Now, from right angled ΔPRS_2, we have,

$$S_2P^2 = S_2R^2 + PR^2 = D^2 + \left(y + \frac{d}{2}\right)^2$$

$$\mathbf{S_2P^2 = D^2 + y^2 + yd + \frac{d^2}{4}} \qquad \ldots (7.23)$$

Also, from right angled ΔPQS_1, we have,

$$S_1P^2 = S_1Q^2 + PQ^2$$

$$= D^2 + \left(y - \frac{d}{2}\right)^2$$

$$\mathbf{= D^2 + y^2 - yd + \frac{d^2}{4}} \qquad \text{. . . (7.24)}$$

Subtracting equation (7.24) from equation (7.23), We get,

$$S_2P^2 - S_1P^2 = 2yd$$

$$(S_2P - S_1P)\ (S_2P + S_2P) = 2yd$$

The path difference, $x = S_2P - S_1P$, So

$$x.\ (S_2P + S_1P) = 2yd$$

As the point P is considered very near to O, we take $S_2P \approx S_1P = D$ so,

$$x.\ (D + D) = 2yd$$

$$\mathbf{x = \frac{yd}{D}} \qquad \text{. . . (7.25)}$$

In the above consideration, you may be surprised that the approximation ($S_2P \approx S_1P$) is applied only for addition of S_2P and S_1P, however it is not applied in the difference (i.e. why $x = S_2P - S_1P$ is not taken zero, but $S_2P + S_1P = D + D$?)

In fact, the difference $S_2P - S_1P$ is comparable with wavelength of light, which provides the meaning full result in the above expression. But, in case of additional part ($S_2P + S_1P$) > > λ so they can be approximated, this approximation does not alter the result significantly.

Alternative way to calculate path difference:

Here path difference (x) = S_2Q

From ΔS_1QS_2, $\sin\theta = \frac{S_2Q}{S_1S_2} = \frac{S_2Q}{d}$ ...(i)

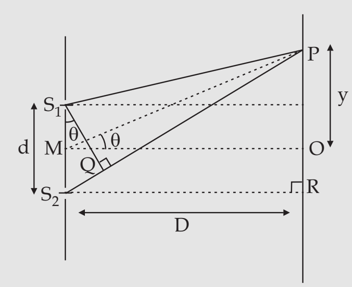
From ΔPOM, $\tan\theta = \frac{PO}{OM} = \frac{y}{D}$ ... (ii)

For very small angle θ, $\tan\theta \approx \sin\theta \approx \theta$

So equating (i) and (ii), we get,

$$\frac{S_2Q}{d} = \frac{y}{D}$$

$S_2Q = \frac{yd}{D}$ i.e. path difference (x) = $\frac{yd}{D}$



Position of bright fringes on the screen

To produce bright fringe on the screen, the path difference of two waves must be integral multiple of wavelength of light (λ).

$$\mathbf{i.e.\ x = n\lambda} \qquad \text{. . . (7.26)}$$

where n = 0, 1, 2, 3, . . . using equation (7.25) in equation (7.26), we get,

$$\frac{yd}{D} = n\lambda$$

$$\therefore \quad \mathbf{y = \frac{n\lambda D}{d}} \qquad \text{. . . (7.27)}$$

For n = 0, $y_0 = 0$ (central bright fringe) For n = 1, $y_1 = \frac{\lambda D}{d}$ (First bright fringe)

For n = 2, $y_2 = \frac{2\lambda D}{d}$ (Second bright fringe) For n = 3, $y_3 = \frac{3\lambda D}{d}$ (Third bright fringe)

. . . For n = n, $y_n = \frac{n\lambda D}{d}$ (n^{th} bright fringe).

Fringe width of bright fringe: The separation of centers of two consecutive bright fringes is called the width of bright fringes. In the above expression, we find, $y_1 - y_0 = y_2 - y_1 = y_3 - y_2 = \ldots = y_n - y_{n-1}$. It means fringe width of every consecutive waves are equal. It is denoted by α.

$$\therefore \quad \alpha = y_n - y_{n-1}$$

$$= \frac{n\lambda D}{d} - \frac{(n-1)\lambda D}{d}$$

$$\alpha = \frac{\lambda D}{d} \qquad \ldots (7.28)$$

Position of dark fringes on the screen

To produce the dark fringe at point P, the path difference of two waves.

$$x = (2n-1)\frac{\lambda}{2} \qquad \ldots (7.29)$$

where n = 1, 2, 3 . . .

It is to be noted that, n = 0 is not used in dark patterns because no dark pattern is produced at the centre of the screen (i.e. at y = 0).

Using equation (7.25) in equation (7.29), we get

$$\frac{yd}{D} = (2n-1)\frac{\lambda}{2}$$

$$y = (2n-1)\frac{\lambda D}{2d} \qquad \ldots (7.30)$$

For n = 1, $y_1 = \frac{\lambda D}{2d}$

For n = 2, $y_2 = \frac{3\lambda D}{2d}$

For n = 3, $y_3 = \frac{5\lambda D}{2d}$

...

For n = n, $y_n = \frac{(2n-1)\lambda D}{2d}$

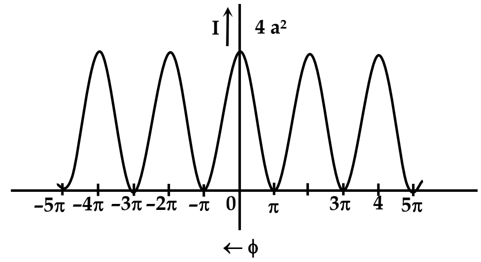


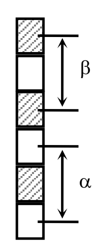
Fig. 7.7: Intensity distribution graph in

Fringe width for dark fringes: The separation of centers of two consecutive dark fringes is called fringe width of dark fringes. In above conditions, every consecutive dark fringes have the equal separation, i.e.

$$y_2 - y_1 = y_3 - y_2 = \ldots = y_n - y_{n-1}$$

The fringe width for dark fringes are denoted by β. So,

$$\beta = \frac{\lambda D}{d} \qquad \ldots (7.31)$$



Comparing equations (7.28) and (7.31), we get

$$\alpha = \beta$$

It concludes that, the fringe width of consecutive bright fringes or consecutive dark fringes is equal in interference fringe patterns.

Angular width of fringe

The angular width is the angle subtended by the centers of two consecutive bright or consecutive dark fringes on the slit. In the Fig 7.8, two bright fringes are taken as the consideration. Here, θ is the angular width of two consecutive fringes.

So, $\tan\theta = \frac{\alpha}{D}$

For very small angle θ, $\tan\theta \approx \theta$

$$\therefore \quad \boldsymbol{\theta = \frac{\alpha}{D}} \qquad \ldots (7.32)$$

Similarly, for dark fringes

$$\boldsymbol{\theta = \frac{\beta}{D}} \qquad \ldots (7.33)$$

$$\text{So,} \quad \boldsymbol{\theta = \frac{\alpha}{D} = \frac{\beta}{D}} \qquad \ldots (7.34)$$

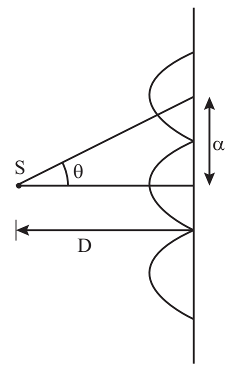


Fig. 7.8: Angular width of fringe

Also, $\alpha = \beta = \frac{\lambda D}{d}$

The angular width,

$$\theta = \frac{\lambda D}{Dd} = \frac{\lambda}{d}$$

$$\therefore \quad \boldsymbol{\theta = \frac{\lambda}{d}} \qquad \ldots (7.35)$$

Conclusions

i. Fringe width is directly proportional to wavelength, (i.e. β α λ), so, the fringe produced by the light of shorter wavelength will be narrow relative to those produced by light of longer wavelength. Since the wavelength of light decreases when whole Young's double slit apparatus is submerged into the water, the fringe width will be smaller.

ii. Fringe width is directly proportional to the screen separation D from the slit. So, farther the screen from the slit, larger the fringe width.

iii. Fringe width is inversely proportional to the slit separation. Smaller the separation between two slits, larger is the fringe width. Therefore, to visualize the interference patterns on the screen, the slit separation must be very small, otherwise the fringe width becomes too small to be detected.

iv. If one of the two slits S_1 and S_2 is covered, the interference pattern will disappear because interference pattern is due to superposition of waves from the two sources S_1 and S_2.

v. If a thin transparent sheet is introduced in the path of one of the two interfering beams, the fringe system gets displaced towards the beam in whose path, the sheet is introduced.

Displacement of Central Bright Fringe

The bright fringe is produced at the center of screen, only when the optical path is same for both waves coming from slits S_1 and S_2 to the center. If a optically denser object is inserted between a slit and the screen, the central bright fringe is displaced from the centre of the screen due to the change in path difference. The displacement of central bright fringe is determined by using following technique.

We know, the distance of any bright fringe from the central fringe is,

$$y = \frac{Dx}{d}, \text{ where } x = \text{path difference}$$

For central bright fringe without inserted object of thickness t is

$$y = 0$$

When the denser object is placed in path, then

$$y' = \frac{Dx'}{d} = \frac{D(\eta - 1)t}{d}$$

The displacement of central bright fringe,

$$\Delta y = y' - y_0 \quad = \frac{D(\eta - 1)t}{d}$$

$$= \left(\frac{D\lambda}{d}\right) \cdot \frac{1}{\lambda} (\eta - 1)t$$

$$\Delta y = \frac{\beta}{\lambda} (\eta - 1)t$$

If Young's double slit apparatus is immersed in a liquid of refractive index η, the wavelength of light decrease to $\lambda' = \frac{\lambda}{\eta}$, so the fringe width reduces to,

$$\beta' = \frac{\lambda' D}{d} = \frac{\lambda D}{\eta d} = \frac{\beta}{\eta}$$

Since $\eta > 1$, $\beta' < \beta$.

7.6 Interference in a thin film

An extremely small thickness of a transparent medium of thickness about the order of 1 wavelength of light in visible region is called a thin film. The film of soap bubble and the oil spread on the water surface are familiar examples of thin film. The light incident from a source when reflected from the upper and lower surface of the thin film acts as the coherent sources. The waves along R_1 and R_2 represent the coherent sources in Fig. 7.9. Due to the interference of such reflected waves from the different surfaces of soap bubble, beautiful colours are produced.

The net path difference of light reflected from thin film can be determined considering a parallel sided thin film of thickness t and refractive index η as shown in Fig. 7.9. Suppose a ray IA of monochromatic light is incident on its upper surface. This wave suffers partial reflection and refraction from the upper surface. The refracted ray, then, reflects from the lower surface of the film and finally emerges out from the upper surface. Let R_1 and R_2 be the path of waves from upper and lower surface of the film respectively as shown in Fig. 7.9.

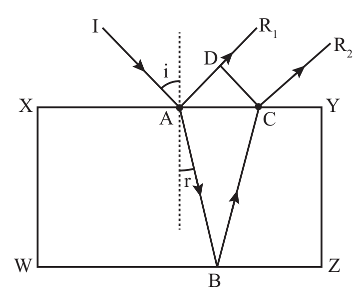


Fig. 7.9: Interference in a thin film

Interference by reflection

In such condition, the path difference, between two successive reflected rays is,

$$x = 2\eta t \cos r + \frac{\lambda}{2}$$

The term $\frac{\lambda}{2}$ is obtained due to the interference by reflection of wave.

In the equation, ( )

i = Angle of incidence on upper surface of thin film

r = Angle of refraction in the refracted medium at same surface

λ = Wavelength of light used.

In Newton's ring experiment, the thin film is the air film of refractive index, $\eta = 1$ enclosed in the space between the plano - convex lens and a glass plate. The light rays fall perpendicularly on the plano - convex less,

so, $\cos r = \cos 0^o = 1$

Hence, the path difference of two reflected light rays, $x = 2t + \frac{\lambda}{2}$

7.7 Newton's Ring

Newton's rings is a phenomenon in which an interference pattern is created by the reflection of light between two surfaces - a spherical surface and an adjacent touching flat surfaces. Newton's rings phenomenon named so after the name of Sir Isaac Newton, who studied the effect in 1717. The interference patterns produced in Newton's rings phenomenon are circular in shape. Therefore, these circular rings are also called Newton's rings. It is to be noted that Newton's rings is a phenomenon and also the name of interference patterns.

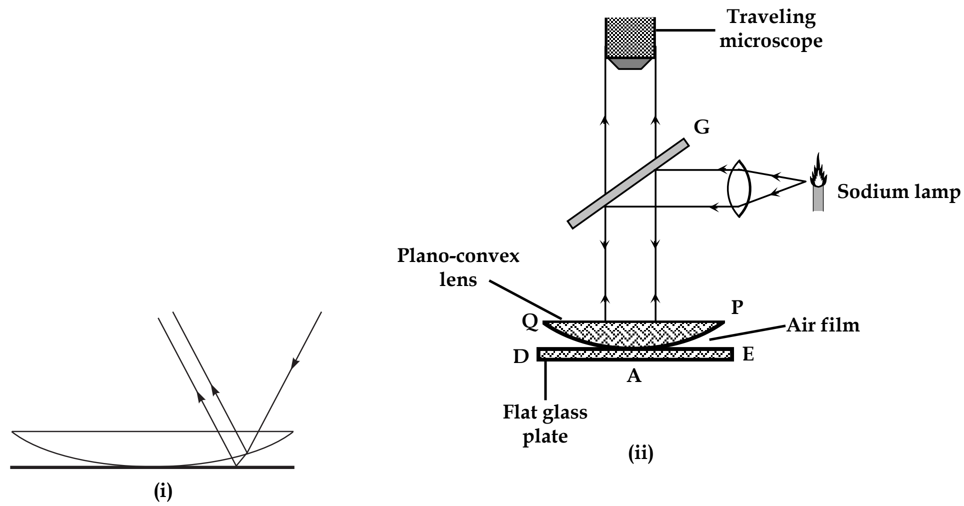


Fig. 7.10: (i) sketch for coherent sources (ii) Arrangement for Newton's Ring Experiment

Newton's rings phenomenon depends on the interference by reflection in a thin film. The layer of air of varying thickness between the Plano - Convex lens and a glass plate acts as the thin film. The path difference between the reflected rays from bottom of Plano – Convex lens and upper surface of glass plate depends upon the thickness of the air gap between them. As the lens is symmetric along its axis, the thickness is constant along its circumference of a ring of a given radius. Hence, Newton's rings are circular in shape.

The apparatus arrangement to produce the Newton's rings consists of a Plano - Convex lens placed on a plane glass plate, facing curve surface of lens towards the plate. A monochromatic light is refracted through a convex lens in such a way that the refracted rays are parallel to each other. These parallel rays then fall upon a glass plate G such that these rays are partially reflected towards the

Plano – Convex lens. The glass plate G is so inclined that the rays reflected from it falls normally on the Plano - convex lens. The interference Phenomenon is observed from the travelling microscope.

When the parallel rays of light fall on the Plano-Convex lens, the reflection takes place from the lower surface of Plano–Convex lens and upper surface of its base plate as shown in Fig. 7.10 (ii). The rays reflected from two different surfaces act as the coherent sources. The superposition of these rays produces the interference patterns that can be observed through the travelling microscope. At the point of contact of Plano - Convex lens and base plate, the thickness of air film is zero. So, there is no geometrical path difference, but phase is reversed by π due to reflection of light at an optically denser medium. Hence, the center of rings is dark spot. Since the thin air film has symmetrically varying thickness from center to edge of the Plano-Convex lens, the bright and dark rings of gradually increasing radii are obtained in the experiment. As the radii of rings increases, the separation of the rings decreases as shown in Fig. 7.10 (iii).

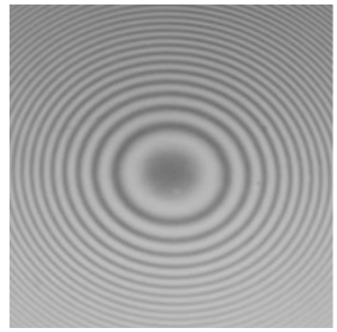


Fig. 7.10(iii): Newton's rings

Radii of Newton's rings

Let the curvature of spherical surface of Plano – Convex lens is completed considering O as the center of the sphere as shown in Fig. 7.11. Let R be the radius of curvature of the lens. The thickness of the air film is zero at the center and 't' at the edge of the lens. Consider P and Q be the two points of the lens. PM and QN be the maximum thickness of air film such that PM = QN = t. Also, OP = CQ = r be the radius of largest ring where air film has thickness t.

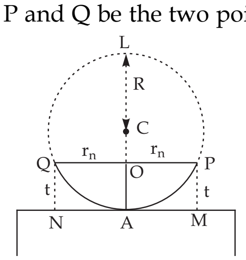


Fig. 7.11 Demonstration of radius of Newton's ring

Let CL = CA = R = Radius of curvature of the lens and PQ = Cord of the sphere

From the theorem of intersecting cord, we have

$$PO \times OQ = LO \times OA$$

$$r \times r = (2R - t) \times t$$

$$r^2 = 2Rt - t^2$$

The value of t^2 is very small as compared to 2Rt. So, it can be neglected.

$$\therefore \quad \mathbf{r^2 = 2Rt} \qquad \text{. . . (7.36)}$$

Also, the path difference of rays in interference by reflection in thin film is,

$$\mathbf{x = 2t + \frac{\lambda}{2}} \qquad \text{. . . (7.37)}$$

i. For bright rings, path difference,

$$\mathbf{x = n\lambda.} \qquad \text{. . . (7.38)}$$

Where n = 1, 2, 3, . . .

So, equating equation (7.37) and equation (7.38), we get,

$$2t + \frac{\lambda}{2} = n\lambda,$$

$$2t = n\lambda - \frac{\lambda}{2}$$

$$\mathbf{2t = \left(\frac{2n - 1}{2}\right)\lambda} \qquad \text{. . . (7.39)}$$

Now, substituting the value of 2t from equation (7.39) in equation (7.36), we get

$$r^2 = R.\left(\frac{2n-1}{2}\right)\lambda$$

$$r^2 = \left(\frac{2n-1}{2}\right)\ \lambda R$$

Therefore the radius of nth bright ring can be determined from.

$$r_n^2 = \left(\frac{2n-1}{2}\right)\lambda R$$

$$\therefore \quad \mathbf{r_n = \sqrt{\left(\frac{2n-1}{n}\right)\lambda R}} \qquad \text{. . . (7.40)}$$

ii. For dark rings, the path difference,

$$\mathbf{n = \left(\frac{2n+1}{2}\right)\lambda} \qquad \text{. . . (7.41)}$$

Where, n = 0, 1, 2, 3, . . .

Now, equating the equations (7.37) and (7.41), we get,

$$\left(\frac{2n+1}{2}\right)\lambda = 2t + \frac{\lambda}{2}$$

$$n\lambda + \frac{\lambda}{2} = 2t + \frac{\lambda}{2}$$

$$\therefore \quad \mathbf{2t = n\lambda} \qquad \text{. . . (7.42)}$$

Now, substituting the value of 2t from questions (7.42) in equation (7.37), we get.

$$r^2 = R \cdot n\lambda$$

$$\therefore \quad \mathbf{r^2 = n\lambda R} \qquad \text{. . . (7.43)}$$

The radius of nth dark ring is determined from

$$r_n^2 = n\lambda R$$

$$\therefore \quad \mathbf{r_n = \sqrt{n\lambda R}} \qquad \text{. . . (7.44)}$$

It shows that, the radius $r_n = 0$ only when n = 0 in dark ring, because $r_n \propto \sqrt{n}$. But $r_n \neq 0$ in case of bright ring because $r_n \propto \sqrt{2n-1}$. Hence, it can be concluded that, the center ring in Newton's rings experiment for interference by reflection is dark in nature. But the situation is different, if the Newton's rings are obtained from the interference by transmission through thin films. In such case, center ring is bright in nature as shown in Fig. 7.10 (ii).

Determination of Wavelength of Light

Newton's ring experiment is performed to determine the wavelength of light.

If D_n be the diameter of the nth dark ring, then $r_n = \frac{D_n}{2}$

$$\therefore \quad \frac{D_n}{2} = \sqrt{n\lambda R}$$

$$\text{or,} \quad \mathbf{D_n^2 = 4n\lambda\ R} \qquad \text{. . . (7.45)}$$

Similarly, if D_{n+m} be the diameter of (n + m)th dark ring then we can write

$$\mathbf{D^2_{n+m} = 4\ (n+m)\ \lambda\ R} \qquad \text{. . . (7.46)}$$

From (7.45) and (7.46), we get,

$$D^2_{n+m} - D^2_n = 4m\lambda R$$

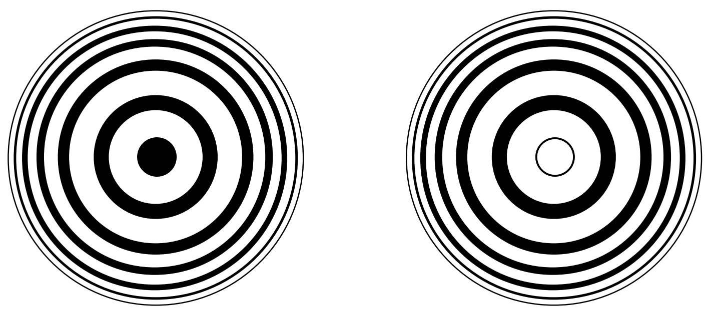
$$\text{or,} \quad \lambda = \frac{D^2_{n+m} - D^2_n}{4mR} \qquad \ldots (7.47)$$

Measuring the diameters of (n + m)th and nth dark fringes, the wavelength of monochromatic light is determined.

Equation (7.47) is also applicable for bright fringes.

Regarding Newton's Ring

1. The nature of centred Newton's ring is different in different situations.
   i. A dark ring is obtained at the centre when interference occurs due to the reflected light as shown in Fig. 7.12 (i).
   ii. A bright ring is obtained at the centre when interference occurs due to the transmitted light as shown in Fig. 7.12. (ii).
2. The centre ring is considered zero order whether the nature of ring is dark or bright.



(i) (ii)

Fig. 7.12: (i) Newton's ring due to reflected light (ii) Newton's ring due to transmitted light

3. The centre ring is dark, although white light is used to produce Newton's ring. However, other rings are seen colourful due to the rings obtained by different wavelengths of light, as different colours have different wavelength.
4. The width of rings decreases in accordance with the increasing order (n) as shown in Fig. 7.12.

Applications of Interference

The phenomenon of interference has a wide range of applications:

i. It is used in optical communications and sending messages. It is used to prevent the waves from overlapping and losing data.
ii. Michelson and Morley disproved the existence of 'ether' in the space of universe in place of vacuum purposed by Huygen by using interference of light.
iii. It is used in holography to produce three dimensional images.
iv. It is used to test the flatness and parallelism of plane surfaces.
v. A lot of optical filters are based on constructive interference.
vi. It is used to determine the wavelength of light precisely.
vii. It is used to determine refractive index or thickness of transparent thin sheets.

Tips for MCQs

1. **About interference**
   i. Interference was discovered by Thomas Young.
   ii. Law of conservation of energy is not violated in interference, although light added to light may give darkness.
   iii. To produce sustainable interference, two light sources must be coherent.
   iv. The separation between two coherent sources should be small.
   v. Not only the light, particles like electrons also show the interference phenomenon, but during that time it behaves like wave.
2. **Constructive and destructive interference**
   i. The resultant amplitude in interference of two waves $a = \sqrt{a_1^2 + a_2^2 + 2a_1a_2 \cos \phi}$
   ii. Intensity of resultant wave, $I = I_1 + I_2 + 2\sqrt{I_1I_2} \cos\phi$
   Where, ϕ is the phase difference between two waves
   iii. Constructive interference is obtained, when $\phi = 2n\pi$ and $x = n\lambda$, where, n = 0, 1, 2, …
   iv. Destructive interference is obtained, when $\phi = (2n - 1)\pi$ and $x = (2n - 1)\frac{\lambda}{2}$, where n = 1, 2, 3, …
3. **Some important relations:**
   i. Relation between optical path and geometric path:
   Optical path (L) = refractive index (η) × geometric path (x)
   ii. $\phi = \frac{2\pi}{\lambda}x$
   iii. The angular width of a fringe produced by Young's double slit experiment is,
   $\theta = \frac{\beta}{D} = \frac{\alpha}{D} = \frac{\lambda D}{dD} = \frac{\lambda}{d}$
   iv. The fringe width for both bright and dark pattern is $\alpha = \beta = \frac{D}{d}\lambda$
   v. The expression for the radius of nth dark Newton's ring is $r_n = \sqrt{n\lambda R}$
   vi. The expression for the radius of nth bright Newton's ring is $r_n = \sqrt{(2n+1)\frac{\lambda}{2} R}$.
   vii. Wavelength of light, $\lambda = \frac{D_{n+m}^2 - D_n^2}{4mR}$
4. The energy is only redistributed during interference between dark and bright fringes but total energy still remains the same. That is why, there is no violation of law of conservation of energy.

Worked Out Problems

1. In young's double slit experiment, the separation of four bright fringes is 2.5 mm. The wavelength of light used is 6.2×10^{-5} cm. Calculate the separation of slits.

SOLUTION

Given

Width of four bright fringe, $4\alpha = 2.5 \text{ mm} = 2.5 \times 10^{-3}$ m

Wavelength of light, $\lambda = 6.2 \times 10^{-5} \text{ cm} = 6.2 \times 10^{-7}$ m

Screen distance, D = 80 cm = 0.80 m

Slits separation, d = ?

We have,

Fringe width, $\alpha = \frac{\lambda D}{d}$

Here, $4\alpha = 2.5 \times 10^{-3}$ m

$\therefore \alpha = \frac{2.5 \times 10^{-3}}{4} = 6.25 \times 10^{-4}$ m

Now, $d = \frac{\lambda D}{\alpha}$

$= \frac{6.2 \times 10^{-7} \times 0.80}{6.25 \times 10^{-4}}$

$= 7.94 \times 10^{-4}$ m

Slit separation is 7.94×10^{-4} m

2. **[NEB 2075] In a Newton' rings experiment, the diameter of 15th ring was found as 0.590 cm and that of 5th ring was 0.336 cm. Calculate the radius of curvature of the plano-convex lens if the wavelength of light used is 5880Å.**

Solution

Given,

Diameter of 15th ring (D_{15}) = 0.590 cm

Diameter of 5th ring (D_5) = 0.336 cm

Wavelength of light (λ) = 5880 Å = 5880×10^{-8} cm

Radius of curvature (R) = ?

We have,

$$R = \frac{D_{15}^2 - D_5^2}{4 \times \lambda \times (15 - 5)}$$

$$= \frac{0.590^2 - 0.336^2}{4 \times 5880 \times 10^{-8} \times 10}$$

= 100 cm

= 1 m

Therefore, the radius of curvature of plano-convex lens is 1 m.

3. **[HSEB 2067] In a two-slit interference pattern, the slits are 0.2 mm apart, and the screen is at a distance of 1 m. The third bright fringe is found at 9.49 mm from the central fringe. Find the wavelength of light used.**

SOLUTION

Given,

Slit separation (d) = 0.2 mm = 2×10^{-4} m

Screen distance (D) = 1 m

y_3 = 9.49 mm = 9.49×10^{-3} m

n = 3

λ = ?

We know that

$$y_n = \frac{n\lambda D}{d}$$

or $\lambda = \frac{y_n d}{nD} = \frac{9.49 \times 10^{-3} \times 2 \times 10^{-4}}{3 \times 1}$

$= 6.3267 \times 10^{-7}$ m = 632.67 nm

$\therefore$ The wavelength of light (λ) = 632.67 nm.

4. **[HSEB 2064] In a Young's double slit experiment, the separation between the first and the fifth bright fringes is 2.5 mm when the wavelength of light used is 6.2×10^{-4} mm. Calculate the separation of the two slits when the distance between slit and screen is 80 cm.**

SOLUTION

Given,

Separation between the first and fifth bright fringes

$= y_5 - y_1 = 2.5$ mm $= 2.5 \times 10^{-3}$ m

Screen distance (D) = 80 cm = 0.80 m

Wavelength of light (λ) = 6.2×10^{-4} mm = 6.2×10^{-7} m

Slit separation (d) = ?

We know that

$$y_n = \frac{n\lambda D}{d}$$

For the first fringe (y_1) = $\frac{1 \times \lambda D}{d} = \frac{\lambda D}{d}$

For the fifth fringe (y_5) = $\frac{5 \times \lambda D}{d}$

Therefore,

$$y_5 - y_1 = \frac{5\lambda D}{d} - \frac{\lambda D}{d} = \frac{4\lambda D}{d}$$

or, $2.5 \times 10^{-3} = \frac{4 \times 6.2 \times 10^{-7} \times 0.80}{d}$

$\therefore d = \frac{4 \times 6.2 \times 10^{-7} \times 0.80}{2.5 \times 10^{-3}} = 7.9 \times 10^{-4}$ m

$\therefore$ The slit separation distance = 7.9×10^{-4} m

5. In Newton's ring experiment, the diameters of the 4th and 12th dark rings are 0.400 cm and 0.700 cm respectively. Find the diameter of the 20th dark ring.

SOLUTION

Given,

For, n = 4

Diameter of 4th ring (D_4) = 0.40 cm

For, n + m = 12,

Diameter of 12th ring (D_{12}) = 0.70 cm

$\therefore$ m = 12 – 4 = 8

D_{20} = ?

We know that

For n = 4 and m = 8,

$$\lambda = \frac{D_{n+m}^2 - D_n^2}{4mR} = \frac{D_{12}^2 - D_4^2}{4 \times 8 \times R}$$

Also, for n = 4, m = 16,

$$\lambda = \frac{D_{20}^2 - D_4^2}{4 \times 16 \times R}$$

From (i) and (ii), we get

$$\frac{D_{12}^2 - D_4^2}{4 \times 8 \times R} = \frac{D_{20}^2 - D_4^2}{4 \times 16 \times R}$$

or $(0.70)^2 - (0.40)^2 = \dfrac{D_{20}^2 - (0.40)^2}{2}$

or $(0.49 - 0.16) \times 2 = D_{20}^2$

or $0.33 \times 2 + 0.16 = D_{20}^2$

or $D_{20} = \sqrt{0.66 + 0.16}$

$= \sqrt{0.82}$

$\therefore$ D_{20} = 0.906 cm

$\therefore$ The diameter of 20th ring is 0.906 cm.

6. [HSEB 2072] In young's double slit experiment, the slits are 0.03 cm apart and the screen is placed 1.5 m away. The distance between the central bright fringe and fourth bright fringe is 1 cm. Calculate the wavelength of light used.

SOLUTION

Given,

Distance between slits (d) = 0.03 cm = 0.03×10^{-2} m

Distance between slit and screen (D) = 1.5 m

$4\alpha = 1$ cm $= 1 \times 10^{-2}$ m, α = width of a bright fringe

or, $\alpha = 0.25 \times 10^{-2}$ m

Wavelength (λ) = ?

Now, we have

$$\alpha = \frac{\lambda D}{d}$$

or, $\lambda = \dfrac{\alpha d}{D} = \dfrac{0.25 \times 10^{-2} \times 0.03 \times 10^{-2}}{1.5} = 5.0 \times 10^{-7}$ m

$\therefore$ Wavelength of light used (λ) = 5.0×10^{-7} m.

7. [HSEB 2072] Two coherent sources A and B of radio waves are 5 m apart. Each source emits waves with wavelength 6 m. Consider points along the line between two sources, at what distances, if any, from A is the interference constructive.

SOLUTION

Given,

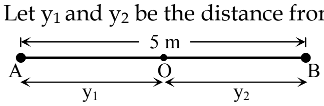
Distance between coherent source A and B (d) = 5 m

Wavelength of source (λ) = 6 m

The waves become constructive only when the path difference of two waves is integral multiple of λ.

i.e. path difference (Δx) = $n\lambda$, where, n = 0, 1, 2, ...

Let y_1 and y_2 be the distance from A and B where constructive wave is formed (at point O in figure)



Let $y_1 = x$

$y_2 = 5 - x$

and $\Delta x = y_1 - y_2$

Now, $y_1 - y_2 = n\lambda$

$x - (5 - x) = n\lambda$

$2x - 5 = n\lambda$

i. For n = 0

$2x - 5 = 0 \times \lambda$

$2x - 5 = 0$

$2x = 5$

$\therefore$ $x = 2.5$ m

ii. For n = 1

$2x - 5 = \lambda$

$2x - 5 = 6$

$2x = 11$

$\therefore$ $x = 5.5$ m

For all $n \geq 1$, the constructive interference is produced beyond point B (i.e. $x > 5$ m).

Hence, the constructive interference must be obtained at 2.5 m away from A.

8. [HSEB 2073] The separation between the consecutive dark fringes in a Young's double slit experiment is 1 mm. The screen is placed at a distance of 2 m from the slits 1.0 mm separation. What is the wavelength of light used in the experiment?

SOLUTION

Given,

Fringe width for dark fringe (β) = 1 mm = 1×10^{-3} m

Slit separation (d) = 1.0 mm = 1.0×10^{-3} m

Screen distance (D) = 2 m

Wavelength of light (λ) = ?

We have,

$$\beta = \frac{\lambda D}{d}$$

$$\lambda = \frac{\beta d}{D} = \frac{1 \times 10^{-3} \times 1 \times 10^{-3}}{2} = 5 \times 10^{-7} \text{ m}$$

$\therefore$ The wavelength of light used is 5×10^{-7} m.

Challenging Problems

1. **[UP]** Coherent light from a sodium–vapor is passed through a filter that blocks every except for light of single wavelength. It then falls on two slits separated by 0.46 mm. In the resulting interference pattern on a screen 2.2 m away, adjacent bright fringes are separated by 2.82 mm. What is the wavelength?

Ans: 590 nm

2. **[UP]** Young's experiment is performed with light from excited helium atoms (λ = 502 nm). Fringes are measured carefully on a screen 1.20 m away from the double slit, and the center of the twentieth fringe (not counting the central bright fringe) is found to be 10.6 mm from the center of the central bright fringe. What is the separation of the two slits?

Ans: 1.14×10^{-3} m

3. **[UP]** Two slits spaced 0.450 mm apart are placed 75.0 cm from a screen. What is the distance between the second and third dark lines of the interference pattern on the screen when the slits are illuminated with coherent light with a wavelength of 500 nm?

Ans: 0.833×10^{-3} m

4. **[UP]** Coherent light with wavelength 400 nm passes through two very narrow slits that are separated by 0.200 mm and the interference pattern is observed on a screen 4.00 m from the slits. (a) What is the width (in mm) of the central interference maximum? (b) What is the width of the first-order bright fringe?

Ans: (a) 8 mm (b) 8×10^{-3} m

5. **[UP]** Coherent light with wavelength 600 nm passes through two very narrow slits and the interference pattern is observed on a screen 3.00 m from the slits. The first-order bright fringe is at 4.84 mm from the center of the central bright fringe. For what wavelength of light will the first-order dark fringe be observed at this same point on the screen?

Ans: 1200 nm

6. **[UP]** White light illuminates two thin slits that are 0.100 mm apart. Calculate the angular width of the first full-color visible spectrum on either side of the central bright line. *(Note: The wavelength range for visible light is 400 nm to 700 nm.)*

Ans: 0.172°

7. **[UP]** Coherent light with wavelength 500 nm passes through narrow slits separated by 0.340 mm. At a distance from the slits large compared to their separation, what is the phase difference (in radians) in the light from the two slits at an angle of 23.0° from the centerline?

Ans: 1670 rad

8. Coherent light that contains two wavelengths, 660 nm (red) and 470 nm (blue), passes through two narrow slits separated by 0.300 mm and the interference pattern is observed on a screen 5.00 m from the slits. What is the distance on the screen between the first-order bright fringes for each wavelength?
9. Two very narrow slits are spaced 1.80 μm apart and are placed 35.0 cm from a screen. What is the distance between the first and second dark lines of the interference pattern when the slits are illuminated with coherent light with λ = 550 nm?

*[**Note:** Hints to challenging problems are given at the end of this chapter.]*

Conceptual Questions with Answers

1. State the essential conditions for two light waves to be coherent.

↳ Following are the basic conditions for two light waves to be coherent.

a. These waves should have same wavelength or frequency.
b. They should have constant phase difference.
c. They must be continuous.
d. Equal amplitudes are mostly preferred.

2. Why are coherent sources necessary to produce a sustained interference pattern?

↳ Coherent sources have equal wavelength and constant phase difference. It ensures that the positions of maxima and minima do not change with time. It means sustained interference is obtained.

3. State two conditions to obtain sustained interference of light.

↳ Two essential conditions for obtaining sustained interference of light are:

a. Two light sources should be coherent.
b. Coherent sources should be narrow and placed close to each other.

4. Two independent light sources cannot act as coherent sources. Why?

↳ Coherent sources must have equal wavelength and constant phase difference. If we closely observe the cause of emission of light, they are produced from the excitation and de-excitation of atoms in a source. Light is emitted by individual atoms, when they return to ground state. Even the smallest source of light contains billions of atoms which cannot emit light waves in the same phase. So, two independent sources cannot act as coherent.

5. Why should we have a narrow source to produce good interference fringes?

↳ A broad source is equivalent to a large number of narrow sources lying close to each other. Different pairs of narrow sources will produce their own interference patterns which will overlap each other. So, visibility is almost similar to all over the screen and the fringe system is lost.

6. No interference pattern is detected when two coherent sources are infinitely close to one another. Why?

↳ The fringe width is, $\beta = \dfrac{\lambda D}{d}$

If the sources are infinitely close to each other i.e. $d \to 0$.

The fringe width β tends to infinity. It means the fringe width is so large that even a single fringe can occupy the entire screen. Hence, the alternate dark and bright fringes are impossible to observe.

7. Why is interference pattern not detected, when the two coherent sources are far apart?

↳ The fringe width of interference pattern is determined from,

$$\beta = \frac{\lambda D}{d}$$

When coherent sources are kept very far to each other, the fringe width β is very small, i.e. for $d \to \infty$. Thus, the fringes are too small to be detected. So, the interference pattern cannot be observed.

8. In young's double slit experiment, if the distance between two slits and the screen are halved and distance between the slits and the screen is doubled, then what will be the effect on fringe width?

➭ The fringe width of interference pattern is $\beta = \frac{\lambda D}{d}$

a. Suppose for initial condition,

$$\beta = \frac{\lambda D}{d} \qquad \text{. . . (i)}$$

b. If the distance between two slits is halved and distance between the slits and screen is double,

$$\beta' = \frac{\lambda D'}{d'} \qquad \text{...(ii)}$$

and $D' = 2D$ and $d' = \frac{d}{2}$

$$\beta' = \frac{\lambda \cdot 2D}{\frac{d}{2}} = 4\left(\frac{\lambda D}{d}\right) = 4\,\beta$$

New fringes have the fringe width four times wider than the original.

9. In young's double slit experiment, light of green, yellow and orange colours are successively used. Compare the fringe widths for the three colours.

➭ For the same apparatus arrangement, d and D are equal, however the fringe widths are varied in accordance with wavelength of different colours. i.e. $\beta \propto \lambda$

As we know, $\lambda_{Green} > \lambda_{Yellow} > \lambda_{Orange}$

The fringe width are also in the order of

$\beta_{Green} > \beta_{Yellow} > \beta_{Orange}$

10. Explain the statement "light added to light can produce darkness".

➭ From the principle of superposition of two waves,

$y = y_1 \pm y_2$

When two waves of equal amplitude meet at a point in opposite phases, the resultant displacement in terms of amplitudes is

$a = a_1 - a_2$

for $a_1 = a_2$

$a = 0.$

It means, the resultant amplitude is zero and hence intensity becomes zero at the point. In such condition, when light added to light undergoes destructive interference, and hence produces darkness.

11. What happens light when light waves interfere destructively at a point? Does this event violate principle of conservation of energy?

➭ Total energy in an interference phenomenon is conserved in one destructive and one consecutive constructive interference pattern. The energy gets transferred from the region of destructive interference to the regions of constructive interference. Hence, this phenomenon does not violate the principle of conservation of energy.

12. What will be the effect on the fringes formed in Young's double slit experiment, if the apparatus is immersed in water?

➭ For the identical apparatus arrangement d and D are same in both conditions. The fringe width is determined by the wavelength of light.

In air, $\beta = \frac{\lambda D}{d}$ . . . (i)

and when the apparatus is immersed into water.

$\lambda' = \frac{\lambda}{\eta}$ ...(ii)

So,

$\beta' = \frac{\lambda}{\eta}\frac{D}{d} = \frac{1}{\eta}\left(\frac{\lambda D}{d}\right) = \frac{\beta}{\eta}$

We know, $\eta > 1$ for water, so,

$\beta' < \beta$

$\therefore$ When the apparatus is immersed into water, fringe width decreases.

---

13. Why is it comparatively difficult to observe interference in light waves as compared to that in water waves?

➪ The wavelength of light waves is much smaller than the water waves. Although the interference phenomenon is studied in the identical apparatus, d and D are equal for both cases. So, the fringe width (β) is directly proportional to wavelength in the given condition, $\lambda_{water} >> \lambda_{light}$

$\beta_{water} >> \beta_{light}$

$\therefore$ The interference fringes have much smaller width in case of light waves than in water waves.

---

14. Why does a soap bubble show beautiful colours when illuminated by white light?

➪ A white light constitutes seven different colours with different wavelengths. In it, the wavelength of red colour is largest and that of violet colour is the shortest. Others lie in between them. Different colours are reflected from the upper and lower surfaces of a film interface and the patterns are produced. Since the conditions for bright and dark fringes are satisfied at different positions for different wavelengths, so coloured fringes are produced.

---

15. What is the effect on the interference pattern observed in a double slit experiment in the following cases:

a. Screen is moved away from the plane of slits.
b. Separation between two slits is increased.
c. Width of slits are doubled.

➪ The fringe width in young's double slit experiment is,

$\beta = \frac{\lambda D}{d}$

a. Taking λ of light and d constants, $\beta \propto D$. So, when screen is moved away from the slits, fringe width increases.

b. For λ and D constants, $\beta \propto \frac{1}{d}$, so, when the separation between two slits is increased, fringe width decreases.

c. When width of slits are doubled, the interference patterns overlap due to the various pairs of two slits. Hence, the contract between the maxima and minima decreases.

---

16. When a thin transparent film is placed just in front of one of the slits in the Young's double slit experiment using white light, what change results in the fringe system?

➪ If thin film is placed just in front of one slit, its optical path changes, so the entire interference pattern gets displaced by a distance $\Delta x = (\eta - 1)t\frac{D}{d}$

As refractive index η depends on λ, the voilet fringe is shifted farther than the red fringe. So, there is a kind of dispersion in the central white fringe.

17. When a low flying aircraft passes overhead, a slight shaking of the picture on TV screen is noticed, why?

↳ A low flying air craft passing overhead can reflect the TV signals that spreading in the space. The direct signal and the reflected signal act as coherent sources, then the superposing of these waves produces the interference. This activity causes the slight shaking of the picture on TV screen.

18. A source emits white light in the double slit experiment and one slit is covered with blue filter and other with red filter. Is it possible to observe interference pattern?

↳ To produce the interference patterns, two sources (i.e. coherent sources) must have equal wavelength. Light filters select the particular colour of light and allow that colour to pass through it. Blue filter and red filters pass the blue colour and red colour respectively. These colours of light have different wavelength. Hence, interference patterns are impossible to obtain.

19. Why is the central fringe bright in Young's double slit experiment?

↳ The kind of fringe (bright or dark) relies on the path difference of two waves at a point from coherent sources. In interference phenomenon, bright fringe is obtained when path difference, $x = n\lambda$, where $n = 0, 1, 2, 3, \ldots$ In Young's double slit experiment, $x = 0$ (i.e. path difference is zero) at the region of centre fringe. Therefore, bright fringe is produced at the center.

20. Why are Newton's rings concentric circles?

↳ Newton's rings are produced from interference by reflection or transmission of light waves. Out of two sources: a plane surface acts as a source and a spherical surface acts as another source. The space between these two surfaces is filled with transparent medium of uniformly varying thickness. Therefore, the interference occurs in two dimensional pattern with varying path difference from all directions. Hence, the interference patterns are obtained circular rings.

21. In Newton's ring experiment, the central ring of the pattern is dark, when viewed by reflected light, why?

↳ When light incident from rarer medium and falls on denser medium, the reflected wave suffers the phase change by π radian. It means, the path difference is $\frac{\lambda}{2}$. Hence, two waves of path difference $\frac{\lambda}{2}$ produce the dark pattern. This happens at the center point of the convex lens while producing the interference patterns. This refers, center ring is dark.

Exercises

Short-Answer Type Questions

1. Write basic requirements to produce interference patterns.
2. What are coherent sources? Can two 100 watt bulbs be connected in parallel circuit to make a coherent sources?
3. Differentiate between constructive and destructive interference.
4. Interference phenomenon supports the wave nature of light. Explain.
5. When crest of wave overlap to trough of identical wave, the location will be dark. What is the reason behind this phenomenon. Does it violate principle of conservation of energy?
6. How does optical path changes when a denser medium is inserted in the path of light?
7. State the path difference between two waves for destructive interference.
8. What is the effect on the interference fringes in Young's double slit experiment if the separation between two slits is increased?
9. "In Young's double slit experiment performed with a source of white light, only back and white fringes are observed". Is this statement true?

10. Why are Newton's ring circular in shape?
11. What types of pattern forms at the centre of Newton's rings?
12. What happens when one of the slits in double slit experiment is covered with opaque material?
13. "A very thin film seen in reflected light shows no colour." Why?
14. "The conditions for the production of interference pattern in a thin film due to reflected light and transmitted light are complementary to each other." Why?
15. In Young's experiment the widths of the two slits are in the ratio 1: 4. What is the ratio of the amplitudes of the two light waves?
16. Is it possible to produce interference using longitudinal wave? Explain.
17. The phase difference between the light wave emitted by two coherent sources is $\pi/2$. If two waves have amplitudes 3 mm and 4 mm, what is the resultant amplitude?
18. If white light is used in Young's double –slit experiment rather than monochromatic light, how does the interference pattern change?
19. Why is it so much easier to perform interference experiments with a laser than with an ordinary light source?

Long-Answer Type Questions

1. What is interference of light? How can you determine the wavelength of light by interference method?
2. What do you understand by interference of light? Distinguish between constructive and destructive interference. Is law of conservation of energy hold good during interference? Explain.
3. What is constructive and destructive interference? Obtain conditions for constructive and destructive interference in Young's double slit experiment.
4. Describe Young's double slits experiment for the interference of light and show that widths of bright and dark fringes are the same. [HSEB 2066]
5. Define coherent sources of light. Prove that the bright and dark fringes in Young's double slit experiment are equally spaced. [NEB 2074]
6. What do you mean by interference of light? Derive the fringe width form Young's double slit experiment. [HSEB 2057, 2062, 2072]
7. Derive an expression for fringe width using Young's double slit experiment for interference of light. What will happen if distance between two slits becomes nearly zero?
8. Describe Young's double slit experiment to determine the wavelength of monochromatic light.
9. What is Newton's ring? How can you determine the wavelength of light using Newton's ring?

Numerical Problems

1. A double slit of 0.5 mm separation is illuminated by light to obtain fringes that are 0.1 cm apart. The wavelength of blue cadmium light is 4800 Å. What is the distance between slits and the screen?

   Ans: 1.04 m

2. In a two-slit interference experiment, the slits are 0.2 mm apart, and the screen is at a distance of 1 m. The third bright fringe is found at 9.49 mm from the central fringe. Find the wavelength of light used.

   Ans: 632.7 nm

3. Two slits are 0.3 mm apart and placed 50 cm from a screen. What is the distance between the second and the third dark lines of the interference pattern when the slit are illuminated with a light of 600 nm wavelength?

   Ans: 10^{-3} m

4. In an experiment using Young's slit, the distance between centre of the interference pattern and the tenth bright fringe on either side is 3.44 cm. Distance between slit and the screen is 2 m. If the wavelength of the light used is 5.89×10^{-7} m, determine the slit separation and angle made by the central bright fringe at the slit.

   Ans: 3.42×10^{-4} m, 3.4×10^{-5} rad

5. In a young's double slit experiment, the wavelength of light used is 5461 Å. How many fringes can be seen in a width of 1 cm if the screen is at a distance of 1 m from the source? The slit separation is 1 mm.

Ans: 18

6. In a Young's double slit experiment, interference fringes were produced on the screen placed at 1.5 m from two slits 0.3 mm apart and illuminated by light of 6400 Å. Find the fringe width.

Ans: 3.2 mm

7. In YDSE, the angular width of a fringe formed on a distance screen is 0.1°. The wavelength of light used is 6000 Å. What is the spacing between the slits?

Ans: 3.44×10^{-4} m

8. Light of wavelength 4800 Å is incident on a double slit. The average thickness of the fringes formed on the screen 150 cm away is 3 mm. Find the distance between the slits.

Ans: 0.24 mm

9. In Young's experiment, two coherent sources are 1.5 mm apart and interference fringes are obtained at a distance of 2.5 m from them. If the sources produces light of wavelength 5893 Å, find the number of fringes in the interference pattern which is 4.9×10^{-3} m long.

Ans: 5

10. In Young's double slit experiment, the two slits 0.12 mm apart are illuminated by monochromatic light of wavelength 420 nm. The screen is 1.0 m away from the slits. Find the distance of the second (i) bright fringe (ii) dark fringe from the central maximum.

Ans: 7 mm, 5.25 mm

11. Two very narrow slits are spaced 1.80 μm apart and are placed 35.0 cm from a screen. What is the distance between the first and second dark lines of the interference pattern when the slits are illuminated with coherent light with λ = 550 nm?

Ans: 10.69 cm

12. If the diameter of two consecutive Newton's rings in reflected light of wavelength 5890 Å are 2.0 and 2.02 cm respectively, what is the radius of curvature of the lens surface in contact with plane glass surface?

Ans: 341.2 cm

13. In a Newton's rings experiment, the diameter of the 5th ring was 0.336 cm and the diameter of the 15th ring was 0.590 cm. Find the radius of curvature of the plano-convex lens if the wavelength of light used is 5890×10^{-8} cm.

Ans: 99.82 cm

14. A Newton's ring arrangement is used with a source emitting two wavelengths $\lambda_1 = 6.0 \times 10^{-5}$ cm and $\lambda_2 = 4.5 \times 10^{-5}$ cm and it is found that the nth dark ring due to λ_1 coincides with (n + 1)th dark ring due to λ_2. If the radius of curvature of the curved surface of the lens is 90 cm, find the diameter of the nth dark ring for λ_1.

Ans: 0.2534 cm

15. Newton's rings are observed in reflected light of $\lambda = 5.9 \times 10^{-5}$ cm. The diameter of the 10th dark ring is 0.50 cm. Find the radius of curvature of the lens and the thickness of the air film.

Ans: R = 1.06 m, t = 0.0003 cm]

16. Newton's rings formed with sodium light between a flat glass plate and a convex lens are viewed normally. What will be the order of the dark ring which will have double the diameter of that of the 40th dark ring?

Ans: 160

Multiple Choice Questions

1. The fringe width interference of monochromatic light produced by double slit experiment is β. The wavelength of light is λ. Then the ratio of the slit separation to the distance between the slits and the screen are:

a. $\beta\lambda$ b. λ/β
c. $1/\lambda\beta$ d. β^2/λ

2. In Young's double slit experiment, 12 fringes are obtained in a certain fragment of the screen when light of wavelength 600 nm is used. If the wavelength of light is changed to 400 nm, number of fringes obtained in the same segment of the screen will be:
   a. 12 b. 18
   c. 24 d. 30
3. Young's experiment is performed inside water, the fringe width will:
   a. Decrease b. Remain same
   c. Increase d. None
4. In Young's double slit experiment distance between slits is 1 mm distance between slits and screen is 1 m and wavelength of wave is 4000 Å. Find the fringe width in meter.
   a. 0.04 b. 0.0004
   c. 4 d. 0.1
5. Young's double slit fringe width is 4.365 mm, separation between slit is 1.35×10^{-4} m and distance between screen and slit is 1 m then wavelength of light used is:
   a. 5890 Å b. 58900 Å
   c. 589 Å d. 8950 Å
6. The double-slit arrangement is illuminated with light from a mercury vapour lamp so filtered that only the string green line (λ 5460Å) is effective. The slits are 0.1 mm apart and the screen on which the interference pattern appears is 20 cm away. The angular position of the first minimum is:
   a. 2730×10^{-5} b. 5460×10^{-7}
   c. 5460×10^{-5} d. 2730×10^{-7}
7. In a double slit experiment, instead of taking slits of equal widths, one slit is made twice as wide as the other. Then, in the interference pattern
   a. The intensities of both the maxima and the minima increase.
   b. The intensity of the maxima increases and the minima has zero intensity.
   c. The intensity of the maxima decreases and that of the minima increases.
   d. The intensity of the maxima decreases and the minima has zero intensity.
8. Two beams of light having intensities I and 4I interfere to produce a fringe pattern on a screen. The phase difference between the beams is $\pi/2$ at point A and π at point B. Then the difference between the resultant intensities at A and B is
   a. 3I b. 4I
   c. 5I d. 7I
9. In a Young's double slit experiment, 12 fringes are observed to be formed in a certain segment of the screen when light of wavelength 600 nm is used. If the wave-length of light is changed to 400 nm, number of fringes observed in the same segment of the screen is given by
   a. 12 b. 18
   c. 24 d. 30
10. Two coherent monochromatic light beams of intensities I and 4I are superposed. The maximum and minimum possible resulting intensities are
   a. 5I and 0 b. 5I and 3I
   c. 9I and I d. 9I and 3I
11. In Young's double slit experiment, if the slit widths are in the ratio 1 : 4, the ratio of the intensities at minima and maxima will be
   a. 1 : 2 b. 1 : 3
   c. 1 : 4 d. 1 : 9
12. The light beams of intensities in the ratio of 9 : 1 are allowed to interfere. What will be the ratio of the intensities of maxima and minima?
   a. 3 : 1 b. 4 : 1
   c. 25 : 9 d. 81 : 1

13. In Young's double slit experimental set-up, if the wavelength alone is doubled, the band width β becomes

a. $\frac{\beta}{2}$ b. 2β

c. 3β d. β

14. Which one of the following statements is correct?

a. Monochromatic light is never coherent.
b. Monochromatic light is always coherent.
c. Two independent monochromatic sources are coherent.
d. Coherent light is sometimes monochromatic.

15. In Young's double slit experiment with slit separation d, a monochromatic light of wavelength λ is used. The angular separation of the fringes is

a. $\frac{d}{\lambda}$ b. $\frac{\lambda}{d}$

c. $\frac{2\lambda}{d}$ d. $\frac{\lambda}{2d}$

16. If the two slits in Young's double slit experiment are of unequal width, then

a. The bright fringes will have unequal spacing.
b. The bright fringes will have unequal brightness.
c. The fringes do not appear.
d. The dark fringes are not perfectly dark.

Answers

1. (b) 2. (b) 3. (a) 4. (b) 5. (a) 6. (b) 7. (a) 8. (b) 9. (b) 10. (c) 11. (d) 12. (b) 13. (b) 14. (d) 15. (b) 16. (d)

Hints to Challenging Problems

HINT: 1

Given,

Distance between two slits, $d = 0.46$ mm $= 0.46 \times 10^{-3}$ m

$D = 2.2$ m

$\alpha = 2.82$ mm $= 2.82 \times 10^{-3}$ m

$\lambda = ?$

Required formula, $\alpha = \frac{\lambda D}{d}$

So, $\lambda = \frac{\alpha d}{D}$

HINT: 2

Given,

$\lambda = 502$ nm $= 502 \times 10^{-9}$ m

$D = 1.2$ m, $n = 20$

$y_{20} = 10.6$ mm $= 10.6 \times 10^{-3}$ m

$d = ?$

Required formula, $y_n = \frac{n\lambda D}{d}$

$\therefore \quad d = \frac{n\lambda D}{y_n}$

HINT: 3

Given,

$d = 0.45$ mm $= 0.45 \times 10^{-3}$ m

$D = 75$ cm $= 75 \times 10^{-2}$ m

$\lambda = 500$ nm $= 500 \times 10^{-9}$ m

$\beta = ?$

Required formula, $\beta = \frac{\lambda D}{d}$

HINT: 4

Given,

$\lambda = 400$ nm $= 400 \times 10^{-9}$ m

$d = 0.2$ mm $= 0.2 \times 10^{-3}$ m

$D = 4$ m

a. width of central maximum, $\alpha_0 = \frac{\lambda D}{d}$

b. The width of bright fringe is independent of order of fringe so the width of the first order bright fringe, will also be equal.

HINT: 5

Given,

$\lambda_1 = 600$ nm $= 600 \times 10^{-9}$ m

$\lambda_2 = ?$, $D = 3$ m

$y_1 = 4.84$ mm $= 4.84 \times 10^{-3}$ m, for n= 1

Required formula, $y_1 = \frac{1 \times \lambda_1 D}{d}$, So, $d = \frac{\lambda_1 D}{y_1}$

First order dark and bright fringes have the same width. Therefore,

$y_1 = y_1'$ or, $y_1 = \frac{\lambda_2 D}{2d}$

So, $\lambda_2 = \frac{y_1 \times 2d}{D}$

HINT: 6

Given,

$d = 0.1$ mm $= 0.1 \times 10^{-3}$ m

$\lambda_v = 400$ nm $= 400 \times 10^{-9}$ m $= 4 \times 10^{-7}$ m

$\lambda_r = 700$ nm $= 700 \times 10^{-9}$ m $= 7 \times 10^{-7}$ m

Angular width of first full colour, $\theta_1 - \theta_2 = ?$

Required formula,

Angular width for first order, n = 1, is $\sin\theta = \frac{\lambda}{d}$

For the violet colour in the spectrum,

$\theta_1 = \sin^{-1}\left(\frac{\lambda_v}{d}\right)$

Similarly, for red colour

$\theta_2 = \sin^{-1}\left(\frac{\lambda_r}{d}\right)$

HINT: 7

Given,

$\lambda = 500$ nm $= 5 \times 10^{-7}$ m

$d = 0.34$ mm $= 0.34 \times 10^{-3}$ m, $\theta = 23°$

Phase difference, $\phi = ?$

We know that

$$\phi = \frac{2\pi}{\lambda} \times (\text{path difference})$$

$$= \frac{2\pi}{\lambda} \times d\sin\theta$$

HINT: 8

Given,

$\lambda_r = 660$ nm $= 660 \times 10^{-9}$ m

$\lambda_b = 470$ nm $= 470 \times 10^{-9}$ m

$d = 0.3$ mm $= 0.3 \times 10^{-3}$ m

$D = 5$ m

Distance between first order bright fringe for each wavelength, $y_r - y_b = ?$

We know that,

$$y_n = \frac{n\lambda D}{d}$$

For red colour

$$\therefore\ y_r = \frac{\lambda_r D}{d}$$

Similarly, for blue colour

$$y_b = \frac{\lambda_b D}{d}$$

HINT: 9

Given,

$d = 1.8$ μm $= 1.8 \times 10^{-6}$ m

$D = 35$ cm $= 35 \times 10^{-2}$ m

Wavelength of light (λ) = 550 nm $= 550 \times 10^{-9}$ m

Distance between the first and second dark lines, $y_2 - y_1 = ?$

We know that

$d\sin\theta_1 = \frac{\lambda}{2}$ for first order dark line

$\therefore\ \theta_1 = \sin^{-1}\left(\frac{\lambda}{2d}\right)$

Now, $y_1 = D\tan\theta_1$

Similarly,

$\sin\theta_2 = \frac{3\lambda}{2d}$ for second order dark line

$\therefore\ \theta_2 = \sin^{-1}\left(\frac{3\lambda}{2d}\right)$

Also, $y_2 = D\tan\theta_2$

Dark fringe separation $= y_2 - y_1$

❖ ❖ ❖

DIFFRACTION OF LIGHT

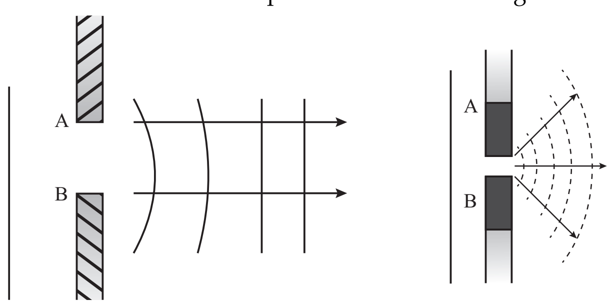
8

CHAPTER

8.1 Introduction

The phenomenon of spreading of light when it is passed through small openings or obstacles is known as diffraction of light. This is the phenomenon that can only be described from the wave aspect of light and is found to violate the rectilinear propagation of light. Diffraction phenomenon occurs also in sound waves, radio waves, x-rays, etc. The fact that light undergoes diffraction is powerful evidence that light has wave properties. Although the diffraction was observed in light before, it could only be explained in detail after, the discovery of wave theory of light purposed by Augustin Fresnel in 1815.

The degree to which waves are diffracted depends upon the size of the obstacle or aperture and the wavelength of the light. The greatest effect occurs when wavelength of light is about same as the aperture. Since the wavelength of visible light is very small (~10^{-7} m), it is very difficult to detect diffraction in our common activities. When the opening is wide compared to the wavelength, the spreading effect is negligibly small. Although the diffraction occurs in wide aperture, it is impossible to detect. The diffraction in wide and narrow aperture are shown in Fig. 8.1.



(i) **(ii)**

Fig. 8.1: Diffraction of light

Frequency modulation (FM) radio waves have shorter wavelength, so they don't diffract as much around buildings, and hills. That is why, they are not received in mountain sides. But amplitude modulation (AM) waves can be received well in big cities and mountains, since the waves have large wavelength. TV signals which are also the radio waves do not spread much around the corners. This is the reason why the antennas are put on the rooftops but not inside the room. This is done to improve signal reception.

Difference between Interference and Diffraction

Interference	Diffraction
1. Interference is the result of superposition of waves starting from two different wave fronts.	1. Diffraction is the result of superposition of waves starting from different portions of the same wave front.
2. All bright fringes in an interference pattern are of same intensity.	2. Intensity of bright fringes in a diffraction pattern decreases as one move away from the central bright fringe.
3. The points of minimum intensity in an interference pattern are perfectly dark.	3. The points of minimum intensity are not perfectly dark in a diffraction pattern.
4. The spacing between fringes is uniform.	4. The spacing between fringes is not uniform.
5. Large number of patterns are produced.	5. A few numbers of patterns are produced.

8.2 Classification of Diffraction

There are two types of diffraction (i) Fresnel diffraction and (ii) Fraunhofer diffraction.

Fresnel Diffraction

i. **Fresnel diffraction:** Fresnel diffraction occurs when a wave passes through a small hole and bends creating a diffraction pattern. It is also called the near-field diffraction, since the source is situated nearer to the slit. The size of Fresnel diffraction pattern depends on the distance between a projection and an aperture. No lens is used to produce this diffraction pattern as shown in Fig. 8.2.

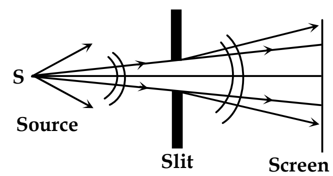


Fig. 8.2: Fresnel diffraction

ii. **Fraunhofer diffraction:** Fraunhofer diffraction occurs when a wave emerges from infinity and passes through a small slit and finally forms the diffraction patterns on the screen. It is also called far-field diffraction, since the source is situated effectively at infinity from the slit. Two converging lenses are used: first lens L_1 converts the spherical wave front into plane wave front as if source is situated at infinity and second lens L_2 converges the diffracted waves on the screen as shown in Fig. 8.3.

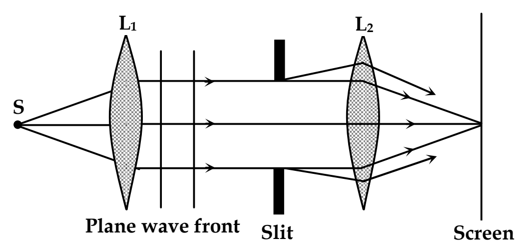


Fig. 8.3: Fraunhofer diffraction

Difference between Fresnel Diffraction and Fraunhofer Diffraction

Fresnel Diffraction	Fraunhofer Diffraction
1. The source and the screen are at finite distance from the obstacle or slit.	1. The source and the screen or both are effectively at infinite distance from the obstacle or slit.
2. Observation of Fresnel diffraction does not require any lens.	2. The conditions required for the Fraunhofer diffraction are achieved using two convex lenses.

3. Cylindrical wave fronts are used.	3. Planar wave fronts are used.
4. The maxima and minima are not well defined.	4. The maxima and minima are well defined.
5. It has less applications in designing the optical instruments.	5. It has many applications in designing the optical instruments.

Huygen's Explanation of Diffraction

Huygen's wave theory provides the firm support to explain diffraction phenomenon in light waves. According to Huygen's principle, every point of a wavefront acts as the secondary source of light, which produces the secondary wavelets. These wavelets spread independently around the corners. As every point of sharp corners acts as the independent source of light, it spreads around. The light wave spreading around the sharp corners in the form of spherical wavefronts is shown in Fig. 8.1 (ii). Huygen's theory is useful to explain diffraction phenomenon in Fraunhofer experiment.

8.3 Fraunhofer Diffraction at a Single Slit

The experimental set up to study the diffraction phenomenon of light from single slit is shown in Fig. 8.4. It consists of a very narrow single slit where plane wave fronts are allowed to fall upon. A screen is placed in front of single slit. According to Huygen's theory, every point of a wavefront acts as secondary source of light and the waves, thus, generated are called wavelets. These wavelets emanate from the wavefront in the same phase. A convex lens is placed between the slit and screen such that the screen is in the focal plane of the lens; a bright image should be obtained at a point O on the screen as shown in Fig. 8.4. Since every point on the slit acts as independent source of light, the wavelets originating from these points are able to meet at any point of the screen. Hence, dark and bright patterns are obtained on the screen. These patterns are called diffraction patterns.

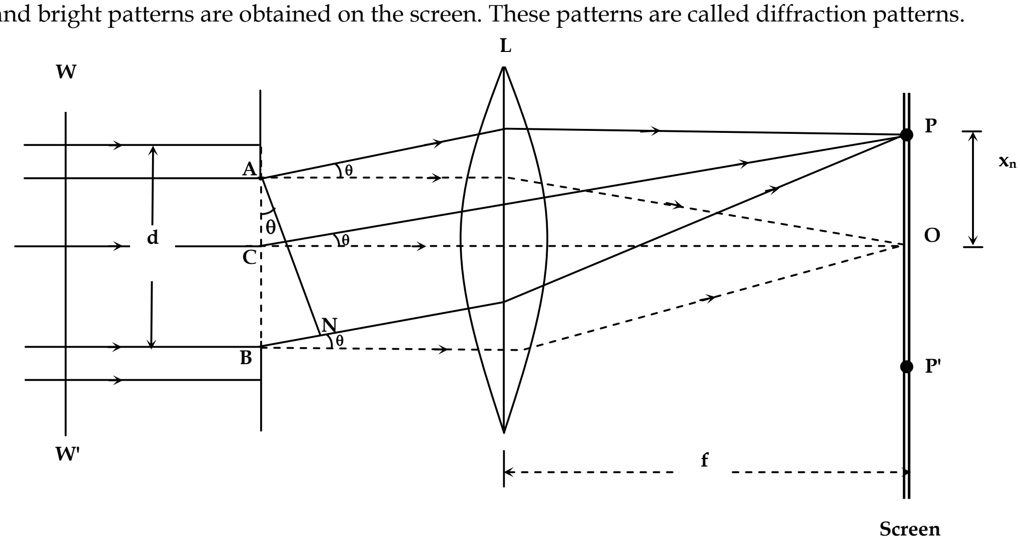


Fig.8.4: Fraunhofer diffraction at a single slit

Formation of Central Maximum

The slit AB is supposed to be divided into a number of very narrow strips of equal width parallel to the slit. All the wavelets originating from the narrow strips have the same phase. These wavelets reach at center point O traversing the equal path having same phase. After then, the wavelets reinforce each other's effect to give maximum intensity at O. Thus, a bright fringe is formed, called the central maximum.

Path difference between extreme wavelets

Consider the diffracted wavelets with angle θ at each strip of slit AB. Let us take a general point P on the screen at finite distance away from the center point O. The wavelets starting from each strip are in the same phase. But, they may not have the same phase at point P because these wavelets travel unequal distance in reaching P.

The wavelets originated from the strips nearer from points A and B are called extreme wavelets because they lie at the farthest distance in the consideration. Let us draw a perpendicular AN on a light ray traversing from point B. Beyond the points A and N, the secondary wavelets cover equal distance. Hence, BN is the path difference of extreme wavelets.

Taking the right angled triangle ANB,

$$\sin\theta = \frac{BN}{AB}$$

$$BN = AB \sin\theta$$

$$BN = d \sin\theta$$

Where d = slit width

$$\therefore \quad \textbf{Path difference, BN = d sin}\ \boldsymbol{\theta} \qquad \ldots (8.1)$$

Formation of secondary minima

Consider a point P, on the screen at which the path difference of extreme wavelets originating from A and B is λ with corresponding diffracted angle θ_1. In this condition, dark pattern appears at P_1 and is called the first secondary minimum. To study such mechanism, let the slit AB be divided into two equal parts AC and BC. So, every strips in the upper half AC, there is a corresponding strips in the lower half BC such that the path difference from every corresponding strip to reach at P_1 is $\frac{\lambda}{2}$ so that the waves superimpose out of Fig. 8.5 phase. Thus, they cancel each other's effect. Hence, a dark pattern is formed at P_1. The condition for the first secondary minimum is written, by using equation (8.1).

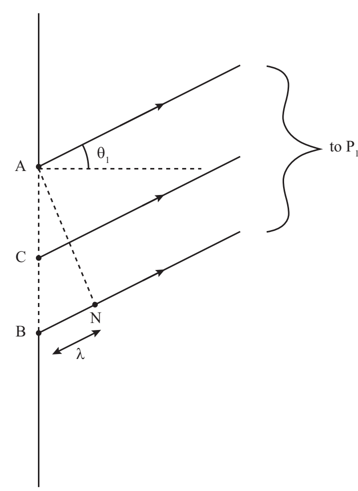


Fig.8.5: Formation of first minimum

$$\mathbf{d \sin\theta_1 = \lambda} \qquad \ldots (8.2)$$

$$\therefore \quad \sin\theta_1 = \frac{\lambda}{2}$$

For very small diffracted angle θ_1 we write, $\sin\theta_1 \approx \theta_1$, So,

$$\boldsymbol{\theta_1 = \frac{\lambda}{d}} \qquad \ldots (8.3)$$

If the path difference of wavelets originated from strips nearer the points A and B, in reaching another point P_2 on the screen is 2λ, second dark pattern is appeared. This is called second secondary minimum. To study this mechanism, the slit AB is to be divided into four equal parts AC_1, $C_1 C_2$, $C_2 C_3$ and $C_3 B$ such that the path difference of each corresponding part should be $\frac{\lambda}{2}$ (i.e. path difference of extreme wavelets passing from A_1C and $C_1 C_2$ is λ and so on) as shown in Fig.8.6. So the effect of A_1C is cancelled by C_1C_2 and, the effect of C_2C_3 is cancelled by C_3B. Thus, the dark pattern is obtained on P_2. The condition for the second secondary minimum is written by using equation (8.1) as,

$$\mathbf{d \sin \theta_2 = 2\lambda} \qquad \ldots (8.4)$$

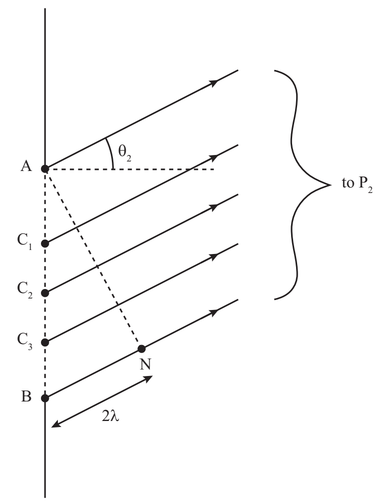


Fig.8.6: Formation of second minimum

Where θ_2 is the diffracted angle at slit to reach at P_2.

$$\sin \theta_2 = \frac{2\lambda}{d}$$

∴ For very small angle of θ_2, $\sin \theta_2 \approx \theta_2$.

$$\boldsymbol{\theta_2 = \frac{2\lambda}{d}} \qquad \ldots (8.5)$$

Proceeding the above processes, the condition for n^{th} secondary minimum is.

$$\mathbf{d \sin \theta_n = n\lambda} \qquad \ldots (8.6)$$

$$\text{or,} \quad \sin \theta_n = \frac{n\lambda}{d}$$

For very small angle θ_n, $\sin \theta_n \approx \theta_n$

$$\therefore \quad \boldsymbol{\theta_n = \frac{n\lambda}{d}} \qquad \ldots (8.7)$$

Formation of secondary maxima

Consider a point P_1' on the screen such that the path difference of extreme wavelets in reaching P_1' is $\frac{3\lambda}{2}$. In this condition, the point P_1' appears bright. To study this mechanism, the slit AB is to be divided into three equal parts: AC_1, $C_1 C_2$, $C_2 B$ as shown in Fig. 8.7 such that the path difference of extreme wavelets of every consecutive part is λ (i.e. path difference of extreme wavelets of AC_1 and C_1C_2 is λ and so on. So, the wavelets reaching P_1' from the consecutive points of the parts AC_1 and C_1C_2 will cancel each other's effect. The wavelets originating from the part C_2B will produce first secondary maximum. The condition for first secondary maximum is,

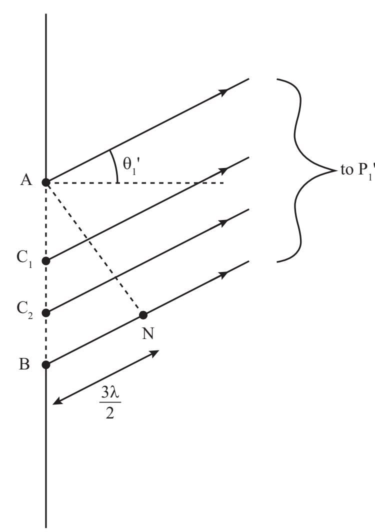


Fig. 8.7: Formation of first maximum

$$\mathbf{d \sin \theta_1' = 3\frac{\lambda}{2}} \qquad \ldots (8.8)$$

Where θ_1' is the, diffracted angle to produce for secondary maximum.

$$\text{or,} \quad \sin \theta_1' = \frac{3\lambda}{2d}$$

For very small angle θ_1', $\sin \theta_1' \approx \theta_1'$, then,

$$\therefore \quad \mathbf{\theta_1' = \frac{2\lambda}{2d}} \qquad \ldots (8.9)$$

Further, suppose the wavelets diffracted at an angle θ_2' and reaching at a point P_2' on the screen produce the second secondary maximum. This occurs when the path difference between the extreme wavelets is $\frac{5\lambda}{2}$. In this condition, the slit AB is divided into five equal parts AC_1, C_1C_2, C_2C_3, C_3C_4, C_4B such that every consecutive part that contains the effect of AC_1 is cancelled by C_1C_2 and the effect of C_2C_3 is cancelled by C_3 C_4 at P_2' as shown in Fig. 8.8. The wavelets from fifth part would produce second secondary maximum at P_2'. The condition for second secondary maximum is,

$$\mathbf{d \sin \theta_2' = \frac{5\lambda}{2}} \qquad \ldots (8.10)$$

$$\text{or,} \quad \sin \theta_2' = \frac{5\lambda}{2d}$$

For very small angle θ_2', $\sin \theta_2' \approx \theta_2'$

$$\therefore \quad \mathbf{\theta_2' = \frac{5\lambda}{2d}} \qquad \ldots (8.11)$$

Proceeding the same process, the condition for second secondary maximum is,

$$d \sin \theta_n' = (2n + 1)\frac{\lambda}{2}, \text{ where } n = 1, 2, 3, \ldots$$

$$\therefore \quad \mathbf{\sin \theta_n' = \frac{(2n + 1)\lambda}{2d}} \qquad \ldots (8.12)$$

For very small angle of θ_n', $\sin \theta_n' \approx \theta_n'$ we get

$$\mathbf{\theta_n' = \frac{(2n + 1)\,\lambda}{2d}} \qquad \ldots (8.13)$$

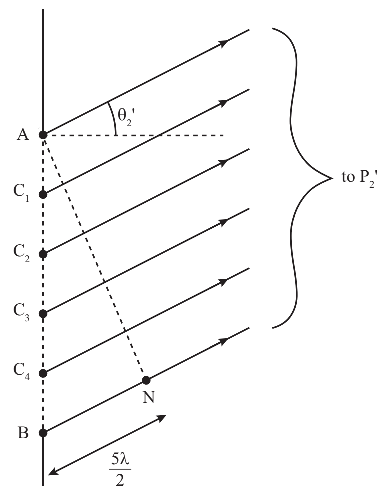


Fig. 8.8: Formation of second maximum

Points to be Considered

1. A distinct diffraction pattern is possible only if the slit is sufficiently narrow.
2. Secondary maxima are less intense than central maximum.
3. The diffraction pattern is symmetrical about the central maximum (i.e. both sides of central maximum have similar pattern).
4. The width of the central maximum is double than that of the secondary maximum.
5. If the slit is replaced by circular aperture, circular dark and bright rings will appear as shown in Fig. 8.9.

6. The conditions of diffraction minima and maxima are exactly reverses of the conditions for interference minima and maxima.

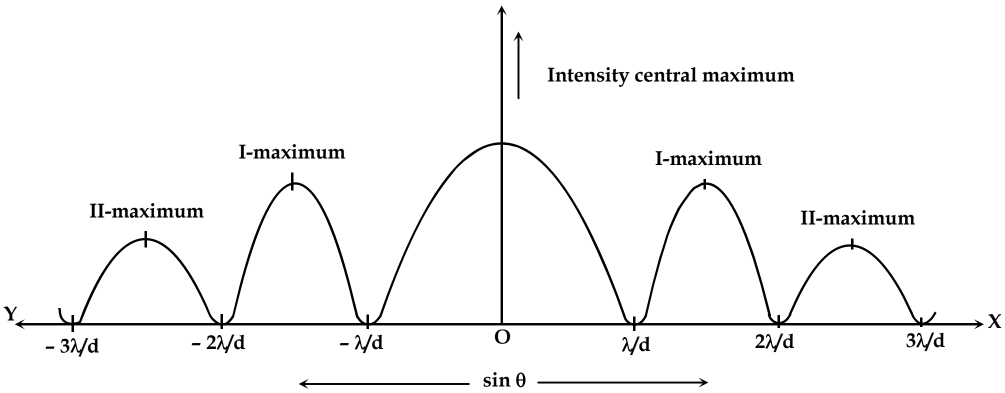


Fig. 8.9: Intensity variation curve in diffraction

Width of central maximum and secondary maximum

Let θ_1 and θ_2 be the angular diffraction for first and second secondary minimum respectively. Suppose the screen is placed at distance D from the slit as shown in Fig. 8.10.

$OP_1 = y_1$ = displacement of first secondary minimum from the center O.

$OP_2 = y_2$ = Displacement of second secondary minimum from the center O.

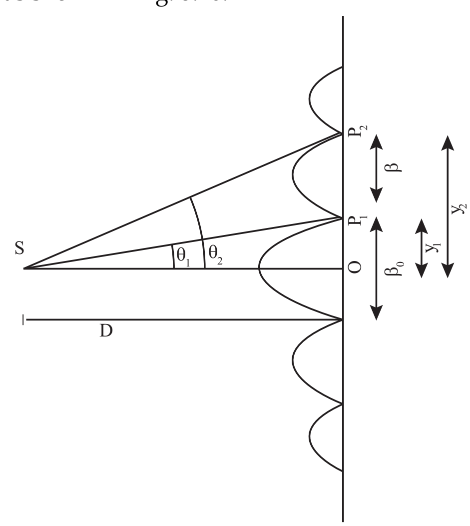


Fig. 8.10: Diffraction pattern

In ΔP_1OC of Fig. 8.10,

$$\tan\theta_1 = \frac{OP_1}{CO} = \frac{y_1}{D}$$

For very small angle of θ_1, $\tan\theta_1 \approx \theta_1$

$$\therefore \quad \theta_1 = \frac{y_1}{D} \qquad \text{. . . (8.15)}$$

Also, from the condition of secondary minima, for very small angle θ_1

$$\theta_1 = \frac{\lambda}{d} \qquad \text{. . . (8.16)}$$

Equating equations (8.15) and (8.16), we get

$$\frac{y_1}{D} = \frac{\lambda}{d}$$

$$\therefore y_1 = \frac{\lambda D}{d} \qquad \text{. . . (8.17)}$$

In the similar manner, from $\Delta P_2 OC$ in Fig 8.10,

$$\theta_2 = \frac{y_2}{D} \qquad \text{. . . (8.18)}$$

and from the condition of secondary minimum for very small angle θ_2,

$$\theta_2 = \frac{2\lambda}{d} \qquad \text{. . . (8.19)}$$

Equating equations (8.18) and (8.19)

$$\frac{y_2}{D} = \frac{2\lambda}{d}$$

$$\mathbf{y_2} = \frac{\mathbf{2\lambda D}}{\mathbf{d}} \quad \text{. . . (8.20)}$$

Now, the width of $OP_1 = y_1 - 0 = \frac{\lambda D}{d}$

The width of central maximum,

$$\beta_0 = 2 \times OP_1$$

$$= \frac{\mathbf{2\lambda D}}{\mathbf{d}} \quad \text{. . . (8.21)}$$

Also, the width of first secondary maximum,

$$(\beta) = P_1P_2 = \frac{2\lambda D}{d} - \frac{\lambda D}{d}$$

$$\beta = \frac{\mathbf{\lambda D}}{\mathbf{d}} \quad \text{...(8.22)}$$

If we proceed for the second secondary maximum the width is β. This shows that, $\beta_0 = 2\beta$. It concludes that the width of central maximum is double than the with of secondary maximum.

Note

i. When white light is used in place of monochromatic light, the central maximum is white but other fringes are coloured.

ii. Most of the energy of the wave lies in the central maximum.

iii. since sin $\theta_n = \frac{n\lambda}{d}$*, for first secondary minimum n = 1 and hence sin* $\theta_1 = \frac{\lambda}{d}$*. This result tells us that the ratio of wavelength (λ) to the size of aperture (d) determines to what extent light or any other wave falls to travel in a straight line. If this ratio is small, the bending of light (diffraction) will be small and vice-versa.*

8.4 Diffraction Grating

As explained earlier, the slit width for the transmission of light must be comparable with the wavelength of corresponding wave to obtain the diffraction pattern. The wavelength of visible light ranges from 400 nm to 700 nm (i.e. in the order of 10^{-7} m). In Fraunhofer single slit diffraction experiment, the mechanism of diffraction of light is explained taking light intensity passing through the slit of width about 10^{-7} m. But in practice, diffraction maxima and minima can not be visualized taking single slit of such very narrow width. For the practical purposes, large number of very narrow equidistant and parallel slits are arranged in a single glass plate such that the superposition of diffracted waves from these slits provides the appropriate intensity to visualize the diffraction patterns on the screen. *This optical device which*

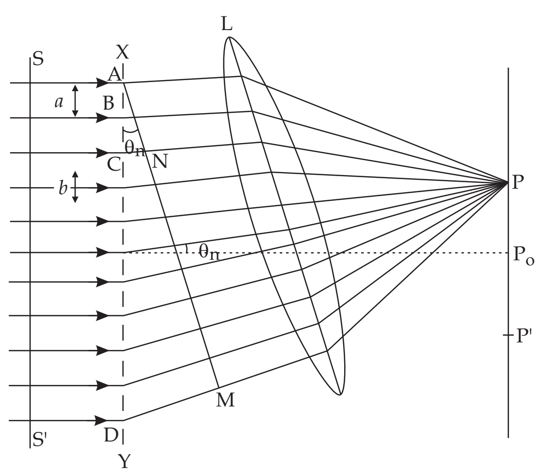


Fig. 8.11: Diffraction grating

contains a large number of narrow equidistant and parallel slits to obtain the diffraction patterns is called diffraction grating. It is also called the transmission grating. An ordinary diffraction grating contains thousands of parallel slits in a millimeter width over a thin glass plate by means of fine sharp diamond points.

The opaque lines of the grating are called rulings and transparent lines are called slits. The opaque and transparent lines are scratched alternately on a thin glass plate. Each slit is separated with equally wide opaque line. The slit width 'd' of the grating is the sum of width of slit 'a' and width of ruling 'b'. i.e. d = a + b. The slit width 'd' is also called grating element or grating spacing.

Mechanism

Suppose a plane wave front SS' incidents on a plane transmission grating XY as shown in Fig. 8.11. When the wave front falls on the grating, the slits allow the light to pass and rulings block the light. As explained by Huygen's principle, every point of a wave front acts as secondary source of light. So, each point of slits acts as independent source of light and produces the wavelets. Finally, every point of screen receives light coming from each slit, so that these waves superimpose and diffraction patterns are obtained.

Theory

Let us take a point P on the screen in which the light waves incident from each slit superimpose after diffracting with an angle θ and traversing through a converging lens as shown in Fig.8.11.

Consider two waves passing through upper two consecutive slits AB and BC. Let θ be the phase difference and x be the path difference of these waves while reaching at P from the slits. To determine the path difference of waves passing through these consecutive slits, a straight line AN is drawn so that the path difference is zero for all waves after crossing line AN to meet at P.

From right angled triangle ANC,

$$\sin\theta = \frac{CN}{AC}$$

$$CN = AC\sin\theta$$

$$= (a+b)\sin\theta$$

Here, AC = slit width = d = a + b

The point P to be maximum, the path difference of two waves must be integral multiple of λ.

$$\therefore \quad CN = n\lambda \quad \text{Where, } n = 0, 1, 2, 3, \ldots$$

$$\text{or,} \quad (a+b)\sin\theta = n\lambda$$

$$\textbf{or,} \quad \mathbf{d\sin\theta = n\lambda} \qquad \ldots(8.23)$$

Let N be the number of lines per unit length of the grating. So,

$$d = \frac{1}{N}$$

$$\mathbf{a + b = \frac{1}{N}} \qquad \ldots(8.24)$$

So, the equation (8.23) becomes,

$$\frac{1}{N}\sin\theta = n\lambda$$

Note

In the young's experiment both interference and diffraction are present. When light waves strike the slits, they get diffracted first and then undergo interference.

Applications of diffraction

i. Diffraction gratings are used for accurate estimation of the wavelengths.

ii. Structure of crystalline solids is determined by x-rays, electron and neutron diffraction measurements.

iii. Velocity of ultrasonics can be measured with diffraction techniques.

iv. The location, size and shape of ulcer, tumours etc can be found by ultrasound scanning.

8.5 Resolving Power of Optical Instruments

Our eyes are unable to distinctly separate two objects which lie very near to each other. They can visualize two objects separately only if the angle subtended by them is greater than one minute (1'). If the angle subtended by them is smaller than 1', two objects appear as a same object. Optical instruments like telescope, microscope, lens are used to visualize such very closely lying objects as separate. *The method of visualizing such very close object as separate is called resolution and the ability of an optical instrument to produce separate images of two objects which lie very close to each other is called resolving power.*

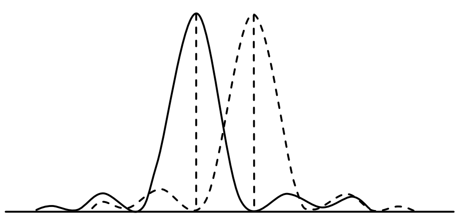


Fig. 8.12: Diffraction pattern upto image

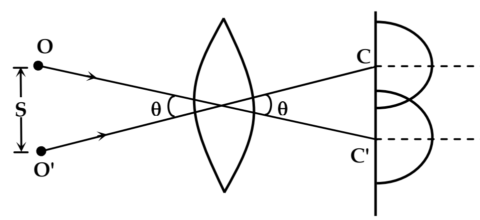


Fig. 8.13: Resolution between two objects

Optical instruments like lens, microscope and telescope act as aperture. For example a lens is considered as a circular aperture. Light passing through that aperture goes on diffraction. Then, image of each point is a set of alternate bright and dark circular fringes with a bright disc at the center. The size of this disc depends on the aperture of the lens and the wavelength of light used. If there are two nearby points, their images may give rise to diffraction patterns which overlap on each other making the resolution of two points.

Diffraction acts as a limit on resolving power of optical instruments. It is termed as limit of resolution. The limit of resolution is defined as "the smallest linear or angular separation between two point objects at which they can be just seen as separate object by an optical instrument. The smaller the limit of resolution of an optical instrument, greater is its resolving power.

Lord Rayleigh experimentally studied how the optical instruments can resolve the very close points as separate. The phenomenon of distinguishing two very near points were described in terms of Rayleigh criterion. According to this criterion, the images of two point objects are just resolved when the central maximum of diffraction pattern of one falls over the first minimum in the diffraction pattern of the other point source, then the point sources are said to have been resolved by optical instrument as shown in Fig. 8.13.

Resolving power of microscope

It is defined as the reciprocal of the smallest distance between two point objects at which they can be just resolved when seen though the microscope.

The limit of resolution of microscope

$$d\theta = \frac{\lambda}{2\eta \sin\theta}$$

and resolving power $= \frac{1}{d\theta} = \frac{2\eta \sin\theta}{\lambda}$

Where, λ = wavelength of light

η = refractive index of medium enclosed between object and lens

θ = half the angle of cone of light from each point object

Resolving power of telescope

It is defined as the reciprocal of the smallest angular separation between two distant objects whose images can be just resolved by it. The limit of resolution for telescope is,

$$d\theta = \frac{1.22\,\lambda}{D}$$

Resolving power $= \frac{1}{d\theta} = \frac{D}{1.22\,\lambda}$

Where, λ = wavelength of light

D = the diameter of telescope objective

Resolving power of our eye

The diameter of our pupil (D) = 2 mm

Suppose we use the light of wavelength, λ = 5000 Å

Then, the smallest angular separation between two distant points that the human eye can resolve will be,

$$d\theta = \frac{1.22\,\lambda}{D} = \frac{1.22 \times 5000 \times 10^{-10}}{2 \times 10^{-3}}$$

$$= 0.305 \times 10^{-3} \text{ rad}$$

$$\approx 1 \text{ minute}$$

Tips for MCQs

1. About diffraction: Diffraction occurs on account of interference of secondary wavelets from portions of the wave front which are allowed to pass through the slits.
2. Conditions of diffraction minima and maxima are exactly reverse of the conditions for interference minima and maxima.
3. Interference and diffraction can be explained on the basis of wave theory of light. These phenomena exist also in longitudinal waves, but polarization phenomenon occurs only in transverse wave.
4. For the diffraction at single slit of width (d),
   i. Condition of n^{th} minimum is, $d \sin\theta = n\lambda$, where n = 1, 2, 3, …
   ii. Condition of n^{th} secondary maximum is, $d \sin\theta = (2n + 1)\frac{\lambda}{2}$, where n = 1, 2, 3, …
   iii. The angular position of n^{th} minimum, $\theta_n = \frac{n\lambda}{d}$
   iv. The distance of n^{th} minimum from the center of the screen, $x_n = \frac{n\lambda D}{d}$

v. The angular position of n^{th} secondary maximum, $\theta_n' = (2n + 1)\dfrac{\lambda}{2d}$

vi. Distance of n^{th} secondary maximum from the center of the screen $x_n' = \left(\dfrac{2n+1}{2}\right)\dfrac{\lambda D}{d}$

5. Width of central maximum,

i. width $\beta_0 = 2\beta = \dfrac{2\lambda D}{d}$

ii. Angular spread of central maximum on either side, $\theta = \pm\dfrac{\lambda}{d}$.

iii. Total angular spread of central maximum $2\theta = \dfrac{2\lambda}{d}$.

6. The resolving power is the reciprocal of limit of resolution. The expression of limit of resolution is $\theta = \dfrac{1.22\lambda}{D}$. The resolving power is $\dfrac{D}{1.22\lambda}$.

7. Grating equation, $\sin\theta_n = Nn\lambda$, where, $N = \dfrac{1}{(a+b)}$.

Worked Out Problems

1. [NEB 2075] How wide is the central diffraction peak on a screen 3.5m behind a 0.01 mm slit illuminated by 500 nm light source?

SOLUTION

Given,

Distance of screen (D) = 3.5 m

Slit width (d) = 0.01 mm = 0.01×10^{-3} m

Wavelength of light = 500 nm = 500×10^{-9} m

Width of central maximum (β_0) =?

We have,

$$\beta_0 = \frac{2\lambda D}{d} = \frac{2 \times 500 \times 10^{-9} \times 3.5}{0.01 \times 10^{-3}} = 0.35 \text{ m}$$

$\therefore$ The width of central maximum is 0.35 m.

2. Light of wavelength 633 nm from a distant source is incident on a slit 0.750 mm wide, and the resulting diffraction pattern is observed on a screen 3.50 m away. What is the distance between the two dark fringes on either side of the central bright fringe?

SOLUTION

Given,

Wavelength of light (λ) = 633×10^{-9} m

Width of slit (d) = 0.75 mm = 0.75×10^{-3} m

Distance of screen (D) = 3.5 m

Distance between two dark fringes on either side of the central bright fringe (β) = ?

We have

$$\beta = \frac{D\lambda}{d} = \frac{3.5 \times 633 \times 10^{-9}}{0.75 \times 10^{-3}} = 2.95 \times 10^{-3} \text{ m}$$

3. [HSEB 2071] A parallel beam of sodium light of wavelength 589.3 nm is incident normally on a diffraction grating. The angle between the two first order spectra on either side of the normal is 27°42'. What will be the number of lines per mm on the grating?

SOLUTION

Given,

Wavelength (λ) = 589.3 nm = 589.3×10^{-9} m

Angle $(2\theta_1) = 27^o\,42' = 27^o + \dfrac{42^o}{60} = 27.27^o$

Number of lines per m (N) = ?

$\therefore\ \theta = 27.7/2 = 13.85^o$

We have,

$d\sin\theta = \lambda$. (for first order)

$$\frac{1}{N}\sin\theta = \lambda$$

$$\text{or, } N = \frac{\sin\theta}{\lambda} = \frac{\sin 13.85}{589.3\times10^{-9}}$$

= 406 lines per mm.

4. **[HSEB 2073] A plane transmission grating having 500 lines per mm is illuminate normally by light source of 600 nm wavelength. How many diffraction maxima will be observed on a screen behind the grating?**

SOLUTION

Given,

Grating lines (N) = $500 \frac{\text{lines}}{\text{mm}} = 500 \times 10^3 \frac{\text{lines}}{\text{mm}}$

Wavelength of light (λ) = 600 nm = 600×10^{-9} m

Number of diffraction maxima (n) = ?

We have,

$$d \sin\theta = n\lambda \quad \ldots(i)$$

For total number of diffraction maxima (θ) = 90°

Also, $d = \frac{1}{N} = \frac{1}{500 \times 10^3} = 2 \times 10^{-6}$ m

From equation (i)

$2 \times 10^{-6} \times \sin 90 = n \times 600 \times 10^{-9}$

$n = \frac{2 \times 10^{-6}}{600 \times 10^{-9}}$

$= 3.33 \approx 3$

$\therefore$ Number of diffraction maxima = 3

5. **A diffraction grating has 400 lines per mm and is illuminated normally by a monochromatic light of wavelength 6000 Å. Calculate the grating spacing, the angle at which first order maximum is seen and the maximum number of diffraction maxima obtained.**

SOLUTION:

Given,

Ruling (N) = $400 \frac{\text{lines}}{\text{mm}} = 400 \times 10^3 \frac{\text{lines}}{\text{m}}$

Wavelength of light (λ) = 6000 Å

$= 6000 \times 10^{-10}$ m

We have,

i. Grating spacing, $d = \frac{1}{N} = \frac{1}{400 \times 10^3}$

$= 2.5 \times 10^{-6}$ m

ii. For first order maximum, n = 1,

we have,

$d \sin\theta = n\lambda$

$\sin\theta = \frac{n\lambda}{d} = \frac{1 \times 6000 \times 10^{-10}}{2.5 \times 10^{-6}} = 0.24$

$\therefore \theta = \sin^{-1}(0.25) = 13.89°$

iii. To find the maximum number of diffraction maxima, θ = 90°.

$d \sin\theta = n\lambda$

for n maximum, $\sin\theta = (\sin\theta)_{max} = 1$

$\therefore n_{max} = \frac{d}{\lambda}$

$= \frac{2.5 \times 10^{-6}}{6000 \times 10^{-10}} \approx 4$

6. **A diffraction grating is set up on a spectrometer table so that, parallel light is incident on it normally. For a light of wavelength 5.5×10^{-7} m, first order reinforcement is observed in directions making angles θ of 18° with the normal on each side of the normal. (i) Find the value of the grating spacing d. (ii) Calculate the wavelength of mono-chromatic light which would give a first order spectral line at θ = 25°, (iii) What would be the values of θ for second order spectral lines for each of these two wavelength? (iv) Is a third order spectrum possible for each? Explain.**

SOLUTION

Given,

Wavelength of light (λ) = 5.5×10^{-7} m

Diffraction angle (θ) = 18°

Number of diffraction pattern (n) = 1

i. Grating spacing (d) = ?

From grating equation,

or, $d \sin\theta_n = n\lambda$

or, $d = \frac{n\lambda}{\sin\theta_n} = \frac{1 \times 5.5 \times 10^{-7}}{\sin(18°)} = \frac{5.5 \times 10^{-7}}{0.28}$

$\therefore d = 1.97 \times 10^{-6}$ m

ii. Wavelength (λ) = ?, if n = 1 at θ = 25°

we have,

$d \sin\theta_n = n\lambda$

$\therefore \lambda = \frac{d \sin\theta_n}{n} = \frac{1.97 \times 10^{-6} \times \sin 25°}{1}$

$= 0.754 \times 10^{-6}$ m

$= 7.54 \times 10^{-7}$ m

iii. θ = ?

n = 2, for above both wavelengths.

Grating equation, $d \sin\theta = n\lambda$

$\therefore \theta = \sin^{-1}\left(\frac{n\lambda}{d}\right)$

for $\lambda = 5.5 \times 10^{-7}$ m

$\theta = \sin^{-1}\left(\frac{2 \times 5.5 \times 10^{-7}}{1.97 \times 10^{-6}}\right)$

$= \sin^{-1}\left(\frac{11 \times 10^{-1}}{1.97}\right)$

$= \sin^{-1}\left(\frac{11}{19.7}\right)$

$= \sin^{-1}(0.56)$

$\therefore\ \theta = 37.84°$

Similarly, for $\lambda = 7.54 \times 10^{-7}$ m

$\theta = \sin^{-1}\left(\frac{2 \times 7.54 \times 10^{-7}}{1.97 \times 10^{-6}}\right) = \sin^{-1}(0.77)$

$\therefore\ \theta = 55.95°$

iv. For n = 3 and $\lambda = 5.5 \times 10^{-7}$ m, we can write,

$\sin\theta = \left(\frac{3 \times 5.5 \times 10^{-7}}{1.97 \times 10^{-6}}\right) = 0.837$

for $\lambda = 7.54 \times 10^{-7}$ m

$\sin\theta = \frac{3 \times 7.54 \times 10^{-7}}{1.97 \times 10^{-6}} = 1.15$

Since, sin θ can not be more than 1, so the third order spectrum is not possible for the wavelength $\lambda = 7.54 \times 10^{-7}$ m but, possible for $\lambda = 5.5 \times 10^{-7}$ m

7. A plane transmission grating is ruled with 400 slit/ cm. Assume normal incidence. The α and β lines emitted by atomic hydrogen have wavelengths 656 nm and 486 nm respectively. Compute the angular separation in degrees between these wavelengths for (i) first order and (ii) second order spectrum.

SOLUTION

Given,

Grating lines (N) = 400 slits/ cm = 4×10^4 slits/m

Wave length of α line (λ_α) = 656 nm = 656×10^{-9} m

Wave length of β line (λ_β) = 486 nm = 486×10^{-9} m

Angular separation in first order = ?

i. For the first order diffraction of α – lines (θ_α) = ?

n = 1

We know that ,

$\sin\theta_\alpha = nN\lambda_\alpha$

or, $\theta_\alpha = \sin^{-1}(1 \times 4 \times 10^4 \times 656 \times 10^{-9}) = \sin^{-1}(0.262)$

$\therefore\ \theta_\alpha = 15.19°$

Similarly, for β -lines (θ_β) = ?

Since, $\sin\theta_\beta = nN\lambda_\beta$

or, $\theta_\beta = \sin^{-1}(1 \times 4 \times 10^4 \times 486 \times 10^{-9}) = 11.18°$

$\therefore$ Angular separation $= \theta_\alpha - \theta_\beta$

$= 15.19 - 11.18 = 4.01°$

ii. Angular separation in second order = ?

Angular separation for α and β line Δθ = ?

n = 2

We can write ,

$\sin\theta_\alpha' = \sin^{-1}(2 \times 0.262) = \sin^{-1}(0.524)$

$\therefore\ \theta_\alpha' = 31.67°$

Similarly, for β – lines

$\sin\theta_\beta' = 2 \times N\lambda_\beta$

or $\theta_\beta' = \sin^{-1}(2 \times 0.194) = \sin^{-1}(0.388)$

$\therefore\ \theta_\beta' = 22.9°$

$\therefore$ angular separation $= \Delta\theta = \theta_\alpha' - \theta_\beta'$

$= 31.67 - 22.9 = 8.7°$

Challenging Problems

1. **[UP]** Monochromatic light from a distant source is incident on a slit 0.750 mm wide. On a screen 2.00 m away, the distance from the central maximum of the diffraction pattern to the first minimum is measured to be 1.35 mm. Calculate the wavelength of the light.

Ans: 506 nm

2. **[UP]** Parallel rays of green mercury light with a wavelength of 546 nm pass through a slit covering a lens with a focal length of 60.0 cm. In the focal plane of the lens the distance from the central maximum to the first minimum is 10.2 mm. What is the width of the slit?

Ans: 32.1×10^{-6} m

3. **[UP]** Parallel rays of light with wavelength 620 nm pass through a slit covering a lens with a focal length of 40.0 cm. The diffraction pattern is observed in the focal plane of the lens and the distance from the center of the central maximum to the first minimum is 36.5 cm. What is the width of the slit? (*Note*: The angle that locates the first minimum is *not* small.)

Ans: 0.92μm

4. **[UP]** Red light of wavelength 633 nm from a helium-neon laser passes through a slit 0.350 mm wide. The diffraction pattern is observed on a screen 3.00 m away. (a) What is the width of the central bright fringe? (b) What is the width of the first bright fringe on either side of the central one?

Ans: (a) 10.9 mm (b) 5.4 mm

5. **[UP]** If a diffraction grating produces its third-order bright band at an angle of 78.4° for light of wavelength 681 nm, find (a) the number of slits per centimeter for the grating; (b) the angular location of the first-order and second-order bright bands, (c) Will there be a fourth-order bright band? Explain.

Ans: (a) 4794 slits/ cm (b) 19°, 40.7° (c) Not possible

6. **[UP]** Monochromatic light is at normal incident on a plane transmission grating. The first order maximum in the interference pattern is at an angle of 8.94°. What is the angular position of the fourth order maximum?

Ans: 38.4°

7. **[UP]** Visible light passes through a diffraction grating that has 900 slits/ cm and the interference pattern is observed on a screen that is 2.5 m from the grating. Is the angular position of the first order spectrum small enough for $\sin\theta \approx \theta$ to be a good approximation?

Ans: Yes, $\sin\theta \approx \theta$

8. **[UP]** Plane monochromatic waves with wavelength 520 nm are incident normally on a plane transmission grating having 350 slits/mm. Find the angles of deviation in the first, second and third orders.

Ans: 10.5°; 21.3°; 33.1°

9. **[UP]** (a) What is the wavelength of light that is deviated in the first order through an angle of 13.5° by a transmission grating having 5000 slits per cm? (b) What is the second order deviation?

Ans: (a) 466.89×10^{-9} m (b) 27.80°

10. **[ALP]** A plane diffraction grating is illuminated by a source which emits two spectral lines of wavelengths 420 nm (420×10^{-9} m) and 600 nm (600×10^{-9} m). Show that the third order line of one of these wavelengths is diffracted through a greater angle than the fourth order of the other wavelength.

11. **[ALP]** A spectral line of known wavelength 5.792×10^{-7} m emitted from a mercury vapour lamp is used to determine the spacing between the lines ruled on a plane diffraction grating. When the light is incident normally on the grating, the third order spectrum, measured using spectormeter, occurs at an angle of 60°19' to the normal. Calculate the grating spacing.

Ans: 20×10^{-7} m

*[**Note:** Hints to challenging problems are given at the end of this chapter.]*

Conceptual Questions with Answers

1. What is diffraction of light?

↳ The phenomenon of bending of light towards the geometrical shadow, when it is passed through a small openings or obstacles is known as diffraction of light. It occurs also in sound waves, radio waves, x-rays etc. This phenomenon is a firm evidence of wave nature of light.

2. Diffraction is common in sound but not common in light waves, why?

↳ The degree to which waves are diffracted depends upon the size of the obstacle or slit and wavelength of light. The greatest effects occur when wavelength of light is about same as slit. As the wavelength of light ($\sim 10^{-7}$ m) is much smaller than the size of the objects around us, diffraction pattern is difficult to see. However, sound waves have large wavelength. They get easily diffracted by the objects around us.

3. If light bends around the obstacles, then why can't we see around a building?

↳ The diffraction of light depends on the size of obstacle. The size of the walls and building are much larger than the wavelength of light. So, the diffraction of light around the edge of building is very poor. So, we cannot see diffraction patterns of light around the wall.

4. A single slit diffraction apparatus is completely immersed in water without changing any other parameter. How is the width of central maximum affected?

↳ The width of central maximum in diffraction pattern is, $\beta = \dfrac{2\lambda D}{d}$

If the diffraction apparatus is completely immersed into water, the wavelength, $\lambda' = \frac{\lambda}{\eta}$.

Hence, the wavelength decreases. So, the width of central maximum decreases.

5. Why are diffraction effects more prominent through a slit formed by two blades than through a slit formed by two fingers?

↳ Diffraction is prominent when narrow slits are used having parallel edges. The slit of blades is perfectly parallel and narrow. But, such type of perfection is impossible in the slit formed by two fingers.

6. How is the width of central maximum affected if (a) the width of slit is double (b) the wavelength of the light used is increased?

↳ a. The width of central maximum is,

$$\beta = \frac{2 \cdot \lambda D}{d}$$

If the width of slit is double, i.e. d' = 2d, then,

$$\beta' = \frac{2 \cdot \lambda D}{2d} = \frac{1}{2}\left(\frac{2 \cdot \lambda D}{d}\right) = \frac{1}{2}\beta$$

It means the width is reduced by half.

b. As $\beta \propto \lambda$, the width of central maximum increases.

7. What two main changes in diffraction pattern of single slit will you observe when monochromatic source of light is replaced by a source of white light?

↳ Following changes are observed when monochromatic light is replaced by a source of white light.

a. In each diffraction order, the diffracted image of the slit gets dispersed into constituent colours of white light. The red fringe with higher wavelength is wider than violet fringe with smaller wavelength.

b. In higher spectra, the dispersion is more and it causes overlapping of different colours.

8. What is diffraction grating?

↳ The optical device which contains a large number of narrow equidistant and parallel slits to obtain the diffraction patterns is called diffraction grating. It is also called transmission grating. An ordinary grating contains thousands of parallel slits per millimeter width over a thin glass plate.

9. What is limit of resolution of an optical instrument?

↳ The smallest linear or angular separation between two point objects at which they can just be seen separately or resolved by an optical instrument is called the limit of resolution of the instrument.

10. Define resolving power of an optical instrument.

↳ The resolving power of an optical instrument is its ability to resolve or separate the images of two nearby point objects so that they can be distinctly seen. It is equal to the reciprocal of the limit of resolution of the optical instrument.

11. On what factors does the resolving power of a telescope depend?

↳ The resolving power of a telescope is,

$$\frac{1}{d\theta} = \frac{D}{1.22\,\lambda}$$

Thus, the resolving power of a telescope depends on the wavelength of light used and the diameter of the telescope objective.

12. Why does the intensity of the secondary maximum become less as compared to the central maximum?

↳ The central maximum is formed due to the constructive interference of wavelets coming from all parts of the slits. But, other maxima are formed due to the constructive interference of only a certain part of total slit width. For example, first maximum is produced by the illumination of one third part of total width of slit.

13. Is it correct to say that diffraction is interference between different parts of the same wave front?

↳ Yes. Actually, light is diffracted from the narrow slit, and the patterns are produced due to the superposition of waves emerging from different points of aperture. According to Huygen's principle, every point of a wave front acts as secondary source of light, called wavelets. The waves spreading as the wavelets interfere on the screen and diffraction patterns are obtained.

14. Although the visible light and radio waves are electromagnetic waves, only radio waves diffract around the buildings, why?

↳ Visible light and radio waves are, though the electromagnetic waves, they have extremely different wavelengths. The wavelength of radio waves is much larger than the visible light. Hence, the radio wave can diffract through the buildings and walls.

15. TV antennas are put at the rooftop, why?

↳ TV antennas should receive the radio waves to operate the television. In many cases, the signal is poorly reached into the building due to the lack of diffraction. For the easy reception of signals, they are kept at rooftops.

Exercises

Short-Answer Type Questions

1. Define diffraction of light.
2. Can sound wave diffract?
3. How does diffraction phenomenon give the wave nature of light?
4. Differentiate between Fresnel and Fraunhofer diffraction.
5. Which principle is appropriate to explain the mechanism of single slit diffraction?
6. The intensity of diffraction maxima gradually reduces towards the edge from central maximum, why?
7. What is diffraction grating?
8. Why is diffraction grating also called transmission grating?
9. How is diffraction phenomenon useful to increase the resolving power of microscope?
10. What is the resolving power of astronomical telescope?
11. What is the reciprocal of resolving power of an optical instrument?
12. What should be the order of size of obstacle or aperture for diffraction of light?
13. Write the feature which distinguishes the diffraction pattern from the double slit interference pattern.
14. A bright patch is observed at the middle of the shadow of one rupee coin placed in the path of light from a distance source. Explain.
15. Why is a diffraction grating preferable to a prism for use in a spectrometer?
16. What is the maximum number of order that can be obtained using a diffraction grating?
17. How will you increase the angular width of the diffraction pattern observed in a grating?

Long-Answer Type Questions

1. Derive an expression for the width of secondary maxima for diffraction of light at single slit. How does this width change with increase in width of the slit?
2. Explain the phenomenon of diffraction of light at a single slit to show the formation of diffraction fringes.
3. What do you mean by diffraction of light? Discuss the Fraunhofer diffraction at a single slit and explain the formation of the diffraction pattern.
4. Define Fraunhofer diffraction. How is transmission grating constructed? Describe necessary theory of diffraction grating. [HSEB 2072]
5. What is resolving power? How is it different from magnifying power of an optical instrument?

6. Define limit of resolution and resolving power of an optical instrument. Discuss the resolving power of an optical instrument with the change of wavelength of light used.
7. What is diffraction of light? How does it differ from interference of light? [HSEB 2058]
8. Describe the diffraction of light at a single slit and find the condition for secondary maxima and minima. [HSEB 2061]
9. What is diffraction of light? Describe diffraction of light through a single slit. [HSEB 2067]
10. What are the differences between interference and diffraction? Explain the theory of diffraction of light through a single slit. [HSEB 2065]
11. What is diffraction of light? How can you determine wavelength of light using Fraunhofer diffraction at a single slit?
12. Explain briefly, what is common to the phenomenon of interference and diffraction.

Numerical Problems

1. The light of wavelength 600 nm is incident normally on a slit of width 3 mm. Calculate the linear width of central maximum on a screen kept 3 m away from the slit.

   ANS: 1.2 mm
2. Red light of wavelength 6500 Å from a distant source falls on a slit 0.50 mm wide. Calculate the distance between the two dark bands on each side of central bright band of the diffraction pattern observed on a screen placed 1.8 m from the slit.

   ANS: 4.68 mm
3. Calculate the resolving power of astronomical telescope, assuming the diameter of the objective lens to be 6 cm and the wavelength of light used to be 540 nm.

   ANS: 9.1×10^4
4. In Fraunhofer diffraction pattern due to a single slit, the screen is placed at a distance of 100 cm from the slit and the slit is illuminated by monochromatic light of wavelength 5893 Å. If the separation between the central maximum and the first secondary minimum is 0.5893 cm, find the width of the slit.

   Ans: 5.89×10^{-3}
5. A parallel beam of monochromatic light is allowed to incident normally on a plane transmission grating having 5000 lines per cm. The angle between the directions of the first and the second order is 15°. Find the wavelength of the light used.

   Ans: 4855×10^{-1}
6. Monochromatic light of wavelength 6.56×10^{-7} m falls normally on a grating 2 cm wide. The first order spectrum is produced at an angle of 18° 15′ from the normal. Deduce the total number of lines on the grating.

   Ans: 9546
7. A plane transmission grating has 5000 lines per cm and is adjusted for normal incidence. At what angle will the second order spectral line be seen using a light of wavelength 5790 Å.

   Ans: 35.38°
8. A plane transmission grating contains 600 slits per mm. Find the angular deviation for 400 nm violet light and 700 nm red light in first order maximum.

   Ans: 10.9°
9. Using a grating of 6000 lines per cm a first order spectral line was seen at certain angle using a light of wavelength 5270 Å. Calculate the angle of diffraction?

   Ans: 18.43°
10. What is the angular separation in the second order spectrum between the two mercury yellow lines of 5790 Å and 5770 Å using a plane diffraction grating of 5000 lines per cm adjusted for normal incidence?

    Ans: 0.14°
11. Light of wave length 5000 Å is incident normally on a plane transmission grating having 6000 lines per cm on the grating surface. Find the difference in the angles of deviation in the first and the third order spectra.

    Ans: 46.7°

12. A parallel beam of light falls normally on a plane grating having 5500 lines per cm. A second order spectral line is observed to be deviated through 30°. Calculate the wavelength of the spectral line?

Ans: 4545Å

13. What is the highest order of the spectrum which may be seen with monochromatic light of wavelength 6000 Å by means of transmission grating of 6000 lines per cm?

Ans: 2.7≈ 3

14. In Fraumhofer single slit diffraction, light of wavelengths λ_1 and λ_2 are used. If first diffraction minimum of λ_1 is to coincide with the second minimum of λ_2, (i) how are the two wavelengths related? (ii) Will any other minima coincide in the diffraction pattern?

Ans: $\lambda_1 = 2\lambda_2$, $2n_1 = n_2$ – under this condition other minima will coincide

15. A parallel beam of sodium light is incident normally on a diffraction grating. The angle between the two first order spectra on either side of the normal is 27° 42′. Assuming that, the wavelength of the light is 5.893×10^{-7} m, find the number of rulings per mm on the grating.

Ans 406 per mm

16. A plane transmission grating contains the slits 4000 lines per cm. Assume normal incidence. The α and β lines emitted by atomic hydrogen have the wavelengths 656 nm and 486 nm respectively. Compute the angular separation in degree between these lines in the record order spectrum.

Ans: 8.77°

17. The resolution limit of eye is 1 minute. At a distance of r km from the eye, two persons stand with a lateral separation of 3 m. Calculate the distance r so that the two persons are just resolved by the naked eye.

Ans: 10.3 Km

18. A parallel beam of white light is incident normally on a diffraction grating with 6000 lines per cm. Calculate the angular separation of the red and violet rays of the first order spectrum. Take the wavelengths of the red and violet light to be 7×10^{-7} m and 4×10^{-7} m respectively.

Ans: 10.94°

Multiple Choice Questions

1. A grating with 4000 lines/inch is given. The number of orders of entire visible spectrum between (4000-7000) are:

   a. One complete order b. Two complete orders
   c. Three complete orders d. Four complete orders

2. Unpolarised light coverts to partially or plane polarised light by many processes. Which of the following does not do that?

   a. reflection b. diffraction
   c. double refraction d. scattering

3. In a Fraunhoffer diffraction experiments at a single slit using a light of wavelength 400 nm, the first minimum is formed at an angle of 30°. The direction θ of the first secondary maximum is given by

   a. $\sin^{-1}\left(\frac{2}{3}\right)$ b. $\sin^{-1}\left(\frac{3}{4}\right)$
   c. $\sin^{-1}\left(\frac{1}{4}\right)$ d. $\tan^{-1}\left(\frac{2}{3}\right)$

4. Consider Fraunhoffer diffraction pattern obtained with a single slit at normal incidence. At the angular position of first diffraction minimum, the phase difference between the wavelets from the opposite edges of the slit is

   a. $\pi/4$ b. $\pi/2$
   c. π d. 2π

Answers

1. (b) 2. (b) 3. (b) 4. (d)

Hints to Challenging Problems

HINT: 1

Given,

Width of slit (d) = 0.75 mm = 0.75×10^{-3} m

Distance of screen from slit (D) = 2 m

Distance of first minima from central maxima (x) = 1.35×10^{-3} m

Wavelength (λ) = ?

Required formula, $x = \frac{D\lambda}{d}$ ($\because$ n = 1)

or $\lambda = \frac{xd}{D}$

HINT:2

Given,

Wavelength of light (λ) = 546×10^{-9} m

Focal length of lens (f) = 60 cm = 0.6 m

$\therefore$ Distance of screen (D) = 0.6 m

($\because$ image is formed in the focal plane of the lens, so f = D)

First minima from central maxima (x) = 10.2 mm

= 10.2×10^{-3} m

Now,

Width of slit (d) = ?

Required formula, $x = \frac{D\lambda}{d}$ ($\because$ n = 1)

or $d = \frac{D\lambda}{x}$

HINT: 3

Given,

Wave length of light (λ) = 620 nm

= 620×10^{-9} m

Focal length of lens (f) = 40 cm = 0.4 m

$\therefore$ Distance of screen (D) = 40 cm = 0.4 m

Distance of first minima from central maximum (x_1) = 36.5 cm = 0.365 m.

Width of slit, d = ?

To find θ, use $\tan\theta = \frac{x}{D}$

Then, use θ in , $d = \frac{\lambda}{\sin\theta}$

HINT: 4

Given,

Wavelength (λ) = 633 nm = 633×10^{-9} m

Width of slit (d) = 0.350 mm = 0.350×10^{-3} m

Distance of screen (D) = 3 m

(a) Width of central bright fringe, $\beta_0 = \frac{2\lambda D}{d}$

(b) Width of first bright fringe on either side of central fringe, $\beta = \frac{\lambda D}{d}$

HINT:5

Given,

Number of order, n = 3

Angle (θ_3) = 78.4°

Wavelength (λ) = 681 nm = 681×10^{-9} m

a. $N = \frac{\sin\theta_n}{n\lambda}$

b. For the first order, $\sin\theta_1 = 1 \times N\lambda$

For the second order, $\sin\theta_2 = 2 \times N\lambda$

c. $\sin\theta_4 = 4 \times N\lambda$

(But $\sin\theta$ cannot be greater than 1, so fourth order bright band will not be possible.)

HINT: 6

For the first order (n = 1), angle (θ_1) = 8.94°

For the fourth order (n = 4), angle (θ_4) = ?

From grating equation,

$\sin\theta_n = Nn\lambda$

Now, taking ratio,

$$\therefore \frac{\sin\theta_1}{\sin\theta_4} = \frac{N \times 1 \times \lambda}{N \times 4 \times \lambda} = \frac{1}{4}$$

$$\therefore \theta_4 = \sin^{-1}\left(\frac{4\sin\theta_1}{1}\right)$$

HINT: 7

Given,

N = 900 slits/ cm = 9×10^4 slits/m

λ = 400 nm = 400×10^{-9} m

n = ?

From grating equation,

(Given, θ is very small so we can say that $\sin\theta \approx \theta$ to be a good approximation.)

HINT: 8

Given,

λ = 520 nm = 520×10^{-9} m

N = 350 slits/mm = 350×10^3 slits/m

i. For the first order, $\sin\theta_n = 1 \times N\lambda$

ii. For the second order, $\sin\theta_2 = 2 \times N\lambda$

iii. For the third order, $\sin\theta_3 = 3 \times N\lambda$

HINT: 9

Given,

Angle for first order, $\theta_1 = 13.5°$

$N = 5000$ slits/ cm $= 5000 \times 10^2$ slits/m

$= 5 \times 10^5$ slits/m

$n = 1$

$\lambda = ?$

a. From grating equation, $\sin\theta_1 = 1 \times N\lambda$

b. Second order deviation (θ_2), n = 2

We have, $\sin\theta_2 = 2 \times N\lambda$

HINT: 10

Given,

$\lambda_1 = 420$ nm $= 420 \times 10^{-9}$ m

$\lambda_2 = 600$ nm $= 600 \times 10^{-9}$ m

i. For third order line of λ_2, $\sin\theta_3 = \dfrac{3\lambda_2}{(a + b)}$

ii. For fourth order line of λ_1, $\sin\theta_4 = \dfrac{4\lambda_1}{a + b}$

Then, find, $\dfrac{\sin\theta_3}{\sin\theta_4}$

HINT: 11

Given,

$\lambda = 5.792 \times 10^{-7}$ m

$n = 3$

$\theta = 60°19' = \left(60 + \dfrac{19}{60}\right)^o = 60.32^o$

Grating spacing $(a + b) = ?$

From grating equation, we have

$(a + b)\sin\theta_n = n\lambda$

or $(a + b) = \dfrac{n\lambda}{\sin\theta_n}$

❖ ❖ ❖

POLARIZATION OF LIGHT

9

CHAPTER

9.1 Introduction

Interference and diffraction phenomena are the evidence of wave nature of light. However, these phenomena do not tell about the nature of wave, i.e., longitudinal or transverse. The nature of longitudinal or transverse wave is confirmed by testing the polarization property. The wave which shows the properties of polarization must travel as transverse wave. Polarization is a phenomenon of wave in which the vibration can be allowed to pass through in one direction only. The fact that light can be polarized was understood in the early years of 1800s. It showed the light wave is a transverse wave motion. Later on, Maxwell described the transverse nature of light considering the light as an electromagnetic wave. According to Maxwell's description of wave theory, the oscillation of electric and magnetic fields are right angles to each other and the direction of motion of wave is perpendicular to both fields. In our discussion, we take the electric field into account because electric field is responsible for the visibility.

9.2 Polarization of waves

The polarization phenomenon is associated with the transverse nature of light. It can be studied experimentally by oscillating a rope up and down or from side by side in one dimensional plane as shown in Fig. 9.1. As the vibration is done only in a plane, the oscillation is called plane polarized and that can be in vertical or horizontal plane.

Suppose a rope is fixed at one end and another end is passed through a vertical slit. Then, the free end is hold with hand and is oscillated up and down. Because the plane of vibration is parallel with the vertical slit, the wave in the oscillating plane travels both sides of the slit as shown in Fig. 9.1(i). If the rope is oscillated horizontally, the wave damps down to zero amplitude by one the slit as shown in Fig. 9.1(ii). In another experiment, if the rope is passed through the horizontal slit and oscillate it, only horizontal oscillation of rope produces the sustainable wave as shown in Fig. 9.1(iii). But, the vertical oscillated damps down to zero amplitude as shown in Fig. 9.1(iv). These experiments show that, only the transverse vibration of wave can be polarized. Longitudinal waves vibrate along the direction of wave travel, and whatever the orientation of the slit, it would make no difference to the transmission of the waves.

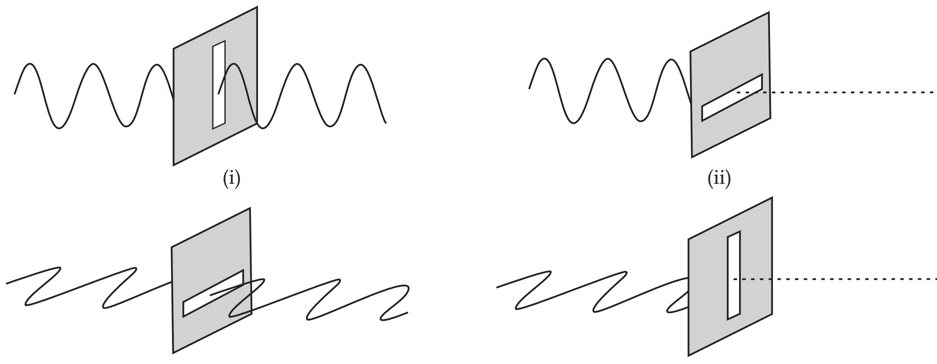


Fig. 9.1: Transverse vibration in horizontal slit

Unpolarized and polarized light

The sun and domestic light bulbs emit the light that possesses transverse vibration in all possible directions. In ordinary beam of light, there are millions of waves travelling in all directions. *These light waves which contain the electromagnetic fields oscillating in all possible directions are called unpolarized light waves.* The vibration of each wave is perpendicular to the direction of propagation of light energy. Unpolarized light is conveniently denoted by a line containing an arrowhead with two rectangular components: one dot and another perpendicular arrowheads line as shown in Fig. 9.2.

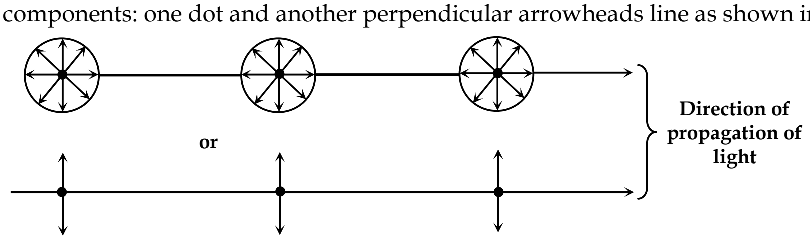
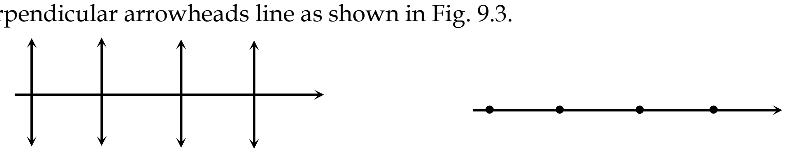


Fig. 9.2: Unpolarized light

The light wave in which the vibration of wave is restricted in a particular plane is known as polarized light. The plane in which the vibrations take place is called the plane of vibration. The plane perpendicular to the plane of vibration is called the plane of polarization. Polarized light can be obtained by passing light through a polaroid. Polarized light is denoted by a straight line containing an arrowhead with a dot or a perpendicular arrowheads line as shown in Fig. 9.3.



(i) Vibrations parallel to the plane of paper **ii) Vibrations perpendicular to the plane of paper**

Fig. 9.3: Polarized light

Differences between unpolarized and polarized light

Unpolarized light	Polarized light
1. The vibration of wave occur symmetrically in all direction.	1. The vibration of wave is restricted in a plane.

2. Unpolarized light contains maximum possible intensity.	2. When unpolarized light is polarized, it is always reduced in intensity.
3. Original light source gives out unpolarized light.	3. It is impossible to create polarized light sources without the usage of a polaroids.
4. In unpolarized light, electric field vector may be directed in all possible directions.	4. In polarized light, the electric field vector is confined in a particular plane.

9.3 Polarization Methods

Light can be polarized by three different methods:

i. **Polarization by scattering:** Polarization of light may occur due to the scattering through molecules of air. When unpolarized light falls on the small particles of air in atmosphere, it is partially polarized. Therefore, sky appears blue in the morning (at the time of sun rise) and red in the evening (at the time of sun set.)

ii. **Polarization by absorption:** When unpolarized light falls on the natural crystals or artificial crystalline material, light is absorbed more in one component than another. So, the light when passes through such crystals, it is found polarized after passing through it. This unequal absorption of components of fields in light is called dichroism and the corresponding material is called dichroic material.

iii. **Polarization by reflection:** When unpolarized light is incident on the surface of transparent medium, the reflected light can be obtained completely polarized at a certain angle of incidence.

9.4 Poloroids

Polaroids are the optical devices which select a part of ordinary light to oscillate only in a particular direction. They are actually thin commercial sheets and have the property of selective absorption. Tourmaline crystal is a natural polaroid. W.H. Herapath discovered a synthetic crystalline material iodo sulphate of quinine in 1852. Later on, in 1932, an American scientist Edwin Land developed a polarizer in the form of large sheets.

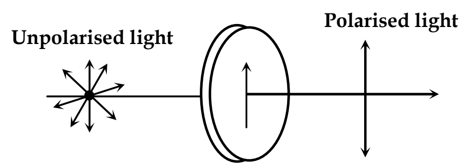


Fig. 9.4: Action of a sheet of Polaroid on unpolarized light

Uses of Polaroids

1. **Polaroid sunglasses:** Polaroid sun glass allows about 50% intensity of incident beam of light. These glasses permit only the vertical oscillation part of wave and horrzontal oscillation is either absorbed or reflected. Hence, it reduces excessive glaring of sunlight.
2. **Polaroid filters:** These filters are used to eliminate the glare of reflected light in photographic camera.
3. **In headlight of automobiles:** Polaroids are used in headlight of automobiles to reduce the glare.

4. **In CD players:** Polarized laser beam acts as a needle for getting sound from compact disc (CD).
5. **In navigation in polar region:** In geographical polar region, the magnetic needle doesn't work properly. In this region, polarized sunlight is used for navigation.
6. **Diabetic patient:** Plane polarized light is used to find the concentration of sugar level in diabetic patients.

Plane of Vibration and Plane of Polarization

When ordinary light is allowed to pass through a polaroid, the light is plane polarized. The direction of vibration of electric field vector is always perpendicular to the direction of propagation of light. If the propagation of light is taken along the horizontal direction, the direction of vibration of electric field vector can be vertical or horizontal plane but perpendicular to propagation. *The plane containing the direction of vibration and the direction of wave propagation is called the plane of vibration.* In Fig. 9.5, ABCD represents the plane of vibration.

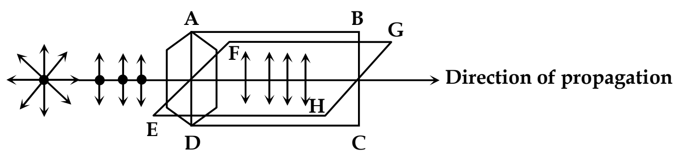


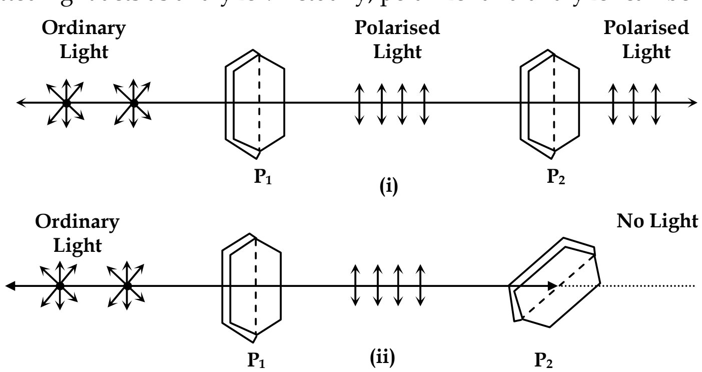
Fig. 9.5: Planes of vibrations and polarizations

The plane passing through the direction of wave propagation and perpendicular to the plane of vibration is called plane of polarization. No vibration occurs in the plane of polarization. In Fig. 9.5, EFGH represents the plane of polarization.

9.5 Experimental Demonstration of Transverse Nature of Light

Polaroid is a device which polarizes the incident beam of light. A natural crystal, tourmaline crystal, acts as a Polaroid. When the unpolarized light arrives at the crystal, the component of electric field of the incident light which is parallel to the molecules is strongly absorbed, whereas the light with its electric field perpendicular to the molecules is transmitted through the space of crystal. Thus, the tourmaline crystal acts as a Polaroid.

To begin the experiment, an original light source and two polaroids are arranged linearly. The unpolarized light emitted by original source is allowed to fall on polaroids. The first Polaroid nearer to the light source acts as the polarizer and the second polaroid which is used to observe the final result of transmitted light acts as analyzer. Actually, polarizer and analyzer can be interchanged.



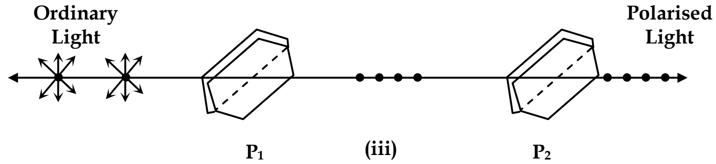


Fig. 9.6: Polarization of a light wave by tourmaline crystal.

In first step, the polarizer P_1 and analyzer P_2 are arranged parallelly, then the plane polarized light transmitted from polarizer can pass through the analyzer. Hence, the polarized light can easily be observed through the analyzer as shown in Fig. 9.6 (i). In the second step, the analyzer is rotated by 90^o so the intensity is crossed. Then, it will be found that, no light is transmitted through the analyzer as shown in Fig. 9.6 (ii). Finally, the polarizer is rotated by 90° such that the polarizer and analyzer again lie parallel to each other. In this condition, the plane polarized light reappears through the analyzer as shown in Fig. 9.6 (iii). These situations are similar to the waves generated in a rope while passing through the parallel and crossed slits as explained in the previous experiment.

It can be concluded from above experiment that,

i. Light can be plane polarized in any components, vertically or horizontally.

ii. The direction of propagation of light is always perpendicular to the oscillation of field vector.

9.6 Malus' law

Statement: When a beam of plane polarized light is incident on an analyzer, the intensity I of light transmitted from the analyzer varies directly as the square of the cosine of the angle θ between the planes of transmission of analyzer and polarizer.

i.e. $I \propto \cos^2 \theta$

Proof: Let A be the amplitude of electric field vector transmitted by a polarizer. Suppose the plane of analyzer make an angle θ with the polarizer as shown in Fig. 9.7. Then, a component of amplitude, i.e. A cos θ will be transmitted by the analyzer. As we know, the intensity of wave is directly proportional to the square of amplitude, we write,

$$\mathbf{I \propto (A \cos \theta)^2}$$

$$\mathbf{I = K A^2 \cos^2 \theta}$$

$$\mathbf{= (KA^2) \cos^2 \theta}$$

$$\mathbf{I = I_0 \cos^2 \theta}$$

Where I_0 (= KA^2) is the maximum intensity of light transmitted from polarizer.

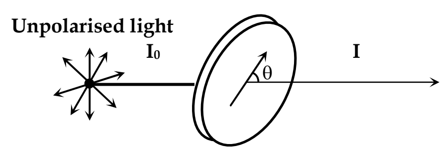


Fig. 9.7 Intensity variation of light

Case I: When polarizer and analyzer are parallel, i.e., θ = 0,

$$I = I_0$$

It means, when analyzer is parallel to the polarizer, the intensity of light transmitted from the analyzer is equal to that which falls on it from the polarizer.

Case II: When polarizer and analyzer are mutually perpendicular, i.e. $\theta = 90°$

$$I = 0$$

So, when analyzer is perpendicular to polarizer, the intensity of light transmitted from the analyzer is zero.

9.7 Polarization by Reflection

In the previous experiments, we have used polaroids to study the Polarization phenomenon of light. In addition to this method, light wave can be polarized by reflection. The phenomenon of polarization by reflection was discovered by Malus in 1908. He discovered that light can be partially or completely polarized by reflection.

Consider an unpolarized light beam falling on a glass–air interface. The beam contains infinitely large number of wavetrains. The electric field vector of each wavetrain can be resolved into two components: perpendicular to the plane of incidence (σ – components) and parallel to the plane of incidence (π – component). When light waves fall on the surface of glass, the components of electric field vectors form certain angles with the reflecting plane. It has been experimentally observed that the reflection coefficient of each component varies as we change the angle of incidence. At a particular angle of incidence, the reflection coefficient for π – component is zero. *This particular angle of incidence at which the reflection coefficient for π – component is zero is called the angle of polarization or polarizing angle.* It is denoted by θ_p. In such condition, the reflected light possesses only the σ – component of electric field vector and is completely free from the π – component. This light beam is completely plane polarized. Thus, the light can be completely polarized by reflection.

As the π – component of field vector is completely absent in the reflected beam, it is completely refracted. But, the σ – component is partially refracted. It is to be noted that σ – component is not completely polarized in reflection therefore, the transmitted beam of light is only partially polarized. It is obvious that the intensity of transmitted beam has greater intensity than the reflected beam. This phenomenon is shown in Fig. 9.8.

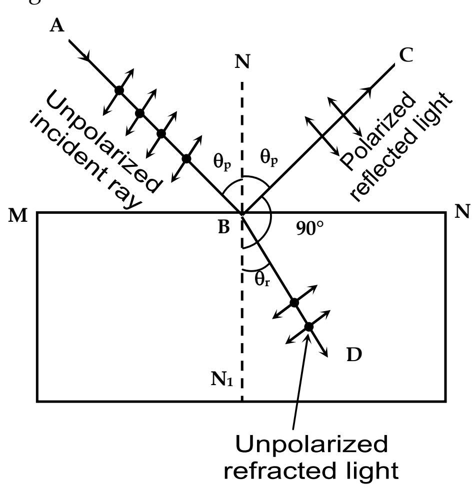


Fig. 9.8: Plane polarization by reflection

9.8 Brewster's law

In 1812, sir David Brewster studied the relation of reflected and refracted angle in a transparent medium at the condition of polarization by reflection. He experimentally found that the sum of these angles, reflected and refracted angles, is 90°. It means the reflected wave and refracted wave are mutually perpendicular to each other at the condition of polarization by reflection. Considering this experimental finding, Brewster formulated a law regarding the condition of polarization by reflection. This law is known as Brewster's law.

This law states that "*at the condition of polarization by reflection, the tangent angle of polarization is equal to the refractive index of the refracting medium*."

Let θ_i and θ_r be the angle of reflection and angle of refraction respectively in a transparent medium (suppose glass). At the condition of polarization by reflection $\theta_i = \theta_p$ = angle of polarization. The required figure to prove this law, is shown in Fig. 9.8.

The experimental result shows that $\theta_p + \theta_r = 90°$,

$$\therefore\ \theta_r = 90° - \theta_p \qquad \text{. . . (9.1)}$$

According to Snell's law,

$$\eta = \frac{\sin \theta_i}{\sin \theta_r} = \frac{\sin \theta_p}{\sin \theta_r} = \frac{\sin \theta_p}{\sin \theta_r} \qquad \text{. . . (9.2)}$$

Where η is the refractive index of transparent medium.

Applying equation (9.1) in equation (9.2), we get

$$\eta = \frac{\sin \theta_p}{\sin (90 - \theta_p)} = \frac{\sin \theta_p}{\cos \theta_p}$$

$$\therefore\ \eta = \tan \theta_p$$

This is the required expression for Brewster's law.

Tips for MCQs

1. Polarization of light confirms its transverse nature.
2. Light can be polarized by passing through certain crystals like tourmaline crystal, Nicol's prism etc. which are called polarizer or Polaroids.
3. Our naked eyes are unable to detect whether a given beam is polarized or not.
4. The intensity of light when transmitted through a polaroid is reduced ideally by 50% of its original intensity.
5. Malus law tells that the resultant intensity (I) transmitted from the analyzer varies directly the square of cosine angle ($\cos \theta$), between plane of transmission of analyzer and polarizer.
   $I \propto \cos^2\theta$
6. Brewster's law states that, the refractive index of a medium is equal to the tangent of the polarizing angle for the given medium. Mathematically,
   $\eta = \tan \theta_P$
7. Polaroids are used to produce plane polarized light and analyser used to analyse (i.e., identify) the given light.
8. In general, light reflected from an amorphous material such as glass is partially polarized.

   The bees can, not only distinguish unpolarized light, from polarized light, but can also determine the direction of polarization.

Worked Out Problems

1. Polarizing angle for a medium is 60°. Calculate the velocity of light in the medium. [HSEB 2072]

SOLUTION:

Given,

Polarizing angle $(\theta_p) = 60^o$

From Brewster's law,

$\eta = \tan\theta_p$

$\theta = \tan 60$

$= \sqrt{3}$

Now, $\eta = \frac{c}{v}$

$\therefore \frac{c}{v} = \sqrt{3}$

$v = \frac{c}{\sqrt{3}} = \frac{3\times10^8}{\sqrt{3}}$

$= \sqrt{3}\times10^8$

$= 1.73\times10^8\ ms^{-1}$

$\therefore$ Velocity of light in that medium is $1.73\times10^8\ ms^{-1}$.

2. Calculate the polarizing angle if light travels from water of refractive index 1.33 to glass of refractive index 1.50.

SOLUTION

Given,

Refractive index of water $(\eta_w) = 1.33$

Refractive index of glass $(\eta_g) = 1.50$

Polarizing angle $(\theta_p) = ?$

From Brewster's law, we can write,

Refractive index of glass with respect to water $({}_w\eta_g) = \tan\theta_p$

or, $\frac{\eta_g}{\eta_w} = \tan\theta_p$

or, $\theta_p = \tan^{-1}\left(\frac{1.50}{1.33}\right) = \tan^{-1}(1.13)$

$\therefore \theta_p = 48.5°$

3. [HSEB 2071] A beam of light is incident at polarizing angle on a piece of transparent material of refractive index 1.62. What is the angle of refraction for the transmitted beam?

SOLUTION

Refractive index $(\eta) = 1.62$

Angel of refraction $(r) = ?$

If θ_p be polarizing angle, then,

$\tan\theta_p = \eta$

or, $\theta_p = \tan^{-1}(\eta) = \tan^{-1}(1.62) = 58.3°$

Again,

$\theta_p + r = 90$

or, $r = 90 - 58.3 = 31.7°$

Challenging Problems

1. The critical angle of light in a certain substance is 45°. What is the polarizing angle?

Ans: 54.7°

2. A parallel beam of liquid is incident at an angle of 58° on a glass surface. The reflected beam is completely plane polarised. Find the critical angle of light in glass.

Ans: 38.68°

3. **[UP]** A parallel beam of unpolarized light in air is incident at an angle of 55° on a plane glass surface. If the reflected beam is completely plane polarized, find the refractive index of the glass and the angle of refraction of the transmitted beam. [HSEB 2067]

Ans: 1.43, 35.5°

4. **[UP]** Light traveling in water strikes a glass plate at an angle of incidence of 53.0°, part of the beam is reflected and part is refracted. If the reflected and refracted portions make an angle of 90.0° with each other, what is the index of refraction of the glass?

Ans: 1.76

5. **[UP]** The refractive index of a certain glass is 1.66. For what incident angle is light reflected from the surface of this glass completely polarized, if the glass is immersed in (a) air, (b) water (η_w = 1.33)?

Ans: (a) 58.9° (b) 51.3°

6. Unpolarized light traveling in a liquid with refractive index η is incident on the surface of the liquid, above which there is air. If the light is incident on the surface at an angle of 31.2° with respect to the normal, the light reflected back into the liquid is completely polarized. (a) What is the refractive index η of the liquid? (b) What angle does the refracted light traveling in air make with the normal to the surface?

Ans: 58.8°

*[**Note:** Hints to challenging problems are given at the end of this chapter.]*

Conceptual Questions with Answers

1. What is polarization of light?

↳ The phenomenon of light in which the vibration of electromagnetic field can be allowed to pass through in one direction only is called polarization of light. Light can be polarized by passing through polariods or by reflection from the surface of transparent medium.

2. Why longitudinal waves cannot be polarized?

↳ Basically, the vibration of particles or fields should be perpendicular to the direction of propagation of wave to polarize the waves. Otherwise, the vibration cannot be restricted in only one direction. This is possible in transverse wave. But, vibrations occur along the direction of propagation in longitudinal waves. So, their polarization is not possible.

3. Which plane is defined as the plane of polarization in a plane polarized electromagnetic wave?

↳ The plane containing the direction of propagation of light and perpendicular to the plane of vibration is called plane of polarization. It contains no vibrations.

4. Which field vector, electric or magnetic is used to represent the polarization of an em wave?

↳ Electric field vector is used to represent the polarization of an electromagnetic wave. It is because electric field is responsible to provide the visibility due to electromagnetic wave.

5. Does the value of polarizing angle of incidence depend on colour of light?

↳ Yes. The polarizing angle (θ_p) depends on the refractive index (η) of a transparent medium. Also, the refractive index depends on wavelength of light. For $\lambda_{red} > \lambda_{voilet}$, $\eta_{voilet} > \eta_{red}$. Hence, value of polarizing angle depends on colour of light.

6. If the polarizing angle for air-glass interface is 56.3°, what is the angle of refraction in glass?

↳ Given, polarizing angle, $\theta_p = 56.3°$.

We know, in the polarizing by reflection, reflected wave and refracted wave are mutually perpendicular to each other. So, $\theta_p + r = 90°$

$\therefore$ $r = 90° - \theta_p$

$= 90° - 56.3° = 33.7°$

7. What is the polarizing angle of a medium of refractive index $\sqrt{3}$?

↳ At the condition of polarization by reflection, the Brewster's law tells us that:

$\tan\theta_p = \eta$

$\tan\theta_p = \sqrt{3}$

$\therefore\ \theta_p = 30°$

8. Sun glasses are made of polariods and not of coloured glasses. Why?

↳ Polariods polarize the light. It means, they absorb only that part of light which produces the dazzling effect in the eye. But the coloured glasses absorb greater quantity of light energy than that of polariods. So, the image formed by coloured glass is dim.

9. Write two uses of polariods?

➯ Two uses of polariods are:

a. They are used in sunglasses and camera filters to reduce the glare of light.

b. In window panes of aeroplanes to control the quantity of light coming in.

10. What is the relation between critical angle and polarization angle?

➯ Critical angle is related to refractive index as,

$\eta = \frac{1}{\sin C}$, where C = critical angle

Also, the relation between refractive index is,

$\eta = \tan\theta_p$, where θ_p = polarization angle

So, $\tan\theta_p = \frac{1}{\sin C}$

11. Which of the following waves can be polarized (a) x – rays (b) sound waves?

➯ a. X-rays are electromagnetic waves. They propagate in transverse form. So, x-rays can be polarized.

b. Sound waves propagate in longitudinal form. So, they cannot be polarized.

12. Name three properties, which are mutually perpendicular to each other in a plane polarized light wave.

➯ The three properties are (a) electric field vector (b) magnetic field vector and (c) direction of propagation of light wave

Exercises

Short-Answer Type Questions

1. Define polarization of light wave.
2. How does the polarization property conform the transverse nature of light?
3. Is light from sodium lamp polarized?
4. Sunglasses reduce the glare of sunlight, why?
5. Which among x-rays, sound waves and radio waves can be polarized?
6. What do you understand by 'crossed polaroids'?
7. What is plane polarized light?
8. Explain σ-polarization and π - Polarization in reflection.
9. Will ultrasonic wave show any polarization?
10. What is the value of refractive index of a medium for which polarizing angle is 60^o?
11. What is Brewster's angle?

Long-Answer Type Questions

1. Describe an experiment to show that, light waves are transverse in nature.
2. What is polarization by reflection? Derive Brewster's law.
3. What do we understand by polarization of a wave? How does this phenomenon help us to decide whether a given wave is transverse or longitudinal in nature?
4. Define polarizing angle. Derive the relation connecting polarizing angle and refractive index of a medium.
5. How can you experimentally distinguish between plane polarized light and unpolarised light?

6. Show that: $\eta = \tan \theta_p$, where η=refractive index of the medium, θ_p = angle of polarization or polarizing angle. [HSEB 2062]
7. Prove that, reflected rays and refracted rays are normal to each other when the light is incident at the angle of polarization.
8. What are polaroids? How do they work? Write down their some applications.

Numerical Problems

1. Light reflected from the surface of a glass plate of refractive index 1.57 is linearly polarized. Determine the angle of refraction in glass.

   ANS: 32.5°
2. Critical angle for a certain wavelength of a light in glass is 40^o. Calculate the polarizing angle and the angle of refraction in glass corresponding to this.

   ANS: 57.3°, 32.7°
3. The polarizing angle for a ray travelling from air to ice is 52°26′. What is the polarizing angle, if the light ray travels from ice to air?

   Ans: 37.5°
4. The sunlight reflected from the surface of water in a pond is completely polarized. If the refractive index of water is 1.33, find the angle between the sun and the horizon?

   Ans: 36.94°
5. At what angle of incidence, sun light reflected from the surface of a lake is fully polarized? The water lake has the refractive index 1.33.

   Ans: 33°
6. A light ray strikes a glass surface at an angle of incidence 59°. The reflected ray and refracted ray are mutually perpendicular. Find the refractive index of glass.

   Ans: 1.66

Multiple Choice Questions

1. The angle of incidence at which reflected light is totally polarized for reflection from air to glass (refractive index n), is

   a. $\sin^{-1}(n)$ b. $\sin^{-1}(1/n)$
   c. $\tan^{-1}(1/n)$ d. $\tan^{-1}(n)$
2. A polaroid is placed at 45^o to an incoming light of intensity I. Now the intensity of light after polarisation would be

   a. I b. $\frac{I}{2}$
   c. $\frac{I}{\sqrt{2}}$ d. zero
3. The numerical operture for a human eye is of the order of

   a. 1 b. 0.1
   c. 0.01 d. 0.001
4. Critical angle for certain medium is $\sin^{-1}(0.6)$. The polarising angle of that medium is

   a. $\tan^{-1}[1.5]$ b. $\sin^{-1}[0.8]$
   c. $\tan^{-1}[1.6667]$ d. $\tan^{-1}[0.6667]$

Answers

1. (d) 2. (b) 3. (d) 4. (c)

Hints to Challenging Problems

HINT: 1

Given,

$C = 45°, \theta_p = ?$

i. $\eta = \frac{1}{\sin C} = 1.41$

ii. From Brewter's law, $\tan \theta_p = \eta$

HINT: 2

Given,

$\theta_p = 58°$

From Brewter's law, $\eta_g = \tan \theta_p$

Then, $C = \sin^{-1}\left(\frac{1}{\eta_g}\right)$

HINT: 3

Given,

$\theta_p = 55°$

$\eta_g = ?$

$r = ?$

From Brewster's law, we know that

i. $\eta_g = \tan \theta_p = \tan 55° = 1.43$

ii. Then use, $\eta_g = \frac{\sin i}{\sin r} = \frac{\sin \theta_p}{\sin r}$

HINT: 4

Given,

$\theta_p = 53^o$

$\theta_p + \theta_r = 90°$

$\theta_r = 90 - 53 = 37°$

$\eta_g = ?$

From Brewster's law, $\frac{\eta_g}{\eta_w} = \tan \theta_p$

HINT: 5

Given,

$\eta_g = 1.66$

a. When the glass is placed in air,

$\therefore \theta_p = \tan^{-1} \eta_g$

b. When the glass is immerged in water, $\theta_p = ?$

${}_w\eta_g = \tan \theta_p$

or $\frac{\eta_g}{\eta_w} = \tan \theta_p$

HINT: 6

Given,

Polarizing angle $(\theta_p) = 31.2°$,

Refractive index $(\eta_l) = ?$

a. From Snell's law,

We can write,

${}_l\eta_a = \frac{\sin \theta_p}{\sin r}$

or, $\frac{1}{{}_a\eta_l} = \frac{\sin \theta_p}{\sin (90 - r)}$

$[\because r + \theta_p = 90°]$

or, $\frac{1}{\eta_l} = \frac{\sin \theta_p}{\cos \theta_p} = \tan \theta_p$

or, $\eta_l = \frac{1}{\tan \theta_p}$

b. $r + \theta_p = 90°$

❖ ❖ ❖

DIRECT CURRENT CIRCUIT

10

CHAPTER

10.1 Introduction

The branch of physics which deals about the motion of electric charges is known as current electricity. When certain amount of charge is given to an insulator, it is deposited at a point, which is named as static charge. The study about the properties of static charge is known as electrostatics. Charge can also be stored in a conductor, when it is surrounded with insulating material. If the charge is added to end of a metallic conductors like silver, copper, aluminium, etc., potential difference is created between two ends, hence, they readily move from one end to another. This dynamics (motion) of charge is dealt in current electricity.

In metallic conductors, the electrons in the outermost orbits are loosely bound to their respective atoms. So, they can easily travel from one atom to neighbouring atoms, these electrons are called free electrons. However, the motion of free electrons in a conductor is completely random. If a certain potential difference is created by any means across two ends of a conductor, the direction of motion of the electrons is specific. This unidirectional flow of charge particles (electrons) creates electric current. The electrons which take part in the electric current are called conduction electrons.

10.2 Electric Circuit

A closed path containing the electric source and other electric components like resistor, switch, etc., is known as electric circuit. If the oppositely charged conducting plates are connected by a metal wire, the charge particles move from higher potential to lower potential. Conventionally, charge flows from positive plate to negative plate as shown in Fig. 10.1 (i). In reality, electrons move from negative charge plate to positive charge plate as shown in Fig. 10.1 (ii), but we describe the direction of movement of charge in conventional way (from positive potential to negative potential). The wire serves as a charge pipe through which the charge can flow (similar to water flow in water pipe). Moreover, charge can flow from positive to positive terminal (also negative to negative terminal), if they have the different electric potential as shown in Fig. 10.1 (iii).

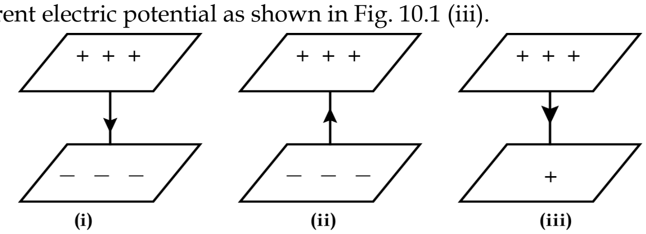


Fig. 10.1: (i) Convention direction of charge flow (ii) Direction of flow of electrons (iii) Flow of charge in similar charged plate

In the above examples, the charge flow continues until the plates have different potentials, and gradually ceases when the plates acquire equal potential. To be a true circuit, charge must continuously flow through the charge pipe (wire) and return back to original position and cycle through again. This can be done forming a conducting path that allows the positive charge from negative plate and back up to the positive plate, then positive charge again flow to the negative plate through wire (charge pipe). The continuous flow of charge generates the electric current in an electric circuit as shown in Fig. 10.2.

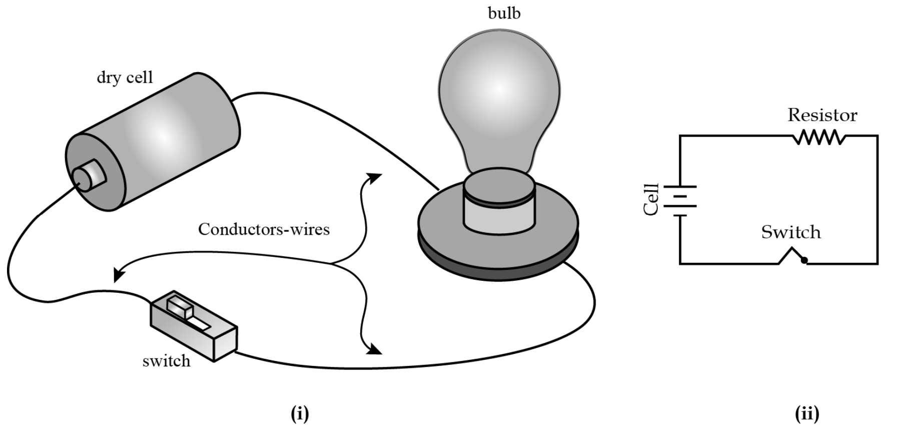
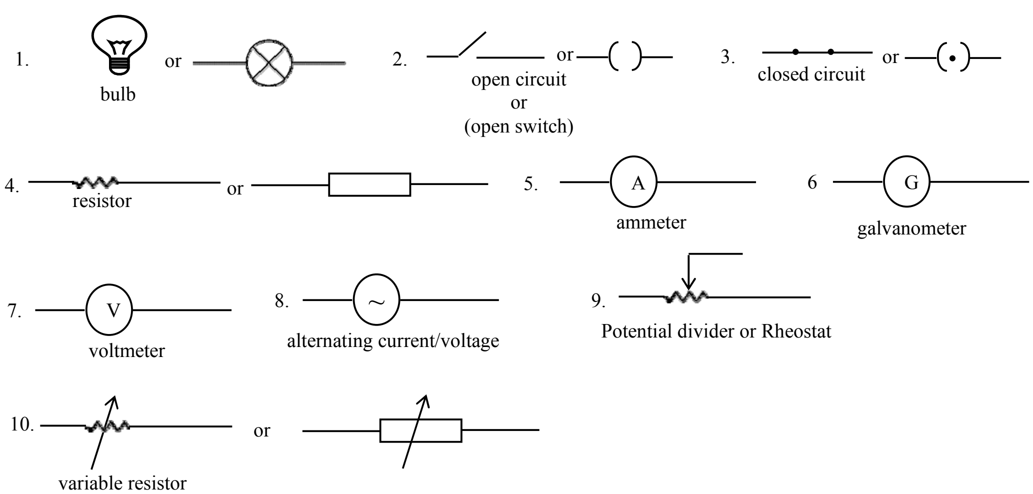


Fig. 10.2: (i) An electric circuit (ii) a symbolic representation of electric circuit

Circuit Symbols



10.3 Electric Current

The electric current is defined as the rate of flow of electric charge through any cross-section of a conductor. It is denoted by I. Its unit is ampere (A). The magnitude of electric current (I) depends on the amount of charge flowing (q) and the time rate (t) at which charge flows.

If the rate of flow of charge is independent with time, the current is said to be steady current. For a charge q flowing over an interval of time 't', the steady electric current is defined as,

$$\text{Electric current (I)} = \frac{\text{Charge flowing (q)}}{\text{Time (t)}}$$

$$\therefore \quad \mathbf{I = \frac{q}{t}} \quad \ldots(10.1)$$

If the rate of flow of charge varies with time, the current is expressed in the differential form of charge with respect to time,

$$\textbf{i.e.} \quad \mathbf{I = \frac{dq}{dt}} \quad \ldots(10.2)$$

This current at any time is called instantaneous current. In equation (10.2), dq is the extremely small charge that flows in any cross-section of a conductor at very short time dt.

Current can consist of any moving charged particles; but most commonly there are electrons. The electric current has particle nature i.e. current relies on the number of charge particles crossing a cross-section of a conductor. If N number of charge particles carrying individual charge e cross the cross-section of a conductor at time interval t, the electric current (I) is written as,

$$I = \frac{Ne}{t} \quad (\because q = Ne)$$

Although the current has both magnitude and direction, it is not a vector quantity. It does not obey the vector addition rules. Hence, current is a scalar quantity.

Electrons flow in a conductor when potential difference is maintained at its two ends. The flow of electrons means the flow of charge. So, the total charge that flows in a circuit can be determined by integrating the electric current with respect to time. i.e.

$$\mathbf{q = \int I\, dt} \quad \ldots(10.3)$$

Unit and Dimension of Current

Electric current is considered as a fundamental physical quantity. Its dimension is [A] or [I]. From the definition of current,

$$I = \frac{q}{t}$$

For, q = 1 coulomb, and t = 1 second (s)

$$I = \frac{1 \text{ coulomb}}{1 \text{ second}}$$

i.e. $I = 1\ C/s = 1$ ampere

ampere (A) is the unit of electric current.

Therefore, the current flowing in a conductor is said to be one ampere, if one coulomb of charge flows across any cross-section of a conductor in one second.

From quantization of charge,

$$q = Ne$$

$$N = \frac{q}{e}$$

For, $q = 1$ C, and $e = 1.6 \times 10^{-19}$ C

Then,

$$N = \frac{1\ C}{1.6 \times 10^{-19}\ C} = 6.25 \times 10^{18}$$

Therefore, 1 A electric current constitutes of 6.25×10^{18} electrons crossing a given cross-section in 1 second.

Some facts about current

i. The flow of charge can be compared with the mechanism of flow of water in a pipe of varying diameter. As the water flows from high pressure to low pressure, charge particles also flow from high potential to low potential. Whatever the diameter of pipe, the volume rate of volume of water is same throughout each cross-section.
ii. The symbol of electric current I or i was taken from the French word "Intensite". Intensite means the intensity.
iii. The magnitude of current at any cross-section of a conductor is same.
iv. The conductor is not charged when current flows through it. Number of electrons that enter into the conductor is equal to the number of electrons that leave from the conductor, while current flows.
v. 'Electric Current' is used for both a phenomenon and a physical quantity.
vi. Current is a scalar quantity.

Types of current in electricity

i. Electric current: The current constituted due to the electrons in a conductor is called electric current. Free electrons are the charge carriers in electric current.
ii. Ionic current: The current constituted due to the motion of positive or negative ions of electrolytes is known as ionic current. For example, when $CuSO_4$ is dissolved in water Cu^{++} ions move towards the negative electrode and SO_4^{--} ions move towards the positive electrode.
iii. Displacement current: The current produced by electric or magnetic fields is called displacement current. The current passing between two capacitor plates is an example of displacement current. Although two capacitor plates are not connected internally with conducting wire, one plate influences the another by electrostatic induction i.e. by electric field. This is the cause of displacement current.

Direct current and alternating current

Direct Current: An electric current whose magnitude and direction does not change with time is known as direct current (d.c.). A dry cell produces the direct current. It repels the living beings. If we touch the high voltage d.c. line, it throws us away. Magnitude of current versus time graph in direct current is shown in Fig. 10.3.

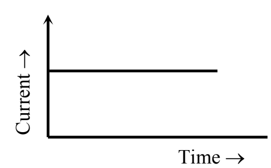


Fig. 10.3: Nature of d.c.

Alternating current: The electric current whose magnitude varies with time and direction reverses periodically is known as alternating current. It is produced by a.c. generator. Its production depends on the Faraday's laws of electromagnetic induction. The magnitude of current versus time graph in alternating current is shown in Fig. 10.4.

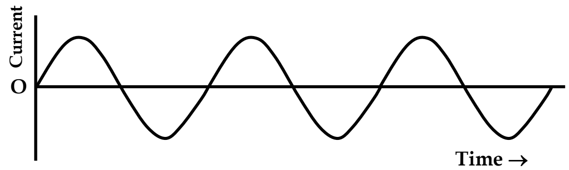


Fig. 10.4.: Nature of a.c.

10.4 Metallic Conduction

An electric conductor contains large number of free electrons. The electrons in outermost orbits of atoms of the conductor are almost free from the nuclear attraction, so they can easily travel from one atom to neighbouring atoms. A conductor contains large number of atoms (in the order of 10^{28} atoms per cubic meter), so that it contains same order of free electrons. These free electrons in the conductor move randomly like the movement of air molecules in the atmosphere. However, the motion of free electrons is not specific, since two ends are at same potential. So, no current is observed, although the charge particles move in such situation. When two ends of the conductor are maintained at different potentials, the net flow of charge can be measured. The free electrons travel towards the positive end of the conductor, and constitute electric current. However, it is a conventional current that is directed from positive end to negative end (i.e. higher potential to lower potential) of conductor and is called the conventional current.

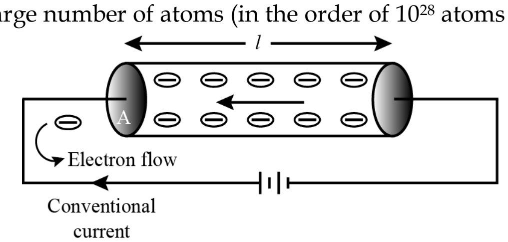


Fig. 10.5: Metallic Conduction

Consider a metallic conductor of length *l* and uniform cross-section A. Two ends of the conductor are maintained at different potentials, connecting it to a dc power supply (a cell) as shown in *Fig. 10.5*. As soon as the ends of conductor become at different potentials, a steady current flows across the conductor. Let q be the net flow of charge at time t, then the net electric current in the conductor is,

$$\mathbf{I = \frac{q}{t}} \qquad \ldots(10.4)$$

Let N be the number of free electrons in the conductor, then from the quantization of charge,

$$\mathbf{q = Ne} \qquad \ldots(10.5)$$

Using equation (10.5) in equation (10.4), we get,

$$\mathbf{I = \frac{Ne}{t}} \qquad \ldots(10.6)$$

Where, e is the charge of an electron.

Suppose n be the number of free electrons per unit volume (V), i.e., $n = \frac{N}{V}$. It is also called electron density. Electron density is constant for a conductor at constant temperature. So, the current expressed in terms of electron density is very useful in the calculation. Now,

$$N = nV$$

Also, volume of conductor (V) = A · *l*. So,

$$\mathbf{N = nA\mathit{l}} \qquad \ldots(10.7)$$

Now, using equation (10.7) in equation (10.6), we get,

$$I = \frac{nAle}{t}$$

$$= nA\left(\frac{l}{t}\right)e$$

$$\mathbf{I = nAv_d e} \qquad \ldots(10.8)$$

Where, v_d is called the drift velocity of an electron in the conductor. Here, $v_d = \frac{l}{t}$, since the electron travels l distance (crosses the conductor) at time t.

In equation (10.8), n and A are constant for a uniform conductor and e is universal constant value of electronic charge. So, the electric current (I) depends on drift velocity (v_d) of electrons, i.e. $I \propto v_d$.

In a conductor, the velocity of electrons is uniform, although the constant potential difference tends to accelerate them from negative potential end to positive potential end, they suffer frequent collisions with other charge particles and interactions with nucleus. The velocity of electron in every instant is almost impossible to observe, so the average velocity is determined to study the flow of charge in the conductor. Therefore, an average velocity of electron in a conductor along a specific direction is known as drift velocity.

The value of drift velocity is practically very small (in the order of mm per second). But, the electric bulb glows as soon as the electric switch is ON, then how is it possible? Actually, all free electrons in the wire are influenced readily when the circuit is ON. So, they form electric wave with velocity about the velocity of light. Thus, the disturbance of charged particles in the filament of electric bulb produces the glow in it.

Current Density

Current density within a conductor is defined as the electric current crossing per unit area of the conductor. The direction of current through the conductor is always perpendicular to the cross-sectional area of that point. Current density is a vector quantity. Its direction is along the direction of current. It is denoted by $\vec{J}$. Therefore,

$$J = \frac{I}{A}$$

i.e. $$\mathbf{I = \vec{J} \cdot \vec{A}} \qquad \ldots(10.9)$$

In magnitude,

$$J = \frac{nev_d A}{A}$$

$$\mathbf{J = nev_d} \qquad \ldots(10.10)$$

The unit of current density is Am^{-2} and its dimensional formula is $[L^{-2} A]$.

10.5 Ohm's Law

Ohm's law provides the basic relation between electric current and potential difference across two ends of a conductor. As explained in metallic conduction, the electric current is detected only when a potential difference is maintained at two ends of a conductor. This was discovered by a school teacher, a German physicist George Simon Ohm in 1828. The law was named after his name 'Ohm'.

Ohm's law states that "Physical conditions like temperature, mechanical strain, etc. remaining the same, the electric current through a conductor is directly proportional to the potential difference across two ends of the conductor".

Let I be the electric current passing through a conductor when potential difference V is maintained at two ends of a conductor, then the law is written as, (provided that the temperature of conductor does not change)

$$V \propto I$$

$$\mathbf{V = RI} \quad \text{... (10.11)}$$

Where, R is proportionality constant, it is called resistance of a conductor.

Equation (10.11) can be compared with straight line equation, $y = mx + c$, with y-intercept, $c = 0$.

This shows that the graph between V versus I is a straight line passing through the origin as shown in Fig. 10.6.

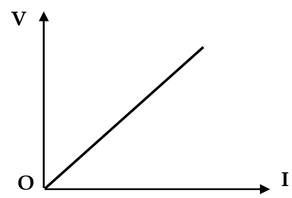


Fig. 10.6: Graph of V versus I

For a conductor, the ratio of potential difference to current at any instant is constant. This type of conductor is called ohmic conductor. All the conductors do not obey Ohm's law.

Experimental Verification of Ohm's Law

Consider an electric circuit containing circuit components: a cell, a resistor, rheostat (a variable resistor), a switch, a voltmeter, and an ammeter, as shown in Fig. 10.7. The ammeter is connected in the series and the voltmeter is connected parallel to the fixed resistor. The rheostat is connected to the series with the fixed resistor. Rheostat varies the resistance in the circuit so that current can be changed to study the relationship between the current and voltage. A standard direct current (dc) source in the circuit provides the constant voltage in the circuit. This voltage can be divided into fixed resistor and the rheostat. Ammeter measures the current and voltmeter measures the potential difference across the resistor.

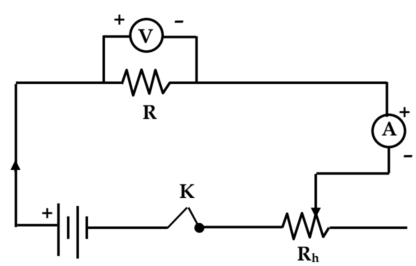


Fig. 10.7: Experimental set-up for ohm's law

To perform the experiment, the deflection of voltmeter and ammeter are set initially at zero, although the circuit is closed. Then, the resistance of rheostat is gradually varied (lowered) so that the current is gradually increased in the circuit. Then, the deflection is observed in the ammeter. As the deflection is observed in ammeter, the deflection in voltmeter needle is also observed increasing gradually. During the procedure, corresponding values of potential difference (V) are noted at different values of current (I). This experiment can be repeated for different conductors of different resistances.

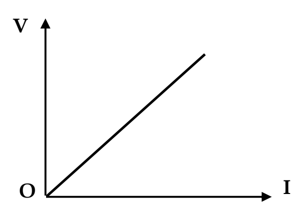


Fig. 10.8: Characteristics curve

Let V_1, V_2, V_3, V_4 and V_5 be the corresponding potential differences for current I_1, I_2, I_3, I_4 and I_5 respectively in a conductor. Then, we can find,

$$\frac{V_1}{I_1} = \frac{V_2}{I_2} = \frac{V_3}{I_3} = \frac{V_4}{I_4} = \frac{V_5}{I_5} = \text{constant}$$

It means, $\frac{V}{I}$ = constant

In this way Ohm's law can be verified experimentally. The graph of V versus I is found as shown in Fig. 10.8. This curve is called characteristics curve of Ohm's law and shows the linear relationship between current and voltage (potential difference).

10.6 Resistance and Resistivity

When an electric current flows through a conductor, it offers opposition in its path due to various factors. Although, the conductor allows electricity to pass through it, it opposes the motion of charge particles (electrons). The opposition offered by the conductor to the flow of electric current through it is known as resistance. It is denoted by R. Its unit is Ohm (Ω).

The value of resistance of a conductor basically depends on two physical dimensions: the length and the cross-sectional area. The resistance of a conductor is,

i. Directly proportional to the length (l) of the conductor,

$$\mathbf{R \propto l} \quad \dots(10.12)$$

ii. Inversely proportional to the cross-sectional area (A) of the conductor,

$$\textbf{i.e.} \quad \mathbf{R \propto \frac{1}{A}} \quad \dots(10.13)$$

Combining the equations (10.12) and (10.13), we get,

$$R \propto \frac{l}{A}$$

$$\therefore \quad \mathbf{R = \rho \frac{l}{A}} \quad \dots(10.14)$$

where, ρ is proportionality constant, it is known as resistivity or specific resistance of a conductor.

For a conductor of length, l = 1 m and cross-section, A = 1 m^2, R = ρ

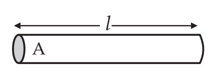
Therefore, *resistivity of a material is the resistance per unit length per unit cross-sectional area of that material.* Its unit is Ohm meter (Ωm). The materials of low resistivity are called conductors. Metals are conductors. The materials of very high resistivity are called non-conductors or insulators. The resistivity of semi-conductors is greater than conductors and smaller than insulators.

The resistivity of a conductor is not universal constant quantity. It primarily depends on the temperature of conductor. It is constant for a conductor whether you change its geometrical dimension such as its length, cross-sectional area, etc. but increases as the temperature increases because collision with the fixed ions increases due to the thermal agitation. Resistance depends on the geometrical dimension, but the resistivity depends on the nature of conductor.

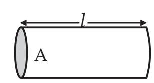
From Ohm's law,

$$R = \frac{V}{I}$$

For, V = 1 V and I = 1 A



For high resistance R
Longer length (l) and
Smaller area (A)



For low resistance R
Shorter length (l) and
Larger area (A)

$$R = \frac{1\ V}{1\ A} = 1\ VA^{-1} = 1\ \Omega$$

Therefore, the resistance of a conductor is said to be 1 Ohm if a current of one ampere flows through the conductor when a potential difference of one volt is applied across its ends.

Mean free path and relaxation time

The free electrons in a metal show same behavior as the gas molecules in a closed vessel. As they move like gas (in atmosphere) within the metal, they are known an 'electron gas'. These free electrons move randomly at sufficiently high speed about $10^5\ ms^{-1}$, however they cannot produce the electric current due to their random motion. During their motion, free electrons collide with ions in the metal and frequently change the direction. *The average distance travelled by a free electron between two successive collisions is called 'mean free path' of that electron and the average time interval between two successive collisions is called its 'relaxation time'*. The relaxation time is denoted by τ.

In a conductor, the electron experiences the electric force when two ends are connected at two terminals of a battery. This force tends to move the electrons in a specific direction, so the current is produced in the conductor. In such motion, the speed of free electrons in a specific direction (i.e. drift velocity) is very small (in the order of mm/s). In such situation, the acceleration of electron into the conductor is determined from drift velocity and relation time,

i.e. $a = \frac{v_d}{\tau}$

$$\therefore \quad \mathbf{v_d = a\tau} \qquad \text{...(10.15)}$$

Resistivity of a conductor

When potential difference V is maintained at two ends of a conductor, the force experienced by free electrons is,

$$F = ma$$

$$\text{or,} \quad \mathbf{a = \frac{F}{m} = \frac{eE}{m} = \frac{eV}{m\mathit{l}}} \qquad \text{... (10.16)}$$

Where,

E = electric field in the conductor

m = mass of electron

V = p.d. across two ends of the conductor.

l = length of conductor

Also, the drift velocity of electrons can be calculated by two different ways,

$$\mathbf{v_d = a\tau} \qquad \text{.... (10.17)}$$

$$\text{and} \quad \mathbf{v_d = \frac{I}{neA}} \qquad \text{... (10.18)}$$

$$\text{So,} \quad \mathbf{a\tau = \frac{I}{neA}} \qquad \text{... (10.19)}$$

Now, substituting the value of 'a' from equation (10.16) to equation (10.19), we get,

$$\frac{eV}{ml}\tau = \frac{I}{neA}$$

$$\therefore \quad \frac{V}{I} = \frac{ml}{ne^2\tau A}$$

$$\mathbf{R} = \left(\frac{\mathbf{m}}{\mathbf{ne^2\tau}}\right)\frac{\boldsymbol{l}}{\mathbf{A}} \qquad \text{... (10.20)}$$

The resistance (R) in terms of resistivity of a conductor is,

$$\mathbf{R} = \rho\frac{\boldsymbol{l}}{\mathbf{A}} \qquad \text{... (10.21)}$$

Comparing equations (10.20) and (10.21), we get,

$$\rho = \frac{m}{ne^2\tau}$$

The value of resistivity for a conductor is constant at a given temperature. The resistivity of alloys are much more than those of pure metals from which they are made.

Unit and dimension of resistivity

$$\text{Since, } R = \rho\frac{l}{A}$$

$$\therefore \quad \rho = R\frac{A}{l}$$

The S.I. unit of resistivity is Ohm meter (Ωm). And,

dimension of resistivity, $[\rho] = [R\frac{A}{l}] = [ML^2T^{-3}A^{-2}\frac{L^2}{L}] = [ML^3T^{-3}A^{-2}]$

Dimensionally, the unit of electrical resistivity is kg $m^3 s^{-3} A^{-2}$.

Electrical Conductance

The reciprocal of resistance of a conductor is called electrical conductance and is denoted by G. Resistance measures the opposition which it offers to the flow of the current. Conductance measures the inducement which it offers to its flow.

$$\therefore \quad \text{Conductance} = \frac{1}{\text{resistance}}$$

$$G = \frac{1}{R}$$

Unit and dimension of Conductance

$$G = \frac{1}{R}$$

The S.I. unit of conductance is inverse Ohm or per Ohm or Ohm^{-1}(Ω^{-1}) or mho or siemen (S). So

$$1\text{ S} = 1\ \Omega^{-1} = 1\text{ mho.}$$

$$\because \quad [G] = \left[\frac{1}{R}\right] = \left[\frac{1}{ML^2T^{-3}A^{-2}}\right] = [M^{-1}L^{-2}T^3A^2]$$

Therefore, the dimensional formula of conductance is $[M^{-1}L^{-2}T^3A^2]$.

Dimensionally, unit of conductance is $kg^{-1} m^{-2} s^3 A^2$ which is equivalent to siemen.

Electrical Conductivity or Specific Conductance

The reciprocal of resistivity of a conductor is called its electrical conductivity and is denoted by σ (small sigma).

$$\therefore \quad \sigma = \frac{1}{\rho}$$

Unit and dimension of electrical conductivity

We know that,

$$\sigma = \frac{1}{\rho}$$

The unit of electrical conductivity is per Ohm per metre ($\Omega^{-1}\ m^{-1}$) or mho meter^{-1} or siemens per metre (Sm^{-1}).

And dimension of conductivity, $[\sigma] = \left[\frac{1}{\rho}\right] = \left[\frac{1}{ML^3T^{-3}A^{-2}}\right] = [M^{-1}L^{-3}T^3A^2]$

The unit of conductivity on the basis of dimension is $kg^{-1}\ m^{-3}s^3\ A^2$.

Relation between Current Density and Electric Field Strength

From the definition of current density, we can write,

$$J = \frac{I}{A} = \frac{1}{A}\frac{V}{R}$$

$$= \frac{1}{A}\frac{V}{\frac{\rho l}{A}} = \frac{V}{\rho l}$$

$$\therefore\ \mathbf{J = \frac{E}{\rho} = \sigma E} \qquad \left[\because \mathbf{E = \frac{V}{\mathit{l}}}\right] \qquad \mathbf{\ldots(10.22)}$$

Variation of Resistance with Temperature

Resistance arises in a conductor due to the collision of moving electrons with fixed ions. As the temperature increases, there is more vibration of ions in the conductor. Therefore, the free electrons at high temperature suffer greater collision than that at lower temperature. Thus, the resistance in a conductor increases, as the temperature increases.

The resistance of a conductor depends on its temperature.

For small temperature variations, the increase in resistance with respect to resistance at 0°C per degree rise in temperature is constant for a conductor. This constant is called temperature coefficient of resistance, α. So,

$$\text{Temperature coefficient of resistance } (\alpha) = \frac{\text{Rise in resistance}}{\text{Resistance at 0°C} \times \text{Rise in temperature}}$$

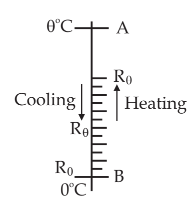
$$\alpha = \frac{R_\theta - R_0}{R_0 \times (\theta - 0)} = \frac{R_\theta - R_0}{R_0\theta}$$

$$R_\theta - R_0 = \alpha R_0\theta$$

$$R_\theta = R_0 + \alpha R_0\theta$$

$$\mathbf{R_\theta = R_0\,(1 + \alpha\theta)} \qquad \mathbf{\ldots(10.23)}$$

Also, $R = \rho\frac{l}{A}$



Temperature coefficient α itself is not constant but depends on the initial temperature on which the increment in resistance is based. When the increment is based on the resistance measured at 0°C, then α has the value of α_0. At any other initial temperature θ, value of α is α_θ and so on.

As temperature changes, the length and the area also change. But, these changes are very small and the factor $\frac{l}{A}$ can be treated as constant. Then, $R \propto \rho$. So,

$$\rho_\theta = \rho_0 (1 + \alpha\theta) \quad \text{...(10.24)}$$

Where, ρ_θ and ρ_0 be the resistivity of the conductor at temperature θ°C and 0°C respectively. In this condition, α is also called temperature coefficient of resistivity. Its unit is $°C^{-1}$ or K^{-1}.

Properties of α

i. The value of temperature coefficient of resistivity of a metal is positive. It means, the resistivity of the metal increases on heating.
ii. The value of temperature coefficient of resistivity of some alloys like managing and constantan is about zero. It means the resistivity of these alloys is almost independent of temperature. Hence, these alloys are used to make the standard resistors.
iii. The value of temperature coefficient of resistivity of semiconductor is negative. It means, resistivity decreases on heating. Also, the resistivity of electrolytes decreases with increase in temperature.

10.7 Colour Code for Resistors

Resistor is an essential component of an electric circuit. Carbon resistors are frequently used in electronic devices, science laboratory and other many parts in electric circuits. To specify the value of resistance in a resistor, colour bands are marked on the surface of resistors as shown in Fig. 10.9. Different colour bands indicates the different numerical values. The numerical values of colour bands are tabulated below.

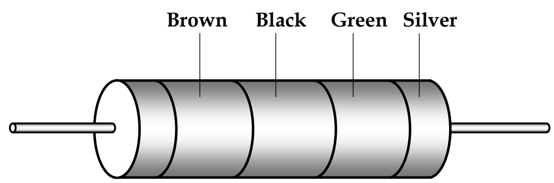


Fig. 10.9: Colour code of resistance

There is a specific rule of reading the value of resistance of a resistor. First two colour bands indicate the resistance value and third one serves as multiplier. For example if a resistor is coded with colours yellow, green and orange, the numerical value of resistance of that resistor is calculated as, YGO = 45 $\times 10^3$ = 45 k Ω. Also, there is one additional colour (i.e. fourth colour) which gives the tolerance. Tolerance is the precision of the resistor and it is given as a percentage. For example, if a resistor of resistance 470 Ω with a tolerance of $\pm$ 10%, then it will have value within 10% of 470, i.e., between 470 – 47 = 423 Ω and 470 + 47 = 517 Ω.

Colour	Colour code	Multiplier	Tolerance %
Black	0	10^0	
Brown	1	10^1	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Gray	8	10^8	
White	9	10^9	
Gold		10^{-1}	5
Silver		10^{-2}	10
No colour			20

Ohmic and non Ohmic conductors

The conductors whose resistance does not depend on the variation of voltage and current in them are called ohmic conductors. These conductors obey Ohm's law. The graph between voltage and current of ohmic conductor is a straight line. So, the resistance of such conductors is also called linear resistance. Metallic conductors like copper, iron, etc. are ohmic conductors. The characteristic curve (I - V curve) for ohmic conductors is shown in Fig. 10.10.

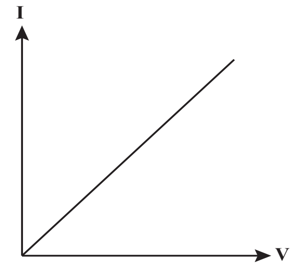


Fig. 10.10: Nature of ohmic resistance

The conductors whose resistance varies with changing voltage and current in them are called non-ohmic conductors. These conductors do not obey Ohm's law. The graph between voltage and current of non-ohmic conductor is a non-linear, so the resistance of such conductors is called non-linear resistance. Semiconductor diodes, triodes and electrolytes are non-ohmic conductors. The characteristic curves (I - V curve) for non-ohmic conductors are shown in Fig.10.11.

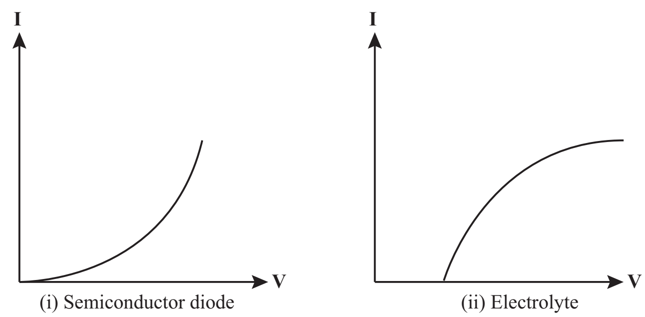


Fig. 10.11: Nature of non-ohmic resistance

10.8 Combinations of Resistors

The connection of two or more resistors in a single circuit is known as combination of resistors. The combination is basically of two types: series combination and parallel combination.

i. Series Combination of Resistors

The combination of resistors one after another in linear chain such that same current passes through each of them is known as series combination of resistors. In this combination, electric current in each resistor is same, and potential difference provided by the source is divided into the resistors in series. So, this combination is also called as voltage divider combination.

Let R_1, R_2 and R_3 be the resistances of three resistors in series combination as shown in Fig. 10.12. It is possible to replace these resistances with a single resistance R in any given electric circuit without changing the potential difference between the terminals of the combination and the current in the circuit. This resistance of single resistor that represents all resistors in the circuit and draws same current from the same source is known as equivalent resistance (R).

Let V_1, V_2 and V_3 be the potential difference across the resistors with resistance R_1, R_2 and R_3 respectively. Also, I be the current in each resistor.

$$\mathbf{V = V_1 + V_2 + V_3} \qquad \ldots (10.25)$$

Now from Ohm's law, we can write,

$V_1 = IR_1, \quad V_2 = IR_2, \quad V_3 = IR_3$

Putting these values in equation (10.25), we get,

$$V = IR_1 + IR_2 + IR_3$$

$$\therefore \quad \mathbf{V = I\,(R_1 + R_2 + R_3)} \qquad \text{. . . (10.26)}$$

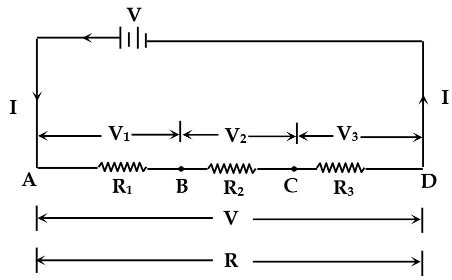


Fig. 10.12: Resistors in series

If these resistors are replaced by a single resistor of resistance R such that same current I flows through it when the same potential difference V is applied across it, then from Ohm's law, we can write,

$$\mathbf{V = IR} \qquad \text{. . . (10.27)}$$

From equations (10.26) and (10.27), we get,

$$IR = I\,(R_1 + R_2 + R_3)$$

$$\therefore \quad R = R_1 + R_2 + R_3$$

In general, for n resistors in series, we have,

$$\mathbf{R = R_1 + R_2 + . . . + R_n} \qquad \text{. . . (10.28)}$$

Thus, if resistors are connected in series, equivalent resistance is equal to the sum of individual resistances. R is more than even the maximum among $R_1, R_2, R_3, ...R_n$.

From (10.28), it is clear that equivalent resistance of series combination is always greater than individual resistance. So, to increase the resistance in the circuit, resistors are connected in series.

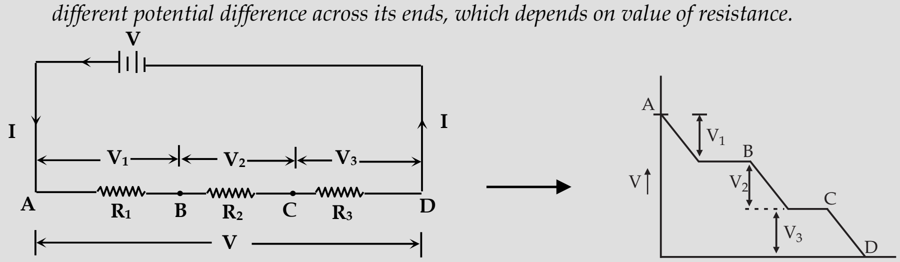
If we consider 'n' number of resistors of equal resistance then, we have,

$R_{equivalent} = R + R + R + . . .$ upto n numbers

$\therefore \quad R_{eq.} = nR$ (maximum)

Note

When a no. of resistors connected in series are joined to a terminal of battery, then each resistance has a different potential difference across its ends, which depends on value of resistance.



ii. Parallel Combination of Resistors

The combination of resistors in which one end of each resistors is connected at a point and another end is connected at another common point such that the combination has common potential difference in each of them is known as parallel combination of resistors. In this combination, potential difference across each of them is the same and current is divided among the resistors.

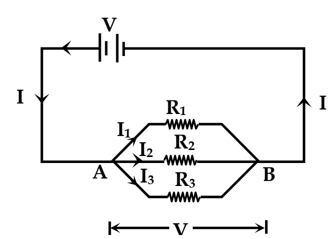


Fig. 10.13: Resistors in parallel

Let R_1, R_2 and R_3 be the resistances of three resistors in parallel combination as shown in Fig. 10.13. It is possible to replace these resistances with a single resistance R in any given electric circuit such that the potential difference across it remains equal to the source, without altering the total current in the circuit. This resistance of single resistor that represents all the resistors is known as equivalent resistance (R).

Let I_1, I_2 and I_3 be the electric current through resistors having resistances R_1, R_2 and R_3 respectively. Since the resistors are combined in parallel, potential difference across each resistor is same.

The total current (I) is the sum of current I_1, I_2 and I_3 through each resistor.

So, $$\mathbf{I = I_1 + I_2 + I_3} \qquad \text{. . . (10.29)}$$

But, the potential difference across each resistance is the same and is equal to the voltage V of battery. So from Ohm's law, we have,

$$I_1 = \frac{V}{R_1}, \; I_2 = \frac{V}{R_2} \text{ and } I_3 = \frac{V}{R_3}$$

Putting these values in equation, (10.29), we get,

$$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$\therefore \quad \mathbf{I = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)} \qquad \text{. . . (10.30)}$$

If parallel combination of resistors R_1, R_2, R_3 is replaced by an equivalent resistor of resistance R in such a way that the same current I flows through it when the same potential difference V is applied across it, then from Ohm's law,

$$\therefore \quad \mathbf{I = \frac{V}{R}} \qquad \text{. . . (10.31)}$$

Here, R is called equivalent resistance of R_1, R_2 and R_3.

From equations (10.30) and (10.31), we have,

$$\frac{V}{R} = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$\text{or,} \quad \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

In general for n resistors in parallel, we have,

$$\therefore \quad \mathbf{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}} \qquad \text{. . . (10.32)}$$

Thus, if resistors are connected in parallel, then reciprocal of equivalent resistance is equal to the sum of the reciprocal of individual resistances. The equivalent resistance is less than the smallest individual resistance, among R_1, R_2, R_3, ...R_n.

From equation (10.32), it is clear that equivalent resistance in parallel combination of resistances is always less than individual resistance. To decrease the resistance in the circuit, resistors are joined in parallel.

If we consider 'n' number of resistors of equal resistance then,

$$\frac{1}{R_{eq.}} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} + \ldots \text{ upto 'n' number.}$$

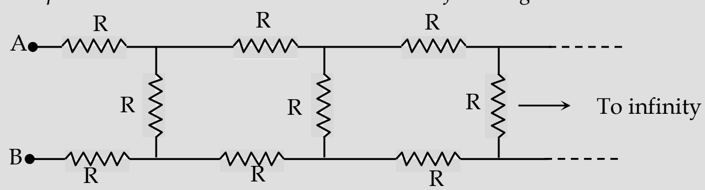
$$\text{or,} \quad \frac{1}{R_{eq.}} = \frac{n}{R}$$

$$\therefore \quad R_{eq.} = \frac{R}{n} \text{ (minimum).}$$

Note

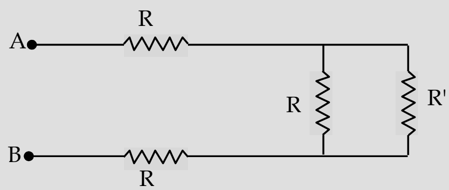
i. *Current does not take the path of least resistance. You may have heard a phrase like "current takes the path of least resistance." This is a reference to a parallel combination of current paths, such that the current can* *take two or more paths. The phrase is incorrect, however, the current takes all paths. Those paths with lower resistance will have large currents, but even very high-resistance paths will carry some of the current.*

ii. *To determine the equivalent resistance between A and B in the following ladder circuit.*



The above circuit is called ladder circuit. In infinitely long ladder circuit, the identical steps of resistors are repeated. If one complete step of ladder is removed from the ladder, remaining part also gives the same value of equivalent resistance.

Let R' be the equivalent resistance of the given circuit. The equivalent circuit diagram for the given circuit is as follows.



Here, equivalent resistance $(R') = R + \dfrac{RR'}{R + R'} + R$

10.9 Voltage Divider Circuit

An electric circuit that contains series combination of resistors is known as *voltage divider circuit*. Let R_1 and R_2 be the resistances of two resistors, connected in series form as shown in Fig. 10.14. In this circuit connection, current I remains constant in each resistor, but the potential difference V provided by the cell is divided into each resistor. Let V_1 and V_2 be the potential differences across resistors with resistances R_1 and R_2 respectively.

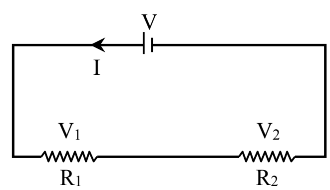


Fig. 10.14: Voltage divider circuit

Here, equivalent resistance = R = $R_1 + R_2$

Total current = I

$\therefore$ Total voltage (V) = I $(R_1 + R_2)$

$$\therefore \quad \mathbf{I = \frac{V}{R_1 + R_2}} \qquad \ldots\mathbf{(10.33)}$$

Now, potential difference across R_1,

$$\mathbf{V_1 = IR_1} \qquad \ldots\mathbf{(10.34)}$$

Using equation (10.33) in equation (10.34), we get,

$$V_1 = \frac{V}{R_1 + R_2} \cdot R_1$$

$$\therefore \quad V_1 = \left(\frac{R_1}{R_1 + R_2}\right) V$$

Similarly, potential difference across R_2,

$$V_2 = \left(\frac{R_2}{R_1 + R_2}\right) V$$

It is to be noted that total voltage (V) is the sum of V_1 and V_2, if internal resistance is negligible.

10.10 Current Divider

An electric circuit that contains parallel combination of resistors is known as *current divider circuit*. Let R_1 and R_2 be the resistances of two resistors, connected in parallel as shown in Fig. 10.15. In this circuit connection, potential difference V in each resistor remains constant, but the current I is divided into each path of parallel circuit. Let I_1 and I_2 be the currents passing through resistors with resistances R_1 and R_2 respectively.

Here, equivalent resistance (R) is calculated as,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2}$$

$$\therefore \mathbf{R} = \frac{\mathbf{R_1 R_2}}{\mathbf{R_1 + R_2}} \qquad \ldots\textbf{(10.35)}$$

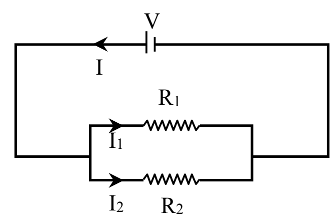


Fig. 10.15: Current divider circuit

Now, current passing through R_1,

$$I_1 = \frac{V}{R_1}$$

$$= \frac{\mathbf{I\,R}}{\mathbf{R_1}} \qquad \ldots\textbf{(10.36)}$$

Using equation (10.35) in equation (10.36), we get,

$$I_1 = \frac{I}{R_1}\left(\frac{R_1 R_2}{R_1 + R_2}\right)$$

$$\therefore I_1 = \left(\frac{R_2}{R_1 + R_2}\right) I$$

Similarly, current passing through R_2,

$$\therefore I_2 = \left(\frac{R_1}{R_1 + R_2}\right) I$$

Total current (I) is the sum of I_1 and I_2.

10.11 Superconductivity

At low temperatures, certain metals and alloys acquire infinite conductivity (zero resistivity to the flow of charge). These materials are superconductors. The property of superconductors is called superconductivity. The temperature below which the special metals exhibit the superconductivity is known as critical temperature (T_c). Above the critical temperature, the resistivity of the material follows trend of normal metal, however the resistivity suddenly drops to zero when temperature approaches to critical temperature. This phenomenon was discovered by Heike Kamerlingh Onnes in 1911. He firstly observed this phenomenon in mercury at a critical temperature of 4.2 K. The variation of resistivity on changing the temperature for normal metals and superconductors are shown in Fig. 10.16.

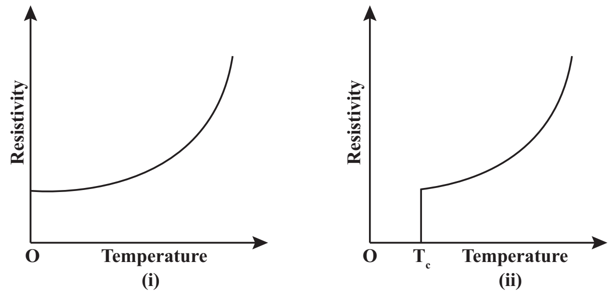


Fig. 10.16: Resistivity versus temperature graph (i) normal metal, (ii) superconductor

Once electric current is established in a superconductor, electrons flow indefinitely, without any applied emf. Steady current can be observed even for several years in a superconductor loops without any observable loss. Since 1987, superconductivity at "high" temperatures (above 100 K) has been found in a variety of non-metallic compounds. Superconductivity property has not been observed in gold, silver and pure ferromagnetic materials.

Superconductors show different behaviour than that of metals in magnetic field. The magnetic lines of force are pushed out from such materials when placed in the magnetic field as shown in Fig.10.17. This behaviour was firstly studied by Meissner and OchsenFeld in 1933, and this effect is called Meissner effect.

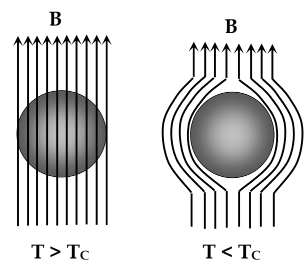


Fig. 10.17: Diagram of the Meissner effect

Properties of superconductors

1. They behave as strong diamagnetic substance in magnetic field. The magnetic lines of force are pushed out from the specimen.
2. The resistivity of the materials drop to zero at the critical temperature and show same value of resistivity below critical temperature.
3. They exhibit Meissner effect.
4. The current in the superconductors persists for very long time.
5. The conductivity of superconductor can be destroyed even below the critical temperature, if it is placed in strong magnetic field.

Applications of superconductors

1. They are used in very strong magnets.
2. They can be used in ultra fast computer switches.
3. They are applicable to construct the transmission of electric power through superconducting power lines.
4. They are used in powerful superconducting electromagnets used in Maglev trains, magnetic resonance imaging (MRI), and nuclear magnetic resonance (NMR).
5. They are used in high sensitive particle detectors like transition edge sensor, superconducting bolometer, etc.
6. They are used in radio frequency and microwave filters.

Perfect conductors

Perfect conductors are ideal conductors which have zero resitivity. Metals can show zero resistivity at 0 K temperature, however we can not achieve exactly zero kelvin temperature experimentally. Hence, the conductor is termed "ideal" conductor. Perfect conductors are not actually super conductors. Superconductivity can be obtained above 0 K temperature. There are some basic differences between perfect conductors and superconductors.

Perfect conductors	Superconductors
1. Magnetic field inside the perfect conductor has non zero value.	1. Magnetic field inside the superconductor is zero.
2. They do not show Meissner effect.	2. They show Meissner effect.
3. Perfect conductivity can be realized in noble metals like gold, silver, etc.	3. Superconductivity is not observed in noble metals like gold, silver, etc.
4. They are not perfect diamagnetic substances.	4. They are perfect diamagnetic substances.
5. Temperature should be decreased to 0 K to get a perfect conductor.	5. Superconductors can be made above 0 K, below critical temperature.

10.12 Electrical Devices

Voltmeter

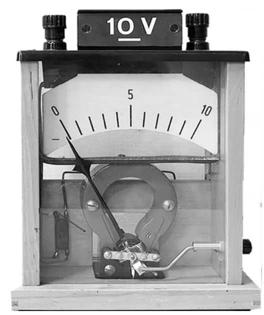


Fig. 10.18: Voltmeter

A voltmeter is an electrical device which is used to measure the electric potential difference between two points in an electric circuit. A voltmeter is connected in parallel with a device to measure the voltage drop across it. A properly calibrated voltmeter can display the accurate value of potential difference of two points in an electric circuit. Display systems of voltmeter are two types: analog system and digital system. In a analog voltmeter, the pointer (or needle) of voltmeter is deflected in proportional to the circuits voltage as shown in Fig. 10.18. A digital voltmeter provides a numerical display. Voltmeter has a high resistance so that it should not draw an appreciable amount of current towards it. Hence, it gives the accurate value.

Ammeter

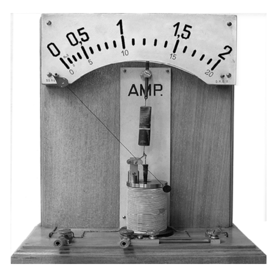


Fig. 10.19: Ammeter

An ammeter is an electrical device which is used to measure the electric current in a circuit. Electric currents are measured in amperes (A), hence the device is named 'ammeter' (or ampere meter). It is connected in series with the circuit. Ammeter can be analog and digital. It measures direct and alternating current. The internal resistance of an ammeter is very small so that it may not change the value of current flowing in the circuit. In ac circuit, a current transformer converts the magnetic field around a conductor into a alternating current. The diagram of ammeter is shown in Fig.10.19. Ammeter should have much small resistance so that by connecting it to the circuit, the current flowing through the circuit should not change.

Rheostat

Rheostat is an electrical instrument used to control current by varying the resistance. The value of resistance can be varied by sliding its key. Rheostat consists of three terminals in which two

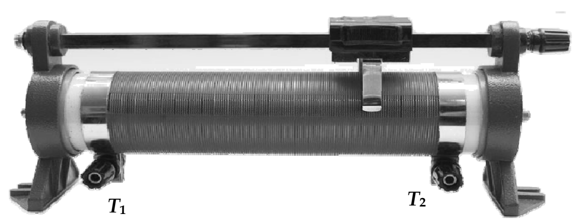


Fig. 10.20: Rheostat

Fig. 10.21: Multimeter

are fixed and connected at two ends of resistive element, called track and another one is the variable terminal connected to the sliding wiper or slider. Resistance of rheostat depends on the length of the resistive track. The wiper that moves along the resistive element varies the resistance of the rheostat. The resistance of the rheostat is changed when the wiper is moved over the resistive path. The resistive element of the rheostat is made of a coil of wire or a thin carbon film as shown in Fig.10.20.

Multimeter

A multimeter is an electronic measuring device that combines several measurement functions in one unit. A typical multimeter measures current, voltage and resistance. Multimeters are also analog and digital. The diagram of an analog multimeter is shown in Fig.10.21.

Galvanometer

Galvanometer is an electrical device which detects the presence of current in a circuit. It does not measure directly the quantity of current and voltage, but can be converted into current and voltage measuring devices after suitable modification. It means, galvanometer can be converted into an ammeter and voltmeter, connecting the suitable resistors in it. It is denoted in circuit by —G—.

Fig. 10.22: Galvanometer

Shunt

A very small resistance connected in parallel to the galvanometer is called a shunt. It is denoted by S. The shunt serves to reduce the internal resistance of an ammeter. Therefore, the ammeter can be connected in series in electric circuit without altering the value of total current. Moreover, it prevents the galvanometer from over heating.

Multiplier

A very high resistance connected in series to the galvanometer is called a multiplier. It is denoted by R. The multiplier serves to increase the internal resistance of a voltmeter.

i. Conversion of a galvanometer into an ammeter

Ammeter is current measuring device. It is connected in the series of an electric circuit.

To convert a galvanometer into an ammeter, a resistor of very small resistance, called shunt, is connected in parallel with the galvanometer. The galvanometer, is a low resistance device, but the resistance is not negligibly small. When large current is passed through the galvanometer, the large amount of heat is generated in it. So, the internal resistance of galvanometer must be reduced so that heat generated in it would not harm it. So, the first requisite is to reduce the heat appreciably. The shunt, very small resistance combined in parallel, when connected parallel to the galvanometer reduces the resistance without appreciably reducing the current in the circuit. The value of resistance in the shunt determines the range of ammeter (0 – I). The value of shunt is so adjusted that most of current passes through the shunt. The circuit diagram of the combination of galvanometer and shunt is shown in Fig. 10.23.

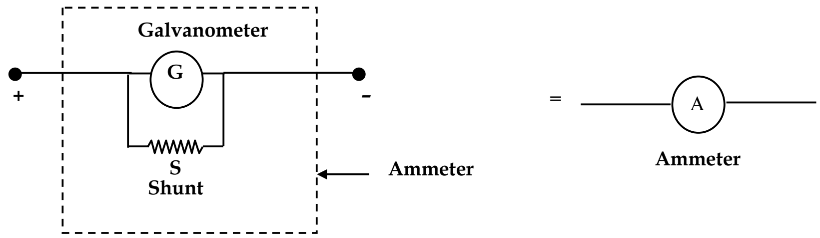


Fig. 10.23: Conversion of galvanometer into ammeter

Let G be the internal resistance of the galvanometer. A suitable resistance having very small value, shunt (S), is connected in parallel with the galvanometer as shown in Fig. 10.23. The total current, I in the circuit is divided into two parts, through the galvanometer and through the shunt. Let I_g be the maximum current through the galvanometer which gives the full scale deflection in it and the remaining current $(I - I_g)$ $(>> I_g)$, is passed through the shunt. Although, very small current I_g passes through the galvanometer, the deflection in it provides the equivalent value of the total current I.

Since, the galvanometer and shunt are connected parallel to each-other, the potential difference across the galvanometer is equal to the potential difference across the shunt. Therefore,

$$(I - Ig)\,S = I_g G$$

$$\therefore \quad \mathbf{S} = \left(\frac{\mathbf{I_g}}{\mathbf{I - I_g}}\right)\mathbf{G} \qquad \ldots\mathbf{(10.37)}$$

Knowing the value of I_g, I and G, the value of shunt can be determined.

i. Internal Resistance of Ammeters

In the conversion of galvanometer into an ammeter, shunt(S) is connected in parallel with galvanometer of resistance (G). So, the equivalent resistance (R_a) for the ammeter is,

$$R_a = S \parallel G$$

$$\mathbf{R_a} = \frac{\mathbf{SG}}{\mathbf{S + G}} \qquad \ldots\mathbf{(10.38)}$$

This shows that, the resistance of ammeter is even smaller than the resistance of shunt (S).

ii. Conversion of Galvanometer into Voltmeter

Voltmeter measures the potential difference across the electric components in an electric circuit. It is connected in parallel with an electric circuit.

To convert a galvanometer into a voltmeter, a very high resistance, called multiplier, is connected in series with the galvanometer. The internal resistance of the voltmeter should be made very high so that very small current is allowed to pass through it. This makes the large current to pass through the load resistance, and hence achieve the high efficiency of the circuit. The circuit diagram of the series combination of galvanometer and multiplier is shown in Fig. 10.24. It should be noted that voltmeter is connected in parallel circuit, but a high resistance is connected in series with galvanometer for the conversion purpose.

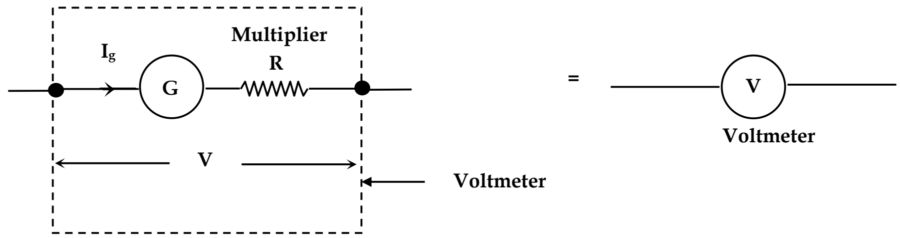


Fig. 10.24: Conversion of galvanometer into voltmeter

Let G be the internal resistance of the galvanometer. A suitable resistance of very high value, multiplier (R), is connected in series with the galvanometer as shown in Fig. 10.24. Let I_g be the maximum value of current through the galvanometer. The potential difference (V) of the combination of G and R is

$$V = I_g (G + R)$$

Where, R = resistance of additional resistor (i.e. multiplier)

$$G + R = \frac{V}{I_g}$$

$$\mathbf{R = \frac{V}{I_g} - G} \qquad \textbf{...(10.39)}$$

Knowing the values of V, I_g and G, the suitable value of R is determined.

Internal resistance of voltmeter

In the conversion of galvanometer into voltmeter, a large resistance R, is connected in series with galvanometer of internal resistance (G). So, the resistance of voltmeter, R_v is,

$$R_v = R + G$$

Obviously, the internal resistance of voltmeter is greater than the resistance of multiplier. In an ideal voltmeter, the internal resistance is considered infinity.

Tips for MCQs

1. i. Electric current (I) $= \frac{dq}{dt} = \frac{Ne}{t}$, where N is total number of charge particles.

   ii. The unit of I is ampere (A), $1\ A = 1\ Cs^{-1}$
   In 1 A current, 6.25×10^{18} electrons flow per unit time which is equivalent to 3×10^9 stat ampere.

   iii. It is a scalar quantity, even though the conventional direction of current is shown from positive terminal to negative terminal of cell.

2. **Electric conduction is defined quantitatively as,**

   $I = nev_dA$

   Where, v_d is drift velocity of electron.

   The drift velocity of electron in a conductor:

   i. is directly proportional to the electric field, E into the conductor, i.e. $v_d \propto E\left(=\frac{V}{l}\right)$, where V is the potential difference across the conductor and l is the length of conductor.

ii. depends upon nature of conductor and electric field applied across the conductor.

iii. is about 10^{-4} ms^{-1} and value of relaxation time is about 10^{-14} second.

3. Electrical conduction is due to the drift of:

i. electrons in a conductor.

ii. free electrons and holes in a semiconductor.

iii. positive and negative ions in an electrolyte.

iv. electrons and ions in gases in gas discharge tubes.

4. Current density (J):

i. Vector quantity, $J = \frac{I}{A} = nev_d$

ii. $I = \vec{J} \cdot \vec{A} = JA \cos\theta$, where θ is the angle made by small cross sectional area A with the direction of current.

5. Resistance and conductance:

i. Resistance (R) = $\rho \frac{l}{A}$ and $\rho = \frac{m}{ne^2\tau}$

Where, τ is relaxation time, m is mass of electron, n is the electron density and e is magnitude of electronic charge.

ii. The resistivity depends on temperature and nature of conductor.

iii. The reciprocal of resistance (R) is conductance (G), $G = \frac{1}{R}$ and the reciprocal of resistivity (ρ) is conductivity (σ), $\sigma = \frac{1}{\rho}$.

iv. The unit of resistance is ohm (Ω) and the unit of conductance is siemen or mho.

v. The unit of resistivity is ohm-meter and the unit of conductivity is $(\text{ohm-meter})^{-1}$ or Siemens per meter.

6. Combination of resistors:

i. Series combination: $R = R_1 + R_2 + R_3 + \ldots$

ii. Parallel combination, $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$

iii. The equivalent resistance of n equal resistors of equal resistance (r) when connected in series is $R = nr$.

iv. The equivalent resistance of n equal resistors of equal resistance (r) when connected in parallel, $R = \frac{r}{n}$.

v. The ratio of n identical resistors of equal resistance when connected in series to parallel is,

$$\frac{R_{series}}{R_{parallel}} = n^2$$

7. Variation of resistance with temperature:

i. The resistance at θ°C is,

$R_\theta = R_0(1 + \alpha\theta)$, where R_0 = resistance of conductor at 0°C

α = temperature coefficient of resistance

ii. The unit of α is $°C^{-1}$ or K^{-1}.

iii. To find the temperature coefficient of resistance, the resistance at 0°C is taken as reference. So,

$$\alpha = \frac{R_\theta - R_0}{R_0(\theta - 0)} = \frac{R_\theta - R_0}{R_0\theta}$$

If R_1 is the resistance at θ_1°C and R_2 is the resistance at θ_2°C, then

$R_1 = R_0 (1 + \alpha\theta_1)$ and

$R_2 = R_0 (1 + \alpha\theta_2)$

Solving, we get, $\alpha = \dfrac{R_2 - R_1}{R_1\theta_2 - R_2\theta_1}$ (Solution is quite tricky)

iv. The value of α is:

a. positive for a conductor

b. negative for a semiconductor

c. for alloys like manganin and constantan, it is negligibly small, $\alpha \to 0$. So, they are used to make standard resistances.

d. zero in superconductors.

8. Conversion of galvanometer into ammeter

i. Internal resistance of ammeter is very small, $R_a = \dfrac{GS}{G + S}$

ii. Value of shunt is, $S = \dfrac{I_g G}{I - I_g}$

9. Conversion of galvanometer into voltmeter:

i. Internal resistance is very large; $R_V = R + G$

ii. Value of multiplier, $R = \dfrac{V}{I_g} - G$

10. Unit and dimension of some physical quantities.

Physical Quantity	Symbol	Dimensions	Unit	Remark
Electric Current	I	[A]	A	SI base unit
Charge	Q or q	[T A]	C	
Voltage, Electric potential difference	V	$[M L^2 T^{-3} A^{-1}]$	V	$\dfrac{\text{Work}}{\text{charge}}$
Electromotive force	E	$[M L^2 T^{-3} A^{-1}]$	V	$\dfrac{\text{Work}}{\text{charge}}$
Resistance	R	$[M L^2 T^{-3} A^{-1}]$	Ω	$R = \dfrac{V}{I}$
Resistivity	ρ	$[M^{-1} T^{-3} A^{-2}]$	Ωm	$R = \dfrac{\rho l}{A}$
Electrical conductivity	σ	$[M^{-1} L^{-3} T^3 A^2]$	S	$\sigma = \dfrac{1}{\rho}$
Electric field	E	$[M L T^{-3} A^{-1}]$	Vm^{-1}	$\dfrac{\text{Electric force}}{\text{charge}}$
Drift speed	v_d	$[L T^{-1}]$	ms^{-1}	$v_d = \dfrac{I}{neA}$
Current density	J	$[L^{-2} A]$	Am^{-2}	$\dfrac{\text{current}}{\text{area}}$

Worked Out Problems

1. A 3 Ω and 6 Ω resistors are connected in parallel and the combination is connected series with 8 Ω resistors. Calculate the equivalent resistance and total current in the circuit if a cell of 2 V is connected in the circuit.

SOLUTION

Using the informations given in the question, an electric circuit can be drawn, which is shown in figure below.

Here,

Potential difference, V = 2 V

$R_1 = 3\ \Omega$

$R_2 = 6\ \Omega$

$R_3 = 8\ \Omega$

Equivalent resistance (R) $= (R_1 \,||\, R_2) + R_3$

$= \dfrac{R_1 R_2}{R_1 + R_2} + R_3$

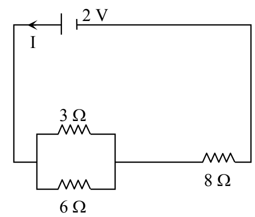
$= \dfrac{3 \times 6}{3 + 6} + 8 = 2 + 8 = 10\ \Omega$

$\therefore$ R = 10 Ω.

Equivalent resistance = 10 Ω

Now, total current (I) $= \dfrac{V}{R} = \dfrac{2}{10} = 0.2$ A

$\therefore$ Total current in the circuit is 0.2 A.



2. **Two resistance of 1000 Ω and 3000 Ω are connected in series with 200 V main supply. What will be the reading in voltmeter of internal resistance 1000 Ω when placed across the 1000 Ω resistance?**

SOLUTION

The appropriate circuit design using the given information is shown in figure below.

Here,

V = 200 V

$R_1 = 1000\ \Omega$

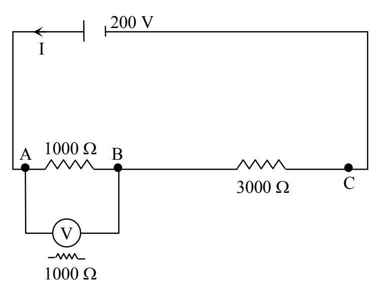
$R_2 = 3000\ \Omega$

$R_V = 1000\ \Omega$

When voltmeter is connected across 1000 Ω resistor, the resistance between A and B is,

$R_{AB} = R_1 \,||\, R_V$

$= \dfrac{R_1 R_V}{R_1 + R_V} = \dfrac{1000 \times 1000}{1000 + 1000} = 500\ \Omega$



Now, equivalent resistance of the circuit,

$R = R_{AB} + R_{BC}$

$= 500 + 3000 = 3500\ \Omega$

Total current (I) $= \dfrac{V}{R} = \dfrac{200}{3500} = 0.057$ A

Now, voltmeter reading gives the voltage across A and B, i.e.

$V_{AB} = IR_{AB} = 0.057 \times 500 = 28.5$ V

3. **A cell of emf 12 V and negligible internal resistance is connected in series with two resistors of resistance 100 Ω and 200 Ω. Calculate the potential drop across each resistor.**

SOLUTION

the electric circuit design in accordance with the given information is given below.

Here,

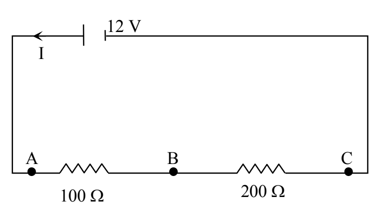
V = 12 V

$R_1 = R_{AB} = 100\ \Omega$

$R_2 = R_{BC} = 200\ \Omega$

Equivalent resistance (R) $= R_1 + R_2 = 100 + 200 = 300\ \Omega$

Total current in the circuit (I) $= \dfrac{V}{R} = \dfrac{12}{300} = 0.04$ A



Now, potential difference across 100 Ω resistor,

$V_{AB} = IR_{AB} = 0.04 \times 100 = 4$ V

The potential difference across 200 Ω resistor

$V_{BC} = IR_{BC} = 0.04 \times 200 = 8$ V

4. A moving coil galvanometer of resistance 10 Ω produces full scale deflection when a current of 25 mA flows through it. How will you convert it into:

a. a voltmeter of range (0 - 120 V)

b. an ammeter of range (0 - 20 A)

SOLUTION

Given,

Resistance of galvanometer (G) = 10 Ω

Current in galvanometer (I_g) = 25 mA = 0.025 A

a. For a voltmeter range (0 – 120 V)

V = 120 V

Series resistance (R) = ?

We have,

$$R = \frac{V}{I_g} - G = \frac{120}{0.025} - 10 = 4800 - 10 = 4790\ \Omega$$

b. For an ammeter range (0 - 20 A)

I = 20 A

Shunt (S) = ?

We have, $S = \frac{I_g}{I - I_g} \times G = \frac{0.025}{20 - 0.025} \times 10 = 0.0125\ \Omega$

5. A Silver wire 2.6 mm in diameter transfers a charge of 420 C in 80 min. Silver contains 5.8 × 10²⁸ free electrons per cubic meter. (a) What is the current in the wire? (b) What is the magnitude of the drift velocity of the electrons in the wire?

SOLUTION

Given,

Diameter (d) = 2.6 mm 2.6×10^{-3} m

Charge (Q) = 420 C

Time (t) = 80 min. = 80 × 60 s.

density of electrons (n) = $5.8 \times 10^{28}\ m^{-3}$

a. Current in the wire (I) = ?

We know that

$$\because\ I = \frac{Q}{t} = \frac{420}{80 \times 60} = 9 \times 10^{-2}\ A.$$

b. Drift velocity (v_d) = ?

We know that

$$v_d = \frac{I}{neA}$$

$$= \frac{I}{ne\left(\frac{\pi d^2}{4}\right)} \qquad \left[\because A = \frac{\pi d^2}{4}\right]$$

$$= \frac{9 \times 10^{-2}}{5.8 \times 10^{28} \times 1.6 \times 10^{-19}\left(\frac{3.14 \times (2.6 \times 10^{-3})^2}{4}\right)}$$

$$= 1.8 \times 10^{-6}\ ms^{-1}$$

6. When a wire carries a current of 1.20 A, the drift velocity is 1.20×10^{-4} ms⁻¹. What is the drift velocity when the current is 6.00 A?

SOLUTION

In the first case,

Current (I) = 1.2 A

Drift velocity of (v_d) = 1.20×10^{-4} ms^{-1}

We know that,

$$v_d = \frac{I}{nAe} \quad \ldots \textbf{(i)}$$

In the second case,

Current (I') = 6 A

Drift velocity (v_d') = ?

Again,

$$v_d' = \frac{I'}{nAe}$$

Dividing (ii) by (i), we get

$$\frac{v_d'}{v_d} = \frac{I'}{I}$$

or $v_d' = \frac{I'}{I} \times v_d = \frac{6}{1.2} \times 1.20 \times 10^{-4} = 6 \times 10^{-4}\ ms^{-1}$

7. The potential difference between points in a wire 75.0 cm apart is 0.938 V when the current density is 4.40×10^7 Am^{-2}. What is (a) the magnitude of $\vec{E}$ in the wire? (b) The resistivity of the material of which the wire is made?

SOLUTION

Given,

Potential difference (V) = 0.938 V

Length (l) = 75.0 cm = 75.0×10^{-2} m

Current density (J) = 4.40×10^7 Am^{-2}

a. **Electric field (E) = ?**

We know that,

$$E = \frac{V}{l} = \frac{0.938}{75.0 \times 10^{-2}}$$

$\therefore$ $E = 1.25\ Vm^{-1}$

b. **Resistivity (ρ) = ?**

We know that

$$E = \rho J$$

$$\therefore \rho = \frac{E}{J} = \frac{1.25}{4.40 \times 10^7}$$

$$\therefore \rho = 2.84 \times 10^{-8}\ \Omega m$$

9. A Copper transmission cable 100 km long and 10.0 cm in diameter carries a current of 125 A. (a) What is the potential drop across the cable? (b) How much electrical energy is dissipated as thermal energy every hour? [$\rho = 1.72 \times 10^{-8}\ \Omega m$]

SOLUTION

Given,

Length (l) = 100 km = 100×10^3 m

Diameter (d) = 10 cm = 10×10^{-2} m

Current (I) = 125 A

Time (t) = 1 hr = 3600 s

a. Potential drop (V) = ?

b. Energy (E) = ?

Now,

a. $V = I R$

$$= I.\ \rho \frac{l}{A} \qquad \left[\because R = \rho \frac{l}{A}\right]$$

$$= I\rho \frac{l}{\left(\frac{\pi d^2}{4}\right)} \qquad \left[\because A = \frac{\pi d^2}{4}\right]$$

$$= 125 \times 1.72 \times 10^{-8} \times \frac{100 \times 10^3}{\frac{\pi}{4}(10 \times 10^{-2})^2}$$

$$= 27.4\ V.$$

And,

b. $E = P t$

$= V I t$ [$\because$ P = V I]

$= 27.4 \times 125 \times 3600 = 12.3 \times 10^6$ J

10. The resistance of a galvanometer coil is 9.36 Ω, and the current required for full scale deflection is 0.0224 A. We want to convert this galvanometer to an ammeter reading 20 A full scale. The only shunt available has a resistance of 0.025 Ω. What resistance must be connected in series with the coil?

SOLUTION

Given,

Galvanometer resistance (G) = 9.36 Ω

Galvanometer current (I_g) = 0.0224 A

Full scale current (I) = 20 A

Shunt resistance (S) = 0.025 Ω

Additional resistance (r) = ?

Now,

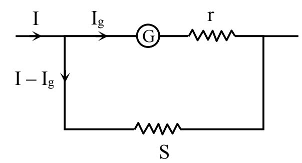
P.D. across (G + r) = P.D. cross (S)

$$I_g (G + r) = (I - I_g) S$$

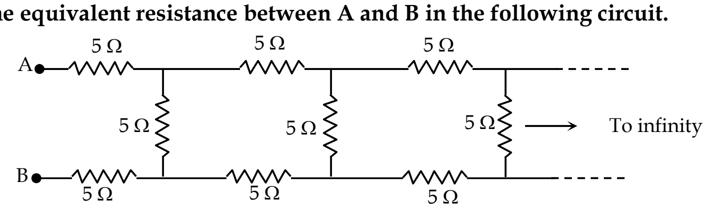
or, $$r = \frac{I - I_g}{I_g} . S - G$$

$$= \frac{20 - 0.0224}{0.0224} \times 0.025 - 9.36 = 12.94\ \Omega$$

∴ 12.94 Ω resistance is needed to connect across coil to make it ammeter.



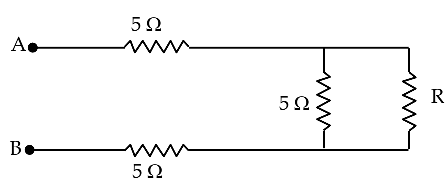
11. Determine the equivalent resistance between A and B in the following circuit.



SOLUTION

This type of circuit is called ladder circuit. In infinitely long ladder circuit, the identical steps of resistances are repeated. If one complete step of ladder is removed from the ladder, remaining part also gives the same value of equivalent resistance.

Let R be the equivalent resistance of the given circuit. The equivalent circuit diagram for the given circuit is as follows.



Here, equivalent resistance (R) = $5 + \frac{5R}{5 + R} + 5$

∴ $$R = 10 + \frac{5R}{5 + R}$$

$$R(5 + R) = 10(5 + R) + 5R$$

$$5R + R^2 = 50 + 15R$$

∴ $R^2 - 10R - 50 = 0$

∴ Using the solution of quadratic equation,

$$R = \frac{-(-10) \pm \sqrt{(-10)^2 - 4 \times 1 \times (-50)}}{2 \times 1}$$

$$R = \frac{10 \pm \sqrt{300}}{2} = \frac{10 \pm 17.32}{2}$$

∴ The valid resistance is, $R = \frac{10 + 17.32}{2}$

$= 13.67\ \Omega$

12. [HSEB 2071] Consider the figure below. The current through 6 Ω resistor is 4 A in the direction shown. What are the currents through the 25 Ω and 20 Ω resistors?

SOLUTION

Since R_1 and R_2 are parallel

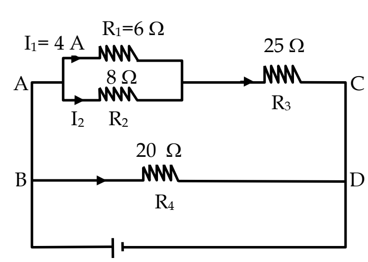
P.D. across R_1 = P.D. across R_2

$I_1 \times R_1 = I_2 \times R_2$

$4 \times 6 = 8 \times I_2$

$I_2 = 3$ A

Total current through path AC = 4 + 3 = 7 A



So, current through 25 Ω is 7 A.

Now, $R_{AC} = R_1 || R_2 + R_3 = \frac{6 \times 8}{6+8} + 25 = \frac{48}{14} + 25 = 28.43\ \Omega$

Since R_{AC} and R_{BD} are parallel.

P.d..across R_{AC} = P.d. across R_4

$28.43 \times 7 = 20 \times I_4$

or, $I_4 = \frac{28.43 \times 7}{20} = 9.95$ A

$\therefore$ $I_4 = 9.95$ A

13. [HSEB 2073] An electric lamp consumes 60 W at 220 V. How many dry cells of emf 1.5 V and internal resistance 1 Ω are required to glow the lamp?

SOLUTION

Given,

Power (P) = 60 W.

E.M. F of cell (E) = 1.5 V

Internal resistance (r) = 1 Ω

No. of cell (n) = ?

We have,

$P = IV$

$60 = I \times 220$

$I = 0.28$ A

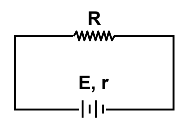
Again,

$V = IR$

$200 = 0.28 \times R$

$R = 806.75\ \Omega$

And,



$I = \frac{nE}{R + nr}$

$0.28 = \frac{n \times 1.5}{806.75 + n}$

or, $220 + 0.28 \times n = n \times 1.5$

or, $220 = 1.23 \times n$

or, $n = \frac{220}{1.23}$

$= 179$

$\therefore$ no. of cell = 179

14. A copper wire has a diameter of 1.02 mm and carries a constant current of 1.67A. If the density of free electrons in copper is $8.5 \times 10^{28}/m^3$, calculate the current density and the drift velocity of the electrons.

SOLUTION

Given,

Diameter of copper wire (d) = 1.02 mm

$= 1.02 \times 10^{-3}$ m

Current (I) = 1.67 A

Electron density (n) = $8.5 \times 10^{28}/m^3$

Current density (J) = ?

Drift velocity (v_d) = ?

The cross-sectional area of wire,

$A = \frac{\pi d^2}{4} = \frac{\pi \times (1.02 \times 10^{-3})^2}{4}$

$= 8.17 \times 10^{-7}\ m^2$

Now,

Current density (J) = $\frac{I}{A}$

$= \frac{1.67}{8.17 \times 10^{-7}} = 2.04 \times 10^6\ A/m^2$

Also, Drift velocity (v_d) = $\frac{J}{ne}$

$= \frac{2.04 \times 10^6}{8.5 \times 10^{28} \times 1.6 \times 10^{-19}}$

$= 1.5 \times 10^{-4}\ ms^{-1}$

Therefore, the current density is 2.04×10^6 A/m^2 and drift velocity is $1.5 \times 10^{-4}\ ms^{-1}$.

15. [HSEB 2069] The resistance of a conductor is 10 ohm at 50°C and 15 ohm at 100°C. Calculate its resistance at 0°C.

SOLUTION

Given,

Temperature (t) = 50° C

Resistance (R_{50}) = 10 Ω

at 100°C, R_{100} = 15 Ω

at 0°C, R_0 = ?

We know,

$R_\theta = R_0 (1 + \alpha\Delta\theta)$ [α = temp. coefficient of resista

so,

$R_{50} = R_0 (1 + \alpha \times 50)$

and

$R_{100} = R_0 (1 + \alpha \times 100)$

$\therefore \frac{R_{100}}{R_{50}} = \frac{1 + 100\alpha}{1 + 50\alpha}$

or, $\frac{15}{10} = \frac{1 + 100\alpha}{1 + 50\alpha}$

or, $15 + 750\alpha = 10 + 1000\alpha$

or, $5 = 250\,\alpha$

or, $\alpha = 0.02\ K^{-1}$

$\therefore R_{50} = R_0 (1 + \alpha\, 50)$

$10 = R_0 (1 + 0.02 \times 50)$

$10 = R_0 (1 + 1)$

$\therefore R_0 = 5\ \Omega$

16. [NEB 2075] Two resistance of 1000 Ω and 2000 Ω are placed in series with 50 V mains supply. What will be the reading on a voltmeter of internal resistance 2000 Ω when placed across the 1000 Ω resistor? What fractional change in voltage occurs when voltmeter is connected?

SOLUTION

Given,

R_1 = 1000 Ω

R_2 = 2000 Ω

E = 50 V

Internal Resistance of Voltmeter (R_V) = 2000 V

Now,

$R_{ab} = \frac{1000 \times 2000}{1000 + 2000} = 666.7\Omega$

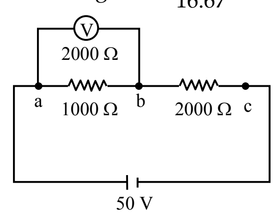
Total resistance, $R = R_{ab} + R_{bc}$
$= 666.7 + 2000$
$= 2666.7\Omega$

Total current in the circuit, $I = \frac{E}{R} = \frac{50}{2666.7}$
$= 0.01875$ A

P.d. across 1000Ω $= I\,.R_{ab}$
$= 0.01875 \times 666.7$
$= 12.5$ V

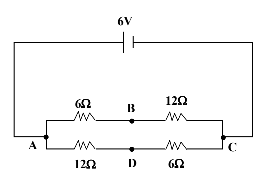
P.d. across 1000 Ω before connecting the voltmeter, $= \frac{1000}{3000} \times 50 = 16.67$

Fraction change in voltage $= \frac{16.67 - 12.5}{16.67} = 0.25 \times 10\% = 25\%$



17. [HSEB 2058] In the given circuit, calculate the potential difference between the points B and D.

SOLUTION



Given,

Emf of cell (E) = 6 V

Potential difference between B and D,

(V_{BD}) = ?

Total resistance R_{AC} = (6 + 12) || (6 + 12)
= 18 || 18

$= \frac{18 \times 18}{18 + 18} = 9\ \Omega$

Total current in the circuit $(I) = \frac{E}{R_{AC}} = \frac{6}{9} = \frac{2}{3}$ A

Then, $I_{AB} = \frac{1}{3}$ and $I_{AD} = \frac{1}{3}$

Now,

$V_{AB} = \frac{1}{3} \times 6 = 2$ V

$V_{AD} = \frac{1}{3} \times 12 = 4$ V

$\therefore V_{BD} = V_{AD} - V_{AB}$
$= 4 - 2 = 2$ V

Challenging Problems

1. **[UP]** Copper has 8.5×10^{28} free electrons per cubic meter. A 71.0 cm length of 12-gauge Copper wire, that is 2.05 mm in diameter, carries 4.85 A current. How much time does it take for an electron to travel the length of the wire?

   Ans: 109.5 min

2. **[UP]** A metallic wire has a diameter of 4.12 mm. When the current in the wire is 8.00 A, the drift velocity is 5.40×10^{-5} ms^{-1}. What is the density of free electrons in the metal?

   Ans: 6.94×10^{28} electrons/m^3

3. **[UP]** A Copper wire has a square cross section 2.3 mm on a side. The wire is 4.0 m long and carries a current of 3.6 A. The density of free electrons is 8.5×10^{28} m^{-3}. Find the magnitudes of (a) the current density in the wire; (b) the electric field in the wire. (c) How much time is required for an electron to travel the length of the wire? [$\rho = 1.72 \times 10^{-8}$ Ωm]

   Ans: (a) 8.67×10^6 Am^{-2} (b) 0.149 V/m (c) 1.75 hr

4. **[UP]** In an experiment conducted at room temperature, a current of 0.820 A flows through a wire 3.26 mm in diameter. Find the magnitude of the electric field in the wire if the wire is made of (a) tungsten; (b) aluminum. [$\rho_t = 5.25 \times 10^{-8}$ Ω m; $\rho_{Al} = 2.75 \times 10^{-8}$ Ω m]

   Ans: (a) 5.16×10^{-3} Vm^{-1} (b) 2.70×10^{-3} N/m

5. **[UP]** What diameter must a Copper wire have if its resistance is to be the same as that of an equal length of aluminum wire diameter 3.26 mm? [$\rho_c = 1.72 \times 10^{-8}$ Ω m, $\rho_a = 2.75 \times 10^{-8}$ Ω m]

   Ans: 2.58×10^{-3} m

6. **[UP]** You need to produce a set of cylindrical Copper wire 3.50 m long that will have a resistance of 0.125 Ω each. What will be the mass of each of these wires? [$\rho_c = 1.72 \times 10^{-8}$ Ωm, Density of Copper wire (D) = 8.9×10^3 kgm^{-3}]

   Ans: 0.015 kg

7. **[UP]** An aluminum cube has a side length of 1.80 m. What is the resistance between two opposite faces of the cube? [$\rho = 2.75 \times 10^{-8}$ Ωm]

   Ans: 1.53×10^{-8} Ω

8. **[UP]** You apply a potential difference of 4.50 V between the ends of a wire that is 2.50 m in length and 0.654 mm in radius. The resulting current through the wire is 17.6 A. What is the resistivity of the wire?

   Ans: 1.37×10^{-7} Ωm

9. **[UP]** A current carrying gold wire has diameter 0.84 mm. The electric field in the wire is 0.49 V/m. What is (a) The current carried by the wire? (b) The potential difference between two points in the wire 6.4 m apart? (c) The resistance of a 6.4 m length of the wire? [$\rho = 2.44 \times 10^{-8}$ Ωm]

   Ans: (a) 11.12 a (b) 3.13 v (c) 0.281 Ω

10. **[UP]** What is the resistance of a Nichrome wire at 0.0°C if its resistance is 100.00 Ω at 11.5°C? [$\alpha = 0.0004°C^{-1}$]

    Ans: 99.54 Ω

11. **[UP]** A strand of wire has resistance 5.60 μΩ. Find the net resistance of 120 such strands if they are (a) placed side by side to form a cable of the same length as a single strand; (b) connected end to end to form a wire 120 times as long as a single strand.

    Ans: (a) 4.67×10^{-8} Ω (b) 6.72×10^{-4} Ω

12. **[UP]** A 32 Ω and a 20 Ω resistor are connected in parallel, and the combination is connected across a 240 V d.c. line. (a) What is the resistance of the parallel combination? (b) What is the total current through the parallel combination? (c) What is the current trough each resistor?

    Ans: (a) 12.3 Ω (b) 19.5 a (c) 7.5 a and 12 a

13. **[UP]**A 150 V voltmeter has a resistance of 30,000 Ω. When connected in series with a large resistance R across a 110 V line, the meter reads 68 V. Find the resistance R.

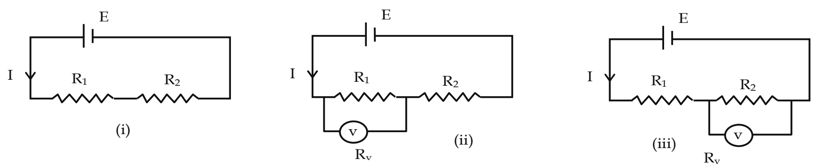
    Ans: 18.6×10^3 Ω

14. **[ALP]** A thin film resistor in a solid-state circuit has a thickness of 1 μm and is made of nichrome of resistively 10^{-6} Ωm. Calculate the resistance available between opposite edges of a 1 mm^2 area of film

a. If it is square shaped

b. If it is rectangular, 20 times as long as it is wide **Ans:(a) 1 Ω (b) 0.05 Ω**

15. **[ALP]** Two resistors of 1200 Ω and 800 Ω are connected in series with a battery of emf 24 V and negligible internal resistance as in figure (i). What is the potential difference across each resistor? A voltmeter V of resistance 600 Ω is now connected firstly across the 1200 Ω resistor as shown, and then across the 800 Ω resistor. Find the potential difference recorded by the voltmeter in each case.



Ans: 14.4 V, 9.6 V, 8 V, 5.33 V

16. **[ALP]** A moving coil meter has a resistance of 25 Ω and indicates full scale deflection when a current of 4.0 mA passes through it. How could this meter be converted to a milliammeter having a full scale deflection for a current of 50 mA? **Ans: 2.17 Ω**

*[**Note:** Hits to challenging problems are given at the end of this chapter.]*

Conceptual Questions with Answers

1. Resistors R_1 and R_2 are connected in parallel to an emf source that has negligible internal resistance. What happens to the current through R_1 when R_2 is removed from the circuit? [NEB 2074]

➪ In the given circuit design, the resistors of resistances R_1 and R_2 are connected parallel and are directly connected to the source of emf E. Since the internal resistance of the cell is negligible, both resistors possess the potential difference individually equal to the emf of the source. So, in first case, total current through R_1 is $\frac{E}{R_1}$. When R_2 is removed from the circuit, total emf across R_1 is still E. Hence, the current through R_1 is again $\frac{E}{R_1}$. Hence the current does not vary, although the circuit design is changed.

2. Ammeters often contain fuses that protect them from large currents while voltmeter seldom do. Explain.

➪ Ammeter is connected in series in an electric circuit. It has very low internal resistance. Hence, large current may flow through the ammeter. Due to these reasons, high heat may be generated in it and may have the chance of burning. Hence, to protect from damaging, fuses are connected in it. However, a voltmeter is connected parallel in an electric circuit and has high resistance, so much less current flows through it. So, it is almost free from such heating effect.

3. A steady current is flowing in a cylindrical conductor, Is there any electric field within the conductor?

➪ Yes. Two ends of a cylindrical conductor acts as two parallel plate capacitors. The free electrons between two ends of that conductor move under the electric field provided by external power supply, connected at two ends of the conductor.

4. Will the drift velocity of electrons change if the diameter of a connecting wire is halved? Why? (HSEB 2073)

➪ The electric current in a conductor is,

$I = nev_d A$

$\therefore \quad v_d = \frac{I}{neA}$

At constant current, for first case,

$v_1 = \frac{I}{neA_1}$ and

For second case,

$v_2 = \frac{I}{neA_2}$

$\therefore \quad \frac{v_1}{v_2} = \frac{A_2}{A_1}$

$$\frac{v_1}{v_2} = \frac{\frac{\pi d_2^2}{4}}{\frac{\pi d_1^2}{4}} = \frac{d_2^2}{d_1^2}$$

When diameter is halved, i.e. $d_2 = \frac{d_1}{2}$

$$\frac{v_1}{v_2} = \frac{\left(\frac{d_1}{2}\right)^2}{d_1^2}$$

$\frac{v_1}{v_2} = \frac{1}{4}$

$\therefore \quad v_2 = 4v_1$

Therefore, drift velocity is increased by 4 times when diameter is halved.

---

5. Why don't we consider the drift velocity of positive ions?

↳ Electric field into the conductor influences not only the free electrons, but also the positive ions into it. But, positive ions are relatively heavier than the electrons and they also bind tightly into the atoms, so that the movement of positive ions is approximately impossible. Therefore, the drift velocity of positive ions is almost zero.

---

6. Two copper wires of different diameters are joined end to end. If a current flows in the wire combination, what happens to the drift velocity of the electrons when they move from the large-diameter to the smaller-diameter wire?

↳ The electric current in metallic conduction, $I = v_d enA$

When two wires are joined in series, they pass equal current, although they have different diameters. So,

$v_d = \left(\frac{I}{en}\right) \cdot \frac{1}{A}$

$\therefore \quad v_d \propto \frac{1}{A}$ and $A = \frac{\pi d^2}{4}$

It means drift velocity is smaller in larger diameter wire. So, the drift velocity of electrons increases when they move from larger-diameter to smaller diameter wire.

---

7. An electric current move along the length of conductor. If so, why it is not the vector quantity?

↳ To be vector quantity, the physical quantity must follow the rules of vector addition and multiplication. However, ordinary algebra is sufficient to add the electric current and laws of vector addition do not apply to add of electric currents.

---

8. What is the cause of resistance of a conductor?

↳ The conductor contains ions and atoms. As the electrons are charge particles, they interacts with electrons, ions and atoms. While drifting, these free electrons collide with the ions, other free electrons and atoms of the conductor. Then, the motion is opposed during the collisions. This is the primary cause of resistance in a conductor.

---

9. Two wires of equal lengths, one of copper and the other of manganin have the same resistance. Which wire will be thicker?

↳ The resistance, $R = \rho \frac{l}{A}$

i. For copper, $R_1 = \rho_1 \frac{l_1}{A_1}$

ii. For manganin, $R_2 = \rho_2 \dfrac{l_2}{A_2}$

Here, given that the wires of equal length have equal resistance, $l_1 = l_2$ and $R_1 = R_2$, so

$$\frac{\rho_1}{A_1} = \frac{\rho_2}{A_2}$$

$$\frac{A_2}{A_1} = \frac{\rho_2}{\rho_1}$$

Since, the resistivity of manganin is greater than copper, $\rho_2 > \rho_1$.

Then, $A_2 > A_1$.

Therefore, manganin wire is thicker than the copper wire.

---

10. Why don't the free electrons is a metal fall to the bottom of the metal due to the gravity?

➯ Free electrons are distributed almost uniformly throughout the conductor due to the electrostatic interactions with the ions and atoms. The electrostatic force between two charge particles is much greater than the gravitational attraction of earth. So, they donot fall from the conductor, although they are free.

---

11. The charges in a conductor are supposed to reside on the surface, then why don't the free electrons all go to the surface?

➯ The charges reside on the surface only when a conductor possesses excess electrons or deficit of electrons i.e. it happens only when the conductor is charged. But conductor is not charged, when current flows through it. The electrons are influenced by the atoms in a conductor, so they do not come on the surface. They can be distributed throughout the volume of conductor.

---

12. Name three materials used for making standard resistance. Give reason, Why they are suitable?

➯ Standard resistances are usually, made with alloys. For examples Manganin, constantan and nicrome. These alloys are suitable because of the reasons that (i) they possess high resistivity (ii) they have low temperature coefficient of resistance and have high melting point.

---

13. When resistors are connected in series the effective resistance is increased. Why?

➯ When resistors are connected in series, there is the increase in "effective length". Since resistance varies directly as the length, effective resistance is increased.

---

14. Write two applications of superconductivity.

➯ Two important applications of superconductivity are:

i. No power is lost when electrical signals are passed through it.

ii. It is used to developed extremely high speed computers.

---

15. Write the dimension of electrical conductivity.

➯ Electrical conductivity (σ) is the reciprocal of resistivity (ρ). i.e.

$$\sigma = \frac{1}{\rho} = \frac{l}{R \times A}$$

Now, dimension, $[\sigma] = \dfrac{[L]}{[ML^2T^{-3}A^{-2}]\,[L^2]} = [M^{-1}L^{-3}T^3A]$

Therefore, the dimension of conductivity is $[M^{-1}L^{-3}T^3A]$.

---

16. What is the difference between resistance and resitivity of a wire?

➯ Resistance is the variable quantity even for a material at the same physical condition. It is determined from the ratio of potential difference applied across two ends of a conductor to the current flowing through it. Resistance depends on shape, size and also on the nature of its material. Resistivity is the resistance offered by a conductor of unit length per unit cross sectional area. It depends on the nature of the material and on the physical conditions like temperature and pressure.

---

17. What is the value of resistance of a resistor of colour coding of red, red and orange,

✎ The colour code of red and orange are 2 and 3 respectively. So, Applying the rule of coding pattern, we have, $22 \times 10^3 = 22\ k\Omega$.

18. The current flowing through a conductor is 2 mA at 50 V and 4 mA at 80 V. Is it an ohmic or non-ohamic conductor?

✎ The resistance of the conductor in two cases are,

i. For I = 2 mA and V = 50 V

$$R = \frac{V}{I} = \frac{50}{2 \times 10^{-3}} = 25 \times 10^3\ \Omega = 25\ k\ \Omega$$

and for I = 4 mA and V = 80 V,

$$R = \frac{V}{I} = \frac{80}{4 \times 10^{-3}} = 20 \times 10^3 = 20\ k\Omega$$

As the resistance changes with current, the given conductor is non ohmic.

19. Though same current flows through the electric line wires, and the bulb filament, yet only the filament glows. Why?

✎ The dissipation of electric energy not only depends on current, it also depends on the resistance of the conductor used, i.e. $H = I^2Rt$. The filament has high resistance, but the electric line wires in the electric circuit have negligibly small resistance (i.e. $R \to 0$). So, the current passing through the high resistance filament produces a large amount of energy into light (and heat also). Hence it makes glow.

20. How will you convert galvanometer into ammeter?

✎ A very small resistance, called shunt, is connected in parallel with the galvanometer to convert it into an ammeter. The very small resistance, when connected with galvanometer, bypass as the current through low resistance region without altering the deflection in galvanometer as it whole current passes through it. So, ammeter can measure the total value of current in the circuit with minimum loss of electric power.

21. Voltmeters are always connected in parallel in an electric circuit, why?

✎ Voltmeter measures the potential difference of two points in an electric circuit, usually across the resistor and terminals of a cell. It is easily understood that this device measures difference of potential, it means two points should be taken as reference to find the difference. As we know, electric potential is considerably difference at two ends of a resistor. Connection at two ends automatically makes the parallel with resistor or cell. For the efficient circuit, the power loss at the voltmeter should be minimized. To minimize the power loss in voltmeter, high resistance is connected in series to the galvanometer so that the current passing through it is negligibly small.

22. Ammeters are always connected in series in an electric circuit. Why?

✎ Ammeters are current measuring device. Actually, deflection in ammeter occurs due to the amount of charge passing through per second in an electric circuit. To detect the total charge flow through a certain cross section of wire, it must be connected in series, otherwise it can detect the partial value. The internal resistance of ammeter is made very small so that there is negligible loss of electric power in it.

23. How do you convert a galvanometer into a voltmeter?

✎ Voltmeter measures the potential different of two points in an electric circuit. To convert a galvanometer into voltmeter, high resistor of large resistance (called multiplier) is connected in series with galvanometer. The high resistance series with galvanometer makes the very high internal resistance of the voltmeter.

The high value of resistance in voltmeter prevents from appreciable loss of electric power in it.

24. In a conductor, large number of electrons are free to move in it, but why no current is detected?

✎ There are many free electrons moving in a conductor even though no electric source is connected across it, but these free electrons move randomly so, net flow of these charge particles (electrons) in a specified direction is zero. Hence, the net current in the conductor is zero. If an electric source is connected across the end of a conductor, the motion of charge particles is specific, hence the current is detected.

25. What is specific resistance (resistively) ? What is its unit?

↳ The resistance of a conductor is

$$R = \rho \frac{l}{A}$$

For $l = 1$ m, and $A = 1$ m², $R = \rho$, ρ is known as specific resistance (resistivity)

Therefore the specific resistance (or resistivity) is defined as the total resistance of a conductor of unit length and unit cross sectional area. It is constant for a conductor at constant temperature, its unit is ohm meter (Ωm).

26. A metallic conductor is stretched to double of its original length. What would be the resistance and resistivity of the conductor?

↳ When a conductor is stretched, its length increases and the diameter decreases, but volume remain same.

In initial condition, $R = \rho\frac{l}{A}$ ... (i)

For $l' = 2l$, $R' = \rho\frac{l'}{A'}$

Since the volume remains unchanged,

$$Al = A'l'$$

$$\therefore \quad A' = \frac{A}{2}.$$

$$\text{So,} \quad R' = \rho\frac{2l}{A/2}$$

$$R' = \rho\frac{4l}{A} \quad \text{... (ii)}$$

Now, dividing equation (ii) by equation (i), we get

$$\frac{R'}{R} = \frac{\rho\frac{4l}{A}}{\rho\frac{l}{A}}$$

$$\frac{R'}{R} = 4$$

$$\therefore \quad R' = 4R$$

This shows that the resistance is increased by four times the initial value.

The resistivity of a material depends on the nature of material and its temperature, it remains constant in the given situation.

27. Differentiate between Ohmic and non Ohmic conductors.

↳ Some important difference between Ohmic and non Ohmic conductors are as follows:

Ohmic Conductors	Non-Ohmic conductors
1. The conductors in which the graph between potential difference (V) and current (I) is linear called the Ohmic conductors.	1. The conductors in which the graph between different (V) and current (I) is non-linear are known as non-Ohmic conductors.
2. They strictly obeys the Ohm's law	2. They do not obey the Ohm's law
3. For example: metallic conductors like copper, silver etc.	3. For example: Semiconductor devices, diode, transistors etc.

28. Is current a scalar or vector quantity?

➪ Current is a scalar quantity. Although we show the direction of current in a electric in an electric circuit, this is done only to show the direction of conversion flow of charge. But, the property of charge flow does not obey the vector addition and multiplication rules.

29. A wire is carrying current. It is charged? Explain.

➪ To charge a wire some excess charge must be deposited in it (either excess positive or excess negative). But in a current carrying wire, the number of charge particle entering the wire is equal to number of these particle leaving it, so no excess charges are deposited. So, the wire carrying current is not charged.

30. Why are copper wires used as connecting wires?

➪ The resistivity of copper wire is very small, so its resistance 'R' tends to zero ($R \to 0$). So, the copper wire does not consume any electric power, rather it easily allow the electricity passing a long distance with out appreciable loss of electric power. Moreover, the resistance of copper wire does not contribute to add up in the calculation of total resistance.

31. Why are constantan and manganin used for making standard resistors?

➪ The temperature coefficient of resistance in constantan and manganin is nearly independent to temperature. The resistivity in them almost remains constant, although the temperature rises or falls. This properly of constantan and managanin makes possible in using in very cold and hot places, moreover in cold and hot season. Hence, they are used for making standard resistors.

Exercises

Short-Answer Type Questions

1. How drift velocity is related with current through a conductor?
2. Is the resistivity of a metal a constant quantity?
3. It is dangerous to operate electrical appliances with wet hands. Why?
4. Is ohm's law applicable to all conductors?
5. Which combination of resistance increases the equivalent resistance?
6. What is the ratio of n-equal resistances when they are connected in series to parallel?
7. What do you mean by the sensitivity of a galvanometer?
8. What is drift velocity?
9. How drift velocity is related with current through a conductor?
10. Does a conductor charge when current flows through it?
11. Is the resistivity of a metal a constant quantity?
12. It is dangerous to operate electrical appliances with wet hands. Why?
13. Why should an ammeter have low resistance?
14. A voltmeter should have high resistance, why?
15. Is ohm's law applicable to all conductors?
16. What is shunt?
17. Which combination of resistance increases the equivalent resistance?
18. Differentiate between ohmic and non ohmic resistance.
19. What is the ratio of n-equal resistances when they are connected in series to parallel?
20. Large amount of current flows through the conductor, why?
21. What do you mean by resistivity of a material? What is its unit?
22. Why do we use connecting wires made of copper?
23. What do you mean by the sensitivity of a galvanometer?
24. A proton beam is going from East to West. Is there an electric current? If yes, in what direction?
25. What are the order of magnitude of thermal velocity and drift velocity of electrons in a current carrying conductor at room temperature.

26. Silver is a homogeneous conductor and it obeys Ohm's law. An electrical device is made-up of pure silver, will it obey Ohm's law for all values of electric field?
27. What are the factors on which resistivity of a material depend?
28. What do you mean by conductivity of a material? Give its SI units.
29. What is temperature coefficient of resistivity? What is its unit?
30. What happens when an ammeter is placed in parallel with a circuit?
31. If a shunt resistance of 0. 001 Ω is connected across a galvanometer, what can we say about the resistance of the resulting ammeter?
32. Suppose we want to increase the range of an ammeter from 1 A to 10 A, what should be done to the shunt resistance?
33. A galvanometer is first converted into a voltmeter of range 0-2 V and then into a voltmeter of range 0-5 V. In which case the resistance will be higher one?
34. What is superconductivity?
35. How superconductor is different from perfect conductor?

Long-Answer Type Questions

1. Describe the mechanism of current flow in a conductor and derive a relation between current density and drift velocity to electrons. [NEB 2074]
2. What is drift velocity of an electron? Derive a relation between the current through a metallic conductor and the drift velocity in terms of the number of free electrons per unit volume of the conductor. [HSEB 2059]
3. What is current density? Derive an expression for drift velocity of electrons in a conductor in term of current density?
4. State Ohm's law. How it is experimentally verified?
5. State and explain Ohm's law. Two resistors are connected in parallel and third resistor be connected in series with the combination of parallel resistors. If this combination be connected with a battery of the negligible internal resistance, find the potential difference across each resistor. [HSEB 2064]
6. What is equivalent resistance of resistors? Derive its expression when the resistors are connected (i) in series (ii) in parallel.
7. What is resistance of a conductor? On what factor does it depends? Give the correspondence relation.
8. What is multiplier? How can you convert galvanometer into voltmeter? [HSEB 2072]
9. Why has an ammeter a very low resistance? How can you convert galvanometer into ammeter?
10. Discuss the mechanism of metallic conduction. Derive J = nev where J is current density, e is electronic charge and v is drift velocity. [HSEB 2060]

Numerical Problems

1. A 2 Ω resistance coil is to be constructed from a constantan wire of diameter 0.315 mm. If the resistivity of constantan is 4.9×10^{-6} Ω cm, find the length of the wire required to construct the coil.

   Ans: 31.8 cm
2. Two resistors 500 Ω and 300 Ω are connected in series with a battery of emf 20 V. A voltmeter of resistance 500 Ω is used to measure the p d across the 500 Ω resistor. Find the error in the measurement.

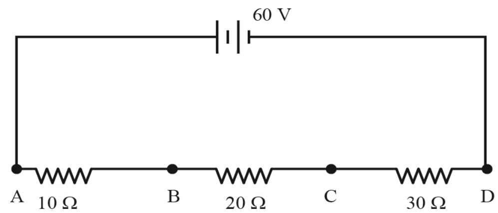
   Ans: 3.4 V
3. Wire A has a resistance of 2 Ω. Wire B, made of the same materials is twice as long and has half the thickness of wire A. Find the resistance of B.

   Ans: 16 Ω

4. When a wire carries a current of 1.20 A, the drift velocity is 1.20 x 10^{-4} m/s. What is the drift velocity when the current is 6.00 A?

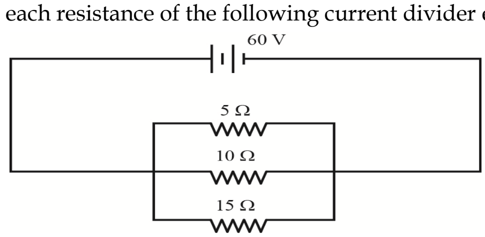
Ans: 6 *X* 10^{-4} m/s

5. Calculate the potential difference in each resistance in the following voltage divider circuit.



Ans: 10 V, 20 V, 30 V

6. Calculate the current in each resistance of the following current divider electric circuit.



Ans: 9 A, 4.5 A, 3 A

7. A silver wire 2.6 mm in diameter transfers a charge of 420 C in 80 min. Silver contains 5.8×10^{28} free electrons per cubic meter. What is the current in the wire? What is the magnitude of the drift velocity of the electrons in the wire?

Ans: 87.5×10^{-3} A, 1.77×10^{-4} ms^{-1}

8. The current density through a conductor is $1Am^{-2}$ where the electric field applied its length is 3 Vm^{-1}. Calculate the resistivity of the conductor. Also calculate its conductivity.

Ans: 3.30 Ωm, 0.33 $\Omega^{-1}m^{-1}$

9. A tungsten coil has a resistance of 12.0 Ω at 15°C. If the temperature coefficient of resistance of tungsten is 0.004 K^{-1}, calculate the coil resistance at 80°C.

Ans: 14.94 Ω

10 A 20 Ω resistor and a resistor X are placed in series with a battery of 10 V and of negligible resistance. If the voltage across X is 2 V, what is the value of X?

Ans: 5 Ω

11. A moving coil meter has a resistance of 25 Ω and indicates full scale deflection when a current of 4.0 mA flows through it. How could this meter be converted (i) to a voltmeter with 0 – 3 V range (ii) to an ammeter with 0 – 1 A range.

Ans: 725 Ω, 0.10 Ω

12. A galvanometer can bear a maximum current of 25 mA and has a resistance of 5 Ω. Find the suitable resistance to convert it into

   a. A voltmeter of range 0 – 2 V
   b. An ammeter of ramge 0 – 10 A.

Ans: R = 80 Ω, S = 0.0125 Ω

13. The earth has a negative surface charge density of 10^{-9} cm^{-2}. The potential difference of 500 kV between the top of the atmosphere and the surface results in a current of 2000 A over the entire earth. How much time is supposed to neutralise the earth's surface? (Radius of earth = 6370 km)

Ans: 255.05 sec

14. The wire of a fuse in an electric circuit melts when the current density increases to 600 A/cm^2. What should be the diameter of the wire so that it may limit the current to 0.4 A?

Ans: 0.29 mm

15. A copper wire of radius 1.0 mm carries a current of 10 A. Calculate the drift velocity of the electrons. Metallic copper has one conduction electron per atom, the atomic mass of copper is 64 u and density of copper is 8900 kgm^{-3}. Given 1u = 1.66×10^{-27} kg.

Ans: $2.37 \times 10^{-4}\ ms^{-1}$

16. A length of copper wire of mass 4.5 kg has a resistance of 14 Ω. Calculate the length and diameter of the wire. Density of copper is 8930 kg m^{-3} and resistivity is 1.8×10^{-8} Ωm.

Ans: 6.26×10^{2} m, 1.013 mm

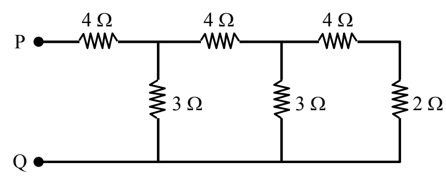
17. At 27.0°C, the resistance of a resistor is 83 Ω. What is the temperature of the resistor if the resistance is found to be 100 Ω and the temperature coefficient of the material of the resistor is $1.7 \times 10^{-4}\ °C^{-1}$?

Ans: 1232°C

18. A tungsten coil has a resistance of 12 Ω at 15°C. If the temperature coefficient of resistance of tungsten is $0.004°C^{-1}$, calculate the resistance of the coil at 80°C.

Ans: 15 Ω

19. Calculate the equivalent resistance between the points P and Q of the network shown in figure given below:



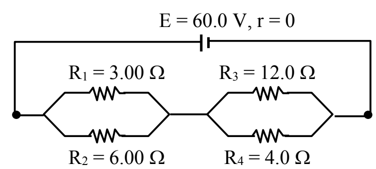
Ans: 6 Ω

20. The resistance between the ends of thick wire of length 50 cm and diameter 0.55 cm is 1.44×10^{-3} Ω. A circular disc of diameter one centimeter and thickness 1.0 mm is this material. Find the resistance between the opposing round face.

Ans: $8.6X10^{-7}$ Ω

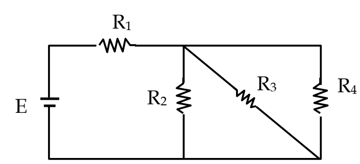
21. Complete the equivalent resistance of the network in figure and find the current in each resistor. The battery has negligible internal resistance.

Ans: 5 Ω, I_1 = 8 A, I_2 = 4 A, I_3= 3 A, I_4 = 9 A



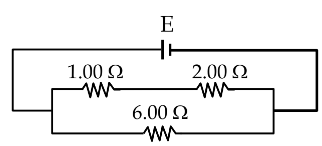
22. Four resistors and a battery of negligible internal resistance are assembled to make the circuit in figure. Let E = 6.00 V, R_1 = 3.50 Ω, R_2 = 8.20 Ω, R_3 = 1.50 Ω and R_4 = 4.50 Ω. Find (a) the equivalent resistance of the network (b) the current in each resistor.

Ans: (a) 4.49 Ω (b) I_1 = 3.4 A, I_2 = 0.162 A, I_3 = 0.884 A, I_4 = 0.294 A



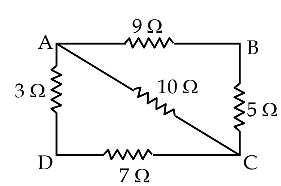
23. In the circuit shown in figure, the voltage across the 2.00 Ω resistor is 12.0 V. What are the emf of the battery and the current through the 6.00 Ω resistor?

Ans: 18 V, 3 A



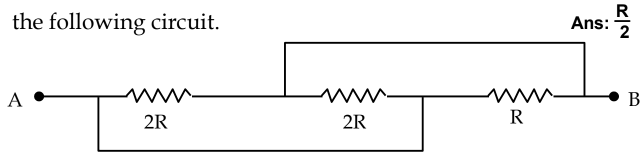
24. Find the equivalent resistance between B and C points.

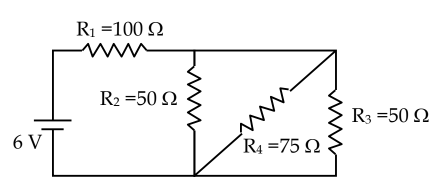
Ans: 3.684 Ω



25. Find the equivalent resistance between the points A and B of the following circuit.

Ans: $\frac{R}{2}$



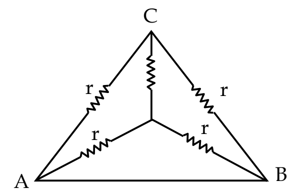


26. Determine the equivalent resistance of following network. Also, find out the currents in each resistor.

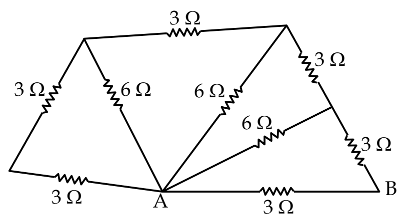
ANs: R = 118.75 Ω, current through R_1= 0.05 A, current through R_2 = 0.02 A, current through R_3 = 0.02 A, current through R_4 = 0.02 A

27. Determine the equivalent resistance between A and B if each resistance is of r Ω.

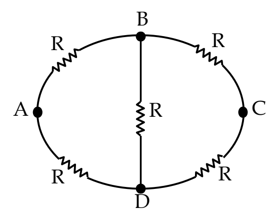
Ans: 0.5 R



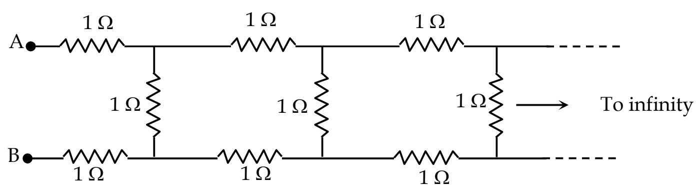
28. Find out the equivalent resistance between A and B in the following circuits.



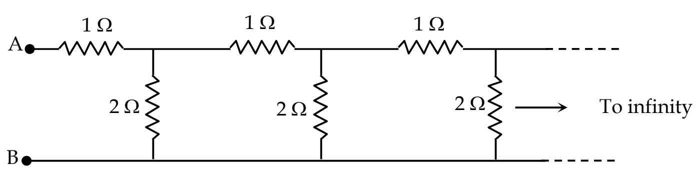
(i)



(ii)



(iii)



(iv)

Ans:(i) 2 Ω, (ii) $\frac{5}{8}$ R (iii) 2.73 Ω (iv) 2 Ω

Multiple Choice Questions

1. When 5.5 ohm and 4.5 ohm resistance are joined together in series and a 10 ohm resistance is joined in parallel the final resistance of the system is:

   a. 2 Ω b. 5 Ω
   c. 2.5 Ω d. 20 Ω

2. A piece of wire of resistance 4 ohm is bent through 180° at mid-point and the two halves are twisted together, their resistance is:

   a. 8 Ω b. 1 Ω
   c. 2 Ω d. 5 Ω

3. The resistance of two wires connected in parallel in 3.43 Ω while the resistance of the same wires connected in series is 14 Ω. The resistance are:

a. 5 and 6
b. 6 and 7
c. 7 and 8
d. 6 and 8

4. When the diameter of a nichrome wire is reduced to half, the resistance changes by:
a. 2
b. 4
c. 8
d. 16

5. The sound travels with speed 30 ms^{-1} in string. Then find the distance between successive nodes. If frequency is 100 Hz
a. 20 cm
b. 30 cm
c. 15 cm
d. 45 cm

6. Three resisters R_1, R_2 and R_3 ohms are connected in a parallel combination. If $R_1 < R_2 < R_3$ the equivalent resistance will be:
a. Less than R_1
b. Equal to R_2
c. Less than R_2
d. Greater than R_1

7. The current flowing through a resistance of 900 Ω is desired to reduce by 90%. What shunt should be joined across it?
a. 100 Ω
b. 90 Ω
c. 900 Ω
d. 10 Ω

8. The resistance of the series combination of two resistance is R. When they are joined in parallel, the total resistance is r. If R = nr, then the minimum possible value of n is
a. 4
b. 3
c. 2
d. 1

9. n resistance, each of r ohm, when connected in parallel give an equivalent resistance of R ohm. If these resistance were connected in series, the combination would have a resistance, in ohm, equal to
a. n^2R
b. R/n^2
c. R/n
d. nR

10. A wire of a certain material is stretched slowly by ten percent. Its new resistance and specific resistance become respectively
a. 1.1times, 1.1 times
b. 1.2 times, 1.1 times
c. 1.21 times, same
d. both remain the same

11. A thin wire of resistance 4 Ω is bent to form a circle. The resistance across any diameter is
a. 4 Ω
b. 2 Ω
c. 1 Ω
d. 8 Ω

12. Dimensions of resistance in an electrical circuit, in terms of dimension of mass M, of length L, of time T and of current I, would be:
a. ML^2T^{-2}
b. $ML^2T^{-1}I^{-1}$
c. $ML^2T^{-3}I^{-2}$
d. $ML^2T^{-3}I^{-1}$

13. A colour coded carbon resistor has the colors orange, blue, green and silver. Its resistance value and tolerance percentage respectively are
a. 36×10^5 Ω and 10%
b. 36×10^4 Ω and 5%
c. 63×10^5 Ω and 10%
d. 35×10^6 Ω and 5%

14. The dimensions of 'resistance' are same as those of .... where h is the Planck's constant, e is the charge.
a. $\frac{h^2}{e^2}$
b. $\frac{h^2}{e}$
c. $\frac{h}{e^2}$
d. $\frac{h}{e}$

15. A coil has resistance 25.00 ohm and 25.17 ohm at 20°C and 35°C respectively. What is the temperature coefficient of resistance?
a. 4.545×10^{-4}/ °C
b. 4.545×10^{-3}/°C

c. $4.545 \times 10^{-2}/\,^{o}C$ d. $4.545 \times 10^{-5}/^{o}C$

Answers

1. **(b)** 2. **(b)** 3. **(d)** 4. **(d)** 5. **(c)** 6. **(a)** 7. **(a)** 8. **(a)** 9. **(a)** 10. **(c)** 11. **(c)** 12. **(c)** 13. **(a)** 14. **(c)** 15. **(a)**

Hints to Challenging Problems

HINT: 1

Given,

Density of electron (n) = 8.5×10^{28} electrons/m^3

Length of wire (l) = 71.0 cm = 71.0×10^{-2} m.

Diameter of wire (d) = 2.05 mm = 2.05×10^{-3} m

Current (I) = 4.85 A

Time (t) = ?

we know that

$$v_d = \frac{I}{neA}$$

$$\text{or } \frac{l}{t} = \frac{I}{ne\left(\frac{\pi d^2}{4}\right)}$$

$$\text{or } t = l \times \frac{\pi d^2}{4} \times \frac{ne}{I}$$

HINT: 2

Given,

Diameter (d) = 4.12 mm = 4.12×10^{-3} m

Current (I) = 8.00 A

Drift velocity (v_d) = 5.40×10^{-5} ms^{-1}

Density of electron (n) = ?

We know that

$$v_d = \frac{I}{nAe}$$

$$\text{or } n = \frac{I}{ev_dA} = \frac{I}{ev_d\left(\frac{\pi d^2}{4}\right)}$$

HINT: 3

Given,

Side length of square cross-section,

L = 2.3 mm = 2.3×10^{-3} m

Cross section area A = L^2

Length of wire, l = 4.0 m

Current, I = 3.6 A

Density of free electrons,

n = 8.5×10^{28} electrons/m^3

Resistivity of copper, $\rho = 1.72 \times 10^{-6}$ Ωm.

a. Current density, $J = \frac{I}{A} = \frac{I}{L^2}$

b. Electric field $E = \rho J$

c. We have, $v_d = \frac{J}{ne}$

$$\text{or } \frac{l}{t} = \frac{J}{ne}$$

$$\text{or } t = \frac{ne \times l}{J}$$

HINT: 4

Given,

Current (I) = 0.820 A

Diameter (d) = 3.26 mm = 3.26×10^{-3} m

a. For tungsten wire electric field, E =?

$\rho = 5.25 \times 10^{-8}$ Ωm.

We know that

$$E = \rho J = \rho \times \frac{I}{A} = \rho \times \frac{I \times 4}{\pi d^2}$$

b. For aluminium, electric field, E = ?

$\rho = 2.75 \times 10^{-8}$ Ω m

We know that, $E = \rho J = \frac{4\rho I}{\pi d^2}$

HINT: 5

Given,

d_C = ?

Length of copper wire = l_c

Length of aluminium wire = l_a

d_a = 3.26 mm = 3.26×10^{-3} m

According to questions,

$R_c = R_a$

$$\text{or } \frac{\rho_c l_c}{A_c} = \frac{\rho_a l_a}{A_a}$$

$$\text{or } \frac{\rho_c}{\left(\frac{\pi d_c^2}{4}\right)} = \frac{\rho_a}{\left(\frac{\pi d_a^2}{4}\right)} \quad (\because l_c = l_a)$$

$$\text{or } \frac{\rho_c}{d_c^2} = \frac{\rho_a}{d_a^2} \qquad \text{or, } d_c^2 = d_a^2 \frac{\rho_c}{\rho_a}$$

HINT: 6

Given,

Length (l) = 3.50 m

Resistance (R) = 0.125 Ω

Resistivity (ρ_c) = 1.72×10^{-8} Ωm

Density (D) = 8.9×10^3 kgm^{-3}
Mass (m) = ?

We know that

$$R = \rho_c \frac{l}{A} = \rho_c \times \frac{l \times l}{A \times l} = \frac{\rho \times l^2}{V}$$

$$\text{or} \quad V = \frac{\rho_c \times l^2}{R}$$

$$\text{or} \quad \frac{m}{D} = \frac{\rho_c \times l^2}{R} \quad \left(\because \text{density (D)} = \frac{\text{mass (m)}}{\text{valume (V)}}\right)$$

$$\text{or} \quad m = \frac{\rho_c \times l^2}{R} \times D$$

HINT: 7

Given,

Length (l) = 1.80 m
Resistance (R) =?
Resistivity (ρ) = 2.75×10^{-8} Ωm

We know that, $R = \rho \frac{l}{A} = \rho \frac{l}{l^2} = \frac{\rho}{l}$

HINT: 8

Given,

Potential difference (V) = 4.50 V
Length (l) = 2.50 m
Radius (r) = 0.654 mm = 0.654×10^{-3} m
Current (I) = 17.6 A
Resistivity (ρ) = ?

We know that

$$\rho = \frac{RA}{l} = \frac{V}{I} \times \frac{\pi r^2}{l} \quad (\because A = \pi r^2 \text{ and } V = IR)$$

HINT: 9

Given,

Diameter (d) = 0.84 mm = 0.84×10^{-3} m
Electric field (E) = 0.49 Vm^{-1},
Resistivity, ρ = 2.44×10^{-8} Ωm

a. Current (I) = ?
We know that

$$E = \rho J = \rho \frac{I}{A}$$

$$\text{or} \quad I = \frac{E \times A}{\rho} = \frac{E}{\rho} \times \left(\frac{\pi d^2}{4}\right)$$

b. Potential difference, V = ?
Length of wire, l = 6.4 m

We know that

$$E = \frac{V}{l}$$

$$\therefore \quad V = E \times l$$

c. Resistance of the wire, $R = \frac{V}{I}$

HINT: 10

Given,

Resistance at 11.5°C, $R_{11.5}$ = 100 Ω
Temperature coefficient of resistance,
α = 0.0004°C^{-1}
Resistance at 0°C, R_0 = ?
We know that

$$R_{11.5} = R_0 (1 + \alpha \Delta\theta)$$

$$\text{or} \quad 100 = R_0 \{1 + 0.0004 \times (11.5 - 0)\}$$

HINT: 11

Given,

Resistance of a strand (R) = 5.60 μΩ
= 5.60×10^{-6} Ω
Number of strands (n) = 120

a. The strands are in parallel, $R_{net} = \frac{R}{n}$

b. The strands are in series, $R_{net} = n \times R$

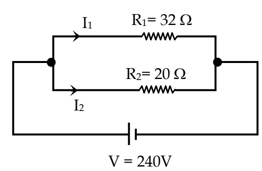
HINT: 12

Given,

Resistor (R_1) = 32 Ω
Resistor (R_2) = 20 Ω
Potential difference (V) = 240 V

a. For parallel combination, $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$

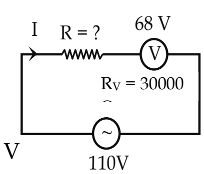
$$\text{or} \quad R_{ed} = \frac{R_1 . R_2}{R_1 + R_2}$$



b. Total current (I) =?
We have,
$V = I R_{eq}$

$$\therefore \quad I = \frac{V}{R_{eq}}$$

c. $I_1 = \frac{V}{R_1}$ and $I_2 = \frac{V}{R_2}$



HINT: 13

Given,

Range of voltmeter = 150 V
R_v = 30,000 Ω
R = ?
emf of the source, E = 110 V
voltmeter reading, V = 68 V
From the figure, we can write
E = potential difference across R + potential difference across voltmeter

$$\text{or} \quad 110 = IR + 68 \quad \text{or} \quad \frac{110 - 68}{I} = R$$

$$\therefore \quad R = \frac{32}{I}$$

But , $I = \dfrac{\text{Voltmeter reading}}{\text{Voltmeter resistance}}$

find the current 'I' and use this value in equation (i)

HINT: 14

Given,

Thickness = t = 1 μm = 10^{-6} m, $\rho = 10^{-6}\ \Omega$m

A = 1 mm^2 = 10^{-6} m^2, R = ?

a. When the film is square shaped.

Length of square, $l = \sqrt{A} = \sqrt{10^{-6}} = 10^{-3}$ m.

Cross-sectional area of the film = thickness × length of square = $10^{-6} \times 10^{-3} = 10^{-9}$ m^2

We have, $R = \rho\dfrac{l}{A}$

b. When the film is rectangular. Let l and b be the length and breadth of the rectangle.

According to question,

l = 20b

∴ A = l × b = 20 b × b = 20b^2

Now, A = 10^{-6} m^2

or $20b^2 = 10^{-6}$

or $b = \sqrt{\dfrac{10^{-6}}{20}} = 2.24 \times 10^{-4}$ m

When the film is placed length wise.

A = thickness × breadth

$= 10^{-6} \times 2.24 \times 10^{-4} = 2.24 \times 10^{-10}$ m^2

∴ $l = 20b = 20 \times 2.24 \times 10^{-4\text{ m}} = 4.48 \times 10^{-3}$ m

∵ $R = \rho\dfrac{l}{A} = 10^{-6}\dfrac{4.48 \times 10^{-3}}{2.24 \times 10^{-10}} = 20\ \Omega$

When the film is placed breadth wise

A = thickness × length

$= 10^{-6} \times 4.48 \times 10^{-3} = 4.48 \times 10^{-9}$ m^2

$l = 2.24 \times 10^{-4}$ m

Now,

$R = \rho\dfrac{l}{A}$

HINT: 15

Given,

$R_1 = 1200\ \Omega$

$R_2 = 800\ \Omega$

$R_v = 600\ \Omega$, E = 24 V

i. From the condition of voltage divider circuit,

a. P.d. across R_1, $V_1 = \left(\dfrac{R_1}{R_1 + R_2}\right)V$

b. P.d. across R_2, $V_2 = \left(\dfrac{R_2}{R_1 + R_2}\right)V$

ii. When voltmeter is connected.

Let R_V be the resistance of voltmeter.

a. For voltmeter connected across R_1

$R' = \dfrac{R_1 \times R_V}{R_1 + R_V}$

∴ $V_1' = \dfrac{R'}{R' + R_2} \times V$

b. For voltmeter connected across R_2

$R'' = \dfrac{R_2 \times R_V}{R_2 + R_V}$

∴ $V_2' = \left(\dfrac{R''}{R_1 + R''}\right)V$

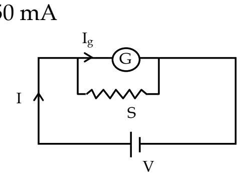
HINT: 16

Let S be the required resistance of shunt which when connected in parallel with the galvanometer, it becomes milliammeter.

G = 25 Ω

I_g = 4 mA

I = 50 mA



Since, potential difference across G = potential difference across S

∴ $I_g \times G = (I - I_g) \times S$

❖ ❖ ❖

HEATING EFFECT OF CURRENT

11

CHAPTER

11.1 Introduction

When potential difference is maintained between two ends of a conductor, current flows through it. Flow of current is the flow of charged particles (usually electrons). In the movement of charged particles, they encounter with the nuclei and ions in their path so that interactions take place between the charge particles and ions and nuclei in their path. Due to such interactions, charged particles cannot move freely through the conductor. To overcome such difficulty in the movement of charge particles, external work should be done by using the external power supply. A part of such work done is converted into the thermal energy of the particles in conductor and eventually produce heat in it. The amount of heat energy produced in the conductor was studied quantitatively by James Prescott Joule. So, the law regarding the thermal energy production due to the current is called Joules law of heating.

11.2 Joules Law of Heating

The production of heat in a resistor basically depends on quantity of current, resistance and time interval for which the current is passing through it. James Joule, in 1841, derived the relation for quantity of heat generated in a conductor of ohmic resistance R, when current I is passed through it for time t. So, this law is known as Joule's law of heating. According to this law, the amount of heat (H) developed in an ohmic conductor by the passage of current is,

i. directly proportional to the square of current flowing through the conductor.

$$\mathbf{H \propto I^2} \quad \ldots (11.1)$$

ii. directly proportional to the resistance of the conductor.

$$\mathbf{H \propto R} \quad \ldots (11.2)$$

iii. directly proportional to the time of current flow.

$$\mathbf{H \propto t} \quad \ldots (11.3)$$

Now, combining equations (11.1), (11.2) and (11. 3), we get,

$$H \propto I^2Rt$$

$$\mathbf{H = kI^2Rt} \quad \ldots (11.4)$$

Where, k is the proportionality constant. The value of k depends on the system of unit of heat.

i. In calorie unit, $k = \frac{1}{J}$

Where, J is the unit conversion factor. Its value is 4.2 J/calorie. It is called mechanical equivalent of heat. It is not a physical quantity.

From equation (11.4), we get,

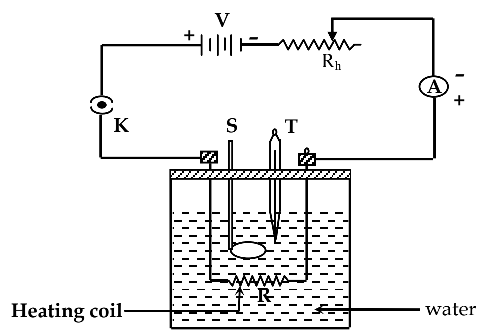
$$H = \frac{I^2Rt}{J} \text{ (calorie)} \quad \dots (11.5)$$

ii. In SI unit, k = 1. So,

$$H = I^2Rt \text{ (joule)} \quad \dots (11.6)$$

Experimental Verification of Joules Law of Heating

The experimental set up to verify the Joules law of heating is shown in the Fig. 11.1. It consists of a voltameter containing water about two third of its inner volume. A heating rod (i.e. resistance coil) of known resistance is dipped into the water. The rod is connected to an electric source. Also, an ammeter, a rheostat and a switch are connected in an electric circuit. A thermometer is used to record the temperature change in water.



S→ Stirrer; R → Resistance of heating coil; T → Thermometer

Fig. 11.1: Arrangement for experimental verification of Joule's laws of heating

To perform the experiment, electric current is supplied to the resistor (heating rod) through a cell. The rheostat is used to vary the current in the circuit. When electric current is passed through the resistor, it gets heated and the produced heat is lost into water. As the resistor is completely immersed into water, the amount of heat lost by it is equal to the amount of heat gained by the water and voltameter. So, the amount of heat produced in the resistor can be determined summing up the heat gained by water and voltameter (i.e. $H = (ms\Delta\theta)_{water} + (ms\Delta\theta)_{voltameter}$). The experiment is performed in three steps.

i. To verify $H \propto I^2$ (at constant R and t)

To verify $H \propto I^2$, steady current is passed through the resistor for a certain interval of time. The current in the circuit is controlled by adjusting the resistance of the rheostat. Then, the amount of heat produced is determined for different values of steady current. When a graph is plotted for heat lost by rod (equivalently heat gained by water and voltameter) versus square of current supplied for each step, a straight line is found passing through the origin as shown in Fig. 11.2.

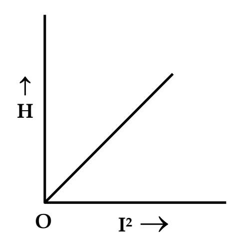


Fig.11.2: Graph of H versus I^2

ii. To verify $H \propto R$ (at constant I and t)

To verify $H \propto R$, many resistors of different resistances are taken to produce the heat. A constant current is passed through these resistors turn by turn for equal interval of time. The amount of heat gained by water and voltameter are calculated for each resistor. When a graph is plotted for heat gained by the water and voltameter (equivalently heat lost by resistor) versus resistance R, a straight line is found passing through the origin as shown in Fig. 11.3.

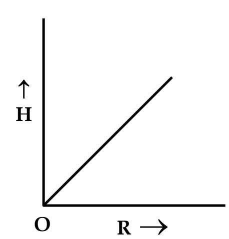


Fig.11.3: Graph of H versus R

iii. To verify H ∝ t (at constant I and t)

To verify H ∝ t, a constant current is passed through heating resistor for different interval of time. The amount of heat gained by water and voltameter are calculated for each time interval. When a graph is plotted for heat gained by the water and voltameter versus the time interval of current supply, a straight line is found passing through the origin as shown in Fig. 11.4.

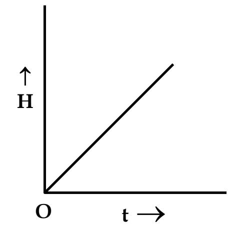


Fig.11.4: Graph H versus t

11.3 Electric Energy and Power

Electric energy is the total energy consumed in an electric circuit in a given time. *In this case, the electric potential carries charge from one point to another in the electric circuit. Thus, the electric source does the work.* Consider a conductor of resistance R in which charge q flows for a time t when potential difference V is maintained across it. Then, the electric workdone is determined by,

$$\begin{aligned} W &= qV \\ &= ItV \\ &= VIt \\ &= IRIt \\ \mathbf{W} &= \mathbf{I^2Rt\ (joule)} \qquad \ldots(11.7) \end{aligned}$$

Therefore, the electric energy produced in such situation is equivalent to heat energy H. So,

$$H = I^2Rt$$

Electric Power

The electric power is defined as the rate at which work is done by an electric charge. Alternatively, the electric power is the rate of electric energy consumption in an electric circuit.

So, electric energy consumption is,

$$W = VIt$$

Now,

$$\text{Electric power, } P = \frac{W}{t} = VI$$

Also,

$$V = IR$$

So, $P = I^2R$

Again,

$$I = \frac{V}{R}$$

$$P = \left(\frac{V}{R}\right)^2 R = \frac{V^2}{R}$$

So, power consumption in an electric circuit can be calculated using only one of the following formula,

$$P = I^2R$$

$$P = IV$$

$$\mathbf{P = \frac{V^2}{R}} \qquad \ldots(11.8)$$

Therefore, if we are given any two of the three quantities I, V and R, the electric power can be calculated. If a device is marked the power rating 40 W – 220 V, the device consumes electric energy at the rate of 40 W when joined to a supply of potential difference 220 V.

Units of Electric Power

The electric power (P) = I V

For I = 1 A and V = 1 V

$$P = 1\ A \times 1\ V$$
$$= 1\ AV$$
$$P = 1\ watt$$

Therefore, electric power dissipated is said to be 1 watt when current of 1 A passes under a potential difference of one volt. In case of household and commercial use, the electric power consumption is measured in kilowatt (kW).

$$1\ kW = 1000\ W$$

However, the electric power is commercially measured in kilowatt hour. One kilowatt hour is called 1 unit electricity i.e. 1 unit = 1kWh.

One kilowatt hour (one unit) is the amount of workdone when a power of one kilowatt is consumed for one hour. i.e.,

$$1\ unit = 1\ kWh = 1000\ W \times 3600\ s$$
$$= 3.6 \times 10^6\ Ws$$
$$= 3.6 \times 10^6 \frac{J}{s}\ s$$
$$1\ unit = 3.6 \times 10^6\ J$$

For example, if a 2000 W - 220 V induction heater is used for 20 minutes in our power supply (i.e. 220 V). Then, the units of electric its used is,

$$E = 2000\ W \times 20\ min$$
$$= \left(\frac{2000}{1000}\right) kW \times \left(\frac{20}{60}\right) h = 0.67\ kWh = 0.67\ unit.$$

If the price of electricity is Rs.10.0 per unit, then 0.67 unit will cost Rs. 6.70.

In many conditions, the electric power is also measured in horse power (HP).

$$1\ HP = 746\ W$$

$$So,\quad 1\ unit = 1\ kWh = \frac{1000}{746}\ HP \times 1\ h$$
$$= 1.34\ HPh\ (horse\ power\ hour)$$

Electricity authority charges the consumption of electric power at homes in the unit of kilowatt hour.

11.4 Electromotive Force

We are familiar with the continuous glow of an electric lamp when connected to an electric cell. The continuous glow is possible only when a sustainable potential difference is maintained across the lamp, i.e. some work must be done to carry the charge in the electric circuit. The work can be done by an electric charge. In a cell, electric energy is liberated by the chemical reaction in the electrolytes. This liberated electric energy does work to maintain the continuous flow of charge in the circuit. This work done by the cell in forcing unit positive charge (1 C charge) to flow in the electric circuit is called electromotive force (emf).

Actually, emf is not a force as it's name suggests, rather it is a work done.

If dW work is done in moving dq charge by an electric source, the emf of that cell is,

$$E = \frac{dW}{dq} \qquad \dots (11.9)$$

In SI system, the unit of E is joule per coulomb (JC^{-1}), which is also called volt (V). It means, the unit of emf is volt.

The emf of a source of current is said to be one volt if one joule of energy is supplied by the source to flow one coulomb of charge in the whole circuit.

To put it on another way, the source of emf provides energy to the circuit. The current in the circuit transfers energy from source of emf to a device. If the device is another battery, then the energy transferred appears as the chemical energy newly stored in the battery. If the device is a resistor, the transferred energy appears as the internal energy (observed perhaps as an increase in temperature) and then can be transferred to the environment as heat. If the device is capacitor, the energy transferred is stored as potential energy in its electric field. In each of these cases, conservation of energy demands that amount of energy lost by battery must be equal to the energy transferred to, dissipated by or stored in the device.

The emf of a source is equal to the potential difference between the terminals of a source when no current is drawn from the source. No current will be drawn when the circuit is open. So, emf is equal to the potential difference in an open circuit.

11.5 Terminal Potential Difference

Positive charge (conventional charge) flows from positive terminal to negative terminal through the external path of an electric circuit, but the positive charge flows from negative terminal to positive terminal inside the cell. Whatever the direction of charge, the work should be done to move the charge particles in the circuit. If a voltmeter is connected across two terminals of a cell at the closed circuit condition, it measures the potential difference of the external circuit, which is called the terminal potential difference. Therefore, *terminal potential difference is defined as the potential difference between two terminals of a cell in closed circuit.*

Emf of a cell is divided into two parts: outside the cell and inside the cell. The potential difference that is developed outside the cell is equal to the terminal potential difference (V) and the potential difference inside the cell is called the internal potential difference V_i. Which is also called as lost volt as this is lost in the source due to its internal resistance.

$$\text{So,} \quad E = V + V_i \qquad \dots (11.10)$$

The unit of terminal potential difference is volt (V) and its dimension is $[ML^2T^{-3}A^{-1}]$

11.6 Internal Resistance of a Cell

When two terminals of a cell are connected with a resistance wire, current flows from positive terminal plate to negative terminal plate outside the cell and negative terminal plate to positive terminal plate in the electrolyte inside the cell. The flow of current in an electric circuit is opposed by the external resistance in the circuit as well as electrolytes inside the cell. The resistance offered by the electrolyte of a cell to the flow of current through it is called the internal resistance of a cell. It is denoted by 'r'.

Due to the internal resistance of a cell, a part of electric energy is converted into the heat energy which makes the cell heated. The internal resistance of a cell depends upon the following factors:

i. It is directly proportional to the separation of two plates of a cell.

ii. It is inversely proportional to the area of plates dipped into the electrolyte.

iii. It depends on the nature, concentration and temperature of the electrolyte.

After long use, the conductivity of electrolytes used into the cell decreases, hence the internal resistance increases. Moreover, the deposition of ions on the terminal plates may increase the internal resistance of a cell.

11.7 Relation of emf, Terminal Potential Difference and Internal Resistance of a Cell

Consider an electric circuit containing a cell, a resistor and a key. Let E be the emf, V be the terminal potential difference and r be the internal resistance of the given cell. The resistance R in exterior circuit is called external resistance. The circuit diagram with necessary components of an electric circuit is shown in Fig. 11.5. The internal resistance of the cell is considered to be connected in series with cell.

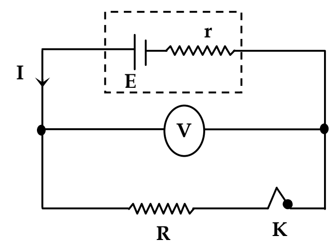


Fig. 11.5: Circuit diagram for internal resistance of a cell

When the circuit is switched on, the electric cell supplies energy to move the charge through the circuit. It means the cell performs work in displacing the charge in the electric circuit. Then, the total workdone to displace the charge q,

$$\mathbf{W = E \cdot q = EIt} \quad \ldots(11.11)$$

The work is done in the circuit is divided into two parts; (i) work done outside the cell against the external resistance (R) (ii) work done inside the cell against the internal resistance (r).

From the principle of conservation of energy,

$$W = W_{external} + W_{internal}$$

$$EIt = I^2Rt + I^2rt$$

$$E = IR + Ir$$

or, $E = V + Ir$, where $V = IR$ is the terminal potential difference across the resistor R

$$\therefore \quad \mathbf{E - V = Ir} \quad \ldots(11.12)$$

The term Ir is the potential drop across the internal resistance which is equal to the difference of emf and terminal potential difference of a cell. At the condition of discharging, the current is taken positive value. For non-zero value of internal resistance $r > 0$, $(E - V) > 0$ i.e. emf is greater than the terminal potential difference.

$$\therefore r = \frac{E - V}{I} = \frac{E - V}{\left(\frac{V}{R}\right)}$$

$$\therefore \mathbf{r = \left(\frac{E - V}{V}\right) R} \quad \ldots(11.13)$$

Thus, the internal resistance of a cell can be determined by using formula in equation (11.13).

Special Cases

i. If internal resistance of a cell is negligibly small i.e. $r \to 0$, then E = V.

ii. During the charging of a cell, the direction of current is taken negative, so we can write,

$$E = V + (-I)\, r$$

$$\therefore \quad \mathbf{E = V - Ir} \qquad \ldots (11.14)$$

It shows that, terminal potential difference can be greater than emf when a cell is charging.

11.8 Combination of Cells

The grouping of two or more cells in a single electric circuit is known is combination of cells. Cells are basically combined in the following ways:

i Series combination of cells.

ii. Parallel combination of cells

iii. Mixed combination of cells.

i. Series Combination of Cells

Cells are said to be connected in series when they are joined end to end so that the same quantity of current flows through each cell. In series combination of cells, the negative terminal of one cell is connected to the positive terminal of the next, the negative of the second to the positive terminal of the third and so on. The series combination of cells is shown in Fig. 11.6.

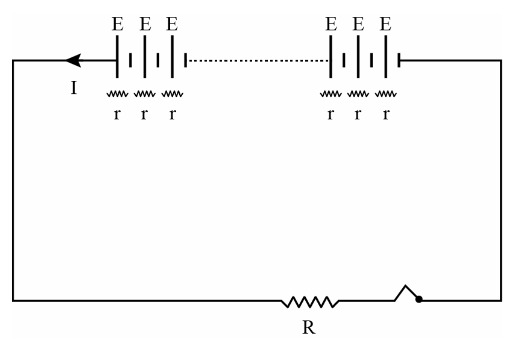


Fig. 11.6: Series combination of cells

Let, emf of each cell = E

Internal resistance of each cell = r

External resistance = R

Total emf of n-cells in series = nE

Total internal resistance of n-cells = nr

So, total resistance of the complete circuit = R + nr

If I be the current flowing through the circuit, then according to Ohm's law, we can write,

$$I = \frac{\text{Total emf}}{\text{Total resistance}}$$

$$\therefore \quad \mathbf{I = \frac{nE}{R + nr}} \qquad \ldots (11.15)$$

Case (i) If R >> nr i.e. if external resistance R is large enough with respect to total internal resistance nr,

$$I = \frac{nE}{R} = n \times \frac{E}{R}$$

= n times the current that can be drawn from a single cell.

Case (ii) If R << nr, then $I = \frac{E}{r}$ = same as that given by one cell.

Conclusion: When internal resistance is negligible in comparison to external resistance, the cells are connected in series to get maximum current.

ii. Parallel combination of cells

Cells are said to be connected in parallel when the current is divided between the various cells. In the parallel connection of cells, all the positive terminals are connected together at one point and all the negative terminals are connected together at another point as shown in Fig. 11.7.

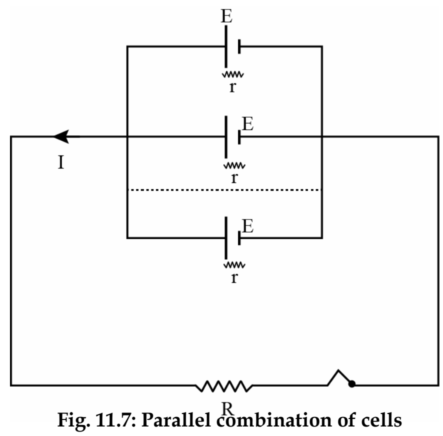


Fig. 11.7: Parallel combination of cells

Let, emf of each cell = E
Internal resistance of each cell = r
External resistance = R
Total internal resistance of n-cells = r/n
(∵ they are in parallel combination)

So, total resistance of the complete circuit = R + r/n

If I be the current flowing in the circuit, according to Ohm's law, we have,

$$I = \frac{\text{Total emf}}{\text{Total resistance}}$$

$$= \frac{E}{R + \frac{r}{n}} \qquad \ldots(11.16)$$

Case (i) If $\frac{r}{n} >> R$ (i.e. internal resistance is extremely high), then $I = \frac{nE}{r}$ = n times current given by one cell.

Case (ii) If $\frac{r}{n} << R$ (i.e. internal resistance of a cell is low), then

$$I = \frac{E}{R} \text{ = same as given by one cell.}$$

Conclusion: When external resistance is negligible in comparison to the internal resistance, the cells are connected in parallel to get maximum current.

iii. Mixed combination of cells

The combination in which the cells are arranged in such a way that some of them are connected in series and others are connected in parallel is known as mixed combination of cells. The circuit diagram for mixed combination of cells is shown in Fig. 11.8.

Now,

Let, emf of each cell = E
Internal resistance of each cell = r
External resistance = R
Number of rows = m
Number of cells in a row = n
Total internal resistance of given cells = nr/m

So, total resistance of the complete circuit
= R + nr/m

Total number of cells = mn

emf of each row = nE

Total emf of circuit = nE

If I be the current in the circuit, by Ohm's law we have,

$$I = \frac{\text{Total emf}}{\text{Total resistance}}$$

$$= \frac{nE}{R + \frac{nr}{m}}$$

$$= \frac{nE}{\frac{mR + nr}{m}}$$

$$= \frac{mnE}{mR + nr} \quad \ldots (11.17)$$

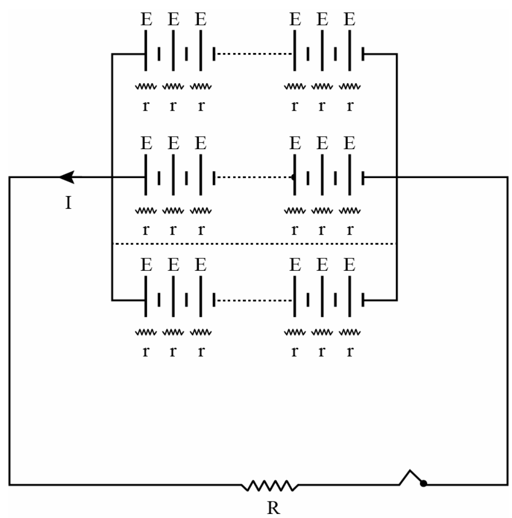


Fig. 11.8: Mixed combination of cells

Condition for Maximum Current

In the above combination, m, n and E are constant. The current in the circuit can be produced maximum only when denominator is minimum. To minimize the denominator,

$$mR + nr = \left(\sqrt{mR} - \sqrt{nr}\right)^2 + 2\sqrt{mnrR}$$

$$\therefore \quad I = \frac{mnE}{\left(\sqrt{mR} - \sqrt{nr}\right)^2 + 2\sqrt{mnrR}}$$

Current, I will be maximum if $\left(\sqrt{mR} - \sqrt{nr}\right)^2 = 0$

or, $\sqrt{mR} - \sqrt{nr} = 0$

or, $\sqrt{mR} = \sqrt{nr}$

or, $mR = nr$

or, $$\mathbf{R = \frac{nr}{m}} \quad \ldots (11.18)$$

i.e. external resistance = total internal resistance of the cells

Conclusion: When the external resistance of the circuit is equal to the internal resistance of cells, we use mixed combination of cells to obtain maximum current.

$$\mathbf{I_{max} = \frac{mnE}{2\sqrt{mnRr}}} \quad \ldots (11.19)$$

Tips for MCQs

1. **Electric power:**
   i. It is defined as the rate at which work is done by the source of emf in maintaining the current in electric circuit.

   $$\therefore \quad P = \frac{qV}{t} = IV = I^2R = \frac{V^2}{R}$$

   ii. Its unit is watt or ampere-volt. It is also expressed into horse power, (1 HP = 746 watt).

iii. In series combination of resistances, the potential difference and power consumed will be more in larger resistance. So, 60 W bulb glows brighter than 100 W bulb when connected in series circuit i.e. $\frac{1}{P_s} = \frac{1}{P_1} + \frac{1}{P_2} + \frac{1}{P_3} + \ldots$

iv. In parallel combination of resistances, the current and power consumed will be more in smaller resistances. Therefore, 100 W bulb glows brighter than 60 W bulb when connected in parallel combination, i.e. $P_p = P_1 + P_2 + P_3 + \ldots$

v. In filament bulb, larger powered bulb is made with smaller resistance.

2. Electric energy:

i. It is defined as the total work done or energy supplied by the source of emf in maintaining the current in an electric circuit for a given time.

Electric energy (E) = $I^2Rt = VIt = Pt = \frac{V^2t}{R}$

ii. It's unit is joule. The commercial unit of electric energy is kilowatt-hour (kWh).

1 kWh = 3.6×10^6 J. This is called one unit of electricity.

iii. Total number of units (n) = $\frac{\text{Total watt} \times \text{Total hour}}{1000}$

3. Emf, terminal potential and internal resistance:

i. Relation, $E = V + Ir$

ii. Efficiency of a source of emf, $\eta = \frac{P_0}{P_i} = \frac{V}{E} = \frac{R}{R + r}$

iii. If R = r, the maximum efficiency of a cell can be obtained, i.e. $\eta = \frac{R}{R + R} = \frac{1}{2} = 50\%$

iv. Lamp used for house lightening are connected in parallel.

4. Emf of a cell depends on (a) nature of two plates (b) nature, temperature and concentration of electrolyte.

5. Terminal potential difference,

a. While charging, $V > E$, $E = V - Ir$

b. While discharging, $V < E$, $E = V + Ir$

c. While short circuited, $R = 0$, $V = 0$

d. In open circuit, $I = 0$

6. Internal resistance depends on:

a. Separation between two electrodes

b. Nature, temperature and degree of dissociation of electrolyte between plates.

Worked Out Problems

1. Two heating coils A and B connected in parallel in a circuit produces power of 12 W and 24 W respectively. What is the ratio $\frac{R_A}{R_B}$ when used?

SOLUTION

Given,

$P_A = 12$ W

$P_B = 24$ W

$\frac{R_A}{R_B} = ?$

We know, in parallel circuit, V remains constant.

$$P = \frac{V^2}{R}$$

$$\text{So, } P_A = \frac{V^2}{R_A} \quad \ldots\text{(i) and}$$

$$P_B = \frac{V^2}{R_B} \quad \ldots\text{(ii)}$$

$$\frac{P_B}{P_A} = \frac{V^2/R_B}{V^2/R_A}$$

$$\frac{P_B}{P_A} = \frac{R_A}{R_B}$$

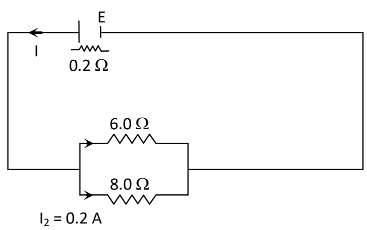
$$\therefore \frac{R_A}{R_B} = \frac{24}{12} = \frac{2}{1}$$

$\therefore$ $R_A : R_B = 2 : 1$

2. A cell of internal resistance of 0.2 Ω is connected two coils of resistance 6.0 Ω and 8.0 Ω joined parallel. There is a current of 0.2 A in the 8.0 Ω coil. Find the emf of cell.

SOLUTION

According to the given information, the electric circuit can be drawn as follows:



Here,

$r = 0.2\ \Omega$

$R_1 = 6.0\ \Omega$

$R_2 = 8.0\ \Omega$

I_2 (in 8.0 Ω) = 0.2 A

Since, 6.0 Ω and 8.0 Ω resistance are in parallel, the potential difference across them are equal so,

$I_1R_1 = I_2R_2$

$I_1 \times 6 = 0.2 \times 8$

$I_1 = \frac{0.2 \times 8}{6}$

$= 0.27$ A

Therefore, total current (I) = 0.2 + 0.27 = 0.47 A

External resistance (R) = $R_1 \,||\, R_2$

$= \frac{R_1 R_2}{R_1 + R_2}$

$= \frac{6 \times 8}{6 + 8} = 3.4\ \Omega$

Now,

Emf (E) = IR + Ir

= 0.47 × 3.4 + 0.47 × 0.2

= 1.7 V

3. A battery of emf 1.5 V has a terminal potential difference of 1.25 V when a resistor of 25 Ω is joined to it. Calculate the current flowing, the internal resistance and terminal p.d. when resistance of 10 Ω is replaces 25 Ω.

SOLUTION

Given,

Emf (E) = 1.5 V

Terminal p.d. (V) = 1.25 V

External resistance (R) = 25 Ω

Now, current (I) = $\frac{V}{R} = \frac{1.25}{25} = 0.05$ A

E = V + Ir

or, 1.5 = 1.25 + 0.05 r

or, 0.05r = 1.5 – 1.25

or, 0.05 r = 0.25

$r = \frac{0.25}{0.05} = 5\ \Omega$

If the 10 Ω resistance replaces 25 Ω, total resistance in the circuit is,

Also,

R + r = 10 + 5 = 15 Ω

Now, total current (I') $= \frac{E}{R + r}$

$= \frac{1.5}{15} = 0.1$ A

Now, new terminal potential difference (V)

= IR

V = 0.1 × 10 = 1.0 V

4. A battery of emf 4 V and internal resistance 2 Ω is joined to a resistor of 8 Ω. Calculate the terminal potential difference. What additional resistance in series with 8 Ω resistor would produce a terminal p.d. of 3.6 V?

SOLUTION

Given,

Internal resistance (r) = 2 Ω

External resistance (R) = 8 Ω

Total current (I) = $\frac{E}{R + r} = \frac{4}{8 + 2} = 0.4$ A

Let x be the additional resistance in series with 8 Ω resistor to produce terminal p.d. of 3.6 V.

Total external resistance (R) = 8 + x

Total current (I) $= \dfrac{E}{R + r}$

$= \dfrac{4}{8 + x + 2} = \dfrac{4}{10 + x}$

Emf (e) = 4 V

Now,

Terminal p.d. = IR

$3.6 = \dfrac{4}{(10 + x)}(8 + x)$

or, $36 + 3.6x = 32 + 4x$

or $4x - 3.6x = 36 - 32$

or, $0.4x = 4$

or, $x = \dfrac{4}{0.4}$

$\therefore$ $x = 10\ \Omega$

$\therefore$ Additional resistance is 10 Ω.

5. **The circuit shown in figure contains two batteries, each with an emf and an internal resistance and two resistors. Find (a) the current in the circuit (magnitude and direction); (b) the terminal voltage V_{ab} of the 16.0 V battery; (c) the potential difference V_{ac} of point a with respect to point c.**

SOLUTION

Given,

Emf of cell 1 (E_1) = 16 V $r_1 = 1.6\ \Omega$

EMF of cell 2 (E_2) = 8 V $r_2 = 1.4\ \Omega$

$R_1 = 5\ \Omega$ $R_2 = 9\ \Omega$

a. Circuit current (I) = ?

Using Kirchoff's voltage law in the given circuit, we can write

$I \times r_1 + I \times R_1 + I \times r_2 + I \times R_2 = E_1 - E_2$

or $I(1.6 + 5 + 1.4 + 9) = 16 - 8$

or $I = \dfrac{8}{17} = 0.47$ A

Its direction is in anticlockwise direction.

b. Terminal voltage (V_{ab}) = ?

We know that

$V = E - Ir$

or $V_{ab} = E_1 - Ir_1$

$= 16 - 0.47 \times 1.6 = 15.25$ volt

c. Terminal voltage (V_{ac}) = ?

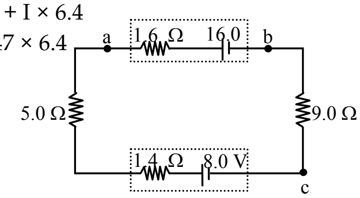
From the figure, we can write

$V_c + 8 + I\,(1.4 + 5) = V_a$

or $V_a - V_c = 8 + I \times 6.4$

or $V_{ac} = 8 + 0.47 \times 6.4$

$\therefore$ $V_{ac} = 11.0$ V



6. **A "540-W" electric heater is designed to operate from 120 V lines. (a) What is its resistance? (b) What current does it draw? (c) If the line voltage drops to 110 V, what power does the heater take? (Assume that the resistance is constant. Actually, it will change because of the change in temperature.)**

SOLUTION

Given,

Power of heater (P) = 540 W

Potential (V) = 120 V

a. Resistance (R) = ?

We know that

$P = \dfrac{V^2}{R}$

$\therefore$ $R = \dfrac{V^2}{P} = \dfrac{(120)^2}{540} = 26.7\ \Omega.$

b. Current (I) = ?

We know that

$P = VI$

$\therefore$ $I = \dfrac{P}{V} = \dfrac{540}{120} = 4.5$ A.

c. For potential difference, V = 110 volt, power (P) = ?

We know that

$P = \dfrac{V^2}{R} = \dfrac{(110)^2}{26.7} = 454$ W

7. **An electrical heating coil is connected in series with a resistance of X Ω across the 240 V mains, the coil being immersed in a kilogram of water at 20°C. The temperature of the water rises to boiling point in 10 minutes. When a second heating experiment is made with the resistance X short-**

circuited the time required to develop the same quantity of heat is reduced to 6 minutes. Calculate the value of X.

SOLUTION

P.d of mains (V) = 240 V,

Mass of water (m_w) = 1 kg,

$\theta_1 = 20°C$

$t_1 = 10$ min, $\theta_2 = 100°C$, $t_2 = 6$ min

Let R be the resistance of the heating coil.

Heat developed in the coil = heat absorbed by water

or $I^2Rt_1 = mc_w\Delta\theta$

or $\left(\frac{240}{R+X}\right)^2 R \times 10 \times 60 = 1 \times 4200\,(100 - 20)$

or $\left(\frac{240}{R+X}\right)^2 R = \frac{4200 \times 80}{600} = 560$ . . . (i)

When resistance x is short circuited, we can write

$I = \frac{240}{R}$

Also, Heat developed in the coil = heat absorbed by water

or $I^2Rt_2 = mc_w\Delta\theta$

or $\left(\frac{240}{R}\right)^2 R \times 6 \times 60 = 1 \times 4200\,(100 - 20)$

or $\frac{240 \times 240}{R} = \frac{4200 \times 80}{6 \times 60}$

or $R = \frac{240 \times 240 \times 6 \times 60}{4200 \times 80} = 61.71\ \Omega$

Putting the value of R in (i), we get

$\left(\frac{240}{61.71 + X}\right)^2 \times 61.71 = 560$

or $\left(\frac{240}{61.71 + X}\right)^2 = \frac{560}{61.71} = 9.075$

or $\frac{240}{61.71 + X} = 3.012$

or $61.71 + X = \frac{240}{3.012}$

or $X = \frac{240}{3.012} - 61.71 = 18\ \Omega$

8. **[NEB 2075] Two lamps rated 25 W – 220 V and 100 W – 220 V are connected to 220 V supply. Calculate the powers consumed by the lamps.**

SOLUTION

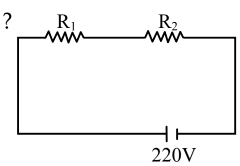
Given,

First lamp = 25 W – 220 V

Second lamp = 100 W – 220 V

Voltage (V) = 220 V

Power consumed (P) = ?



Now,

For first lamp,

$P_1 = 25$ W

$V_1 = 220$ V

$R_1 = \frac{V_1^2}{P_1} = \frac{220 \times 220}{25} = 1936\ \Omega$

For second lamp,

$P_2 = 100$ W

$V_2 = 220$ V

$R_2 = \frac{V_2^2}{P_2} = \frac{220 \times 220}{100} = 484\ \Omega$

If two lamps are connected in series and joined to 220 V mains, the current in the circuit, I is given as,

$I = \frac{V}{R_1 + R_2} = \frac{220}{1936 + 484} = 0.091\text{A}$

Power consumed by first lamp,

$I^2R_1 = (0.091)^2 \times 1936 = 16$ W

Power consumed by second lamp,

$I^2R_2 = (0.091)^2 \times 484 = 4$ W

9. **[HSEB 2062] Twelve cells each of e.m.f. 2 V and of internal resistance 0.5 ohm are arranged in a battery of n rows and an external resistance 0.4 ohm is connected to the poles of the battery. Estimate the current flowing through the resistance in terms of n.**

SOLUTION

Given,

No. of cells (N) = 12

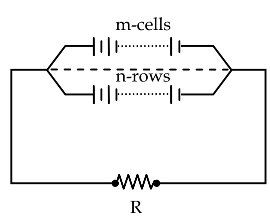
Emf (E) = 2 V

Internal resistance (r)= 0.5 ohm

No. of rows = n

External resistance (R) = 0.4 Ω

Current (I) = ?



Let, ‘m’ be the no. of cells in each row;

Then, we have;

$m \times n = 12$

$\therefore \quad m = \frac{12}{n}$

Now, we have;

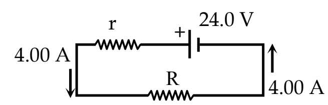
$$I = \frac{mE}{\frac{mr}{n} + R} = \frac{mnE}{mr + nR} = \frac{12 \times 2}{\frac{12}{n} \times 0.5 + n \times 0.4} = \frac{24\,n}{6 + 0.4n^2} = \frac{\frac{24n}{0.4}}{\frac{6 + 0.4n^2}{0.4}} = \frac{60n}{15 + n^2}\,A$$

Hence, the required current flowing through the resistance is $\frac{60n}{15 + n^2}$ Ampere.

Challenging Problems

1. **[UP]** The open-circuit terminal voltage of a battery is 12.6 V, when a resistor $R = 4.00\ \Omega$ is connected between the terminals of the battery, the terminal voltage of the battery is 10.4 V. What is the internal resistance of the battery?
Ans: 0.857 Ω

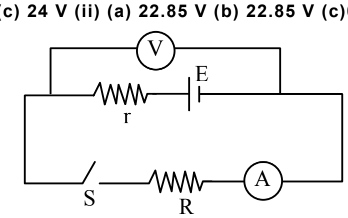
2. **[UP]** The terminal voltage of the 24.0 V battery is 21.2 V. What is (a) the internal resistance r of the battery; (b) the resistance R of the circuit resistor?



Ans: (a) 0.7 Ω (b) 5.3 Ω

3. **[UP]** A complete circuit consists of a 24.0 V battery, a 5.60 Ω resistor and a switch. The internal resistance of the battery is 0.28 Ω. The switch is opened. (i) What does an ideal voltmeter read when placed (a) across the terminals of the battery? (b) across the resistor? (c) across the switch? (ii) Repeat parts (a), (b) and (c) for the case when the switch is closed.
Ans: (I) (a) 24 V (b) 0 (c) 24 V (ii) (a) 22.85 V (b) 22.85 V (c)0

4. **[UP]** When switch S in figure is open, the voltmeter V of the battery reads 3.08 V. When the switch is closed, the voltmeter reading drops to 2.97 V, and the ammeter A reads 1.65 A. Find the internal resistance of the battery, and the circuit resistance R. Assume that the two meters are ideal, so they don't affect the circuit.



Ans: 0.07 Ω and 1.8 Ω

5. **[UP]** A resistor with a 15.0 V potential difference across its ends develops thermal energy at a rate of 327 W. (a) What is its resistance? (b) What is the current in the resistor?
Ans: (a) 0.688 Ω (b) 21.8 A

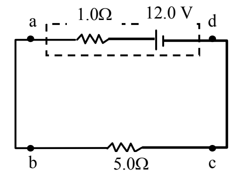
6. **[UP]** To stun its prey, the electric eel electrophorus electrius generates 0.8 A pushes of current along its skin. This current flows across a 650 V potential difference. At what rate does electrophorus deliver energy to its prey?
Ans: 520 W

7. **[UP]** A battery-powered global positioning system (GPS) receiver operating on 9.0 V draws a current of 0.13 A. How much electrical energy does it consume during $1\frac{1}{2}$ h?
Ans: 6.3×10^3 J

8. **[UP]** A capacity of a storage battery, such as those used in automobile electrical systems, is rated in ampere-hours (A.h). A 50 A.h battery can supply a current of 50 A for 1.0 h, or 25 A for 2.0 h and so on. What total energy can be supplied by a 12 V, 60 A.h battery if its internal resistance is negligible?
Ans: 2.6×10^6 J

9. **[UP]** In the circuit in figure, find (a) the rate of conversion of internal (chemical) energy to electrical energy within the battery; (b) the rate of dissipation of electrical energy in the battery; (c) the rate of dissipation of electrical energy in external resistor.

Ans: (a) 24 W (b) 4 W (c) 20 W

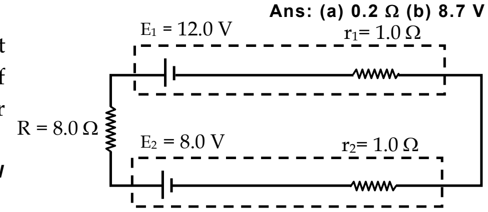


10. **[UP]** The potential difference across the terminals of a battery is 8.4 V when there is a current of 1.50 A in the battery from the negative to the positive terminal. When the current is 3.50 A in the reverse direction, the potential difference becomes 9.4 V. (a) What is the internal resistance of the battery? (b) What is the emf of the battery?

Ans: (a) 0.2 Ω (b) 8.7 V

11. **[UP]** In the following circuit, find (a) the current through the 8.0 Ω resistor; (b) the total rate of dissipation of electrical energy in the 8.0 Ω resistor and in the internal resistance of the batteries.

Ans: (a) 0.40 A (b) 1.6 W



12. **[ALP]** A surge suppressor is made of a material whose conducting properties are such that the current passing through is directly proportional to the fourth power of the applied voltage. If the suppressor dissipates energy at a rate of 6.0 W when the potential difference across it is 240 V, estimate the power dissipated when the potential difference rises to 1200 V.

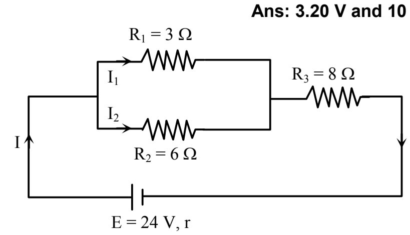
Ans: 18.75 kW

13. **[ALP]** A battery of emf 4 V and internal resistance 2 Ω is joined to a resistor of 8 Ω. Calculate the terminal potential difference. What additional resistance in series with the 8 Ω resistor would produce a terminal potential difference of 3.6 V?

Ans: 3.20 V and 10 Ω

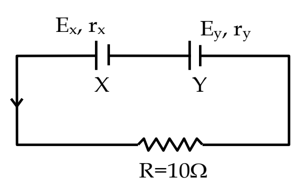
14. **[ALP]** As shown in the figure, a battery of emf 24 V and internal resistance r is connected to a circuit containing two parallel resistors of 3 Ω and 6 Ω in series with an 8 Ω resistor. The current flowing in the 3 Ω is 0.8 A. Calculate (i) the current in the 6 Ω resistor, (ii) r and (ii) the terminal potential difference of the battery. [HSEB 2061]

Ans: (i) 0.4 A (ii) 10 Ω (iii) 12 V



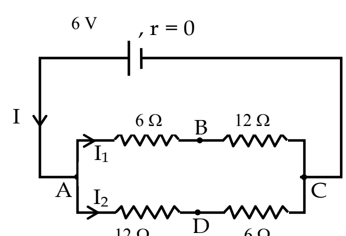
15. **[ALP]** A battery X of emf 6 V and internal resistance 2 Ω is in series with a battery Y of emf 4 V and internal resistance 8 Ω so that the two emfs act in the same direction. A 10 Ω resistor is connected to the batteries. Calculate the terminal potential difference of each battery. If Y is reversed so that the emf now opposes each other, what is the new terminal potential difference of X and Y?

Ans: 5 V, 0 V, 5.8 V, 3.2 V



16. A voltmeter having a resistance of 1800 Ω is used to measure the potential difference across a 200 Ω resistance which is connected to the terminals of a.d.c. power supply having an emf of 50 V and an internal resistance of 20 Ω. Determine the percentage change in the potential difference across the 200 Ω resistor as a result of connecting the voltmeter across it.

Ans: 1%



17. **[ALP]** Two heating coils A and B, connected in parallel in a circuit, produce power of 12 W and 24 W respectively. What is the ratio of their resistances, R_A/R_B, when use?

Ans: 2 : 1

18. **[ALP]** A heating coil of power rating 10 W is required when the potential difference across it is 20 V. Calculate the length of nichrome wire needed to make the coil if the cross-sectional area of the wire used is 1×10^{-7} m^2 and the resistivity of nichrome is 1×10^{-6} Ωm. What length of wire would be needed if its diameter was half that previous used?

Ans: 4 m and 1 m

19. **[ALP]** The temperature of 0.3 kg of oil in a vacuum flask rises 10°C per minute with an immersion heater of 12.3 watts input. On repeating with 0.4 kg of oil the temperature rises by 1.2°C per minute for an input of 19.2 watts. Find the specific heat capacity of the oil and the thermal capacity of the flask.

Ans:2220 $Jkg^{-1}K^{-1}$, 72 J K^{-1}

20. **[ALP]** An electric hot plate has two coils of managing wire, each 20 m in length and 0.23 mm^2 cross sectional area. Show that it will be possible to arrange for three different rates of heating, and calculate the wattage in each case when the heater is supplied from 200 V mains. The resistivity of manganin is 4.6×10^{-7} Ωm.

Ans: 1 kW, 0.5 kW, 2 kW

21. **[ALP]** An electric fire dissipates 1 kW when connected to a 250 V supply. Calculate to the nearest whole number the percentage change that must be made in the resistance of the heating element in order that it may dissipate 1 kW on a 200 V supply. What percentage change in the length of the heating element will produce this change of resistance if the consequent increase in the temperature of the wire causes its resistivity to increase by a factor 1.05? The cross sectional area may be assumed constant.

Ans: 36%, 39%

*[**Note:** Hints to challenging problems are given at the end of this chapter.]*

Conceptual Questions with Answers

1. Five bulbs are connected in series across 220 volt line. If one bulb is fused, the remaining bulbs are again connected across the same line. Which one of the arrangements will be more illuminated? Justify you answer.

➷ Let R be equal resistance of filament of each bulb. The total resistance of the circuit as 5 bulbs are connected in series, $R_1 = 5R$

Therefore, total current $(I_1) = \frac{V}{5R} = \frac{220}{5R}$

Similarly, when only four bulbs are connected in series, $I_2 = \frac{V}{4R}$

Therefore, Power dissipation in first case, $P_1 = I_1^2\ (5R)$

$$P_1 = \left(\frac{220}{5R}\right)^2 5R$$

$$P_1 = \frac{(220)^2}{5R} \quad \text{... (i)}$$

Similarly power dissipation in the second case,

$$P_2 = \frac{(220)^2}{4R} \quad \text{... (ii)}$$

Dividing (ii) by (i), we get

$$\frac{P_2}{P_1} = \frac{5}{4}$$

$\frac{P_2}{P_1} = \frac{5}{4} > 1$. Simply, as one bulb gets fused, the total series resistance of four bulbs is less than the series resistance of five bulbs. By more current flows in second case giving more illumination.

Therefore, power dissipation is more in second case, hence, the four bulbs in series illuminate brighter.

---

2. Batteries are always labelled with their emf, for instance, an A flashlight battery is labelled '1.5 volt'. Would it also be appropriate to put a label on batteries stating how much current they provide? Why or why not?

✎ Emf is the property of electric source, but not the components connecting in the circuit. However, the electric current depends on both the emf and combination of resistances in the circuit. Since, the current depends on value of resistance in the circuit, its value varies, although the source is same. Hence, the labelling of current is not appropriate.

---

3. Why does an electric bulb nearly always burn out just as you turn on the light, almost never while the light is shining? (HSEB 2070)

✎ In the beginning, the filament has relatively low temperature. When current flows through it, it gets heated. The resistance of a conductor increases on heating. Hence, the electric current is high initially and decreases as the filament is heated. Due to the sudden change of temperature, the wire suffers differential expansion at different cross section and it burns.

---

4. Why an electric bulb becomes dim when an electric heater in parallel circuit is switched on?

✎ In parallel combination, low resistance draws more power. Low resistance heater draws a higher current. In such combination, some current from the bulb is diverted into heater. So, the bulb becomes dim.

---

5. How does the internal resistance of a cell vary with temperature?

✎ Internal resistance of a cell decreases when temperature increases. Following are the main reasons of decreasing internal resistance due to the increase in temperature.

i. The coefficient of viscosity of electrolyte decreases on heating. So, ions can move freely.

ii. Kinetic energy of ion increases on heating. Therefore, ions can move with the speed.

---

6. Deduce dimensional formula for potential difference.

✎ Potential difference is the amount of work done in bringing a charge from one point to another in an electric field.

$\therefore$ Potential difference (V) = $\dfrac{\text{Work done}}{\text{charge}}$

The dimension of work done = $[ML^2T^{-2}\}$

The dimension of charge = [AT]

so, $[V] = \dfrac{[ML^2T^{-2}]}{[AT]} = [ML^2T^{-3}A^{-1}]$

Therefore, the dimensional formula of potential differences is, $[ML^2T^{-3}A^{-1}]$.

---

7. Why heat is generated in a conductor, when current flows through it?

✎ When charge particles (electrons) move in a conductor, they encounter with nuclei and with ions which oppose the free movement of electrons. To overcome such difficulty in the movement of charge particles, external works should be done by using the external power supply. Collision of electron with ions is like collision of small stone with much larger stone. A part of such workdone is converted into the thermal energy of the particles in conductor and eventually produced heat in it.

---

8. At what condition, current can be multiplied in series combination of cells?

✎ When external resistance (R) is very much greater than net internal resistance (nr) of cells, the current drawn in the circuit is,

$I = \dfrac{nE}{R} = n\left(\dfrac{E}{R}\right)$ = n × current drawn from every cell.

---

9. Define emf and terminal potential difference.

✎ **emf:** The potential difference between two terminals of a cell while current is not drawn from it is known as electromotive force (emf). It is the maximum possible voltage that the cell can provide in the circuit. It is denoted by E. Its unit is volt (V)

Terminal potential difference: The potential difference between two terminals of a cell while current is drawn from it is known as terminal potential difference. Terminal potential difference is equivalent to voltage drop on the external resistance of an electric circuit. It is denoted by V. Its unit is volt (V).

---

10. Can terminal potential difference be greater than emf of a cell?

✎ Yes. This condition is possible when a battery is being charged. In a common circuit, conventional current flows form positive terminal to negative terminal of a circuit, but the current flows form negative to positive terminal inside the cell. However, in charging a cell, the convensional current flows form positive to negative terminal of the cell. In such condition, the magnitude of current is assigned negative (i.e. – I). So, in the relation of emf and terminal potential difference.

$E = V + (-I)r$

i.e. $E = V - Ir$

Since r has some 'positive value,

$V > E$

---

11. What is internal resistance? Write the relation between E,V and r.

✎ The electrolytes of a cell offers the resistance for the current in it. This resistance provided by the electrolytes of the cell is known as internal resistance of a cell. If the internal resistance of a cell is non zero. Then the total resistance in an electric circuit is, R + r, where,

R = external resistance

r = internal resistance

Now, emf (E) = I (R + r)

I = total current in an electric circuit.

$\therefore E = IR + IR$

$\boxed{E = V + Ir}$

This is the relation between E, V and r.

---

12. What is one unit electricity?

✎ When 1 kW electric power is consumed for 1 hour, the electricity so used is called one unit electricity. So, 1 unit = 1 kWh.

If ten bulbs of each 100 W are lighted regularly for one hour, the consumed electricity is equal to 1 unit electricity.

---

13. Two bulbs 60 W and 100 W are connected (i) in series (ii) in parallel, which bulb glows brightly? explain.

✎ Resistance of lower power bulb is made with greater resistance than high power, i.e. resistance of 60 W bulb has greater resistance than the resistance of 100 W (i.e. $R_{60} > R_{100}$).

i. In series, current remains constant in both bulbs, So, $P = I^2R$, i.e $P \propto R$.

It means, $P_{60} > P_{100}$ (for $R_{60} > R_{100}$).

So, the 60 W bulb consumes more power, hence it glows brighter.

ii. In parallel, potential difference remains constant in both bulbs, so, $P = \frac{V^2}{R}$ i.e. $P \propto \frac{1}{R}$

It means $P_{60} < P_{100}$ (for $R_{60} > R_{100}$)

So, in this case, 100 W bulb consumes more power, hence it glows brighter.

---

14. Batteries are labelled with their emf. For example the dry cell which we use is labelled 1.5 V. Would it be appropriate to put a label on the batteries stating how much current they provide?

✎ Resistors, inductors, capacitors are the variable components of an electric circuit. But the current provided by the cell is depends on the external components of the circuit. Hence, the current varies in the circuit, although the circuit contains the constant emf. Hence, the value of current is not appropriate to specify in a cell.

---

15. Which resistance, internal or external, should be greater to draw the maximum current the series combination of the cells?

➥ In this condition, total current in the circuit,

$$I = \frac{nE}{R + nr}$$

To be maximum current, R > nr. Hence external resistance must be greater than the internal resistance of a cell.

---

16. A heater wire is heated to red hot but not the conducting wire to it from electric power supply. Why?

➥ The amount of heat produced in a conductor is determined from the formula of joules law of heating, $H = I^2Rt$, i.e. $H \propto R$.

i. In case of heater wire, R is very high.
so, the heat (H) is significantly very high

ii. In case of connecting wire, R tends to zero, so $H \rightarrow 0$. So, it can not red hot.

---

17. Can we measure emf by voltmeter? Explain.

➥ No, it is impossible. Emf is measured only when current drawn from the source is zero. But Voltmeter does not work if no current is drawn from the source. A potentiometer is used to measure the emf.

---

18. Which combination is set in household wiring, series or parallel?

➥ In household wiring, parallel combination is set into practice. As the parallel combination is set, every component of electric appliances like, bulb, fan etc in every room achieves the equal potential difference. Also, individual switches for individual appliance is possible.

---

19. Three bulbs 40 W, 60 W and 100 W are connected to 220 V mains. Which bulb will glow brightly, if they are connected in series?

➥ The resistance of filament bulb is designed in accordance with the power dissipation,

$P = \frac{V^2}{R}$. So, $R = \frac{V^2}{P}$

For the constant voltage supply, $R \propto \frac{1}{P}$. It means the bulb of greater power has lower resistance. So, $R_{100} < R_{60} < R_{40}$.

If these bulbs are connected in series, the power consumed by the bulb, $P = I^2R$.

i.e. $P \propto R$.

It means greater resistance consumes more power, So, 40 W bulb consumes maximum power and 100 W bulb consumes minimum power. Hence, 40 W bulb glows more brightly.

Exercises

Short-Answer Type Questions

1. The same current is passed through the line wire and filament of a bulb, the filament becomes hot but not the line wire, why?
2. How are the electric lamps connected in houses?
3. On an electric bulb, it is written 100 W and 220 V, what does it mean?
4. What do you mean by electric power?
5. When electric circuits are shorted light spark appears, why?

6. Which of the combination will you prefer if you have to draw large current if external resistance is negligible compared to internal resistance of a cell? Also, internal resistance is negligible compared to external resistance?
7. Define watt, kilowatt, kilowatt-hour and horsepower.
8. What do you mean by emf of a cell?
9. What do you mean by the terminal potential difference?
10. What do you mean by the internal resistance of a cell?
11. What are the factors on which the internal resistance of a cell depends?
12. What is the relation between "emf "and "terminal potential difference"?
13. Though the same current flows through the electric line wires and the bulb filament, yet only the filament glows, why?
14. In what respect does a heater wire differ from a fuse wire?

Long-Answer Type Questions

1. Derive an expression for the heat produced in the resistor when the current flows through it and hence state Joule's law of heating.
2. What is internal resistance of a cell? On what factors does it depend? Derive circuit formula for a cell in a circuit.
3. Define emf of a cell. Show that the voltage drop across a resistor connected in parallel with a cell is different from the emf of the cell.
4. What is a cell? Two identical cells, each of emf (E) and internal resistance(r) are connected in series to an external resistance(R). Find the expression for total current in the circuit.

Numerical Problems

1. The maximum power dissipated in a 10000 Ω resistor is 1 W. What is the maximum current?

   Ans: 0.01 A
2. Eight cells, each of emf 1.5 V, are connected in series. If a current of 3 A flows through an external resistance of 2 Ω, calculate the internal resistance of a cell.

   Ans: 0.25 Ω
3. A resistor with a 15.0 V potential difference across its ends develops thermal energy at a rate of 327 W. (a) What is its resistance? (b) What is the current in the resistor?

   Ans: (a) 0.688 Ω (b) 21.8 A
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   Ans: 2.6×10^6 J
7. The potential difference across the terminals of a battery is 8.4 V when there is a current of 1.50 A in the battery from the negative to the positive terminal. When the current is 3.50 A in the reverse direction, the potential difference becomes 9.4 V. (a) What is the internal resistance of the battery? (b) What is the emf of the battery?

   Ans: (a) 0.2 Ω (b) 8.7 V
8. An electric hot plate has two coils of manganin wire, each 20 m in length and 0.23 mm^2 cross sectional area. Show that it will be possible to arrange for three different rates of heating, and calculate the

wattage in each case when the heater is supplied from 200 V mains. The resistivity of manganin is 4.6×10^{-7} Ωm.

Ans: 1 kW, 0.5 kW, 2 kW

9. The wire of a fuse in an electric circuit melts when the current density increases to 600 A/cm^2. What should be the diameter of the wire so that it may limit the current to 0.4 A?

Ans: 0.29 mm

10. At 27.0°C, the resistance of a resistor is 83 Ω. What is the temperature of the resistor if the resistance is found to be 100 Ω and the temperature coefficient of the material of the resistor is 1.7×10^{-4} °C^{-1}?

Ans: 1232°C

11. An electric heating element to dissipate 400 W on 220 V mains is to be made from a wire 1 mm wide and 0.05 mm thick. Calculate the length of the wire required if the resistivity of material is 1.1×10^{-6} Ωm.

Ans: 5.5 m

12. A battery of emf 10 V and internal resistance 0.5 Ω is charged by a d.c. source of 100 V with the help of a series resistor of 10.0 Ω. Find the terminal voltage of the battery when it is being charged.

Ans: 14.3 V

13. Find the minimum number of cells required to produce an electric current of 1.5 A through a resistance of 30 Ω. Given that the emf of each cell is 1.5 V and internal resistance of each cell is 1.0 Ω.

Ans: 120

14. An electric heater is marked 1000 W, 220 V. How long will it take to heat 1 litre of water at 20°C to its boiling point?

Ans: 5.6 minutes

15. A fuse of lead wire has an area of cross-section 0.2 mm^2. On short circuiting, the current in the fuse wire reaches 30 A. How long the short circuiting, will the fuse begin to melt? For lead, specific heat capacity = 0.032 cal g^{-3} °C^{-1}. Melting point = 327°C, density = 11.34 g cm^{-3} and resistivity = 22×10^{-6} Ω cm. The initial temperature of wire is 20^{0}C, Neglect heat losses.

Ans: 0.945 sec

Multiple Choice Questions

1. 4 bulbs is rated at 100 V, 200 W, when the voltage drops by 2%, then change in power of bulb is:
   a. Increased by 2%
   b. Increased by 4%
   c. Decreased by 2%
   d. Decreased by 4%
2. The power of a bulb is 100 watt at 200 V. When the voltage is 110 V, power of the bulb is:
   a. 150 W
   b. 50 W
   c. 120 W
   d. 25 W
3. The power of two heater coils is P_1 and P_2. If they are connected in series, the resultant power is:
   a. $P_1 + P_2$
   b. $\dfrac{P_1P_2}{P_1 + P_2}$
   c. 0
   d. $\sqrt{P_1P_2}$
4. In order to light a 6 W, 6 V bulb at rated power a battery of emf 6 V and internal resistance 2 Ω is used. The bulb will light at power:
   a. 6 W
   b. 27/8 W
   c. 4 W
   d. 16/3 W
5. 5 cells each of emf 'E' and internal resistance 'r' are connected in series, by mistake one of the cell was connected wrongly; then equivalent emf and internal resistance will be:
   a. 5E, 3r
   b. 3E, 5r
   c. 3E, 3r
   d. 5E, 5r
6. In a dynamo, voltage is 6 V current 0.5 A. What is the power generated?
   a. 12
   b. 1.5
   c. 3
   d. 5

Answers

1. **(d)** 2. **(d)** 3. **(b)** 4. **(b)** 5. **(b)** 6 **(c)**

Hints to Challenging Problems

HINTS:1

The open-circuit terminal voltage of a battery is 12.6 V which is its emf.

$\therefore$ E = 12.6 V

R = 4 Ω

Terminal voltage (V) = 10.4 V

$r = \frac{E - V}{V} R$

HINTS: 2

Given,

emf of battery, E = 24.0 V

Current, I = 4.00 A

Terminal voltage, V = 21.2 V

a. $r = \frac{E - V}{V} R$

b. $R = \frac{V.r}{E - V}$

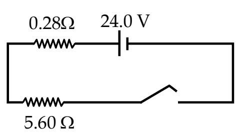
HINTS: 3

Given,

emf, E = 24 V

Internal resistance, r = 0.28 Ω

Resistance, R = 5.60 Ω



i. When switch is opened,

a. terminal voltage,

V = E – Ir

b. V = IR = 0 × R

c. V= E – IR = E – 0 × R = E

ii. When switched is closed

a. terminal voltage, V = ?

Firstly, find $I = \frac{\text{Total emf}}{\text{total resistance}} = \frac{E}{R + r}$

Then use I in V = E – IR

b. Voltage a cross resistor V = IR

c. No potential is dropped across the ideal switch, so V = 0

HINT: 4

Given,

(i) For open switch, voltmeter reading is equal to emf of the battery. So, E = 3.08 V

(ii) For closed switch, voltmeter reading is equal to terminal potential difference so,

Terminal potential difference, V = 2.97 V

Ammeter reading, I = 1.65 A

$r = \frac{E - V}{I}$ and $R = \frac{V}{I}$

HINT: 5

Given,

Potential difference (V) = 15.0 V

Electric power developed (P) = 327 W

(a) Resistance (R) = $\frac{V^2}{P}$

(b) $I = \frac{P}{V}$

HINT: 6

Given,

Current, I = 0.80 A

Potential difference, V = 650 V

Power, P = I V

HINT: 7

Given,

Potential, V = 9.0 V

Current, I = 0.13 A

Time taken, $t = 1\frac{1}{2}$ h = 1.5 × 3600 sec.

Energy consumed, E = $P \times t$ $[\because P = V I]$

HINT: 8

Given,

Voltage, V = 12 V

Current, I = 60 A

Time, t = 1 h = 3600 s

Total energy supplied, E = Pt = IVt

HINT: 9

Given,

Internal resistance (r) = 1.0 Ω

Emf, E = 12.0 V

Resistance (R) = 5.0 Ω

(a) Rate of conversion of energy in battery indicates power of battery. So,

First find, I from

$I = \frac{E}{R + r}$

Then, P = EI

b. Rate at dissipation of energy in the battery is the power dissipation. So, $P = I^2 r$

c. Rate of dissipation of energy in external resistance i.e., power dissipation in the external resistor

$P_{ext} = I^2 R$

HINT: 10

Given,

Potential difference across the battery, V = 8.4 V

Current, I = 1.50 A

Current, I_1 = – 3.50 A

Potential difference across the battery(V_1)=9.4 V

a. Internal resistance of the battery $(r) = \frac{E - V_1}{I_1}$

b. Emf of the battery $(E) = V + I\,r$

HINT: 11

Given,

Emf $(E_1) = 12.0$ V

Internal resistance $(r_1) = 1.0\ \Omega$

Emf $(E_2) = 8.0$ V

Internal resistance $(r_2) = 1.0\ \Omega$

Resistance $(R) = 8.0\ \Omega$

a. Current through 8 Ω, $I = \frac{E_1 - E_2}{r_1 + R + r_2}$

b. Power dissipation, $P = I^2 (r_1 + R + r_2)$

HINT: 12

Given,

$P_1 = 6$ W, $V_1 = 240$ V

$P_2 = ?$ $\quad V_2 = 1200$ V

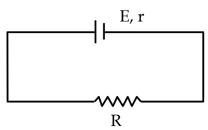
According to question, we can write

$I \propto V^4$

or $I = KV^4$

Now,

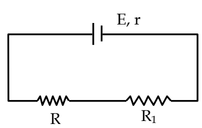
$\because\ P = VI = V.KV^4 = KV^5$



So, $P_1 = KV_1^5$, $P_2 = KV_2^5$

or $P_2 = \left(\frac{V_2}{V_1}\right)^5 .P_1$

HINT: 13



Given,

$E = 4$ V

$r = 2\ \Omega$

$R = 8\ \Omega$

To find, terminal potential difference, V

Use formula, $r = \frac{E - V}{V} . R$

After addition of resistance, use formula

$r = \frac{E - V}{V}(R + R_1)$

HINT: 14

Given,

i. $I_1 = 0.8$ A, $R_1 = 3\ \Omega$, $E = 24$ V

$I_2 = ?$

$R_2 = 6\ \Omega$

Use, $I_1 \times R_1 = I_2 \times R_2$

ii. Internal resistance of a cell, r = ?

Total current in the circuit,

$I = I_1 + I_2$

The equivalent resistance of R_1 and R_2 is

$R = \frac{R_1 \times R_2}{R_1 + R_2}$

Total resistance in the circuit = $R + R_3 + r$

We know that

Total current = $\frac{\text{net emf}}{\text{total resistance}}$

or $I = \frac{E}{10 + r}$

iii. terminal potential difference of the cell, V = ?

we know that

$V = E - Ir$

HINT: 15

Given,

$E_x = 6$ V $\quad E_y = 4$ V

$r_x = 2\ \Omega$ $\quad r_y = 8\ \Omega$

$R = 10\ \Omega$

Terminal potential difference across each battery $(V_x, V_y) = ?$

i. Firs case, the total current in the circuit is given by

$I = \frac{\text{net emf}}{\text{total resistance}} = \frac{E_x + E_y}{r_x + r_y + R}$

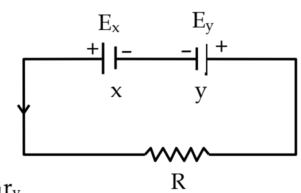
Use I in, $V_x = E_x - Ir_x$

Also, $V_y = E_y - Ir_y$

ii. In the second case, the terminal of y is reversed. The current in the circuit is given by

$I_1 = \frac{\text{net emf}}{\text{total resistance}}$

$= \frac{E_x - E_y}{r_x + r_y + R}$



Now, use I_1 in ,

$V'_x = E_x - I_1 r_x$

Also, $V'_y = E_y - I_1 r_y$

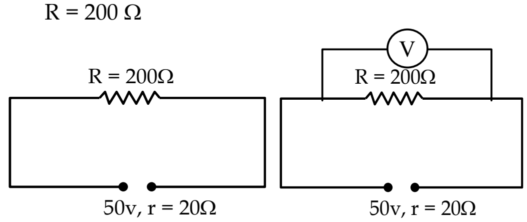
HINT: 16

Given,

E = 50 V,

r = 20 Ω,

R = 200 Ω



(i) $\qquad$ (ii)

Current in the circuit,

$I = \frac{E}{R + r}$

Potential difference across R before using voltmeter as in figure (i),

$V = IR$

When voltmeter is connected across R, then

$E = 50\ V,\ r = 20\ \Omega,\ R = 200\ \Omega,\ R_v = 1800\ \Omega$

The combined resistance of the resistor and voltmeter is given by

$$R' = \frac{R_v \times R}{R_v + R}$$

Then, $I = \frac{E}{R' + r}$

$\therefore$ Potential difference across R after using voltmeter as in figure (ii), $V' = IR'$

$\therefore$ % change potential difference

$$= \left|\frac{V' - V}{V}\right| \times 100$$

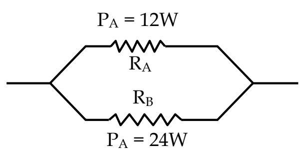
HINT: 17

Given,

$P_A = 12\ \Omega$

$P_B = 24\ \Omega$

$\frac{R_A}{R_B} = ?$



$\because$ A and B are connected in parallel, the voltage applied across them is the same.

$$\therefore \quad P_A = \frac{V^2}{R_A} \quad \ldots (i)$$

and

$$P_B = \frac{V^2}{R_B} \quad \ldots (ii)$$

$$\therefore \quad \frac{P_A}{P_B} = \frac{R_B}{R_A}$$

HINT: 18

Given,

$P = 10\ W$ $\qquad V = 20\ V$

$A = 10^{-7}\ m^2$ $\qquad \rho = 10^{-6}\ \Omega\ m$

$l = ?$

a. $P = \frac{V^2}{R}$

But,

$R = \rho \frac{l}{A}$, where A = cross sectional area.

So,

$$P = \frac{V^2}{\rho l} . A$$

or $l = \frac{V^2}{\rho \times P} . A$

b. $l_1 = ?$ when diameter was half

$d_1 = \frac{d}{2}$

We know that

$$A = \frac{\pi d^2}{4}$$

$$\therefore \quad A_1 = \frac{\pi d_1^2}{4} = \frac{\pi}{4} \times \left(\frac{d}{2}\right)^2 = \frac{\pi}{4} \times \frac{d^2}{4}$$

$$\therefore \quad A_1 = \left(\frac{\pi d^2}{4}\right)\frac{1}{4} = \frac{A}{4}$$

$$\therefore \quad P = \frac{V^2}{R} = \frac{V^2}{\rho l_1} . A_1$$

or $l_1 = \frac{V^2}{\rho \times P} A_1$

HINT: 19

Given,

$m_1 = 0.3\ kg,\ P_1 = 12.3\ W,\ \frac{\Delta\theta_1}{t} = \frac{1}{60}\ °C/s$

$m_2 = 0.4\ kg,\ P_2 = 19.2\ W,\ \frac{\Delta\theta_2}{t} = \frac{1.2}{60}\ °C/s$

Let S be the specific heat capacity of paraffin oil and its thermal capacity be C.

In the first case, we can write

$m_1 S\Delta\theta_1 + C\Delta\theta_1 = Q_1$

or $(m_1 S + C)\frac{\Delta\theta_1}{t} = \frac{Q_1}{t} = P_1$

or $0.3S + C = 738 \quad \ldots (i)$

In the second case, we can write

or $(m_2 S + C)\frac{\Delta\theta_2}{t} = \frac{Q_2}{t} = P_2$

or $(0.4S + C)\frac{1.2}{60} = 19.2$

or $0.4S + C = 960 \quad \ldots (ii)$

Solve (i) and (ii) and find S and C.

HINT: 20

Given,

$l = 20\ m$

$A = 0.23\ mm^2 = 0.23 \times 10^{-6}\ m^2$

$V = 200\ V,\ \rho = 4.4 \times 10^{-7}\ \Omega m.$

Resistance of the coil, $R = \rho \frac{l}{A}$

a. When only one coil is used, $P = \frac{V^2}{R}$

b. When both the coil are connected in series, total resistance of the circuit, $R = 40 + 40 = 80\ \Omega$

Wattage of the circuit,

$P = \frac{V^2}{R}$

c. When both coils are connected in parallel, the total resistance of the circuit R is given by

$$R = \frac{40 \times 40}{40 + 40}$$

Wattage of the circuit, $P = \frac{V^2}{R}$

HINT: 21

Given,

$P_1 = 1\text{ kW} = 10^3\text{ W}$

$V_1 = 250\text{ V}$

$P_2 = 1\text{ kW} = 10^3\text{ W}$

$V_2 = 200\text{ V}, \dfrac{\rho_1}{\rho_2} = 1.05$

% change in resistance = ?

i. First case, $R_1 = \dfrac{V_1^2}{P_1}$ and $R_2 = \dfrac{V_2^2}{P_2}$

$\therefore$ % change in resistance $= \dfrac{R_1 - R_2}{R_1} \times 100\%$

Also, we have

$$R = \rho \frac{l}{A}$$

or $l = \dfrac{RA}{\rho}$

$\therefore\ l_1 = \dfrac{R_1 A}{\rho_1}$

$\therefore\ l_2 = \dfrac{R_2 A}{\rho_2}$

Thus, % change in length $= \dfrac{l_1 - l_2}{l_1} \times 100\ \%$

❖ ❖ ❖

ELECTRIC CIRCUITS

12

CHAPTER

12.1 Introduction

In simple circuits consisting of a single source of emf, the relationship between the current and the voltage drop across each resistor could be calculated by using Ohm's law. However, for any circuit consisting of more than one source of emf, the situation becomes quite complicated. For such circuits, finding the current flowing through each component and hence the voltage drop across each component is very much important to study their relationships.

12.2 Kirchhoff's Laws

Kirchhoff has given two rules commonly known as Kirchhoff's laws for the calculation of current and voltage across any electrical component which are discussed below.

i. Kirchhoff's first law or Junction law or current law

A junction is a common point at which all the conductors are joined. Kirchhoff's first law, states that , "in any network algebraic sum of currents at any junction is zero".

i.e. $\Sigma I = 0$ **... (12.1)**

If positive sign is assigned for the currents arriving and negative sign is assigned for the currents leaving, the law may also be stated as "sum of currents entering the junction must be equal to the sum of currents leaving the junction".

i.e. $\Sigma I_{in} = \Sigma I_{out}$ **... (12.2)**

Let us consider a network of conductors as shown in Fig. 12.1 in which the current points along the direction of arrow and O is the junction point.

Then, from Kirchhoff's first law at junction O,

Sum of current in = Sum of current out

$$\Sigma I_{in} = \Sigma I_{out} \quad \text{... (12.3)}$$

Here, $\Sigma I_{in} = I_1 + I_2 + I_3 + I_4$.

and $\Sigma I_{out} = I_5 + I_6$

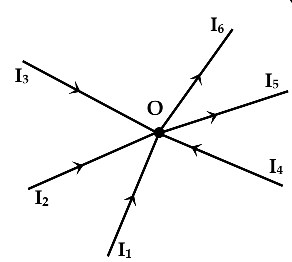


Fig. 12.1: A network of conductors shown with currents entering and leaving the junction O

From equation (12.3), we can write,

$$\therefore \quad I_1 + I_2 + I_3 + I_4 = I_5 + I_6$$

This law is based on the principle of conservation of charge. According to this law, when steady current flows in a circuit, the electric charges don't get accumulated or destroyed at any point. This means, if billions of charges particles enter a junction in 1 second, then, same number of charges must come out of it in 1 second.

ii. Kirchoff's second law

This law also known as mesh or loop law basically deals with p.ds and emfs. Before discussing this law, let us know what mesh and loop refer to? In a complicated circuit, a mesh refers to a closed path with no other paths inside it where as loop refers to a closed path that contain two or more meshes in it.

For examples: In the given circuit in Fig. 12.2, ABEFA and BCDEB are the meshes where as ACDFA is a loop as it contains two meshes ABEFA and BCDEB in it.

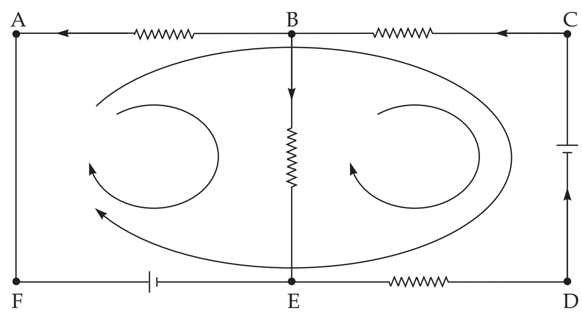


Fig. 12.2: Mesh and loop

The mesh or loop here is referred to mean a closed path of a circuit where a junction point comes only once in the consideration.

Kirchoff's second law, states that, "*the directed sum of emfs and p.ds round a closed loop is zero*".

i.e. $\Sigma E + \Sigma IR = 0$, When proper direction of emfs and p.ds are chosen.

This can also be stated as "around any closed circuit or loop the algebraic sum of emf (ΣE) is equal to the sum of p.ds ($\Sigma V = \Sigma IR$) around that closed loop.

$$\textbf{i.e. } \Sigma E = \Sigma IR \qquad \textbf{... (12.4)}$$

The sign conventions in direction adopted for applying Kirchoff's second law are discussed below.

1. The direction emfs are taken positive if we pass from negative terminal to positive terminal of the source. In the source of emf, a charge particle gains electric potential energy as it moves from negative terminal to positive terminal. So, in this direction of motion i.e. from negative terminal to positive terminal, the potential difference is positive. So, this sign convention is justified.

2. The p.ds across any component are taken negative, if we move along the direction of current and positive, if we move opposite to the direction of current. The charge particles move from higher potential region to lower potential region. This means current flows from higher potential region to lower potential region. So, potential difference across a conductor in the direction of current is negative. For the same reason, the potential difference across a conductor in the direction opposite to current must be negative. So, this sign convention is also justified.

Illustration:

Let us consider an electric circuit shown in Fig. 12.3. It consists of three closed paths ABCDFA, ABEFA and BCDEB. Consider a loop, BCDEB first.

A reference point B is chosen and from this point let us move towards the direction of I_1 along BC.

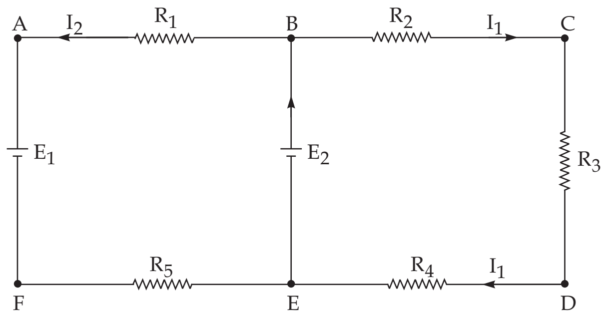


Fig. 12.3: Applying Kirchoff's rule in a circuit

Since, we are along the direction of current,

Potential difference across $R_2 = -I_1R_2$

Potential difference across $R_3 = -I_1R_3$

Potential difference across $R_4 = -I_1R_4$

Now, moving along EB to reach B, we move from negative to positive terminal of sources E_2, so, E_2 is positive. Thus, Kirchoff's second law for the loop can be written as,

$$-I_1R_2 - I_1R_3 - I_1R_4 + E_2 = 0$$

$$\therefore \quad \mathbf{E_2 = I_1R_2 + I_1R_3 + I_1R_4} \qquad \text{... (12.5)}$$

Consider another loop AFEBA and choosing A as reference point let us move along AF to reach at A again.

Since, we are moving from positive to negative terminal of source E_1, it must be taken negative.

Again, along FE, we are in the direction of current. So, potential difference across $R_5 = -I_2R_5$

For the reason stated previously, E_2 is positive and p.d across $R_1 = -I_2R_1$.

Thus, for this loop, Kirchoff's second law gives,

$$-E_1 - I_2R_5 + E_2 - I_1R_1 = 0$$

$$\mathbf{E_2 - E_1 = I_2R_5 + I_1R_1} \qquad \text{.. (12.6)}$$

Finally, chosing reference point A again for closed loop ABCDEFA and moving along AC to reach at A again, we get,

P.d across $R_1 = +I_2R_1$ (Here, we move opposite to direction of current)

P.d across $R_2 = -I_1R_2$ (we move along the direction of current)

P.d across $R_3 = -I_1R_3$ (we move along the direction of current)

P.d across $R_4 = -I_1R_4$ (we move along the direction of current)

P.d across $R_5 = +I_2R_5$ (we move opposite to direction of current)

The emf E_1 is positive.

So, from Kirchoff's second law,

$$I_2R_1 - I_1R_2 - I_1R_3 - I_1R_4 + I_2R_5 + E_1 = 0$$

$$\mathbf{E_1 = -I_2R_1 + I_1R_2 + I_1R_3 + I_1R_4 - I_2R_5} \qquad \text{... (12.7)}$$

This is how we apply sign conventions in Kirchoff's second law.

Sometimes, the direction of currents may not be shown in the circuit. In such case, it is always wise to choose the direction of current originating from the positive terminal of larger emf source. If the direction you assumed is opposite, you will get negative value of current but the magnitude must not alter. You may alter the polarity of battery in such case.

Kirchhoff's second law and conservation of energy

When a charge, moves around a circuit, it gains energy as it moves through each source of emf and loses energy as it passes through each potential difference. So, if a charge moves all way round the circuit, so that it ends up where it started, it must have same energy at the end as it initially had. Otherwise, we would be able to create energy from nothing simply by moving charges around circuits. Thus, Kirchhoff's second law is a direct consequence of principles of conservation of energy.

So, energy gained in passing through sources of e.m.f. = energy lost in passing through components with potential differences

Remember that an emf in volt is simply the energy gained per one coulomb of charge as it passes through source, and a potential difference (p.d.) is the energy lost per one coulomb as it passes through a component.

1 volt = 1 joule per coulomb

Thus, Kirchhoff's second law can be thought of as:

So, energy gained per coulomb around loop = energy lost per coulomb around loop

12.3 Wheat Stone Bridge

A bridge network that operates on the principle of comparison by a null deflection method is wheat stone bridge. A Wheat stone bridge consists of a network of four resistances out of which one is unknown. The value of unknown resistance can be determined accurately in terms of three known resistances. This method was devised by British physicist Prof. Charles, F. Wheatstone.

A Wheat stone bridge consists of four resistances P, Q, R and X which form the four arms of rectangle as shown in Fig. 12.4.

The resistances P and Q are the ratio arms and R is standard arms. X is the resistor whose resistance is to be determined. A source of emf E is connected between points A and C and a sensitive galvanometer is connected between B and D as shown in Fig. 12.4.

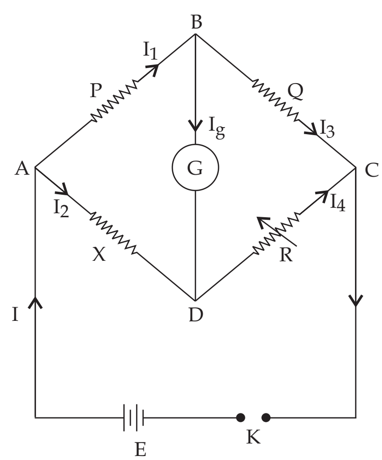


Fig. 12.4: Wheatstone bridge

When the resistance of R is so adjusted that, there is no deflection of the galvanometer, the bridge is said to be balanced and there is no potential difference across the galvanometer. Under such condition,

$$\frac{P}{Q} = \frac{X}{R} \quad \text{... (12.8)}$$

$$X = \frac{P}{Q} \times R \quad \text{... (12.9)}$$

Thus, knowing the ratio $\frac{P}{Q}$ and value of R, we can calculate X from equation (12.9).

The wheat stone bridge is said to be sensitive if the galvanometer shows large variation for small change in resistance of R and it is most sensitive when the resistances of four arms are of the same order. In balanced condition, the effective resistance (R_w) offered by the bridge is,

$$R_w = (P + Q) \,||\, (R + X)$$

$$R_w = \frac{(P + Q)\,(R + X)}{P + Q + R + X}$$

Under balanced condition the position of galvanometer and battery can be interchanged without affecting the balance of bridge. So, arms BD and AC are also called as conjugate arms.

Derivations of balanced condition from Kirchhoff's law

Let the direction of currents in various arms be as shown in Fig. 12.4.

Then, from Kirchhoff's second law in loop ABDA

$$\mathbf{-I_1P - I_gG + I_2X = 0} \qquad \text{... (12.10)}$$

Also, from Kirchhoff's second law in loop BCDB,

$$\mathbf{-I_3Q + I_4R + I_gG = 0} \qquad \text{... (12.11)}$$

Again, from Kirchhoff's first law at junctions B and D,

$$\mathbf{I_1 = I_3 + I_g} \qquad \text{... (12.12)}$$

$$\mathbf{I_4 = I_2 + I_g} \qquad \text{... (12.13)}$$

When the galvanometer shows null deflection, $I_g = 0$.

So, equations (12.12) and (12.13) yield,

$$\mathbf{I_1 = I_3} \qquad \text{... (12.14)}$$

$$\mathbf{I_4 = I_2} \qquad \text{... (12.15)}$$

And, also from equations (12.10) and (12.11)

$$\mathbf{I_1P = I_2X} \qquad \text{... (12.16)}$$

$$\mathbf{I_3Q = I_4R} \qquad \text{... (12.17)}$$

Dividing (12.16) by equations (12.17) and using values from equations (12.14) and (12.15), we get,

$$\frac{P}{Q} = \frac{X}{R}$$

This is the required balance condition of Wheat stone bridge.

12.4 Meter Bridge

It is an electrical instrument which works on the principle of Wheat stone bridge and is used for measuring the unknown resistance.

It consists of a meter long wire AC of uniform cross-section and composition usually made of an alloy such as constantan or magnanin. The uniformity in composition and cross-section ensures that, resistance per unit length remains constant throughout the wire. The wire is stretched over a meter scale fixed on a flat wooden plank by two L-shaped copper strip of low resistance as shown in Fig. 12.5. Another flat copper strip is fixed a little above the meter scale providing two gaps, where a known resistance (R) and unknown resistance X are kept. The known resistance is usually a resistance box A jockey (J) which can slide over the wire AC and a galvanometer (G) is connected from the midpoint D of the flat copper strip by the help of a wire. When the jockey is slided over the resistance wire, the length of the resistance wire left to contact point serves as a resistor P and that to the right serves as another resistor Q. A source of emf E is connected across AC as shown in Fig. 12.5

with the help of four ways key (K). The overall construction now serves as a wheat stone bridge for null deflection in G with P and Q as standard arms R known resistance and X unknown resistance.

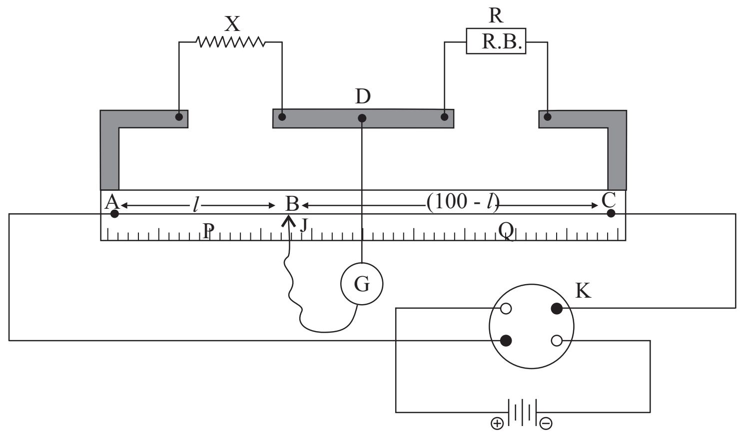


Fig. 12.5: Meter Bridge

Working: When a suitable known resistance is plucked out of a resistance box, the jockey (J) is slided over wire AC until there is no deflection in the galvanometer G. This condition of zero deflection is Galvanometer is called balanced condition of the bridge and the point B on the wire AC is called balance point.

Let l cm be the length of resistance wire (AB = P) then length of resistance wire (BC = Q) is (100 – l) cm.

Since resistance is directly proportional to length, we can write,

$$\mathbf{P \propto l} \qquad \text{... (12.18)}$$

$$\textbf{and } \mathbf{Q \propto (100 - l)} \qquad \text{... (12.19)}$$

From equations (12.18) and (12.19) we can write,

$$\frac{P}{Q} = \frac{l}{(100 - l)} \qquad \text{... (12.20)}$$

Also, from balanced condition of wheat stone bridge,

$$\frac{P}{Q} = \frac{X}{R} \qquad \text{... (12.21)}$$

From equations (12.20) and (12.21), we get,

$$\frac{X}{R} = \frac{l}{(100 - l)} \qquad \text{... (12.22)}$$

$$\therefore \quad X = \frac{l}{(100 - l)} R$$

Thus, knowing the values of R and l, we can calculate the value of unknown resistance X.

Further, if r be the radius of the wire X and ρ be its resistivity, then,

$$X = \frac{\rho l}{A}$$

$$\therefore \quad \rho = \frac{AX}{l} \qquad \text{... (12.23)}$$

Here, $A = \frac{\pi d^2}{4}$ can be calculated by measuring the diameter (d) of wire with the help of micrometer.

Thus, the resistivity of X can be easily determined by using equation (12.23).

In the two gaps above, if both the resistors R and X are unknown, then equation (12.22) provides a suitable comparison of their resistance.

Thus, we can say that, the meter bridge measures or compares the resistance of unknown wires in terms of length.

12.5 Post office box (P.O. Box)

Post office box is an instrument that works on the Wheat stone bridge principle. It is so called because, this instrument was used to find the resistance of broken telegraph wires in the post office.

P.O Box is a compact form of Wheat stone bridge more specially a resistance box in which the resistances with in the box serve as three arms of Wheat stone bridge and value of unknown resistance can be found by inserting it in the fourth arm. This box consists of two ratio arms AB and BC which serve on P and Q of the wheat stone bridge and a third long arm CD which serves as the known resistance. An unknown resistance is inserted between A and D whose value is to be determined. Each ratio arm consists of three coils of resistance marked 10 Ω, 100 Ω and 1000 Ω inside the box where as the known resistance arm has the coils resistances ranging from 1 to 5000 Ω inside the box. A source of emf is connected between A and C' which is connected to C when the key K_1 is pressed. A galvanometer is connected to D and B' is connected to B by a key K_2 (Galvanometer key) while noting the readings, battery key (K_1) is pressed first and then K_2 is pressed in order to avoid inductive effects in the galvanometer coil.

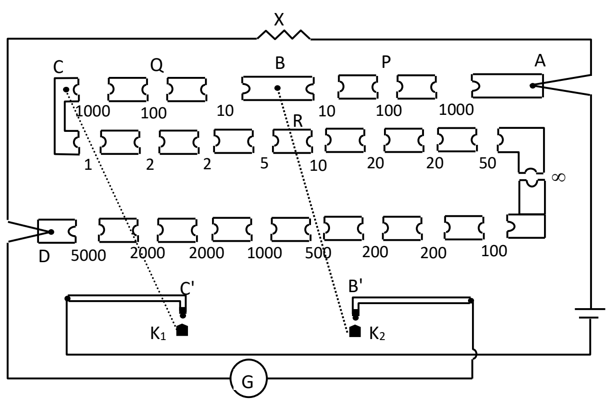


Fig. 12.6: Circuit arrangement for determining unknown resistance using P.O.Box

Procedure

Let an unknown resistance X be connected between A and D.

At first, 10Ω resistors are plucked out of each ratio arms so that, the ratio $\frac{P}{Q} = \frac{1}{1}$

Now, the known resistors from arm CD are plucked out so as to observe the deflection in the galvanometer. By changing the value of known resistance gradually, the deflection in the galvanometer changes it direction. If the galvanometer shows left (say) deflection for 3 Ω and rigal deflection for 4Ω resistor taken out of CD, then, we say that the resistance lies between 3 Ω and 4 Ω.

In second step, the ratio of arms P and Q are maintained at, P:Q = 10:100. In this case the value of X now must increase 10 times and hence. We check its resistance directly between 30 Ω and 40 Ω by plucking suitable resistors. In this case also, we try to find opposite deflections of galvanometer for two consecutive value of R. If, the galvanometer shows opposite deflection for 36 Ω and 37 Ω then we say, the resistance lies between 3.6 Ω to 3.7 Ω. Sometimes, we may get a null deflection for particular value say 36 Ω. In such case we can directly say the value of unknown resistance is 3.6 Ω.

Finally the ratio P:Q is maintained at 10:1000 by plucking 1000 Ω key from Q. The same procedure is repeated as this time by plucking the resistance key between 360 and 370Ω. If a null deflection is obtained for certain value of R, say 361 Ω then we say the resistance of the wire is 361Ω. If the null deflection is not obtained and still opposite deflections exist, then we adopt following method to calculate exact value of X. Suppose, for, R = 361 Ω, galvanometer deflects by 2 divisions left and for R = 362 Ω galvanometer deflects by 1 division right.

Total deflection of galvanometer for 1 Ω change in resistance = 2 + 1 = 3 divisions.

Here, 3 division corresponds to 1 Ω resistance

or, 1 division corresponds to $\frac{1}{3}$ Ω resistance

or, 2 division corresponds to $\frac{2}{3}$ Ω resistance.

Therefore, the value of resistance R required for null deflection = $\left(361 + \frac{2}{3}\right)\Omega = 361.66\ \Omega$.

Thus, the value of unknown resistance from wheat stone bridge principle is, $\frac{P}{Q} = \frac{X}{R}$.

$$\text{or,}\quad X = \frac{P}{Q} \times R$$

$$\text{or,}\quad X = \frac{10}{1000} \times 361.66$$

$$\therefore\quad X = 3.616\ \Omega$$

In this way, we can determined, the value unknown resistance X using P.O. Box.

12.6 Potentiometer

A potentiometer is an arrangement which measures the emf of a cell or potential difference accurately. The voltmeter used for such purpose draws some current from the cell and hence some of the volt is lost in its internal resistance. So, it can't measure the emf accurately. However, a potentiometer measures emf accurately without drawing any current from the cell as this method of measurement follows null deflection. When the galvanometer shows null deflection, there is no current in it, equivalently no current is drawn from cell. This implies the infinite resistance of potentiometer wire. Thus a potentiometer is also called as ideal voltmeter.

Principle

The basic principle of potentiometer is that, the voltage drop across the length of the conductor having uniform cross-section and composition due to a steady current is directly proportional to its length. If V be the p.d across a conductor of length l, then,

$$V \propto l$$

$$\text{or,}\quad V = K.l$$

$$\text{or,} \quad K = \frac{V}{l} \qquad \text{... (12.24)}$$

Here, K is a constant known as potential gradient and is simply the voltage drop per unit length of the conductor.

Construction

A potentiometer consists of a long resistance wire of length 1 m to 10 m stretched horizontally between two points A and B alongside a meterscale fitted over a wooden plank. A steady current is maintained in the wire by a cell connected across A and B which is known as standard cell. As a result there is p.d between two points on the wire which is proportional to their distance from one another.

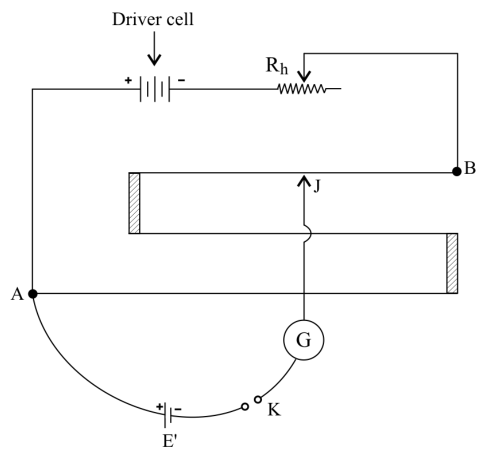


Fig. 12.7: Potentiometer

This arrangement can be used for various purpose which are discussed below.

12.7 Comparison of Emfs of two cell

The experimental circuit for the comparision of emfs of two cells is an shown in Fig. 12.8. The figure shows a driving cell E connected to points A and B. The cells whose emfs are to be compared are called the primary cells and their emfs must be less than that of driver cell. The positive terminals of E_1 and E_2 are respectively connected to point A whereas their negative terminals are connected to a galvanometer through key K_1 and K_2 respectively. A jockey which can slide over the wire is also connected to the galvanometer. A steady current I is maintained in the potentiometer wire by adjusting the rheostate (R_h).

At first, key K_1 is closed so that E_1 is introduced in the circuit. The jockey is now slided over AB to obtain a null deflection of galvanometer. Let, the corresponding length of AB for null deflection of

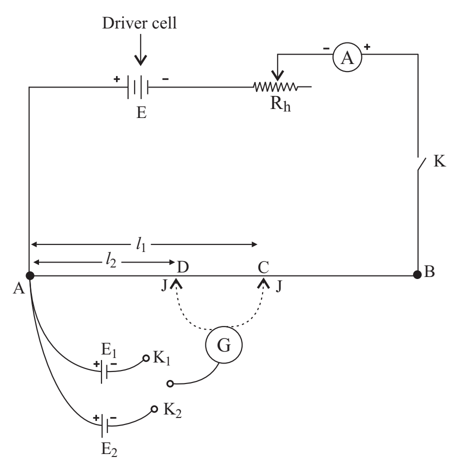


Fig. 12.8: Potentiometer arrangement for comparison of emf of two cells

galvanometer be l_1, which is called the balancing length, then,

$$\mathbf{E_1 = K}\boldsymbol{l_1} \qquad \text{... (12.25)}$$

Similarly, the key K_2 is closed and K_1 is open. Now, the cell E_2 gets introduced in the circuit. The jockey is again slided over AB to obtain null deflection of galvanometer for this case. If l_2 be the corresponding length of AB in this case, then,

$$\mathbf{E_2 = K}\boldsymbol{l_2} \qquad \text{... (12.26)}$$

From equations (12.25) and (12.26), we get,

$$\frac{E_1}{E_2} = \frac{l_1}{l_2} \qquad \text{... (12.27)}$$

Thus, measuring the l_1 and l_2 the ratio $\frac{E_1}{E_2}$ can be calculated.

Also, from equation (12.27),

$$E_1 = \frac{l_1}{l_2} \cdot E_2 \qquad \text{... (12.28)}$$

If the emf of one cell is known, the emf of another cell can determined from equation (12.28).

12.8 Measurement of internal Resistance of the cell

The circuit arrangement for the measurement of internal resistance of cell is as shown in Fig. 12.9.

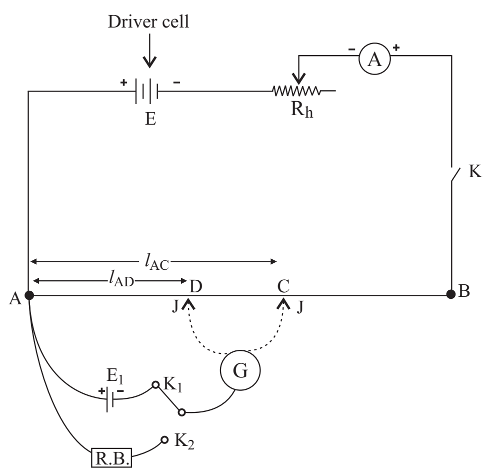


Fig.: 12.9: Potentiometer arrangement for measurement of internal resistance of a cell

The cell of emf E_1, whose internal resistance r is to be determined is introduced in the circuit with its positive terminal at A and the negative terminal is connected to a galvanometer provided with a jockey. A resistance box (R.B) is connected across the cell through a key K_2.

At first the key K_2 is opened and jockey is slided over AB to get a balance point C(say). Let the balancing length of AB be l_{AC}.

In this case, the p.d. across AC (= V_{AC}) is equal to emf of cell. So,

$$E_1 = V_{AC} = K l_{AC} \qquad \text{... (12.29)}$$

Now, the key K_2 is closed so that R. B is introduced in the circuit. The jockey is again slided over AB to get a balance point say Δ for a particular resistance R in R_B. Let, the balancing length be l_{AD}. Then terminal p.d

$$V = K\,l_{AD}. \quad \text{... (12.30)}$$

There is no current from lower circuit to the potentiometer at balance point. However, current flows in the lower circuit through R.B driven by cell E_1. So, V is the terminal p.d.

From Ohm's law in lower circuit,

$$E_1 = I\,(R + r) \text{ and } V = IR.$$

$$\textbf{Here, } \frac{E_1}{V} = \frac{(R + r)}{R} \quad \text{... (12.31)}$$

From equations (12.29), (12.30) and (12.31),

$$\frac{K l_{AC}}{K l_{AD}} = \frac{(R + r)}{R}$$

$$\text{or,} \quad \frac{l_{AC}}{l_{AD}} = 1 + \frac{r}{R}$$

$$\text{or,} \quad \frac{r}{R} = \frac{l_{AC}}{l_{AD}} - 1$$

$$\therefore \quad r = R\,\frac{l_{AC} - l_{AD}}{l_{AD}} \quad \text{... (12.32)}$$

Knowing the values of R, l_{AC} and l_{AD} we can calculate the value of r. One must note that, ($l_{AC} > l_{AD}$).

Tips for MCQs

1. **Kirchhoff's first law:**

   i. Sum of incoming currents – sum of out going currents = 0, i.e. $\Sigma I = 0$

   ii. If the current reaching to the junction is taken positive, the current leaving from the junction is taken negative.

   iii. It is based on conservation of charge.

2. **Kirchoff's second law:**

   i. $\Sigma E - \Sigma IR = 0$

   ii. It is based on conservation of energy

   iii. Guide-lines for sign convention in the second law.

   a. Choose any closed loop in the network and designate a direction (clockwise or counter clockwise) to traverse the loop.

   b. Go around the loop in the designated direction, adding emfs and potential differences.

   c. A potential drop across a resistance is considered positive or negative depending on whether one moves in the same sense as the current or in the opposite sense.

3. **Wheat stone bridge circuit.**

   i. When current through the galvanometer is zero, the potential difference at two connecting junction (B and D) is zero.

   ii. Wheat stone's bridge method is not suitable for the measurement of very low and very high resistances.

   iii. In balanced conditions, $\frac{P}{Q} = \frac{X}{R}$

   iv. If the positions of galvanometer and battery are interchanged, bridge remains same.

   v. Meter bridge, post office box are practical applications of wheat stone bridge.

4. Meterbridge

i. The meterbridge wire is generally made of manganin or constantan or nicrome because these materials have low temperature coefficient of resistance and high resistivity.

ii. If R_1 and R_2 be the resistance of same order, and R_1 is kept at left gap and R_2 is kept at right gap, in balancing condition.

$$\frac{R_1}{R_2} = \frac{l_1}{l_2}$$

iii. The resistance wire of **1 m long** is used in this device, and **wheat stone bridge principle** is applied in its working, so it is named meterbridge.

5. Potentiometer

i. Potentiometer is an ideal voltmeter. It measures the emf of a cell accurately and does not draw any current from the cell.

ii. The working principle: The potential drop at any section of the wire is directly proportional to the length of wire of that section, provided that the wire is of uniform cross sectional area and constant current is passing.

i.e. $V \propto l$ (for I, A and ρ constant)

iii. The sensitivity of potentiometer can be increased by increasing the length of potentiometer wire. In laboratory potentiometer, total length of is 10 m.

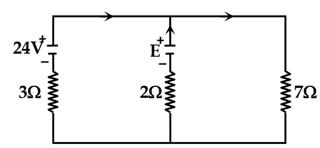
iv. As the potentiometer length is increased, its potential gradient decreases.

v. Comparison of emf: $\frac{E_1}{E_2} = \frac{l_1}{l_2}$

vi. Determination of Internal resistance: $r = \left(\frac{E}{V} - 1\right) R = \left(\frac{l_1}{l_2} - 1\right) R$

Worked Out Problems

1. What must be the emf E in the circuit so that the current flowing through the 7 Ω resistor is 1.80 A? Each emf source has negligible internal resistance.



SOLUTION

Given,

Current (I) = 1.80 A (in R = 7 Ω)

Let I_1 and I_2 be the current passing through path AB and EB respectively.

From closed circuit, ABCDEFA

or, $24 = 1.8 \times 7 + 3 \times I_1$

or, $3I_1 = 24 - 12.6$

or, $I_1 = \frac{11.4}{3} = 3.8$ A

Also, Total current, $I_1 + I_2 = I$

or, $3.8 + I_2 = 1.8$

or, $I_2 = -2$ A

Now, from closed circuit, BCDEB,

or, $E = 2 \times (-2) + 1.8 \times 7$

or, $E = -4 + 12.6$

or, $E = 8.6$ V

∴ The value of E is 8.6 V.

2. [HSEB 2059] The emf of a battery A is balanced by a length of 75 cm on a potentiometer wire. The emf of a standard cell, 1.02 V, is balanced by a length of 50 cm. What is the emf of A? Calculate the new balance length if A has an internal resistance of 2 Ω and a resistor of 8 Ω is joined to its terminals.

SOLUTION

Given,

emf of a cell A (E_A) = ?

Balancing length for cell A (l_A) = 75.0 cm emf of a standard cell (E) = 1.02 V

Balancing length of standard cell (l) = 50 cm

According to the principle of potentiometer, we have

$$\frac{E_A}{E} = \frac{l_A}{l}$$

or $$E_A = \frac{l_A}{l} \times E = \frac{75}{50} \times 1.02 = 1.53 \text{ V}$$

In the second case, if l_1 be the balancing length, we can write

$$r = \frac{l_A - l_1}{l_1} \times R$$

or $$2 = \frac{75 - l_1}{l_1} \times 8$$

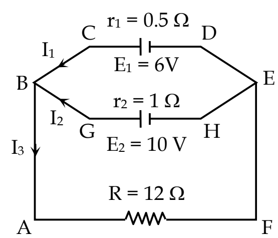
or $$\frac{1}{4} = \frac{75}{l_1} - 1$$

or $$\frac{1}{4} + 1 = \frac{75}{l_1}$$

or $$l_1 = \frac{75 \times 4}{5} = 60 \text{ cm}$$

3. [HSEB 2072] A battery of 6 V and internal resistance 0.5 Ω is joined in parallel with another of 10 V and internal resistance 1 Ω. The combination sends a current through an external resistance of 12 Ω. Find the current through each battery.

SOLUTION



Given,

Emf of cells (E_1) = 6V

(E_2) = 10 V

Internal resistances (r_1) = 0.5 Ω

(r_2) = 1 Ω

External resistances (R) = 12 Ω

Currents; I_1 = ?, I_2 = ?

From Kirchhoff's voltage law in a closed loop ABCDEFA.

$E_1 = I_1r_1 + I_3R$

$= I_1r_1 + (I_1 + I_2) R = I_1 \times 0.5 + 12I_1 + 12I_2$

$6 = 12.5I_1 + 12I_2$ …(i)

Again in a closed loop ABGHEFA,

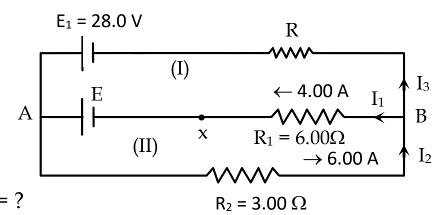
$E_2 = I_2r_2 + I_3R = I_2 \times 1 + (I_1 + I_2) \times 12$

$10 = 12I_1 + 13I_2$ …(ii)

Solving (i) and (ii) we get, I_1 = –2.27 A and I_2 = 2.86 A

The negative value of current depicts that current flows from positive to negative terminal into the cell E_1.

4. In the circuit shown in figure, find (a) the current in resistor *R*; (b) the resistance *R*; (c) the unknown emf *E*. (d) If the circuit is broken at point x, what is the current in resistor *R*?



SOLUTION

Given,

E_1 = 28 V

R_1 = 6 Ω, I_1 = 4A

R_2 = 3 Ω, I_2 = 6A

a. Current through R (I_3) = ?

Using Kirchoff's current law at junction B, we have,

$I_2 = I_1 + I_3$

or $I_3 = I_2 - I_1$

$= 6 - 4$

$= 2A$

b. E = ?

Using Kirchhoff's voltage law in loop (II), we get

$E = I_1R_1 + I_2R_2$

$= 4 \times 6 + 6 \times 3$

∴ E = 42 V

c. R = ?

Using Kirchhoff's voltage law in loop (I), we get

$E_1 - E = -I_1R_1 + I_3R$

or $28 - 42 = -4 \times 6 + 2 \times R$

or $-14 = -24 + 2R$ or $R = 5\ \Omega$

d. If circuit is broken at x,
Current through R (I_4) = ?
Using Kirchoff's voltage law in above complete circuit, we can write,

$$E_1 = I_4 \times R_2 + I_4 \times R$$

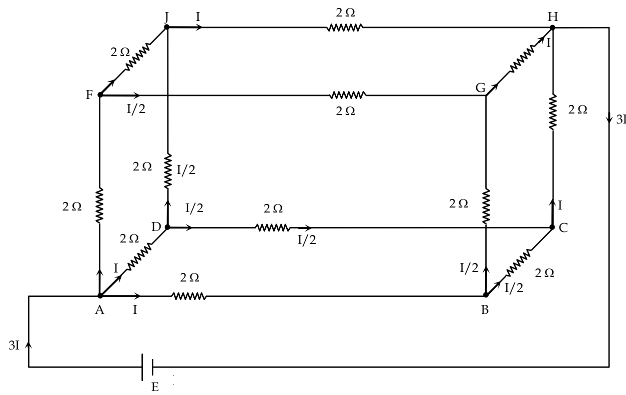
$$\text{or } I_4 = \frac{E_1}{R_2 + R} = \frac{28}{3+5} = \frac{28}{8}$$

$$\therefore\ I_4 = 3.5\text{A}$$

5. A battery of emf 24 V and negligible internal resistance is connected across the diagonally opposite corners of a cubical network consisting of 12 resistors each of resistance 2Ω. Determine the equivalent resistance of the network and the current through each edge of the cube.

SOLUTION

The necessary figure is shown below.



The given circuit is not easily reducible, so it can be solved by the method of "network symmetry". Here, the paths AB, AD and AF are placed symmetrically in the network. So, current through each of above path is same. If we take total current 3I, the current in each three paths AB, AD and AF is I. Making use of 'symmetry considerations' and Kirchhoff's first law, we can write current in terms of I in all edge of the cube.

Applying Kirchhoff's second law to the loop ABCHEA, we get

$$E - I \times 2 - \frac{I}{2} \times 2 - I \times 2 = 0$$

$$E - 5I = 0$$

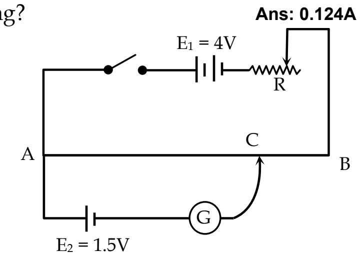
$$I = \frac{E}{5} = \frac{24}{5} = 4.8 \text{ A}$$

And the equivalent resistance $(R) = \dfrac{E}{3I} = \dfrac{24}{3 \times 4.8} = 1.67\ \Omega$

Challenging Problems

1. **[ALP]** A 1.0 Ω resistor is in series with an ammeter M in a circuit. The p.d. across the resistor is balanced by a length of 60.0 cm on a potentiometer wire. A standard cell of emf 1.02 V is balanced by a length of 50 cm. If M reads 1.10 A, what is the error in the reading?
Ans: 0.124A

2. **[ALP]** A simple potentiometer circuit is set up as in figure using a uniform wire AB, 1.0 m long, which has a resistance of 2.0 Ω. The resistance of the 4 V battery is negligible. If the variable resistor R were given a value of 2.4 Ω , what would be the length AC for zero galvanometer deflection?



If R were made 1.0 Ω and the 1.5 V cell and galvanometer were replaced by a voltmeter of resistance 20 Ω, what would be the reading of the voltmeter if the contact C were placed at the midpoint of AB?

Ans: 0.825 m and 1.29 V

3. **[ALP]** A potentiometer consists of a fixed resistance of 2030 Ω in series with a slide wire of resistance 4 Ω meter^{-1}. When a constant current flows in the potentiometer circuit a balance is obtained when (a) a Weston cell of emf 1.018 V is connected across the fixed resistance and 150 cm of the slide wire and also when (b) a thermocouple is connected across 125 cm of the slide wire only. Find the current in the potentiometer circuit and the emf of the thermocouple.

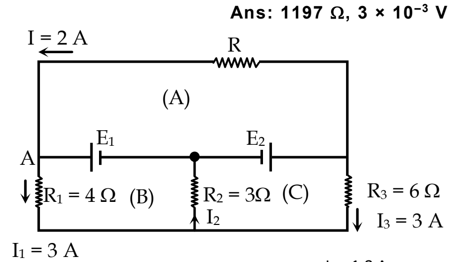
Ans: 0.5×10^{-3} A, 2.5×10^{-3} V

4. **[ALP]** The driving cell of a potentiometer has an emf 2 V and negligible internal resistance. The potentiometer wire has a resistance of 3 Ω. Calculate the resistance needed in series with the wire of p.d. of 5 mV is required across the whole wire. The wire is 100 cm long and a balance length of 60 cm is obtained for a thermocouple of emf E. What is the value of E? [HSEB 2056, 2063]

Ans: 1197 Ω, 3×10^{-3} V

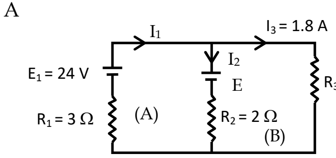
5. **[UP]** In the circuit shown in figure, find (a) the current in the 3.00 Ω resistor; (b) the unknown emf's E_1 and E_2; (c) the resistance *R*. Note the three currents are given.

Ans: (a) 8A (b) 36 V, 54 V (c) 9 Ω



6. **[UP]** What must the emf E in the figure be in order for the current through the 7.00 Ω resistor to be 1.80 A? Each emf source has negligible internal resistance.

Ans: 8.6 V



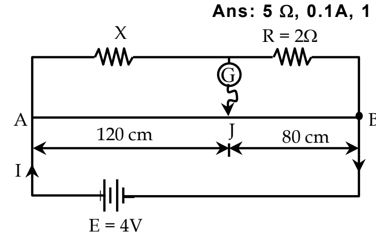
7. A battery of emf 1.5 V has a terminal p.d. of 1.25 V when a resistor of 25 Ω is joined to it. Calculate the current flowing, internal resistance and terminal p.d. when a resistance of 10 Ω replaces 25 Ω resistor. [HSEB 2060]

Ans: 5 Ω, 0.1A, 1 V

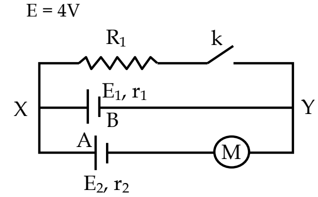
8. Calculate the current and unknown resistance (X) if no current flows in the galvanometer as in figure alongside:

Assuming resistance per unit length of the wire AB is 0.01 Ωm^{-1}.

Ans: 2.8A, 3 Ω



9. **[ALP]** In figure A has an emf of 3 V and negligible internal resistance and B has an emf of 4 V and internal resistance 2.0 Ω. With the switch K open, what current flows in the meter M? When K is closed, no current flows in M. Calculate the value of R.



Ans: 0.5 A, 6 Ω

*[**Note:** Hits to challenging problems are given at the end of this chapter.]*

Conceptual Questions with Answers

1. How Kirchhoff's first law is based on conservation of charge?

➪ The charge does not accumulate at a junction of electric circuit. The number of charge particles that arrives at a junction in a certain time must leave the junction in the same time. This is in accordance with the conservation of charge. Mathematically, $\Sigma I = 0$.

2. How Kirchhoff's second law based on conservation of energy?

➪ While current flows in the circuit, the charge particles carries the energy. The charge particles draw energy from the electric source so that net change of energy of a charge after completing a closed path must be zero. i.e. $(\Sigma E - \Sigma IR) = 0$.

3. Differentiate the Kirchhoff's first law and second law.

➪ Same major differences are:

i. First law is in accordance with the law of conservation of charge, whereas the second law is based on the principle of conservation of energy.

ii. First law is applicable to junction of conductors. The second law is applicable to closed circuit.

4. Why are the connections between resistors in a meter bridge made of thick copper strips?

➪ The resistance of connectors (copper strips) is not accounted for in the bridge formula, so it is advisable to minimize the resistance of these connections. If the connectors are made with high resistivity wires, the experimental results would be wrong.

5. What advantage is there in measuring resistance by using wheatstone bridge?

➪ If the resistance of a wire is measured by using voltmeter-ammeter combination, i.e. $R = \frac{V}{I}$. It is somehow affected by the fluctuation in battery voltage and current in the circuit. But, in wheat stone bridge circuit, the measurement of resistance is free from these fluctuations in V and I.

i.e. $\frac{P}{Q} = \frac{X}{R}$

6. Why meter bridge is also called slide wire bridge?

➪ A meter bridge consists of 1 m long resistance wire. It works on the principle of wheat stone bridge. In meter bridge, the wire acts as the resistance for two arms of wheat stone bridge, dividing from the jockey point. To find the balance condition in measuring the resistance, the jockey is to be slided over bridge wire. Hence it is named slide wire bridge.

7. Why Wheat stone bridge is not suitable to measure the very low and very high resistance?

➪ The sensitivity of bridge is ensured only when all other resistances used in the circuit have low value. This requires a galvanometer of very low internal resistance which itself would be very sensitive. However, the resistance of galvanometer has fixed value in its construction. So, the wheat stone bridge cannot measure very low resistance.

For measuring high resistance all other resistance forming the bridge should also be high so as to ensure the sensitivity of the bridge. But this reduces the current through the galvanometer which becomes insensitive.

8. Why slider should not be run continuously on the slide wire?

➪ If the slider is continuously run on the slide wire, scraping may occur on the wire which can affect the uniformity of the wire. As the uniformity is lost, the resistance of different portion of the wire changes and the measurement of resistance in terms of length will be inaccurate due to the variation of cross-sectional area.

9. Why potentiometer is called ideal voltmeter?

➪ Ideal Voltmeter measures the original potential difference. For this, the internal resistance of voltmeter should be infinite so that it does not draw any current. Potentiometer is a device which does not draw any current from the given circuit and still measures the potential difference. Thus, the potentiometer is equivalent to an ideal voltmeter.

10. What is the main advantage of comparing emfs of cells by potentiometer?

➪ The resistance of galvanometer and internal resistances of cells cause no voltage drop because no current flows through them. Hence, the potentiometer can compare the emfs without any external effect in the circuit.

11. Sometimes, we face the problem of single side deflection in galvanometer in the experiment on potentiometer. What is its main cause?

➪ If the potential difference across two ends of potentiometer wire is smaller than the emf of the cell to be measured, the balance point will not be obtained on the potentiometer wire. Then, the galvanometer shows single side deflection.

12. Why is voltmeter less accurate in measuring emf than a potentiometer?

➪ Potentiometer method is null method. It doesnot draw any current from the source whose emf is to be measured. However, voltmeter draws some current and consequently measures a slightly less value of emf.

13. Can we measure the internal resistance of a car battery with the help of wheat stone bridge?

➪ The car battery consists of accumulators whose internal resistance is very small (of the order of 0.01Ω). Such a small resistance cannot be determined by wheat stone bridge method because the bridge would then be insensitive.

14. Why do we prefer a potentiometer with a longer bridge wire?

➪ When the bridge wire is larger, the potential gradient is smaller (i.e. $\frac{V}{l}$ is smaller for large l). Smaller the potential gradient, more is the sensitivity of the potentiometer wire.

15. Why should the current be not passed through potentiometer wire for a long time?

➪ If the current is passed for a long time, the potentiometer wire heats up, which ultimately changes the resistance of the wire. Then the potential drop per unit length of the wire will also change.

16. Of which material the potentiometer wire normally made and why?

➪ The potentiometer wire is generally made of manganin, constantan or nicrome because these materials have low temperature coefficient of resistance and high resistivity. A material having low temperature coefficient of resistance ensures that its resistance does not change appreciably due to heating.

17. Why should the potentiometer wire be of uniform cross-section and composition?

➪ Potentiometer wire works in the principle that, the potential drop across a portion of its length is directly proportional to its length. This principle holds only when the cross-section of the wire is uniform and the resistivity is same throughout the wire. Uniform composition ensures the same resistivity throughout the wire.

Exercises

Short-Answer Type Questions

1. Can Kirchhoff's laws be applied both to direct current and alternating current circuits? Explain.
2. When is a Wheatstone bridge said to be balanced?
3. What do you mean by null deflection?

4. What do you mean by potential gradient?
5. What are the uses of a potentiometer?
6. Would the galvanometer show any current if the galvanometer and cell are interchanged at the balance point of the bridge?
7. On what principle post office box depends?
8. Post office box contains the resistors of only discrete values of resistance, then how it measures the resistance in decimal?
9. What is the advantage of measuring internal resistance of a cell by using potentiometer?
10. What is the principle of Meter bridge?
11. What is meant by the sensitivity of a potentiometer?

Long-Answer Type Questions

1. What is Wheatstone bridge? Using Kirchhoff's law, derive the principle of Wheatstone bridge. [HSEB 2059]
2. Apply Kirchhoff's law to derive the expression for an unknown resistance in a Wheatstone bridge.
3. Obtain the balanced condition of Wheatstone bridge circuit.
4. What is a potentiometer? Explain how you compare the emfs of two cells using potentiometer. [HSEB 2058, 2061]
5. Discuss the principle of potentiometer and use it to determine the internal resistance of a cell. [HSEB 2060, 2062, 2065, 2067]
6. What is a meter bridge? How can you measure an unknown resistance using Meter Bridge? [HSEB 2069]
7. Describe with a complete circuit to compare two resistances using Meter Bridge.
8. What is a P.O. Box? How can you measure an unknown resistance using P.O. Box?
9. How would you use P.O. Box to verify the laws of series and parallel combinations of resistors?

Numerical Problems

1. Find the emf of a cell which balances against a length of 180 cm of a potentiometer wire. Given potential difference per cm of wire as 0.006 V.

   Ans: 1.08 V

2. A potentiometer wire of length 300 cm has a resistance of 20 Ω. It is connected in series with a resistance and a cell of emf 4 volts of negligible internal resistance. A source of emf 20 mV is balanced against a length of 60 cm of the potentiometer wire. What is the value of the external resistance?

   Ans: 780 Ω

3. Two batteries of 7 V and 13 V with the internal resistances 1 Ω and 2 Ω respectively are connected in parallel with a resistance of 12 Ω. Find the current through each branch of the circuit and the potential difference across 12 Ω resistance.

   Ans: 1.526 A, 2.237 A, 0.711 A, 8.53 V

4. A 1.0 Ω resistor is in series with an ammeter M in a circuit. The p.d. across the resistor is balanced by a length of 60.0 cm on the potentiometer wire. A standard cell of emf 1.02 V is balanced by a length of 50.0 cm. If M reads 1.10 A. What is the error in the reading?

   Ans: 0.124

5. A driver cell of a potentiometer has an emf of 2 V and negligible internal resistance. The potentiometer wire has a resistance of 3 ohm. Calculate the resistance needed in series with the wire if a p.d. of 5 mV is required across the whole wire. The wire is 100 cm long and a balance length of 60 cm is obtained for a thermocouple of e.m.f. E. What is the value of E?

   Ans: 3 mV

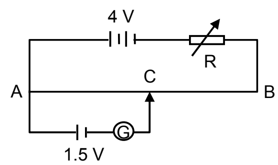
6. The length of 600 cm of a potentiometer wire is required to balance the emf of a cell. When a 20 Ω resistor is connected across the terminals of the cell, the length required for balance is 550 m. Calculate the internal resistance of the cell.

   Ans: 1.82 Ω

7. Using a wheat stone circuit, a coil of wire was found to have a resistance of 5 Ω in melting ice. When the coil was heated to 100°C, a 100 Ω resistance had to be connected in parallel with the coil in order to keep the bridge balanced at the same point. Calculate the temperature coefficient of resistance of the coil.

Ans: 5.26×10^{-4} K^{-1}

8. A simple potentiometer circuit is setup as in figure below, using a uniform wire AB, 1.0 m long, which has a resistance of 2 Ω. The resistance of the 4 V battery is negligible. If the variable resistor R were given a value of 2.4 Ω, what would be the length AC for zero galvanometer deflection?



Multiple Choice Questions

1. In a potentiometer experiment, the balancing with a cell is at length 240 cm. On shunting the cell with a resistance of 2 Ω, the balancing length becomes 120 cm. The internal resistance of the cell is
   a. 4 Ω　　b. 2 Ω
   c. 1 Ω　　d. 0.5 Ω
2. In a Wheatstone's bridge, all the four arms have equal resistance R. If the resistance of the galvanometer arm is also R, the equivalent resistance of the combination as seen by the battery is
   a. R　　b. 2R
   c. $\frac{R}{4}$　　d. $\frac{R}{2}$
3. To draw maximum current from a combination of cells, how should the cells be grouped?
   a. Series　　b. Parallel
   c. Mixed　　d. Depends upon the relative values of external and internal resistance
4. A cell can be balanced against 100 cm and 110 cm of potentiometer wire, respectively with and without being short-circuited through a resistance of 10 Ω. Its internal resistance is
   a. zero　　b. 1.0 Ω
   c. 0. 5 Ω　　d. 2.0 Ω
5. The current in the primary circuit of a potentiometer is 0.2 A. The specific resistance and cross-section of the potentiometer wire are 4×10^{-7} Ω m and 8×10^{-7} m^2 respectively. The potential gradient will be equal to
   a. 0.2 Vm^{-1}　　b. 1 Vm^{-1}
   c. 0.3 Vm^{-1}　　d. 0.1 Vm^{-1}
6. A current of 2 A flows through a 2 Ω resistor when connected across a battery. The same battery supplies a current of 0.5 A when connected across a 9 Ω resistor. The internal resistance of the battery is
   a. 0.5 Ω　　b. 1/3 Ω
   c. 1/4 Ω　　d. 1 Ω
7. Two batteries of emfs 2 V and 1 V of internal resistance 1 Ω and 2 Ω respectively are connected in parallel. The effective emf of the combination is
   a. $\frac{3}{2}$ V　　b. $\frac{5}{3}$ V
   c. $\frac{3}{5}$ V　　d. 2 V

Answers

1. (b) 2. (a) 3. (d) 4. (b) 5. (d) 6. (b) 7. (b)

Hints to Challenging Problems

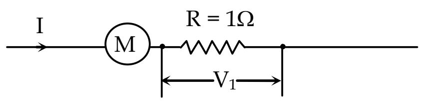
HINT: 1

Given,

Current in the circuit (I) = 1.10A

Error in reading of the ammeter = ?

For potential difference (V_1) across R = 1 Ω, balancing length (l_1) = 60 cm



For emf (E = 1.02 V) of a standard cell, balancing length (l_2) = 50 cm

According to the principle of potentiometer, we can write

$$\frac{V_1}{E} = \frac{l_1}{l_2} \quad (\because V \propto l)$$

$$\text{or} \quad \frac{I_1 \times R}{E} = \frac{l_1}{l_2}$$

$$\text{or} \quad I_1 = \frac{l_1}{l_2} \times \frac{E}{R}$$

Hence, error in the reading = (1.224 – 1.10) A

= 0.124 A

HINT: 2

Given,

l_{AB} = 1 m $\quad$ R_{AB} = 2 Ω

R = 2.4 Ω $\quad$ E_1 = 4 V

E_2 = 1.5 V $\quad$ l_{AC} = ?

In balanced condition,

Potential difference across AC = E_2

$IR_{AC} = 1.5$

$$\text{or} \quad \frac{4}{R + R_{AB}} \times R_{AC} = 1.5$$

Now,

∴ 2 Ω wire has length 1 m.

∴ 1 Ω wire has length $\frac{1}{2}$ m.

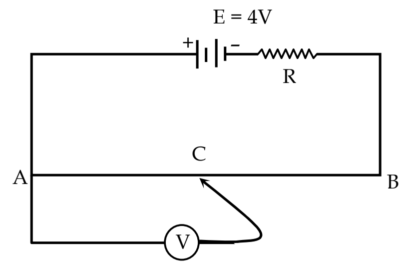
∴ 1.65 Ω wire has length $\frac{1}{2} \times 1.65$ m = 0.825 m

Hence, required length = 0.825 m

According to the second part of the question, the following circuit is drawn.

R_v = 20 Ω

R = 1 Ω



Since C is at the middle point of AB so

$R_{AC} = R_{BC} = 1\ \Omega$

Voltmeter reading across AC = ?

Now, Voltmeter reading across AC = potential difference across AC

= current × resistance

$$= I \times \frac{R_{AC} \times R_V}{R_{AC} + R_V}$$

$$= \frac{E}{R + R_{BC} + \frac{R_{AC} \times R_V}{R_{AC} + R_V}} \times \frac{R_{AC} \times R_V}{R_{AC} + R_V}$$

HINT: 3

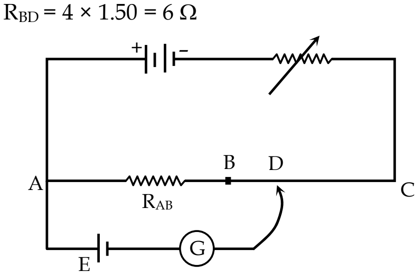
Let AB the potentiometer wire of resistance 2030 Ω which is connected in series with the slide wire BC.

$$R_{AB} = 2030\ \Omega, \frac{R_{BC}}{l_{BC}} = 4\ \Omega m^{-1}$$

a. E_2 = 1.018 V

l_{BD} = 150 cm = 1.50 m

R_{BD} = 4 × 1.50 = 6 Ω



Current in the potentiometer, I =?

According to questions,

E_2 = potential difference across the fixed resistance and 150 cm of the slide wire.

or 1.018 = I ($R_{BD} + R_{AB}$)

b. In the second case, the balanced condition is achieved only for 125 cm slide wire for thermocouple connected of emf E.

So, emf of thermocouple = potential difference across 125 cm slide wire

or $E = I \times$ resistance of 125 cm

HINT: 4

Given,

$R_{AB} = 3\ \Omega$ $\quad$ $l_{AB'} = 60$ cm

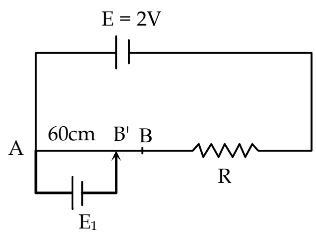
$l_{AB} = 100$ cm, $\quad$ $E = 2$ V

$V_{AB} = 5$ mV $= 5 \times 10^{-3}$ V $\quad E_1 = ?$

Let I be the current in the circuit and R be the resistance needed in series with the potentiometer wire AB.

$\therefore\ I = \dfrac{E}{R + R_{AB}}$

$\therefore\ I = \dfrac{2}{R + 3}$



Now, according to the questions,

V_{AB} = potential difference across whole wire AB

or $5 \times 10^{-3} = I \times 3$

Now, in balanced condition

E_1 = potential difference across wire AB'

$= I \times R_{AB'}$

$= \dfrac{2}{R+3} \times \dfrac{9}{5}$, here, $R_{AB'} = \dfrac{9}{5}\ \Omega$

HINT: 5

a. Current through 3 Ω, $I_2 = ?$

From given circuit,

$I_2 = I_1 + I_3$

$= 3 + 5$

$\therefore\ I_2 = 8$A

b. from loop B,

$E_1 = I_1R_1 + I_2R_2$

$\therefore\ E_1 = 36$ V

from loop C,

$E_2 = I_2R_2 + I_3R_3$

c. from loop A,

$E_2 - E_1 = IR$

HINT: 6

emf, E = ?

from loop A,

$E_1 - E = I_1 \times R_1 + I_2 \times R_2$

or $24 - E = I_1 \times 3 + I_2 \times 2$

or $E = 24 - (3I_1 + 2I_2)$ . . . (i)

from loop B,

$E = I_3 \times R_3 - I_2 \times R_2$

$= 1.8 \times 7 - I_2 \times 2$

$\therefore\ E = 12.6 - 2I_2$ . . . (ii)

Equating (i) and (ii), we get,

$24 - (3I_1 + 2I_2) = 12.6 - 2I_2$

From figure, we can write

$I_1 = I_2 + I_3$

or $I_2 = I_1 - I_3$

From (ii), we get

$E = 12.6 - 2 \times 2$

$\therefore\ E = 8.6$ V

HINT: 7

Given,

$E = 1.5$ V

Terminal potential difference; $V_1 = 1.25$ V

$R_1 = 25\ \Omega$, $I_1 = ?$, $r_1 = ?$, $R_2 = 10\ \Omega$

Terminal potential difference, $V_2 = ?$

In the first case, $r_1 = \dfrac{E - V_1}{V_1} R_1$

Find I, from $I = \dfrac{E}{R_2 + r_1}$

and use I in equation, $V_2 = E - Ir_1$

HINT: 8

Unknown resistance (X) = ?

Current (I) = ?

$R = 2\ \Omega$

$l_1 = 120$ cm $= 1.20$ m

$l_2 = 80$ cm $= 0.80$ m

For no galvanometer deflection, we can write

$\dfrac{P}{Q} = \dfrac{X}{R}$

$\therefore\ X = \dfrac{P}{Q} \times R$ . . . (i)

find P and Q and then,

from (i) we can have

$X = \dfrac{1.20 \times 0.01}{0.80 \times 0.01} \times 2 = 3\ \Omega$

In the above circuit, $(X + R) = 3 + 2 = 5\ \Omega$ is parallel to $(P + Q) = 1.2 + 0.8 = 2\ \Omega$. So,

Total resistance, $R_1 = \dfrac{5 \times 2}{5 + 2}$

$\therefore \quad R_1 = \frac{10}{7}\,\Omega$

Total current in the circuit, $I = \frac{E}{R_1} = \frac{4 \times 7}{10}$

$\therefore \quad I = 2.8$ A

HINT: 9

Given,

$E_1 = 4$ V $\qquad r_1 = 2\,\Omega$

$E_2 = 3$ V $\qquad r_2 = 0$

With switch k open,

Current through meter (M), I = ?

Applying Kirchoff's voltage law in the circuit XAYBX, we get,

$Ir_1 = E_1 - E_2$

or $I \times 2 = 4 - 3$

or $I = \frac{1}{2} = 0.5$ A

With k closed, no current through M. In this case

R = ?

Now,

potential difference across XY = p.d across A.

or $\frac{E_1 R}{r + R} = E_2 \qquad \left(\because r = \frac{E - V}{V}.R \quad \therefore V = \frac{ER}{r + R} \right)$

or $\frac{4 \times R}{2 + R} = 3$

❖ ❖ ❖

THERMOELECTRICITY

13 CHAPTER

13.1 Introduction

In the previous chapters, an electric circuit was designed using an electric cell as a source of electricity. The cell maintains the potential difference at two points of an electric circuit so that net displacement of charge particles is directed in a specific direction. This process constitutes the electric current. There is another method of electricity generation. When a temperature difference exists between the junctions of two dissimilar conductors, electricity can be produced. This method is called thermoelectric effect. This thermal effect on generating the electricity was discovered by German physicist, Thomson Johann Seebeck, in 1821.

13.2 Thermoelectric Effect

The direct conversion of temperature differences to electric voltage is known as thermoelectric effect. When two conducting wires of different materials are joined to form a closed circuit and two junctions (joined end) are placed at different temperatures, then a small emf is developed at the junctions of that coupled circuit and small current flow through the closed circuit. This effect is known as thermoelectric effect.

Thermocouple

A couple of wires of dissimilar metals forming a loop and producing thermoelectricity is called a thermocouple. The magnitude of thermo emf produced and direction of current depends on the pair of metals selected from the thermo electric series and temperature of the junction as shown in Fig. 13.1.

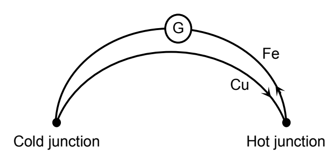


Fig. 13.1: Demonstration of Seebeck

Thermoelectric Series

In an electric circuit containing a cell, showing the conventional direction of current flow is much easier i.e. from positive terminal to negative terminal. However, in the production of thermo-emf, the thermocouple produces current without a cell so that, it is quite difficult to know the direction of current flow. To resolve this difficulty, Seebeck made an arrangement of selected metals in a series form, called the thermoelectric series. The arrangement of metals in thermoelectric series are listed below.

Antimony, Iron, Zinc, Silver, Gold, Tin, Lead, Copper, Platinum, Nickel, Bismuth.

This series has two main advantages: (a) to know the direction of current flow in the couple, (b) to find the thermo-emf in the thermo-couple.

a. **To know the direction of current flow in the couple:** The knowledge of direction of current in the couple is obtained from letters A, B and C. A denotes Antimony, B – Bismuth and C denotes cold. That is, in the couple Antimonly-Bismuth the current will pass from antimony to Bismuth at cold junction. This effect is reversible i.e., if the hot and cold junctions are interchanged, the emf changes sign, its magnitude remaining unchanged.

b. **To find the thermo-emf in the thermo-couple:** The magnitude of thermo-emf depends on the separation of members in thermoelectric series. For example, the magnitude of thermo-emf produced by iron zinc pair is smaller than that of iron-copper pair because iron-copper separation is greater than iron-zinc separation in the thermoelectric series. In this series, Antimony-Bismuth (Sb–Bi) couple produces the maximum thermo-emf among any possible couple in the given series. Therefore, thermocouple is usually made up of Antimony and Bismuth.

13.3 Seebeck Effect

The conversion of heat to electricity in a thermocouple was discovered by Estonian Physicist Thomas Seebeck, in 1821. So, this effect is called seebeck effect. Actually, Seebeck effect is a phenomenon in which temperature difference between two dissimilar electrical conductors, called thermocouples, produces a voltage difference between two ends of the conductors. The emf generated by Seeback effect is due to the temperature gradient along the wire.

Causes of Seebeck Effect

Different metals possess different electron densities. If two metals of different electron densities are connected at the ends, free electrons diffuse from the metal with higher electron density to metal of lower electron density as their average velocity varies from one metal to another. Due to the diffusion of free electrons, potential difference is created setting opposing electric field across two metals at the junction which is called the contact potential. If two junctions are at same temperature, the diffusion process is continuous for a moment and stops because equal and opposite potentials are generated at the ends. If two junctions are maintained at different temperatures, the diffusion rate of free electrons is different at two ends and creates a different contact potential. So, the current flows continuously through the thermocouple. The Seeback voltage does not depend on the distribution of temperature among the metals between the junction.

13.4 Variation of Thermo-emf (E) with Temperature (θ)

The experimental set up to study the variation of Thermo-emf with temperature is shown in *Fig. 13.2* (i) It consists of a thermocouple with a galvanometer. One end of the thermocouple is put into oil bath and another end is put into a melting ice. Suppose both of them have the same temperature, θ_c. At this condition, galvanometer does not show any deflection. It means, no emf is developed across the ends of the thermocouple. Then, the oil bath is heated keeping cold junction at constant temperature. As soon as the temperature of oil bath is increased, the deflection in galvanometer also takes place. On increasing the temperature of the oil bath continuously by keeping cold junction at constant temperature, the deflection of galvanometer is also increased upto a certain point on scale and then is found decreasing. This shows that Thermo-emf increases until it attains maximum value and the temperature at which Thermo-emf becomes maximum is called neutral temperature. Therefore, *the temperature of hot junction at which the Thermo-emf becomes maximum is called neutral temperature* (θ_n). When the temperature is further increased, the Thermo-emf is gradually decreased and becomes zero and reverses the direction. The temperature of the hot junction at which the Thermo-emf is zero and reverses the direction is called temperature of inversion (θ_i). The variation of Thermo-emf with temperature is shown graphically in Fig. 13.2 (ii).

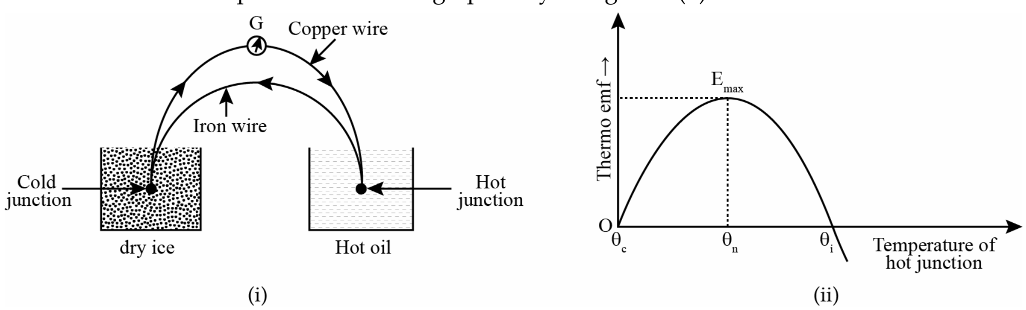


Fig. 13.2: (i) Demonstration of Seebeck effect; (ii) variation of thermo emf with temperature

The graph 13.2 (ii) shows that variation of Thermo-emf and temperature forms a parabolic curve, beginning from the origin. In such condition, the relation between Thermo-emf and temperature is given by,

$$\mathbf{E = \alpha\theta + \frac{1}{2}\beta\theta^2} \qquad \textbf{...(13.1)}$$

In equation (13.1), α and β are constants which are called thermoelectric coefficients, whose value depends upon the material of conductor and temperature difference of two junctions.

At neutral temperature: the emf is maximum at neutral temperature so, the first order derivative of emf with respect to neutral temperature must be zero, i.e.

$$\frac{dE}{d\theta_n} = 0$$

$$\frac{d\left(\alpha\theta_n + \frac{1}{2}\beta\theta_n^2\right)}{d\theta} = 0$$

$$\alpha + \beta\theta_n = 0$$

$$\boldsymbol{\theta_n = -\frac{\alpha}{\beta}} \qquad \textbf{... (13.2)}$$

At temperature inversion: The thermo emf is zero at the temperature of inversion (θ_i). Beyond, the temperature of inversion, thermo emf changes sign and direction current reverses. At the point of temperature of inversion, E = 0. So, from equation (13.1),

$$\alpha\theta_i + \frac{1}{2}\beta\theta_i^2 = 0$$

$$\alpha + \frac{\beta\theta_i}{2} = 0$$

$$\boldsymbol{\theta_i = -\frac{2\alpha}{\beta}} \qquad \textbf{...(13.3)}$$

Comparing the equations (13.2) and (13.3), we get the temperature of inversion is twice the neutral temperature.

Therefore, the neutral temperature can be determined by taking average of temperature of inversion (θ_i) and temperature of cold junction (θ_c)

$$\text{i.e.} \quad \theta_n = \frac{\theta_i + \theta_c}{2} \qquad \ldots(13.4)$$

In Cu-Fe thermocouple, neutral temperature is about 270°C when cold junction is maintained at 0°C.

13.5 Peltier Effect

In 1834, Jean Peltier fund that an electric current would produce a temperature gradient a the junction of two dissimilar metals.When current flows through the junctions of a thermocouple (coupling of two dissimilar metals) in the form of closed circuit, heat is absorbed at one junction and evolved at the another end. This effect is called Peltier effect. Peltier effect shows the counter effect of Seebeck effect, hence it is also called reverse Seebeck effect. The peltier effect can be used to make a refrigerator which is compact and has no circulating fluid.

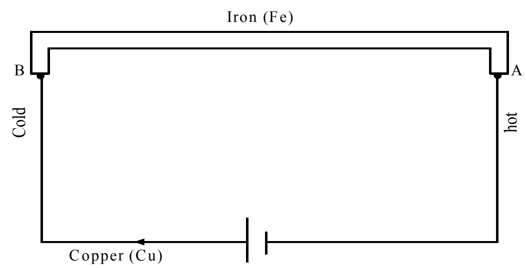


Fig. 13.3: Peltier Effect

Causes of Peltier Effect

Two dissimilar metals have different electron densities. When they are connected at two ends, potential difference is created at two ends called the contact potential. So, electrons tend to diffuse from higher potential metal to lower potential metal. If a dc source is connected across a metal, the current flows from higher potential metal to lower potential metal at one junction and from low potential metal to high potential metal at the other junction. For example, in Cu-Fe thermocouple, iron (Fe) has greater potential than the copper (Cu). So, current flows from Fe to Cu at junction A and from Cu to Fe at junction B as shown in Fig. 13.3. At the junction where the current flows from lower to higher potential metal (Cu to Fe), some work has to be done to flow the current. Hence, this end becomes cool. Conversely, at the other end where the current passes from higher potential to lower potential (Fe to Cu), energy is released. Hence, this junction evolves the heat and becomes hot.

Differences between Peltier effect and Seebeck effect

The differences between Peltier and Seebeck effects are given below:

Seebeck effect	Peltier Effect
1. This effect is the conversion of heat energy into electrical energy when the two junctions of a thermocouple are kept at different temperatures.	1. This effect is the generation or absorption of heat at two junctions of a thermocouple due to the passage of current. It is reverse Seebeck effect.
2. One end is cooled and another end is heated to generate electricity.	2. A potential difference is maintained at two ends so that heat is absorbed or evolved.
3. Temperature difference at two junctions of a thermocouple produces the thermo emf.	3. Heat is evolved at one junction and absorbed at the another junction due to the passage of current in thermocouple.

13.6 Thomson's Effect

When two ends of a metal conductor are maintained at different temperatures and current is passed through it, heat is evolved from one end and heat is absorbed at another end. This effect is called Thomson's effect. Thomson's effect is the combined effect of Seebeck effect and Peltier effect. The evolution and absorption of heat in a metal depends upon the direction of current through it. There are two types of Thomson's effect: positive Thomson's effect and negative Thomson's effect.

Let us consider a thick copper rod with its ends maintained at the same constant temperature and the centre (O) maintained at a much higher temperature as shown Fig. 13.4.

If no current flows in this conductor, the points P and Q, equidistant from the centre, would be at the same temperature due to thermal conduction alone but the point O is at much higher temperature than P and Q. Now, if a current is sent through the rod in the direction as shown in the *Fig. 13.4*, then clearly, current flows from lower temperature point P to higher temperature point O and then to lower temperature point Q. It is observed that, the temperature at P is less than that at Q. This means that heat energy has been transferred from P to Q (in the direction of the current). To be more clear, heat is absorbed when current flows from cold region to hot region (i.e. P to Q) and heat is evolved when current flows from hot region to cold region (i.e. O to Q). Thus, the part PO of the conductor becomes cooler whereas the part OQ of conductor becomes hotter.

Positive Thomson effect: The evolution of heat in the part of conductor along which current flows in the direction of temperature fall, is called positive Thomson's effect. This effect is observed in Cu, Cd, Zn, Ag and Sb.

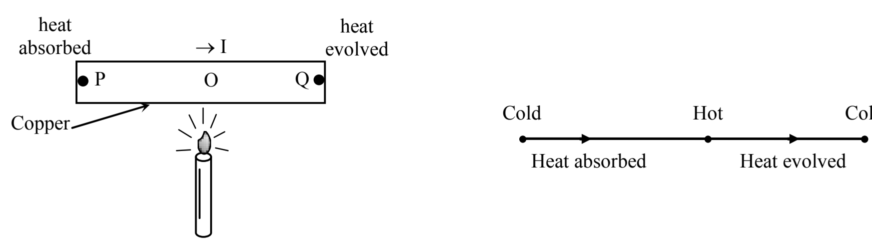


Fig. 13.4: Demonstration of positive Thomson's effect

Negative Thomson effect: When a current is sent in the iron rod in the direction from P to Q, the point P becomes hotter than point Q i.e., heat energy is transferred from Q to P (in a direction opposite to that of the current). This is called negative Thomson effect. This effect is observed in Fe, Pt, Bi, Co, Ni and Hg.

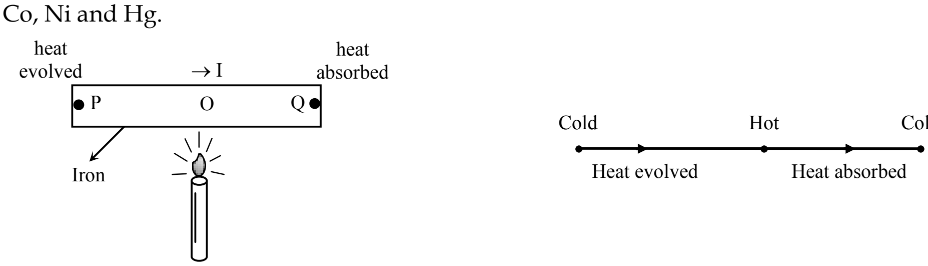


Fig. 13.5: Demonstration of negative Thomson's effect

If the direction of current in either of the above cases is reversed, the Thomson effect is also reversed.

In lead, the Thomson effect is zero. It is for this reason that, the thermoelectric behaviour of metals is studied by taking lead as the second element.

Differences between Thomson effect and Joule's effect

The differences between Thomson and Joule's effects are given below:

Thompson effect	Joule's Effect
1. This effect is produced when different sections of a conductor are maintained at different temperatures.	1. This effect is produced when current flows through a resistor whatever the temperature is.
2. It is both heating and cooling effect.	2. It is basically heating effect.
3. This effect depends on the current passing through a conductor.	3. This effect is directly proportional to the square of current through the conductor.
4. This effect basically depends on nature of conductor and temperature difference of different parts of conductor.	4. This effect depends on the resistance of the conductor.

13.7 Thermopile

Thermopile is an electrical device that uses Seebeck effect to detect and measure the intensity of thermal radiation. It works on the principle of thermoelectric effect. It is constructed with the series combination of thermocouple made up of Antimony (Sb) and Bishmuth (Bi). One side of the thermopile is coated with lampblack to absorb the thermal radiation and another side is covered with cotton kept at constant temperature. A galvanometer is connected at two ends of the device as shown in Fig. 25.8. When heat radiation is exposed to the black coated end, it absorbs heat and temperature rises. This instrument is very sensitive to even a small difference of temperature between two faces. Hence the Thermo-emf is generated across the ends of thermopile. The magnitude of thermal intensity is measured from the deflection in galvanometer. Thermopile is used for measurement of solar radiation and comparison of distribution of heat energy in a spectrum.

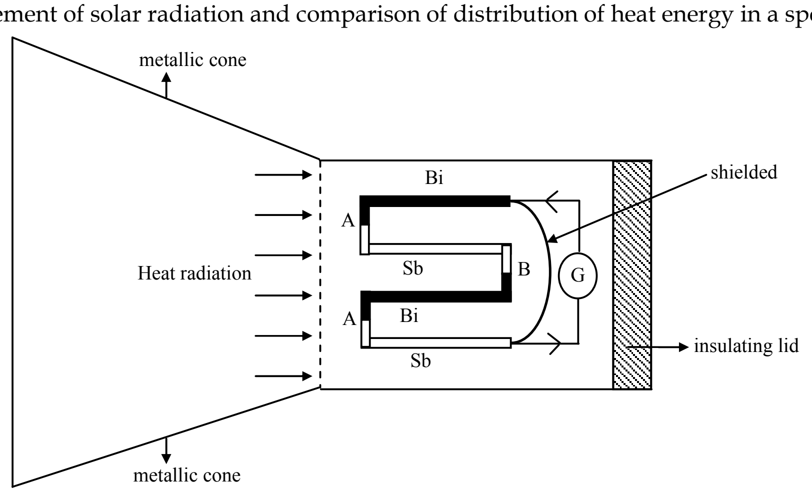


Fig.13.6: Thermopile

Tips for MCQs

1. Thermoelectric effect is discovered by Seebeck effect. So, it is also called Seebeck effect.
2. Peltier effect is the reverse effect of Seebeck effect.
3. Antimony is the first member and bismuth is the last member of thermo-electric series. So, they are used to produce maximum thermo emf than that of any other combination in thermoelectric series.
4. The neutral temperature, $\theta_n = \frac{\theta_c + \theta_i}{2}$, where θ_c is the temperature of cold junction and θ_i is the temperature of inversion.
5. The thermo emf, $E = \alpha\theta + \frac{1}{2}\beta\theta^2$, where, α and β are constants.

Worked Out Problems

1. The temperature of the cold junction of a thermocouple is kept at 0°C. The temperature inversion is 550°C. Find the neutral temperature.

SOLUTION

Given,

Temperature of cold junction (θ_c) = 0°C

Temperature of hot junction (θ_i) = 550°C

Neutral temperature (θ_n) = ?

We have,

$$\theta_n = \frac{\theta_c + \theta_i}{2} = \frac{0 + 550}{2} = 275^\circ C$$

2. A thermocouple has cold junction at 0°C and when the hot junction is at θ°C, the thermo-emf is given by $E = (20\theta + 0.02\theta^2)$ μV. What is the temperature of the hot junction if the thermo-emf produced is 7.5 mV?

SOLUTION

Given,

$E = (20\theta + 0.02\,\theta^2)\ \mu V$

Temperature of hot junction (θ) = ?

for $E = 7.5\text{ mV} = 7.5 \times 10^{-3}$ V.

Now,

$\because\ E = (20\,\theta + 0.02\,\theta^2)\ \mu V$

$\therefore\ E = (20\,\theta + 0.02\,\theta^2) \times 10^{-6}$ V

or $7.5 \times 10^{-3} = (20\theta + 0.02\,\theta^2) \times 10^{-6}$

or $7.5 \times 10^{-3} = 20\,\theta + 0.02\,\theta^2$

or $0.02\,\theta^2 + 20\,\theta - 7.5 \times 10^3 = 0$

$$\therefore\ \theta = \frac{-20 \pm \sqrt{(20)^2 - 4 \times 0.02 \times (-7.5 \times 10^3)}}{2 \times 0.02}$$

$$= \frac{-20 \pm \sqrt{400 + 0.6 \times 10^3}}{0.04}$$

$$= \frac{-20 \pm 31.6}{0.04}$$

$$= \frac{-51.6}{0.04} \text{ or } \frac{11.6}{0.04}$$

Negative temperature is not possible.

So, $\theta = \frac{11.6}{0.04} = 290^\circ C$

3. In a thermocouple the thermo-emf is related to the temperature of hot junction θ when cold junction is at 0°C as $E = a\theta + \frac{1}{2}b\theta^2$ where $a = 14\ \mu V°C^{-1}$ and $b = -0.04\ \mu V/°C^2$. Find (i) the neutral temperature and (ii) the temperature of at which the thermo-emf changes sign.

SOLUTION

Given, $E = a\theta + \frac{1}{2}b\theta^2$

$a = 14\mu\ V°C^{-1}$, $b = -0.04\ \mu V\ °C^{-1}$

i. neutral temperature (θ_n) = ?

The neutral temperature is that temperature at which Thermo-emf becomes maximum. So, if

or $a + \frac{1}{2} \times b \times 2\theta = 0$

or $a + b\theta = 0$

or $\theta = -\frac{a}{b}$

$= -\left(\frac{14}{-0.04}\right) = 350°C$

i.e., $\theta_n = \theta = 350°C$

$\theta = \theta_n$ then $\frac{dE}{d\theta} = 0$ (As E is maximum constant value).

or $\frac{d}{d\theta}(a\theta + \frac{1}{2}b\theta^2) = 0$

or $\frac{d}{d\theta}(a\theta) + \frac{1}{2}\frac{d}{d\theta}(b\theta^2) = 0$

ii. The temperature at which thermo-emf changes sign is called temperature inversion (θ_i).

$\therefore$ θ_i = ?

We know that

$\theta_n = \frac{\theta_i + \theta_c}{2}$

or $\theta_i = 2\theta_n - \theta_c$

$= 2 \times 350 - 0$ $\quad [\because \theta_c = 0°C]$

$\therefore$ $\theta_i = 700°C$

4. [HSEB 2073] The thermo-emf $\in$ and the temperature of hot junction θ satisfy the relation $\in = a\theta + b\theta^2$, where $a = 4.1 \times 10^{-5}\ V(°C)^{-1}$ and $b = -4.1 \times 10^{-8}\ V(°C)^{-1}$.
If the cold junction temperature is 0°C, find the neutral temperature.

SOLUTION

Given equation is,

$\in = a\theta + b\theta^2$ ...(i)

Also, $a = 4.1 \times 10^{-5}\ V(°C)^{-1}$

$b = -4.1 \times 10^{-8}\ V(°C)^{-2}$

To find the neutral temperature, $\in = \in_{max}$, at $\theta = \theta_n$

So, $\frac{d\in_{max}}{d\theta_n} = 0$ for $\in_{max}$

$\therefore$ $\frac{d}{d\theta_n}(a\theta_n + b\theta_n^2) = 0$

$a + 2b\theta_n = 0$

$\theta_n = -\frac{a}{2b} = -\frac{4.1 \times 10^{-5}}{2 \times (-4.1 \times 10^{-8})} = 500°C$

Conceptual Questions with Answers

1. What is seeback effect? OR What is thermoelectric effect?

➪ The conversion of heat energy into electric energy when two ends of a thermocouple are maintained at different temperatures is knows as thermoelectric effect. A pair of conductors when connected at two ends is called a thermocouple. This effect was firstly discovered by seebeck, hence it is also called seebeck effect.

2. What is peltier effect?

➪ When a d.c electric source is connected to a thermocouple heat is evolve at one point and absorbed at another end. This phenomenon is called peltier effect. Peltier effect is reverse effect of seebeck effect. Peltier effect shows the counter effect of Seebeck effect, hence it is called reverse Seebeck effect.

3. Differentiate between Joule's effect and peltier effect.

➪ The differences between joule's and peltier effects are given below:

Joule's Effect	Peltier Effect
1. It occurs in a conductor.	1. It occurs in a thermocouple.
2. Heat is evolved throughout the conductor.	2. Heat is evolved at one junction and absorbed at the another junction.
3. The quantity of heat evolved is directly proportional to the square of current.	3. The quantity of heat evolved or absorbed is directly proportional to the current.
4. This effect does not depend on direction of	4. This effect depends on direction of current.

current.	
5. The quantity of heat evolution in conductor depends on the resistance of metals used.	5. The quantity of heat absorption or evolution in thermocouple depends on the resistance of metals used.

4. Thomson's effect is the combined effect of seebeck effect and peltier effect, why?

↳ In seebeck effect, electricity is generated when two ends of a thermocouple are maintained at different temperatures, whereas the heat is evolved or absorbed when a dc electric source is connected one of the arm of thermocouple in peltier effect.

If an electric current is supplied to a conductor maintaining temperature gradient one end absorbs and another end evolves the heat. This phenomenon of absorption or evolution of heat energy due to the flow of current in an unequally heated single conductor is called Thomson's effect. Thus, both seebeck effect and peltier effect are incorporated in Thomson's effect.

5. What is neutral temperature?

↳ The temperature of hot junction of thermocouple at which thermoelectric depends on the separation of places of members in thermoelectric series. Higher the separation, higher the magnitude of thermo emf in thermocouple. Antimony is the first member and Bismuth is the last member of thermoelectric series. So, maximum thermoelectric emf can be produced form this combination. So, Antimony and Bismuth are usually used to form a thermocouple.

6. What are the factors on which the thermo emf produced in a thermocouple depends? [HSEB 2061]

When two junctions of a thermocouple are maintained at different temperatures, thermo emf is produced across these two ends. The magnitude of thermo emf basically depends on two factors:

i. The nature of materials chosen to make the thermocouple. If the first and last members of thermoelectric series are used to make thermocouple, the thermoemf is maximum than any two others in combination in the same temperature difference.

ii. The temperature of hot junction with respect to the cold junction.

7. Define temperature inversion. On what factors does it depend?

In thermoelectric effect, the thermo-emf increases initially, as the temperature of hot junction is raised. However, it does not rise continuously. At a certain high temperature, the emf becomes maximum and starts decreasing and becomes zero. If the temperature is further increased the polarity of emf reverses. The temperature of hot junction in thermocouple at which the polarity of thermo emf is reversed is known as temperature of inversion. It depends on the nature of materials used to form thermocouple and temperature of cold junction.

8. Does the thermoelectric effect obey the law of conservation of energy?

Yes. Three laws are basically dealt in thermoelectric effect.

i. **Seebeck effect:** in this effect, the heat energy absorbed by the hot junction is converted into electric energy in thermocouple.

ii. **Peltier effect:** One junction absorbs the heat, whereas the other junction evolves the heat.

iii. **Thomson's effect:** The electric energy provided by external electric source is converted into heat energy.

Thus, law of conservation of energy is obeyed in thermoelectric effect.

9. What are the uses of thermoelectric effect?

Thermoelectric effect has several uses in electricity. Some of them are as follows:

i. It is used in generating emf without electric source like battery.

ii. It is used in measuring high temperature.

iii. It is used in detecting heat radiation for example, the device; thermopile.

iv. It is used in refrigeration.

Exercises

Short-Answer Type Questions

1. What is meant by thermoelectric series?
2. Is seebeck effect reversible?
3. What is inversion temperature and upon what factors does temperature of inversion depend?
4. What is neutral temperature? Upon what factor does the neutral temperature depend?
5. Write the mathematical relation for thermoelectric emf of a thermocouple in terms of temperature of hot junction.
6. What is seebeck effect?
7. Upon what factors does the magnitude of e.m.f. depend?
8. What is inversion temperature and upon what factors does temperature of inversion depend?
9. What is Thomson's effect?
10. For a given temperature of the hot junction (the cold junction being kept at 0^0C) in which thermocouple in the Seebeck series is the thermo-emf the maximum?
11. If the temperature of the cold junction of a thermo-couple is lowered, what will be the effect on its neutral temperature?
12. How does the thermo-electric series help to predict the direction of flow of current in a thermo-couple?

Long-Answer Type Questions

1. Explain what do you mean by Seedbeck Effect? How does thermoelectric e.m.f. vary with temperature? [HSEB 2056, 2063, 2057]
2. What is thermoelectric effect? Discuss the variation of thermo-emf with the change in temperature of the hot junction. [HSEB 2067]
3. What is Peltier's effect? Discuss its cause.
4. What is Thomson's effect? Discuss its cause.
5. What is a thermo pile? Describe its construction and working.
6. What is thermoelectric effect? How does the thermo-emf of a thermocouple vary with increase in temperature of hot junction, keeping cold junction at 0°C? Explain. [HSEB 2072]

Numerical Problems

1. The temperature of the cold junction of the thermocouple is kept at 10°C. The temperature of inversion is 560°C. Find neutral temperature.

   Ans: 285°C

2. In a given thermocouple if the neutral temperature is 270°C and temperature of inversion is 520°C then find the temperature of cold junction.

   Ans: 20°C

3. The thermo-emf of a copper-iron thermo-couple whose junction is at 0°C is (-1179) μV. If the thermoelectric constants for this thermocouple are a = –13.89 $\mu V°C^{-2}$ α = –13.89 $\mu V^0 C^{-2}$ and b = 0.042 $\mu V°C^{-2}$, find the neutral temperature of the hot junction.

   Ans: 100°C

4. The emf E of a Cu-Fe thermocouple varies with the temperature 'θ' of the hot junction keeping cold junction at 10°C given by E (μV) = $14\theta - 0.02\theta^2$. Find the neutral temperature and the temperature of inversion.

   Ans: 350°C, 690°C

Multiple Choice Questions

1. Thomson effect is the combination of which following effects?
   a. Seebeck effect and Peltier effect
   b. Seebeck effect and Joule's effect
   c. Joule's effect and Peltier effect
   d. Ohm's effect and Joule's effect
2. What type of current is produced in thermoelectric effect?
   a. ionic current
   b. conduction current
   c. hole current
   d. electric current
3. Which combination of metals in thermocouple gives the maximum thermoemf in equal temperature difference at two ends?
   a. Iron and copper
   b. Zinc and iron
   c. Antimony and bishmuth
   d. Antimony and copper
4. If the cold junction of a thermocouple is kept at 0ºC and hot junction is kept at θ°C, then the relation between neutral temperature θ_n and temperature of inversion θ_i is
   a. $\theta_n = \theta_I$
   b. $\theta_n = \frac{\theta}{2}$
   c. $\theta_n = \theta_I$
   d. $\theta_i = \alpha\theta_n + \beta\theta_n^2$
5. The neutral temperature of a thermocouple is 300°C. What is the inversion temperature if the temperature of cold junction is 1.10°C?
   a. 590°C
   b. 610°C
   c. 310°C
   d. 290°C

Answers

1. (a) 2. (d) 3. (c) 4. (b) 5.(a)

❖ ❖ ❖

CHEMICAL EFFECT OF CURRENT

14

CHAPTER

14.1 Introduction

In the previous chapter, we have studied about electric current in which charge is carried by the moving electrons in a wire. Besides electrons, charge can also be carried by ions of some chemical substances, called electrolytes. The current carried by ions is called ionic current. The ionic current is generated due to the dissociation of electrolytes in presence of electric field. The effect of electric current in the electrolytes is called chemical effect of current.

Some Important Terms Related to Electrolysis

1. **Electrolytes:** The substance that provides an electrically conducting solution when dissolved in a polar solvent like water is called electrolyte.
2. **Electrolysis:** The process by which ionic substances are decomposed (broken down) into simpler substances when an electric current is passed through it is known as electrolysis.
3. **Voltameter:** An instrument which is used for measuring the electric charge is called voltameter.
4. **Electrode:** An electric conductor used to make contact with non-metallic part of circuit is known as electrode. In electrolysis, electrodes are partially dipped into the electrolyte to conduct the ions in the solution.
5. **Electrochemical equivalent:** The electrochemical equivalent of a chemical element is the mass of that element (in grams) transported by 1 coulomb of electricity. It is denoted by ece. The ece of an element is measured with a voltameter.
6. **Chemical equivalent:** The gram equivalent weight divided by its valency is called the chemical equivalent of a chemical substance. It is denoted by E.

$$\therefore \quad \text{Chemical equivalent (E)} = \frac{\text{Gram equivalent weight}}{\text{Valency}}$$

Why Electrolytes Decomposes in Presence of Electric Current?

An electrolyte is a substance that produces an electrically conducting solution when dissolved in water. The electrolyte, for example $CuSO_4$ is composed of two ions Cu^{++} and SO_4^{--} combined to form a copper sulphate ($CuSO_4$). When $CuSo_4$ is dissolved in water, the electrostatic attraction between the ions decreases due to the high dielectric constant of liquid water (i.e. $\varepsilon_r = 81$) entering the space of these ions. If two terminals of a dc are set up into the solution, positive ions are attracted towards the negative terminals and negative ions are attracted towards positive terminals. Thus, the electrolytes decompose in presence of electric current.

Conductivity of Electrolytes

Electrolytes conduct electricity. They conduct electricity in ionic form; therefore the current so produced is called ionic current. The conductivity of electrolytes is much smaller than that of electric current in metals (about 10^{-5} to 10^{-6} times at room temperature). There are several reasons behind low conductivity of electrolytes than the metals. The main reasons are:

i. The ionic density (the number of ions per unit volume) is smaller than the electron density in metals.
ii. The mass of individual ion in electrolytes is several thousands times greater than the mass of individual electron in metal. Hence, the drift velocity of ions is many times smaller than those of electrons.
iii. The resistance offered by the solution to the ions is much greater than the resistance offered y the metals to the electrons.

14.2 Electrolysis

Electrolysis is the process of decomposition of ionic substance into simpler substances when electric current is passed through it. It is the chemical effect of current. This process can be studied experimentally in copper sulphate solution, silver nitrate solution, etc. For instance, the electrolysis process in silver nitrate solution is described below:

Let us take a solid silver nitrate ($AgNO_3$) in a voltameter. The solid $AgNO_3$ is dissolved into water to make the $AgNO_3$ solution. This solution conducts electricity in the suitable connection in electric circuit. Two silver plates, called electrodes, are partially immersed into the solution and are connected at two terminals of a battery via a rheostat and an electric switch as shown in Fig. 14.1. Then, the electrolyte ($AgNO_3$) is dissociated in the form of silver ions (Ag^+) and nitrate ion (NO_3^-).

$$AgNO_3 \longrightarrow Ag^+ + NO_3^- \text{ (in the solution)}$$

In presence of steady current in the circuit, the following process takes place.

i. In the beginning, electrons flow from negative terminal of battery to the cathode place C via the connecting wire. It develops the negative potential in cathode plate.
ii. Since the cathode plate possesses negative potential, it is at a lower potential than another plate, anode plate A. Therefore, the positively charged silver ions (cations) move towards C, while negatively charged nitrate ions (anions) move towards A. As the cations are deposited at C, it is called cathode. Similarly, A is called anode, as the anions are deposited on it of the voltameter.

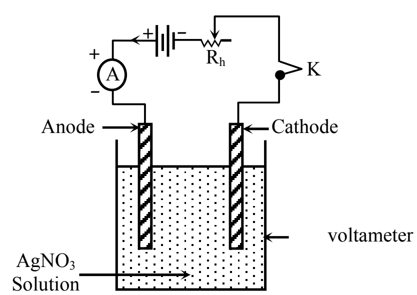


Fig. 14.1: Electrolysis in $AgNO_3$

iii. At the cathode, the Ag^+ ions get neutralized by the incoming electrons from the external circuit. Thus, the reduction reaction takes place at the cathode and the oxidation reaction takes place at the anode, as,

$$Ag^+ + e^- \longrightarrow Ag \text{ (at cathode)}$$

$$Ag + NO_3^- \longrightarrow AgNO_3 + e^- \text{ (at anode)}$$

iv. The silver ions get into the solution, while released electrons flow back to the positive terminal of the battery via connecting metallic wire. The flow of appropriate ions inside the battery then completes the circuit.

Thus, the deposition of silver (Ag) at the cathode plate takes place, while the anode loses an equivalent amount of silver. The number of electrons (or the amount of charge) passed through the circuit is equal to the number of electrons given by the cathode or taken by the anode. In overall process, the concentration of $AgNO_3$ in the solution remains unchanged.

14.3 Faraday's Law of Electrolysis

Michael Faraday studied about the ionic current in electrolytes and deduced the quantitative measurement of the ionic mass deposition in an electrolyte due to the passage of current. After the series of experiments on electrolysis, he derived two laws regarding chemical effect of current, called Faraday's laws of electrolysis.

Faraday's First Law of Electrolysis

This law states that "*mass of a substance liberated (or deposited) on an electrode during electrolysis is directly proportional to the amount of charge passed through the electrolytes*".

Let m be the mass of substance deposited on an electrode during electrolysis when amount of charge q is passed through the electrolyte. From first law of electrolysis,

$$\mathbf{m \propto q} \qquad \dots (14.1)$$

$$\text{or,} \quad \mathbf{m = Zq} \qquad \dots (14.2)$$

Where, Z is a proportionality constant and it is called electrochemical equivalent of a substance.

Its SI unit is kg /C.

Electrochemical equivalent of a substance may be defined as the amount of substance deposited or liberated when 1 coulomb of electric charge is passed through an electrolyte.

We have,

$$I = \frac{q}{t}$$

$$\therefore \quad \mathbf{q = It} \qquad \dots (14.3)$$

Using equation (14.3) in equation (14.2), we get,

$$\mathbf{m = ZIt} \qquad \dots (14.4)$$

Faraday's Second Law of Electrolysis

This law states that "*when same quantity of electricity is passed through several electrolytes, the mass of substance deposited are proportional to their respective chemical equivalent*".

Let m_1 and m_2 be the masses of substance deposited on different electrodes having chemical equivalents E_1 and E_2 respectively. Then, according to the second law of electrolysis,

$$m_1 \propto E_1 \text{ and } m_2 \propto E_2$$

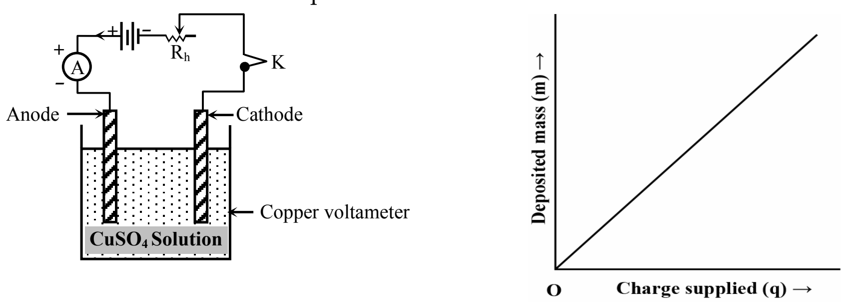
$$\text{Then, } \frac{\mathbf{m_1}}{\mathbf{m_2}} = \frac{\mathbf{E_1}}{\mathbf{E_2}} \qquad \dots (14.5)$$

$$\text{Equivalent mass of an element} = \frac{\text{Atomic mass of element}}{\text{Valency of the element}}$$

Verification of Faraday's First Law of Electrolysis

The experimental set up for the verification of first law of electrolysis is shown in Fig. 14.2. It consists a voltameter containing dilute copper sulphate (dil. $CuSO_4$) solution. Copper sulphate solution acts as the electrolyte for the experiment. Two copper electrodes are partially dipped into the electrolyte. The electric circuit containing a cell, a switch and a rheostat are connected across two electrodes. The rheostat is used to vary the current in the circuit.

To begin with, the mass of negative electrode is measured and noted. Then, current I_1 is supplied to the electrolyte for a certain interval of time t and the negative electrode is taken out for measurement of its mass. The deposited mass of electrolyte is measured. The same process is repeated for different values of current passed for same interval of time. The deposited mass of substance for each step is taken for different values of current at equal interval of time.



(i) (ii)

Fig. 14.2: (i) Experimental arrangement of first law of electrolysis
(ii) Relation of deposited mass of electrolyte and charge supplied

Let Δm_1, Δm_2, Δm_3, Δm_4 and Δm_5 be the masses of electrolyte deposited when currents I_1, I_2, I_3, I_4 and I_5 and hence the charges q_1, q_2, q_3, q_4 and q_5 are passed through the electrolyte, then we get,

$$\frac{\Delta m_1}{q_1} = \frac{\Delta m_2}{q_2} = \frac{\Delta m_3}{q_3} = \frac{\Delta m_4}{q_4} = \frac{\Delta m_5}{q_5} \quad \ldots (14.6)$$

If the graph is plotted between the deposited mass (m) and charge (q = It), a straight line graph is found passing through the origin. Thus, the first law of electrolysis is verified experimentally.

Verification of Faraday's Second Law of Electrolysis

The experimental set up to verify the Faraday's second law of electrolysis is shown in Fig. 14.3. Suppose two voltameters with different electrolytes are taken. An electric circuit is connected in such a way that the electrodes in electrolytes would be in the series combination. Let $CuSO_4$, $AgNO_3$ be the electrolytes in these different voltameters. A direct current (dc) source is used to supply the current and a rheostat is used to vary the current in the electrolytes.

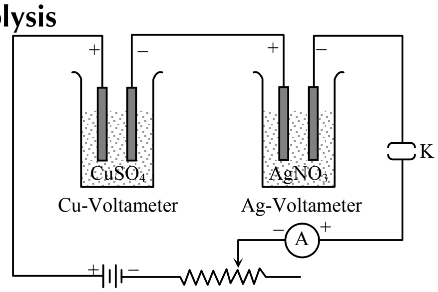


Fig. 14.3: Experimental arrangement of second law of electrolysis

In the beginning, the negative electrodes of each voltameter nt is supplied to electrolytes for a certain interval of time. Then, the electric current is switched off and negative electrodes are again weighed. By Subtracting the mass of negative electrodes before electrolysis from that after electrolysis, the deposited masses are determined.

Let Δm_1 and Δm_2 be the deposited masses of two electrolytes copper and silver respectively, and E_1 and E_2 be their chemical equivalents respectively. Then, we get,

$$\frac{\Delta m_1}{E_1} = \frac{\Delta m_2}{E_2} \qquad \text{... (14.7)}$$

This experiment verifies the Faraday's second law of electrolysis.

Applications of Electrolysis

1. **Electrometallurgy:** It is the process of reduction of metals from metallic compounds to obtain the pure form of metal using electrolysis.
2. **Electroplating:** It is the process where a thin film of metal is deposited over a substrate material.
3. **Production of metal compounds:** Electrolysis process is carried out to produce the sodium chlorate and potassium chlorate.
4. **Production of cheap energy:** Electrolysis is done for the production of hydrogen gas.
5. **Production of oxygen:** In air craft, oxygen is produced using electrolysis process.

14.4 Faraday's Constant

Suppose one mole of substance of atomic mass A is deposited on an electrode. It means the number of atoms deposited on the electrode is equal to Avogadro number N_A. Let V be the valency of deposited atom on the electrode, then the charge per atom of the electrolyte = Ve.

(For example V = 2 for copper, V = 1 for silver)

Now, the total charge flowing through the electrolyte for one mole of electrolyte,

$$q = N_A Ve \qquad \text{... (14.8)}$$

From first law of electrolysis, the mass liberated in one mole of electrolyte,

$$A = Z(N_A Ve)$$

$$\therefore \quad Z = \frac{1}{N_A e}\left(\frac{A}{V}\right) \qquad \text{... (14.9)}$$

N_A and e are constants, so equation (14.9) gives the relation,

$$\text{or,} \quad Z \propto \frac{A}{V}$$

$$\text{or,} \quad Z \propto E$$

The quantity $\frac{A}{V}$ = E is constant for an electrolyte and is known as chemical equivalent or equivalent mass.

Now, the total mass deposited in the cathode plate is,

$$m = ZIt$$

$$m = \frac{1}{N_A e}\left(\frac{A}{V}\right) It \qquad \text{... (14.10)}$$

In the equation (14.10), the quantity $N_A e$ is a fundamental constant known as Faraday's constant, i.e. $F = N_A e$.

$$\text{So,} \quad m = \frac{1}{F}\left(\frac{A}{V}\right). It$$

$$\frac{m}{It} = \frac{1}{F}. E \qquad \text{.... (14.11)}$$

$$Z = \frac{E}{F}$$

$$\therefore \quad \mathbf{F} = \frac{\mathbf{E}}{\mathbf{Z}} \qquad \textbf{... (14.12)}$$

From equation (14.11), we get,

$$\frac{m}{q} = \frac{1}{F}E$$

$$\mathbf{m} = \frac{\mathbf{1}}{\mathbf{F}}\mathbf{Eq} \qquad \textbf{... (14.13)}$$

For m = E, we get F = Q. Therefore, *Faraday's constant is defined as the quantity of charge required to liberate the mass of substance equivalent to its gram equivalent*. It represents the magnitude of electric charge per mole of electrons.

Value of Faraday's Constant

From equation (14.12),

$$F = \frac{E}{Z}$$

$$= \frac{A}{VZ}$$

where, A = gram equivalent weight

V = valency

Considering the silver,

$$A = 107.88 \text{ g}$$

$$V = 1$$

and $Z = 0.001118$ g/C

then,

$$F = \frac{107.88}{1 \times 0.001118} \approx 96500 \text{ C}$$

$$\therefore \quad F = 96500 \text{ C.}$$

Faraday's constant (F) is also defined as the magnitude of electric charge per mole of electrons. So,

$$F = N_A e$$

$$= 6.023 \times 10^{23} \times 1.6 \times 10^{-19}$$

$$\approx 96500 \text{ C}$$

One Faraday refers charge carries by one mole of electron, which is equal to 96500 C charge.

Tips for MCQs

1. Electrolysis is performed in a device called voltameter.
2. Cations are positive ions, they drift towards the cathode plate.
3. Anions are negative ions, they drift towards the anode plate.
4. Faraday's first law of electrolysis, m = Zq = ZIt

   Z is called electrochemical equivalent. Its unit is gC^{-1}.

5. Faraday's second law of electrolysis, $\frac{m_1}{m_2} = \frac{E_1}{E_2}$

   Where, E is chemical equivalent and m is the deposited mass of electrolytes.

6. Chemical equivalent (E) = $\frac{\text{Atomic weight}}{\text{Valency}}$

7. Faraday's second law of electrolysis is alternately written as,

   $Z \propto E$

   $Z = \frac{1}{F} E$

   $\therefore \quad F = \frac{E}{Z} = \frac{E}{m/q} = \frac{Eq}{m}$

   Where, F is called Faraday's constant.

   a. $1\ F = 96500 \frac{C}{\text{gram equivalent}}$

      It means 96500 C charge is required to liberate or deposit 1 gram equivalent of the substance.

   b. $F = N_A e$

8. Faraday's laws are consequence of conservation of energy.
9. Alternating current can't be used for electroplating or electrolysis because of change in polarity.
10. Nature of electrolyte determines the emf between the two metals placed in an electrolyte.
11. Unit and dimensions of some physical quantities:

Quantity	Symbol	Dimensions	Units	Remarks
Chemical equivalent	E	$[M\ mol^{-1}]$	$kg\ mol^{-1}$	Molar mass/ valency
Electrochemical equivalent	Z	$[ML^0T^{-1}A^{-1}]$	$kg\ C^{-1}$	$Z = m/It$
Faraday constant	F	$[M^0L^0TAmol^{-1}]$	$C\ mol^{-1}$	$F = N_A e$

Worked Out Problems

1. A current 5 A is passed through a silver voltmeter for 1 h; the mass of silver deposited is 15.10 g. What is the e.c.e. of silver?

SOLUTION

Given,

Current (I) = 5 A

Time (t) = 1 h = 60 × 60 = 3600 s

Mass (m) = 15.10 g = 15.10×10^{-3} kg

ece (Z) = ?

We know,

$m = ZIt$

$Z = \frac{m}{It} = \frac{15.10 \times 10^{-3}}{5 \times 3600} = 8.4 \times 10^{-7}\ Cmol^{-1}$

$\therefore\ Z = 8.4 \times 10^{-7}\ kg/C$

2. A metal plate weighing 200 g is to be electroplated with 5% of its weight in silver. If the electroplated in 12 h, what is the value of current to be passed through the electrolyte? (e.c.e of silver = 0.001118 g/C)

SOLUTION

Given,

Mass of metal (m) = 200 g

Electroplated mass (m') = 5% of m = $\frac{5}{100} \times 200$

= 10 g

Time (t) = 12 h = 12 × 60 × 60 = 43,200 s

e.c.e of silver (Z) = 0.00111 g/C

Current (I) = ?

We have,

$m' = ZIt$

$I = \frac{m'}{Zt} = \frac{10}{0.00111 \times 43{,}200} = 0.2\ A$

3. Assuming the Faraday constant is 96500 C mol^{-1}, calculate (i) the charge needed to deposit 1.6 gm of oxygen in the electrolysis of water, (ii) the time required if a steady current of 2.5 A is used, (iii) the mass of hydrogen deposited at the end of this time, (Relative molecular masses of hydrogen and oxygen are 2 and 32 respectively.)

SOLUTION

Given,

1 F = 96,500 C mol^{-1}

i. Charge required (q) = ?

mass of oxygen deposited, m = 1.6 g = 1.6×10^{-3} kg

From laws of electrolysis, we have,

$$m = \frac{1}{F} qE$$

$$\text{or, } q = \frac{mF}{E} = \frac{mFV}{A} = \frac{1.6 \times 10^{-3} \times 96500 \times 2}{16 \times 10^{-3}}$$

$$\therefore q = 19300 \text{ C}$$

ii. Time required (t) = ?

Here, I = 2.5 A

$$\because I = \frac{q}{t}$$

$$\therefore t = \frac{q}{I} = \frac{19300}{2.5} = 7720 \text{ s}$$

iii. mass of hydrogen deposited, m = ?

molar mass of hydrogen, M = 2 g = 2×10^{-3} kg

$$\text{Atomic mass, } A = \frac{M}{2} = \frac{2}{2} = 1$$

Valency of hydrogen, V = 1

Chemical equivalent of hydrogen,

$$E = \frac{A}{V} = \frac{1}{1} = 1$$

From laws of electrolysis, we have,

$$m = \frac{1}{F} qE$$

$$= \frac{1}{F} \times It \times E$$

$$= \frac{1}{96500} \times 2.5 \times 7720 \times 1 = 0.2 \text{ gm}$$

$$\therefore m = 0.2 \times 10^{-3} \text{ kg}$$

Challenging Problems

1. **[ALP]** Assuming the Faraday constant is 96500 C mol^{-1} and that the relative atomic masses of copper and Silver are 63 and 108 respectively, calculate:
   a. the number of atoms of Copper, Cu^{2+} and of Silver which are liberated respectively by the Faraday,
   b. the masses of these two elements liberated respectively by 0.5 A in 10 min.

   (Electronic charge, $e = -1.6 \times 10^{-19}$ C.)

   Ans: (a) 3×10^{23}, 6×10^{23}(b) 0.098 g and 0.34 g

2. **[ALP]** (a) If 1 mole of electrons contains 6.02×10^{23} electrons, calculate a value of *F*. (b) In a copper plating system an electrolysis current of 3.0 A is used. How many atoms of Cu^{2+} are deposited in 1.5 hr? (Electronic charge, e = -1.6×10^{-19} C.)

   Ans: (a) 96320 Cmol^{-1} (b) 5.06×10^{22}

3. **[ALP]** Calculate the volume at s.t.p. of hydrogen formed when a current of 0.5 A passes for 2 h in electrolysis of H_2SO_4. (Given, $N_A = 6.0 \times 10^{23}$ mol^{-1}, Volume of 1 mole of gas at STP = 2.24×10^{-2} m^3)

   Ans: 4.18×10^{-4} m^3

4. **[ALP]** If the mass of hydrogen deposited per coulomb is 1.04×10^{-8} kg C^{-1} and if 1 g of hydrogen on burning to form water liberates 147000 J, calculate the back e.m.f. produced in a water voltameter when it is connected to a 2 V accumulator.

   Ans: 1.5 V

5. **[ALP]** A battery of accumulators of e.m.f. 50 V and internal resistance 2 Ω, is charged on a 100 V direct-current mains. What series resistance will be required to give a charging current of 2 A? If the price of electrical energy is 1p per kilowatt-hour, what will it cost to charge the battery for 8 hours, and what percentage of the energy supplied will be wasted in the form of heat?

   Ans: 23 Ω, 1.6 P and 50%

*[**Note:** Hints to challenging problems are given at the end of this chapter.]*

Conceptual Questions with Answers

1. State Faraday's laws of electrolysis.

↳ There are a couple of laws in electrolysis of Faraday.

i. First law: "The mass of ions liberated at an electrode in electrolysis is directly proportional to the quantity of charge passing through the electrolyte." In mathematical form, this law is stated as,

$m = Zq = ZIt.$

ii. Second law: "When the same amount of electricity is passed through a number of electrolytes placed in series, then the mass of ions liberated at the electrodes is directly proportional to their chemical equivalents." In mathematical form, this law is stated as,

$\frac{m}{E}$ = Constant.

2. What is meant by Faraday's constant?

↳ Faraday's constant is defined as the quantity of charge required to liberate the mass of substance equal to its gram equivalent. Its value is, F = 96500 $Cmol^{-1}$. Faraday's constant is quantitatively determined from the ratio of chemical equivalent (E) to electrochemical equivalent (Z).

i.e. $F = \frac{E}{Z}$

3. Why electrolytes have lower conductivity than metallic conductors?

↳ Electrolytic current is also called ionic current. In this condition, the current is produced due to the flow of ions. Ions are massive than electrons. Hence, they offer high resistance while flowing in the solution. But the metallic current is produced due to the flow of free electrons. Hence, it is also called electric current. Electrons are much lighter and smaller than the ions. Hence, they offer low resistance while moving in metallic conductor.

Therefore, electrolytes have lower conductivity than metallic conductors. Moreover, the free electron density (n) of metal is more than that of electrolyte. In equation,

$I = nev_dA$,

i.e. $I \propto n$.

4. Electrolysis is possible by d.c. not by a.c. why?

↳ The alternating current changes its direction periodically in very short interval of time (in our electric domestic lines, 50 times per second). So, the ions of the solution are unable to flow to any specific direction and impossible to deposit on a plate. But, the polarity is fixed in d.c., so, the deposition is possible in a electrode containing opposite natured ions.

5. What type of current flows in a voltameter?

↳ Ionic current flows inside a voltameter. In voltameter a chemical compound dissociates and ions move to the plates containing opposite potential. Hence, ions flow in the voltameter. Electronic current flows in conducting wire.

6. What is voltameter?

↳ The vessel containing electrodes and electrolytes in which the electrolysis is performed is called voltameter. Voltameter contains two electrodes and the electrodes are connected to two terminals of a d.c. power supply. The electrodes attract the oppositely charged ions from the electrolyte.

7. What are the uses of electrolysis?

↳ There are many uses of electrolysis. Some of them are as follows:

a. It is used in electroplating. Electroplating of gold in cheap metals looks like expensive ornaments.

b. It is used in purifying metals. Pure metals can be deposited at on electrode in electrolysis.

c. It is used in chemical analysis.

d. It is used in manufacturing chemicals.

e. It is used in printing industries.

8. What is the difference between ionization and electrolysis?

↳ The dissociation of an electrolyte into its constituents when a passage of electric current is passed through it is known as ionization. Anions and catations are produces in ionization.

Electrolysis the conduction of ions through the electrolyte is known as electrolysis. Electrolysis occurs due to the motion of ions.

9. Why is the conductivity of a electrolyte low as compared to that of a metal?

↳ In metallic conduction, free electrons moves through the conductor, whereas the ions are moved in the conduction of electrolyte. Due to the following reasons, the conductivity of electrolyte is smaller than the conductivity of a metal.

i. The mass of ions is much more greater than free electrons. So, the drift velocity of electrons is greater than ions in electrolyte.

ii. The density of free electrons in a metallic conductor is relatively very high than the ions in electrolyte.

10. A voltameter measures current more accurately than an ammeter does. Explain why?

↳ From first law of electrolysis, the ionic current is measured as, $I = \frac{m}{Zt}$. In such process, the value of m, Z and t can be measured more precisely, upto 3 - 4 decimal numbers, whereas the ammeter cannot measure the current so accurately. In ammeter, the value of shunt greatly affects the amount of current measurement.

Exercises

Short-Answer Type Questions

1. What is the difference between ionization and electrolysis?
2. How do you distinguish between the passage of electric current through metal and that through electrolyte?
3. Why electrolytes have low conductivity than metallic conductors?
4. What is meant by thermoelectric series?
5. Is seebeck effect reversible?
6. What is inversion temperature and upon what factors does temperature of inversion depend?
7. What is neutral temperature? Upon what factor does the neutral temperature depend?
8. Write the mathematical relation for thermoelectric emf of a thermocouple in terms of temperature of hot junction.
9. What do you mean by thermoelectric effect?
10. What is seebeck effect?
11. Upon what factors does the magnitude of e.m.f. depend?
12. What is inversion temperature and upon what factors does temperature of inversion depend?
13. What is neutral temperature? Upon what factor does the neutral temperature depend?
14. Distinguish between peltier effect and joule effect.
15. Write the mathematical relation for thermoelectric emf of a thermocouple in terms of temperature of hot junction.
16. Can alternating current be used for electrolysis? Explain.
17. Define electrochemical equivalent of a substance.
18. What is the difference between ionization and electrolysis?
19. Define chemical equivalent.

20. What do you mean by faraday's constant?
21. Write faradays first law and second law of electrolysis.
22. How do you distinguish between the passage of electric current through metal and that through electrolyte?
23. Why electrolytes have low conductivity than metallic conductors?

Long-Answer Type Questions

1. State Faraday's laws of electrolysis. How will you verify Faraday's second law experimentally? [NEB 2075]
2. What is faraday's constant? Explain the verification of Faraday's laws of electrolysis. [HSEB 2058]
3. What is neutral temperature? Describe the variation of thermoelectric emf with temperature.
4. What is Thomson's effect? Describe the construction and working of thermopile.

Numerical Problems

1. Calculate the current following through a electrolytic tank if 0.95 g of copper is deposited in the half an hour on the cathode. (e.c.e. of Cu = 0.000329 g/col)
   (Ans: 1.6 A)
2. The cold junction of a thermocouple is maintained at 10°C. No thermo-emf is developed when the hot junction is maintained at 530°C. Find the neutral temperature.
   (Ans: 270°C)
3. If 1 mole if electron contains 6.02×10^{23} electrons, calculate the value for F.
   (Ans: 96320 C/mol)
4. In a copper plating system an electrolysis current of 3.0 A is used. How many atoms of Cu^{2+} are deposited in 1.5 h?
   (Ans: 5.06×10^{22})
5. A current of 2 A flows through an electroplating solution for 40 min deposits 1.6 g of copper. (Atomic mass 63.5, valency 2 on the cathode. Compute the faraday.
   (Ans: 95250 C/mol)
6. How long will it take to deposit electrolytically 10.79 g of silver on the cathode of a silver voltameter by a current of 25 A? [e.c.e. of silver = 0.001118 gC^{-1}] **Ans: 6.4 min**
7. A copper voltameter is connected in series with a heating coil of resistance 10 Ω. A steady current flows in the circuit for 20 minutes and deposits 0.99 g of copper. Calculate the amount of heat generated in the coil in this time. e.c.e. of copper is 0.00033 gC^{-1}.
   Ans: 75 kJ
8. A copper voltameter and an ammeter are connected in series with a battery through a resistance. In 50 minutes, 0.99 g of copper is deposited on the plates. The ammeter reads 0.95 A. Calculate the error in its reading. [e.c.e. of copper = 0.00033 gC^{-1}]
   Ans: 0.05 A

Multiple Choice Questions

1. A current of 10 A, deposits 10.8 g of silver in 900 s. The mass of copper deposited by 9 A of current in 1200 s will (E_{cu} = 31.5 and E_{Ag} = 108):
   a. 3.78 g b. 6.35 g
   c. 7.56 g d. 10.80 g
2. Electrolysis is possible in:
   a. Only a.c. b. Only d.c.
   c. Both d. Depends on Voltage, not a.c. or d.c.
3. In electrolytes, the conduction of electricity is due to,
   a. Free electrons b. bound electrons
   c. ions d. atoms

4. A current of 1 A is possible through a dilute H_2SO_4 for some time to liberate 1 g of oxygen. How much hydrogen is liberated during this period? (Faraday's constant = 96,500 C/mol):
   a. 2 g b. 1 g
   c. 0.5 g d. 0.125 g
5. A current of 1.6 A is passed through $CuSO_4$ solution. The number of Cu^{++} ions liberated per minute are:
   a. 6×10^{19} b. 6×10^{13}
   c. 3×10^{20} d. 3×10^{17}

Answers

1. (a) 2. (b) 3. (c) 4. (d) 5. (c)

Hints to Challenging Problems

HINT: 1

Here,

1 F = 96,500 C mol^{-1}
Atomic mass of copper = 63
Atomic mass of silver = 108

a. Q = 96,500 C
Copper is bivalent element so charge carried by each atom of copper (Cu^{2+}) is 2e.

∴ Number of atoms librated by 1 F = $\frac{1\text{ F}}{2e}$

Silver is monovalent, so charge carried by each atom of silver (Ag^+) is e.

∴ Number of atoms liberated by 1 F = $\frac{1\text{ F}}{e}$

b. I = 0.5 A
t = 10 min = 600 s

i. For copper, $E_c = \frac{A}{V}$

Now, $m_c = \frac{1}{F} qE = \frac{1}{F} \times It \times E$

ii. For silver, $E_s = \frac{A}{V}$

Now, $m_s = \frac{q}{F} E_s = \frac{1}{F} \times It \times E_s$

HINT: 2

a. $1\text{ F} = N_A \times e$
$= 6.023 \times 10^{23} \times 1.6 \times 10^{-19}$
$= 96320\text{ Cmol}^{-1}$

b. I = 3 A, t = 1.5 h = 1.5 × 3600 s = 5400 s
Number of atoms of Cu^{2+} deposited = ?
Total charge passed, Q = It
Since copper is diatomic so charge curried by each atom of Cu^{2+} is q = 2e

Now, Number of atoms of Cu^{2+} deposited = $\frac{Q}{q}$

HINT: 3

Given,

I = 0.5 A
t = 2 h = 2 × 3600 s = 7200 s

mass of hydrogen deposited, $m = \frac{1}{F} qE = \frac{1}{F} \times It \times E$

Number of moles of hydrogen deposited,

$N = \frac{\text{total mass}}{\text{molar mass}}$

∵ 1 mole has volume $2.24 \times 10^{-2}\text{ m}^3$

∴ 1.86×10^{-2} moles have volume,
$(2.24 \times 10^{-2} \times 1.86 \times 10^{-2})\text{ m}^3$
Hence,
Volume of hydrogen formed = $4.18 \times 10^{-4}\text{ m}^3$

HINT: 4

Given, Mass of hydrogen deposited by 1 C charge
$= 1.04 \times 10^{-8}$ kg

∴ Charge due to 1 g (10^{-3} kg) hydrogen deposited,

$q = \frac{10^{-3}}{1.04 \times 10^{-8}} = 9.61 \times 10^{-4}\text{ C}$

Energy liberated = 147000 J

Now, Back emf = $\frac{\text{Energy liberated (E)}}{\text{charge (q)}}$

HINT: 5

Given, emf, E = 50 V

Internal resistance, r = 2 Ω
Voltage of source , V = 100 V
Series resistance required, R = ?
Current, I = 2A
We know that

Total current = $\frac{\text{net emf}}{\text{total resistance}} = \frac{V - E}{R + r}$

Energy taken from mains in 8 hours = VIt

∵ 1 kWh requires 1 P for electrical energy

∴ 1.6 kWh requires 1.6 P

Now,

Energy wasted = $(I^2R + I^2r)\,t$

∴ % energy wasted = $\frac{\text{Energy wasted}}{\text{Energy taken}} \times 100\%$

MAGNETIC EFFECTS OF CURRENT

15

CHAPTER

15.1 Introduction

It is a well established fact that electricity and magnetism are interrelated fields of study. A current carrying conductor has magnetic field associated with it. Though this fact came as a surprise to the people who discovered it, this feature has become enormously important in the field of science and technology.

We will try to find the magnetic field associated with different types of current carrying conductor in this chapter.

15.2 Oersted Discovery

Hans Christian Oersted, one of the leading scientists of 19th century, found that a magnetic field is established around a current carrying conductor. In 1820, he accidently found that a compass needle got deflected when an electric current was passed through a metallic wire placed nearby as shown in Fig. 15.1. His experiment showed that electricity and magnetism are linked to each other and then, paved the way to study the several laws regarding the electromagnetism. His research later created technologies such as radio, television, fibre optics etc.

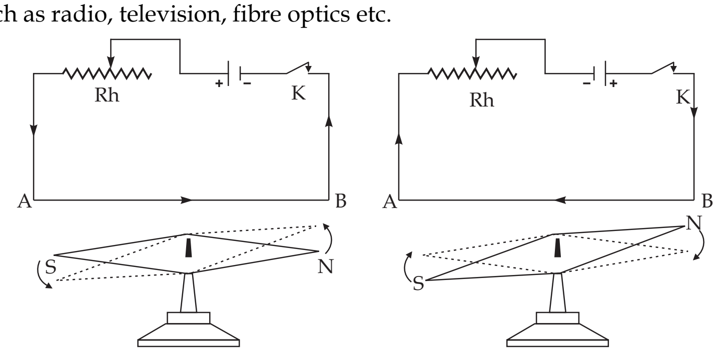


Fig. 15.1: Magnetic effect of current

15.3 Rules of Finding the Direction of Magnetic Field

Magnetic field being a vector quantity, possesses both magnitude and direction. The magnitude of the field can be found out by the implementation of various mathematical formulation which shall be discussed later in this chapter. However, the direction of the field can be determined by using any of the following rules as per the convenience.

i. Maxwell's Cork Screw Rule

According to this rule, if the direction of forward movement of a right handed screw gives the direction of current, direction of rotation of screw shows the direction of magnetic lines of force. The direction of field at any point is then along the direction of tangent at that point drawn along the direction of motion. Fig. 15.2 (i) shows right handed screw advancing forward by rotating it in clockwise direction. So, direction of rotation gives magnetic lines of force. It is also known as right handed cork screw rule.

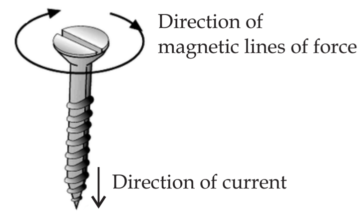


Fig. 15.2(i): Maxwell's cork screw rule

ii. Right Hand Thumb Rule

According to this rule, if a current carrying conductor is held by right hand keeping the thumb straight such that electric current is in the direction of thumb then the direction in which the fingers curl (bend) shows the direction of magnetic lines of force as shown in Fig. 15.2(ii). And again the direction of field at any point is along the direction of tangent at that point drawn along the direction of curled fingers.

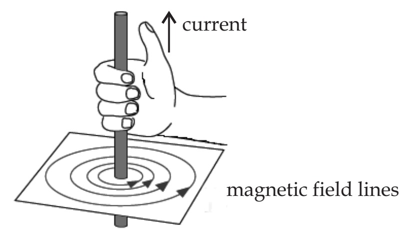


Fig. 15.2 (ii): Right hand thumb rule

For a circular conductor, if the curled fingers have the direction of current, then thumb points along the direction of magnetic field.

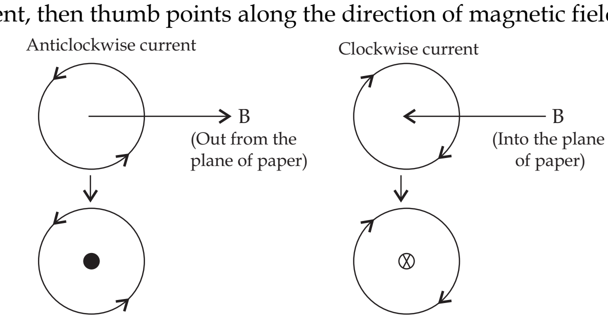


Fig. 15.3: Right hand rule for circular conductor carrying current

iii. Fleming's Left Hand Rule

This rule is used to predict the direction of force and hence the direction of motion of a current carrying conductor or any charged particle moving in the magnetic field. According to this rule, if thumb, first finger and second finger of left hand are held mutually perpendicular to each other as shown in Fig. 15.4 such that,

- **F**irst finger points the direction of **F**ield. (F to F)
- Se**C**ond finger points the direction of **C**urrent. (C to C)

Then, **T**hu**M**b points the direction of **T**hrust (force) and hence the **M**otion (T to T and M to M).

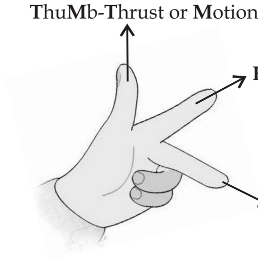


Fig. 15.4 Fleming's left hand rule

15.4 Lorentz Force

A charge particle held stationary in the magnetic field does not experience any force. But, the situation is different, if the charge is in motion. Experimentally, it has been found that, a moving charge particle experiences magnetic force which is, proportional to the magnitude of charge (q), its velocity (v) and the strength of the magnetic field (B). The direction of this force is such that, it points in the direction perpendicular to the velocity and magnetic field, mathematically it can be expressed as,

$$\vec{F}_m = q\,(\vec{v} \times \vec{B}) \qquad \text{... (15.1)}$$

or, $\vec{F}_m = Bqv \sin\theta\, \hat{n}$ (Where $\hat{n}$ is unit vector along the perpendicular to the plane containing $\vec{v}$ and $\vec{B}$ and θ is the angle between them.

In magnitude,

$$|\vec{F}_m| = Bqv \sin\theta \qquad \text{... (15.2)}$$

If the electric field is also present, then force on q due to electric field is,

$$\vec{F}_e = q\,\vec{E} \qquad \text{... (15.3)}$$

The sum of forces on moving charge q due to electric and magnetic field is called Lorentz force and is given as,

$$\vec{F} = \vec{F}_e + \vec{F}_m = q\,[\vec{E} + (\vec{v} \times \vec{B})] \text{ using equations (15.1) and (15.3)}$$

In equation (15.2), we see that,

i. If q is at rest, $v = 0$, So, $F_m = 0$.
   So, a stationary charge particle a magnetic field does not experience any force.

ii. If $\theta = 0$ or 180^o, $F_m = 0$. If charge moves parallel or antiparallel to the direction of magnetic field, then $F_m = 0$.

iii. If $\theta = 90^o$, $F_m = Bqv$ which is the maximum value of force experienced by charge q. It shows that a charge particle moving along a line perpendicular to the direction of magnetic field experiences maximum force. Since the force is always perpendicular to velocity vector $\vec{v}$, the force is centripetal in nature. Therefore, the particle moves in a circular path.

15.5 Magnetic Force on a Current Carrying Conductor

Whenever a current carrying conductor is placed in magnetic field, the moving charges within the conductor experience magnetic force and this force is transmitted to the material of the conductor as a whole. Thus, conductor experiences force distributed along its length.

Let us consider a straight segment of a conductor of length l and cross-sectional area A carrying a steady current I from bottom to top as shown in Fig. 15.5. Let it be placed in a region of uniform magnetic field $\vec{B}$ which is parallel to the plane of paper and θ be its inclination with the direction of field. We have assumed the conventional direction of current and hence the moving charges are positive.

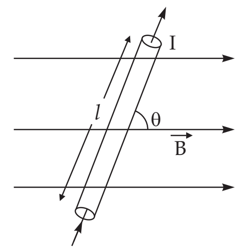


Fig. 15.5: Conductor moving in a uniform magnetic field

Let v_d be the drift velocity of these charges which also must have an inclination of θ with the magnetic field. The magnetic Lorentz force experienced by each charge of magnitude q is given by,

$$\vec{F} = q(\vec{v_d} \times \vec{B})$$

$$\mathbf{F = Bqv_d \sin\theta} \quad \text{...(15.4)}$$

The direction of this force is into the plane of paper as defined by Fleming's left hand rule.

If n be the number of charge per unit volume of the conductor, the total number of charge in the conductor is,

$$\mathbf{N = nV = nA}l \quad \text{...(15.5)}$$

So, the total magnetic force experience by all the charges in the conductor is,

$$\mathbf{F = NBqv_d \sin\theta} \quad \text{...(15.6)}$$

From equations (15.5) and (15.6), we get,

$$F = (nAl)\, Bq\, v_d \sin\theta$$

$$\therefore \quad \mathbf{F = (nqAv_d)(B}l\,\mathbf{\sin\theta)} \quad \text{...(15.7)}$$

We know, the drift velocity of the charge particle is given by,

$$v_d = \frac{I}{nAq} \quad (\because \text{ in metallic conduction, } I = nev_dA\text{, here, } e = q)$$

$$\mathbf{I = nAqv_d} \quad \text{...(15.8)}$$

So, from equations (15.7) and (15.8), we get,

$$\mathbf{F = IB}l\,\mathbf{\sin\theta} \quad \text{...(15.9)}$$

$$\vec{F} = I(\vec{l} \times \vec{B}) \quad \text{...(15.10)}$$

Thus, total force experienced by the conductor placed in uniform magnetic field is perpendicular to both length and field (perpendicularly inward into the plane of paper which contains both l and B in this case.)

Case I : If the conductor is parallel or antiparallel to the field, then

$$\theta = 0^\circ \text{ or } 180^\circ$$

$$\therefore \quad F = IBl \sin 0^\circ \text{ or } IBl \sin 180^\circ$$

$$F = 0$$

Thus, a conductor placed parallel to uniform magnetic field experiences no force.

Case II : When the conductor is placed perpendicular to magnetic field

$$\theta = 90^\circ$$

$$\therefore \quad F = IBl \sin 90^\circ = BIl$$

This is the maximum force experienced by the conductor.

15.6 Torque on Rectangular Current Loop and Magnetic Moment

When a current carrying wire is placed in external magnetic field $\vec{B}$, the wire experiences a magnetic force. Besides the values of B, I and *l*, the magnitude of the force depends also on the orientation of wire in the magnetic field. If a wire of closed current loop is placed in the uniform magnetic field, the magnitude of force relies on the orientation of area covered by the loop.

i. If the plane of enclosed area by the current loop is perpendicular to the direction of uniform magnetic field, the net force on it is zero. In this condition, the force on one half of the loop is equal and directed exactly in opposite direction of the next half part, and the line of action of these forces is the same. So, no net torque is produced.
ii. If the current carrying loop is orientated at some angle to the magnetic field, two pairs of forces do not pass through the same line of action and hence constitute a couple. The moment of this couple, called torque τ, tends to rotate the loop in the field.

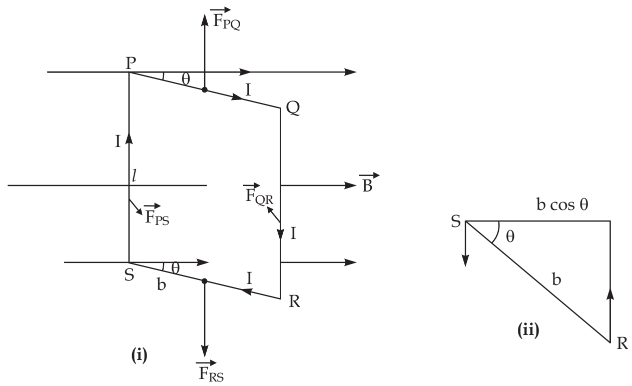


Fig. 15.6: (i) Magnetic torque on rectangular coil (ii) Showing perpendicular distance between couple

Consider a rectangular current loop PQRS of a conducting wire with linear dimension *l* and b carrying current I through it, and is placed in a uniform magnetic field $\vec{B}$. Suppose the field is directed along the horizontal direction parallel to plane of paper. The rectangular wire is so adjusted that its sides PS and QR are vertical, and sides PQ and RS are horizontal but not parallel with the field as shown in Fig. 15.6 (i). Let the plane of the loop forms an angle θ with the magnetic field $\vec{B}$.
The force acting on each side of the loop is described below.

$F_{PQ} = BIb \sin\theta$, (upward)

$F_{RS} = BIb \sin\theta$, (downward)

Here, PQ = RS = b (into the plane of paper)

In such condition, the line of action of F_{PQ} and F_{RS} is the same, but are directed in exactly opposite direction. As the magnitude of these forces is also equal, they cancel each other. Therefore, there is no displacement along the vertical direction.

Also, $F_{QR} = BIl \sin 90^o$ (perpendicularly out from the plane of paper)

$= BIl$

and $F_{PS} = BIl \sin 90^o$ (perpendicularly in from the plane of paper)

These two forces, F_{QR} and F_{PS}, have equal magnitude and are also directed in exactly opposite direction, but they do not pass through the same line of action and therefore make a couple. Thus, the wire tends to rotate about the vertical axis. Then, the moment of couple (τ) is,

τ = magnitude of either force × perpendicular distance between them,

Here, $F_{QR} = F_{PS} = BIl$

Perpendicular distance between these forces = $b \cos\theta$ as shown in Fig. 15.6 (ii)

$\therefore \quad \tau = BIl \times b \cos\theta$

$\tau = BIA \cos\theta \; (\because l \times b = A)$

If the rectangular loop contains N number of turns, total torque will be N times the value of torque by one turn. So,

$\tau = NBIA \cos\theta$

$$\boldsymbol{\tau = BINA \cos\theta} \qquad \textbf{... (15.11)}$$

The magnitude of torque is sometimes studied in terms of angle between the field $\vec{B}$ and normal to the plane of the loop as shown in Fig. 15.6(iii). Let α be the angle between $\vec{B}$ and normal ($\hat{n}$) to the plane of loop, then from geometry of figure,

$\alpha = 90^o - \theta$

or, $\theta = 90^o - \alpha$

Hence, $\tau = BINA \cos(90^o - \alpha)$

$$\boldsymbol{\tau = BINA \sin\alpha} \qquad \textbf{... (15.12)}$$

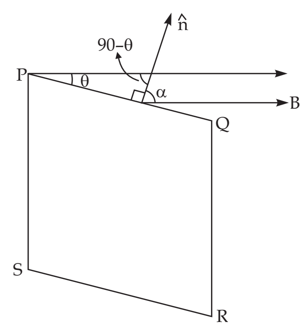


Fig. 15.6 (iii): Magnetic torque showing the plane of coil

Special cases

i. When plane of loop is perpendicular to the field $\vec{B}$,
($\theta = 90^o$, $\alpha = 0^o$)
$\tau = BINA \cos 90^o = 0$. (No torque is acted)

ii. When plane of loop is parallel to the field $\vec{B}$, ($\theta = 0^o$, $\alpha = 90^o$)
$\tau = BINA \cos 0^o = BINA$ (The torque is maximum.)

15.7 Magnetic Moment

When current flows in a closed loop, it acts as a tiny magnet in which the magnetic field is directed axially outward as shown in Fig. 15.7. In such condition, the current loop acts as a dipole. This produces the magnetic moment, $\mu = IA$. The magnetic moment of a coil containing N turns is,

$\mu = NIA$

Also, the torque, $\vec{\tau} = \vec{\mu} \times \vec{B}$

So, $\tau = \mu B \sin\theta$

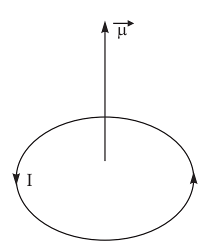


Fig. 15.7: Magnetic moment in a circular loop

The maximum torque, $\tau_{max} = \mu B$. ($\because \sin\theta = 1$ is the maximum value)

In vector form,

$$\vec{\tau} = \vec{\mu} \times \vec{B} = NI\,\vec{A} \times \vec{B}. \quad \text{... (15.13)}$$

The unit of magnetic moment is ampere square meter (Am^2)

15.8 Moving Coil Galvanometer

Moving coil galvanometer is an example of torque experienced by a coil in magnetic field that can measure small value of current. Most of the current measuring devices are made from the moving coils, which rotate due to the torque generated in it. It is also called moving coil meter or wetson galvanometer.

Principle: This galvanometer works in the principle of torque generated by the magnetic field in a current loop. When a current carrying coil is placed in magnetic field, it experiences a net torque.

Construction: It consists of a narrow rectangular coil ABCD consisting of a large number of turns of fine insulated copper wire wound over a frame of light, non-magnetic metal. A cylindrical shaped soft iron (P) known as core is placed within the coil. The coil is suspended between the two cylindrical pole-pieces of magnet by a thin flat phosphor bronze strip. The upper end of the strip is connected to a movable torsion head H. The lower end of the coil is connected to a hair-spring of phosphor bronze having only a few turns as shown in Fig. 15.8. The upper and lower end of the coil are connected to two terminals T_1 and T_2 for the connection to the external circuit.

The pole-pieces of magnet are made cylindrical so that the angle between the plane of the coil and the magnetic field is zero in all orientation of the coil. This makes the magnetic field in the small air gap between the cylindrical pole pieces radial and hence constant torque is experienced by the coil. A plane mirror is rigidly attached to the phosphor-bronze strip. This helps to measure the deflection of coil.

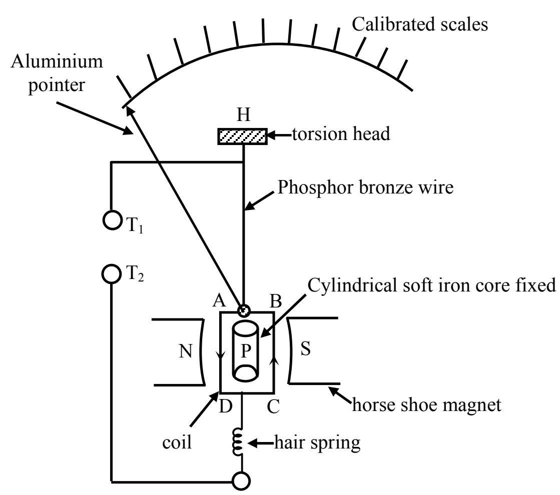


Fig. 15.8: Moving coil galvanometer

Theory: When terminals T_1 and T_2 are connected to the external electric circuit, current I flows through the coil kept into the space of pole-pieces, hence the coil experiences torque. Since the field is radial, the plane of coil remains parallel to the magnetic field in all the orientations. The sides AB and DC do not experience any force. The sides BC and AD remain perpendicular to the direction of the magnetic field and experience forces perpendicular to the plane of the coil. Force on BC is,

$$F_{BC} = NIBl \sin 90^{\circ} = NIBl$$

Where,

N = number of turns in the coil

I = Current flowing through the coil

B = magnetic field strength

l = length of BC = length of AD

The forces F_{BC} and F_{AD} are equal in magnitude and exactly opposite in direction, but the line of action is different. So, two forces constitute a couple. This couple tends to deflect the coil and is known as deflecting couple.

The moment of deflecting couple,

$$\tau_1 = NIBl \times b$$

Where, b = perpendicular distance between points of action of two forces.

$$\therefore\ \boldsymbol{\tau_1 = NIBA} \qquad \textbf{... (15.14)}$$

When the coil rotates, the suspension fibre gets twisted. Then, the restoring couple is set up in the fibre, which opposes the deflection of the coil. The restoring couple is directly proportional to the twist. Let θ be the angular twist and k be the torsion constant (restoring couple per unit angular twist) of the suspension fibre, then,

The moment of restoring couple,

$$\boldsymbol{\tau_2 = k\theta} \qquad \textbf{... (15.15)}$$

The deflection of the coil stops, when deflecting couple (τ_1) is equal to the restoring couple (τ_2). This condition is called the equilibrium condition.

In equilibrium,

$$NIBA = k\theta$$

$$\textbf{or,}\quad \mathbf{I} = \left(\frac{\mathbf{k}}{\mathbf{NBA}}\right)\theta \qquad \textbf{... (15.16)}$$

$$\textbf{or,}\quad \mathbf{I = K\theta} \qquad \textbf{...(15.17)}$$

Where, $K = \frac{k}{NBA}$ is called galvanometer constant because the value of k, N, B and A for a galvanometer remains constant.

So, from equation (15.17), we can write,

$$I \propto \theta$$

This shows, the deflection of the coil is proportional to the current flowing through it. The variation of current (also the voltage) can be measured directly from the angular deflection of galvanometer needle over a properly calibrated linear scale.

Current sensitivity

The current sensitivity of a galvanometer is defined as the angular deflection of the meter needle per unit current. It is denoted by $\frac{\theta}{I}$.

Therefore, from equation (15.16), we get,

$$\frac{\boldsymbol{\theta}}{\mathbf{I}} = \frac{\mathbf{NBA}}{\mathbf{k}} \qquad \textbf{... (15.18)}$$

The sensitivity can be increased by increasing N, A and B, and decreasing the value of torsion constant k.

Voltage Sensitivity

Voltage sensitivity of a galvanometer is defined as the angular deflection of the meter needle per unit voltage. It is denoted by $\frac{\theta}{V}$.

We have, from equation (15.16),

$$\frac{\theta}{I} = \frac{NBA}{k}$$

Dividing both sides by R (of galvanometer coil),

$$\text{or,} \quad \frac{\theta}{IR} = \frac{NBA}{kR}$$

$$\therefore \quad \mathbf{\frac{\theta}{V} = \frac{NBA}{kR}} \qquad \text{... (15.19)}$$

The sensitivity can be increased by increasing N, A and B and decreasing k and R.

Variation of N, A, B, k and R

i. N can be increased by increasing the number of turns of the coil.
ii. Magnetic field B can be increased by using a strong magnet.
iii. A is the area enclosed by the coil, which can be increased by winding the coil over a larger frame.
iv. k can be decreased by using the material of low torsion constant, so phosphor-bronze for suspension is used.
v. R can be decreased by using the low resistivity coil like copper.

15.9 Biot-Savart Law

Biot-Savart Law is a mathematical tool devised to calculate the magnitude of magnetic field due to steady current distribution. This law was first discovered by Jean-Baptiste Biot and Felix Savart towards the beginning of 19th century. This law is used to find net magnetic field due to any distribution of currents by first writing differential magnetic field ($\overrightarrow{dB}$) due to a current-length element and summing the contributions of $\overrightarrow{dB}$ from all the elements. For this, we mentally divide the current carrying conductor into differential (small) length dl and then define a length vector $\overrightarrow{dl}$ for each element which we call length element. Obviously, this is a vector that has length dl and direction is the direction of current in dl. The product of current (I) and length element ($\overrightarrow{dl}$) i.e ($I\overrightarrow{dl}$) is called a differential current-length element (or current element).

Thus, net field at any point is the superposition of differential magnetic field due to all such current-length element of the conductor. Experimentally, it has been found that, the magnitude of differential magnetic field (dB) at point P at distance r due to current length element $I\overrightarrow{dl}$ is,

i. Directly proportional to the current element Idl,

$$dB \propto Idl$$

ii. Inversely proportional to the square of radial distance (r),

$$dB \propto \frac{1}{r^2}$$

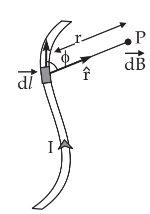


Fig. 15.9: Magnetic field by current element

iii. Directly proportional to the sine angle between $\vec{dl}$ and $\vec{r}$,

$$dB \propto \sin\phi$$

Combining above conditions (i), (ii) and (iii), we get,

$$dB \propto \frac{\mu_0}{4\pi} \frac{Idl \sin\phi}{r^2}$$

$$dB = k\frac{Idl \sin\phi}{r^2}$$

where k is proportionality constant. In SI system, $k = \frac{\mu_0}{4\pi}$ and in CGS system, $k = \frac{1}{c}$, c is speed of light. We use SI system throughout the book. So,

$$dB = \frac{\mu_0}{4\pi} \frac{Idl \sin\phi}{r^2} \text{ (Magnitude)}$$

$$\vec{dB} = \frac{\mu_0}{4\pi} \frac{I\,\vec{dl} \times \hat{r}}{r^2} \text{ (with direction)} \qquad \text{... (15.20)}$$

Where $\hat{r}$ is a unit vector that points from dl towards P. θ is the angle between the directions of $\vec{dl}$ and $\hat{r}$, and μ_0 is the absolute permeability and its value is $4\pi \times 10^{-7}$ Hm^{-1}.

If medium is other than vacuum, then,

$$\vec{dB} = \frac{\mu_0\mu_r}{4\pi} \frac{I\,\vec{dl} \times \hat{r}}{r^2} \text{ Where } \mu_r = \frac{\mu}{\mu_0} \text{ is the relative permeability.}$$

In equation (15.20), $\vec{dB}$ is a vector which is perpendicular to plane containing $\vec{dl}$ and $\hat{r}$. So, the direction of $\vec{dB}$ is along the direction of perpendicular to the plane containing $\vec{dl}$ and $\hat{r}$, in this case, perpendicularly outward from the plane of paper shown by ⊙.

15.10 Applications of Biot-Savart's Law

i. Magnetic field due to an infinitely long straight conductor carrying current

Let us consider an infinitely long straight conductor carrying steady current I as shown in Fig. 15.10. Let P be a point, at a perpendicular distance x from the conductor where the magnetic field $\vec{B}$ is to be determined. The calculation of magnetic field due to this conductor explicitly exploits the application of Biot-Savart's law.

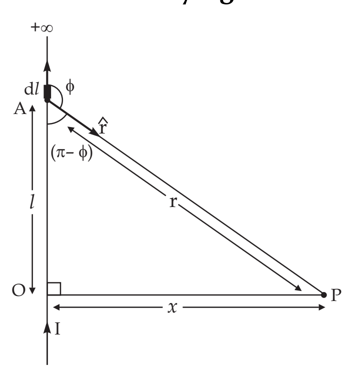


Fig. 15.10: Magnetic field by straight conductor

For this, we consider an element of conductor of length dl = dy at point A as shown in Fig. 15.10 which is r distance away from point P and x distance away from point O. Also, ϕ be the angle between length dl and unit vector $\hat{r}$ along the line AP (= r) which joins the length element and point P.

The magnetic field $\overrightarrow{dB}$ at point P due to this elemental length can be written from Biot-Savart's law as,

$$\overrightarrow{dB} = \frac{\mu_0}{4\pi} \frac{I\overrightarrow{dl} \times \hat{r}}{r^2} \quad \text{...(15.21)}$$

In equation (15.21), the vector $\overrightarrow{dl} \times \hat{r}$ shows that direction of $\overrightarrow{dB}$ is perpendicular to the plane of paper.

The magnitude of this magnetic field can be written as,

$$dB = \frac{\mu_0}{4\pi} \frac{Idl \sin\phi}{r^2} \quad \text{...(15.22)}$$

Now, from Fig. 15.10 in right angled triangle AOP,

$$r^2 = x^2 + l^2 \quad \text{...(15.23)}$$

$$\text{and} \quad \sin(\pi - \phi) = \frac{x}{r}$$

$$\sin\phi = \frac{x}{(x^2 + l^2)} \quad \text{...(15.24)}$$

From equations (15.22), (15.23) and (15.24), we can have,

$$dB = \frac{\mu_0 I}{4\pi} \frac{x dl}{(x^2 + l^2)^{3/2}} \quad \text{...(15.25)}$$

Since, the conductor can be imagined to consist of infinite number of length elements each of length dl and each contributing magnetic field $\overrightarrow{dB}$ along the same direction (perpendicular to the plane of paper), the net magnetic field P due to entire wire can be obtained by superposition (sum) of fields due to individual length elements.

Thus, magnitude of total magnetic field is,

$$B = \int_{-\infty}^{\infty} dB = \int_{-\infty}^{\infty} \frac{\mu_0 I}{4\pi} \frac{x dl}{(x^2 + l^2)^{3/2}} \quad \text{...(15.26)}$$

Further, referring to Fig. 15.10, we get,

$$\cot(180 - \phi) = \frac{l}{x}$$

$$\text{or,} \quad -\cot\phi = \frac{l}{x} \quad \text{...(15.27)}$$

Differentiating above expression with respect to ϕ, we get

$$dl = x \operatorname{cosec}^2\phi \, d\phi \quad \text{...(15.28)}$$

Also, from equation (15.27), we get,

$$l = -x \cot\phi$$

Squaring both sides,

$$l^2 = x^2 \cot^2\phi$$

Adding x^2 both sides,

$$x^2 + l^2 = x^2 + x^2 \cot^2\phi = x^2 \operatorname{cosec}^2\phi \quad \text{...(15.29)}$$

And, from equation (15.27), we can write,

$$\tan\phi = -\frac{x}{l}$$

Thus, when $l \to \infty$, $\phi \to \pi$ and when $l \to -\infty$, $\phi \to 0$

Using equations (15.28) and (15.29) in equation (15.26) and changing the limiting values, we can write,

$$B = \frac{\mu_0 I}{4\pi}\int_0^{\pi} \frac{x \cdot x\,\text{cosec}^2\phi\, d\phi}{(x^2\,\text{cosec}^2\phi)^{3/2}} = \frac{\mu_0 I}{4\pi}\int_0^{\pi} \frac{\sin\phi\, d\phi}{x}$$

$$\text{or,}\quad B = \frac{\mu_0 I}{4\pi x}[-\cos\phi]_0^{\pi} = \frac{\mu_0 I}{4\pi x}[\cos\pi + \cos 0]$$

$$\text{or,}\quad B = \frac{\mu_0 I}{4\pi x}[-(-1) + 1]$$

$$\therefore \quad \mathbf{B = \frac{\mu_0 I}{2\pi x}} \qquad \textbf{...(15.30)}$$

This is the required expression for the magnitude of the magnetic field due to an infinitely long conductor at any point x distance away from it. The result implies that, even though the conductor is infinite, magnetic field due to it is not infinite and tends to be infinite when x tends to zero. The field is directly proportional to the current through it and inversely proportional to the distance from the conductor.

It is interesting to note from above expression that, the magnetic field $\vec{B}$ has same magnitude at all points on a circle of radius x centred on the conductor and lying in the plane perpendicular to conductor itself. The direction of field at any point on this circle is along the tangent to it at that point. Thus, direction of field must be everywhere along the tangents to such circle. We can simply find the direction of magnetic field by using Right-hand rule.

ii. Magnetic field due to a circular coil

a. **At the centre of the coil:** Let us consider a narrow circular coil carrying steady current I. Let O be its centre and 'a' be its radius such that OP = a as shown in Fig. 15.11.

To find magnetic field due to the coil at its centre, let us assume the coil to consist of small length elements each of length d*l*. The magnetic field at O due to any such length element d*l* at P is then given by Biot-Savart's law as,

$$\mathbf{d\vec{B} = \frac{\mu_0}{4\pi}\frac{I d\vec{l} \times \hat{a}}{a^2}} \qquad \textbf{...(15.31)}$$

Where, $\hat{a}$ is a unit vector along PO.

In the above equation (15.31), the vector product $\vec{dl} \times \hat{a}$ shows that, direction of field is perpendicular to the plane of paper (shown by ⊙ at the centre O).

The magnitude of this magnetic field can be written as,

$$dB = \frac{\mu_0 I dl \sin\phi}{4\pi a^2} = \frac{\mu_0 I dl \sin 90^{\circ}}{4\pi a^2}$$

Here, $\phi = 90^{\circ}$ is the angle between length element and radius a.

$$\therefore \quad \mathbf{dB = \frac{\mu_0}{4\pi} \frac{Idl}{a^2}} \qquad \ldots(15.32)$$

Similarly, the magnetic field due to all other length elements are considered along the circumference of the circular coil acts along the same direction (perpendicularly out of the plane of paper as shown by ⊙ in Fig. 15.11). So, the total magnetic field is the sum of magnetic fields due to individual length elements. Thus, magnitude of total magnetic field is,

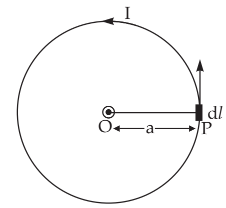


Fig. 15.11: Magnetic field at the centre of circular coil

$$\mathbf{B = \int_0^{2\pi a} \frac{\mu_0}{4\pi} \frac{Idl}{a^2}} \qquad \ldots(15.33)$$

Here, total length of circular coil = circumference of coil = $2\pi a$

$\therefore$ From equation (15.33), we can have,

$$B = \frac{\mu_0 I}{4\pi a^2} \int_0^{2\pi a} dl = \frac{\mu_0 I}{4\pi a^2} \cdot (2\pi a) = \frac{\mu_0 I}{2a}$$

If the coil consists of N number of turns,

$$B = \frac{\mu_0 NI}{2a}$$

Note

Magnetic field at the center of hydrogen atom

In neutral hydrogen atom, an electron revolves around the nucleus. The nucleus of hydrogen atom contains one proton and is much heavier than an electron. So, the proton is taken relatively at rest. The magnetic field intensity at the center of the atom can be considered as the sole contribution of motion of electron in its orbit. In this situation, the motion of electron in its orbit is compared to the electric current in a current loop.

The magnetic field at the centre, given by current through a circular coil is,

$$B = \frac{\mu_0 IN}{2a} \qquad \ldots(i)$$

Where, a is the radius of electronic orbit of hydrogen atom. N is the total number of revolution of electron around the nucleus. This can be considered as that there are N turns of coil around the nucleus of hydrogen atom and each turn contains one electron.

Also, $q = e$

So, $I = \frac{q}{t} = \frac{e}{t} \qquad \ldots(ii)$

Using equation (ii) in equation (i), we get

$$B = \frac{\mu_0 e}{2at} \cdot N$$

$$= \frac{\mu_0}{2a}\left(\frac{N}{t}\right) \cdot e$$

$$B = \frac{\mu_0}{2a} f e$$

Where $f = \frac{N}{t}$ *is the frequency of revolution of electron around the nucleus.*

b. **At any point on the axis of the coil:** Let us consider a circular coil of mean radius 'a' carrying a steady current I in anti-clockwise direction. Let the coil lies in the Y-Z plane such that its axis is along the X-axis. Let 'P' be a point at a distance x on the x-axis from centre O, where the magnetic field is to be determined as shown in Fig. 15.12.

Let us consider an elemental length 'dl' on the upper half of the coil such that the current element $I\vec{dl}$ at C is along Z-axis out of the plane of the paper and perpendicular to it. Let r be the distance between elemental length and the point P such that the unit vector $\hat{r}$ is perpendicular to the $I\vec{dl}$ and on the plane paper.

Then, from Biot-Savart's law, the magnetic field at 'P' due to this length element is,

$$\vec{dB} = \frac{\mu_0}{4\pi}\frac{I\vec{dl} \times \hat{r}}{r^2} \quad \text{...(15.34]}$$

The vector product $(\vec{dl} \times \hat{r})$ shows that, the direction of magnetic field is along PQ perpendicular to $\hat{r}$ and $\vec{dl}$ and lies in the plane of paper.

The magnitude of magnetic field at P is given by,

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl \sin\alpha}{r^2}$$

But, the angle between $\vec{dl}$ and $\hat{r}$ is $\alpha = 90^o$. So,

$$dB = \frac{\mu_0 I dl}{4\pi r^2} \quad \text{...(15.35)}$$

From geometry of Fig. 15.12, the angle made by dB with X-axis is θ. So, the components of dB are,

$$dB_x = dB\cos\theta = \frac{\mu_0}{4\pi}\cdot\frac{Idl\cos\theta}{r^2} \text{ (along X-axis)} \quad \text{...(15.36)}$$

and $$dB_y = dB\sin\theta = \frac{\mu_0}{4\pi}\cdot\frac{Idl}{r^2}\sin\theta \text{ (along Y-axis)} \quad \text{...(15.37)}$$

Since, the problems has rotational symmetry, we can find an opposite current element at D in the diametrically opposite half as shown in Fig. 15.12. The current element at D will be into the plane of paper and perpendicular to it. The field due to this current element will have the same magnitude dB but its direction will be along PR as shown in Fig.15.12. Similarly, for every length element dl, we can always find an opposite length element carrying current in opposite direction.

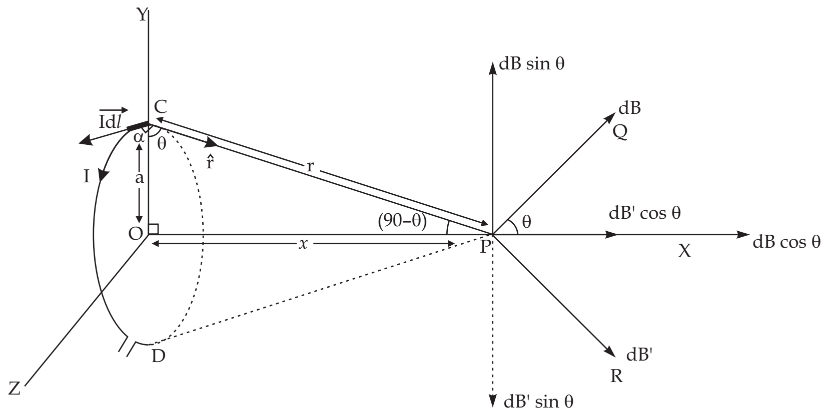


Fig. 15.12: Magnetic field at the axis of circular coil

These two elements give equal contributions to the X-components of $\overrightarrow{dB}$ but opposite components perpendicular to X-axis. So, these perpendicular components cancel each other so that the net field at P will be due to the fields along x-axis.

The net field at P, due to the whole coil is given by,

$$B = \int_0^{2\pi a} dB_x = \frac{\mu_0}{4\pi}\int_0^{2\pi a} \frac{Idl \cos\theta}{r^2} = \frac{\mu_0 I}{4\pi r^2}\cos\theta \int_0^{2\pi a} dl$$

$$= \frac{\mu_0 I}{4\pi r^2}\cos\theta \cdot [2\pi a - 0] = \frac{\mu_0 I}{4\pi r^2}\cos\theta \cdot 2\pi a$$

$$\therefore \quad \mathbf{B = \frac{\mu_0 I\, a}{2 r^2}\cos\theta} \qquad \text{...(15.38)}$$

Further, from Fig. (15.12) in triangle COP, we can have,

$$\cos\theta = \frac{a}{r} \text{ and } r = (a^2 + x^2)^{1/2}$$

∴ Equation (15.38) can be written as,

$$B = \frac{\mu_0 I a^2}{2(a^2 + x^2)^{3/2}}$$

If the coil consists of N-turns, then,

$$\mathbf{B = \frac{\mu_0\, NI\, a^2}{2(a^2 + x^2)^{3/2}}} \qquad \text{...(15.39)}$$

If A is the area of each turn, then,

$$A = \pi a^2 \Rightarrow a^2 = \frac{A}{\pi}$$

$$\therefore \quad \mathbf{B = \frac{\mu_0\, NIA}{2\pi\,(a^2 + x^2)^{3/2}}} \qquad \text{...(15.40)}$$

Special cases

1. When point P lies at the centre of coil, x = 0, so from equation (15.39)

$$B = \frac{\mu_0\, NI}{2a} \quad \text{[Maximum value]}$$

2. When P lies on the axis of coil at a distance equal to the radius of the coil, x = a, again, from equation (15.39), we can have

$$B = \frac{\mu_0 NI}{\sqrt{2^5}\, a}$$

3. When the point P lies on the axis at a distance far away from the centre of coil, such that x >> a, then $a^2 + x^2 \approx x^2$ (a can be neglected as compared to x). From equation (15.39), we can have,

$$B = \frac{\mu_0 NIa^2}{2x^3}$$

iii. Magnetic Field due to Solenoid

A solenoid can be thought of a being made up of many narrow identical circular coils placed side by side. We know, for any narrow circular coil of radius a the magnitude of the magnetic field at a point on its axis is given by equation (15.13),

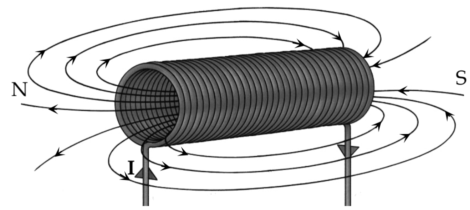


Fig. 15.13 Solenoid

$$\mathbf{dB = \frac{\mu_0 Ia^2}{2r^3}} \quad \ldots(15.41)$$

Where, r is the radius vector from coil to the point on axis.

Consider a transverse section of a long solenoid in which the ⊙ on the side PQ shows the current coming out of the paper and ⊗ on side RS shows the current entering into the plane of paper. Let its length be L and it is long in the sense that its diameter is small as compared to length. Let us consider a point A on the axis YY' of the solenoid as shown in Fig. 15.14 at which the magnetic field is to be determined.

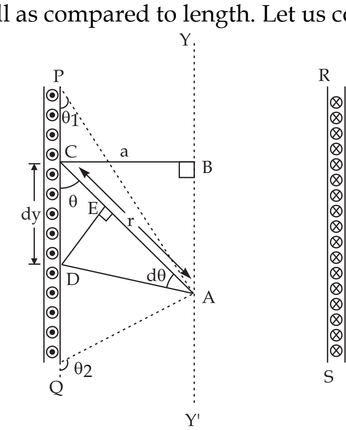


Fig. 15.14: Magnetic field at the axis of Solenoid

Now, let us consider an elemental portion CD of thickness dy and dθ be the angle subtended by this portion at A as shown in Fig. 15.14. If 'n' be the number of turns per unit length of solenoid, then magnetic field at point A due to this elemental portion is,

$$\mathbf{dB = \frac{\mu_0 Ia^2}{2r^3}\, ndy} \quad \ldots(15.42)$$

In Fig. 15.14, AC ≈ AD = r is the radius vector and ∠DCA = θ is the angle between radius vector and elemental portion DC. From geometry,

$$\angle DCA = \angle CAB = \theta$$

In ΔDCE,

$$\sin\theta = \frac{DE}{CD} = \frac{DE}{dy}$$

$$\mathbf{dy = \frac{DE}{\sin\theta}}$$

$$\mathbf{DE = dy \sin\theta} \quad \ldots(15.43)$$

Again, in ΔDAE,

$$\sin d\theta \approx d\theta = \frac{DE}{DA} = \frac{DE}{r}$$

$$\mathbf{DE = rd\theta} \qquad \text{...(15.44)}$$

From equations (15.43) and (15.44),

$$\mathbf{rd\theta = dy \sin\theta}$$

$$\therefore \quad \mathbf{dy = \frac{rd\theta}{\sin\theta}} \qquad \text{...(15.45)}$$

Now from D, a perpendicular DE to line AC is drawn.

So, from ΔCAB,

$$\sin\theta = \frac{a}{r}$$

$$\mathbf{a = r\sin\theta} \qquad \text{...(15.46)}$$

Using the values from (15.45) and (15.46) in equation (15.42) we get,

$$dB = \frac{\mu_0 I}{2r^3}(r\sin\theta)^2 \cdot \frac{nrd\theta}{\sin\theta}$$

$$\therefore \quad \mathbf{dB = \frac{1}{2}\mu_0 \cdot nI\sin\theta \cdot d\theta} \qquad \text{...(15.47)}$$

In a solenoid, as we move from one end to another end, there is variation of θ subtend at A by each elemental portion of thickness dy. So, total magnetic field due to solenoid can be obtained by integrating equation (15.47) between limits θ_1 to θ_2.

$$\therefore \quad B = \int_{\theta_1}^{\theta_2} \frac{1}{2}\mu_0 nI\sin\theta \cdot d\theta = \frac{\mu_0 nI}{2}[-\cos\theta]_{\theta_1}^{\theta_2}$$

$$= \frac{\mu_0 nI}{2}[-\cos\theta_2 - (-\cos\theta_1)]$$

$$\therefore \quad B = \frac{\mu_0 nI}{2}(\cos\theta_1 - \cos\theta_2]$$

If the solenoid is infinitely long, then θ varies from 0^o to 180^o as we move from one extreme end to another extreme end. So,

$$B = \frac{1}{2}\mu_0 nI(\cos 0^o - \cos 180^o) = \frac{1}{2}\mu_0 nI[1 - (-1)]$$

$$\therefore \quad B = \mu_0 nI$$

If N be the total number of turns, then, $n = \frac{N}{L}$

$$\therefore \quad B = \frac{\mu_0 NI}{L}$$

iv. Helmholtz Coil

A Helmholtz coil is a parallel pair of identical circular coils spaced one radius apart and arranged co-axially such that same current flows through both the coils in same direction. The basic principle of Helmholtz coil is that, it produces a homogenous magnetic field in its centre which is directly proportional to the number of turns in the coils and the current applied to them.

Let us consider two identical coils each of radius a and carrying steady current I which are arranged co-axially separated by a distance a from each other as shown in Fig. 15.15.

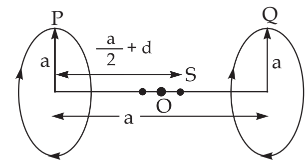


Fig. 15.15: Magnetic field by Helmholtz coil

Let S be any point on the axis d distance away from mid-point O where magnetic field is to be determined.

If N be the number of turns in each coil, then magnetic field at S due to coil P is

$$B_P = \frac{\mu_o N I a^2}{2\left[\left(\frac{a}{2}+d\right)^2 + a^2\right]^{\frac{3}{2}}}$$

Similarly, magnitude field at S due to coil Q is,

$$B_Q = \frac{1}{2}\frac{\mu_o N I a^2}{\left[\left(\frac{a}{2}-d\right)^2 + a^2\right]^{\frac{3}{2}}}$$

The resultant field at S due to both the coils is the sum of B_P and B_Q as they are directed along the same direction. Because they carry current is same direction.

$$\text{So, } B = B_P + B_Q = \frac{\mu_o N I a^2}{2}\left[\frac{1}{\left\{\left(\frac{a}{2}+d\right)^2 + a^2\right\}^{\frac{3}{2}}} + \frac{1}{\left\{\left(\frac{a}{2}-d\right)^2 + a^2\right\}^{\frac{3}{2}}}\right]$$

$$= \frac{\mu_0 N I a^2}{2}\left[\frac{1}{\left\{\left(\frac{a^2}{4}+ad+d^2\right) + a^2\right\}^{\frac{3}{2}}} + \frac{1}{\left\{\left(\frac{a^2}{4}-ad+d^2\right) + a^2\right\}^{\frac{3}{2}}}\right]$$

$$= \frac{\mu_0 N I a^2}{2}\left[\frac{1}{\left\{\frac{a^2}{4}\left(1+\frac{4d}{a}+\frac{4d^2}{a^2}\right) + a^2\right\}^{\frac{3}{2}}} + \frac{1}{\left\{\frac{a^2}{4}\left(1-\frac{4d}{a}+\frac{4d^2}{a^2}\right) + a^2\right\}^{\frac{3}{2}}}\right]$$

Here d << a, so, neglecting higher power of $\left(\frac{2d}{a}\right)$ we get,

$$B = \frac{\mu_0 N I a^2}{2}\left[\frac{1}{\left\{\frac{a^2}{4}\left(1+\frac{4d}{a}\right) + a^2\right\}^{\frac{3}{2}}} + \frac{1}{\left\{\frac{a^2}{4}\left(1-\frac{4d}{a}\right) + a^2\right\}^{\frac{3}{2}}}\right]$$

$$= \frac{\mu_0 N I a^2}{2}\left[\frac{1}{\left(\frac{a^2}{4}+ad+a^2\right)^{\frac{3}{2}}} + \frac{1}{\left(\frac{a^2}{4}-ad+a^2\right)^{\frac{3}{2}}}\right]$$

$$= \frac{\mu_0 N I a^2}{2}\left[\frac{1}{\left(\frac{5a^2}{4}+ad\right)^{\frac{3}{2}}} + \frac{1}{\left(\frac{5a^2}{4}-ad\right)^{\frac{3}{2}}}\right]$$

$$= \frac{\mu_0 N I a^2}{2}\left[\frac{1}{\left(\frac{5a^2}{4}\right)^{\frac{3}{2}}\left(1+\frac{4d}{5a}\right)^{\frac{3}{2}}} + \frac{1}{\left(\frac{5a^2}{4}\right)^{\frac{3}{2}}\left(1-\frac{4d}{5a}\right)^{\frac{3}{2}}}\right]$$

$$= \frac{\mu_0 N I a^2}{2}\left(\frac{4}{5a^2}\right)^{\frac{3}{2}}\left[\left(1+\frac{4d}{5a}\right)^{\frac{-3}{2}} + \left(1-\frac{4d}{5a}\right)^{\frac{-3}{2}}\right]$$

$$= \frac{\mu_0 N I a^2}{2}\left(\frac{4}{5}\right)^{\frac{3}{2}} \cdot \frac{1}{\left(a^2\right)^{\frac{3}{2}}}\left[1 - \frac{3}{2}\left(\frac{4d}{5a}\right) + ... + 1 + \frac{3}{2}\left(\frac{4d}{5a}\right) - ......\right]$$

($\because$ Using binomial expansion and neglecting higher powers of $\frac{4d}{5a}$)

$$= \frac{\mu_0 N I}{2a}\left(\frac{4}{5}\right)^{\frac{3}{2}} [1 + 1]$$

$$= \frac{\mu_0 N I}{2a}\left(\frac{4}{5}\right)^{\frac{3}{2}} = 0.72\frac{\mu_0 N I}{a} \text{ (approx.)} \qquad ... (15.48)$$

Now, at mid-point 0,

$$B = 2 \times \text{magnetic field due to each} = 2.\frac{\mu_0 N I a^2}{2\left(\frac{a^2}{4} + a^2\right)^{\frac{3}{2}}}$$

$$= \frac{\mu_0 N I a^2}{\left(\frac{5a^2}{4}\right)^{\frac{3}{2}}} = 0.72\frac{\mu_0 N I}{a} \qquad ...(15.49)$$

Thus, from equations (15.48) and (15.49) we see that field over small region around mid-point is nearly uniform.

15.11 Statement of Ampere's Circuital Law

Ampere's law is an useful mathematical tool which relates net magnetic field along a closed loop to the electric current passing through the loop. This law is more convenient while calculating the magnetic fields of current distribution with a high degree of symmetry.

This law states that, "*the line integral around a closed path of the component of the magnetic field tangent to the direction of the path equals μ_o times the current intercepted by the area within the path.*"

i.e. $\oint \vec{B} . \vec{dl} = \mu_0 \times$ (net current)

To state it in more simpler scalar form,

$$\oint B_{||}\, dl = \mu_0 \times \text{(net current)} \qquad ... (15.50)$$

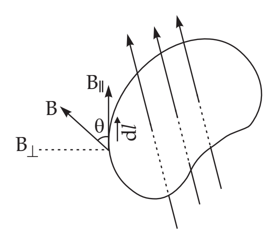


Fig. 15.16: Magnetic field through closed surface

The closed path over which the integration is carried out is called amperian surface.

In order to apply ampere's law, all current should be steady. Currents have to be taken with their algebraic sign for example, if currents going out of the surface are positive, then those going in are negative. The net magnetic field is zero only when net current is zero or when magnetic field is normal to the selected path at any point.

15.12 Application of Ampere's circuital law

i. **Magnetic field due to a long straight conductor carrying current:** let us consider an infinitely long straight conductor which carries a steady current I in upward direction as shown in Fig. 15.17. Let us consider an arbitrary point A, which is r distance away from the conductor where the magnetic field is to be determined.

Consider an Amperian loop in the form of a circle of radius r passing through point A. Also, dl be an elemental length of this Amperian loop as shown in Fig. 15.17. The direction of the magnetic field at all points of the circular loop is along the direction of tangent to that point. So, magnetic field $\vec{B}$ and length element $\vec{dl}$ are parallel to each other at the point of consideration i.e. the angle θ between them is 0°.

To calculate, the magnitude of magnetic field, applying Ampere's law in the closed loop,

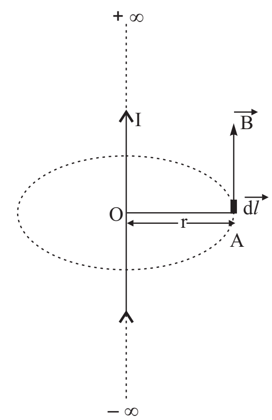


Fig. 15.17: Magnetic field by Straight conductor

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I$$

$$\oint B\, dl \cos\theta = \mu_0 I$$

$$\oint \mathbf{B}dl \cos 0^\circ = \mu_0 \mathbf{I}$$

$$\oint \mathbf{B}dl = \mu_0 \mathbf{I} \qquad \text{...(15.51)}$$

At all points at a distance r from the conductor, the magnitude of magnetic field is constant. So, equation (15.51) can be written as,

$$B\oint dl = \mu_0 I$$

or, $$\mathbf{B}l = \mu_0 \mathbf{I} \qquad \text{...(15.52)}$$

But, $l = 2\pi r$ is the circumference of Amperian loop.

$$B \cdot 2\pi r = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

This is the required expression for magnetic field due to a long straight conductor carrying steady current.

ii. **Magnetic field due to a solenoid:** Let us consider a transverse section of a solenoid as shown in Fig. 15.18 in which the upper view of dots ⊙ represents the current coming out of paper and lower view of cross ⊗ represents the current going into the plane of paper.

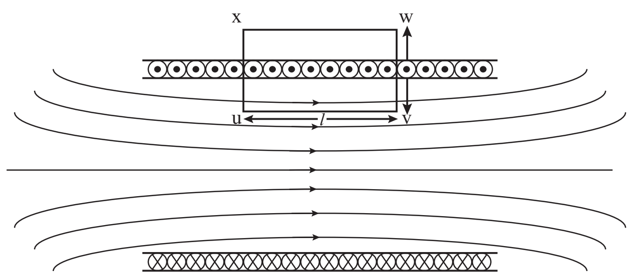


Fig. 15.18 Transverse section of solenoid

The magnetic field (B) inside the solenoid is uniform but at places outside it, magnetic field is negligible (zero for ideal solenoid).

Let us consider a rectangular Amperian loop uvwx as shown in Fig. 15.18 such that uv = wx = l and vw = xu = h.

Applying Ampere's law in closed rectangular loop,

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 \times \text{net current enclosed by the loop} \qquad \ldots(15.53)$$

Here,

$$\oint \vec{B} \cdot \vec{dl} = \int_u^v \vec{B} \cdot \vec{dl} + \int_v^w \vec{B} \cdot \vec{dl} + \int_w^x \vec{B} \cdot \vec{dl} + \int_x^u \vec{B} \cdot \vec{dl}$$

Evaluating these integrals individually, we get,

$$\int_u^v \vec{B} \cdot \vec{dl} = \int_u^v B\, dl \cos 0^o = B\int_u^v dl = Bl \quad (\because \text{Angle } \theta \text{ between B and d}l \text{ is zero.})$$

$$\int_v^w \vec{B} \cdot \vec{dl} = \int_v^w B\, dl \sin 90^o = 0 \quad (\because \text{Angle } \theta \text{ between B and d}l \text{ (along vw) is } 90^o)$$

$$\int_{w}^{x} \vec{B} \cdot \vec{dl} = 0 \qquad (\because \text{Field is almost zero outside for ideal solenoid})$$

$$\int_{x}^{u} \vec{B} \cdot \vec{dl} = \int_{x}^{u} B\, dl \cos 90^{o} = 0$$

Again, if n be the number of turns per unit length of the solenoid, number of turns in length l = nl

If I be the current in each turn, net current enclosed by the loop = nIl

So, equation (15.53) becomes,

$$\oint B\, dl = \mu_0\, nIl$$

or, $\quad Bl = \mu_0\, nIl$

$\Rightarrow \quad B = \mu_o\, nI$

iii. **Magnetic field due to toroid:** A toroid is a hollow circular ring consisting of large number of turns of insulated wire which are tightly wound so as to form a dough-nut shape. A toroid can be thought of as a solenoid which has been bent to close on itself forming circular shape.

If the turns are closely spaced then we call it as ideal toroid. For such toroid, the magnetic field is confined within the hollow region inside it and is constant in magnitude. However, the magnetic field outside this hollow region of ideal toroid is zero. The magnetic field lines inside the toroid are close circles concentric with toroid itself. Hence, direction of field is along the tangent at any point of such circles.

Consider transverse section of toroid as shown in Fig. 15.19 in which dots ⊙ on the outer part represent the current coming out and crosses ⊗ in the interior side represent the current going into the plane of paper. The direction of magnetic field is thus clockwise based on right hand thumb rule for circular loops.

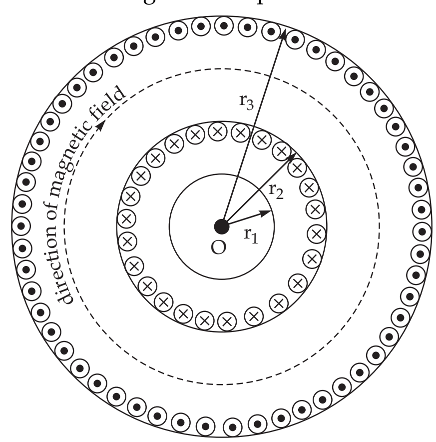


Fig. 15.19: Magnetic field due to toroid

Let us consider three Amperian circular loops each of radius r_1, r_2 and r_3 as shown in Fig. 15.19. For each loop the magnetic field should be tangential to them and constant in magnitude. Let B_1 be the magnetic field for innermost loop of radius r_1. Applying Ampere's law in this loop,

$$\oint B_1\, dl = \mu_0 \times \text{net current enclosed by the loop}$$

or, $B_1 \oint dl = \mu_0 \times$ net current enclosed by the loop

Here,

$$\oint dl = 2\pi r_1$$

and net current enclosed by this loop is zero. So,

$$B_1 = 0$$

This means, magnetic field at such points in the open space inside the toroid is zero.
Similarly, considering outermost circular amperian loop of radius r_3,

$$\oint B_3\, dl = \mu_0 \times \text{net current enclosed}$$

Here, B_3 is the magnetic field for outermost loop and is constant in magnitude.

$$\therefore \quad B_3 \oint dl = \mu_0 \times \text{net current enclosed}$$

Now, $\oint dl = 2\pi r_3$ (circumference of outermost loop) and

net current enclosed = $\Sigma I = 0$

[$\because$ Current coming out of paper is cancelled exactly by current going into the plane paper]

$$\therefore \quad B_3 = 0$$

Thus, magnetic field outside the toroid is also zero. This is the case of ideal toroid. However, small field exists outside the toroid in reality.

To find, magnetic field inside the toroid, consider an amperian loop of radius r_2 as shown in Fig. 15.19.

If B_2 be the magnetic field for this loop, applying Ampere's law,

$$\oint B_2 \cdot dl = \mu_0 \times \text{net current enclosed}$$

Here, B_2 is constant in magnitude and B_2 and dl are parallel at each point of the circular loop. So,

$$\oint B_2\, dl = \oint B_2\, dl \cos\theta = B_2 \oint dl \cos 0^o = B_2 \oint dl$$

$$\therefore \quad B_2 \oint dl = B_2\, 2\pi r_2 = 2\pi B_2 r_2$$

If n be the number of turns per unit length of the toroid and I be the current through each turn, then

$$\begin{aligned} \text{Total current enclosed} &= \text{current in each turn} \times \text{total number of turns} \\ &= I \times n \times 2\pi r_2 \\ &= 2\pi n I r_2 \end{aligned}$$

Therefore, equation (15.54) can be written as,

$$2\pi B_2 r_2 = \mu_0\, 2\pi n I r_2$$

$$\mathbf{B_2 = \mu_0\, nI} \quad \text{...(15.54)}$$

Thus, magnetic field inside the toroid is independent of the radius

15.13 Force between two conductors carrying current

It is now the well established fact that, every current carrying conductor has a magnetic field of its own. So, whenever two conductors carrying current are placed near to one another, they exert magnetic force on each other. The magnitude of these forces depends upon the magnitude of current carried by each and their distance of separation. The nature of the force may be attractive or repulsive, again depending upon the direction of current in each.

15.14 Magnetic force between two parallel conductors

i. **When currents are in the same direction:** Fig. 15.20 shows two infinitely long straight conductors A and B in the form of wires placed parallel, r distance away from each other. Let I_A and I_B be the steady currents flowing on wires A and B respectively from bottom to top (i.e., along same direction).

   For each of the wire, the field lines are the circles centred at the conductor itself and the plane of these circles are perpendicular to the plane of paper.

   The current I_A in the wire A sets up magnetic field B_A and at any distance r away from it (say at point R) the magnitude of this field is given by,

$$\mathbf{B_A = \frac{\mu_0 I_A}{2\pi r}} \qquad \textbf{...(15.55)}$$

The direction of magnetic field at this point is perpendicularly inward from the plane of paper and is shown by cross ⊗, whereas at point P, the direction of field is perpendicularly outward from the plane of paper and is shown by ⊙.

Another conductor B carrying current I_B placed at distance r and passing through point R experiences force F_{BA} due to B_A, whose magnitude is,

$$\mathbf{F_{BA} = B_A I_B \mathit{l} \sin\theta} \qquad \textbf{...(15.56)}$$

Where, l is a segment of wire B

The direction of force F_{BA} is perpendicular to wire B and parallel to plane of paper towards wire A as defined by Fleming's left hand rule.

The angle θ between length segment and direction of field is 90° so,

$$\mathbf{F_{BA} = B_A I_B \mathit{l}} \qquad \textbf{...(15.57)}$$

From equation (15.55) and (15.57), we get

$$\mathbf{F_{BA} = \frac{\mu_0 I_A I_B \mathit{l}}{2\pi r}} \qquad \textbf{...(15.58)}$$

Similarly, the magnetic field B_B set up by wire B at any point Q which is r distance away from it is,

$$\mathbf{B_B = \frac{\mu_0 I_B}{2\pi r}} \qquad \textbf{...(15.59)}$$

The direction of this field is perpendicularly outward from the plane of paper and is shown by ⊙. So, the force F_{AB} due to B_B on any conductor A carrying current I_A and passing through point Q which is r distance away from it is,

$$F_{AB} = B_B I_A l \sin\theta$$

Where, l is a length segment of wire A equivalent to that of wire B.

Again, this force is perpendicular to wire A and is parallel to plane of paper towards wire B. So,

$$\mathbf{F_{AB} = B_B I_A \mathit{l}} \qquad \textbf{...(15.60)}$$

From equations (15.59) and (15.60), we get,

$$\mathbf{F_{AB} = \frac{\mu_0 I_A I_B \mathit{l}}{2\pi r}} \qquad \textbf{...(15.61)}$$

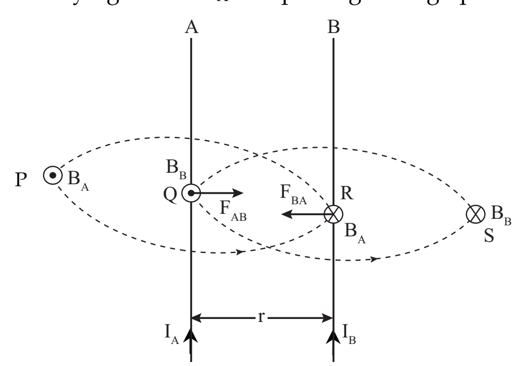


Fig. 15.20: Force between parallel conductors with current in same direction

From equation (15.58) and (15.61), we get,

$$F_{AB} = F_{BA}$$

Thus, the force exerted by the conductors on one another is equal in magnitude and act towards one another. This means, force is attractive when currents are in the same direction.

ii. When currents are in opposite direction:

In the above case, if the direction of the current say I_A is reversed such that it flows from top to bottom, the force between them turns out to be repulsive. The magnitude B_A of the magnetic field at any point R at distance r away from conductor A carrying current I_A is given by,

$$\mathbf{B_A = \frac{\mu_0 I_A}{2\pi r}} \qquad \text{...(15.62)}$$

The direction of this field is perpendicularly outward from the plane of paper at point R and is shown by ⊙. Another conductor B is placed at this distance and passing through point R experiences force F_{BA} due to B_A whose magnitude is,

$$\mathbf{F_{BA} = B_A I_B \mathit{l} \sin\theta} \qquad \text{...(15.63)}$$

Where, l is the length of wire B.

The direction of this force is perpendicular to wire B and parallel to plane of paper away from wire A as defined by Fleming's left hand rule.

Here, $\theta = 90^o$ is the angle between the length segment and direction of magnetic field

$$\mathbf{F_{BA} = B_A I_B \mathit{l}} \qquad \text{...(15.64)}$$

So, equation (15.62) and (15.64), we get,

$$\mathbf{F_{BA} = \frac{\mu_0 I_A I_B \mathit{l}}{2\pi r} y} \qquad \text{...(15.65)}$$

In similar manner, the force F_{AB} or an equivalent length segment l of A due to field B_B of wire B can be shown to be,

$$\mathbf{F_{AB} = \frac{\mu_0 I_A I_B \mathit{l}}{2\pi r}} \qquad \text{...(15.66)}$$

The direction of this force is perpendicular to wire A and parallel to plane of paper away from B.

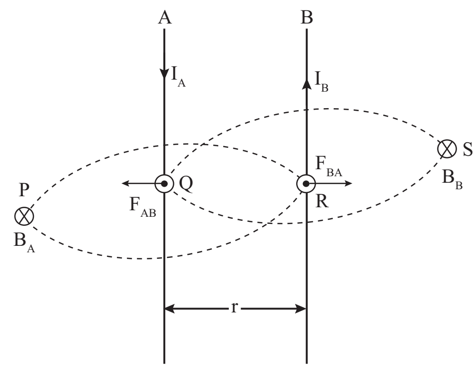


Fig. 15.21: Force between parallel conductors with current in opposite direction

From equations (15.65) and (15.66),

$$F_{AB} = F_{BA}$$

Though the forces are equal in magnitude, they act away from one-another and are repulsive in nature.

Definition of one Ampere

The force acting between two wires placed parallel to each other is the basis for the definition of one of the seven S.I. base units, known as ampere.

We know, the force exerted by either conductors carrying current placed parallel to each other is,

$$F = \frac{\mu_0 I_A I_B l}{2\pi r}$$

$$\text{or,} \quad \mathbf{\frac{F}{\mathit{l}} = \frac{\mu_0 I_A I_B}{2\pi r}} \qquad \text{...(15.67)}$$

Here, $\frac{F}{l} = f$ is called force per unit length of the conductor.

In above equation (15.67), if $I_A = I_B = 1$ A, $l = 1$ m and r = 1 m then,

$$F = \frac{4\pi \times 10^{-7}}{2\pi} = 2 \times 10^{-7}\ \text{N}$$

Thus, the ampere is the constant current, which if maintained in two straight parallel conductors of infinite length and negligible cross-section placed 1 m apart in vacuum would produce on each other a force of magnitude 2×10^{-7} N per meter of wire.

15.15 Hall Effect

It is a well established fact that, a beam of electrons projected in vacuum can be deflected by externally applied magnetic field. This kind of deflection by the externally applied magnetic field is possible also for the drifting conduction electrons in a conducting wire. And this effect was discovered in 1879 by Edwin H. Hall, then a 24 year old graduate student while working on his doctoral degree at the John Hopkin's University, USA and is known as Hall effect in his honor.

Let us consider a copper strip of width d and thickness t carrying current I from top to bottom (conventional direction). The charge carries in such conductors are electrons and hence these drift in the direction opposite to current, that is from bottom to top. Let this copper strip be placed in a uniform magnetic field $\vec{B}$ which is perpendicularly inward from the plane of paper as shown in Fig. 15.22 A soon as the magnetic field is turned on, each drifting electrons will experience a deflecting magnetic force $\vec{F_B}$ pushing it towards the right edge of the strip (Flemming's left hand rule) and start to pile up there. The left edge will now have uncompensated positive charges (ions) in the fixed positions. This separation of positive and negative charges within the strip sets up an electric field E, pointing from left to right. This electric field now exerts electric force F_E on each electron tending to push it to left. This electric force on the electrons opposes the magnetic force on them and an equilibrium is reached soon at which these forces balance each other.

i.e. $F_E = F_B$ **...(15.68)**

These electric and magnetic fields are mutually perpendicular to each other and constitute a cross-field.

If V_H is the potential difference associated with electric field also known as hall voltage, then,

$$\mathbf{E = \frac{V_H}{d}} \quad \text{...(15.69)}$$

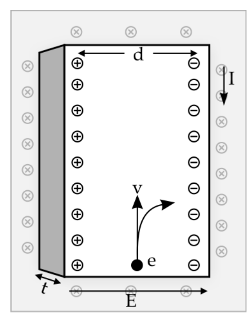


Fig. 15.22: Hall effect

The electric force F_E experienced by each electron of charge e is,

$$\mathbf{F_E = eE} \quad \text{...(15.70)}$$

The magnetic force experienced by each electron is

$$\mathbf{F_B = Bev_d} \quad \text{...(15.71)}$$

Where, v_d is drift velocity of electron.

From equations (15.68), (15.69), (15.70) and (15.71) we get,

$$Bev_d = eE$$

or, $$Bev_d = \frac{e\,V_H}{d}$$

$$\Rightarrow \quad \mathbf{v_d = \frac{e\,V_H}{Bed}} \quad \text{...(15.72)}$$

Also, if n be the number of conduction electrons per unit volume of the conductor whose cross-sectional area is A, then

$$\mathbf{v_d = \frac{I}{nAe}} \quad \text{...(15.73)}$$

From equations (15.72) and (15.73), we get,

$$\frac{I}{nAe} = \frac{e\,V_H}{Bed}$$

or, $V_H = \frac{BId}{nAe}$

But, $A = d \times t$

so, $\mathbf{V_H = \frac{BI}{net}}$ **...(15.74)**

From above equation (15.74), we see that hall voltage is inversely proportional to charge density. So, hall voltage is greater in semi-conductor than in conductor.

In equation (15.74), $\frac{1}{ne} = H_c$ is called hall coefficient.

$\therefore$ $\mathbf{V_H = \frac{BIH_c}{t}}$ **...(15.75)**

If R_H be the hall resistance then, $V_H = IR_H$

so, $R_H = \frac{BH_c}{t}$

If J be the current density in the strip then,

$$J = \frac{I}{A}$$

$$I = JA$$

So, from equation (15.75), we get,

$$V_H = \frac{BJAH_c}{t} = BJdH_c$$

$$H_c = \left(\frac{V_H}{d}\right) \times \frac{1}{BJ} = \frac{E_H}{BJ}$$

Significances of Hall Effect

i. **Measuring the drift velocity of charge carries:** The drift speed of electrons in any conductor is very small usually of the order of 10^{-4} m/s. So, if we move the entire conductor in the direction opposite to the direction of current with a speed equal to drift speed, relative drift speed of electron with respect to magnetic field is zero. i.e. electrons are at rest. Electrons at rest do not experience magnetic force. This means, in situations as discussed above, the hall voltage must disappear. Thus, the conductor speed needed to vanish out hall voltage is equal to drift speed. In this way, we can measure the drift speed.

ii. **Detecting nature of charge carriers:** In the case discussed above, consider that the current is due to positive charge carriers and these move from top to bottom in the strip. Now, due to the applied magnetic field these positive charges must be pushed to right hand side of the strip and hence this side must be at higher potential. But, a voltmeter connected across the extreme edges will show that right edge is at lower potential. This means, voltmeter reading contradicts our assumption that the charge carriers are positive and hence these must be negative.

iii. Hall effect permits the direct measurement of the concentration of the charge carriers (n) in the material.

Tips for MCQs

1. **Magnetic field:**

   i. When electric current passes through a conductor, magnetic field is associated with it, but not the electric field because the conductor does not acquire any charge.

ii. Magnetic field strength is represented by $\vec{B}$ and its unit is Tesla (T) or Weber per square metre (Wb/m^2) in SI system, and in CGS system its unit is Gauss (G) or Maxwell per square centimeter (Mx/cm^2).

$1\ G = 10^{-4}\ T$.

iii. A current in straight conductor produces circular magnetic field, whereas the current in a circular coil produces straight magnetic field at the center of the circular coil.

2. Lorentz Force:

i. The magnitude of magnetic Lorentz force is,

$|\vec{F}| = q\,|\vec{v} \times \vec{B}| = Bqv \sin\theta$

ii. The direction of Lorentz force can be determined by Fleming's left hand rule. For mutually perpendicular set of thumb, fore finger and middle finger in left hand, Fore finger- magnetic field; middle finger - electric current and thumb - force. This rule is taken for conventional current (i.e. flow of charge from positive terminal to negative).

iii. If the charge particle moves parallel to the direction of magnetic field, it does not experience any force. In such condition, velocity, kinetic energy and momentum of charge particles remain same.

iv. ⊗ symbol represents the magnetic field inward perpendicular to the plane of paper and ⊙ symbol represents the magnetic field outward perpendicular to the plane of paper.

v. If a current I flows in a straight wire of length l in a magnetic field $\vec{B}$, it experiences the force, $F = I(\vec{l} \times \vec{B}) = BIl \sin\theta$.

vi. Total Lorentz force on a charged particle due to both electric and magnetic fields.

$\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})$.

3. Magnetic torque and moving coil meters

i. The magnetic torque in a rectangular coil

$\tau = BINA \cos\theta$, where θ is the direction of the current with respect to uniform magnetic field.

ii. The magnetic moment of a current loop,

$\mu = IA$, I = current, A = Area of loop.

iii. The angular displacement of needle in moving coil meter, $\theta = \left(\frac{BNA}{k}\right) I$

Where k is restoring torque per units twist.

iv. The current sensitivity of coil meter, $\frac{\theta}{I} = \frac{BNA}{k}$

v. The voltage sensitivity of coil meter, $\frac{V}{I} = \frac{BNA}{kR}$.

4. Biot and Savart law:

i. Biot Savart law is analogous to Coulomb's law of electrostatics.

ii. The magnitude of magnetic field $\vec{dB}$ due to a current element $(I\vec{dl})$ is,

$|\vec{dB}| = \frac{\mu_0}{4\pi} \frac{Idl \sin\theta}{r^2}$

In vector form, $\vec{dB} = \frac{\mu_0}{4\pi} \frac{I\vec{dl} \times \vec{r}}{r^3}$ also, $\vec{dB} = \frac{\mu_0}{4\pi} \frac{I\vec{dl} \times \hat{r}}{r^2}$

In terms of current density, $\overrightarrow{dB} = \frac{\mu_0}{4\pi}\frac{\vec{J} \times \vec{r}}{r^3} dV$.

iii. In circular coil of N terns carrying current I,

Value of B at the center, $B = \frac{\mu_0 NI}{2r}$

and the direction of B is perpendicular to the plane of circular coil.

iv. In circular coil of radius R and having N turns carrying current I, value of B along its axis is,

$\vec{B} = \frac{\mu_0 NIR^2}{2(R^2 + x^2)}$

v. Value of B due to an infinite long straight current carrying conductor, $B = \frac{\mu_0 I}{2\pi r}$.

vi. Value of B due to an infinite long current carrying solenoid,

$B = \mu_0 nI$ (at the center) and $B = \frac{\mu_0 nI}{2}$ (at the end).

5. **Amperes law:**

i. Statement: The line integral $\oint Bdl = \mu_0 I_{net}$.

ii. It is the alternative method to Biot Savart law.

iii. If a closed path does not encircle the wire, then the line integral $\oint \vec{B}.\vec{dl}$ is zero.

6. **Magnetic force per unit length, due to two current carrying conductor,**

i. $\frac{F}{l} = \frac{\mu_o I_1 I_2}{2\pi d}$, d = distance between two wires.

The magnitude of force per unit length is same for both conductor, although current is different in them.

$\left(\frac{\vec{F}}{l}\right)_1 = -\left(\frac{\vec{F}}{l}\right)_2$

ii. Parallel currents attract, and antiparallel current repel.

7. **Hall effect:**

i. Hall voltage, $V_H = \frac{BI}{net}$

iii. Hall coefficient $H_C = \frac{1}{ne} = \frac{E_H}{BJ}$

ii. Hall resistance, $R_H = \frac{BH_C}{t} = \frac{BH_c d}{A}$

Worked Out Problems

1. **An α-particle of mass 6.62×10^{-27} kg and charge twice that of an electron but of positive sign travels at right angles to a magnetic field with a speed of 6×10^5 ms^{-1}. Strength of magnetic field is 0.2 T. (i) Calculate the force on α-particle. (ii) Also calculate its acceleration.**

SOLUTION

Given,

Mass of α-particles (m) = 6.62×10^{-27} kg

Charge (q) = +2e = $2 \times 1.6 \times 10^{-19}$ C = 3.2×10^{-19} C

Speed (v) = 6×10^5 ms^{-1}

and $\theta = 90^o$, magnetic field (B) = 0.2 T

Then, Force (F) = ?

We have,

$F = Bqv \sin\theta$

$= 0.2 \times 3.2 \times 10^{-19} \times 6 \times 10^5 \times \sin 90^o = 3.84 \times 10^{-14}$ N

Now, acceleration (a) $= \dfrac{F}{m}$

$$= \frac{3.84 \times 10^{-14}}{6.62 \times 10^{-27}} = 5.77 \times 10^{12}\ \text{ms}^{-2}.$$

Therefore, the acceleration of α-particle is 5.77×10^{12} ms^{-2}.

2. The plane of a 5.0 cm × 8.0 cm rectangular loop of wire is parallel to a 0.19 T magnetic field. The loop carries a current of 6.2 A. (a) What torque acts on the loop? (b) What is the magnetic moment of the loop? (c) What is the maximum torque that can be obtained with the same total length of the wire carrying the same current in this magnetic field?

SOLUTION

Given,

Area of loop (A) = 5 × 8 cm^2 = 4×10^{-3} m^2

Magnetic field (B) = 0.19 T

Current (I) = 6.2 A

Angle between plane of loop and field (θ) = 0°

Number of turns (N) = 1

a. Torque on the loop (τ) = ?

We know that,

$\tau = BINA \cos\theta$

$= 0.19 \times 6.2 \times 1 \times 4 \times 10^{-3} \times \cos 0$

$\therefore \tau = 4.7 \times 10^{-3}$ Nm

b. Magnetic moment (M) = ?

We have,

$M = IA\,N = 6.2 \times 4 \times 10^{-3} \times 1$

$\therefore M = 24.8 \times 10^{-3}$ Am^2

c. Maximum torque (τ_{max}) = ?

We have,

$\tau = BINA \cos\theta$

For maximum torque, $\cos\theta = 1$

$\therefore \tau_{max} = BINA = 0.19 \times 6.2 \times 1 \times 4 \times 10^{-3}$

$\therefore \tau_{max} = 4.7 \times 10^{-3}$ Nm

3. In the Bohr model of the hydrogen atom, the electron circulates around the nucleus in a path of radius 5.1 × 10^{-11} m at a frequency of 6.8 × 10^{15} rev/s. What value of B is set up at the center of the orbit?

SOLUTION:

Given,

Radius (r) = 5.1×10^{-11} m

Frequency (f) = 6.8×10^{15} rev/s

Magnetic field (B) = ?

We have,

Magnetic field at the center of circular coil, B $= \dfrac{\mu_o NI}{2r}$

$$= \left(\frac{\mu_0}{2r}\right) . N\frac{q}{t} = \left(\frac{\mu_0}{2r}\right) . N\left(\frac{e}{t}\right) = \frac{\mu_0}{2r} . \frac{N}{t} . e$$

$$= \left(\frac{\mu_0}{2r}\right) . \text{ f.e} \quad \text{Where, } f = \frac{N}{t}$$

$$= \frac{4\pi \times 10^{-7} \times 6.8 \times 10^{15} \times 1.6 \times 10^{-19}}{2 \times 5.1 \times 10^{-11}}$$

$$= 13.40 \text{ T.}$$

The value of B at the center of orbit is 13.40 T.

4. The coil of a moving coil galvanometer has 50 turns and its resistance is 10 Ω. It is replaced by a coil having 100 turns and resistance 50 Ω. Find the factor by which the current and voltage sensitivities change.

SOLUTION

Given,

In first case,

Number of turns (N_1) = 50 turns

Resistance (R_1) = 10 Ω

In second case,

Number of turns (N_2) = 100 turns

Resistance (R_2) = 50 Ω

a. Current sensitivity change = ?

We have, $\left(\frac{\theta}{I}\right) = \frac{BNA}{k}$

∴ Now, current sensitivity change

$$\frac{(\theta/I)_2}{(\theta/I)_1} = \frac{BN_2A/k}{BN_1A/k} = \frac{N_2}{N_1} = \frac{100}{50} = 2 : 1$$

b. Voltage sensitivity change?

We have, $\left(\frac{\theta}{V}\right) = \frac{BNA}{kR}$

∴ Now, current sensitivity change

$$\frac{(\theta/V)_2}{(\theta/V)_1} = \frac{BN_2A/kR_2}{BN_1A/kR_1}$$

$$= \frac{N_2R_1}{N_1R_2} = \frac{100 \times 10}{50 \times 50} = \frac{2}{5} = 2 : 5$$

5. A copper wire has 1 × 10^{29} free electrons per cubic meter and cross-sectional area 2 mm^2 carries a current of 6 A. Calculate the force acting on each electron if the wire is now placed in uniform magnetic field of flux density 0.1 T perpendicularly.

SOLUTION

Given,

No. of free electrons (n) = 1 × 10^{29} /m^3

Cross section area (A) = 2 mm^2 = 2 × 10^{-6} m^2

Current (I) = 6 A

Flux density (B) = 0.1 T

Force on electron (F) = ?

Now,

The drift velocity (v_d) $= \frac{I}{enA}$

and $F = Bev_d = Be\frac{I}{enA} = \frac{BI}{nA} = \frac{0.1 \times 6}{1 \times 10^{29} \times 2 \times 10^{-6}} = 3 \times 10^{-24}$ N

6. A solenoid is designed to produce a magnetic field of 0.0270 T at its center. It has radius 1.40 cm and length 40.0 cm and the wire can carry a maximum current of 12.0 A. (a) What minimum number of turns per unit length must the solenoid have? (b) What total length of wire is required?

SOLUTION

Given,

Magnetic field (*B*) = 0.0270 T

Radius (r) = 1.40 cm = 1.40 × 10^{-2} m

Length of each turn(*l*) = 40.0 cm = 40.0 × 10^{-2} m

Current (I) = 12.0 A

Now,

Total length of the wire (L) = circumference of one turn × Number of turns

or, L = 2π r × N

Number of turns per unit length (n) = ?

Total length of the wire (L) = ?

The magnetic field due to solenoid is given by,

$B = \mu_0 nI$

or, $n = \dfrac{B}{\mu_0 I} = \dfrac{0.0270}{4\pi \times 10^{-7} \times 12.0}$

$\therefore$ n = 1790 turns/m

$= 2\pi r \times n\,l \qquad \left[\because n = \dfrac{N}{l}\right]$

$= 2\pi \times 1.40 \times 10^{-2} \times 1790 \times 40.0 \times 10^{-2}$

= 63.0 m

7. A closed curve encircles several conductors. The line integral $\oint \vec{B} \cdot \vec{dl}$ around this curve is 3.83×10^{-4} Tm. What is the net current in the conductors?

SOLUTION

Given,

$\oint \vec{B} \cdot \vec{dl} = 3.83 \times 10^{-4}$ Tm

According to Ampere's law, we have,

$\oint \vec{B} \cdot \vec{dl} = \mu_0 I$

or, $3.83 \times 10^{-4} = 4\pi \times 10^{-7}$ I

$\therefore$ I = 305 A

8. [HSEB 2072] A horizontal wire, of length 5 cm and carrying a current of 2 A, is placed in the middle of a long solenoid at right angles to its axis. The solenoid has 1000 turns per meter and carries a steady current I. Calculate I, if the force on the wire is equal to 10^{-4} N. ($\mu_0 = 4\pi \times 10^{-7}$ H m^{-1}).

SOLUTION

Given,

Length of wire (l) = 5 cm = 5×10^{-2} m

Current on wire (I_1) = 2 A

No. of turns of solenoid (n) = 1000 turns/m

Force on wire (F) = 10^{-4} N

Current on solenoid (I_2) = ?

We have,

$F = B.I_1.\,l$

or, $F = \mu_0\, n\, I_2.\, I_1\, l \quad [B = \mu_0\, n\, I_2]$

or, $10^{-4} = 4\pi \times 10^{-7} \times 1000 \times I_2 \times 2 \times 5 \times 10^{-2}$

or, $I_2 = 0.8$ A

9. An electron of kinetic energy 10 eV is moving in a circular orbit of radius 11 cm, in a plane at right angles to a uniform magnetic field. Determine the value of flux density. (Mass of an electron = 9.1×10^{-31} kg, e = 1.6×10^{-19} C.

SOLUTION

Given,

Kinetic energy (E_k) = 10 eV = $10 \times 1.6 \times 10^{-19}$ J = 1.6×10^{-18} J

Radius (r) = 11 cm = 11×10^{-2} m

Mass of electron (m) = 9.1×10^{-31} kg

Charge of electron (e) = 1.6×10^{-19} C

We know,

$E_k = \dfrac{1}{2} mv^2$

$v = \sqrt{\dfrac{2E_k}{m}} = \sqrt{\dfrac{2 \times 1.6 \times 10^{-18}}{9.1 \times 10^{-31}}} = 1.88 \times 10^6 \text{ ms}^{-1}$

As the electron revolves in a circular orbit, it experiences the centripetal force,

$$\frac{mv^2}{r} = Bev$$

$$\therefore \quad \frac{mv}{re} = \frac{9.1 \times 10^{-31} \times 1.88 \times 10^6}{11 \times 10^{-2} \times 1.6 \times 10^{-19}} = 9.69 \times 10^{-4}\ \text{T}.$$

$\therefore$ The flux density is 9.69×10^{-4} T.

10. [HSEB 2072] A 60 cm long wire of mass 10 g is suspended horizontally in a transverse magnetic field of flux density 0.4 T, through two springs at its two ends. Calculate the current required to pass through the wire so that, there is zero tension in the springs.

SOLUTION

Given,

Length of wire (l) = 60 cm = 0.6 m

Mass of wire (m) = 10 g = 10×10^{-3} kg

Magnetic flux density (B) = 0.4 T

Current in the wire (I) = ?

For no tension in the wire, the weight of wire must be equal to the magnetic force in upward direction. i.e.

Weight of wire = Magnetic force

or, $mg = BIl$

or, $10 \times 10^{-3} \times 10 = 0.4 \times I \times 0.6$

or, $0.1 = 0.24\ I$

or, $I = \frac{0.1}{0.24} = 0.42$ A

11. [HSEB 2067] A slice of indium antimonide is 2.5 mm thick and carries a current of 150 mA. A magnetic field of flux density 0.5 T, correctly applied, produces a maximum Hall voltage of 8.75 mV between the edges of the slice. Calculate the number of free charge carriers per unit volume, assuming they each have a charge of -1.6×10^{-19} C.

SOLUTION

Given,

Thickness (t) = 2.5 mm = 2.5×10^{-3} m

Current (I) = 150 mA = 150×10^{-3} A

Magnetic field (B) = 0.5 T

Hall voltage (V_H) = 8.75 mV = 8.75×10^{-3} V

Electronic charge (e) = -1.6×10^{-19} C

Electron density (n) = ?

Now,

We have,

Hall voltage (V_H) = $\frac{BI}{net}$

$\therefore \quad n = \frac{BI}{V_H et}$

or, $n = \frac{BI}{V_H et} = \frac{0.5 \times 150 \times 10^{-3}}{8.75 \times 10^{-3} \times (1.6 \times 10^{-19}) \times 2.5 \times 10^{-3}}$

$= \frac{75}{35 \times 10^{-22}}$

$\therefore \quad n = 2.14 \times 10^{22}$

Hence, the required number of free charge carriers is $2.14 \times 10^{22}\ \text{m}^{-3}$.

12. [HSEB 2061] A horizontal straight wire 5 cm long weighing 1.2 gm^{-1} is placed perpendicular to a uniform horizontal magnetic field of flux density 0.6 T. If the resistance of the wire is 3.8 Ωm^{-1}, calculate the p.d. that has to be applied between the ends of the wire to make it just self supporting.

SOLUTION

Given,
Length of wire (l) = 5 cm = 5×10^{-2} m
Weight per meter = 1.2 g
∴ Total mass of wire (m) = $1.2 \times 5 \times 10^{-2}$g = 0.06 g
Flux density (B) = 0.6 T
Resistance per meter = 3.8 Ω m^{-1}
∴ Total resistance of wire (R) = $3.8 \times 5 \times 10^{-2}\,\Omega$ = 0.
Potential difference (V) = ?
Now,
To make the wire just self supporting,

Weight of wire = Magnetic force

or, mg = BIl sin 90° [∵ θ = 90]

or, $0.06 \times 10^{-3} \times 9.8 = 0.6 \times \frac{V}{R} \times 5 \times 10^{-2}$

or, $0.588 \times 10^{-3} = 0.03 \times \frac{V}{0.19}$

∴ V = 3.7×10^{-3} V

Hence, the required potential is 3.7×10^{-3} V.

Challenging Problems

1. **[UP]** A horizontal rod 0.200 m long is mounted on a balance and carries a current. At the location of the rod a uniform horizontal magnetic field has magnitude 0.067 T and direction perpendicular to the rod. The magnetic force on the rod is measured by the balance and is found to be 0.13 N. What is the current?

Ans: 9.7 A

2. **[UP]** A circular coil of wire 8.6 cm in diameter has 15 turns and carries a current of 2.7 A. The coil is in a region where, the magnetic field is 0.56 T. (a) what orientation of the coil gives the maximum torque on the coil, and what is the maximum torque? (b) for what orientation of the coil is the magnitude of the torque 71% of that found in part (a)?

Ans: (a) θ = 0°, 0.13 Nm (b) 45°

3. **[UP]** An electron experiences a magnetic force of magnitude 4.60×10^{-15} N when moving at an angle of 60.0° with respect to a magnetic field of magnitude 3.50×10^{-3} T. Find the speed of the electron.

Ans: 9.49×10^{6} ms^{-1}

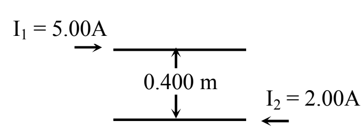
4. **[UP]** An electromagnet produces a magnetic field of 0.550 T in a cylindrical region of radius 2.50 cm between its poles. A straight wire carrying a current of 10.8 A passes through the center of this region and is perpendicular to both the axis of the cylindrical region and the magnetic field. What magnitude of force is exerted on the wire?

Ans: 0.297 N

5. **[UP]** Two long, parallel transmission lines, 40.0 cm a part, carry. 25.0 A and 75.0 A currents. Find all locations where, the net magnetic field of the two wires is zero if these currents are in (a) the same direction; (b) the opposite direction.

Ans: (a) 10 cm (b) 20 cm

6. **[UP]** Two long, parallel wires are separated by a distance of 0.400 m. The currents I_1 and I_2 have the directions shown. Calculate the magnitude of the force exerted by each wire on a 1.20 m length of the other. Is the force attractive or repulsive?



Ans: 6.00×10^{-6} N

7. **[UP]** Two long, parallel wires are separated by a distance of 2.50 cm. The force per unit length that each wire exerts on the other is 4.00×10^{-5} N/m, and the wires repel each other. The current in one wire in 0.600 A. (a) What is the current in the second wire? (b) Are the two currents in the same direction or in opposite direction?

Ans: (a) 8.3 A (b) yes, currents are in opposite directions

8. **[UP]** A closely wound, circular coil with radius 2.40 cm has 800 turns. (a) What must the current in the coil be, if the magnetic field at the center of the coil is 0.0580 T? (b) At what distance x from the center of the coil, on the axis of the coil, is the magnetic field $\frac{1}{2}$ of its value at the center?

Ans: (a) 2.77 A (b) 0.0184 m

9. **[UP]** A closely wound coil has a radius of 6.00 cm and carries a current of 2.50 A. How many turns must it have, if at a point on the coil axis 6.00 cm from the center of the coil, the magnetic field is 6.39×10^{-4} T?

Ans: 69

10. **[UP]** A 15.0 cm long solenoid with radius 2.50 cm is closely wound with 600 turns of wire. The current in the windings is 8.00 A. Compute the magnetic field at a point near the center of the solenoid.

Ans: 40.2×10^{-3} T

11. **[UP]** As a new electrical technician, you are designing a large solenoid to produce a uniform 0.150 T magnetic field near the center of the solenoid. You have enough wire for 4000 circular turns. This solenoid must be 1.40 m long and 20.0 cm in diameter. What current will you need to produce the necessary field?

Ans: 41.8 A

12. **[UP]** A wooden ring whose mean diameter is 14.0 cm is wound with a closely spaced toroidal windings of 600 turns. Compute the magnitude of the magnetic field at the center of the cross-section of the windings when the current in windings is 0.650 A.

Ans: 1.11×10^{-3} T

13. **[UP]** A current of 0.5 A is passed through a rectangular section of a semiconductor 4 mm thick which has majority carriers of negative charges or free electrons. When a magnetic field of 0.2 T is applied perpendicular to the section, a Hall voltage of 6.0 mV is produced between the opposite edges. Calculate the number of charge carriers per unit volume?

Ans: 2.6×10^{22} m^{-3}

14. A piece of germanium has dimensions 10 mm × 5 mm × 1 mm. When it is carrying a current of 150 mA, the Hall voltage is 57 mV. The number of charge carriers per unit volume in germanium is 4.3×10^{21} m^{-3}. What is the magnetic field strength?

Ans: 0.26 T

*[**Note:** Hints to challenging problems are given at the end of this chapter.]*

Conceptual Questions with Answers

1. How many ways are there to produce a magnetic field?

➪ There are four basic ways of producing magnetic field. They are:
   i. by a magnet
   ii. by a current carrying conductor
   iii. by a moving charge
   iv. by changing electric field.

2. A charge q is moving in a region where both the magnetic field $\vec{B}$ and electric field $\vec{E}$ are simultaneously present. What is the Lorentz force acting on the charge?

➪ The Lorentz force due to magnetic field is $F_m = q(\vec{v} \times \vec{B})$ and the Lorentz force due to the electric field is $F_e = q\vec{E}$. Then, the total Lorentz force acting on the charge is,

$$F = F_B + F_e = q(\vec{v} \times \vec{B}) + q\vec{E}.$$

3. Write the condition under which an electric charge does not experience a force in a magnetic field.

➪ The force experienced by an electric charge in a magnetic field is,

$$F = Bqv\sin\theta$$

When charge q moves in a magnetic field $\vec{B}$, the magnitude of force is affected by velocity (v) of charge particle and direction of motion (θ) of charge particle. Here, the force can be zero when (i) velocity is zero, i.e. charge particle is at rest or (ii) direction of motion of charge particle is parallel or antiparallel to the field B, i.e. $\theta = 0^o$ or 180^o.

4. Under what condition does an electron moving through a magnetic field experience maximum force?

✎ For an electron, when moving in a magnetic field B with speed v, the force on it is, $F = Bev\sin\theta$, where e = electronic charge.

The value of F is maximum, at $\theta = 90^o$.

Therefore, an electron experiences maximum force, when it moves perpendicular to the direction of magnetic field.

5. A charge particle carrying a charge q moves in an electric field E, if its specific charge is S, write an expression for its acceleration in terms of above entities. [HSEB 2073]

✎ The force experienced by a charge particle in an electric field is, $F = qE$.

So, $ma = qE$,

where, m = mass of charge particle

a = acceleration of charge particle

$$\therefore \quad a = \frac{qE}{m}$$

The specific charge is the charge to mass ratio of a charge particle. Here, $\frac{q}{m} = S$.

So, acceleration, $a = SE$.

6. Explain how the direction of Lorentz force is determined? [HSEB 2065]

✎ The direction of Lorentz force is determined by Fleming's left hand rule. According to this rule if first finger, second finger and thumb of our left hand are stretched mutually perpendicular to each other, then first finger shows the direction of magnetic field, second finger shows the direction of current and thumb shows the direction of Lorentz force.

7. What is the work done by the magnetic field on a moving charge?

✎ The power supplied by the magnetic field is $P = F.v$ where F is Lorentz force and v is the velocity of charge particle. We know,

$$F = q(\vec{v} \times \vec{B})$$

So, $P = q(\vec{v} \times \vec{B}).\ \vec{v} = 0$

Since the power of magnetic field is zero. The work done by magnetic field, ($W = P.t$) is also zero.

8. What are the unit and dimensional formula of magnetic field B?

✎ The SI unit of B is tesla (or weber per square meter, Wb/m^2). Its CGS unit is gauss (G). Its dimensional formula is $[ML^0T^{-2}A^{-1}]$.

9. If an electron is not deflected in passing through a certain region of space, can we be sure that there is no magnetic field in that region?

✎ No. The magnitude of force experienced by the electron depends on the direction of its velocity (v) with respect to magnetic field B.

i.e. $F = q(\vec{v} \times \vec{B})$

$= q\, vB \sin\theta\, \hat{n}$.

If the electron travels parallel ($\theta = 0$) or anti-parallel ($\theta = 180^o$) with the field, it does not experience any force. In this case, the electron does not experience any force, even though it is moving in magnetic field. So, no deflection condition of electron does not necessarily mean the absence of magnetic field in that region.

10. What is the principle of moving coil galvanometer?

➯ The principle of moving coil galvanometer is, "When a current-carrying coil is placed in magnetic field, it experiences a torque." In the working of moving coil galvanometer, moment of deflecting couple ($\tau = NBIl$) is equal to the moment of restoring couple ($\tau_2 = k\theta$)

i.e. $NBIl = k\theta$.

11. What is meant by high current sensitivity of moving coil galvanometer? How can we increase it?

➯ A galvanometer is said to be sensitive, if it gives a large deflection for a small current. The current

$$\text{sensitivity} \left(\frac{\theta}{I}\right) = \frac{NBA}{k}.$$

To increase the current sensitivity of galvanometer, number of turns of coil (N), magnetic field (B) and area covered by coil (A) can be increased and torsion constant (k) should be decreased. But experimentally, N and A cannot be increased much because this will increase the length and correspondingly the resistance of the coil.

12. What is voltage sensitivity of moving coil galvanometer? How can we increase it?

➯ It is defined as the angular deflection of galvanometer needle per unit voltage. It is denoted by $\frac{\theta}{V}$.

$$\frac{\theta}{V} = \frac{NBA}{kR}.$$

To increase the voltage sensitivity, N, B and A can be increased and torsion constant (k) and resistance of coil (R) should be decreased. However, N and A cannot be increased much which ultimately increases the length and so, increases the resistance.

13. Why are pole pieces of galvanometer made cylindrical?

➯ Cylindrical pole pieces provide the radical magnetic field in the air gap between them. The magnetic lines of force within the air gap are along the radii. On the account of this, the plane of the coil remain always parallel to the direction of the magnetic field, which provides the constant moment of deflection couple at any position.

14. Define 1 ampere of current in terms of force between two parallel conductors carrying current.

➯ If two parallel conductors carrying currents I_1 and I_2 respectively, which are separated with a distance d, the force per unit length between them is,

$$F = \frac{\mu_0}{2\pi}\frac{I_1I_2}{d}$$

For $\mu_0 = 4\pi \times 10^{-7}$ Hm^{-1}, $I_1 = I_2 = 1$ A and d = 1 m

$$F = \frac{4\pi \times 10^{-7}}{2\pi} \times \frac{1 \times 1}{1} = 2 \times 10^{-7} N.$$

Hence, 1 A current can be defined as that amount of current which flows through each of long parallel wires separated by 1 m distance when force per unit length between them is 2×10^{-7} N.

15. What is a current element? Give its significance.

➷ The product of magnitude of current (I) and the elementary length (dl) of current carrying conductor is called the current element. Current element is represented by $I\vec{dl}$. Current element Idl is the source of magnetic field in current carrying conductor, as the charge particle is a source of electric field.

16. A charge particle moves through a region of uniform magnetic field. Is the momentum of the particle affected?

Magnetic force deflects the charged particle continuously from its path, so its momentum changes due to the change in its direction of motion.

17. What is the basic difference between magnetic field and electric field?

➷ Whether a charged particle is at rest or in motion, an electric field always exerts a force on it and the changes its speed and hence changes its kinetic energy. A magnetic field exerts a force only when the charge particle is in motion. In magnetic field, there is no change in speed and hence, no change in kinetic energy.

18. Why does a current carrying conductor experience a force in a magnetic field?

➷ When current flows, electrons drift through the conductor. A moving electron experiences magnetic force in a definite direction in a magnetic field. The force on moving electrons of the current carrying conductor gets transmitted to the conductor as a whole.

19. A stream of protons is moving parallel to a stream of electrons. Do the two streams tend to come closer to move apart?

➷ The path of beam depends on the speed of charge particles. Protons and electrons are opposite charge particles. In the small speed, the electrostatic force between them is larger than the magnetic force. So, they tend to come closer. But, they gradually diverge as their speed increases. They behave as if two parallel current carrying conductor with current in opposite direction are taken closer. In such condition, magnetic repulsion dominates the electrostatic attraction between the beam of electrons and protons.

20. Why do we prefer phosphor-bronze alloy for the suspension wire of a moving coil galvanometer?

➷ Because of the following reasons, we prefer phosphor-Bronze alloy for suspension wire of a moving coil galvanometer:

i. It has small restoring torque per unit twist. This makes galvanometer highly sensitive.

ii. It is rust resisting.

iii. It has high tensile strength so that even a thin fibre does not break under the weight of the suspended coil.

21. What is the importance of radial magnetic field in a moving coil galvanometer?

➷ In radial magnetic field, the plane of the coil remains always parallel to the direction of the field. Hence, the torque on the coil is always same in all positions of the coil in the magnetic field. This provides a linear current scale.

22. A current carrying solenoid tends to contract. Explain Why?

➷ A solenoid consists of large number of turns closely compact to each other. When a current flows through it, the direction of current of adjacent turns is in the same direction. So, the current flowing in the adjacent turns in considered as if two current carrying straight conductors are placed nearer to each other. Since the direction of current is same, the magnetic force tends to pull them inward. Thus, the solenoid tends to contract.

23. What is the magnetic field at the center of a current carrying cube?

↳ The cube can be regarded as a set of six current carrying pairs. The contribution of each pair is zero. So, the net magnetic field strength at the center is zero.

24. A parallel electron beam moving with a uniform velocity is gradually diverging. When it is accelerated to a high velocity, it starts converging. Explain.

↳ A beam of electrons experiences both electrostatic repulsive force and magnetic attractive force. In the low velocity, electrostatic repulsion dominates the magnetic attraction in the beam. So, the beam diverges. If the velocity is increased sufficiently large value, the magnetic force dominates the electrostatic force. Then, the beam tends to converge. However, the electrons should be accelerated in such a high speed (about velocity of light) which is very difficult to achieve practically.

25. Explain the magnetic field around a current carrying solenoid.

↳ Solenoid is a long cylindrical coil of insulated copper wire. The magnetic field around a solenoid is made up of the magnetic fields of a large number of narrow circular coils joined in series. All these magnetic fields add up to give a strong magnetic field. This field resemble the field of a bar magnet. The strength of magnetic field is strong into the coil. But, the field towards the equatorial region is very weak because, lines of force are distributed in infinitely large space in long solenoid.

26. What is the advantage of ampere's law over Biot-Savart law?

↳ Ampere's law and Biot -Savart law are useful to determine the magnetic field due to current carrying conductor. Ampere's law is superior than Biot-Savart law in determining B for highly symmetrical systems. It is easier method. But, it is not applicable for complicated shaped wire.

27. What is the principle of Hall effect?

↳ When a conductor or semiconductor with current flowing in one direction is introduced perpendicular to a magnetic field, a voltage can be measured at right angles to the current path and magnetic field.

28. What are the use of Hall effect?

↳ Major uses of Hall effect are:

i. The Hall effect is relevant to a variety of sensor applications.

ii. Hall probes are often used as magnetometer to measure the magnetic fields.

Exercises

Short-Answer Type Questions

1. What is the magnetic field at a point on the axis of the current element?
2. Show the magnetic line of force around a straight current carrying conductor.
3. What is the nature of magnetic field in a moving coil galvanometer?
4. Why a current carrying rectangular coil experiences a torque in magnetic field? Is there any condition, that the coil experiences no torque?
5. What is the advantage of using pole pieces magnets in moving coil galvanometer?
6. What type of galvanometer is commonly used in experiments?
7. What is the advantage of Biot-Savart law over Amperes circuital law?
8. Find of ratio of magnetic field intensity produced by a long solenoid at the center to the end.
9. Magnetic field strength is taken almost zero at the equatorial region of a long solenoid. Why?
10. Toroid is special case of long solenoid. Explain.

11. When does a current carrying conductor placed in a magnetic field experience maximum and minimum forces?
12. Does the force between two current carrying conductors depend on the medium? Explain.
13. If the field strength of the radial field of a galvanometer is increased, does its sensitivity increase or decrease?
14. If an electron beam in a cathode-ray tube travels in a straight line, Can one be sure that, there is no magnetic field present?
15. An electron beam moving with uniform speed is gradually diverging, and when it is accelerated to high speed it starts converging, why?
16. A loop of wire is placed in a uniform magnetic field. For what, orientation of the loop is the magnetic flux a maximum? For what orientation is the flux zero?
17. How does a current carrying coil behave as a bar magnet?

Long-Answer Type Questions

1. Explain in brief, the motion of an electron moving normal to a magnetic field. [HSEB 2056]
2. Obtain the expression for the emf induced in the conductor moving in a magnetic field. [HSEB 2052]
3. Derive the formula for the magnetic field at a point due to a long straight current carrying conductor using Biot and Savart law. [HSEB 2061, 2062, 2064, 2067]
4. State Biot and Savart law and obtain the expression for the magnetic field at the centre of the circular coil. [HSEB 2055]
5. Derive the formula for the magnetic field at a point on the axis of a current carrying circular coil using Biot and Savart law.
6. What is a solenoid? Derive the formula for the magnetic field at a point on the axis of long current carrying solenoid using Biot and Savart law.
7. State and explain Biot and Savart law with a case of its application. [HSEB 2065]
8. State and explain Ampere's theorem and hence, use it to find the magnetic field intensity due to a long current carrying solenoid. [HSEB 2060, 2067]
9. What is a toroid? Find the magnetic field at a point inside a current carrying toroid.
10. Derive an expression for the magnitude of the magnetic flux density at the centre of a narrow circular coil. [HSEB 2056]
11. Derive an expression for the magnetic field at a point due to a long straight conductor carrying current. [HSEB 2057]
12. What is a Helmholtz Coil? Derive an expression for the magnetic field due to this coil. [HSEB 2058, 2063]
13. Derive the formula for the magnetic field at the centre of a circular coil carrying current. Explain why the magnetic field at the center of the coil disappears when the circular coil is made infinitely large. [HSEB 2059]
14. Derive the expression for the force experienced by current carrying conductor placed in a uniform magnetic field.
15. Derive an expression of force per unit length between two parallel conductors separated by a distance 'r' and carrying currents I_1 and I_2 in the same direction. [HSEB 2066]
16. Find an expression for torque on rectangular coil in a uniform magnetic field. [HSEB 2063]
17. Describe the principle, construction and working of moving coil galvanometer.
18. What is Hall effect? Obtain an expression for the Hall emf in terms of area of cross section, width of strip, magnitude of magnetic field and the current carried by the strip.

Numerical problems

1. A horizontal rod 0.200 m long is mounted on a balance and carries a current. At the location of the rod a uniform horizontal magnetic field has magnitude 0.067 T and direction perpendicular to the rod. The magnetic force on the rod is measured by the balance and is found to be 0.13 N. What is the current?

Ans: 9.7 A

2. The plane of a 5.0 cm × 8.0 cm rectangular loop of wire is parallel to a 0.19 T magnetic field. The loop carries a current of 6.2 A. (a) What torque acts on the loop? (b) What is the magnetic moment of the loop? (c) What is the maximum torque that can be obtained with the same total length wire carrying the same current is this magnetic field?

Ans: 4.7×10^{-3} Nm, 24.8×10^{-3} Am^2, 4.71×10^{-3} Nm

3. A circular coil of wire 8.6 cm in diameter has 15 turns and carries a current of 2.7 A. The coil is in the region where the magnetic field is 0.56 T. (a) What orientation of the coil gives the maximum torque on the coil, and what is the maximum torque? (b) For what orientation of the coil is the magnitude of the torque 71% of that found in part (a)?

Ans: 0.13 Nm, 45^{O}

4. Two long, parallel wires are separated by a distance of 2.50 cm. The force per unit length that each wire exerts on the other is 4.00 x 10-5 N/m, and wires repel each other. The current in the wire is 0.600 A. (a) What is the current is the second wire? (b) Are the currents in the same direction or in opposite directions?

Ans: 8.3 A, opposite direction

5. A closely wound, circular coil with radius 2.40 cm has 800 turns. (a) What must the current in the coil be if the magnetic field at he center of the coil is 0.0580 T? (b) At what distance from the center is the coil on the axis if the coil, is the magnetic field ½ its value at the center?

Ans: 2.77 A, 0.0184 m

6. A straight conductor X, of mass 50 g and length 0.5 m, is placed in a uniform magnetic field of 0.2 T perpendicular to X. Calculate the current in X if the force acting on it just balances its weight.

Ans: 5 A

7. A closely wound coil has a radius of 6.00 cm and carries a current of 2.50 A. How many turns must it has if at a point on the coil axis 6.00 cm from the center of the coil, the magnetic field is 6.39 x 10-4 T?

Ans: 69 turns

8. The electron in hydrogen atom revolves in a circular path of radius 0.5 × 10-10 m with a uniform speed of 4.0×10^6 m/s. Calculate the magnetic field produced by the electron at the nucleus.

Ans: 25.6 T

9. Calculate the magnetic field produced at the center of a square coil of sides 4 m and carrying current 5A.

Ans: 3.54×10^{-5} T

10. A current of 1 A is flowing in the sides of an equilateral triangle of side 2 m, find the magnetic field at the centroid of the triangle.

Ans: 9×10^{-7} T

11. A magnetic field of 37.2 T has been achieved at the MIT Francis Bitter National Magnetic Laboratory. Find the current needed to achieve such a field (a) 2.00 cm from a long straight wire. (b) At the center of the circular coil of radius 42.0 cm that has 100 turns.

Ans: 3.72×10^6 A, 2.49×10^5 A

12. A wire 28 m long is bent into N turns of circular coil of diameter 14 cm forming a solenoid of length 60 cm. Calculate the flux density inside it when a current of 5 amp passes through it.

Ans: 6.67×10^{-4} T

13. A wooden ring of diameter 14 cm is wound with a closely spaced toroidal winding of 600 turns. Compute the magnetic field at the center of the cross section of the windings when a current of 0.65 m flows through it.

Ans: 1.11×10^{-3} T

14. Two galvanometers which are otherwise identical are fitted with different coils. One has a coil of 50 turns and resistance of 10 Ω while the other has 500 turns and a resistance of 600 Ω. What is the ratio of the deflections when each is connected in turns to a cell of emf 2.5 V and internal resistance of 50 Ω?

Ans: 13:12

15. If the coil if moving coil galvanometer having 10 turns and of resistance 4 ohm removed and replaced by second coil having 100 turns and of resistance 160 ohm. Calculate (i) The factor by which the current sensitivity changes and (ii) The factor by which the voltage sensitivity changes.

Ans: 10:1, 1:4

16. A proton is moving northwards with a velocity of 5×10^6 m/s in a magnetic field of 0.1 T directed eastwards. Find the force on proton.

Ans: 8×10^{-14} N

17. A circular coil of 50 turns and area 1.25×10^{-3} m^2 is pivoted about a vertical diameter in a uniform horizontal magnetic field and carries a current of 2 A. When the coil is held with its plane with a north south direction. It experiences a corresponding couple is 0.03 Nm. Calculate magnetic flux density.

Ans: 0.4 T

18. A horizontal straight wire 5 cm long weighing 1.2 gm^{-1} is placed perpendicular to a uniform horizontal magnetic field of flux density 0.6 T. If the resistance of the wire is 3.8 Ω m^{-1}, Calculate the p.d. that has to be applied between the ends of the wire to make it just self-supporting.

Ans: 3.72×10^{-3} V

19. A rectangular coil of 50 turns hangs vertically in a uniform magnetic field of magnitude 0.01 T so that plane of the coil is parallel to the field. The mean height of the coil is 5 cm and its mean width is 2 cm. Calculate the strength of the current that must pass through the coil in order to deflect it 30° if the torsional constant of the suspension is 10^{-9} Nm/degree.

Ans: 69 μA

20. Two parallel straight conductors carrying currents in the same direction 12 A and 8 A are 10 cm apart. Calculate the position of a third conductor placed in between the two conductors so that the force experienced by it is zero.

Ans: 0.06 m

21. A 2 MeV proton is moving perpendicular to uniform magnetic force field of 2.5 T. What is the magnetic force on the proton? (Mass of proton = 1.6×10^{-27} kg)

Ans: 8.0×10^{-12} N

22. A wire of length 20 cm and mass 50 mg lies in a direction 30° east of north. The earth's magnetic field at this site is horizontal and has a magnitude of 8.0×10^{-3} T. What current must be passed through the wire, so that it may float in air? (g = 10 ms^{-2})

Ans: 0.63 A

23. A coil of 10 turns and area 5 cm^2 has a magnetic moment of 4×10^{-8} Am^2 and experiences a maximum torque of 2×10^{-8} Nm when placed in a uniform magnetic field. Calculate the magnetic induction in the coil.

Ans: 0.5 T

24. A horizontal wire 0.1 m long carries a current of 5 A. find the magnitude and direction of the magnetic field which can support the weight of the wire, assuming it's mass to be 3×10^{-4} kg?

Ans: 5.88×10^{-3} T. B is horizontal and perpendicular to the wire

25. A block of semiconductor of size 10.0 mm wide and 2.0 mm thick has a Hall voltage of 60mV in the magnetic field of 0.09 T. The number of charge carriers in the material is 8.0×10^{20}. What is the current in the semiconductor?

Ans. 0.85 A

26. A slice of indium antimonide is 2.5 mm thick and carries a current of 150 mA. A magnetic field of flux density 0.5 T, correctly applied, produces a maximum Hall voltage of 8.75 mV between the edges

of the slice. Calculate the number of free charge carriers per unit volume, assume they each have a charge of -1.6×10^{-19} C.

Ans: 2.1×10^{22}

27. A flat silver strip of width 1.5 cm and thickness 1.5 mm carries a current of 150 A. A magnetic field of 2.0 T is applied perpendicular to the flat face of the strip. The voltage developed across the strip is measured to be 17.9 μV. Estimate the number density of free electrons in the metal.

Ans: 6.983×10^{28} m^{-3}

Multiple Choice Questions

1. A long wire carries a steady current. It is bent into a circle of one turn and the magnetic field at the centre of the coil is B. It is then bent into a circular loop of n turns. The magnetic field at the centre of the coil will be

   a. nB b. n^2B
   c. 2nB d. $2n^2B$

2. Two parallel beams of positrons moving in the same direction will

   a. repel each other. b. will not interact with each other.
   c. attract each other. d. be deflected to the plane containing the two beams.

3. A circular coil of radius R carries an electric current. The magnetic field due to the coil at a point on the axis of the coil located at a distance r from the centre of the coil, such that r >> R, varies as

   a. 1 : 2 b. 1 : 4
   c. 1 : 16 d. 4 : 1

4. A charged particle of mass m and charge q travels on a circular path of radius r that is perpendicular to a magnetic field B. The time taken by the particle to complete one revolution is

   a. $\frac{2\pi qB}{m}$ b. $\frac{2\pi m}{qB}$
   c. $\frac{2\pi mq}{B}$ d. $\frac{2\pi q^2B}{m}$

5. A straight wire of mass 200 g and length 1.5 m carries a current of 2 A. It is suspended in mid-air by a uniform horizontal magnetic field B. The magnitude of B (in tesla) is (Assume g = 9.9 m s^{-2})

   a. 2 b. 1.5
   c. 0.55 d. 0.66

6. A horizontal overhead powerline is at a height of 4 m from the ground and carries a current of 100 A from east to west. The magnetic field directly below it on the ground is

   ($\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1})

   a. 2.5×10^{-7} T southward b. 5×10^{-6} T northward
   c. 5×10^{-6} T southward d. 2.5×10^{-7} T northward

7. A particle of mass m, charge Q and kinetic energy T enters a transverse uniform magnetic field of induction $\vec{B}$. After 3 s, the kinetic energy of the particle will be

   a. 3T b. 2T
   c. T d. 4T

8. A long straight wire of radius a carries a steady current i. The current is uniformly distributed across its cross-section. The ratio of the magnetic fields at $\frac{a}{2}$ and 2a is

   a. $\frac{1}{4}$ b. 4
   c. 1 d. $\frac{1}{2}$

9. A change q coulomb makes n revolutions in one second in a circular orbit of radius r. The magnetic field at the centre of the orbit in N A^{-1} m^{-1} is

a. $\frac{2\pi rn}{q} \times 10^{-7}$ b. $\frac{2\pi q}{r} \times 10^{-7}$

c. $\frac{2\pi q}{nr} \times 10^{-7}$ d. $\frac{2\pi nq}{r} \times 10^{-7}$

10. A galvanometer has a coil of resistance 100 ohm and gives a full scale deflection for mA current. If it is to work as a voltmeter of 30 volt range, the resistance required to be added will be

a. 900 Ω b. 1800 Ω

c. 500 Ω d. 1000 Ω

11. For a current I along positive z-direction, what is the magnitude of magnetic field at (a, 0, 0)?

a. $\frac{\mu_0 I}{a^2}$ b. $\frac{\mu_0 I}{a}$

c. $\frac{\mu_0 I}{\pi a}$ d. $\frac{\mu_0 I}{2\pi a}$

12. A proton and an alpha particle enter a uniform magnetic field with the same velocity perpendicular to the field. What is the ratio of time periods of alpha particle to that of proton?

a. 2 b. 1/2

c. 1 d. 4

13. Magnetic field at the centre of a circular coil of radius R due to current I flowing through it is B. The magnetic field at a point along the axis at distance R from the centre is

a. $\frac{B}{2}$ b. $\frac{B}{4}$

c. $\frac{B}{\sqrt{8}}$ d. $\sqrt{8}$ B

14. A current I flows in an infinitely long wire with cross-section in the form of a semi-circular ring of radius R. The magnitude of the magnetic induction along its axis is

a. $\frac{\mu_0 I}{\pi^2 R}$ b. $\frac{\mu_0 I}{2\pi^2 R}$

c. $\frac{\mu_0 I}{2\pi R}$ d. $\frac{\mu_0 I}{4\pi R}$

15. An electron moving around the nucleus with an angular momentum *l* has a magnetic moment

a. $\frac{e}{m} l$ b. $\frac{e}{2m} l$

c. $\frac{2e}{m} l$ d. $\frac{e}{2\pi m} l$

16. The magnetic field at a perpendicular distance of 2 cm from an infinite straight current carrying conductor is 2×10^{-6} T. The current in the wire is

a. 0.1 A b. 0.2 A

c. 0.4 A d. 0.8 A

17. The shunt resistance required to allow 4% of the main current through the galvanometer of resistance 48 Ω is

a. 1 Ω b. 2 Ω

c. 5 Ω d. 4 Ω

18. Biot-Savart law can be expressed alternatively as

a. Coulomb's law b. Ampere's circuital law

c. Ohm's law d. Gauss' law

Answers

1. (b) 2. (a) 3. (d) 4. (b) 5. (d) 6. (c) 7. (c) 8. (c) 9. (d) 10. (a) 11. (d) 12. (a) 13. (c) 14. (a) 15. (b) 16. (b) 17. (b) 18. (b)

Hints to Challenging Problems

HINT:1

Given,

Length, $l = 0.200$ m

Magnetic field, B = 0.067 T

Force, F = 0.13 N

Current, I = ?

Angle between field and rod, $\theta = 90°$

We have,

$F = B I l \sin\theta$

or $I = \dfrac{F}{lB \sin\theta}$

HINT:2

Given,

Diameter (d) = 8.6 cm = 8.6×10^{-2} m

Number of turns (N) = 15

Current (I) = 2.7 A

Magnetic field (B) = 0.56 T

a. What orientation gives maximum torque = ?

Maximum torque, τ_{max} = ?

We have,

$\tau = BINA \cos\theta$

For τ to be maximum, $\cos\theta = 1$ i.e., $\theta = 0°$. Hence, if the plane of coil is oriented along the magnetic field, the torque is maximum given by

$$\tau_{max} = BINA = BIN \times \frac{\pi d^2}{4}$$

b. For what orientation of the coil is the torque 71% of that found in part (a) ?

$\tau = 71\% \times \tau_{max}$

or $BINA \cos\theta = \dfrac{71}{100} \times 0.13$

Find θ,

HINT: 3

Given,

Force (F) = 4.60×10^{-15} N

Angle (θ) = 60°

Magnetic field (B) = 3.50×10^{-3} T

We know that

$\because$ $F = qvB \sin\theta$

or $v = \dfrac{F}{qB\sin\theta}$

HINT: 4

Given,

B = 0.550 T

r = 2.50 cm = 2.50×10^{-2} m

I = 10.8 A

Force exerted on the wire, F = ?

$\theta = 90°$

We know that

$F = BIl \sin\theta$

In this case, $l = 2r$ (diameter of cylindrical region is equal to the length of wire)

$F = BI \times 2\,r \sin 90$

HINT: 5

Given,

Separation between to wires,

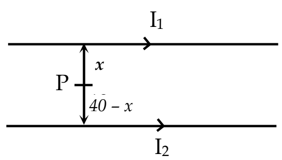
$d = 40\ cm = 40 \times 10^{-2}\ m$

Current in one wire, $I_1 = 25\ A$

Current in another wire, $I_2 = 75\ A$

a. Let x be the distance of point of zero magnetic field from first wire,

$x \ = ?$



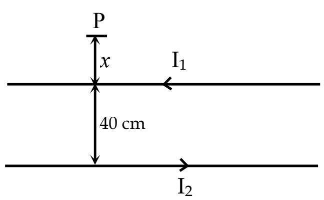
The distance of point P from the second wire is (40 – x) cm when net field is zero. So, we can write

$B_1 = B_2$

or $\frac{\mu_0}{2\pi} \cdot \frac{I_1}{x} = \frac{\mu_0 I_2}{2\pi\,(40 - x)}$

b. Currents are in opposite directions as in figure

In this case, P be the point nearby the wire carrying current I_1 at a distance x where net field is zero.



So, we can write

$B_1 = B_2$

or $\frac{\mu_0}{2\pi}\frac{I_1}{x} = \frac{\mu_0}{2\pi}\frac{I_2}{x + 40}$

HINT: 6

Given,

Separation between wires, $r = 0.400\ m$

Current, $I_1 = 5.00\ A$

Current, $I_2 = 2.00\ A$

Length of each wire, $l = 1.20\ m$

We know that

$F = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{r}\, l$

The force is repulsive due to opposite direction of current.

HINT: 7

Given,

Distance between two wires, $r = 2.50\ cm$

$= 2.50 \times 10^{-2}\ m$

Force per unit length $\left(\frac{F}{l}\right) = 4 \times 10^{-5}\ N/m$

Current in one wire, $I_1 = 0.60\ A$

a. Use formula, $\frac{F}{l} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{r}$

b. The two wires carrying current repel each other, so the currents in the wires must be in the opposite directions.

HINT: 8

Given,

$a = 2.40\ cm = 2.40 \times 10^{-2}\ m$

Number of turns, $N = 800$ turns.

$B = 0.058\ T$

a. Current in the coil, I = ?

We know that

$B = \frac{\mu_0 NI}{2a}$

or $I = \frac{2a \times B}{\mu_0 N}$

b. At what distance x from the centre of the coil is the magnetic field $\frac{1}{2}$ its value at the centre? i.e, $x = ?$

From given condition,

magnetic field, $B' = \frac{B}{2}$

or $\frac{\mu_0 NIa^2}{2(x^2 + a^2)^{3/2}} = \frac{1}{2} \times \frac{\mu_0 NI}{2a}$

or $\frac{a^2}{(x^2 + a^2)^{3/2}} = \frac{1}{2a}$

HINT: 9

Given,

$a = 6\ cm = 6 \times 10^{-2}\ m$

$I = 2.50\ A$

Number of turns, N = ?

Distance of a point from the centre of the coil, $x = 6\ cm = 6 \times 10^{-2}\ m$.

Magnetic field, $B = 6.39 \times 10^{-4}\ T$.

The magnetic field at a point on the axis of the coil is given by

$B = \frac{\mu_0 NIa^2}{2(x^2 + a^2)^{3/2}}$

or $N = \frac{B \times 2(x^2 + a^2)^{3/2}}{\mu_0 Ia^2}$

HINT: 10

Given,

Length of solenoid (l) = 15.0 cm = 15.0×10^{-2} m

Number of turns (N) = 600

Current (I) = 8.00 A

Magnetic field (B) = ?

The magnetic field due to a solenoid is given by

$B = \mu_0 nI = \mu_0 \frac{N}{l} I$ $(\because n = \frac{N}{l})$

HINT: 11

Given,

Magnetic field (B) = 0.150 T
Length (l) = 1.40 m
Number of turns (N) = 4000
Diameter (d) = 20.0 cm

Current needed, I = ?

$\because$ The magnetic field due to a solenoid is given by

$B = \mu_0 nI$

or $B = \mu_0 \frac{N}{l} I$

or $I = \frac{Bl}{\mu_0 N}$

HINT: 12

Given,

Diameter (d) = 14.0 cm = 14×10^{-2} m

$\therefore$ Radius (r) $= \frac{d}{2} = \frac{14.0}{2} = 7.0 \times 10^{-2}$ m

Number of turns (N) = 600
Magnetic field (B) = ?
Current (I) = 0.650 A

The magnetic field due to a toroidal solenoid is given by

$B = \frac{\mu_0 NI}{2\pi a}$

HINT:13

Given,

Current (I) = 0.5 A
Thickness (t) = 4 mm = 4×10^{-3} m
Magnetic field (B) = 0.2 T
Hall voltage (V_H) = 6.0 mV = 6.0×10^{-3} V
n = ?

We know that

$V_H = \frac{BI}{net}$

$n = \frac{BI}{V_H et}$

HINT: 14

Given,

Thickness (t) = 1 mm = 10^{-3} m
Current (I) = 150 mA = 150×10^{-3}A
Hall voltage (V_H) = 57 mV = 57×10^{-3} V
n = 4.3×10^{21} m^{-3}, B = ?

We know that

$V_H = \frac{BI}{nte}$

or $B = \frac{nteV_H}{I}$

❖ ❖ ❖

MAGNETISM

16
CHAPTER

16.1 Introduction

A freely suspended bar magnet rests showing north and south direction roughly. If the magnet is displaced from its equilibrium position and observed repeatedly, it comes to line up at N-S direction in each case. This activity was an interesting query in the scientific society, in the beginning of study about the magnetism of earth. Later on, many experimental results that were done at different locations of the earth proved that the earth is a huge magnet and behaves as if a huge bar magnet is laid at interior of the earth. This branch of physics which deals with the study of earth's magnetic field is called terrestrial magnetism. It is also called geomagnetism.

Although many scientists studied about the magnetic property of the earth, William Gilbert, in 1600, is considered as the pioneer of research on this field, when he published a paper 'De Magnete'. The cause behind the terrestrial magnetism has not yet been clarified. Out of many purposed theories, some of them are given below:

i. Consideration of a permanent magnet at the interior of the earth.
ii. Effect of external magnetic field whose source lies beyond the earth.
iii. Due to the cosmic rays.
iv. Due to the thermoelectric current that generates in the metallic part on the surface of the earth.

Despite these proposed theories none of them have been able to fully convince the researchers and general public.

Facts about the Geomagnetism

There are many observed facts about the geomagnetism. Some of them are given below:

i. The north and south pole of earth's magnet (i.e. terrestrial magnet) lie at the opposite directions of geographical poles.
ii. The geographical poles and earth's magnetic poles do not coincide.
iii. The earth's magnetic poles change over time. They do not lie always at a fixed location of the earth.
iv. Geomagnetic field strength varies from 3.0×10^{-5} T (at the equator) to 6.0×10^{-5} T (at the poles).
v. Over the long time, the magnetic poles are supposed to be reversed.
vi. The distance between the geomagnetic poles is considerably shorter than the diameter of the earth.

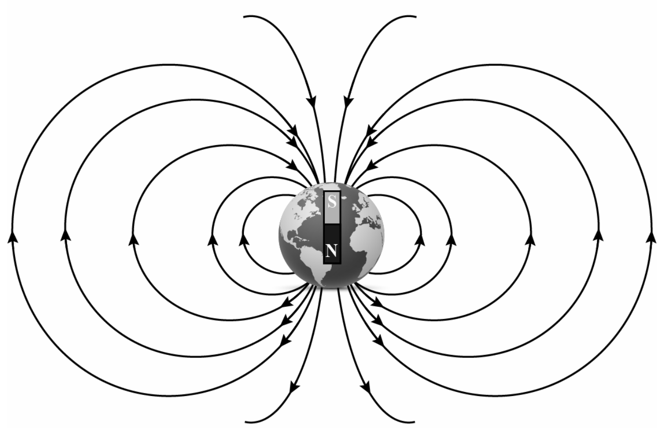


Fig. 16.1: Geomagnetism

16.2 Geographical Meridian and Magnetic Meridian

The north pole of terrestrial magnet lies towards the geographical south pole and vice-versa, but they do not coincide at a particular location. They are significantly far, about 1800 km, to each other. To study the geographical locations and magnetic field strength of the earth, imaginary planes are considered on the earth's surface, they are called meridians. Two types of meridian are drawn on the earth, they are geographical meridian and magnetic meridian.

Geographical Meridian

The imaginary lines passing through the geographical north pole and south pole of the earth is called geographical axis and the plane passing vertically through the geographical axis is known as geographical meridian.

Magnetic Meridian

The imaginary line passing through the earth's magnetic south pole and north pole is called magnetic axis and the plane passing vertically through the axis is known as magnetic meridian.

The geographical meridian and magnetic meridian do not coincide with each other. They have approximately 17^o angular separation at the equator.

16.3 Magnetic Elements of the Earth

The physical quantities which are useful to study the magnitude and direction of earth's magnetic field are known as magnetic elements of the earth. These elements are also applicable to determine the geographical locations. There are three types of magnetic elements of the earth.

i. Angle of declination
ii. Angle of dip or angle of inclination
iii. Horizontal components of earth's magnetic field

Angle of Declination

As explained previously, the magnetic meridian and geographical meridian do not coincide with each other. They lie at certain angles apart. The angle between the magnetic meridian and the geographical meridian at a place is known as the angle of declination. It varies from varies place to place on the surface of the earth. The declination angle is shown in Fig. 16.2.

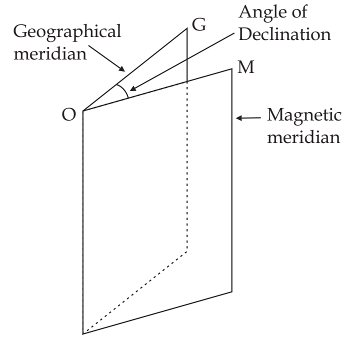


Fig.16.2: Angle of declination

Angle of Dip or Angle of Inclination

A magnetic needle supported with a horizontal frictionless axle and placed at magnetic meridian does not remain in horizontal direction at all places; one end dips down and another end rises up. The direction shown by the needle at a place gives the direction of resultant magnetic field of the earth. The inclination of the needle differs from place to place. i.e. magnetic field strength of the earth is different at different places. *The angle made by the resultant magnetic field with its horizontal components is known as an angle of dip or angle of inclination.* It is denoted by δ.

If the magnetic needle is taken towards the magnetic poles of the earth, through the path of magnetic meridian, one end of the needle dips down gradually. If we go towards north holding the needle, the north pole of the needle dips down. Conversely, the south pole of the needle dips down if we bring the needle along the south. The needle aligns perfectly vertical at the magnetic poles. This shows that angle of dip is 0^o at the equatorial region and 90^o at the earth's magnetic poles. In between equator and pole, angle of dip lies between 0^o to 90^o (i.e. $0 < \delta < 90°$).

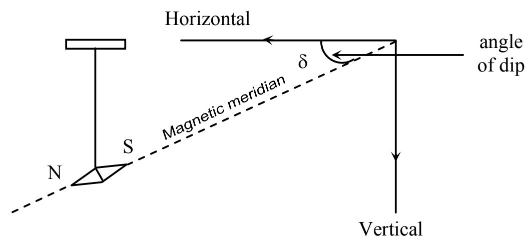


Fig. 16.3: Angle of dip

Consider a magnetic needle that can be rotated freely about the horizontal axis as shown in Fig.16.3. The angle between the direction of needle and horizontal line drawn from the point of rotation gives the angle of dip (δ).

Dip angle is measured with a device, called the dip circle.

Horizontal Components of Earth's Magnetic Field

The resultant intensity of earth's magnetic field can be resolved into two mutually perpendicular components, horizontal component and vertical component. The component parallel to the horizontal plane of earth is called horizontal components of earth's magnetic field, the horizontal component is taken as the reference to measure the angle of dip. *This component of the earth's magnetic field at which the angle of dip is zero is known as horizontal component of earth's magnetic field.* It is denoted by B_H. Another component of earth's magnetic field which is perpendicular to horizontal one is known as vertical component of earth's magnetic field. It is denoted by B_V. The horizontal component and vertical component of earth's magnetic field are shown in Fig. 16.4.

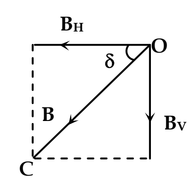


Fig. 16.4: Components of magnetic field

Let B be the resultant intensity of earth's magnetic field and δ be the angle of dip, then from Fig. 16.4,

$$\tan \delta = \frac{B_V}{B_H} \qquad \text{...(16.1)}$$

$$\text{or,}\quad \mathbf{B_V = B \sin \delta} \qquad \text{...(16.2)}$$

$$\text{or,}\quad \mathbf{B_H = B \cos \delta} \qquad \text{... (16.3)}$$

Squaring and adding equations (16.2) and (16.3), we get,

$$B_V^2 + B_H^2 = B^2 \sin^2\delta + B^2 \cos^2\delta$$

$$\text{or,}\quad B_V^2 + B_H^2 = B^2$$

$$\therefore\quad \mathbf{B = \sqrt{B_V^2 + B_H^2}} \qquad \text{...(16.4)}$$

16.4 Apparent Dip

The plane of scale of dip circle must be positioned along the magnetic meridian to determine the true value of dip angle. If the dip circle is rotated at certain angle (usually horizontal) from the magnetic meridian, the needle of dip circle does not indicate the correct direction of earth's magnetic field. The angle of dip at the position except the magnetic meridian is known as apparent dip. It is denoted by δ'.

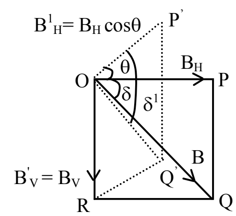


Fig. 16.5: Real dip and apparent dip

Let the plane of scale of dip circle is set at an angle θ with the magnetic meridian as shown in Fig. 16.5. If the dip circle is rotated only along the horizontal plane, and vertical plane remains unchanged. Then,

New component along horizontal, $B'_H = B_H \cos\theta$

and component along vertical = B_V

Then, the tangent angle of apparent dip,

$$\tan\delta' = \frac{B_V}{B'_H}$$

$$= \frac{B_V}{B_H \cos\theta}$$

$$\therefore\quad \mathbf{\tan\delta' = \frac{\tan\delta}{\cos\theta}} \qquad \left(\because \tan\delta = \frac{B_V}{B_H}\right) \qquad \text{... (16.5)}$$

This is the expression for the relation of true dip and apparent dip.

Dip circle

Dip circle is an instrument which measures the angle of dip at different locations of the earth. It is used in surveying, mining and study of earth magnetism. It consists of a magnetic needle pivoted at the centre of a vertical circular scale that can rotate in the plane of scale about the horizontal axis passing through its centre of gravity. The circular scale is divided into four quadrants with 0-90°, 90°-0, 0-90° and 90°-0 as shown in Fig. 16.6. The deflection of magnetic needle measures the angle of dip. Also, the rotation of vertical scale about the horizontal circular scale fixed on its base gives the angle of rotation of circle.

Measurement of dip angles

The dip circle measures the angle of dip with two different methods.

i. Bringing the needle into the magnetic meridian

ii. Without bringing the needle into the magnetic meridian

i. **Bringing the needle into the magnetic meridian:** The angle of dip is directly measured by using this method. First, zero point vernier scale on the support position of dip circle is coincided with the zero of horizontal scale at its base. Then, whole the apparatus is rotated upto a position that the magnetic needle of dip circle points 90°-90° on the vertical scale. In this position, the plane of vertical scale is perpendicular to the magnetic meridian (i.e. along the east-west) direction. Then, keeping the horizontal scale fixed, the plane of vertical scale is rotated by 90°. Now, the plane of vertical circular scale aligns along the magnetic meridian. In this condition, the deflection of magnetic needle is read, which gives the true angle of dip.

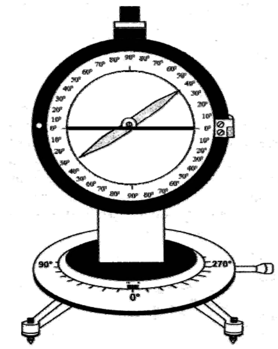


Fig. 16.6: Dip circle

Errors associated with this measurement

a. The centre of gravity of needle may not lie at the axis of rotation, which gives the erroneous value of dip angle.

b. The centre of needle may not coincide with the centre of circular scale. This produces the excessive rotation in one direction only.

c. 0° - 0° line or 90°- 90° line may not lie in a same line, which may deviate the value of dip angle from the true value.

ii. **Without bringing the needle into the magnetic meridian:** The angle of dip can be determined by using the values of apparent dip. In this method, two apparent dips are measured by rotating the scales of dip circle 90° one another.

Let δ_1 be the apparent dip in a plane which makes angle θ with the magnetic meridian. In this plane, the vertical component of earth's magnetic field is not changed but the horizontal component will be changed to $B_H' = B_H \cos\theta$. Then, the apparent dip,

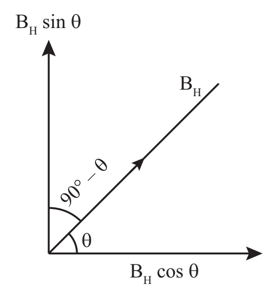


Fig. 16.7: Components of horizontal magnetic field

$$\tan\delta_1 = \frac{B_V}{B_H'} = \frac{B_V}{B_H \cos\theta}$$

$$\text{or,}\quad \cos\theta = \frac{B_V}{B_H \tan\delta_1}$$

$$\therefore\quad \mathbf{\cos\theta = \frac{B_V}{B_H}\cot\delta_1} \qquad \text{...(16.6)}$$

Then, the scale of dip circle is rotated by 90°, so that the angle made by this plane with the magnetic meridian will be (90° – θ). let δ_2 be the apparent dip in the new plane. Now, the horizontal component of earth magnetic field in this plane is,

$$B_H'' = B_H \cos(90° - \theta)$$
$$= B_H \sin\theta$$

Now, the apparent dip is,

$$\tan\delta_2 = \frac{B_V}{B_H''} = \frac{B_V}{B_H \sin\theta}$$

$$\text{or,}\quad \sin\theta = \frac{B_V}{B_H \tan\delta_2}$$

$$\therefore \quad \mathbf{\sin\theta = \frac{B_V \cot\delta_2}{B_H}} \qquad \text{...(16.7)}$$

Squaring and adding equations (16.6) and (16.7), we get,

$$\cos^2\theta + \sin^2\theta = \left(\frac{B_V}{B_H}\cot\delta_1\right)^2 + \left(\frac{B_V}{B_H}\cot\delta_2\right)^2$$

$$\text{or,} \quad 1 = \frac{B_V^2}{B_H^2}(\cot^2\delta_1 + \cot^2\delta_2)$$

$$= \tan^2\delta(\cot^2\delta_1 + \cot^2\delta_2)$$

$$= \frac{1}{\cot^2\delta}(\cot^2\delta_1 + \cot^2\delta_2)$$

$$\therefore \quad \mathbf{\cot^2\delta = \cot^2\delta_1 + \cot^2\delta_2} \qquad \text{... (16.8)}$$

This is the appropriate expression of determining true dip by using two apparent dips.

16.5 Domain Theory of Ferromagnetism

Magnetic substances contain molecular magnets. Molecular magnets are tiny magnets, but they do not show the magnetism independently, rather a group of such molecules produces the magnetism. *This group of molecular magnets which form a tiny but a complete magnet are called domains*. All ferromagnetic substances contain domains. All ferromagnetic substances have a characteristics "domain character".

In the absence of external magnetic field, domains are oriented randomly, so the magnetic substance does not show its magnetism as shown in Fig. 16.8 (i). When an external magnetic field is applied, the domains align along the direction of applied field as shown in Fig. 16.8(ii). Thus, the ferro-magnetic substance shows the magnetic behaviour. In ferromagnetic substance, the magnetism is retained even after the, external magnetic field is removed.

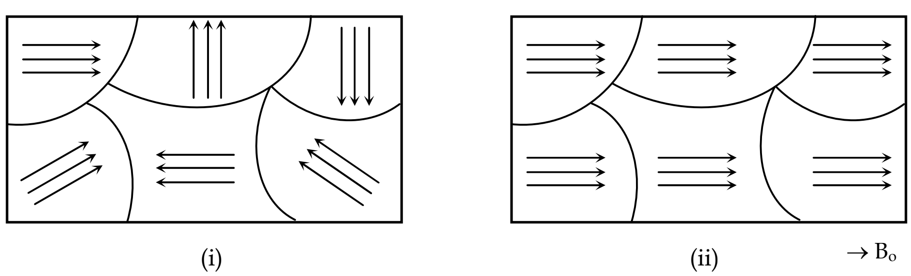


Fig. 16.8 : Domains (i) absence of external magnetic field; (ii) presence of external magnetic field

16.6 Magnetic Properties of Materials

Every matter is made up of atoms. In atoms, electrons revolve around the nuclei that form the current loop as if the current flows in a circular coil as explained in chapter (15). The revolution of electrons in an orbit generates the magnetic field along the axial direction, which forms the magnetic moment. Thus, each current loop acts as the tiny bar magnet. Similarly, spin of particles also generates the magnetic moment.

The magnetic moment due to a current loop is added to magnetic moment due to spins. Also, the magnetic moments due to the spin of protons and neutrons are added up in the magnetic moment generated by electrons. The vector sum of all above magnetic moments provides the net magnetic moment of an atom. In a substance, the direction of magnetic moment is random, so net magnetic moment is observed zero in the absence of external magnetic field. When the magnetic substance is placed into the external magnetic field, tiny magnets start orienting along the direction of applied field. However, the thermal agitation attempts to break the alignment of atoms. So, to prevent from the deformation of such alignment of such tiny magnets, the strength of external magnetic field should be increased. At a certain value of magnetic field, the alignment is completed. This condition is called saturation of magnetism.

Some important terms associated with the magnetic materials are explained below.

i. **Intensity of magnetization:** *It is defined as the net magnetic moment per unit volume of a magnet along the direction of applied magnetic field.* It is denoted by I. It is a vector quantity.

$$\vec{I} = \frac{\text{Magnetic moment } (\vec{M})}{\text{Volume (V)}} = \frac{\vec{M}}{V}$$

The unit of I is ampere per meter (Am^{-1}). In a bar magnet, $M = m \cdot 2l$, where, m is magnetic pole strength and $2l$ is the effective length of the magnet.

Also, the volume of bar magnet (V) = $A \cdot 2l$, where A is the cross-sectional area of magnet

Therefore, the intensity of magnetization,

$$I = \frac{m \cdot 2l}{A \cdot 2l} = \frac{m}{A}$$

Therefore, the intensity of magnetization is also defined as the pole strength per unit cross-sectional area of a magnet.

ii. **Magnetic intensity ($\vec{H}$):** It can also be defined as the force experienced by a unit positive charge flowing with unit velocity in a direction normal to magnetic field. *The degree to which a magnetic field can magnetize a material is called magnetic intensity.* It is denoted by $\vec{H}$. It is a vector quantity. Its unit is ampere per meter (Am^{-1}). It is also called magnetic force. Magnetic intensity magnetize the magnetic substance when placed in this field.

$$H = \frac{B}{H}$$

iii. **Total magnetic field ($\vec{B}$):** When a magnetic substance is placed in an external magnetic field, it gets magnetized. The resultant magnetic field within the magnetic material is the vector sum of magnetic field due to applied field and the induced magnetic field due to the material itself.

Its SI unit is weber m^2 or Tesla (T). Its CGS unit is gauss (G)

Therefore, total magnetic field $\vec{B}$, is written as,

$$\vec{B} = \text{Applied field } (\vec{B}_0) + \text{magnetic field due to the magnetization of material } (\vec{B}_H)$$

$$\textbf{i.e. } \vec{\mathbf{B}} = \vec{\mathbf{B}}_0 + \vec{\mathbf{B}}_H \qquad \textbf{... (16.9)}$$

The terms $\vec{B}_0$ and $\vec{B}_H$ can be expressed into $\vec{H}$ and $\vec{I}$ respectively.

The strength of applied field due to the magnetic intensity $\vec{H}$, $\vec{B}_0 = \mu\vec{H}$ and $\vec{B}_M = \mu_0 I = \mu_0\chi\vec{H}$

Where, χ is dimensionless quantity and is known as magnetic susceptibility. The magnetic susceptibility indicates the degree of magnetization of a material in response to an applied magnetic field.

Therefore,

$$\vec{B} = \mu_0\vec{H} + \mu_0\chi\vec{H}$$

$$= \mu_0 (1 + \chi)\ \vec{H} \qquad \text{... (16.10)}$$

Also, in the material medium, total magnetic field $\vec{B}$ is directly proportional to the magnetic intensity $\vec{H}$.

i.e. $\vec{B} \propto \vec{H}$

$$\vec{B} = \mu\vec{H}$$

Where, μ is the absolute permeability of a medium. So, we write,

$$\mu = \mu_0 (1 + \chi) \qquad \text{... (16.11)}$$

Magnetic Susceptibility

Magnetic susceptibility of a magnetic substance is the ratio of intensity of magnetization to the magnetic intensity. It is denoted by χ. It has no unit. It is the property of substance.

$$\therefore \quad \chi = \frac{\text{intensity of magnetization (I)}}{\text{magnetic intensity (H)}}$$

$$\chi = \frac{I}{H}$$

Magnetic susceptibility χ measures how much extent the materials can be magnetized. The magnetic materials which can be magnetized strongly, have the value of χ high positive value. This type of materials are called ferromagnetic materials. The magnetic materials which are weakly magnetized have the value of χ small positive value. This type of materials are called paramagnetic materials. Likewise, the magnetic materials which are weakly magnetized in the opposite of applied field are called diamagnetic materials. The value of χ is small negative for these type of materials.

Relative Permeability

Absolute permeability of a material medium is a measure of the amount of resistance encountered when forming a magnetic field in that medium. It is denoted by μ. If the absolute permeability is taken for the free space (or vacuum), it is denoted by μ_0. It is also called the permeability constant. The ratio of absolute permeability of a medium to permeability constant is called relative permeability. It is denoted by μ_r. Therefore,

$$\text{Relative permeability } (\mu_r) = \frac{\mu}{\mu_0}$$

It is a dimensionless quantity. The relation between absolute permeability and magnetic susceptibility is,

$$\mu = \mu_0 (1 + \chi)$$

$$\frac{\mu}{\mu_0} = 1 + \chi$$

$$\mu_r = 1 + \chi \qquad \text{... (16.12)}$$

This expression gives the relation between relative permeability of a medium and its magnetic susceptibility.

Curie Law

The intensity of magnetisation (I) of a paramagnetic substance is

i. directly proportional to the magnetic induction produced by the magnetizing field H in free space,

i.e. $I \propto B_0$ and $B_0 = \mu_0 H$

ii. Inversely proportional to the absolute temperature T of the material

i.e. $I \propto \frac{1}{T}$

Combining above equations,

$I \propto \frac{B_0}{T} \propto \frac{\mu_0 H}{T}$

This law was first discovered by Madam Curie and hence the law is called Curie law in magnetism. Since, μ_0 is constant.

$$I \propto \frac{H}{T}$$

$$\frac{I}{H} \propto \frac{1}{T}$$

$$\chi = \frac{1}{T}$$

$$\text{Hence, } \chi \propto \frac{1}{T}.$$

Curie temperature for iron is 1000 K, for cobalt 1400 K and for nicked 600 K.

16.7 Magnetic Substances

The substances which are influenced by the magnetic field are known as magnetic substances. Magnetic substances are divided into three categories:

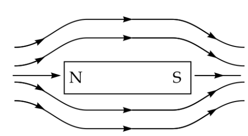
i. Diamagnetic substances
ii. Paramagnetic substances
iii. Ferromagnetic substances

Diamagnetic Substances

When placed in magnetic field, the lines of force tend to avoid substance.

The magnetic substances which have the tendency to move from stronger to weak part of the external magnetic field are known as diamagnetic substances. They are feebly repelled by the magnet. Some examples of diamagnetic substances are: antimony, bismuth, copper, lead, gold, mercury, water, air, zinc, silver, etc.

Some important properties of diamagnetic substances are given below:



1. They are feebly repelled by the magnet and tend to move from stronger field to weaker field.
2. If a diamagnetic rod is suspended in a uniform magnetic field, the rod aligns itself in a direction perpendicular to the direction of the magnetic field.

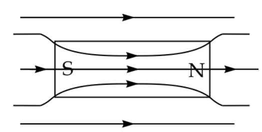
3. When a diamagnetic substance is placed in a magnetic field, the lines of force are rarefied in the substance and pass through the surrounding air.
4. They lose their magnetism as soon as the magnetization field is removed.
5. When a diamagnetic substance is placed in a magnetic field, it develops weak magnetization in a direction opposite to the direction of the magnetizing field.
6. The permeability of a diamagnetic substance is less than one.
7. The magnetic susceptibility (χ) of a diamagnetic substance has a small negative value and it is temperature independent.
8. The intensity of magnetization (I) is small, negative and varies linearly with field.
9. Induced dipole moment (m) is a small –ve value.
10. They do not obey Curie's law.

Paramagnetic Substances

The magnetic substances which are weakly magnetized by the external magnetic field is known as paramagnetic substances. They are weakly attracted to a magnet. They slowly move from weak to strong external magnetic field direction. Some examples of paramagnetic substances are: aluminium, sodium, potassium, calcium, magnesium, oxygen (at STP), platinum, etc.

Important properties of paramagnetic substances are given below:

1. They are feebly attracted by a magnet and have tendency to move from weak to strong magnetic field direction.
2. If a paramagnetic rod is freely suspended in a uniform magnetic field, the rod aligns parallel to the field direction. The lines of force prefer to pass through substance rather than air.



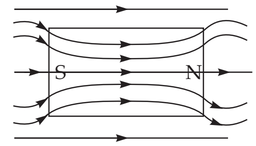
3. As soon as the magnetizing field is removed, they lose their magnetic properties.
4. The magnetic lines of force prefer to pass through a paramagnetic material rather than air.
5. When a paramagnetic substance is placed in a magnetizing field, it is weakly magnetized in the direction of external magnetic field.
6. The magnetic permeability of a paramagnetic substance is slightly greater than one.
7. The magnetic susceptibility (χ) of paramagnetic substance is less than one but has positive value. Value of magnetic susceptibility is inversely proportional to temperature $\chi \propto \frac{1}{T}$ (i.e. curie law).
8. The intensity of magnetization (I) is positive, small and varies linearly with field.
9. Induced dipole moment (m) is a small positive value.
10. They obey Curie's law.

Ferromagnetic Substances

The substances which get strongly magnetized in the direction of external magnetic field are known as ferromagnetic substances. They have strong tendency to move from weaker magnetic field region to strong magnetic field region. These substances are strongly attracted by a magnet. Some examples of ferromagnetic substances are: Iron, Cobalt, Nickel, Gadolinium, Dysprosium, etc.

Some properties of ferromagnetic substances are given below:

1. They are strongly attracted by the magnet and have the tendency to move from weak to strong external magnetic field.
2. When they are placed in a magnetic field, the magnetic lines of force tend to crowd into them.
3. When a ferromagnetic substance is freely suspended in a uniform magnetic field, it aligns itself parallel to the direction of magnetic field.



4. Ferromagnetic substances retain their magnetism even after the magnetizing field is removed.
5. When ferromagnetic substances are placed in external magnetic field, they get magnetized strongly along the direction of field.
6. The permeability of ferromagnetic substance is extremely large as compared to the permeability of free space.
7. They obey Curie's law. At certain temperature called Curie point, they lose ferromagnetic properties and behave like paramagnetic substances.
8. They have large positive susceptibility(χ). The magnetic susceptibility of ferromagnetic substance obeys Curie-Weiss law $\chi \propto \frac{1}{T - T_C}$ where T_C is Curie temperature.
9. The intensity of magnetization (I) is very large, positive and varies non-linearly with field.
10. Induced dipole moment is a large positive value.

16.8 Magnetic Hysteresis

Magnetic substances behave differently in the external magnetic field. The diamagnetic substance and paramagnetic substance have the simple linear relationship between magnetizing field (external magnetic field) $\vec{H}$ and total magnetic field into the substance ($\vec{B}$) or intensity of magnetization (I). However, the ferromagnetic substance exhibits different behaviour than that of dia and para-magnetic substance, $\vec{B}$ and $\vec{H}$ have the non-linear functional relationship, B = f(H). The behaviour of ferromagnetic substance in external magnetic field is explained below.

In the absence of external magnetic field, the arrangement of magnetic domains of ferromagnetic substance are random, so it does not show any magnetic strength around it. When it is kept into the external magnetic field (i.e. inducing field $\vec{H}$), the magnetic domains of the substance tend to orient along the direction of applied magnetic field and hence it becomes magnetized. If the external magnetic field is turned off, the magnetic strength of ferromagnetic substance will not be zero. This persistence of magnetic field strength of magnetic substance although the inducing field is reduced to zero is known as magnetic hysteresis. Hysteresis is lagging (phase) of magnetic induction of ferromagnetic and ferromagnetic materials with respect to be cyclic variation of an applied magnetic

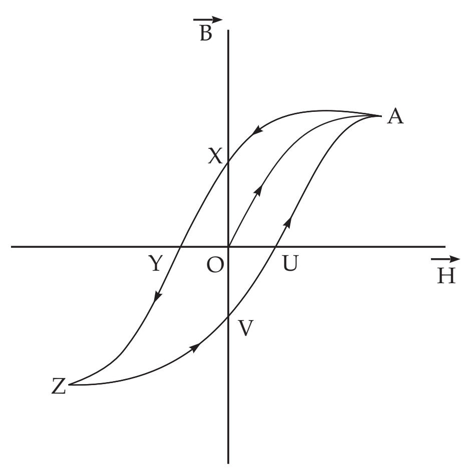


Fig. 16.9: Magnetic hysteresis curve

field, when the specimen is at a temperature below its Curie temperature. The non-linear closed curve between the magnetic intensity $\vec{H}$ and total magnetic field ($\vec{B}$) (or intensity of magnetization (I)) is known as hysteresis curve or hysteresis loop. This behaviour of ferromagnetic substance in the presence of external magnetic field is explained in the following steps. The hysterisis curve is shown in Fig. 16.9.

In the Fig. 16.9, X = retentivity, Y = coercivity, A = Saturation, OY = H_C and OX = Br.

i. In the absence of magnetic intensity ($\vec{H}$), total magnetic field ($\vec{B}$) into the substance is also zero. So, the curve begins from origin O of hysteresis loop.
ii. When the external magnetic field is turned on, the molecular magnets starts showing magnetic behaviour. When the applied field strength is gradually increased, the substance is also induced towards saturation value. It means the molecular magnets gradually orient along the external magnetic field. In figure, the curve beyond point A shows the saturation line. In this condition, almost all of the magnetic domains are aligned along the direction of applied field. If the applied field is further increased, there is no more magnetic domains left to add up the magnetic strength in the substance. So, the saturation is obtained. Graph OA shows the nature of magnetization in ferromagnetic substance from zero to saturation point.
iii. If the applied field (i.e. magnetic intensity $\vec{H}$) is, then, reduced gradually to zero, surprisingly, the total magnetic field $\vec{B}$ in the substance will not be zero, rather it has certain positive value OX in the graph. It means the induced magnetic field retains in the ferromagnetic substance once magnetized, although the applied field is zero. This value OX in graph is called remenance (or retentivity).
iv. If the direction of magnetic intensity $\vec{H}$ is reversed and gradually increased, the value of $\vec{B}$ decreases and becomes zero at a certain reverse value of $\vec{H}$. The graph OY shows the value of H at which the value of B in ferromagnetic substance is zero. This value of reversed magnetic intensity which makes the total magnetic field in the substance is zero is called coercivity.
v. If the magnetic intensity $\vec{H}$ is further increased the substance begins magnetizing as explained in (ii) but in the opposite direction as shown in curve. At certain maximum value of magnetic intensity, the value of $\vec{B}$ remains constant. This is the saturation value of magnetization in the reverse direction. Point Z shows the saturation value in the curve.
vi. If the magnetic intensity ($\vec{H}$) is decreased then the value of $\vec{B}$ is also decreased, but can not be zero, even though $\vec{H}$ becomes zero. This nature is also similar as explained in (iii.)
vii. If the magnetic intensity is then, increased along positive direction (i.e. as explained in ii) value of $\vec{B}$ in magnetic substance initially decreases and becomes zero as shown in point V. Further increasing the value of $\vec{H}$, the curve meets at point A and so, forms a closed loop as shown in Fig. 16.9.

Thus, hysteresis loop shows the relationship between magnetic intensity ($\vec{H}$) and total magnetic field ($\vec{B}$) into the substance. It is always referred to as the B-H loop. It is also noted that the value of $\vec{B}$

depends on the intensity of magnetization ($\vec{I}$) of the substance. Therefore, Hysteresis loop can be drawn between $\vec{I}$ and $\vec{H}$. The area enclosed by the curve gives the energy loss per unit volume of the material per cycle.

Significances of Hysteresis Loop

1. Hysteresis loop is obtained in ferromagnetic materials.
2. This loop determines the capacity of magnetic material how strongly and how permanently the material can be magnetized.
   i. Soft iron has low coercivity and large retentivity. Such types of materials can be magnetized quickly and also demagnetized quickly. This type of material is used in transformer core, moving coil galvanometer and electromagnets.
   ii. Steel has low retentivity and large coercivity. The loop occupies large area in B-H graph. The magnetic property of such type of materials remains relatively long time after once magnetized. So, steel is used for permanent magnets.

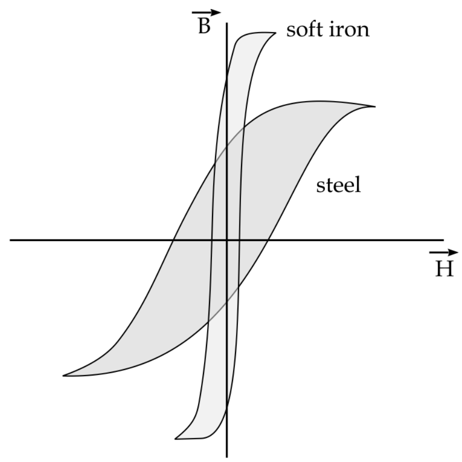


Fig. 16.10: Hysteresis loop for soft iron and steel

Tips for MCQs

1. The strength of earth magnetic field is in the order of 10^{-4} T.
2. The value of angle of declination of the earth at the equator is 17°, its value is measured by kew magnetometer.
3. Angle of dip varies between the value of 0° at equator and 90° at the magnetic poles of the earth.
4. Three types of magnetic materials are classified on the basis of magnetization intensity I, susceptibility χ and relative permeability (μ_r)

Dia-magnetic	Paramagnetic	Ferrromagnetic
$-1 \le \chi_m < 0$	$0 < \chi_{,m} < \varepsilon$	$\chi_m >> 1$
$0 \le \mu_r < 1$	$1 < \mu_r < 1 + \varepsilon$	$\mu_r >> 1$
$\mu < \mu_0$	$\mu > \mu_0$	$\mu >> \mu_0$

Where, ε small positive value.

5. Only ferromagnetic materials show hysteresis behaviour.
6. Comparison of hysteresis loop between soft iron and steel.

Soft iron	Steel
1. Low coercivity, high retentivity, narrow hysteresis loop, susceptibility is high, permeability is high	1. Coercivity is high, low retentivity, wide hysteresis loop, susceptibility is low and permeability is low.
2. Soft iron is used in transformer, moving coil galvanometer, electromagnets, etc.	2. Steel is used to make the permanent magnets.

8. Some important relations

   a. $\tan\delta = \dfrac{B_v}{B_H}$

   b. Apparent dip, $\tan\delta' = \dfrac{\tan\delta}{\cos\theta}$

   c. True dip from two apparent dips, $\cot^2\delta = \cot^2\delta_1 + \cot^2\delta_2$

   d. $B_V^2 + B_H^2 = B^2$

   e. $\mu_r = 1 + \chi$

   f. $\mu = \mu_0\mu_r = \dfrac{B}{H} = \mu_0 (1 + \chi)$

   g. $I = \dfrac{M}{V} = \dfrac{m}{A}$

   h. $H = \dfrac{B_0}{\mu_0}$

   i. $\chi_m = \dfrac{I}{H}$

Worked Out Problems

1. Find the dip angle where the vertical and horizontal components are equal.

SOLUTION

Given,

$B_V = B_H = B$ (let)

The tangent angle of dip is,

$$\tan\delta = \frac{B_V}{B_H}$$

or, $\tan\delta = \dfrac{B}{B}$

or, $\tan\delta = 1$

$\therefore\ \delta = 45^o$

At dip angle 45^o, the vertical and horizontal components are equal.

2. A magnetic needle suspended in a vertical plane at 30° from the magnetic meridian makes an angle of 45° with the horizontal. Find the true angle of dip.

SOLUTION

Given,

Apparent dip (δ) = 45^o

Orientation of needle from the horizontal (θ) = 30^o

True dip (δ) = ?

We know, $\tan\delta' = \dfrac{\tan\delta}{\cos\theta}$

$\therefore\ \tan\delta = \cos\theta \cdot \tan\delta'$

$= \cos 30^o \cdot \tan 45^o$

$= \dfrac{\sqrt{3}}{2} \cdot 1 = 0.866$

$\therefore\ \delta = 41^o$

3. What will be the value of vertical component and total intensity of earth's magnetic field at a place where dip is 60°? Horizontal component of earth's magnetic field is 0.34×10^{-4} T.

SOLUTION

Given,

Horizontal component of earth's magnetic field (B_H) = 0.34×10^{-4} T

Angle of dip (δ) = 60^o

Vertical component of earth's magnetic filed (B_v) = ?

Total intensity (B) = ?

We know,

$$\tan\delta = \frac{B_v}{B_H}$$

$\therefore \quad B_v = B_H \tan \delta$

$= 0.34 \times 10^{-4} \times \tan 60^{o} = 0.59 \times 10^{-4}$ T

and $\quad B = \sqrt{B_H^2 + B_v^2}$

$= \sqrt{(0.34 \times 10^{-4})^2 + (0.59 \times 10^{-4})^2} = 0.68 \times 10^{-4}$ T

4. Find the magnetization of a bar magnet of length 5 cm and cross sectional area 1 cm² if magnetic moment is 0.9 Am².

SOLUTION

Given,

Magnetic moment (M) = 0.9 Am²

l = 5 cm = 5×10^{-2} m

A = 1 cm² = 10^{-4} m²

Volume of the magnet (V) = $l \times A$

$= 5 \times 10^{-2} \times 10^{-4}$

$\therefore$ V $= 5 \times 10^{-6}$ m³

We know that

$$\text{Magnetization (I)} = \frac{M}{V} = \frac{0.9}{5 \times 10^{-4}}$$

$\therefore$ I $= 1.8 \times 10^{5}$ A/m

Challenging Problems

1. The value of dip at a place is 45°. If the plane of the dip circle is turned through 60° from the meridian, what will be the apparent dip?

   Ans: 63.4°

2. The vertical and horizontal components of the earth's magnetic field at a place are 0.2 oersted and 0.3464 oersted respectively. Calculate the angle of dip and the total intensity of earth's magnetic field at that place.

   Ans: 0.4×10^{-4} T

3. The apparent dip at a certain position of the dip circle is 66.1°. The dip circle is rotated through an angle of 90° and the apparent dip is found to be 69.94°. What is the dip at that place?

   Ans: 57.67°

4. Calculate the vertical component of earth's field at a place where the dip is 60° and the horizontal component is 0.2×10^{-4} Wb/m².

   Ans: 0.346×10^{-4} Wb/m²

5. The horizontal component of earth's magnetic field at a place is 0.25 gauss and its vertical component is 0.35 gauss. Calculate angle of dip and earth's total magnetic field at that place.

   Ans: 54.46°, 0.43×10^{-4} T

6. The needle of a dip circle shows an apparent dip of 45° in a particular position and 53° when the circle is rotated through 90°. Find true dip. (HSEB 2062)

   Ans: 38.6°

*[**Note:** Hints to challenging problems are given at the end of this chapter.]*

Conceptual Questions with Answers

1. What do you mean by terrestrial magnetism?

↳ The earth acts as a huge magnet. It behaves as an ordinary magnet. The magnetic behaviour of the earth's magnet is known as terrestrial magnetism. The magnetic polarities of terrestrial magnetism are opposite of geographical polarity, but they do not coincide.

2. What are the three basic components of terrestrial magnetism?

↳ Three basic components of terrestrial magnetism are (a) angle of declination (b) angle of dip and (c) horizontal component of earth's magnetic field.

3. What is angle of declination?

↳ The magnetic meridian and geographical meridian do not coincide at earth. They lie certain distance apart. The angle between the magnetic meridian and the geographical meridian at a place is known as the angle of declination which varies from place to place on the surface of the earth.

4. What is angle of dip?

↳ The angle made by the resultant magnetic field with its horizontal components is known as an angle of dip or angle of inclination. It is denoted by δ.

5. What are the angle of dip at the earth's magnetic poles and equator?
Or, What are the maximum and minimum value of angle of dip?

↳ Angle of dip is different at different locations on the earth. Its value is maximum at the earth's magnetic poles which is equal to 90^o. Value of dip angle is minimum at the equatorial line, where the dip angle is 0^o.

6. How does the knowledge of declination at a place help in navigation?

↳ Angle of declination at a place of the earth gives the angle between the geographical and magnetic meridians. So, the knowledge shall help is sailing the ship in the required direction so as to reach the destination.

7. What is diamagnetic substance? Write some examples.

↳ The magnetic substances which have the tendency to move from stronger to weak part of the external magnetic field are known as diamagnetic substances. They are feebly repelled by the magnet. Some examples of diamagnetic substances are: antimony, bismuth, copper, lead, gold, mercury, water, air, zinc, silver, etc.

8. Write any five important properties of paramagnetic substances.

↳ Some important properties of paramagnetic substances are given below:

a. They are feebly attracted by a magnet and have tendency to move from weak to strong magnetic field direction.
b. If a paramagnetic rod is freely suspended in a uniform magnetic field, the rod aligns parallel to the field direction.
c. As soon as the magnetizing field is removed, they lose their magnetic properties.
d. The magnetic lines of force prefer to pass through a paramagnetic material rather than air.
e. When a paramagnetic substance is placed in magnetizing field, it is weakly magnetized in the direction of external magnetic field.

9. What are magnetic domains?

↳ Magnetic substances contain molecular magnets. Molecular magnets are tiny magnets, but they do not show the magnetism independently, rather a group of such molecules produces the magnetism. This group of molecular magnets which form tiny but complete magnets are called domains.

10. What is apparent dip?

↳ The plane of scale of dip circle must be positioned along the magnetic meridian to determine the true value of dip angle. If the dip circle is rotated certain angle (usually horizontal) from the magnetic meridian, the needle of dip circle does not indicate the correct direction of earth's magnetic field. The angle of dip at the position except the magnetic meridian is known as apparent dip. It is denoted by δ'.

11. A ferromagnetic substance becomes paramagnetic above Curie temperature. Explain why?

↳ The atoms in magnetic substance act as tiny magnets in ferromagnetic substance, these tiny magnets in a domain are strongly bind and align in a specific direction. When the temperature is raised above the curie temperature, no domain exists so that forces due to the magnetic moment of domains disappear. Then, the tiny magnets orient almost randomly, hence, the ferromagnetic substance becomes paramagnetic.

12. What is Curie law in magnetism?

↳ The intensity of magnetisation (I) of a paramagnetic substance is

i. directly proportional to the magnetic induction produced by the magnetizing field H in free space,

i.e. $I \propto B_0$ and $B_0 = \mu_0 H$

ii. Inversely proportional to the absolute temperature T of the material

i.e. $I \propto \frac{1}{T}$

Combining above equations,

$I \propto \frac{B_0}{T} \propto \frac{\mu_0 H}{T}$

This law was first discovered by Madam Curie and hence the law is called Curie law in magnetism. Since, μ_0 is constant.

$I \propto \frac{H}{T}$

$\frac{I}{H} \propto \frac{1}{T}$

$\chi = \frac{1}{T}$

Hence, $\chi \propto \frac{1}{T}$.

13. Define the terms retentivity and coercivity.

✎ The value of intensity of magnetization of a ferromagnetic material when the magnetizing field is reduced to zero is called retentivity or remanance or residual magnetism of the material. It tells that the magnetism retains in a ferromagnetic substance, even though applied field is zero.

The value of magnetizing field required to reduce residual magnetism to zero in a material is called coercivity.

14. Distinguish between dia and para-magnetic substances.

✎ Some important relations between dia-magnetic and para-magentic substances are as follows:

Dia-magnetic substance	Paramagnetic substance
1. It is feebly magnetized in a direction opposite to the direction of the magnetizing field.	1. It is feebly magnetized in the direction of magnetizing field.
2. The value of χ has small negative value.	2. The value of χ has small positive value.
3. For example: copper	3. For example: aluminium

15. What is the basic use of hysteresis loop?

✎ This loop provides the information in the selection of suitable materials for different purposes. For instance, if the retentivity of a material is high, the substance is strongly magnetized, such type of magnetic materials are used in iron cores of transformer. If the area of hysteresis loop of substance is large, such type of materials can be used to form permanent magnet.

16. How does the knowledge of declination at a place help in navigation?

✎ Declination at a place of the earth is the angle between the geographical meridian and magnetic meridian. It is different at different locations of the earth. The information about declination help in steering the ship in the required direction so as to reach the destination.

17. What is the susceptibility and permeability of a perfectly diamagnetic substance?

✎ For perfectly diamagnetic substance,

$B = \mu_0 (H + I) = 0$

or, $H + I = 0$

$\therefore$ I = H

i. Susceptibility, $\chi = \frac{I}{H} = \frac{-H}{H} = -1$

ii. Relative permeability, $\mu_r = 1 + \chi = 1 - 1 = 0$

$\therefore$ Absolute permeability, $\mu = \mu_0\mu_r = 0$

---

18. Why transformer cores are made of soft iron?

✎ Transformer transfers the signals of alternating current. In this process, the transformer core has to be magnetized and demagnetized periodically. In this condition, the material is so chosen that it does loss the relatively low energy i.e. the material has to be made from the material of low hysteresis. Soft iron meets this requirement.

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19. Why soft irons is used to make the electromagnets?

✎ The hysteresis loop of soft iron is narrow. So, loss of energy per unit volume per cycle of its magnetization is small. It also have light permeability. Therefore, it can be magnetized and demagnetized easily. Hence, soft iron is used for making electromagnets.

Exercises

Short-Answer Type Questions

1. What are the geomagnetic elements?
2. How does dip vary on earth's surface from place to place?
3. What do you mean by angle of declination and angle of inclination?
4. Classify ferro-, para-, and dia-magnetic substance in accordance with attraction with an iron rod.
5. Why does not the compass needle point to the true north?
6. What is Curie point?
7. Would the maximum possible magnetization of paramagnetic sample be of the same order of magnitude as the magnetization of a ferromagnet?
8. A certain region of space is to be shielded from magnetic fields. Suggest a method.
9. How many neutral points are obtained when a magnet is placed vertically?
10. Which way would you a compass point if you were at earth's north magnetic pole?
11. A small magnet pivoted at the centre is free to rotate in magnetic meridian. At what place, will it stand vertical?

Long-Answer Type Questions

1. What are magnetic substances? How are they classified? Describe in brief.
2. Distinguish between ferromagnetic, paramagnetic and diamagnetic materials. Write down their some properties.
3. What are the magnetic elements of the earth? Describe them in brief.
4. What are real and apparent dips? Establish the relation between them.
5. If δ is the true dip in the magnetic meridian; and δ_1 and δ_2 be apparent dips at different planes perpendicular to each other then prove that $\cot^2\delta = \cot^2\delta_1 + \cot^2\delta_2$.
6. What is hysteresis? Describe the formation of hysteresis loop in ferromagnetic substance.

Numerical Problems

1. At a given place, the horizontal component of the earth's field is 0.38 oersted and the vertical component is 0.134 oersted. Calculate the total field and dip at that place.

   Ans: 0.36×10^{-4}T, $19^0 25'$

2. At a given place, the horizontal component of the earth's magnetic field is 0.3×10^{-4} Wm^{-2} and the angle of dip is 30^o. What will be the vertical components of the earth's magnetic field?

   Ans: 1.73×10^{-5} T

3. The total magnetic intensity at a place is 0.4 oersted and the angle of dip is 30°. Calculate the horizontal and vertical components.

Ans: 0.34×10^{-4} T, 0.2×10^{-4} T

4. The vertical component of earth's magnetic field at a place is 0.16×10^{-4} tesla. Calculate the value of horizontal component of earth's magnetic field, if angle if dip at the place is 60^0.

Ans: 0.32×10^{-4} T

5. A magnet suspended at 60° with the magnetic meridian makes angles of 45^0 with the horizontal. What shall be the actual value of the angle of dip?

Ans: δ = 26.56°

6. The true value of the dip at a place is 45°. If the plane of the dip circle is turned through 30° from the meridian, what will be the apparent dip?

Ans: 33.7°

Multiple Choice Questions

1. At what angle of dip, the vertical component and horizontal component of geomagnetic field are equal?
   a. 0° b. 30°
   c. 45° d. 90°
2. The susceptibility of a diamagnetic substance
   a. does not vary with temperature.
   b. first decreases and then increases with rise of temperature.
   c. increases with rise to temperature.
   d. decreases with rise of temperature.
3. The magnetic moment of a magnet is 5 Am^2. If the pole strength is 25 Am, what is the length of the magnet?
   a. 10 cm b. 20 cm
   c. 25 cm d. 1.25 cm
4. The ratio of magnetic fields due to a small bar magnet in the end on position to the broad side on position at equal distance is
   a. 1:4 b. 1:2
   c. 1:1 d. 2:1
5. A bar magnet is placed in north-south direction with its north pole to the north. In which direction from the centre of the magnetic field will be points of zero magnetic field lie.
   a. north and south. b. east and west.
   c. north-east and south-east. d. north-west and south-east.

Answers

1. (c) 2. (a) 3. (b) 4. (d) 5. (b)

Hints to Challenging Problems

Hint:1

Given,

True dip (δ) = 45°

θ = 60°

Apparent dip (δ') = ?

We know that

$$\tan \delta' = \frac{\tan \delta}{\cos \theta} = \frac{\tan 45°}{\cos 60°}$$

Hint:2

Given,

B_V = 0.2 oersted

B_H = 0.3464 oersted

True dip (δ) = ?

$$\tan \delta = \frac{B_V}{B_H} = \frac{0.2}{0.3464}$$

$\therefore \quad \delta = 30°$

Now, total intensity, $B = \sqrt{B^2_V + B_H^2}$

Hint:3

Given,

$\delta_1 = 66.1°$

$\delta_2 = 69.94°$

$\delta = ?$

We know that

$\cot^2\delta = \cot^2\delta_1 + \cot^2\delta_2$

Hint:4

Given,

$\delta = 60°$

$B_H = 0.2 \times 10^{-4}$ Wb/m^2

$B_V = ?$

We know that

$$\tan\delta = \frac{B_V}{B_H}$$

or $B_V = B_H \times \tan\delta$

Hint:5

Given,

$B_H = 0.25$ G $= 0.25 \times 10^{-4}$ T [$\because$ 1 G $= 10^{-4}$ T]

$B_V = 0.35$ G $= 0.35 \times 10^{-4}$ T

dip $(\delta) = ?$

Total magnetic field, B = ?

We know that

$$\tan\delta = \frac{B_V}{B_H}$$

or $\tan\delta = \dfrac{0.35 \times 10^{-4}}{0.25 \times 10^{-4}} = 1.4$

$\therefore\ \delta = \tan^{-1}(1.4) = 54.46°$

Also,

$$B = \sqrt{B_V^2 + B_H^2}$$

Hint:6

Given,

$\delta_1 = 45°$

$\delta_2 = 53°$

true dip $(\delta) = ?$

We know that

$\cos^2\delta = \cot^2\delta_1 + \cot^2\delta_2$

❖ ❖ ❖

ELECTROMAGNETIC INDUCTION

17

CHAPTER

17.1 Introduction

The current carrying conductor produces magnetic field of its own. This fact came as a surprise, when Hans Christian Oersted was preparing for a lecture. He discovered that, a magnetic compass kept nearby a current carrying conductor showed deflection in particular direction. When the direction of current was reversed, the compass showed an opposite deflection. After intensive research of months, he came to a conclusion that electric current could generate magnetic field.

Even more surprisingly, a reverse of this effect was discovered i.e. magnetic field (changing) could produce an electric field that could drive a current in the electric circuit. This link between changing magnetic field and the electric field induced by it is called as electromagnetic induction. This effect was observed by Michael Faraday and Joseph Henry independently.

These discoveries formed the basis that electricity and magnetism are no longer the independent fields, they are inseparable. The unification between these two fields was mathematically formulated by James Clerk Maxwell and this unified theory is now known as electromagnetism.

17.2 Electromagnetic Induction

Consider a circular conducting loop provided with a sensitive ammeter as shown in Fig. 17.1. Since, there is no any source of emf, there is no current in the loop initially. Now, if the bar magnet is gradually moved with its north pole (N) facing towards the loop, the ammeter shows deflection indicating the current in the loop. This current disappears suddenly if we stop the motion of the bar magnet. Again, if the bar magnet is moved away from the loop, there is again deflection of ammeter but in opposite direction indicating the reverse current in the loop.

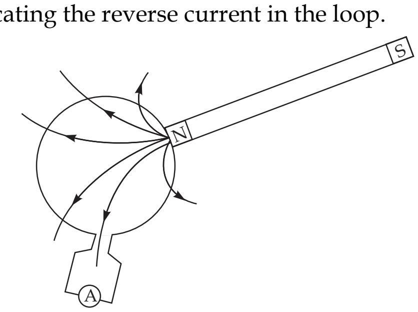
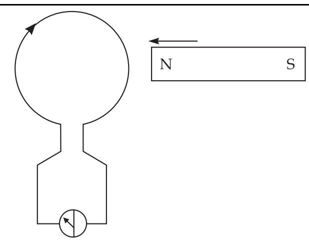
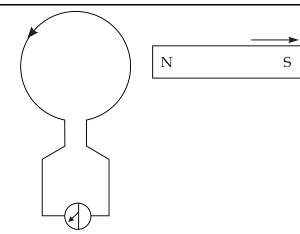
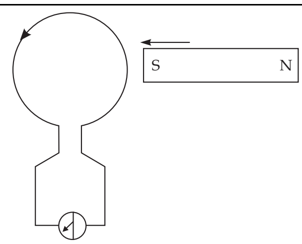
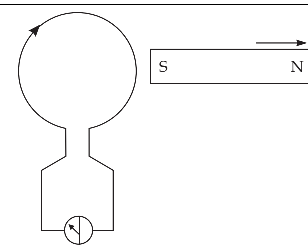


Fig. 17.1: Electromagnetic induction due to motion of bar magnet.

If the magnet is moved faster, the current in the ammeter is greater.

If the magnet is moved away or towards the loop facing its south pole towards it, there is current in the loop but the directions of the current are exactly opposite to the case where north pole of the magnet faces the loop.

The overall experimental observations can be summarized as follows:

	
a. When the magnet is moved towards coil with its north pole facing the coil, the current is clockwise (say) in the coil.	b. When the magnet is moved away from the coil and its north pole still faces the coil, the current is anticlockwise in the coil.
	
c. When the magnet is moved towards coil with its south pole facing the coil, current is again anti-clockwise in the coil.	d. When the magnet with its south pole facing the coil is moved away from the coil, current is in clockwise direction.

From these observations, following results were concluded.

1. A necessary condition for the current to appear in the coil is that, there must be relative motion between the coil and the magnet (flux linkage on coil must change).
2. Faster motion produces greater current and vice-versa.
3. The direction of current produced in the coil due to the movement of the magnet towards it, is exactly opposite to that due to movement of magnet away from it.

17.3 Magnetic Flux and Induction Explained

A magnetic source modifies the space around it in some manner so that any other magnetic material experience force due to it. This region of space is called magnetic field and can broadly be represented in terms of field lines. These are the closed lines (imaginary) emanating from the north pole towards south pole and again towards north forming continuous closed loop.

The number of these field lines crossing per unit area of the surface held perpendicular to its path is called magnetic flux (ϕ_B).

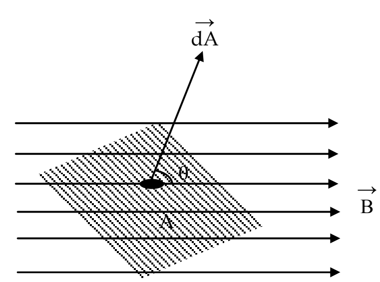


Fig. 17.2(i): Magnetic flux for arbitrary angle θ

If A be the area of loop placed in a magnetic field B, then mathematically, magnetic flux is defined as,

$$\phi_B = \int \vec{B} \cdot \vec{dA} \quad \text{...(17.1)}$$

Here, $\vec{dA}$ is a vector of magnitude dA that is perpendicular to the differential area dA.

$$\text{or,} \quad \phi_B = \int B\, dA \cos\theta$$

Here, θ is the angle between the area vector and the direction of magnetic field. For uniform magnetic field B is constant and the integration is carried for dA only and this gives just the area A of the surface.

$$\text{i.e.} \quad \phi_B = B\cos\theta \int dA = BA\cos\theta \quad \text{...(17.2)}$$

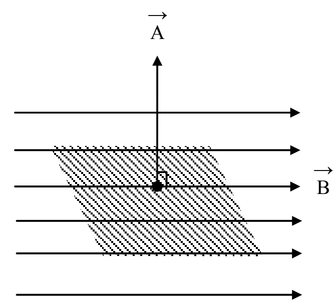


Fig. 17.2(ii): Magnetic flux for θ = 90°

Case I: If $\theta = 90^o$, $\phi_B = BA\cos 90^o = 0$ (minimum)

If the direction of magnetic field is perpendicular to the direction of area vector, i.e. when the surface lies parallel to the direction of field, the magnetic flux crossing the surface is zero as shown in Fig. 17.2 (ii).

Case II: If $\theta = 0^o$, $\phi_B = BA\cos 0^o = BA$ (maximum)

If the direction of magnetic field is parallel to direction of area vector i.e. when surface lies perpendicular to direction of field, magnetic flux crossing the surface is maximum as shown in Fig. 17.2 (iii).

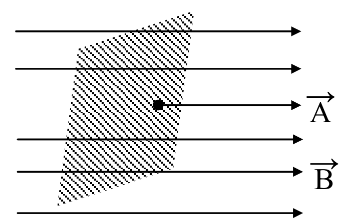


Fig. 17.2(iii): Magnetic flux for θ = 0°

Case III: If $\theta = 180^o$, $\phi_B = BA\cos 180^o = -BA$

If the direction of magnetic field is anti-parallel to area vector. maximum flux is linked with the surface but in opposite direction of the surface.

The current induced in a closed loop can now be explained in terms of flux through the area enclosed by the loop and we call this flux as flux linked with the coil. So, whenever we move magnet away or towards the coil, the flux linked with it changes and this change in magnetic flux induces the current in the coil.

17.4 Faraday's Laws of Electromagnetic Induction

After a series of careful investigations, Faraday was able to visualize the cause of emf and current induced in the closed loop and deduce mathematical expression relating the emf and flux linked with the closed loop. His findings can be summarized both qualitatively and quantitatively in following two statements which are known as Faraday's laws of electromagnetic induction. This law explains the working principle of electric motors, generators and transformers.

Qualitative statement

"An emf and hence the current is induced in a closed loop when the number of magnetic field lines that pass through the loop is changing i.e. when the magnetic flux linked with the coil changes."

It should be understood that, the number of magnetic lines of force passing through the coil has nothing to do with induced emf. But, the change in number of field lines passing through the loop induces emf. The magnitude of induced emf depends only on the rate at which number of field lines changes.

So, more precisely, Faraday's law can be stated in the form of a mathematical expression known as the quantitative statement.

Quantitative statement

"The magnitude of the emf E induced in a closed conducting loop is equal to the rate at which the magnetic flux through that loop changes with time".

$$\text{i.e.} \quad E = -\frac{d\phi}{dt} \qquad \ldots(17.3)$$

The negative sign is used to indicate the opposition to the change of flux by induced emf.

If we seek only the magnitude, we omit the negative sing in equation (17.3).

If the closed loop is a coil of N turns, induced emf appears in every turn. The total emf induced is the sum of these individual emfs.

For a tightly wound coil of N turns, same flux passes through every turn and hence the flux changes at same rate. So, total emf induced is written as,

$$E = -N\frac{d\phi}{dt} \qquad \ldots(17.4)$$

Suppose a coil has N turns and flux through it changes from an initial value of ϕ_1 to the final value of ϕ_2 in time t. Then,

initial flux linkages = $N\phi_1$

final flux linkages = $N\phi_2$

Now, the induced emf (E) = $\dfrac{N\phi_2 - N\phi_1}{t}$

$$E = N\left(\frac{\phi_2 - \phi_1}{t}\right)$$

In differential form, $E = N\dfrac{d\phi}{dt}$ (in magnitude)

General ways of changing magnetic flux

The magnetic flux linked with the coil can be changed by any one or all of the following methods.

1. Changing the magnitude of the magnetic field within the closed loop.
2. Changing either the total area or the portion of the area that lies within the magnetic field.
3. Changing the angle between the direction of field and plane of closed loop. For example, by rotating it.

Note

i. Induced current requires a change in flux. We must remember that the existence of magnetic flux through an area is not sufficient to create an induced emf. A change in that magnetic flux must occur for an emf to be induced.

ii. Expression for induced charge: *From Faraday's law of electromagnetic induction,*

$$I = \frac{E}{R} = \frac{1}{R}\left(-\frac{d\phi}{dt}\right)$$

$$I = -\frac{1}{R}\frac{d\phi}{dt}$$

$$\frac{dq}{dt} = -\frac{1}{R}\frac{d\phi}{dt}$$

$$dq = -\frac{d\phi}{R}$$

Integrating, $q = -\frac{1}{R}\int_{\phi_1}^{\phi_2} d\phi$

$$q = -\frac{1}{R}[\phi_2 - \phi_1]$$

For a coil with N number of turns.

$$q = -\frac{N}{R}[\phi_2 - \phi_1]$$

17.5 Lenz law and direction of induced emf

Michael Faraday showed that the emf induced in the closed loop (conducting) was due to the changing magnetic flux. Determining the direction of induced emf and hence current in the loop is a theoretical rule devised by Henrich Friedrich Lenz most popularly known as Lenz law.

It states that, "*the induced emf (current) has a direction such that the magnetic field due to this current opposes the change in magnetic flux that induces the current*". In short, it can be stated as, "the induced current has a direction such as to oppose the cause producing it". Lenz law is the direct consequence of conservation of energy.

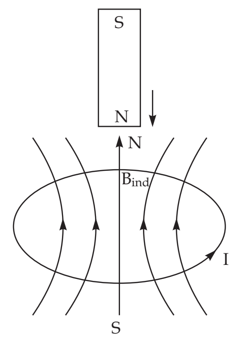


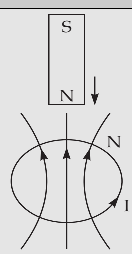
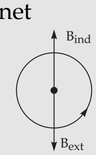
Fig. 17.3: (i) Direction of induced magnetic field

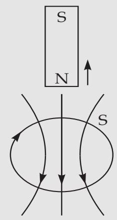
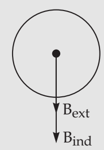
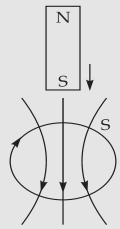
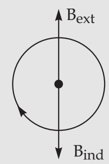
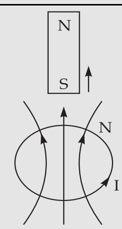
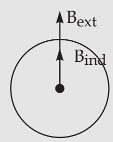
More clearly, whenever magnetic flux linked with closed loop changes, current is induced in it. This in turn induces its own magnetic field. This induced magnetic field opposes the change in the flux linkage due to externally applied field.

Explanation

Consider a bar magnet and a closed conducting loop as shown in Fig. 17.3, which are in relative motion. As the bar magnet is moved toward the closed loop, the magnetic flux linked with the coil increases which induces current in it in the direction shown. The induced current produces the magnetic field (B_{ind}) of its own. According to Lenz law, this induced magnetic field (B_{ind}) must have a direction such as to oppose the approaching bar magnet. This means, the induced magnetic field is directed upwards as if the loop itself acts as a bar magnet with its north pole in upward direction and south pole in downward direction. This direction of magnetic field is possible if the current (I) is in anticlockwise direction as defined by right hand rule.

Lenz law can be better understood by the help of following tabulated experimental observations.

Experiment	Illustration	Direction of B_{ext} and B_{ind}	Direction of current	Remarks
1. A bar magnet is moved towards coil with its N pole facing coil.		B_{ind} must act upward so as to repel approaching magnet 	Anticlockwise	B_{ext} and B_{ind} act opposite so as to oppose the increasing down ward flux

2. A bar magnet is moved away from coil and N-pole faces coil.		B_{ind} and B_{ext} have same direction 	Clockwise	B_{ext} and B_{ind} act in same direction and B_{ind} tend to oppose the decrease in downward flux linkage.
3. Magnet moves towards coil with its s-pole facing the coil.		B_{ext} and B_{ind} are in opposite direction 	Clockwise	B_{ext} and B_{ind} act in opposite direction and B_{ind} opposes the increasing upward flux.
4. Magnet moved away from coil with its s-pole facing the coil		B_{ext} and B_{ind} are in same direction 	Anticlockwise	B_{ext} and B_{ind} act in same direction B_{ind} tend to oppose the decreasing upward flux.

Lenz Law and Conservation of Energy

Let us consider a bar magnet be moved towards closed conducting loop with its north pole facing the coil as shown in Fig. 17.3. According to the Lenz's law the direction of induced emf must be such that it opposes the motion of magnet of the approaching magnet. Let's see why this has to happen.

When the magnet is moved towards the coil, the flux linked with it changes. This induces the emf in the coil as defined by Faraday's law of electromagnetic induction. This emf drives current in the closed loop and hence the loop induces the magnetic field of its own. Now, if this induced field develops the south pole in the region near to north pole of bar magnet, there is attraction between them. This attraction accelerates the bar magnet towards the coil thereby increasing the kinetic energy and without expending the equivalent amount of energy. This is clear violation of conservation of energy.

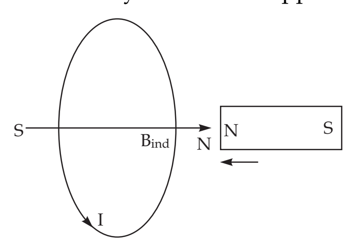
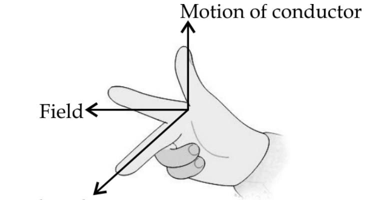


Fig. 17.3: (ii) Direction of induced magnetic field

But, if the induced field develops north pole in the region near the north pole of the bar magnet, there is repulsion between them. Thus, external work has to be done to overcome this repulsion. It is this mechanical energy due to external work done that appears as the electrical energy in the form of induced current. Similarly, when the bar magnet is moved away from the coil, the flux linked with the coil decreases. And again, induced field in the coil has its south pole in the region near the north pole of the bar magnet, so that the force now turns out to be attractive. In order to move bar magnet away from the coil, external work has to be done. This external work done is converted into the electrical energy in the coil. Thus, either the bar magnet is moved towards or away from the coil; external work has to be done which appears as the electrical energy in the coil. For this region the direction of induced emf must be opposite to the changing flux and hence Lenz law is in total agreement with the conservation of energy.

Fleming Right Hand Rule

This rule is used to define the direction of induced current especially for generator. According to this rule, if thumb, first finger and second finger of right hand as held mutually perpendicular to each other such that,



Thu**M**b points in the direction of **M**otion and **F**irst finger points in the direction of **F**ield then, se**C**ond finger points in the direction of induced **C**urrent.

The direction of induced current may be found easily by applying Fleming right hand rule or Lenz law. Fleming rule is used where induced emf is due to flux cutting (dynamically induced emf) and Lenz law is used when flux linkage is for statically induced emf.

17.6 Motional emf

Let us consider a stationary U-shaped conductor placed in a region of uniform magnetic field $\vec{B}$ directed perpendicularly inward into the plane of paper shown by crosses ⊗ as in Fig. 17.4. Also, consider a movable conducting rod AB which can slide over the U-shaped conductor forming a closed loop.

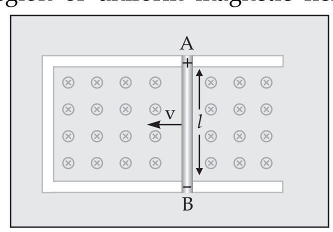


Fig. 17.4: Motional emf of a conductor

Let the rod AB be moved inward with velocity v, then all the charge particles contained in the rod also have the same velocity v along the same direction. Let q be one such charge situated anywhere at the instant when the motion is initiated. The magnitude of Lorentz force experience by this charge is,

$$F = Bqv \sin\theta$$

Here, the angle θ between v and B is 90°. So,

$$\mathbf{F = Bqv} \qquad \ldots\mathbf{(17.5)}$$

According to Flemming's left hand rule this charge experiences force in upward direction along the length of the conductor towards A. The same is applied to all other charges in the conductor so that the end A is more positive than end B. This creates an electric field within the rod directed from A to B. Because of this field, the charge particles flow towards the ends unless the force due to electric field with in the conductor is sufficient enough to cancel magnetic force on charge q.

In this situation, if E be the electric field, then

$$qE = Bqv$$

$$\mathbf{E = Bv} \qquad \ldots\mathbf{(17.6)}$$

The charges will now be in equilibrium.

The stationary U-shaped conductor too has free charge carrier in it but these do not experience any magnetic force. However, the charge particles near end A and B in Fig. 17.4, start to redistribute along the length of stationary conductor creating an electric field within it. This establishes electric current along its length from A to B. Thus, the moving rod has become source of emf, within the rod charges flow from B to A (lower potential to higher potential as in battery) and in the stationary U-shaped conductor current flows from A to B (higher to lower potential as in rest of the circuit). This emf induced in the rod AB which drives the current through the closed circuit is known as motional emf as it is developed in the conductor due to its motion in the magnetic field. Now, if a charge q moves from B to A along the rod, due to non-electrostatic force (magnetic), then work done is,

$$W = Bqv \cdot l$$

From definition of emf,

$$E = \frac{\text{Workdone to move charge from lower potential to higher potential}}{\text{Charge}}$$

$$= \frac{Bqvl}{q} = Bvl$$

$$\therefore E = Bvl$$

This is the required expression for motional emf.

If the conductor moves at an angle θ with the direction of flux, then the induced emf is

$E = Blv \sin\theta$

In vector form, $\vec{E} = l(\vec{v} \times \vec{B})$

i.e. cross product of vector $\vec{v}$ and $\vec{B}$. Generators work on the production of dynamically induced emf in the conductor.

Note

1. ***Induced emf across the wheel spoke:***

   If a bicycle wheel of radius r is rotated in a plane normal to the external uniform magnetic field $\vec{B}$, the induced emf between the axle and rim of the wheel (across the length of spoke) is calculated as follows:
   In this case, two ends of spoke are not in same speed, the end which lies at the center of wheel has zero speed and the end which lies at its rim has maximum speed, v. So, the average speed $v_{av} = \frac{0+v}{2} = \frac{v}{2}$

$$\therefore \text{Induced emf } (E) = B\left(\frac{v}{2}\right) l$$

$$= B\frac{r\omega r}{2} \qquad (\because \text{Here, } v = r\omega, l = r)$$

$$E = \frac{B\omega r^2}{2}$$

2. ***emf induced due to geomagnetic field:***
   If an aircraft flies horizontally at the certain height in the sky, its metallic wing intersects the vertical component of geomagnetic lines of force. So, the emf is induced across two ends of the wings. In such case, net $\vec{B}$ is taken from only the vertical component of geomagnetic field. i.e., $B = B_V = B_H \tan\delta$
   Hence, induced emf (E) $= B_V lv = (B_H \tan\delta)\, lv$, *where δ is angle of dip.*

17.7 Emf induced in a rotating coil in uniform magnetic field

Consider a rectangular coil of surface area A and having N number of turns. The coil is rotated in the uniform magnetic field of $\vec{B}$ so that the direction between the field and area vector changes continuously in its rotation. Then, the magnetic flux linked with the coil at any instant is given by:

$$\boldsymbol{\phi = NBA\cos\theta} \qquad \text{…(16.11)}$$

As the coil rotates about an axis perpendicular to the magnetic field, it keeps on changing its relative orientation with respect to the field as shown in Fig.17.5. So, the magnetic flux linked in coil also changes continuously with time. This change in flux induces the emf in the coil. In every half cycle of its rotation, the direction of emf reverses.

Let ω be the angular velocity of the rotating coil. For time t, the angular displacement of the coil, $\theta = \omega t$. Therefore,

$$\boldsymbol{\phi = NBA \cos \omega t} \quad \text{...(17.7)}$$

Differentiating with respect to time,

$$\frac{d\phi}{dt} = NBA \frac{d \cos \omega t}{dt}$$

$$\boldsymbol{\frac{d\phi}{dt} = -NBA\, \omega \sin \omega t} \quad \text{...(17.8)}$$

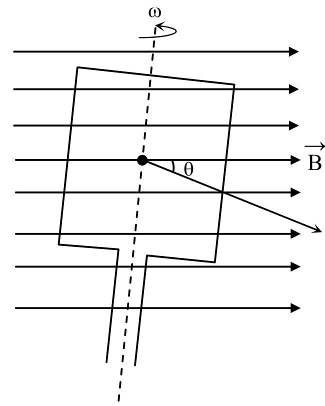


Fig. 17.5: Rotating coil in a uniform magnetic field

The induced emf, $E = -\frac{d\phi}{dt}$

$$\therefore \quad E = -(-NBA\, \omega \sin \omega t)$$

$$\boldsymbol{= NBA\, \omega \sin \omega t} \quad \text{...(17.9)}$$

Since the induced emf E depends on periodic function sin ωt, the emf is periodic in nature. The value of induced emf E is maximum, when $\sin \omega t = 1$, i.e. $\omega t = 90°$. In this condition,

$$\boldsymbol{E_{max} = E_o = NBA\, \omega} \quad \text{...(17.10)}$$

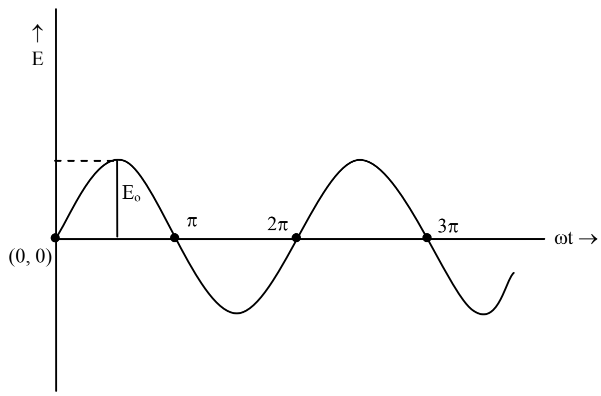


Fig. 17.6: Nature of alternating emf

The value E_o (= NABω) is the peak value (peak value of emf). This condition is achieved when the plane of the coil is parallel to the magnetic field.

$$\therefore \quad \boldsymbol{E = E_o \sin \omega t} \quad \text{...(17.11)}$$

If we plot the graph between E and ωt, the graph is sine curve as shown in Fig.17.6.

The emf represented by equation (17.11) is called alternating emf. Also,

$$\boldsymbol{E = IR} \quad \text{...(17.12)}$$

So, the equation (17.11) is written as,

$$IR = I_o R \sin \omega t$$

$$\boldsymbol{I = I_o \sin \omega t} \quad \text{...(17.13)}$$

The equation represented by equation (17.13) is called alternating current (a.c.). The graph of I versus ωt is shown in Fig.17.7.

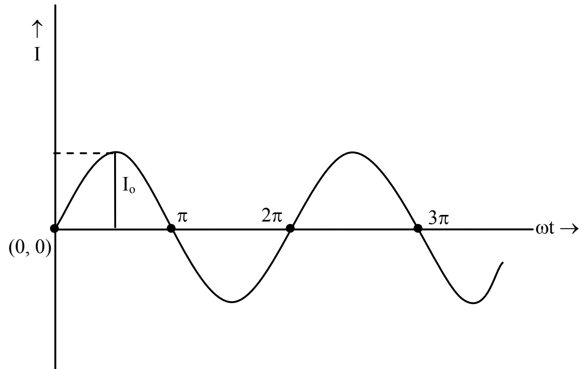


Fig. 17.7: Nature of alternating current

17.8 Inductor and Inductance

Inductor is a circuit element that is designed to oppose any variations in the current through the circuit. It is also called as choke, and the usual circuit symbol for inductor is . Basically, it is a coil of conducting wire.

17.9 Self inductance

In any circuit, the current flowing through it sets up magnetic field of its own and causes magnetic flux. If there is any variation in the current flowing through the circuit, the magnetic flux changes, This change induces an emf in it. This induced emf in any circuit due to change in current in itself is called self induced emf and the phenomenon is known as self induction. According to Lenz law, this induced emf always opposes the current that caused this emf and hence tends to make it more difficult for the variations in current to occur.

In more simple way, when current increases in the circuit, the magnetic flux linked with circuit increases. This induces emf in the direction tending to oppose the increase of current. Similarly, when current deceases in the circuit, the magnetic flux linked with circuit decrease. This induces emf in the direction tending to oppose the decrease of current.

Thus, in both the situations, the self induced emf tends to oppose the variations of the current in the circuit.

Let us consider, an inductor in the form of coil having N number of turns and I be the current flowing through it. As stated earlier, the magnetic flux (ϕ) linked with it is directly proportional to current flowing through it.

i.e. $\phi \propto I$

or, $$\phi = LI \qquad \text{...(17.14)}$$

Where, L is proportionality constant known as coefficient of self induction or self inductance. Rearranging equation (17.14),

$$L = \frac{\phi}{I} \qquad \text{...(17.15)}$$

Thus, self inductance is mathematically defined as the ratio of total magnetic flux linked to the current flowing in the coil. Its value depends upon the number of turns, size, shape and material of the coil and permeability of the medium inside the coil.

Differentiating equation (17.14) with respect to time, we get,

$$\frac{d\phi}{dt} = L\frac{dI}{dt} \quad \text{...(17.16)}$$

According to Faraday's law of induction, the induced emf E due to change in flux is,

$$E = -\frac{d\phi}{dt} \quad \text{...(17.17)}$$

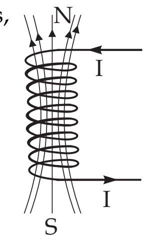


Fig. 17.8: Self inductance

So, from equations (17.16) and (17.17), we get

$$E = -L\frac{dI}{dt} \quad \text{...(17.18)}$$

In equation (17.18), if, $\frac{dI}{dt} = 1\ As^{-1}$, then $E = -L$

Thus, self inductance is also defined as the emf induced in the coil when the rate of change of current in it is unity.

In equation (17.17), writing the dimensions of quantities on R.H.S. we get,

$$[L] = \left[\frac{M^1L^2T^{-3}A^{-1}}{A^1} \times T^1\right] = [M^1L^2T^{-2}A^{-2}]$$

Which is the dimensional formula for self inductance.

Also, in equation (17.17), the unit of E is volt (V) and that of $\frac{dI}{dt}$ is As^{-1}. Thus, S.I. unit of inductance is, $VA^{-1}s$ which is known as Henry, in the honour of Joseph Henry. On the other hand, the equation $\phi = LI$ gives the unit of inductance as weber per ampere (WbA^{-1})

$$\therefore \quad WbA^{-1} = VA^{-1}s = \text{Henry}$$

Finally, one henry is defined as the induced emf of 1 volt when the current changes at the rate of 1 ampere per second in the circuit.

17.10 Energy stored in an inductor

When current flowing through inductor increases, the magnetic flux increases and emf is induced in the coil. This opposes rising current in the coil and the external field does work in it for this purpose. This work is done till the current attains a maximum steady value and is stored in the form of energy in the inductor. After this steady value of current no work is done. The energy acquired in this process is not dissipated if the inductor is ideal (zero resistance) but is stored in it as magnetic potential energy. This energy is released only when the current in the circuit decreases. If $\frac{dI}{dt}$ be the rate at which the current I in the circuit increases, then magnitude of induced emf E is related to self inductance as,

$$E = L\frac{dI}{dt} \quad \text{...(17.19)}$$

If P be the rate at which energy is delivered (i.e. power) to the inductor, then we can write,

$$P = I \cdot E \quad \text{...(17.20)}$$

as (P = IV)

From equation (17.19) and (17.20), we get,

$$P = L\frac{dI}{dt} \cdot I$$

$$\text{or,} \quad \mathbf{P \cdot dt = L\, I\, d\, I} \qquad \text{...(17.21)}$$

The total power delivered to raise the value of current from 0 to a steady value I_0 in time t is obtained by integrating equation (17.21) as,

$$\int_0^t P \cdot dt = \int_0^{I_0} LI \cdot dI$$

Here, $\int_0^t P \cdot dt = W$ = total work done at time 't'

$$\therefore \quad W = L \int_0^{I_0} IdI$$

$$W = \frac{1}{2} LI_0^2$$

Thus, magnetic energy stored in the inductor is,

$$\mathbf{U = W = \frac{1}{2} LI_0^2} \qquad \text{...(17.22)}$$

Self inductance of plane coil

Consider a plane coil of radius r having current I passing through it. The magnetic field at the centre of the coil,

$$\mathbf{B = \frac{\mu_0 NI}{2r}} \qquad \text{...(17.23)}$$

Where, N is the number of turns in the plane coil. The magnetic flux linked at the centre of the coil,

$$\phi = NBA$$

$$= N\left(\frac{\mu_0 NI}{2r}\right)\pi r^2$$

$$\mathbf{\phi = \frac{\pi \mu_0 N^2 Ir}{2}} \qquad \text{...(17.24)}$$

Also,

$$\mathbf{\phi = LI} \qquad \text{...(17.25)}$$

Equating equation (17.24) and (17.25), we get,

$$LI = \frac{\pi \mu_0 N^2 Ir}{2}$$

$$\mathbf{L = \frac{\pi \mu_0 N^2 r}{2}} \qquad \text{...(17.26)}$$

Self Inductance of a Solenoid

Consider an infinitly long solenoid of radius r and cross section A which carries current I. The uniform magnetic field at the centre of solenoid,

$$\mathbf{B = \mu_0 nI = \frac{\mu_0 NI}{\mathit{l}}} \qquad \text{...(17.27)}$$

Now, the flux linked,

$$\phi = NBA$$

$$= N\left(\frac{\mu_0 N I}{l}\right) A$$

$$\boldsymbol{\phi = \frac{\mu_0 N^2 A I}{l}} \qquad \ldots\textbf{(16.28)}$$

Also,

$$\boldsymbol{\phi = LI} \qquad \ldots\textbf{(17.29)}$$

Equating (17.28) and (17.29), we get,

$$LI = \frac{\mu_0 N^2 A}{l} I$$

$$L = \frac{\mu_0 N^2 A}{l}$$

$$= \frac{\mu_0 N^2 A l}{l^2}$$

$$\boldsymbol{L = \mu_0 n^2 A l} \qquad \ldots\textbf{(17.30)}$$

Where, n is the number of turns per unit length, $n = \frac{N}{l}$. From equation (17.30), we conclude that self inductance can be increased by increasing the number of terns per unit length (n), Area enclosed (A) and length (l) of solenoid.

Non-inductive Winding

If a wire is bended two folds and then wounded as shown in Fig.17.8, the coil is said to be non-inductive. Each coil is in close contact with a similar turn carrying same current in the opposite direction. In this condition, the magnetic field produced by one coil is neutralized by another coil. So, the resultant magnetic flux and the net self inductance in non-inductive winding are negligible.

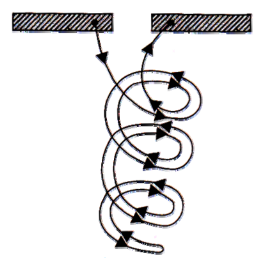


Fig. 17.9: Non-inductive winding

17.11 Mutual Induction

The phenomenon of inducing voltage (emf) in a coil due to variation of current in another coil placed nearby is called mutual induction. The coil in which current is varied is called primary coil and the coil in which emf is induced is called secondary coil.

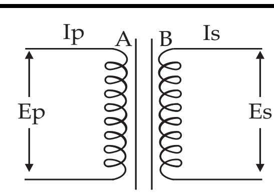


Fig: 17.10: Mutual induction

Let us consider two coils A and B as shown in Fig. 17.10 in which coil A acts as primary and B acts as secondary coil. The coil A has magnetic field of its own due to the current flowing through it and hence has the magnetic flux linked to itself and the coil B near to it. When the current in coil A is gradually increased, there is change in magnetic flux linked with both the coils. This change in magnetic flux induces emf in the coil B and this phenomenon is called mutual induction.

Here, the emf is induced in the primary coil as well due to self induction which tends to oppose the increase in current in it. The emf induced in secondary coil B also opposes the growth of current in primary coil A. It is found that, greater the current flowing in primary greater will be the flux linkage with the secondary. So, if I_p be the current through primary and ϕ_s be the flux linked with the secondary, then

$$\phi_s \propto I_p$$

$$\boldsymbol{\phi_s = MI_p} \qquad \ldots\textbf{(17.31)}$$

Here, M is a proportionality constant known as coefficient of mutual induction of mutual inductance . Rearranging equation (17.31) we get,

$$\mathbf{M = \frac{\phi_s}{I_p}} \qquad \text{...(17.32)}$$

Thus, coefficient of mutual induction is defined as the ratio of magnetic flux linked with secondary to the current in the primary coil. It's value depends upon the number of turns in the secondary coil, shape and size of the coil, distance between two coils, orientation of the coils and permeability of core material.

Differentiating equation (17.31) with respect to time,

$$\mathbf{\frac{d\phi_s}{dt} = M\frac{dI_p}{dt}} \qquad \text{...(17.33)}$$

According to Faraday's law, the emf (E_s) induced in secondary is given as,

$$\mathbf{\frac{d\phi_s}{dt} = -E_s} \qquad \text{...(17.34)}$$

From equations (17.33) and (17.34), we get,

$$\mathbf{E_S = -M\frac{dI_p}{dt}} \qquad \text{...(17.35)}$$

If the rate of change of current in primary, $\frac{dI_p}{dt} = 1\ As^{-1}$ then, $E_s = -M$

Thus, *coefficient of mutual induction is defined as the emf induced in the secondary coil when the current in the primary coil changes at the rate of 1 ampere per second.* It's unit is henry.

In equation (17.35), if $E_S = 1$ volt and $\frac{dI_p}{dt} = -1\ As^{-1}$.

then, M = 1 henry

Thus, coefficient of mutual induction of two coils is said to be 1 henry when the current changing at the rate of 1 As^{-1} in the primary coil induces an emf of 1 volt in the secondary coil.

Mutual inductance of two concentric plane coils

Consider two concentric circular coils of different radii r_1 and r_2 (for $r_2 > r_1$) which are arranged coaxially as shown in Fig. 17.11.

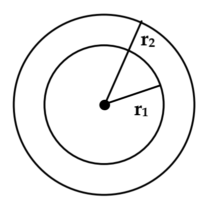


Fig. 17.11: Mutual induction of two concentric plane coils

Let I_1 and I_2 be the current in inner circular coil and outer circular coil respectively.

The magnetic field produced by outer circular coil at the centre,

$$\mathbf{B_2 = \frac{\mu_0 N_2 I_2}{2r_2}} \qquad \text{...(17.36)}$$

The magnetic flux linked across the inner coil is,

$$\phi_1 = N_1 B_2 A_1$$

$$= N_1\left(\frac{\mu_0 N_2 I_2}{2r_2}\right)\pi r_1^2$$

$$\mathbf{\phi_1 = \frac{\pi \mu_0 N_1 N_2 r_1^2}{2r_2} I_2} \qquad \text{...(17.37)}$$

Also,

$$\mathbf{\phi_1 = M_{12} I_2} \qquad \text{...(17.38)}$$

Equating (17.37) and (17.38), we get,

$$M_{12} I_2 = \frac{\pi \mu_0 N_1 N_2 r_1^2}{2r_2} I_2$$

$$\therefore \mathbf{M_{12}} = \frac{\pi \mu_0 \mathbf{N_1 N_2 r_1^2}}{\mathbf{2r_2}} \qquad \text{...(17.39)}$$

Generally, uniform magnetic field B is appropriate while calculating the mutual inductance. Now, when we consider the current through larger coil, then, the magnetic field near the centre of coil becomes uniform. So, the flux linked and hence, the magnetic field of the smaller co-axial coil also becomes uniform. In such case, it is easier to calculate the mutual inductance on smaller coil due to larger coil. But, when we consider the flow of current through inner coil, though the field is more or less uniform inside this cross-section, it becomes non-uniform outside, in the area between larger coil and smaller coil owing to large radius of larger coil. So, calculation of mutual inductance on larger coil due to small, becomes difficult **(Ref. Halliday, Resnik and Walker)**. In such situation, the reciprocity theorem becomes helpful. According to the reciprocity theorem,

$$M_{12} = M_{21} = \frac{\pi \mu_0 N_1 N_2 r_1^2}{2r_2}$$

For $N_1 = N_2 = 1$

$$M_{12} = M_{21} = \frac{\pi \mu_0 r_1^2}{2r_2}$$

Mutual Induction between Two Solenoids

Consider two very long coaxial solenoids of equal length 'l'. Let r_1 and r_2 be the radius of interior and exterior solenoid respectively. If current I_2 flows through the exterior solenoid, magnetic field B_2 is generated inside it. The magnetic field B_2 so produced induces the interior solenoid and emf is induced in it. It is noted that the magnetic flux which is enclosed into the interior solenoid is responsible to create the induced emf (and hence, induced current) in this solenoid.

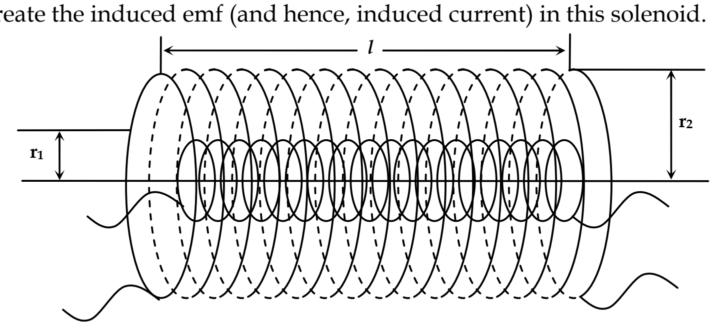


Fig.17.12: Mutual Induction between two solenoids

In this condition, exterior solenoid acts as primary solenoid and interior solenoid acts as secondary solenoid. Let N_1 and N_2 be the number of turns in primary solenoid and secondary solenoid respectively.

The magnetic field at any point inside the primary solenoid is,

$$\mathbf{B_2} = \frac{\mu_0 \mathbf{N_2 I_2}}{l} \qquad \text{...(17.40)}$$

The magnetic flux passing through the secondary solenoid is,

$$\phi_1 = \mathbf{N_1 A_1 B_2} \qquad \text{...(17.41)}$$

where, A_1 is the cross-sectional area of secondary coil and $A_1 = \pi r_1^2$.

Using (17.40) in (17.41), we get,

$$\phi_1 = N_1 r_1^2 \left(\frac{\mu_0 N_2 I_2}{l}\right)$$

$$= \frac{\mu_0 N_1 N_2 \pi r_1^2}{l} \cdot I_2 \quad \text{...(17.42)}$$

Also, the flux linkage in the secondary coil is,

$$\phi_1 = M_{12} I_2 \quad \text{...(17.43)}$$

Equating (17.42) and (17.43), we get,

$$M_{12} I_2 = \frac{\mu_0 N_1 N_2 \pi r_1^2}{l} \cdot I_2$$

$$\therefore \; M_{12} = \frac{\mu_0 N_1 N_2 \pi r_1^2}{l} \quad \text{...(17.44)}$$

As explained in concentric plane coils, the reciprocity theorem can be applied here. So, we get,

$$M_{12} = M_{21} = \frac{\mu_0 N_1 N_2 \pi r_1^2}{l} = \frac{\mu_0 N_1 N_2 A}{l} \quad \text{...(17.45)}$$

where, A = area of smaller solenoid.

17.12 A.C. Generator

A closed loop rotating in a uniform magnetic field in a convenient fashion induces a voltage between the loop terminals. This effect can be used to build on electric power generator. This device actually converts mechanical energy into electric energy. An a.c. generator also called as alternator, converts mechanical energy into sinosuidally varying electrical energy.

Operation Principle

Principle of operation of a.c. generator is electromagnetic induction. When a closed conducting loop rotates in a uniform magnetic field, the flux linked with the loop changes continuously. This induces emf that varies sinosuidally as defined by Faraday law of electromagnetic induction.

Construction and working

The basic elements of an electric generator are shown in Fig. 17.13. A wire loop ABCD rotates about a vertical axis YY' within the uniform magnetic field generated by the pole pieces N and S as in Fig. 17.13(i). This rectangular loop is known as armature. Each terminal of the loop is connected to a metal ring R_1 and R_2 usually made of copper which rotates along with the armature. The contact with the rings is made by means of fixed brushes B_1 and B_2. If the brushes are connected to an electrical load R_L, an alternating current will be established in the circuit. When the coil is rotated anticlock wise (say), from its initial position with its plane parallel to field, the flux linkage increases continuously which induces current in the circuit, and becomes maximum when the orientation of loop is perpendicular to field. The current in this case is from B_2 to B_1 according to Fleming right hand rule. When the coil is rotated further this orientation, the flux starts

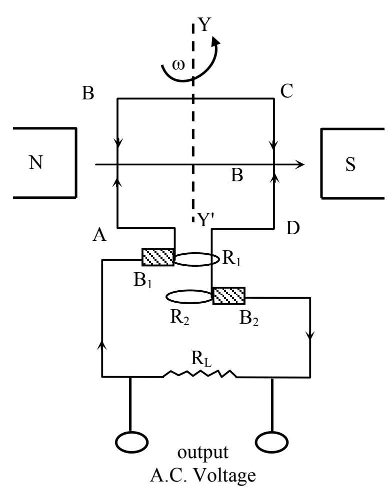


Fig. 17.13 (i): A.C. generator

to decrease and again the induced current in the external circuit is from B_1 to B_2 as determined by Fleming's right hand rule. Thus, the continuous rotation of coil produce current which varies sinosuidally with time which is because of the change of position of coil relatively to the magnetic poles. The amplitude of the current/voltage produced depends upon the magnetic field strength and rotational speed of loop. The frequency is equal to the number of revolutions per second executed by the loop.

Let at any instant of time, the normal to the plane of coil makes angle θ with the field B. If N be the number of turns in the loop each of area A, then,

magnetic flux linkage (ϕ) = $N(\vec{B}.\vec{A}) = NBA\cos\theta$.

Since, ω is the angular frequency, θ = ωt, Thus,

$$\boldsymbol{\phi = NBA\cos\omega t} \qquad \textbf{... (17.46)}$$

The change in θ due to rotation produces change in flux and according to Faraday's law emf (E) is induced which is given by,

$$E = \frac{-d\phi}{dt} = \frac{-d}{dt}(NBA\cos\omega t)$$

$$= -BNA(-\sin\omega t).\,\omega$$

$$\mathbf{E = BNA\omega\sin\omega t} \qquad \textbf{... (17.47)}$$

For maximum emf, sin ωt = 1. So, $E_{max} = E_o = BNA\omega$.

Thus, equation (17.47) can be written as,

$$\mathbf{E = E_o = \sin\omega t} \qquad \textbf{... (17.48)}$$

Cases: If ωt = 0, E = 0

If $\omega t = \frac{\pi}{2}$ $\quad E = E_o\sin\frac{\pi}{2} = E_o$

If $\omega t = \pi$ $\quad E = E_o\sin\pi = 0$

$\omega t = \frac{3\pi}{2}$ $\quad E = E_o\sin\frac{3\pi}{2} = E_o$

$\omega t = 2\pi$ $\quad E = E_o\sin 2\pi = 0$

The variation of emf with time is thus as shown in Fig. 17.13 (ii).

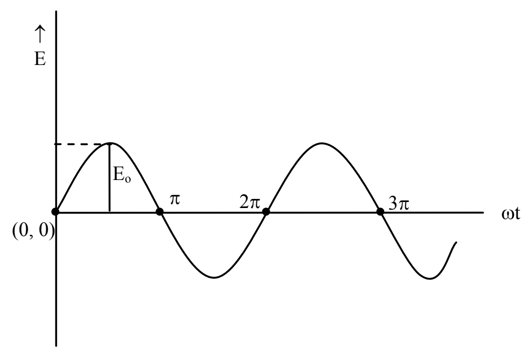


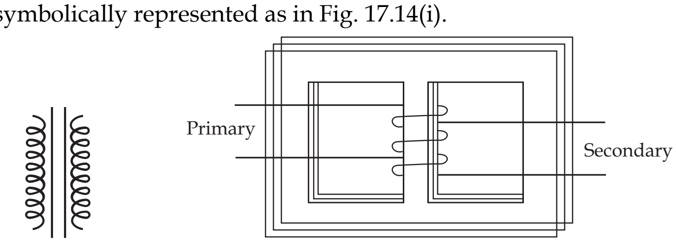
Fig. 17.13(ii) : Nature of alternating emf

The current is given by, $I = \frac{E}{R} = \frac{E_o}{R}\sin\omega t = I_o\sin\omega t$

17.13 Transformer

An electrical device which transforms (changes) an alternating voltage from one value to another of greater or smaller value by using the principle of mutual induction is called transformer.

A transformer is symbolically represented as in Fig. 17.14(i).



(i) (ii)

Fig. 17.14 (i): Symbol of transformer (ii) Single limb transformer

It essentially consists of two coils called primary and secondary separated from each other and coiled around a soft iron core either one on top of other as shown in Fig. 17.14 (ii) or on separate limbs as shown in Fig. 17.14 (iii).

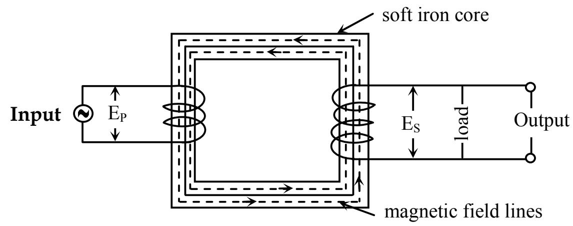


Fig. 17.14 (iii): Schematic diagram for a transformer

When, an alternating voltage is applied to the primary coil, the resulting current produces a large alternating magnetic flux which links the secondary and induces emf in it due to mutual induction. If $\frac{d\phi}{dt}$ be the rate at which the flux linked with secondary changes, then from Faraday's law, emf induced (E_s) in the secondary is,

$$\mathbf{E_s = -N_s \frac{d\phi}{dt}} \quad \text{...(17.49)}$$

Here, N_s = number of turns in the secondary due to change in current in primary with emf (E_p) is induced in due to its self induction. Therefore,

$$\mathbf{E_p = -N_p \frac{d\phi}{dt}} \quad \text{...(17.50)}$$

Here, N_p = number of turns in the primary.

Dividing equation (17.49) by equation (17.50), we get,

$$\mathbf{\frac{E_s}{E_p} = \frac{N_s}{N_p}} \quad \text{...(17.51)}$$

This is known as transformer equation. We see from above equation that emf (E_s) induced in secondary depends upon the number of turns on it. So, we can categorize transformed on the basis of number of turns it has in secondary in comparison to primary coil. A transformer having more number of turns in secondary than primary coil is called step up transformed. This is so called because output voltage is greater than the supplied input voltage.

A transformer having less number of turns in secondary as compared to primary is called step-down transformer. It is so called because, the output voltage is smaller than the supplied input voltage.

For an ideal transformer, applied voltage V_p is nearly not quite equal to E_p and V_s (terminal voltage across secondary) is also equal to E_s to a good approximation.

Thus, from equation (17.51),

$$\mathbf{\frac{V_s}{V_p} = \frac{N_s}{N_p}} \quad \text{...(17.52)}$$

Now, if the transformer is 100% efficient, then from energy considerations,

Input power = output power

$V_p I_p = V_s I_s$

$$\mathbf{\frac{V_s}{V_p} = \frac{I_p}{I_s}} \quad \text{...(17.53)}$$

From equations (17.52) and (17.53), we get

$$\frac{N_s}{N_p} = \frac{I_p}{I_s}$$

From this relation we see that, in a step up transformer ($N_s > N_p$) so, $I_p > I_s$. This implies $V_s > V_p$. The gain in voltage in secondary and loss in current in it are at same ratio. So that I_sV_s is more or less constant. This means we obtain high voltage at low current in step up transformer. The reverse happens in case of step down transformer.

Efficiency of Transformer

The efficiency of transformer is defined as the ratio of output power to input power. It is denoted by η. Transformers are the most highly efficient electrical devices. Most of the transformer have efficiency between 99% to 98.5%.

Output power is developed in secondary coil and input power is supplied from primary coil. Therefore,

$$\text{Output power } (P_{out}) = I_sV_s$$

$$\text{Input power } (P_{in}) = I_pV_p$$

So, $$\eta = \frac{I_sV_s}{I_pV_p}$$

It is usually expressed in percentage, So

$$\eta = \frac{I_sV_s}{I_pV_p} \times 100\%$$

Eddy Current

Instead of a closed conducting loop, if a conducting sheet or block is subjected to time varying magnetic flux, a voltage is induced in this body which gives rise to currents circulating in appropriate paths as shown in Fig. 17.15. These currents are referred to as eddy currents or Foucault current. These currents may be large even for small induced emf because the resistance of conductor is quite low.

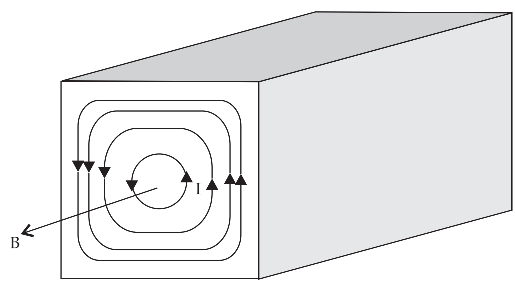


Fig. 17.15: Eddy current

Applications of Eddy Current

i. Eddy currents are used in electric brake.
ii. The heating effect of eddy current is employed in the construction of induction furnace.
iii. It is used a dead-beat galvanometer. The electromagnetic damping principle is used in such galvanometer.
iv. High frequency a.c. (~ 50 MHz) is used in deep heat treatment.
v. It is used in inductive motor.
vi. It is used in car speedometer.

Energy Losses in Transformer

Although transformers are very efficient devices, some energy losses do occur because of the following four factors.

i. **Resistance of the windings:** The copper wire used as winding has resistance so heat loses (I^2R) occur in them. This is called copper loss.

ii. **Eddy currents:** The alternating magnetic flux induces eddy currents in soft iron core and causes heating. This is called as iron loss. This effect can be minimized by using laminated iron core.

iii. **Hysteresis loss:** The magnetization of the core is repeatedly reversed by alternating magnetic field. The resulting expenditure of energy in core appears as heat and this is known as hysteresis loss. This effect can be minimized by using silicon iron core.

iv. **Flux linkage:** The flux due to primary may not all link the secondary if the core is badly designed or has air gaps in it. This causes flux leakage and is known as magnetic loss.

Tips for MCQs

1. **Magnetic Flux:**
   i. It is scalar quantity. The scalar product of magnetic field strength $\vec{B}$ and area vector $\vec{A}$ gives the magnetic flux $\phi = \vec{B}.\vec{A}$ = BA cos θ. (where θ is the angle between $\vec{B}$ and $\vec{A}$)
   ii. The direction of area vector $\vec{A}$ is always taken perpendicular to the plane of A.
   iii. The magnetic flux through a closed surface is zero. i.e. $\phi = \oint \vec{B}.d\vec{A} = 0$
2. **Electromagnetic induction:**
   i. **Faraday's first law:** Whenever the magnetic flux linked with a closed circuit changes, an emf induced in it. This law gives the cause of emf.
   ii. **Faraday's second law:** The magnitude of the induced emf is equal to the rate of change of magnetic flux linked with the closed circuit. This law gives the magnitude of emf.
3. **Lenz law:** The direction of induced current is such that it opposes the cause which produces it. It gives the direction of emf. It is based on conservation of energy.
4. **Information about induced emf, current, charge and power.**

Induced emf	Induced current	Induced charge	Induced power
$E = -N\frac{d\phi}{dt}$	$I = \frac{E}{R} = -\frac{N}{R}\cdot\frac{d\phi}{dt}$	$q = Idt = -\frac{E}{R}d\phi$	$P = IE = \frac{E^2}{R} = N^2\left(\frac{d\phi}{dt}\right)^2\cdot\frac{1}{R}$
Exists in both open and closed circuits	Exists only when circuit is closed	Exists only when circuit is closed	Exists in both open or closed circuit

5. **Fleming Right hand rule:** This rule gives direction of induced current, magnetic field and motion of conductor. In stretched fingers of right hand in mutually perpendicular direction.
   a. Thumb shows direction of motion of conductor.
   b. First finger shows direction of magnitude field.
   c. Second finger shows direction of current.
6. **Magnitude of induced emf:**
   i. For a conducting rod moving in a uniform magnetic field, $E = Blv \sin\theta$, where θ is the angle between $\vec{l}$ and $\vec{B}$.
   ii. For a conducting rod rotating with angular velocity ω in a uniform magnetic field, $E = \frac{1}{2}Bl^2\omega = \frac{1}{2}B.2\pi f.\,l^2 = B\pi l^2 f = BAf$
   Here, $A = \pi l^2$ is the area swept by the rod in one rotation.
   iii. For a disc of radius r rotating in uniform magnetic field, $E = \frac{B\omega r^2}{2} = BAf$.
   iv. For a rectangular coil rotating in a uniform magnetic field, $E = BA\omega N \sin\omega t = E_0 \sin\omega t$

7. **Self induction:**
   i. Magnetic flux, $\phi = LI$
   ii. Induced emf in the coil, $E = -L\dfrac{dI}{dt}$
   iii. The SI unit of L is Henry or weber/ampere.
   iv. For a solenoid, $L = \dfrac{\mu_0 N^2 A}{l}$
   v. For a plane circular coil, $(L) = \dfrac{\mu_0 N^2 A}{2r}$
   vi. Self inductance of straight wire is zero.
   vii. The energy stored by a inductor, $U = \dfrac{1}{2}LI^2$
   viii. Magnetic energy density, $U = \dfrac{B^2}{2\mu_0}$
8. **Mutual induction**
   i. The emf induced by neighbouring coil, $E = -M\dfrac{dI}{dt}$
   ii. The mutual inductance of two long coaxial solenoids each of length l, area of cross-section A wound in air is, $M = \dfrac{\mu_0 N_1 N_2 \pi r_1^2}{2r_2}$, r_2 is radius of bigger coil or $r_2 > r_1$
9. **Eddy Current:**
   i. Transformer do not make the use of eddy current.
   ii. It is produced in conducting sheet or block due to time varying magnetic field.
10. **Transformer:**
   i. It is based on principle of mutual induction.
   ii. Relation of alternating voltage, alternating current and number of turns in two coils. $\dfrac{V_s}{V_p} = \dfrac{N_s}{N_p} = \dfrac{I_p}{I_s}$
   iii. Frequency of a.c. is not altered by the transformer.
   iv. Efficiency, $\eta = \dfrac{\text{output power}}{\text{intput power}} \times 100\%$

Worked Out Problems

1. **[HSEB 2067] A coil of 100 turns each of area $2 \times 10^{-3}m^2$ has a resistance of 12 Ω. It lies in horizontal plane in a vertical magnetic flux density of 3×10^{-3} Wbm^2. What charge circulates through the coil of its end are short circuited and the coil is rotated through 180° about a diameter.**

SOLUTION

Given,

No. of turns (N) = 100

Area (A) $= 2 \times 10^{-3}\ m^2$

Resistance (R) = 12 Ω

Magnetic flux density (B) $= 3 \times 10^{-3}$ T

Charge (q) = ?

We have,

Magnetic Flux $(\phi) = NAB\cos\theta$

Then,

$$\Delta\phi = \phi_1 - \phi_2 = NAB\cos\theta_1 - NAB\cos\theta_2$$
$$= NAB\cos 0° - NAB\cos 180°$$
$$= NAB + NAB = 2\,NAB$$
$$= 2 \times 100 \times 2 \times 10^{-3} \times 3 \times 10^{-3}$$
$$= 0.0012$$
$$= 1.2 \times 10^{-3}\ \text{Wb}$$

Then, we have;

$$\text{Charge (q)} = \frac{\Delta\phi}{R} = \frac{1.2 \times 10^{-3}}{12}$$

$\therefore$ q $= 10^{-4}$ C

Hence, the required charge is 10^{-4} C.

2. **[HSEB 2073] A straight conductor of length 15 cm is moving with uniform speed of 10 ms^{-1} making an angle of 30° with uniform magnetic field of 10^{-4} Tesla. Calculate the emf induced across the length.**

SOLUTION

Given,

Length of conductor (l) = 15 cm = 0.15 m

Velocity of conductor (v) = 10 ms^{-1}

Angle (θ) = 30°

Magnetic field (B) = 10^{-4} T

Induced emf (E) = ?

We have,

$E = Blv \sin\theta$

$= 10^{-4} \times 0.15 \times 10 \times \sin 30°$

$= 7.5 \times 10^{-5}$ V/m

3. **A metal aircraft with a wing span of 40 m files with a ground speed of 1000 kmh^{-1} in a direction due east at constant altitude in a region of the northern hemisphere where the horizontal component of the earth's magnetic field is 1.6×10^{-5} T and the angle of dip is 71.6°. Find the potential difference in volts that exists between the wing tips.**

SOLUTION

Given,

$B_H = 1.6 \times 10^{-5}$ T

Angle of dip (δ) = 71.6°

E = ?

Length of wing span (l) = 40 m

$v = 1000 \text{ km/h} = \frac{1000 \times 1000}{60 \times 60} = \frac{2500}{9}$ m/s

$\because \tan\delta = \frac{B_V}{B_H}$

$\therefore B_V = B_H \tan\delta$

$= 1.6 \times 10^{-5} \times \tan(71.6°) = 4.8 \times 10^{-5}$ T

$\therefore$ p.d. between wing tips,

In this case, the wing of aircraft intersects the vertical component of geomagnetic lines of force, so,

$E = B_V l v = 4.8 \times 10^{-5} \times 40 \times \frac{2500}{9} = 0.53$ V

4. **A square copper loop, 10.0 cm on a side, is located in a region of changing magnetic field. The direction of the magnetic field makes an angle 37° with the plane of the loop. The time-changing field has the following time dependence: $B(t) = 0.10\text{ T} + (1.00 \times 10^{-3}\text{ T/s})\,t$. Find the induced emf in the copper loop for times $t > 0$.**

SOLUTION

Given,

Side of the square (l) = 10.0 cm = 0.1 m

$\therefore$ Area (A) = $l^2 = (0.1)^2 = 0.01\ m^2$

Angle between the field and normal of the loop,

$\theta = 90° - 37° = 53°$

Magnetic field (B) = $0.10 + 1.00 \times 10^{-3}\, t$

Induced emf (E) = ?

Here $\frac{dB}{dt} = \frac{d}{dt}(0.10 + 1.00 \times 10^{-3} t)$

$\frac{dB}{dt} = 1.00 \times 10^{-3}\ Ts^{-1}$

From Faraday's law of electromagnetic induction, we have

$E = \frac{d\phi}{dt}(BA\cos\theta) = A\cos\theta\frac{dB}{dt}$

$= A\cos\theta\,(1 \times 10^{-3})$

$= 0.01 \times \cos 53° \times 1 \times 10^{-3}$

$\therefore E = 6.02 \times 10^{-6}$ V

5. **A satellite, orbiting the earth at the equator at an altitude of 400 km, has an antenna that can be modeled as a 2.0 m long rod. The antenna is oriented perpendicular to the earth's surface. At the equator, the earth's magnetic field is essentially horizontal and has a value of 8.0×10^{-5} T; ignore any changes in B with altitude. Assuming the orbit is circular; determine the induced emf between the tips of the antenna.**

SOLUTION

Given,

Height (h) = 400 km = 400×10^3 m

Length of antenna (L) = 2.0 m

Magnetic field (B) = 8.0×10^{-5} T,

Mass of earth (M) = 5.97×10^{24} kg.

Induced emf (E) = ?

We know that

$\because$ E = vBL . . . (i)

The gravitational force between the satellite and the earth provides necessary centripetal force i.e.,

$$\frac{GMm}{x^2} = \frac{mv^2}{x}$$, where M be the mass of earth, m be the mass of satellite and x = h + R

or $v = \sqrt{\frac{GM}{x}}$

$$= \sqrt{\frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{400 \times 10^3 + 6.38 \times 10^6}}$$

$= 7664\ ms^{-1}$.

From (i) we get

$E = 7664 \times 8.0 \times 10^{-5} \times 2.0 = 1.2$ V

7. **When a wheel with metal spokes 1.2 m long is rotated in a magnetic field of flux density 5×10^{-5} T normal to the plane of wheel, an emf of 10^{-2} V is induced between the rim and axle. Find the rate of rotation of the wheel.** **[4]**

SOLUTION

Given,

Length of metal spokes, r = 1.2 m

Magnetic flux density, B = 5×10^{-5} T

Induced e.m.f., E = 10^{-2} V

Rate of rotation, f = ?

In such case, we have

$$E = B.l.\ v_{av.} = B.\ r.\left(\frac{0 + v}{2}\right)$$

$$= B.\ r.\ \frac{\omega r}{2} = B.\ r.\ \frac{2\pi f r}{2}$$

$$= B.\ \pi r^2\ f = BA\ f$$

$$f = \frac{E}{BA} = \frac{E}{B\ \pi\ r^2}$$

$$= \frac{10^{-2}}{5 \times 10^{-5} \times \pi \times (1.2)^2} = 44.2 \text{ rev/sec}$$

$\therefore$ f = 44.2 rev/s

8. **Find the emf induced in a straight conductor of length 25 cm, on the armature of a dynamo and 12 cm from the axis, when the conductor is moving in a uniform radial magnetic field of 0.5 T. The armature is rotating at 1000 revolutions per minute.** **[4]**

SOLUTION

Induced emf (E) = ?

Length of conductor (l) = 25 cm = 0.25 m

Distance from axis (r) = 12 cm = 0.12 m

Magnetic flux density (B) = 0.5 T

Frequency (f) = 1000 rev/min

= 1000/60 rev/sec

We have,

$E = Blv = B.l.\omega.r$

$= B.l.\ 2\pi f.\ r$

$= 0.5 \times 0.25 \times 2\pi \times 1000/60 \times 0.12$

= 1.57 V.

9. **The current in an inductor of self inductance 40 mH is to be increased uniformly from 1A to 11 A in 4 milliseconds. What is the emf produced in the inductor during this process?**

SOLUTION

Given,

I_1 = 1A, I_2 = 11 A

dt = 4 ms = 4×10^{-3} s

L = 40 mH = 40×10^{-3} H

The change of current with respect to time,

$$\frac{dI}{dt} = \frac{I_2 - I_1}{dt} = \frac{11 - 1}{4 \times 10^{-3}} = 2.5 \times 10^3\ As^{-1}$$

Now, induced emf is,

$$E = -L\frac{dI}{dt} = -40 \times 10^{-3} \times 2.5 \times 10^3 = -100\ V$$

The negative sign tells that it opposes the changes. The induced emf has the magnitude 100 V.

10. **Two plane coils having number of turns 1000 and 2000 and radii 5 cm and 10 cm respectively are placed coaxially in the same plane. Calculate their mutual inductance. ($\mu_0 = 4\pi \times 10^{-7}$ H/m)**

SOLUTION

Given,

Number of turns of interior coil (N_1) = 1000 Number of turns of exterior coil (N_2) = 2000

Radius of interior coil (r_1) = 5 cm = 5× 10^{-2} m Radius of exterior coil (r_2) = 10 cm = 10 × 10^{-2}m

For experimental physibility, outer coil is considered as the primary coil. The fluctuation of current in it induces emf in the inner coil. So, inner coil is considered as the secondary. So, mutual induction is taken as,

$$M = \frac{\mu_0 N_1 N_2 \pi r_1^2}{2r_2} = \frac{4\pi \times 10^{-7} \times 1000 \times 2000 \times \pi \times (5 \times 10^{-2})^2}{2 \times 10 \times 10^{-2}} = 0.099 \text{ H}$$

11. An air-filled torodial solenoid has a mean radius of 15.0 cm and a cross-sectional area of 5.00 cm². When the current is 12.0 A, the energy stored is 0.390 J. How many turns the winding have?

SOLUTION

Given,

Mean radius (r) = 15 cm = 15 × 10^{-2} m

Cross – sectional area (A) = 5 cm² = 5 × 10^{-4} m²

I = 12 A

Energy stored (U) = 0.390 J

Number of turns (N) = ?

We know that

$$U = \frac{1}{2} LI^2$$

or $U = \frac{1}{2}\left(\frac{\mu_0 N^2 A}{2\pi r}\right) I^2$

or $N^2 = \frac{4\pi r U}{4\pi \times 10^{-7} \times A\, I^2}$

$\therefore$ $N = \left(\frac{r \times U}{10^{-7} \times A\, I^2}\right)^{1/2}$

$= \left(\frac{15 \times 10^{-2} \times 0.390}{10^{-7} \times 5 \times 10^{-4} \times (12)^2}\right)^{1/2}$

$\therefore$ N = 2850

12. [HSEB 2052] A step down transforms, a supply line voltage 220 volts into 100 volts. Primary coil has 500 turns. The efficiency and power transmitted by the transformer are 80% and 80 kW. Find (a) the number of turns in the secondary coil; (b) power supplied.

SOLUTION

Given,

E_p = 220 V E_s = 100 V

N_p = 500, η = 80 %,

P_{out} = 80 kW = 80 × 10^3 W

(a) Number of turns in secondary coil (N_s) = ?

We know that

$$\frac{N_s}{N_p} = \frac{E_s}{E_p}$$

$\therefore$ $N_s = \left(\frac{E_s}{E_p}\right) N_p = \left(\frac{100}{220}\right) \times 500$

or, N_s = **227**

(b) Power supplied (P_{in}) = ?

We know that

$$\eta = \left(\frac{P_{out}}{P_{in}}\right) \times 100\%$$

or, $80\% = \frac{80 \times 10^3}{P_{in}} \times 100\%$

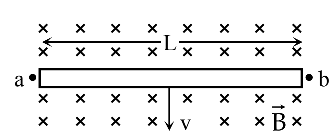
$\therefore$ $P_{in} = 10^5$ W

Challenging Problems

1. **[UP]** The armature of a small generator consists of a flat, square coil with 120 turns and sides with a length of 1.60 cm. The coil rotates in a magnetic field of 0.0750 T. What is the angular speed of the coil if the maximum emf produced is 24.0 mV?

Ans: 10.4 rad/s

2. **[UP]** In figure a conducting rod with length L = 30.0 cm moves in a magnetic field $\vec{B}$ of magnitude 0.450 T directed into the plane



of the figure. The rod moves with speed v = 5.00 ms^{-1} in the direction shown. What is the motional emf induced in the rod?

Ans: 0.675 V

3. **[UP]** Two coils have mutual inductance M = 3.25 × 10^{-4} H. The current I_1 in the first coil increases at a uniform rate of 830 As^{-1} (a) What is the magnitude of the induced emf in the second coil? Is it constant? (b) Suppose that the current described is in the second coil rather than the first. What is the magnitude of the induced emf in the first coil?

Ans: (a) 0.270 V, Yes (b) 0.270 V

4. **[UP]** Two toroidal solenoids are wound around the same form so that the magnetic field of one passes through the turns of the other. Solenoid 1 has 700 turns and solenoid 2 has 400 turns. When the current in solenoid 1 is 6.52 A, the average flux through each turn of solenoid 2 is 0.0320 Wb. (a) What is the mutual inductance of the pair of solenoids? (b) When the current in solenoid 2 is 2.54 A, What is the average flux through each turn of solenoid 1?

Ans: (a) 1.96 H (b) 7.11 × 10^{-3} Wb

5. **[UP]** When the current in a toroidal solenoid is changing at a rate of 0.0260 As^{-1}, the magnitude of the induced emf is 12.6 mV. When the current equals 1.40 A, the average flux through each turn of the solenoid is 0.00285 Wb. How many turns does the solenoid have?

Ans: 238

6. **[UP]** At the instant when the current in an inductor is increasing at a rate of 0.0640 As^{-1}, the magnitude of the self-induced emf is 0.0160 V. (a) What is the inductance of the inductor? (b) If the inductor is a solenoid with 400 turns, what is the average magnetic flux through each turn when the current is 0.720 A?

Ans: (a) 0.25 H (b) 4.5 × 10^{-4} Wb

7. **[UP]** An inductor used in a d.c. power supply has an inductance of 12.0 H and a resistance of 180 Ω. It carries a current of 0.300 A.
   a. What is the energy stored in the magnetic field ?
   b. At what rate is thermal energy developed in the inductor ?

Ans: (a) 0.540 J (b) 16.2 W

8. **[UP]** It has been proposed to use large inductors as energy storage devices.
   a. How much electrical energy is converted to light and thermal energy by a 200 W light bulb in a day?
   b. If the amount of energy calculated in part (a) is stored in a inductor in which the current is 80.0 A, what is the inductance?

Ans: (a) 1.73 × 10^7 J (b) 54.1 × 10^2 H

9. **[UP]** An inductor with an inductance of 2.50 H and a resistance of 8.00 Ω is connected to the terminals of a battery with an emf of 6.00 V and negligible internal resistance. Find
   a. the initial rate of increase of current in the circuit;
   b. the rate of increase of current at the instant when the current is 0.500 A.

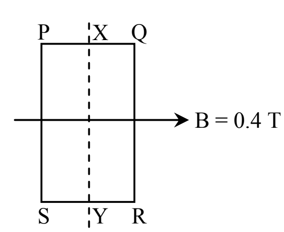
Ans: (a) 2.40 As^{-1} (b) 0.8 As^{-1}

10. A transformer has 500 turns in the primary coil and 100 turns in the secondary coil. What is the output voltage if the input voltage is 4000 V? If the transformer is assumed to have an efficiency of 100%, what primary current is required to draw 2000 watts from the secondary? [HSEB 2060]

Ans: 800 V, 0.5A

11. **[ALP]** In figure the coil PQRS is rotated about the axis XY at 50 rev s^{-1}. Calculate the maximum emf induced in the coil. What is the instantaneous emf in the coil when its plane is (i) parallel to the direction of B (ii) 60° to B (iii) 90° to B ? *(N=5 turns, l=10 cm and b = 5 cm)*

Ans: (i) 3.14 V (ii) 1.57 V (iii) 0



12. **[ALP]** A circular metal disc of area 3.0×10^{-3} m^2 is rotated at 50 rev/s about an axle through its center perpendicular to its plane. The disc is in a uniform magnetic field of flux density 5.0×10^{-3} T in the direction of the axle. Between which points on this disc is the maximum emf induced? What is the value of this emf?

Ans: 7.5×10^{-4} V

13. **[ALP]** A flat search coil containing 50 turns each of area 2.0×10^{-4} m^2 is connected to a galvanometer; the total resistance of the circuit is 100 Ω. The coil is placed so that its plane is normal to a magnetic field of flux density 0.25 T. (a) What is the change in magnetic flux linking the circuit when the coil is moved to a region of negligible magnetic field? (b) What charge passes through the galvanometer?

Ans: (a) 2.5×10^{-3} Wb (b) 2.5×10^{-5} C

14. **[ALP]** A 2.0 H solenoid is connected in series with a resistor, so that resistance is 0.5 Ω, to a 2.0 V d.c. supply. What is (i) the final current? (ii) the initial rate of current with time, (iii) the rate of change of current with time when the current is 2.0 A?

Ans: (i) 4 A (ii) 1 A s^{-1} (iii) 0.5 As^{-1}

*[**Note:** Hints to challenging problems are given at the end of this chapter.]*

Conceptual Questions with Answers

1. State the Faraday's law of electromagnetic induction.

➥ Faraday's laws of electromagnetic induction are:

i. Induced emf is produced due to the change of magnetic flux linked with a circuit and,

ii. The magnitude of induced emf is proportional to rate of change of magnetic flux linked the circuit.

i.e. $\varepsilon = -N\dfrac{d\phi}{dt}$

2. What is electromagnetic induction? What are its uses?

➥ It is the production of electromotive force across an electrical conductor in a changing magnetic field. Its discovery was credited by Michael Faraday in 1831. It has many applications in technology including electrical components such as inductors and transformers, and devices such as electric motors and generators.

3. What is magnetic flux?

➥ The magnetic flux linked with a surface held in a magnetic field is defined as the number of magnetic lines of force crossing the surface normally. It is a scalar quantity and is denoted by ϕ. Quantitatively, the magnetic flux through a plane area A placed in a uniform magnetic field $\vec{B}$ is the dot product of magnetic field vector $\vec{B}$ and area vector $\vec{A}$.

∴ $\phi = \vec{B}.\vec{A} = BA\cos\theta$, where θ is the angle between $\vec{B}$ and $\vec{A}$.

4. What is the elementary idea of electromagnetic induction?

➥ When magnetic flux changes through a coil, a current is induced in the coil. faster the relative motion between the magnet and the coil, greater is the rate of change of magnetic flux through the coil and larger is the current induced in it.

5. Does electromagnetic induction occur, if both coil and magnet move with same velocity along same direction?

➥ If both the coil and the magnet move with the same velocity in the same direction, there is no change in flux linked with the coil. Hence, electromagnetic induction doesnot occur. Hence, no emf is induced.

6. Does emf induce in open circuit?

↳ Yes. Induced emf is set up whenever the magnetic flux linked with a circuit changes even if the circuit is open. However, the induced current flows only when the circuit is closed.

7. Induced emf is also called back emf. Why?

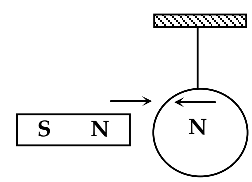
↳ According to Lenz's law the direction of induced current in a circuit is such that it opposes the causes or the change which produces it. Because of the opposing nature of induced emf in any change in applied emf, it is called back emf.

8. A student asserted that if a permanent magnet is dropped down a vertical copper pipe, it eventually reaches a terminal velocity even if there is no air resistance. Why should this be? [HSEB 2068]

↳ If a permanent magnet is dropped down a vertical copper pipe, currents are induced in pipe which ultimately induces the magnetic field opposite of the field of dropping magnet. So, the copper pipe opposes the motion of the magnet (by Lenz law) and the speed ultimately decreases. At a certain condition, the weight of the magnet and the induced magnetic force become equal and opposite, hence attain terminal velocity.

9. A copper ring is suspended by a thread in a vertical plane. One end of magnet is brought horizontally towards the ring. How will the position of the ring be affected? [HSEB 2070]

↳ The ring will move away from the magnet, when a copper ring is suspended in a vertical plane and one end of the magnet is brought horizontally towards the ring. As the magnetic flux changes in the circular coil, current is induced and so magnetic field is produced in such a direction that opposes the direction of motion of its original source. Thus, the coil is repelled out due to the repulsive force between magnet and induced coil.



10. Two closely wound circular coils have the same number of turns, but one has twice the radius of the other. What is the ratio of self inductances of the two coils? [HSEB 2059]

↳ The self-inductance of a closely wounded circular coil (i.e. toroidal coil) is given by, $L = \mu_0 N^2 \frac{A}{l} = \mu_o N^2 \frac{(\pi r^2)}{l}$ ...(i)

where N is the number of turns, $A = \pi r^2$ is the area of the coil, I is the current flowing in the windings and l is the mean length.

As all other quantities are constant for a case in equation (i), $L \propto r^2$. As one has twice the radius of the other, the ratio of the self-inductions of these two coils is 4:1.

11. A sheet of copper is placed between the poles of an electromagnet with the magnetic field perpendicular to the sheet. When it is pulled out, a considerable force is required, and the force required increases with speed, why?

↳ Currents is induced in the copper sheet, when a sheet of copper is placed between the poles of an electromagnet with the magnetic field perpendicular to the sheet and pulled out. Induced current also produces the magnetic field. Induced field, thus produced, opposes the motion of sheet and a considerable force is required to pull it. When the motion or velocity of sheet increases, the magnetic field produced also increases and more force is required to pull it.

12. Pairs of conductors carrying current into or out of the power supply components of electronic equipments are twisted together. Why? [HSEB 2070]

↳ Twisting of pairs of conductors carrying current into or out of the power supply components of electronic equipments becomes non-conductive. Hence it reduces the self induction of the coil significantly, ideally zero. The current in every part of coil is equal and opposite. So, the magnetic field around the one part is cancelled by the magnetic field around the opposite part. Since the self-induction is reduced significantly, the power loss will be minimum.

13. A bar magnet falls through copper ring. Will its acceleration be equal to 'g'? Justify.

✎ No, the acceleration of a bar magnet will not equal to acceleration due to gravity 'g' when it falls through copper ring. When bar magnet falls through copper ring, at first its acceleration increases due to gravity but after some time. As the falling speed increases, the flux linked with the copper ring increases. The current induced in the ring opposes the downward motion of the manet. Hence downward acceleration can not reach equal to g.

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14. Mention two types of loss in a transformer.

✎ Two types of loss in a transformer are:

a. Copper losses: Energy lost in winding the wire of transformer is known as copper loss. This is due to the resistance R of copper wire, when current flows through these wires, power loss ($I^2 R$) takes place. This loss appears as heat produced in the primary and secondary coils. Copper losses can be reduced by using thick wires for the windings.

b. Flux losses: In actual transformer, the coupling between primary and secondary coils is not perfect. It means the magnetic flux linked with the primary coil is not equal to the magnetic flux linked with the secondary coil. So, certain amount of electrical energy supplied to the primary coil is wasted. It is minimized by designing the core for maximum linkage between the primary and secondary coils.

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15. Why can't a transformer be used to change the value of d.c. voltage? [HSEB 2061]

✎ A transformer is device for converting ac current at low voltage to high voltage or vice-versa. It works on the principle of mutual induction. When primary coil of the transformer is connected to a dc current, this produces constant magnetic flux. Consequently, the flux linked with the secondary coil of the transformer is not changed. If there is no change in magnetic flux, emf is not induced in the secondary coil. That's why a transformer can't be used to change the value of dc voltage. However, it is to be noted that d.c. is passed through the transformer at the condition of switch is ON or OFF rapidly.

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16. What is eddy current? Write down its uses.

✎ When a metallic piece is placed in a changing magnetic field, the induced currents are set up in the metal piece. These currents are called eddy currents or Foucault's currents. The direction of eddy currents is given by Lenz law. Eddy currents produce heat due to which there is loss of power which can be reduced by use of laminated core.

Following are some useful uses of eddy currents:

- Eddy current damping
- Induction heating
- Energy meters
- Electromagnetic brakes
- Induction motors.

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17. Birds sitting on a high tension line wire fly off when current is switched on. Why? [HSEB 2075]

✎ When a high tension current is switched on, induced current is set up in the body system of the bird, the nerve and circulatory systems being conducting. Then, the birds experience repulsive force due to electromagnetic induction, consequently the birds fly off.

---

18. What are step up and step down transformers?

✎ Step up transformer: It is a transformer that increases the voltage in the alternating current circuit. Number of turns in secondary coil is greater than the number of turns in primary coil. In this transformer,

Ns > Np, So, $E_s > E_p$.

Step down transformer: It is the transformer that decreases the voltage in alternating current circuit. In this transformer, number of turns in secondary coil is lower than the number of turns in primary coil, i.e. $N_s < N_p$, so, $E_s > E_p$.

19. Does a transformer work in dc?

↳ Transformer does not work in d.c. It works in the principle of electromagnetic induction. The voltage is induced in secondary coil due to the change in magnetic flux in primary coil. The current in the primary coil produces the magnetic flux around it. The flux varies in the secondary coil only when the flux produced in the primary coil fluctuates. To fluctuate the flux in primary coil, it should be connected to alternating current source, not by the direct current source.

Exercises

Short-Answer Type Questions

1. Faraday's laws of electromagnetic induction is reverse effect of Oersted discovery on magnetic effect of current. Justify.
2. What is magnetic flux density? Write its unit and dimensional formula.
3. What is the major consequence of Lenz's law?
4. Write Flemings right hand rule.
5. What is the need of non-inductive coils?
6. Write down the most general formula of the self-inductance of a coil.
7. Write three factors on which the mutual inductance between a pair coils depends.
8. A train is moving with uniform velocity from north to south. Will any induced emf appear across the ends of the axle?
9. Is eddy current usually unwanted? Write your view.
10. When a fan is switched off a spark is produced in the switch. Why?
11. What is the major source of alternating current?
12. Define one tesla.
13. What do you mean by flux linkage?
14. A circular loop is located in a uniform and constant magnetic field. Describe how an emf can be included in the loop in this situation.
15. An electric bulb joined in series with an inductor does not light up fully just after the current is switched on. Why?
16. Why are the primary and secondary coils of a transformer wrapped on an iron core that passes through both coils?
17. How are the energy losses reduced in a transformer?.
18. A loop of wire is placed in a uniform magnetic field. For what orientation of the loop is the magnetic flux a maximum? For what orientation is the flux zero?
19. When a small magnet is moved toward a solenoid, an emf is induced in the coil. However, if the magnet is moved around inside a toroid, there is no induced emf. Explain.
20. Could a current be induced in a coil by rotating a magnet inside the coil? If so, how?

Long-Answer Type Questions

1. State and give the mathematical proof of Faraday's law of electromagnetic induction.
2. State the laws of electromagnet induction. Derive an expression for the emf induced in a conductor moving in a uniform magnetic field. (HSEB 2053)
3. State and explain Lenz's law. (HSEB 2059)
4. State Lenz's law and explain how this law leads to the conservation of energy principle. (HSEB 2062, 2072)
5. State and explain Faraday's law of electromagnetic induction. Derive an expression for the emf induced in a coil rotating in a uniform magnetic field. (HSEB 2057, 2064, 2067)
6. Derive an expression for energy stored in an inductor.
7. What is electromagnetic machine? Describe the principle, construction and working of a.c. generator.
8. Describe the principle, construction and working of d.c. generator.

9. What are eddy currents? Write down the advantages and disadvantages of eddy currents.
10. What is an ideal transformer? Describe principle, construction and working of a transformer.
11. Describe the construction and explain the action of a simple form of a transformer. What are transformer losses? How these losses are minimized?

Numerical Problems

1. A metal rod 0.5 m long is perpendicular to a field of flux density 0.6 T and moves at right angles to the field with a speed of 2 m/s. calculate the emf induced in the rod.

Ans: 0.6 V

2. A coil of area 50 cm^2 is perpendicular to a uniform field of flux density 10^{-3} Wm^{-2}. (i) What is flux passing through the coil.(ii) if the magnetic field drops to zero in 3 s, what is the induced emf.

Ans: 5×10^{-6} weber, 1.67×10^{-6} V

3. A horizontal rod PQ of length 1.5 m is perpendicular to a uniform horizontal field B of 0.1 T. Calculate the induced emf if any in PQ when the rod is moved through the field with a uniform velocity of 4 m/s (a) in the direction of B, (b) perpendicular to B and upwards. Which end of PQ has higher potential?

Ans: 0, 0.6 V, P has higher potential

4. When a wheel of metal spokes 1.2 m long is rotated in a magnetic field of flux density 5×10^{-5} T normal to the plane of wheel, an emf of 10^{-2} V is induced between the rim and the axle. Find the rate of rotation of wheel.

Ans: 44.2 rev/s

5. A coil of 100 turns and cross sectional area 2×10^{-3} m^2 is placed in a field of 8 x 10^{-3}T so that the flux enters all the turns normally. Calculate the average induced emf the field is reversed in 1/50 sec.

Ans: 0.16 V

6. A rectangular coil of 100 turns and area 4×10^{-2} m^2 is rotated about a horizontal axis at a constant rate of 50 rev/ s in a horizontal magnetic field of 0.2 T perpendiculars to the axis. Calculate (i) the maximum value of the induced emf in the coil, (ii) the induced emf when the plane of the coil is 30^o to the horizontal (iii) the induced emf when the plane of the coil is vertical.

Ans: 251.3 V, 217.6 V, 0

7. A jet plane is travelling due west at the speed of 1800 kmh^{-1}. What is voltage difference developed between the ends of the wings 25m long, if the earth's magnetic field at the location is 5×10^{-4} T and the dip angle is 30^o.

Ans: 3.5 V

8. A rectangular coil of area 20 cm^2 and containing 50 turns rotates at 3000 rpm in a uniform magnetic field 0.2 T perpendicular to the axis of rotation. Calculate the peak value and average value of the emf induced in the coil.

Ans: 6.28 V

9. Calculate the emf induced in a straight conductor of length 20 cm in the armature of a dynamo at 10 cm from the axis of rotation when the dynamo is rotating at 1000 rpm in a radial magnetic field of 0.5 T.

Ans: 1.05 V

10. A straight conductor of length 15 cm is moving perpendicular to its length with a uniform speed of 10 ms^{-1} making an angle of 30^o with a uniform magnetic field of 10^{-4} T. Calculate the emf induced across its length.

Ans: 7.5×10^{-5} V

11. Five turns of wire wound closely about the center of a long solenoid of radius 20 mm. If there are 500 turns per meter in the solenoid. Calculate the mutual inductance of the coils.

Ans: 4×10^{-6} H

12. An air filled toroidal solenoid has a mean radius of 15.0 cm and a cross sectional area of 5.00 cm^2. When the current is 12.0 A, the energy stored is 0.390 J. How many turns do the winding have?

Ans: 2850

13. A rod of iron and length of 1 m fixed at one end is rotating with angular velocity 200 rad s^{-1} about the fixed end in a magnetic field of 0.2 T. Find the induced emf between the centre and far end of the rod.

Ans: 20 V

14. A coil has an inductance of 5 mH, in it current changes from 0.1 A to 4.1 A in 0.5 s. Calculate the induced emf

Ans: 40 mV

15. In a car spark coil, emf of 4×10^4 V is induced in the secondary when the primary current changes from 4 A to 0 A in 10 μs. Find the mutual inductance between the primary and secondary windings of this spark coil.

Ans: 0.1 H

16. A step up transformer is used on a 120 V line to provide a potential difference of 2400 V. If the primary has 75 turns, how many turns must the secondary have?

Ans: 1500

17. A step down transformer converts a voltage of 2200 V into 220 V in the transmission line. Number of turns in primary coil is 5000. Efficiency of transformer is 90% and its output power is 8 kW. Calculate (i) number of turns in secondary coil (ii) input power.

Ans: (i) 500 (ii) 8.9 kW

18. A transformer having efficiency 90% is working on 100 V and at 2.0 kW power. If the current in the secondary coil is 5 A, calculate (i) the current in the primary and (ii) voltage across the secondary coil.

Ans: (i) 20 A (ii) 360 volt

19. A step-down transformer is used on 220 V to provide a current of 0.5 A to a 15 W lamp. If the secondary has 20 turns, find the number of turns in the primary coil and also the current flowing through it.

Ans: 147, 0.068 A

20. A coil of inductance 0.4 H is connected to a 6 V battery. Find the rate of growth of current in the coil.

Ans: 15 A/s

21. When a current of 20 A is passed through a coil of 1000 turns, a total magnetic flux of 100 Weber is produced. Calculate the energy stored in the magnetic field of the coil.

Ans: 106 J

Multiple Choice Questions

1. When the current changes from + 2 A to – 2 A in 0.05 second, an emf of 8 V is induced in a coil. The coefficient of self-induction of the coil is

a. 0.8 H
b. 0.1 H
c. 0.2 H
d. 0.4 H

2. The magnetic flux through a circuit of resistance R changes by an amount $\Delta \phi$ in a time Δt. Then the total quantity of electric charge Q that passes any point in the circuit during the time Δt is represented by

a. $Q = \frac{1}{R} \cdot \frac{\Delta \phi}{\Delta t}$
b. $Q = \frac{\Delta \phi}{R}$
c. $\frac{\Delta \phi}{\Delta t}$
d. $Q = R \cdot \frac{\Delta \phi}{\Delta t}$

3. A train is moving on a parallel rail track in a vertical magnetic field of 2×10^{-5} tesla. If speed of the train is 60 km h^{-1}, then induced emf between the axles when distance between the rail track is 1.5 m is

a. 5×10^{-4} V
b. Zero
c. 5 V
d. 10 V

4. A coil of resistance 400 Ω is placed in a magnetic field. If the magnetic flux ϕ (W) linked with the coil varies with time t (in second) as

$\phi = 50\,t^2 + 4$,

The current in the coil at t = 2 second is:

a. 1 A
b. 0.5 A
c. 0.1 A
d. 2 A

5. The magnetic flux linked with a coil of N turns of area of cross-section A held with its plane parallel to the field B is

a. $\frac{NAB}{2}$
b. NAB
c. $\frac{NAB}{4}$
d. 0

6. The rate of change of current of 10 A s^{-1} in a coil produces an emf of 5 V. Then, the self-inductance of the coil in henry is

a. 0.5
b. 0.25
c. 1
d. 1.25
e. 2

7. The magnetic flux (in weber) linked with a coil of resistance 10 Ω is varying with respect to time t as $\phi = 4t^2 + 2t + 1$. Then the current in the coil at time t = 1 second is

a. 0.5 A
b. 2 A
c. 1.5 A
d. 1 A

8. According to the Faraday's law of electromagnetic induction, which of the following is true?

a. Conservation of charge
b. Conservation of magnetic flux
c. Conservation of energy
d. Newton's law of equal and opposite forces

9. The core of a transformer is laminated to reduce

a. flux leakage
b. output power
c. hysteresis
d. eddy current

10. In an ideal transformer the number of turns of primary and secondary coil is given as 100 and 300 respectively. If the power input is 60 W, the power output is

a. 100 W
b. 300 W
c. 180 W
d. 60 W

11. A transformer is used to light a 100 W and 10 V lamp using a 220 V main supply. If the supply current is 0.5 A, then the efficiency of the transformer is

a. 100 %
b. 99 5
c. 90.9 %
d. 87.7 %

12. In an ac generator, a coil with N turns, all of the same area A and total resistance R, rotates with frequency to in a magnitude field B. The maximum value of emf generated in the coil is

a. NABR
b. NABω
c. NABRω
d. NAB

Answers

1. (b) 2. (b) 3. (a) 4. (b) 5. (d) 6. (a) 7. (d) 8. (c) 9. (d) 10. (d) 11. (c) 12. (b)

Hints to Challenging Problems

HINT: 1

Given,

Number of turns, $N = 120$

Length, $l = 1.60$ cm $= 1.60 \times 10^{-2}$ m

$B = 0.0750$ T, $A = l^2$

$= (1.60 \times 10^{-2})^2 = 2.56 \times 10^{-4}$ m^2

$E_{max} = 24$ mV $= 24 \times 10^{-3}$ V

i. $E = NBA\,\omega \sin \omega t$

ii. For E to be maximum, $\sin \omega t = 1$.

So, $E_{max} = NBA\,\omega$

$$\omega = \frac{E_{max}}{NBA}$$

HINT: 2

Given,

Length, $l = 30.0$ cm $= 30.0 \times 10^{-2}$ m

Magnetic field, $B = 0.450$ T

Speed, $v = 5.00\ ms^{-1}$
Induced emf, $E = Blv$

HINT: 3

Given,

$M = 3.25 \times 10^{-4}$ H

$\frac{dI}{dt} = 830\ As^{-1}$

a. Induced emf in the second coil, $E_2 = M\frac{dI}{dt}$

b. $\frac{dI}{dt} = 830\ As^{-1}$

Induced emf in the first coil, $E_1 = M\frac{dI}{dt}$

HINT: 4

Given,

Number of turns in solenoid 1, $N_1 = 700$
Number of turns in solenoid 2, $N_2 = 400$
Current in solenoid 1, $I_1 = 6.52$A
Magnetic flux through solenoid 2, $\phi_2 = 0.032$ Wb

a. Mutual inductance, $M = \frac{N_2\phi_2}{I_1}$

b. Current in solenoid 2, $I_2 = 2.54$ A

Then, use I_2 in $\phi_1 = \frac{MI_2}{N_1}$

HINT: 5

Given,

$\frac{dI}{dt} = 0.0260\ As^{-1}$

$E = 12.6\ mV = 12.6 \times 10^{-3}$ V

$I = 1.40$A,

$\phi = 0.00285$ Wb

find L from $L = \frac{E}{\left(\frac{dI}{dt}\right)}$

and use L in $N = \frac{L \times I}{\phi}$

HINT: 6

Given,

$\frac{dI}{dt} = 0.064\ As^{-1}$

$E = 0.016$ V

a. Inductance of inductor, $L = \frac{E}{\left(\frac{dI}{dt}\right)}$

b. $N = 400$ turns
$I = 0.720$ A
$\phi = ?$

We know that

$$L = \frac{N\phi}{I}$$

or $\phi = \frac{I \times L}{N}$

HINT: 7

Given,

$L = 12$ H
$R = 180\ \Omega$
$I = 0.30$ A

a. Energy stored in the magnetic field, $U = \frac{1}{2}LI^2$

b. Thermal power developed in the inductor,
$P = I^2R$

HINT: 8

Given,

Power, $P = 200$ W
Time, $t = 24$ hrs $= 24 \times 3600$ s.
Current, $I = 80.0$ A

a. We know that

$$P = \frac{U}{t}$$

or $U = Pt = 200 \times 24 \times 3600 = 1.73\times10^7$ J

b. Now, $U = \frac{1}{2}LI^2$

So, $L = \frac{2U}{I^1}$

HINT: 9

Given,

Inductance, $L = 2.50$ H
Resistance, $R = 8\ \Omega$
Emf, $E = 6$ V

a. Initial rate of increase of current, $\frac{dI}{dt} = ?$

The circuit is L –R circuit in which emf of the source is given by

$$E = IR + L\frac{dI}{dt}$$

Initially, I = 0. So, we can write

$$E = L\frac{dI}{dt}$$

or $\frac{dI}{dt} = \frac{E}{L}$

b. $I = 0.5$ A

We have,

$$E = IR + L\frac{dI}{dt}$$

or $\frac{dI}{dt} = \frac{E - IR}{L}$

HINT: 10

Given,

$N_p = 500$, $\eta = 100$ %, $P_{out} = 2000$ W, $I_P = ?$

$N_s = 100$

$E_p = 4000$ V

$E_s = ?$

We know that

$$\frac{E_s}{E_p} = \frac{N_s}{N_p}$$

$$\therefore \quad E_s = \left(\frac{N_s}{N_p}\right) E_p = \left(\frac{100}{500}\right) 4000 = 800 \text{ V}$$

For 100% efficient transformer, we have

Input power = Output power

or, $P_{out} = E_P \times I_P$

HINT: 11

Given,

N = 5 turns,

$A = l \times b = (10 \times 10^{-2}) \times 5 \times 10^{-2}$ m^2

$= 5 \times 10^{-3}$ m^2,

B = 0.4 T

f = 50 rev/s

We have,

$\because \quad E_{max} = E_0 = \omega NAB = 2 \pi f NAB$

i. Plane of coil is parallel to B, so $\theta = 90°$

$\therefore \quad E = E_0 \sin 90°$

ii. $\theta = 90 - 60° = 30°$ (Angle between B and normal of the plane of coil)

$E = E_0 \sin 30°$

iii. Plane of coil makes 90° with B then $\theta = 0°$

$\because \quad E = E_0 \sin 0° = 0$

HINT: 12

Given,

$A = 3 \times 10^{-3}$ m^2

f = 50 rev/s

$B = 5 \times 10^{-3}$ T

$\theta = 0°$

Maximum emf is between centre and rim of the disc and $E_{max} = ?$

We know that

$E_{max} = B \pi r^2 f$

$= B A f \; (\because A = \pi r^2)$

HINT: 13

Given,

N = 50 turns $\qquad$ $A = 2 \times 10^{-4}$ m^2

R = 100 Ω $\qquad$ B = 0.25 T

(a) Change in flux, $\Delta \phi = \phi_1 - \phi_1$

$\phi_1 = NAB \cos \theta$

$\phi_2 = NA \times 0 \times \cos \theta$

(b) Charge passed, $\Delta q = ?$

$\because \quad$ Induced emf $= \frac{\Delta\phi}{dt}$

or $\quad I \times R = \frac{\Delta\phi}{\Delta t}$

or $\quad \frac{\Delta q}{\Delta t} . R = \frac{\Delta\phi}{\Delta t}$

or $\quad \Delta q = \frac{\Delta \phi}{R}$

HINT: 14

Total resistance (R) = 0.50 Ω

L = 2 H

E = 2 V

(i) Final current (I) $= \frac{E}{R}$

(ii) Initial rate of change of current $\left(\frac{dI}{dt}\right) = ?$

$\because E = L\frac{dI}{dt}$

$$\therefore \quad \frac{dI}{dt} = \frac{E}{L} = \frac{2}{2} = 1 \text{ As}^{-1}$$

(iii) Rate of change of current $\left(\frac{dI}{dt}\right) = ?$ for I = 2 A

For current I = 2 A, E = ?

$E = I \times R = 2 \times 0.5 = 1$ V

Now,

$\because E = L\frac{dI}{dt}$

$$\therefore \quad \frac{dI}{dt} = \frac{E}{L}$$

❖ ❖ ❖

ALTERNATING CURRENTS

18
CHAPTER

18.1 Introduction

Many electrical devices that we come across in our daily life need electric energy source to operate. A simple example of such source is a dry cell, which finds its extensive use in the operation of many electrical devices such as radio receiver, torch light, remote controller, etc. If the dry cell is not connected with proper polarity, the device does not function at all. However, we do not really need specific polarity while operating devices such as electric rice cooker, electric fan etc. This means, the polarity plays a vital role for the proper functioning of some electrical devices while it is of no importance in some other electrical devices. The electric source which has fixed polarity i.e. marked with positive and negative signs at the terminals is known as direct current (d.c.) source. The electric source which does not have fixed polarity and reverses periodically is called alternating current (a.c.) source.

The current in a circuit which flows unidirectionally and has almost constant value is known as direct current. Most of our electric devices work with direct current. While, alternating current is one, whose magnitude varies continuously and reverses the polarity periodically. Same property is true for the alternating voltage. Resistor is a basic element to get the output from d.c. circuit, while resistor, inductor and capacitor are the basic elements for a.c.

Here are some basic properties of alternating current.

i. The magnitude of current or voltage varies continuously with time.

ii. The polarity reverses periodically.

iii. It has certain frequency of oscillation.

The symbol of a.c. source is —(~)—. Note that, polarity is not marked on a.c. source because the polarity of the a.c. source changes periodically.

18.2 Alternating Current

We have discussed in chapter 17 that a coil rotating in a magnetic field induces an emf which varies sinusoidally with time. Such emf leads to an sinusoidally varying current with time and is known as alternating current. This means the current flows alternately one way and then the other in the wire

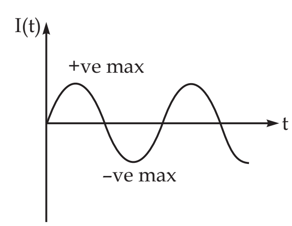


Fig 18.1: Alternating current vs time

in which it is flowing. The plot of such current versus time is as shown in Fig. 18.1.

The graph shows that the current increases from zero to a positive maximum value and then starts to decrease, reaches to zero again and then starts to increase to a negative maximum value and back to zero and the cycle continues. This variation of current from zero to positive maximum and back to zero again and from zero to negative maximum and back to zero again is called one complete cycle of the alternating current. The variation of current from zero to positive maximum and back to zero is called positive half cycle and that from zero to negative maximum and back to zero is called negative half cycle. The time taken by the alternating current to complete one complete cycle is called time period (T). If ω be the angular frequency then,

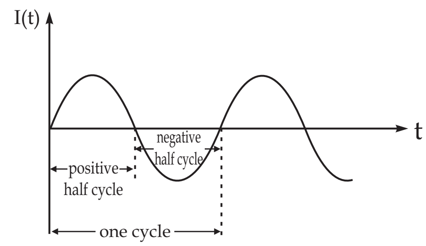
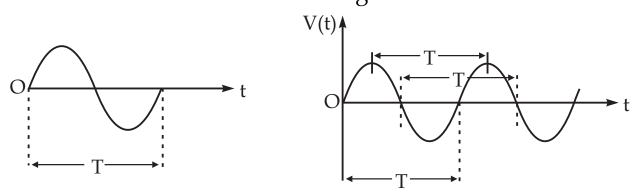


Fig. 18.2: Different cycles of a.c.

$$T = \frac{2\pi}{\omega}$$

The period of the alternating quantity (current, voltage) can be measured between any two corresponding points on the waveform as shown in Fig. 18.3



(i) **(ii)**

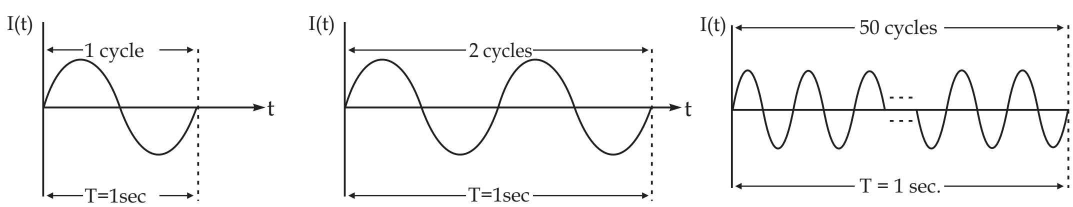
Fig. 18.3: Time period of alternating quantity

The number of cycles completed per unit time by the a.c. is called its frequency its unit is hertz (Hz).

1 Hz = 1 cycle per second

The period and frequency are related by formula,

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$



(a) 1 cycle per second = 1 Hz (b) 2 cycles per second = 2 Hz (c) 50 cycles per second = 50 Hz

Fig. 18.4: Relation between frequency and cycle

Instantaneous A.C.

Since the alternating current (quantity) changes its value continuously, we are interested on a particular value of the quantity at a particular instant of time. The value of alternating quantity (current/voltage/power) at a particular instant of time in the cycle is called instantaneous value of a.c. There are uncountable number of instantaneous values that exist in a cycle.

At any instant, the instantaneous voltage of a alternating source is represented as,

$$V(t) = V_o \sin \omega t$$

where, V_o = peak value of voltage

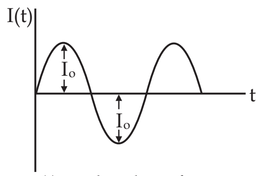
and instantaneous alternating current is given by

$$I(t) = I_o \sin \omega t$$

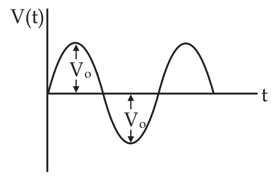
where, I_o = peak of current

Peak value of A.C.

The peak value of alternating quantity is the value of voltage or current at the positive or negative maximum with respect to zero. The peak value for alternating voltage is denoted by V_o and that for alternating current is denoted by I_o. These are also called as amplitudes of alternating quantity.



(i) Peak value of current



(ii) Peak value of alternating voltage

Fig. 18.5: Peak value of ac

Average or mean value of A.C.

A sinusoidally alternating quantity has its value changing from zero to positive maximum and back to zero from zero to negative maximum and back to zero over a cycle. So, if we take the average of all the values over a cycle, it would be zero. So, for such quantities their average is defined over a half cycle only and is calculated as the ratio of sum of all the values of instantaneous a.c. over half cycle to the half-time period.

If I be the alternating current whose instantaneous value is given by $I(t) = I_o \sin \omega t$ then,

$$\text{Average of } I_{inst} = I_{av} = \frac{\int_0^{T/2} I(t) \cdot dt}{\int_0^{T/2} dt}$$

Here T = time period

$$\therefore \quad I_{av} = \frac{\int_0^{T/2} I_o \sin \omega t \, dt}{\int_0^{T/2} dt}$$

$$= \frac{I_o \int_0^{T/2} \sin \omega t \, dt}{\frac{T}{2}}$$

$$= \frac{2I_o}{T}\left[\frac{-\cos \omega t}{\omega}\right]_0^{\frac{T}{2}}$$

$$= \frac{2\,I_o}{T\omega}\left[-\cos\frac{\omega T}{2} + \cos 0\right] \qquad [\because \omega T = 2\pi]$$

$$= \frac{2I_o}{T\omega}[-(-1)+1]$$

$$= \frac{2I_o}{2\pi} \times 2$$

$$\therefore \quad \mathbf{I_{av} = \frac{2I_o}{\pi} = 0.637\, I_o} \qquad \textbf{... (18.1)}$$

Similarly, the average alternating voltage is,

$$\mathbf{V_{av} = 0.637\, V_o} \qquad \textbf{... (18.2)}$$

This average value is the approximate output of d.c. power supply. And thus, average value of a.c. can also be defined as the value of d.c. which when passes through a circuit sends same amount of charge as is done by a.c. current in the same circuit in same time.

18.3 RMS Value of A.C.

The value of an alternating quantity say current changes continuously from zero up to a positive peak, through zero to negative peak and back to zero again. So, for most of the time it is less than the peak value. Thus, using peak value for the calculation of its effect is not a good measure.

Further, we could use average value of a.c. to measure the effect. However, the measurement done using such values does not match with the actual value. For example: the power calculated using the average value does not match the experimental value. For this reason, rms value of a.c. is defined. This value of a.c. is equivalent to a steady d.c. that produces same effect as is done by the d.c. in any circuit component. RMS value refers to root mean square value of alternating quantity. It is the effective value of a.c.

RMS current (I_{rms})

RMS value of alternating current is defined as the steady d.c. value which dissipates energy in a given resistor at the same time as is done by a.c. It is denoted by I_{rms}.

The instantaneous current through a resistor of resistance R fed with a.c. source is,

$$\mathbf{I = I_o \sin \omega t} \qquad \textbf{... (18.3)}$$

Heat produced in the resistor in small time dt is,

$$dH = I^2 R\, dt \text{ (Joule's law of Heating)}$$

Total heat produced over a cycle is,

$$H = \int_0^T dH = \int_0^T I^2 R\, dt$$

$$= \int_0^T I_o^2 \sin^2\omega t\, Rdt = I_o^2 R \int_0^T \sin^2\omega t\, dt$$

$$= I_o^2 R \int_0^T \left(\frac{1-\cos 2\omega t}{2}\right) dt$$

$$= \frac{I_o^2 R}{2}\left(\int_0^T dt - \int_0^T \cos 2\omega t\, dt\right)$$

$$= \frac{I_o^2 R}{2}\left[T - \left(\frac{\sin 2\omega t}{2\omega}\right)_0^T\right]$$

$$= \frac{I_o^2 R}{2}\left[T - \frac{1}{2\omega}\{\sin 2\omega T - \sin 0\}\right]$$

$$= \frac{I_o^2 R}{2}\left[T - \frac{1}{2\omega}\{\sin 4\pi - \sin 0\}\right] \qquad \left(\because T = \frac{2\pi}{\omega}\right)$$

$$= \frac{I_o^2 R}{2} T$$

$\therefore$ Power delivered over a cycle = Rate of heat produced over a cycle.

$$\therefore \quad \mathbf{P = \frac{H}{T} = \frac{I_o^2 R}{2}} \qquad \text{... (18.4)}$$

For steady d.c. current, equivalent power is,

$$\mathbf{P = I_{rms}^2 R} \qquad \text{... (18.5)}$$

Comparing equations (18.4) and (18.5), we get,

$$\mathbf{I_{rms} = \frac{I_o}{\sqrt{2}} = 0.707\, I_o} \qquad \text{... (18.6)}$$

This current which is 0.707 times the peak current is called rms current. Similarly, root mean square voltage is,

$$V_{rms} = \frac{V_o}{\sqrt{2}} = 0.707\, V_o$$

18.4 Phasors

Any sinusoidal wave form can be represented by a vector rotating anti-clockwise with an angular frequency ω, which is known as phasor. Here, the angular frequency of the phasor is equal to the angular velocity of the alternating quantity. This type of phasor representation is used to study the phase relationship between two sinusoidally varying quantities having same frequency.

Consider a sine wave given by

$$\mathbf{I = I_o \sin \omega t} \qquad \text{... (18.7)}$$

The wave form for such wave is as follows:

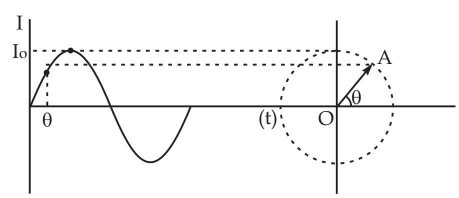


Fig. 18.6: Representing a.c. by phasor

In order to draw phasor, a point is maked on the wave form which represents the value of alternating quantity at that instant of time.

The angle traversed by the alternating quantity at that instant of time can be calculated by simple formula, $\theta = \omega t$

Then on a cartesian co-ordinate axis, a line whose inclination on x-axis is θ is drawn as shown in Fig. 18.6. Since θ changes with time, such lines drawn can rotate from 0^o to 360^o about x-axis.

In figure OA is the phasor for the wave defined by equation (18.7) In Fig. 18.7, $\sin\theta = \frac{OC}{OA}$

$\therefore$ OC = OA sin θ

Here, OC is the projection of OA on y-axis and this projection of OA defines the value of alternating quantity at that instant of time. The angle made by phasor with x-axis at any instant represent its phase angle. The length of phasor gives amplitude of alternating quantity and projections gives the value of alternating quantity at that instant of time.

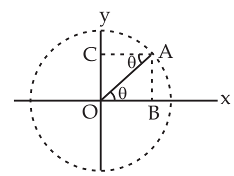


Fig. 18.7: Phasor diagram

Phase shift between two wave forms

When two alternating quantities have same frequency but having a constant phase difference are plotted against time, we often get confused to define which wave form leads or lags the other. In such case, one wave form is considered the reference wave form and other wave form leads or lags that wave form can be found by following simple logic.

The wave form which passes positively through y-axis first leads or, the wave form which passes positively through y-axis last lags.

For example, consider following two wave forms A and B which are plotted against time as in Fig. 18.8 (i) and (ii)

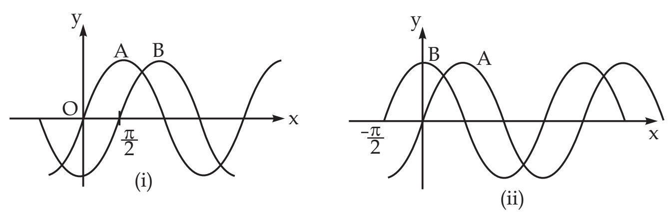


Fig. 18.8: Phase shift between two waves

a. Here, if A is reference, B lags A by 90 or $\frac{\pi}{2}$ because B passes through +ve y-axis later than A.

b. If B is reference, A leads B by 90 because A passes through +ve y-axis first.

Similarly in Fig.18.8(ii)

If A is reference, B leads A by 90 or $\frac{\pi}{2}$ because B passes through +ve y-axis first.

If B is reference, A lags B by 90 because A passes through positive y-axis later than B.

18.5 A.C. Through Resistor

Let us consider a pure resistive circuit consisting of a resistor of resistance R which is supplied with a sinusoidally varying voltage source as shown in Fig.18.9.

Let instantaneous voltage supplied by the source be given by,

$$\mathbf{V(t) = V_o \sin \omega t} \qquad \text{... (18.8)}$$

Where V_o = Peak value of a.c. voltage

Applying Kirchhoff's rule in the above circuit,

$$\mathbf{V(t) - V_R(t) = 0} \qquad \text{... (18.9)}$$

Where $V_R(t)$ is the instantaneous voltage drop across the resistor.

If $I_R(t)$ be the current through resistor at any instant of time, then,

$$V(t) = I_R(t) \cdot R$$

$$\text{or,} \quad I_R(t) = \frac{V(t)}{R} = \frac{V_o \sin \omega t}{R}$$

$$\text{or,} \quad \mathbf{I_R(t) = I_o \sin\omega t} \qquad \text{... (18.10)}$$

$$\text{Where, } I_o = \frac{V_o}{R}$$

From equations (18.8) and (18.10), we see that,

For $\omega t = 0$, $V = 0$ and $I = 0$

$\omega t = \frac{\pi}{2}$, $V = V_o$ and $I = I_o$

$\omega t = \pi$, $V = 0$ and $I = 0$

and so on.

A plot of voltage and current as a function of time is as shown in Fig. 18.10.

Notice that in above graph at t = 0, the voltage V_R and current I_R across the resistor is zero. And Both V_R and I_R reach maximum V_{oR} and I_{oR} respectively at the same time at $\omega t = \frac{\pi}{2}$ (i.e. $t = \frac{\pi}{2} \times \frac{T}{2\pi} = \frac{T}{4}$; one quarter time of its complete cycle) . Thus, we say that current and voltage in a purely resistive circuit are in same phase.

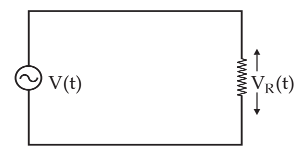


Fig. 18.9: A.C. through purely resistive circuit

The behaviour of $I_R(t)$ and $V_R(t)$ can also be represented with a phase or diagram as shown in Fig. 18.10.

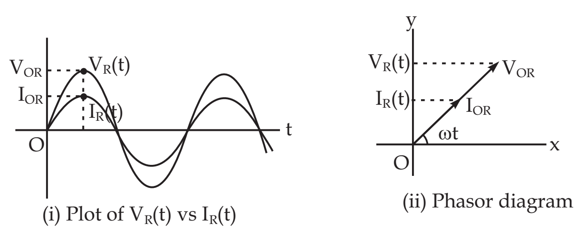


Fig. 18.10: Phase relation between V_R (t) and I_R (t)

In the diagram, OA represents $V_R(t)$ with its peak value V_{oR} and OB represents $I_R(t)$ with its peak value I_{oR}.

18.6 A.C. Through Inductor

Let us consider a purely inductive circuit with an inductor of inductance L connected to an a.c. generator as shown in Fig. 18.11.

Since the inductor is connected to an a.c. source, an emf is induced in the inductor which varies sinusoidally.

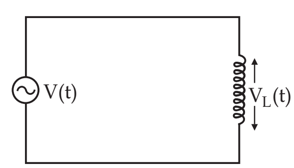


Fig. 18.11: A.C. through purely inductive circuit

Let the instantaneous voltage supplied by source is,

$$\mathbf{V(t) = V_o \sin \omega t} \qquad \text{... (18.11)}$$

If V_L is the induced emf then, at any instant the magnitude of induced emf is,

$$\mathbf{V_L(t) = L\frac{dI}{dt}} \qquad \text{... (18.12)}$$

From Kirchhoff's voltage rule,

$$V(t) = V_L(t)$$

or, $$V_o \sin \omega t = L\frac{dI}{dt}$$

or, $$\mathbf{V_o \sin \omega t\, dt = L\, dI} \qquad \text{... (18.13)}$$

Integrating equation (18.13) we get,

$$\int V_o \sin \omega t.\, dt = \int L dI$$

$$\frac{V_o}{L}\int \sin \omega t.\, dt = \int dI$$

or, $$I = \frac{V_o}{L}\left(\frac{-\cos \omega t}{\omega}\right)$$

or, $$I = \frac{-V_o}{\omega L}\cos \omega t$$

or, $$\mathbf{I = \frac{-V_o}{X_L}\cos \omega t} \qquad \text{... (18.14)}$$

or, $$I = I_o(-\cos \omega t)$$

where, $\frac{V_o}{X_L} = I_o$ is the peak value of current and $X_L = \omega L$ is the inductive reactance.

$$\therefore \quad I = I_o(-)\sin\left(\frac{\pi}{2} - \omega t\right)$$

$$\mathbf{I = I_o \sin\left(\omega t - \frac{\pi}{2}\right)} \qquad \text{... (18.15)}$$

From equations (18.11) and (18.15) we see that,

when, $\omega t = 0$, $V = V_o \sin 0 = 0$

$$I = I_o \sin\left(\omega t - \frac{\pi}{2}\right) = -I_o$$

when $\omega t = \frac{\pi}{2}$, $V = V_o \sin\frac{\pi}{2} = V_o$

$$I = I_o \sin\left(\frac{\pi}{2} - \frac{\pi}{2}\right) = 0$$

when $\omega t = \pi$, $V = V_o \sin\pi = 0$

$$I = I_o \sin\left(\pi - \frac{\pi}{2}\right) = I_o \text{ and so on.}$$

A plot of voltage and current as a function of time is as shown in Fig. 18.12.

From graph it is seen that, voltage is ahead of current by $\frac{\pi}{2}$. This means, voltage leads the current by a phase factor of $\frac{\pi}{2}$. The corresponding phasor diagram for inductive circuit is shown in Fig. 18.12.

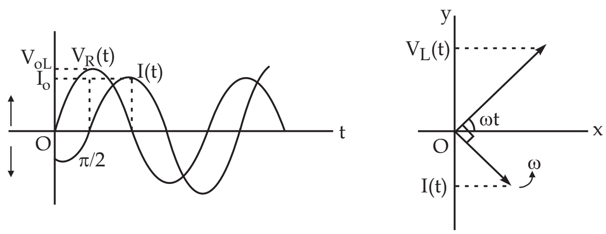


Fig. 18.12: Phase relation between $V_L(t)$ and I(t)

18.7 A.C. Through Capacitor

Let us consider a purely capacitive circuit as shown in Fig. 18.13 with a capacitor of capacitance C connected to an a.c. source which supplies alternating voltage given by,

$$\mathbf{V(t) = V_o \sin \omega t} \qquad \text{... (18.16)}$$

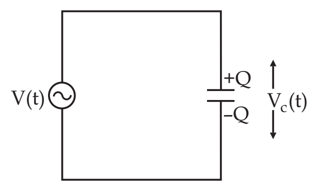


Fig. 18.13: A.C. through purely capacitive circuit

Let $V_C(t)$ be the voltage across the plates of capacitor and Q(t) be the charge on it at any instant of time t. In capacitor the voltage changes at a rate equal to rate of change of electrical charge on the plates.

Using Kirchhoff's voltage rule in the above circuit, we get,

$$V(t) - V_C(t) = 0$$

or, $V(t) = V_C(t)$

$\therefore \quad V_C(t) = V_o \sin \omega t$

But, $V_C(t) = \dfrac{Q(t)}{C}$

$$\therefore \quad \mathbf{Q(t) = C\,V_o \sin \omega t} \qquad \text{... (18.17)}$$

Differentiating equation (18.17) with respect to time,

$$\frac{dQ}{dt} = CV_o \frac{d(\sin \omega t)}{dt}$$

or, $I(t) = CV_o\,\omega \cos \omega t.$

or, $I(t) = V_o\,(\omega C) \cos \omega t.$

But $X_C = \dfrac{1}{\omega C}$ is the capacitive reactance of the capacitor.

$$\therefore \quad I(t) = \frac{V_o}{X_C} \cos \omega t$$

$$\text{or,} \quad \mathbf{I(t) = I_o \sin\left(\omega t + \frac{\pi}{2}\right)} \qquad \text{... (18.18)}$$

where, $\dfrac{V_o}{X_C} = I_o$ is the peak value of current.

From equations (18.16) and (18.18) we see that,

for, $\omega t = 0$, $V_C = V_o \sin \omega t = 0$

$$I = I_o \sin\left(\frac{\pi}{2}\right) = I_o$$

$$\text{for,} \quad \omega t = \frac{\pi}{2} \qquad V_C = V_o \sin\frac{\pi}{2} = V_o$$

$$I = I_o \sin\left(\frac{\pi}{2}+\frac{\pi}{2}\right) = 0$$

$$\text{for} \quad \omega t = \pi \qquad V_C = V_o \sin\pi = 0$$

$$I = I_o \sin\left(\pi+\frac{\pi}{2}\right) = -I_o \text{ and so on.}$$

A plot voltage and current as a function of time is as shown in Fig. 18.14(i) and the corresponding phasor is as shown in Fig. 18.14 (ii).

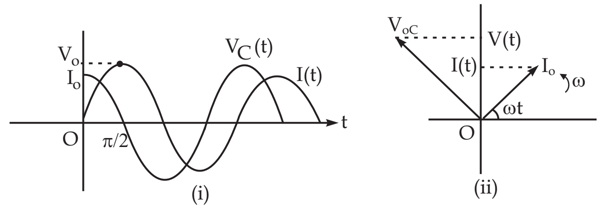


Fig. 18.14: (i) Plot of V_C(t) and I(t) and (ii) Phasor diagram

It is seen from graph that, the current is ahead of the voltage across the capacitor by $\frac{\pi}{2}$. This means, current leads the voltage by a phase factor of $\frac{\pi}{2}$.

The corresponding phasor diagram for capacitive circuit is as shown in Fig. 18.14 (ii).

Notice in above graph that at t = 0, the voltage across the capacitor is zero while the current in the circuit is at a maximum. In fact, I(t) reaches maximum before V_C(t) by one quarter of a cycle $\left(\phi = \frac{\pi}{2}\right)$.

Thus we say that, current leads the voltage by $\frac{\pi}{2}$ is capacitive circuit.

18.8 A.C. Through R–L Series Circuit

Suppose an alternating voltage V(t) is applied across a resistor of resistance (R) is series with an inductor of inductance L as shown in Fig. 18.15 (i).

The applied a.c. voltage is given by

$$\mathbf{V(t) = V_o \sin \omega t} \qquad \text{... (18.19)}$$

Due to this alternating voltage, the current I(t) also takes alternating form which flows through each component and let it be given by,

$$I(t) = I_o \sin(\omega t - \phi)$$

Where, I_o is the amplitude of alternating current and ϕ is the phase factor and so, $I_o = I_{oR} = I_{oL}$.

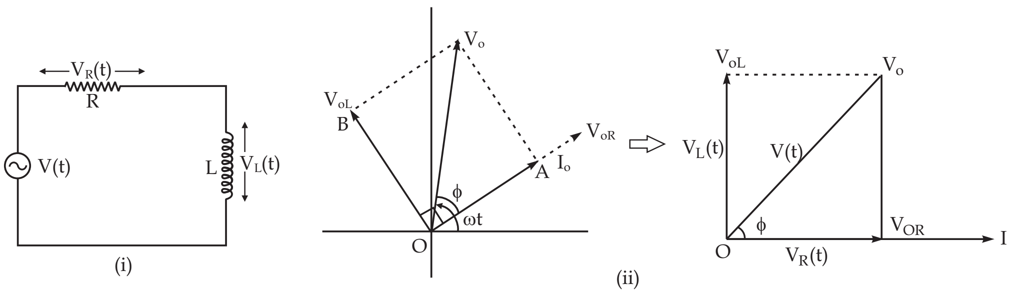


Fig. 18.15: (i) A.C. through R-L circuit (ii) Phasor diagram

Let $V_R(t)$ and $V_L(t)$ be the instantaneous voltage (alternating) across resistor and inductor with their respective amplitudes V_{oR} and V_{oL}. As discussed in preceding topics, $V_R(t)$ across R is in phase with I(t) and $V_L(t)$ leads the current by $\frac{\pi}{2}$. So the phasor diagram for such circuit is as shown in Fig. 18.15 (ii).

As the voltage alternates, both V_R and I_R change along the same direction. In phasor diagram, OA represents such case. But V_L is ahead of I(t) by $\frac{\pi}{2}$. So, in phasor, OA and OB representing I(t) and V_L are perpendicular to each other.

Here, $V(t) = V_R(t) + V_L(t)$

But $V_o \neq V_{oR} + V_{oL}$

Referring to Fig. 18.15 (ii), vector sum of V_{oL} and V_{oR} equals V_o.

i.e. $V_o^2 = V_{oL}^2 + V_{oR}^2$

But, $V_{oR} = I_oR$ and $V_{oL} = I_o X_L$

$\therefore \quad . \ V_o^2 = I_o^2 (R^2 + X_L^2)$

$$\textbf{or,} \quad \mathbf{I_o = \frac{V_o}{\sqrt{R^2 + X_L^2}} = \frac{V_o}{\sqrt{R^2 + \omega^2L^2}}} \qquad \textbf{... (18.20)}$$

The quantity $\sqrt{R^2 + \omega^2L^2}$ is total effective resistance offered by the LR circuit and is known as the impedance of LR circuit.

Thus, impedance $(Z) = \sqrt{R^2 + \omega^2L^2}$

From our assumption, ϕ is the angle between V_o and I_o.

Thus, from Fig. 18.15(iii) $\tan\phi = \frac{V_{oL}}{V_{oR}} = \frac{I_o X_L}{I_oR} = \frac{X_L}{R}$

$\therefore \quad \tan\phi = \frac{\omega L}{R}$

$\therefore \quad \phi = \tan^{-1}\left(\frac{\omega L}{R}\right).$

In terms of impedance, the instantaneous current through circuit can be written as,

$$I(t) = \frac{V_o}{\sqrt{R^2 + \omega^2L^2}} \sin(\omega t - \phi)$$

$$\textbf{or,} \quad \mathbf{I(t) = \frac{V_o}{Z} \sin(\omega t - \phi)} \qquad \textbf{... (18.21)}$$

A plot of voltage and current as a function of time is as shown in Fig. 18.16.

From graph it is seen that, voltage is ahead of current by ϕ. This means voltage leads the current by a phase factor of ϕ. The corresponding phasor diagram for inductive circuit is shown in Fig. 18.15(ii).

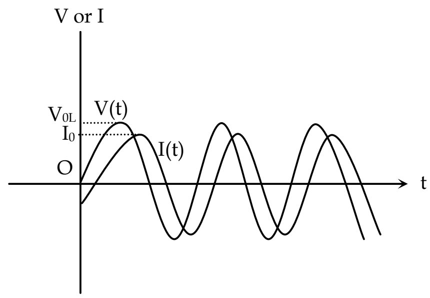


Fig. 18.16: Plot of V_C(t) and I(t)

18.9 A.C. Through R-C Circuit

Suppose an alternating voltage V(t) is applied across a resistor of resistance (R) in series with a capacitor of capacitance (C) as shown in Fig. 18.17(i).

The applied a.c. voltage is given by,

$$\mathbf{V(t) = V_o \sin \omega t} \qquad \text{... (18.22)}$$

Due to this alternating voltage, the current I(t) is also alternating and same current flows through each component which is given by

$$\mathbf{I(t) = I_o \sin (\omega t + \phi)} \qquad \text{... (18.23)}$$

Where I_o is the amplitude of alternating current and ϕ is the phase factor.

Let $V_R(t)$ and $V_C(t)$ be the instantaneous voltage across the resistor and capacitor with their respective amplitudes V_{oR} and V_{oC}.

Here, $V(t) = V_R(t) + V_C(t)$

But $V_o \neq V_{oR} + V_{oC}$

The voltage V_R across R is in same phase with I(t) but V_C across C lags behind I(t) by $\frac{\pi}{2}$. So, the corresponding phasor diagram for the case is as shown Fig.18.17(ii).

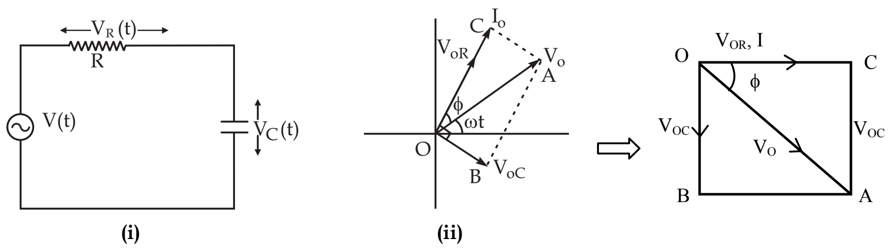


Fig. 18.17: (i) A.C. through R-C circuit (ii) phasor diagram

In above phasor diagram, OC represents the phasor for V_{oR} and $I_R = I_o$ because both of them are in same phase. But, V_C lags behind I(t) by $\frac{\pi}{2}$. So, OB representing V_{oC} must be perpendicular to OC representing I(t).

Referring to Fig. 18.17(ii), vector sum of V_{oR} and V_{oC} equals V_o.

$$\therefore \quad V_o^2 = V_{oC}^2 + V_{oR}^2 = I_o^2 X_C^2 + I_o^2 R^2$$

$$\text{or,} \quad I_o^2 = \frac{V_o^2}{(R^2 + X_C^2)}$$

$$\therefore \quad I_o = \frac{V_o}{\sqrt{R^2 + X_C^2}}$$

The quantity $\sqrt{(R^2 + X_C^2)}$ is the effective resistance offered by the circuit and is known as impedance of the R-C circuit.

Thus, impedance $(Z) = \sqrt{(R^2 + X_C^2)}$

$$\text{But, } X_C = \frac{1}{\omega C}$$

$$\therefore \quad Z = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$$

Also, as per our supposition, ϕ is angle between V_o and I_o.

$$\text{Thus, } \tan\phi = \frac{V_{oC}}{V_{oR}} = \frac{I_o X_C}{I_o R} = \frac{X_C}{R}$$

$$\mathbf{\tan\phi = \frac{1}{\omega CR}} \qquad \textbf{... (18.24)}$$

In terms of impedance, the instantaneous current through circuit can be written as,

$$I(t) = \frac{V_o}{Z} \sin(\omega t + \phi)$$

$$I(t) = \frac{V_o}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \sin(\omega t + \phi)$$

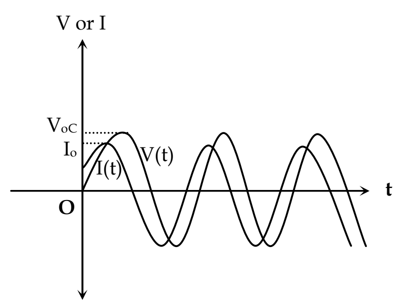


Fig. 18.18: Plot of V_C(t) and I(t)

It is seen from graph that, the current is ahead of the voltage across the capacitor by ϕ. This means current leads the voltage by a phase factor of ϕ.

The corresponding phasor diagram for capacitive circuit is as shown in Fig. 18.17 (ii).

Notice in above graph that at t = 0, the voltage across the capacitor is zero while the current in the circuit is at a maximum. In fact, $I_o(t)$ reaches maximum before $V_C(t)$ by ϕ. Thus we say that, current leads the voltage by ϕ in this circuit.

18.10 L-C-R Series Circuit in A.C.

Fig. 18.19(i) shows a resistor of resistance R, capacitor of capacitance C and inductor of inductance L connected in series with an a.c. source.

The instantaneous voltage supplied by the source is,

$$\mathbf{V(t) = V_o \sin \omega t} \qquad \textbf{... (18.25)}$$

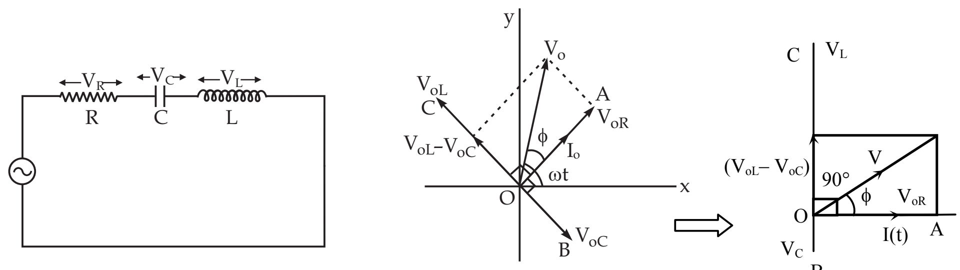
The instantaneous current in the circuit is given by,

$$I(t) = I_o \sin(\omega t + \phi).$$

Let $V(t) = V_R(t) + V_L(t) + V_C(t)$

But, $V_o \neq V_{oR} + V_{oC} + V_{oL}$.

The voltage V_R across R is in phase with I(t), so the corresponding phasor is denoted by OA in Fig. 18.19(ii). Similarly, V_C across C lags behind I(t) by $\frac{\pi}{2}$ so, corresponding voltage and current in phasor are OA = I and OB = V_C which must be perpendicular to each other (i.e. OA⊥OB). Finally, V_L across L leads I(t) by $\frac{\pi}{2}$, so their corresponding lengths in phasor are represented by OA and OC and again, OA and OC are perpendicular to each other. The phasor diagram for the circuit is shown in Fig. 18.19 (ii).



(i) (ii)

Fig. 18.19 (i) A.C. through L-C-R circuit and (ii) phasor diagram

The phase relationship between the supply V(t) and the circuit current I(t) depends on the relative value of inductance and capacitance and whether the inductive reactance (X_L) is greater or less than capacitive reactance (X_C). The phasor above shows the case in which $X_L > X_C$. In this case, since L and C carry same current, it follows that $V_{oL} > V_{oC}$. So, length of V_{oL} is greater than V_{oC} in above phasor.

In Fig. 18.19(ii) above, V_{oL} and V_{oC} are 180° out of phase, so their resultant is, $(V_{oL} - V_{oC})$ and is in the direction of V_L (in this case).

The vector sum of $(V_{oL} - V_{oC})$ and V_{oR} equals the applied voltage. i.e.

$$V_o^2 = (V_{oL} - V_{oC})^2 + V_{oR}^2$$

But, $V_{oL} = I_oX_L$, $V_{oC} = I_oX_C$ and $V_{oR} = I_oR$

$$\therefore \quad V_o^2 = I_o^2\,[R^2 + (X_L - X_C)^2]$$

$$\text{or,} \quad V_o^2 = I_o^2\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]$$

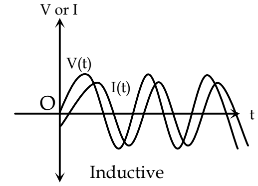
$$\therefore \quad \mathbf{I_o = \frac{V_o}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}}} \qquad \textbf{... (18.26)}$$

In this equation, the factor, $\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}$ = Z is the impedance of the LCR circuit.

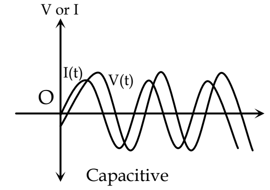
Also, from Fig. 18.19(ii), the phase factor is obtained as,

$$\tan\phi = \frac{V_{oL} - V_{oC}}{V_{oR}} = \frac{I_o\,(X_L - X_C)}{I_oR} = \frac{X_L - X_C}{R}$$

$$\therefore \quad \boldsymbol{\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{\omega L - \dfrac{1}{\omega C}}{R}\right)} \qquad \textbf{... (18.27)}$$



Inductive



Capacitive

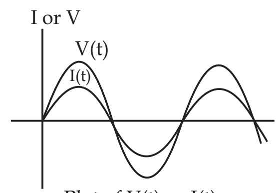
In above equation,

i. If $X_L > X_C$, then tan ϕ will be +ve and ϕ is also a positive angle between 0 to 90°. In this case I_o lags behind V by ϕ. Such circuit is called inductive circuit as shown in figure aside.

ii. If $X_L < X_C$, then tan ϕ will be negative and ϕ will have a negative value. In this case, I leads ahead V by ϕ. Such circuit is called capacitive circuit as shown in figure aside.

iii. If $X_L = X_C$, then from equation (18.26)

$I_o = \dfrac{V_o}{R}$ and is the maximum current in the circuit.



Plot of V(t) vs I(t)

The impedance is minimum and is given by Z = R. This means, the circuit is purely resistive only.

Also, the phase factor $\phi = \tan^{-1} \dfrac{(X_L - X_C)}{R} = 0$

This means V and I are in same phase. This condition in which the inductive reactance will be equal to capacitive reactance and the circuit offers minimum resistance allowing maximum current is known as electrical resonance in LCR circuit as shown in figure aside.

Thus, At resonance, $X_L = X_C$

or, $\omega L = \dfrac{1}{\omega C}$

or, $\omega^2 = \dfrac{1}{LC}$

or, $4\pi^2 f^2 = \dfrac{1}{LC}$

or, $f^2 = \dfrac{1}{4\pi^2} \dfrac{1}{LC}$

$$\therefore \quad \mathbf{f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}} \qquad \textbf{...(18.28)}$$

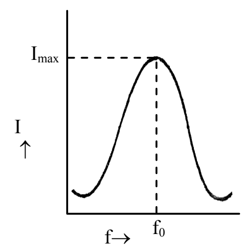


Fig. 18.20 Resonance curve

This frequency is known as resonant frequency.

18.11 Power in LCR Circuit

In series LCR circuit, the instantaneous power delivered by a.c. source is given by

$P(t) = I(t)\,V(t)$

$P(t) = I_o \sin(\omega t + \phi)\, V_o \sin \omega t.$

But, $I_o = \dfrac{V_o}{Z}$

So, $P(t) = \dfrac{V_o}{Z} \sin(\omega t + \phi)\, V_o \sin \omega t.$

$P(t) = \dfrac{V_o^2}{Z} [\sin(\omega t + \phi) \sin \omega t]$

$$\therefore \quad P(t) = \frac{V_o^2}{Z} \cdot \frac{1}{2} \cdot 2\sin(\omega t + \phi) \cdot \sin \omega t$$

Using the identity 2 sin A sin B = cos (A – B) – cos (A + B)

We get,

$$P(t) = \frac{V_o^2}{2Z} [\cos(\omega t + \phi - \omega t) - \cos(\omega t + \phi + \omega t)]$$

$$\mathbf{P(t) = \frac{V_o^2}{2Z} [\cos \phi - \cos(2\omega t + \phi)]} \qquad \text{... (18.29)}$$

If dW be the small workdone is small time dt, then instantaneous power is defined as,

$$\mathbf{P = \frac{dW}{dt}}$$

$$\textbf{or,} \quad \mathbf{dW = P.dt} \qquad \text{.... (18.30)}$$

Thus, total workdone over a cycle of a.c. is given by $W = \int_0^T dW = \int_0^T P.dt.$

$$W = \int_0^T \frac{V_o^2}{2Z} [\cos \phi - \cos(2\omega t + \phi)]\, dt.$$

$$= \frac{V_o^2}{2Z} \left[\left(\int_0^T \cos \phi\, dt - \int_0^T \cos(2\omega t + \phi)\, dt\right) \right]$$

$$= \frac{V_o^2}{2Z} \left[\cos \phi\, (t)_0^T - \left\{\frac{\sin(2\omega t + \phi)}{2\omega}\right\}_0^T \right]$$

$$= \frac{V_o^2}{2Z} \left[\cos \phi. (T - 0) - \left\{\frac{\sin(2\omega T + \phi) - \sin(2\omega \times 0 + \phi)}{2\omega}\right\} \right]$$

$$= \frac{V_o^2}{2Z} \left[T\cos \phi - \left\{ \frac{\sin\left(2\omega \times \frac{2\pi}{\omega} + \phi\right)}{2\omega} - \frac{\sin \phi}{2\omega} \right\} \right]$$

$$= \frac{V_o^2}{2Z} \left[T\cos \phi - \frac{\sin(4\pi + \phi)}{2\omega} + \frac{\sin \phi}{2\omega} \right]$$

$$= \frac{V_o^2}{2Z} \left[T\cos \phi - \frac{\sin \phi}{2\omega} + \frac{\sin \phi}{2\omega} \right] \quad \because \sin(4\pi + \phi) = \sin \phi$$

$$W = \frac{V_o^2}{2Z} \cdot T\cos\phi$$

The average power of the a.c circuit is given by

$$P_{av} = \frac{W}{T} = \frac{V_o^2}{2Z} \cdot \frac{T\cos\phi}{T}$$

$$\mathbf{P_{av} = \frac{V_o^2 \cos\phi}{2Z}} \qquad \textbf{... (18.31)}$$

$$P_{av} = \frac{V_o}{2}\frac{V_o}{Z}\cos\phi$$

$$= \frac{V_o}{2} I_o \cos\phi$$

$$= \frac{I_o}{\sqrt{2}}\frac{V_o}{\sqrt{2}}\cos\phi$$

$$\mathbf{P_{av} = I_{rms} V_{rms} \cos\phi} \qquad \textbf{... (18.32)}$$

Special cases

i. **Power consumption across resistor:** In this circuit, I and V are in same phase i.e. $\phi = 0$,
so, $P_{av} = I_{rms} V_{rms} \cos 0° = I_{rms} V_{rms}$ (maximum power is consumed)
This is the case across the resistor.
In resistor, Z = R, So, $P_{av} = \frac{V_o^2}{2R}$.

ii. **Power consumption across inductor:** In pure inductor, $\phi = 90^o$
So, $P_{av} = I_{rms} V_{rms} \cos 90^o = 0$
No power is consumed in pure inductor.

iii. **Power consumption in capacitor**: In capacitor, $\phi = 90^o$
So, $P_{av} = I_{rms} V_{rms} \cos 90^o = 0$
No power is consumed in capacitor.

18.12 Q Factor in LCR Circuit

Q factor is defined as the voltage magnification of the circuit at resonance.

Current resonance is given by, $I_m = \frac{V}{R}$, where I_m = maximum current in resonant circuit.

$V = I_m R$

and voltage across inductance or capacitor is given by: $I_m X_L$ or $I_m X_C$ respectively.

$$\text{Voltage magnification} = \frac{\text{Voltage across L or C}}{\text{Applied voltage}}$$

$$= \frac{V_L}{V_R} \text{ or } \frac{V_C}{V_R} = \frac{I_m X_L}{I_m R} \text{ or } \frac{I_m X_C}{I_m R}$$

$$= \frac{X_L}{R} \text{ or } \frac{X_C}{R} = \frac{\omega_r L}{R} \text{ or } \frac{1}{\omega_r CR}$$

$$= \frac{2\pi f_r L}{R} \text{ or } \frac{1}{2\pi f_r CR}$$

$$= \frac{2\pi L}{2\pi\sqrt{LC}R} \text{ or } \frac{1}{2\pi \times \frac{1}{2\pi\sqrt{LC}} CR} \quad (\because f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}})$$

$$= \frac{1}{R}\sqrt{\frac{L^2}{LC}} \text{ or } \frac{1}{R}\sqrt{\frac{LC}{C^2}}$$

In both cases, voltage magnification is found equal. So,

$$\therefore \quad \text{Q factor} = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{2\pi f_r L}{R} = \frac{1}{2\pi f_r CR}$$

18.13 Wattless Current

The average power over a cycle of a.c. is given by

$$\mathbf{P_{av} = V_{rms} I_{rms} \cos \phi} \quad \text{... (18.32)}$$

Here, $\cos \phi$ is the power factor and ϕ is the phase factor.

In a resistive circuit, I_{rms} and V_{rms} are in same phase so that $\phi = 0$.

Thus, from equation (18.32),

$$P_{av} = V_{rms} I_{rms}.$$

In a purely inductive circuit, and purely capacitive circuit, V_{rms} and I_{rms} are 90^o out of phase.

And from equation (18.32), $P_{av} = V_{rms} I_{rms} \cos 90^o = 0$ (for reactive circuits)

In inductive circuits, the energy taken from source is stored in the magnetic field of inductor in one quarter cycle and the stored energy is returned to the source in second quarter cycle. So, the mean power over a cycle is zero. Similarly, in capacitive circuits too, V_{rms} and I_{rms} are 90^o out of phase and hence $P_{av} = 0$.

In capacitive circuits, energy taken from the source is stored in the electric field due to the p.d. between the charged plates of capacitor in one quarter cycle. And, during the next quarter cycle, the capacitor discharges and the energy is returned to source. So, mean power over a cycle is zero.

Suppose, V_{rms} leads I_{rms} by a phase factor ϕ. The phasor for this situation is as shown in Fig. 18.21.

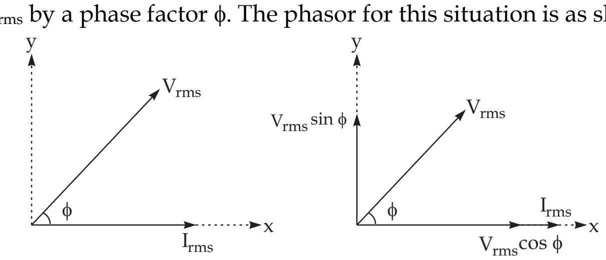


Fig. 18.21: Phasor diagram between I_{rms} and V_{rms}

V_{rms} can be resolved into two components, $V_{rms} \cos \phi$ along x-axis and $V_{rms} \sin \phi$ along y-axis. Referring to Fig. 18.21, we see $V_{rms} \cos \phi$ and I_{rms} are in phase and hence ($V_{rms} \cos \phi$) correspond to purely resistive part of circuit and causes power loss. But $V_{rms} \sin \phi$ and I_{rms} are 90^o out of phase and hence $V_{rms} \sin \phi$ correspond to purely reactive part of the circuit and do not cause power loss. So, $V_{rms} \sin \phi$ is called as wattless component of voltage and the current which is 90^o out of phase with the voltage is called wattless current. Thus, the current in an a.c. circuit is said to be wattless current when the average power consumed in such circuit corresponds to zero.

18.14 Choke Coil

A choke is an inductor used to block high frequency alternating current (a.c.) in an electrical circuit while passing low frequency or direct current (d.c.). It is preferred over resistance in an a.c. circuit because it has large value of self inductance and hence power dissipation is zero for choke coil.

Choke coil is used in series with the fluorescent lamp in a.c. circuit. It decreases the voltage across the lamp to make it safe without any appreciable loss of power. The schematic circuit for the choke coil in series with the resistor is shown in Fig. 18.22.

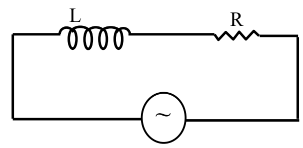


Fig. 18.22 Choke coil

Tips for MCQs

1. Alternating current and alternating voltage
   i. Magnitude changes continuously and direction is reversed periodically.
   ii. Instantaneous value of alternating current is, $I = I_0 \sin \omega t$
   iii. Instantaneous value of alternating voltages , $V = V_0 \sin \omega t$
   iv. Relation between peak value and rms value,

   a. $I_{rms} = \frac{I_0}{\sqrt{2}} = 0.707\, I_o$

   b. $V_{rms} = \frac{V_0}{\sqrt{2}} = 0.707\, V_o$.

2. Parameters of Various a.c. circuits

S.N.	Parameter	R circuit	L circuit	C circuit	RL circuit	RC circuit	LCR circuit
1.	Alternating voltage and current	$V=V_o\sin\omega t$ $I=I_o\sin\omega t$	$V=V_o\sin\omega t$ $I = I_o \sin\left(\omega t - \frac{\pi}{2}\right)$	$V=V_o\sin\omega t$ $I = I_o \sin\left(\omega t + \frac{\pi}{2}\right)$	$V= V_o\sin\omega t$ $I =I_o\sin(\omega t-\phi)$	$V= V_o\sin\omega t$ $I = I_o\sin(\omega t+ \phi)$	$V= V_o \sin\omega t$ $I = I_o \sin (\omega t \pm \phi)$
2.	Phase difference of current w.r.t. voltage	zero	lags by $\frac{\pi}{2}$	Leads by $\frac{\pi}{2}$	Lags by ϕ, $\tan \phi = \frac{X_L}{R}$	Leads by ϕ, $\tan \phi = \frac{X_C}{R}$	Leads by ϕ, $\tan\phi = \frac{X_L - X_C}{R}$ or, $\frac{X_C - X_l}{R}$
3.	Reactance	Zero	$X_L = \omega L$	$X_C = \frac{1}{\omega C}$	$X_L = \omega L$	$X_C = \frac{1}{\omega C}$	$X_L - X_C$ or, $X_C - X_L$
4.	Impedance	$Z = R$	$Z = X_L$	$Z = X_C$	$Z =\sqrt{R^2 + X_L^2}$	$Z = \sqrt{R^2 + X_C^2}$	$Z =\sqrt{R^2 + (X_L - X_C)^2}$
5.	Power factor ($\cos\phi$)	1	Zero	Zero	$\frac{R}{\sqrt{R^2 + X_L^2}}$	$\frac{R}{\sqrt{R^2 + X_C^2}}$	$\frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$
6.	Average Power	$V_{rms}I_{rms}$	Zero	Zero	$V_{rms}I_{rms} \cos \phi$	$V_{rms}I_{rms}\cos \phi$	$V_{rms}I_{rms}\cos \phi$

3. Information regarding a.c.

Physical quantity	Symbols	Dimensions	Units	Remarks
rms voltage	V_{rms}	$[ML^2T^{-3}A^{-1}]$	V	$V_{rms} = \frac{V_o}{\sqrt{2}}$
rms current	I_{rms}	[A]	A	$I_{rms} = \frac{I_o}{\sqrt{2}}$
Reactance: Inductance Capacitance	 X_L X_C	 $[ML^2T^{-3}A^{-2}]$ $[ML^2T^{-3}A^{-2}]$	 Ω Ω	 $X_L = \omega L$ $X_C = 1/\omega C$
Impedance	Z	$[ML^2T^{-3}A^{-2}]$	Ω	Depends on elements present in the circuit.
Resonant frequency	f_o	$[T^{-1}]$	Hz	$f_o = \frac{1}{2\pi\sqrt{LC}}$ for L-C-R series circuit.
Quality factor	Q	Dimensionless		$Q = \frac{\omega_o L}{R} = \frac{1}{\omega_o CR}$ for L-C-R series circuit.
Power factor	cos ϕ	Dimensionless		Power factor = cos ϕ, ϕ is the phase difference between voltage applied and current in the circuit.

Worked Out Problems

1. The voltage across the terminals of an a.c. power supply varies with time according to equation $V = V_0 \sin \omega t$. The voltage amplitude is V_o = 45.0 V. What is (a) the root-mean-square potential difference V_{rms}? (b) the average potential difference V_{av} between the two terminals of the power supply?

SOLUTION

Given,

Voltage amplitude (V_o) = 45.0 V

rms potential difference (V_{rms}) = ?

Average potential difference (V_{av}) = ?

We know that,

$$\because \quad V_{rms} = \frac{V_o}{\sqrt{2}} = \frac{45.0}{\sqrt{2}}$$

$$\therefore \quad V_{rms} = 31.8 \text{ V}$$

The voltage is alternating in nature, so the average value is zero, i.e., $V_{av} = 0$.

2. [HSEB 2071] A circuit consists of a capacitor of 2 μF and a resistor of 1000 Ω. An alternating emf of 12 V (rms) and frequency 50 Hz is applied. Find the current flowing, the voltage across capacitor and the phase angle between the applied emf and current.

SOLUTION

Given,

Capacitance of capacitor (C) = 2 μF = 2×10^{-6} F

Resistance of resistor (R) = 1000 Ω

emf (V) = 12 V (rms)

Frequency (f) = 50 Hz

Current (I) = ?

Voltage across capacitor (V_C) = ?

Phase angle (ϕ) = ?

We have, current,

$$I = \frac{V}{\sqrt{R^2 + X_C^2}} = \frac{V}{\sqrt{R^2 + \left(\frac{1}{2\pi fC}\right)^2}}$$

$$= \frac{12}{\sqrt{1000^2 + \left(\frac{1}{2\pi \times 50 \times 2 \times 10^{-6}}\right)^2}}$$

$= 6.38 \times 10^{-3}$ A

$$V_C = IX_C = I\frac{1}{2\pi fC} = \frac{6.38 \times 10^{-3}}{2\pi \times 50 \times 2 \times 10^{-6}} = 10.2 \text{ V}$$

$$\phi = \tan^{-1}\left(\frac{V_C}{V_R}\right) = \tan^{-1}\left(\frac{V_C}{I_R}\right)$$

$$= \tan^{-1}\left(\frac{10.2}{6.38 \times 10^{-3} \times 1000}\right) = 57.9°$$

3. a. **Compute the reactance of a 0.450 H inductor at frequencies of 60.0 Hz and 600 Hz.**
   b. **Compute the reactance of a 2.50 μF capacitor at frequencies of 60.0 Hz and 600 Hz.**
   c. **At what frequency is the reactance of a 0.450 H inductor equal to that of a 2.50 μF capacitor?**

SOLUTION

a. Given,

Inductance (L) = 0.450 H

Inductive reactance (X_L) = ?

We know that

$X_L = \omega L$

or, $X_L = 2\pi f L$

For f = 60 Hz,

$X_L = 2\pi \times 60.0 \times 0.450 = 170\ \Omega$.

For f = 600 Hz, we have,

$X_L = 2\pi \times 600 \times 0.450 = 1700\ \Omega$.

b. Given,

Capacitance (C) = 2.50 μF = 2.50×10^{-6} F

Capacitive reactance (X_C) = ?

We know that,

or $X_C = \frac{1}{2\pi f C}$

For f = 60.0 Hz, we have,

$$X_C = \frac{1}{2\pi \times 60 \times 2.50 \times 10^{-6}}$$

∴ $X_C = 106\ \Omega$.

for f = 600.0 Hz

$$X_C = \frac{1}{2\pi \times 600 \times 2.5 \times 10^{-6}} = 106.1\ \Omega$$

c. Given,

Inductance (L) = 0.450 H

Capacitance (C) = 2.50 μF = 2.50×10^{-6} F

Frequency (f) = ?

Given condition, $X_C = X_L$

or, $\frac{1}{2\pi f C} = 2\pi f L$

or, $f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.45 \times 2.5 \times 10^{-6}}}$

∵ f = 150 Hz

4. **[HSEB 2059] The maximum capacitance of a variable capacitor is 33 pF. What should be the self-inductance to be connected to this capacitor for the natural frequency of the LC circuit to be 810 kHz. Corresponding to A.m. broadcast band of Radio Nepal?**

SOLUTION

Given,

Capacitance (C) = 33 pF = 33×10^{-12} F

Let, L be the self-inductance connected with the capacitor,

The natural frequency of this LC circuit is f = 810 kHz = 810×10^3 Hz.

Resonant frequency $= \frac{1}{2\pi\sqrt{LC}}$

or, $L = \frac{1}{4\pi^2 f^2 C} = \frac{1}{4\pi^2 (810 \times 10^3)^2 \times 33 \times 10^{-12}}$

$= 1.17 \times 10^{-3}$ H

Hence, the required inductance is 1.17×10^{-3} H.

5. [HSEB 2072] A coil having inductance and resistance is connected to an oscillator giving a fixed sinusoidal output voltage of 5 V rms. With the oscillator set at a frequency of 50 Hz, the rms current in the coil is 1 A and at a frequency of 100 Hz, the rms current is 0.625 A. Determine the inductance of the coil.

SOLUTION

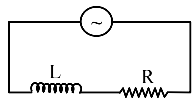
Given,

Voltage (V) = 5.0 V

Frequencies (f_1) = 50 Hz, f_2 = 100 Hz

Currents (I_1) = 1 A, I_2 = 0.625 A

Inductance (L) = ?



Now, Impedance at frequency, 50 Hz

$$Z_1 = \frac{V}{I_1} = \frac{5}{1} = 5\ \Omega$$

Impedance of frequency, 100 Hz

$$Z_2 = \frac{V}{I_2} = \frac{5}{0.625} = 8\ \Omega$$

Again, $Z_1 = \sqrt{R^2 + \omega_1^2 L_1^2}$

or, $Z_1^2 = R^2 + 4\pi^2 f_1^2 L^2$

Similarly,

$Z_r^2 = R^2 + 4\pi^2 f_r^2 L^2$

Subtracting equation (i) from (ii), we get,

$Z_2^2 - Z_1^2 = 4\pi^2 (f_2^2 - f_2^1{}^2) L^2$

or, $L^2 = \dfrac{Z_2^2 - Z_1^2}{4\pi^2(f_2^2 - f_1^2)} = \dfrac{8^2 - 5^2}{4\pi^2(100^2 - 50^2)} = 0.0001317$

$\therefore$ L = 0.0114 H

6. [HSEB 2063] An inductor, a resistor and capacitor are connected in series across an a.c. circuit. A voltmeter reads 60 V when connected across the inductor, 16 V across the resistor and 30 V across the capacitor:

i. What will the voltmeter read when placed across the series circuit?

ii. What is the power factor of the circuit?

SOLUTION

Given,

V_L = 60 V $\quad$ V_C = 30 V

i. Voltmeter reading across series circuits (V) = ?

Voltmeter reading (V) = $\sqrt{(V_L - V_C)^2 + V_R^2}$

$= \sqrt{(60-30)^2 + (16)^2} = 34$ V

ii. Power factor (cos ϕ) = ?

We know that,

Power factor $(\cos\phi) = \dfrac{V_R}{V} = \dfrac{16}{34} = 0.47$

6. A coil connected to an a.c. source of frequency 50 Hz, draws a current of 4.0 A with 240 W of power loss. If the voltage across the coil is 100 V. Calculate its inductance.

SOLUTION

Given,
Frequency (f) = 50 Hz
Current (I_{rms}) = 4.0 A
Power loss (P) = 240 W
Voltage (V_{rms}) = 100 V
Inductance (L) = ?

When there is power loss in a circuit, it must contain resistance. In this condition, the inductor is not ideal, it possesses the Ohmic resistance (R) too.

So,

$$P = I^2R$$

$$\therefore\ R = \frac{P}{I^2} = \frac{240}{(4)^2} = 15\ \Omega$$

Now, the impendence of the circuit with inductance L,

$$Z = \sqrt{R^2 + X_L^2} \quad \text{... (i)}$$

Also,

$$Z = \frac{V_o}{I_o} = \frac{V_{rms}}{I_{rms}} = \frac{100}{4} = 25\ \Omega$$

From (i),

$$25^2 = R^2 + X_L^2$$

$$\therefore\ X_L^2 = 625 - R^2 = 625 - 225 = 400$$

$$X_L = 20\ \Omega$$

$$2\pi fL = 20$$

$$L = \frac{10}{2\pi f} = \frac{20}{2\pi \times 50} = 0.064\ H$$

7. [HSEB 2068] An alternating voltage 10 V (rms) and 4 kHz frequency is applied to a resistor of resistance 5 Ω in series with a capacitor of capacitance 10 μF. Calculate the rms potential differences across the resistor and the capacitor.

SOLUTION

Given,
Voltage (V_{rms}) = 10 V
Frequency (f) = 4 kHz = 4 × 1000 Hz
Resistance (R) = 5 Ω
Capacitance (C) = 10 μF = 10×10^{-6} F
p.d. across resistor (V_R) = ?
p.d. across capacitor (V_C) = ?

Now, we have,

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 4 \times 1000 \times 10 \times 10^{-6}} = 3.98\ \Omega$$

Then, in case of R-C circuit, we have

$$I = \frac{V_{rms}}{\sqrt{R^2 + X_C^2}} = \frac{10}{\sqrt{5^2 + (3.98)^2}} = \frac{10}{6.39} = 1.56\ A$$

Then,
p.d. across resistor (V_R) = I × R = 1.56 × 5 = 7.8 V

Also, p.d. across capacitor (V_C) = I × X_C = 1.56 × 3.98 = 6.2 V

Here, the required p.d across resistor and capacitor are 7.8 V and 6.2 V respectively.

8. [HSEB 2068] A circuit consists of an inductor of 200 μH and resistance of 10 Ω in series with a variable capacitor and a 0.10 V (r.m.s.), 1.0 MHz supply. Calculate (i) the capacitance to give resonance (ii) the quality factor of the circuit at resonance.

SOLUTION

Given,
Inductance (L) = 200 μH = 200×10^{-6} H
Resistance (R) = 10 Ω
Emf (E) = 0.10 V
Frequency (f) = 1 MHz = 10^6 Hz

Now, For (i); we have

$$X_L = \omega L$$

$$= 2\pi fL = 2\pi \times 10^6 \times 200 \times 10^{-6} = 1256.6\ \Omega$$

In resonance condition; we have

$$X_C = X_L$$

or, $\frac{1}{\omega C} = 1256.6$

or, $\frac{1}{2\pi fC} = 1256.6$

or, $\frac{1}{2\pi \times 10^6 \times C} = 1256.6$

$\therefore$ C = 1.26×10^{-10} F

Hence, the required capacitance is 1.26×10^{-10} F.

Again, we have,

$$\text{Quality factor (Q)} = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{200 \times 10^{-6}}{1.26 \times 10^{-10}}} = 125.97 \simeq 126$$

Hence, required value of quality factor is 126.

9. [HSEB 2070] An iron cored coil of 2 H and 50 Ω resistance placed in series with a resistor of 450 Ω and 200 V, 50 Hz a.c. supply is connected across the arrangement, find

i. The current flowing the coil,

ii. Its phase angle relative to the voltage supply

iii. The voltage across the coil.

SOLUTION

Given,

Inductance (L) = 2 H

Resistance of wire (R) = 450 Ω

Frequency of a.c. (f) = 50 Hz

Phase angle (ϕ) = ?

Resistance of coil (r) = 50 Ω

Emf (V) = 200 V

Current (I) = ?

Voltage a cross coil (V_L) = ?

Now,

$$I = \frac{V}{\sqrt{(R + r)^2 + X_L^2}} = \frac{V}{(R + r)^2 + (2\pi fL)^2} = \frac{200}{\sqrt{(450 + 50)^2 + (2\pi \times 50 \times 2)^2}} = 0.25 \text{ A}$$

Now,

$$V_L = IX_L = 0.25 \times 2\pi \times 50 \times 2 = 156.5 \text{ V}$$

$$\phi = \tan^{-1}\left(\frac{V_L}{V_R}\right) = \tan^{-1}\left(\frac{156.5}{0.25 \times 450}\right) = 54.3°$$

10. A 150 Ω resistor is connected in series with a 0.250 H inductor. The voltage across the resistor is V_R = (3.80 V) cos [(720 rad/s) *t*]. (a) Derive an expression for the circuit current. (b) Determine the inductive reactance of the inductor. (c) Derive an expression for the voltage V_L across the inductor.

SOLUTION

Given,

a. Circuit current (I) = ?

Given,

$V_R = 3.80 \cos (720 t)$ … (i)

We know that

$V_R = V_o \cos \omega t$ … (ii)

Comparing (i) and (ii), we get,

$V_o = 3.80$ V

$\omega = 720$ rad/s

Now,

$$I = I_o \cos \omega t = \frac{V_o}{R} \cos (720 t) = \frac{3.80}{150} \cos (720 t)$$

$\therefore$ I = 0.0253 cos (720 t)

b. Inductive reactance (X_L) = ?

we know that,

$X_L = \omega L = 720 \times 0.250$

$\therefore$ $X_L = 180\ \Omega$

c. Voltage across inductor (X_L) = ?

Since, in inductor the voltage leads the current by 90^o so, we can write,

$V_L = V_o \cos (720 t + \pi/2) = -I_0 X_L \cos (720 t)$

$= 2.0253 \times 180$

$\therefore$ $V_L = -4.50 \sin (720 t)$

11. In an L–R–C series circuit, *R* = 300 Ω, *L* = 0.400 H and *C* = 6.00 × 10^{-8} F. When the a.c. source operates at the resonance frequency of the circuit, the current amplitude is 0.500 A. (a) What is the voltage amplitude of the source? (b) What is the amplitude of the voltage across the resistor, across the inductor, and across the capacitor? (c) What is the average power supplied by the source?

SOLUTION

Given,

Resistance (R) = 300 Ω

Inductance (L) = 0.4 H

Capacitance (C) = 6 × 10^{-8} F

Current amplitude (I_o) = 0.5 A

a. Voltage amplitude of the source (V_o) = ?

At the resonance frequency (Z) = R

Now,

$V_o = I_oZ = I_oR = 0.5 \times 300 = 150$ volt

b. Voltage amplitude across resistor (V_R) = ?

Voltage amplitude across inductor (V_L) = ?

Voltage amplitude across capacitor (V_C) = ?

$V_R = I_oR = 0.5 \times 300 = 150$ V

$$V_L = I_o X_L = I_o \omega L = I_o L\left(\frac{1}{\sqrt{LC}}\right) = I_o\sqrt{\frac{L}{C}}$$

$$= 0.5 \times \sqrt{\frac{0.4}{6 \times 10^{-8}}}$$

∴ $V_L = 1290$ V

Now,

$$V_C = I_o X_L = I_o \frac{1}{\omega C}$$

$$= I_o \frac{1}{C\frac{1}{\sqrt{LC}}} = I_o\sqrt{\frac{L}{C}}$$

$$= 0.5 \times \sqrt{\frac{0.4}{6 \times 10^{-8}}}$$

∴ $V_C = 1290$ V

c. Average power supplied by source (P_{av}) = ?

We know that,

$$P_{av} = \frac{1}{2} VI \cos\phi$$

At resonance, $\cos\phi = 1$ and $V = IR$

So, $P_{av} = \frac{1}{2} \times I\,R \times I = \frac{1}{2} I^2 R = \frac{1}{2} \times (0.50)^2 \times 300$

∴ $P_{av} = 37.5$ W

12. [HSEB 2054] A constant a.c. supply is connected to a series circuit consisting of a resistance of 300 Ω in series with a capacitance 6.67 μF, the frequency of the supply being 3000/2π Hz. It is desired to reduce the current in the circuit to half its value. Show how this could be done by placing an additional resistance.

SOLUTION

Given,

Resistance (R) = 300 Ω

Capacitance (C) = 6.67 μ F = 6.67×10^{-6} F

Frequency (f) = $\frac{3000}{2\pi}$ H z

Let r be the additional resistance that can reduce the current to half of its original value. If I be the original current and I' be the current when additional resistance is added in the circuit so that $I' = \frac{I}{2}$, then we can write,

or, $\frac{I}{2} = \frac{V}{\{(R+r)^2 + X_C^2\}^{1/2}}$

or, $\frac{1}{2} \times \frac{V}{(R^2 + X_C^2)^{1/2}} = \frac{V}{\{(R+r)^2 + X_C^2\}^{1/2}}$

or, $(R + r)^2 + X_C^2 = 4\,(R^2 + X_C^2)$

or, $(R + r)^2 = 4\,R^2 + 3X_C^2$

or, $R + r = (4R^2 + 3\,X_C^2)^{1/2}$

or, $r = (4R^2 + 3\,X_C^2)^{1/2} - R$

$$I = \frac{V}{Z}$$

$$= \frac{V}{(R^2 + X_C^2)^{1/2}}$$

Also,

$$I' = \frac{V}{\{(R+r)^2 + X_C^2\}^{1/2}}$$

$$= \left\{4 \times (300)^2 + 3 \times \left(\frac{1}{2\pi fC}\right)^2\right\}^{\frac{1}{2}} - 300$$

$$= \left\{4 \times 9 \times 10^4 + 3 \times \left(\frac{1}{2\pi \times \frac{3000}{2\pi} \times 6.67 \times 10^{-6}}\right)^2\right\}^{\frac{1}{2}} - 300$$

$$= \{36 \times 10^4 + 3 \times (50)^2\}^{1/2} - 300$$

∴ $r = 306.1\ \Omega$

Challenging Problems

1. **[UP]** A 5.00 H inductor with negligible resistance is connected across an a.c. source whose voltage amplitude is kept constant at 60.0 V but whose frequency can be varied. Find the current amplitude when the angular frequency is (a) 100 rad/s; (b) 1000 rad/s; (c) 10,000 rad/s.

Ans: (a) 0.12 A (b) 1.2 × 10^{-2} A (c) 1.2 × 10^{-3} A

2. **[UP]**
   a. What is the reactance of a 3 H inductor at frequency 80 Hz?
   b. What is the inductance of an inductor whose reactance is 120 Ω at 80 Hz?
   c. What is the reactance of a 4 μF capacitor at a frequency of 80 Hz?

d. What is the capacitance of a capacitor whose reactance is 120 Ω at 80 Hz?

Ans: (a) 1508 Ω (b) 0.239 H (c) 497 Ω (d) 16.6 × 10^{-6} F

3. **[UP]** The wiring for a refrigerator contains a starter capacitor. A voltage of amplitude 170 V and frequency 60.0 Hz applied across the capacitor is to produce a current amplitude of 0.850 A through the capacitor. What capacitance *C* is required?

Ans: 13.3 μF

4. **[UP]** You want the current amplitude through a 0.450 mH inductor (part of the circuitry for a radio receiver) to be 2.60 mA when a sinusoidal voltage with amplitude 12.0 V is applied across the inductors. What frequency is required?

Ans: 1.63 × 10^6 Hz

5. **[UP]** A 250 Ω resistor is connected in a series with a 4.80 μF capacitor. The voltage across the capacitor is V_C = (7.60 V) sin [(120 rad/s)t].

Determine the capacitive reactance of the capacitor. Derive an expression for the voltage across the resistor.

Ans: 1736 Ω, V_R = 1.10 sin (120 t) V

6. **[UP]**You have a 200 Ω resistor, a 0.400 H inductor, and a 6.00 μF capacitor. Suppose you take the resistor and inductor and make a series circuit with a voltage source that has voltage amplitude 30.0 V and an angular frequency 250 rad/s. (a) What is the impedance of the circuit? (b) What is the current amplitude? (c) What are the voltage amplitudes across the resistor and across the inductor? (d) What is the phase angle ϕ of the source voltage with respect to the current? Does the source voltage lag or lead the current?

Ans: (a) 224 Ω (b) 0.134 A (c) 26.8 V, 13.4 V (d) 26.6°

7. **[UP]** In a series L–R–C circuit, the components have the following values: *L* = 20.0 mH, *C* = 140 nF and *R* = 350 Ω. The generator has an rms voltage of 120 V and a frequency of 1.25 KHz. Determine (a) power supplied by the generator; (b) the power dissipated in the resistor.

Ans: (a) 9.06 W (b) 7.32 W

8. **[UP]** An L-R-C series circuit consists of a source with voltage amplitude 120 V and angular frequency 50.0 rad s^{-1}, a resistor with *R* = 400 Ω, an inductor with *L* = 9.00 H, and a capacitor with capacitance *C*. (a) For what value of *C* will the current amplitude in the circuit be a maximum? (b) When C has the value calculated in part (a), what is the amplitude of the voltage across the inductor?

Ans: (a) 4.44 × 10^{-5} F (b) 135 V

9. **[ALP]** An iron cored coil of inductance 3 H and 50 Ω resistance is placed in series with a resistor of 550 Ω, and a 100 V, 50 Hz a.c. supply is connected across the arrangements. Find the current following in the coil and the voltage across the coil. [HSEB 2053]

Ans: 0.089 A, 84 V

10. A circuit consists of a capacitor of 10 μF and a resistor of 1000 Ω. An alternating emf of 12 V (rms) and frequency 50 Hz is applied. Calculate the current flowing and voltage across the capacitor.[HSEB,2057]

Ans: 0.01 A, 31.8 V

11. **[ALP]** In a series L-C-R circuit, R = 25 Ω. L = 30 mH and C = 10 μF and these elements are connected to 240 a.c. (rms) 50 Hz source. Calculate the current in the circuit and voltmeter reading across a capacitor . [HSEB, 2062]

Ans: 0.77 A and 245.3 V

12. **[ALP]** A 50 V, 50 Hz, a.c. supply is connected to a resistor of resistance 40 Ω in series with a solenoid having inductance 200 mH with same resistance. The potential difference across the ends of the 40 Ω resistor is found to be 20 V. Find the resistance of the wire of the solenoid. [HSEB, 2064]

Ans: 37.83 Ω

13. **[ALP]** A coil of self inductance of 0.20 H and a resistance of 50.0 Ω is to be supplied with current of 1.00. A form a 240 V, 50 Hz, supply and it is desired to make the current in phase with the potential difference. Find the value of the components that must be up in series with the coil.

Ans: 190 Ω and 50.6 μ F

14. The maximum capacitance of a variable capacitor is 33 pF. What should be the self-inductance to be connected to this capacitor for the natural frequency of the L-C circuit to be 810 kHz corresponding to A.M. broadcast band of Radio Nepal? [HSEB, 2059]

Ans: 1.17×10^{-3} H

*[**Note:** Hints to challenging problems are given at the end of this chapter.]*

Conceptual Questions with Answers

1. Differentiate between a.c. and d.c.

↳ Some important differences between d.c. and a.c. are as follows.

Direct Current (d.c.)	Alternating Current (a.c.)
1. The magnitude of direct current is constant in a circuit over time continuously.	1. The magnitude of alternating current varies periodically over time.
2. The positive and negative terminals of d.c. are fixed. They are not altered for a source.	2. The polarities of a.c. reverse periodically. So, no polarity is fixed in a source.
3. d.c. is symbolized by a cell.	3. a.c. is symbolized by sine curve,

2. Which one is more dangerous a.c. or dc. Why?

↳ Alternating current (a.c.) is more dangerous than direct current (dc). The direct current has constant magnitude of current (or voltage) in the circuit, whereas the magnitude of current (or voltage) varies in a.c.. Suppose the 220 V d.c. means the value is constant over time. However 220 V a.c. means it is the rms value. In such condition a.c. voltage fluctuates from 0 V to $220\sqrt{2}$ (= 311 V). Hence, a.c. provides the greater shock for same magnitude of dc.

3. Define rms value of a.c..

↳ RMS refers to the root mean square. The average value of a.c. is zero over one complete cycle. The direct average of positive and negative cycle of alternating magnitude of a.c. is zero, so the mean value is taken by squaring its magnitude. rms means the square root of the mean of squares of the instantaneous current over a complete cycle.

$I_{rms} = \sqrt{\overline{I^2}}$ or, $I_{rms} = \dfrac{I_o}{\sqrt{2}}$.

4. What are the basic properties of a.c.?

↳ The basic properties of a.c. are:

i. The magnitude of current or voltage varies continuously with time.

ii. The polarity reverses periodically.

iii. It has certain frequency of oscillation.

5. What do you mean by rms value of an A.C. current? [HSEB 2056]

↳ The rms value of a.c. is the steady current (dc), which on passing through a resistance for a given time will produce the same amount of heat as the alternating current does in the same resistance for the same time. It is also called as virtual or effective value. If I_o be the peak value of a.c., then its rms value is given by $I_{rms} = \dfrac{I_o}{\sqrt{2}}$

Similarly the r.m.s value of a.c. voltage is $V_{rms} = \dfrac{V_o}{\sqrt{2}}$.

6. What is meant by impedance of an a.c. circuit? [HSEB 2054]

↳ The net opposition offered by the a.c. circuit to the alternating current is called *impendence* of an a.c. circuit. When an a.c. current passes through a LCR circuit i.e., circuit containing inductor, capacitor and resistor, then each of the components offers different resistances which differ from each other. Inductor (L) offers reactance X_L which is directly proportional to the frequency (f) of an ac. Similarly, capacitor (C) offers reactance X_C which is inversely proportional to the frequency (f) of an ac, and resistance offered by resistor does not depend upon frequency.

Hence, the net resistance offered by LCR circuit is given by,

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \text{ (where } \omega = 2\pi f)$$

which is known as impedance of an a.c. circuit. It plays same role as played by resistance in d.c. circuit. Similarly, the impedance of LR circuit is $Z = \sqrt{R^2 + X_L^2}$ and CR circuit is $Z = \sqrt{R^2 + X_C^2}$.

7. Fluorescent tubes often use an inductor to limit the current through the tube. Why is it better to use inductor than a resistor for this purpose? [HSEB 2059]

➪ The electrical power consumed in an a.c. circuit is given by

$P_{av} = I_{rms} \times V_{rms} \times \cos\theta$,

The value of phase angle in pure inductor is 90°, the power consumed is $P_{av} = I_{rms} \times V_{rms} \times \cos 90° = 0$. Due to this reason, an inductor is used to limit the current through the tube of fluorescent light. But, resistor consumes the power in an electric circuit. If a resistor is used for this purpose, there will be wastage of power ($P = I^2R$) by Joule's law of heating due to zero phase difference between the voltage and current. Thus, unlike resistor, inductor plays a significant role for controlling the current without loss of any power. That's why, inductor is better to use than a resistor in fluorescent tubes.

8. The emf of an a.c. source is given by the expression, E = 300 sin 314 t volts. Write the values of peak voltage and frequency of source. [HSEB 2074]

➪ Given E = 300 sin(314t)

Peak voltage (E_o) = ?

Frequency (f) = ?

Comparing this equation with

$E = E_o \sin\omega t$

Here, E_o = 300 V and ω = 314

Here, $2\pi f = 314$

$\therefore$ f = 50 Hz

9. At high frequencies, a capacitor becomes a short-circuit and an inductor becomes an open circuit. Explain.

➪ The capacitive reactance X_C is given by, $X_C = \frac{1}{2\pi fC}$, where f is frequency and C is capacitance of a capacitor.

If $f \to \infty$ (very high), $X_C \to 0$, So, capacitor acts as short circuit.

The inductive reactance for inductor X_L is given by, $X_L = 2\pi fL$, L = inductance of inductor.

If, $f \to \infty$ (very high), $X_L \to \infty$, so inductor act as open circuit.

10. Why do we prefer a choke coil to rheostat in an a.c. circuit? [HSEB 2053]

➪ Choke coil is a coil of wire with high inductance and low resistance. It is used in a.c. circuit to control the current without significant loss of electrical energy in the form of heat. The power consumed in choke coil is $P_{av} = I_{rms} \times V_{rms} \times \cos\theta$. Since, $\theta = 90°$, then $P_{av} = I_{rms} \times V_{rms} \times \cos 90° = 0$. It means the current in a choke is wattless. If a resistor is used for limiting current in a circuit, there is always wastage of power by Joule's law of heating. The power loss is given by $P = I^2R$. Also, there is gradual potential drop across the resistor. Hence, choke coil is preferred to a resistance in an a.c. circuit to change the magnitude of the current without consuming power from the source.

11. A 220 V a.c. is more dangerous than 220 V d.c. Why? [HSEB 2070]

➪ The root mean square voltage and peak value of voltage are related as

$$V_{rms} = \frac{V_o}{\sqrt{2}}$$

or, $V_o = \sqrt{2}\ V_{rms}$

12. Alternating current passes through a capacitor whereas direct current does not. Explain this fact on the basis of capacitive reactance.

↳ The capacitive reactance of a capacitor is given as,

$X_C = \frac{1}{2\pi fC}$, where f = frequency of current and C is capacitance of a capacitor. So, the peak value of 220 V a.c. is $\sqrt{2}$ times greater than 220 V d.c. Hence, a 220 V a.c. is more dangerous than 220 V d.c.

For a d.c. current, f = 0, $X_C = \infty$ i.e., for d.c. current the reactance of a capacitor becomes infinite and acts as insulator which blocks current. For alternating current, f is more and X_C is less i.e., the capacitor has low reactance for alternating current. So, a.c. easily passes through it.

13. What is meant by wattles current? [NEB 2075]

↳ The electrical power consumed in an a.c. circuit is given by,

$P_{av} = I_{rms} \times V_{rms} \times \cos\theta$,

$\cos\theta$ is called the power factor and θ is the phase difference between the current and voltage. Since, the value of phase angle in pure capacitor and inductor is 90°, the power consumed is $P_{av} = I_{rms} \times V_{rms} \times \cos 90° = 0$, no power is consumed, even though a.c. flows through them. Hence, current flowing in pure capacitor and inductor is called wattles current. Since the electric circuit containing pure inductor or pure capacitor does not consume any power, current of any such a.c. circuit is said to be wattles current.

Exercises

Short-Answer Type Questions

1. What are the properties of alternating current?
2. Define time period and frequency of a.c.
3. What are the advantages of a.c. over d.c.?
4. Why do we find rms value of a.c.?
5. What does 50 Hz a.c. mean?
6. What are reactance and impendence? Give its S. I. units.
7. What do you mean by power factor?
8. A bulb connected in series with a solenoid and a.c. source glows. If a soft iron core is introduced in the solenoid, will the bulb glow more? Explain.
9. What is a phasor diagram?
10. A low power factor of power transmission circuit implies large power loss. How?
11. Some electrical appliances operate equally well on a.c. or d.c., and other work only on a.c. or only on d.c. Give examples of each, and explain the differences.
12. A bulb and a capacitor are joined in series to an a.c. source. What will happen if frequency of the source is increased?
13. Why is alternating current usually transmitted at high practicable voltage?
14. Why is the long distance transmission of a.c. economical?
15. Does capacitor allow d.c.to pass through?
16. What is the unit and dimensional formula of LC?
17. What will be the shape of graph between inductive reactance and frequency of a.c. supply?

18. Can we use 20 cycle per second alternating current for lighting purpose? What happens if frequency is decreased below 10 Hz?
19. What does mean that voltage in LR circuit lead current by 90°?
20. When does LCR series circuit have minimum impedance?
21. What is the frequency of direct current?
22. Why alternating current measuring instruments have a non-linear scale?
23. What are the uses of choke coil?

Long-Answer Type Questions

1. Define peak value and mean value of a.c. and derive the relation between them.
2. Define peak value and root mean square value of a.c. and derive the relation between them.
3. Discuss the phase relationship between the voltage and current in the a.c. circuit containing inductance and resistance. What is power factor of the circuit? (HSEB 2052)
4. An alternating emf is applied across a capacitor. Show that the current in it leads to the applied emf by 90°. (HSEB 2056)
5. An alternating emf is applied across a inductor. Show that the current in it lags the applied emf by 90°.
6. An alternating emf is applied across a resistor. Show that the current and the applied emf are in the same phase.
7. An a.c. passes through a circuit containing a resistor and an inductor in series. Derive an expression for the phase relation between the current and the voltage. (HSEB 2058)
8. Discuss the phase relationship between the voltage and current in the a.c. circuit containing capacitor and resistor in series and hence derive an expression for the impedance of the circuit. (HSEB 2060)
9. Find an expression for impedance of an a.c. circuit containing a resistance and a capacitor in series. Also discuss the phase relation of current and emf in that circuit. (HSEB 2061)
10. Derive the condition for resonant frequency of an L-C-R alternating current series circuit. (HSEB 2065)
11. Derive an expression of current flowing through an a.c. circuit containing a resistor and capacitor in series combination. What is the power factor of this circuit? (HSEB 2066)
12. Discuss the phase relationship between the current and voltage in the a.c. circuit containing capacitor and resistor in series and hence derive an expression for the impedance of the circuit. (HSEB 2067)
13. Find the impendence of L-C-R circuit in series. (HSEB 2055)
14. Obtain an expression for average power consumption in L-C-R series circuit.
15. What is resonance? Obtain an expression for resonance frequency in L-C-R series circuit.

Numerical Problems

1. A 50 mH inductor is connected to 200 V, 50 Hz a.c. supply. Calculate the rms value of the current in the circuit.

   Ans: 12.74 A

2. A series circuit contains a resistor of 20 Ω, a capacitor and an ammeter of negligible resistance. It is connected to a source of 200 V, 50 Hz. If the reading of ammeter is 2.5 A, calculate the reactance of the capacitor.

   Ans: 77.46 Ω

3. A 10 μF capacitor is in series with a 50 Ω resistance and the combination is connected to a 220 V, 50 Hz line. Calculate (i) the capacitive reactance, (ii) the impedance of the circuit and (iii) the current in the circuit.

   Ans: (i) 318.5 Ω (ii) 322.4 Ω (iii) 0.68 A

4. A resistance of 10 Ω, an inductance of 0.2 H and a capacitance of 100 μF are connected in series across a 200 V, 50 Hz supply main. Determine (i) impedance (ii) current (iii) voltage across R, L and C, (iv) power factor and angle of lag and (v) power consumed in watts.

Ans: (i) 32.6 Ω, (ii) 6.13 A, (iii) 61.3 V, 386 V, 196 V, (iv) 0.31, 72° and (v) 380 W

5. An alternating current I is given by I = 5 sin 314 t. Find (i) the maximum value of current, (ii) frequency, (iii) time period, and (iv) instantaneous value of a.c. when time t = 4 ms.

Ans: (i) 5 A (ii) 50 Hz (iii) 0.02 s (iv) 4.76 A

6. A coil of inductance 1 H and resistance 50 Ω is connected to 230 V, 50 Hz a.c. supply. (i) Find the maximum current in the coil, and (ii) find the time lag between the voltage maximum and minimum.

Ans: (i) 1.02 A, (ii) 4.49 × 10^{-3} s

7. A 60 μF capacitor, 0.3 H inductor and a 50 Ω resistor are connected in series with 120 V, 60 Hz source. Calculate the (i) impedance, (ii) current, and (iii) power dissipated in the circuit.

Ans: (i) 85.13 Ω (ii) 1.41 A (iii) 99.4 W

8. An L-R-C series circuit is connected to a 120 Hz a.c. source that has Vrms = 80.0 V. The circuit has a resistance of 75.0 Ω and an impedance at this frequency of 105 Ω. What average power is delivered to the circuit by the source?

Ans: 43.5 W

9. A voltage across the terminals of an a.c. power supply varies with time. The voltage amplitude is 45.0 V. what is (a) the root mean square potential difference V_{rms}? (b) the average potential difference V_{av} between two terminals of the power supply?

Ans: 31.0 V, 0

10. An alternating voltage of 10 V rms and frequency 50 Hz is supplied (i) a resistor of 5 ohm, (ii)an inductor of 2 H and (iii) a capacitor of 1 μF. Calculate the rms current flowing in each case.

Ans: 2 A, 0.016 A, 0.0031 A

11. An alternating current of 0.2 A rms and frequency 100/2π Hz flows, if a circuit considering of series arrangement of a resistor R of 20 Ω an inductor L of 0.15 H and a capacitor of 500 μF. Calculate the a.c. voltage (i) across each component, (ii) across R and L together, (iii) across R and C together (iv) total voltage across R, L, and C. What power is dissipated in each component?

Ans: 4.0V, 3.0 V, 5.0 V, 1.0 V, 4.1 V, 0.8 W

12. A 50 V, 10 W lamp is to run 100 V, 50 Hz a.c. mains. Calculate the inductance of the choke coil required.

Ans: 1.38 H

13. A series circuit contains a resistor of 20 Ω a capacitor and an ammeter or negligible resistance. It is connected to a source of 200 V, 50 Hz. If the reading of ammeter is 2.5 A. Calculate the reactance of the capacitor.

Ans: 77.46 Ω

14. A coil of inductance 0.50 H and resistance 100 Ω is connected to 200 V, 50 Hz a.c. supply. Find the maximum current in the coil. Also, find the lag between the maximum voltage and maximum current.

Ans: 1.52 A, 3.194 × 10^{-3} s

15. In a series LCR circuit, R = 25Ω, L = 30 mH and C = 10 μF. These elements are connected to 240 V (rms) 50 Hz a.c. supply. Calculate the current in the circuit and voltmeter reading across the capacitor.

Ans: 0.774 A, 246.37 V

16. A 50 Ω resistance, 3 mH inductor and 2 μF capacitor are connected in series to a 110 V, 5000 Hz a.c. source. Calculate the value of the current in the circuit.

Ans: 0.44 A

17. An inductor, a resistor and a capacitor are connected in series across an a.c. circuit. A voltmeter reads 60 V when connected across the inductor, 16 V across the resistor and 30 V across the capacitor: (i) what will the voltmeter reads when placed across the series circuit? (ii) What is the power factor of the circuit?

Ans: 34 V, 0.47

18. The wiring of a refrigerator contains a starter capacitor. A voltage of amplitude 170 V and frequency 60 Hz is applied across the capacitor to produce a current amplitude of 0.85 A. What capacitance C is required?

Ans: 13.3 μF

19. A radio tuning LCR circuit contains an inductor of 0.5 mH and a variable capacitor. What should be the value of the variable capacitor to tune a radio signal transmitted at 90 MHz of frequency.

Ans: 6.25×10^{-15} F

20. A 150 Ω resistor is connected is connected if series with a 0.250 H inductor. The voltage across the resistor is V_R = (3.80V) cos [(720 rad/s) t]. (a) Derive the expression for the circuit (b) Determine the inductive reactance is the inductor. (c) Determine the expression for the voltage, V_L across the inductor.

Ans: I = (0.0253 A) cos 720 t, 180 Ω, V_L= - (4.56V) sin720 t

21. In an LCR series circuit, R = 300 Ω, L = 0.400 H, and C = 6.00 μF. When the a.c. source operates at the resonance frequency of the circuit, the current amplitude is 0.500 A. (a) what is power factor? (b) What is the amplitude if the voltage across the resistance, across the inductor and across the capacitor? What is the average power supplied by the source?

Ans: 150 V, 1290 V 1290 V, 37.5 W

22. A coil of self inductance of 0.20 H and a resistance 50.0 ohm is to be supplied with current of 1.00 A from a 240 V, 50 Hz, supply and it is desired to make the current in phase with the potential difference of the source. Find the values of the components that must be put in series with the coil.

Ans: 190 ohm, 50.6 μF

Multiple Choice Questions

1. An alternative current of 1.5 mA rms and angular frequency ω = 100 rads^{-1} flows through a 10 kΩ resistance and a 0.50 μF capacitor in series. The rms potential difference across the capacitor is
   a. 4.8 V b. 15 V
   c. 30 V d. 34 V
2. In a LCR series circuit, the a.c. voltages across R, L and C come out as 10 V, 10 V and 20 V respectively. The voltage across the entire combination will be
   a. 30 V b. $10\sqrt{3}$ V
   c. 20 V d. $10\sqrt{2}$ V
3. In a series resonant circuit, the a.c. voltages across resistance R, inductance L and capacitance are 5 V, 10 V and 10 V respectively. The a.c. voltage applied to the circuit will be
   a. 25 V b. 20 V
   c. 10 V d. 5 V
4. The time taken by a.c. of 50 Hz in reaching from zero to the maximum value is
   a. 50×10^{-3} s b. 5×10^{-3} s
   c. 1×10^{-2} s d. 2×10^{-2} s
5. In an LCR series a.c. circuit, the voltage across each of the components L, C and R is 50 V. The voltage across the LC combination will be
   a. 50 V b. $50\sqrt{2}$ V
   c. 100 V d. 0 V (zero)
6. In a LCR circuit, capacitance is changed from C to 2C. For the resonant frequency to remain unchanged, the inductance should be changed from L to
   a. 4 L b. 2 L
   c. L/2 d. L/4

7. In a.c. voltage V is applied across a series combination of R, L and C. If V_{RL}, V_{LC} and V_{RC} be the voltage drops across resistor and inductor and capacitor and resistor and capacitor respectively, then

a. $V_{RL} < V$ b. $V_{RC} < V$
c. $V_{LC} < V$ d. $V_{LC} = V$

8. A circuit has a resistance of 12 ohm and an impedance of 15 ohm. The power factor of the circuit will be

a. 1.25 b. 0.125
c. 0.8 d. 0.4

9. The self-inductance of the motor of an electric fan is 10 H. In order to impart maximum power at 50 Hz, it should be connected to a capacitance of

a. 1 μF b. 2 μF
c. 4 μF d. 8 μF

10. Antenna is

a. Inductive b. Capacitive
c. Resistive above its resonant frequency d. Resistive at resonant frequency

11. In an a.c. circuit, the emf (e) and the current (i) at any instant are given respectively by:

$e = E_o \sin \omega t$

$i = I_o \sin (\omega t - \phi)$

The average power in the circuit over one cycle of a.c. is:

a. $E_o I_o$ b. $\frac{E_o I_o}{2}$
c. $\frac{E_o I_o}{2} \sin \phi$ d. $\frac{E_o I_o}{2} \cos \phi$

12. A resistor and a capacitor are connected in series with an a.c. source. If the potential drop across the capacitor is 5 V and that across resistor is 12 V, the applied voltage is

a. 13 V b. 17 V
c. 5 V d. 12 V

13. A charged capacitor C = 30 μF is connected to an inductor L = 27 mH. The angular frequency of their oscillations is

a. 9.1×10^3 rad s^{-1} b. 3.0×10^3 rad s^{-1}
c. 1.1×10^3 rad s^{-1} d. 0.3×10^3 rad s^{-1}

Answers

1. (c) 2. (d) 3. (d) 4. (b) 5. (d) 6. (c) 7. (d) 8. (c) 9. (a) 10. (d) 11. (d) 12. (a) 13. (c)

Hints to Challenging Problems

HINT: 1

Given,

Inductance (L) = 5 H

Voltage amplitude (V_o) = 60 V

a. Current amplitude (I_o) = ?

$\omega = 100$ rad/s

We know that

$V_o = I_o X_L$

or $I_o = \frac{V_o}{X_L} = \frac{V_o}{\omega L}$

b. For ω = 1000 rad/s, I_o = ?

$\because \; I_o = \frac{V_o}{X_L} = \frac{V_o}{\omega L}$

c. For ω = 10000 rad/s, I_o = ?

$\because \; I_o = \frac{V_o}{X_L} = \frac{V_o}{\omega L}$

HINT: 2

a. Given,

Inductance (L) = 3 H

Frequency (f) = 80 Hz

Inductive reactance (X_L) = ?

We know that

$X_L = \omega L$

or $X_L = 2\pi f L$

b. Given,

Inductive reactance (X_L) = 120 Ω

Frequency (f) = 80 Hz

Inductance (L) = ?

We know that

$X_L = \omega L$

or $X_L = 2\pi f L$

or $L = \frac{X_L}{2\pi f}$

c. Given,

Capacitance (C) = 4 μF = 4×10^{-6} F

Frequency (f) = 80 Hz

Capacitive reactance (X_C) = ?

We know that

$X_C = \frac{1}{\omega C}$

or $X_C = \frac{1}{2\pi f C}$

d. Given,

Capacitive reactance (X_C) = 120 Ω

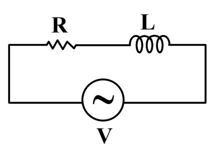
Frequency (f) = 80.0 Hz

Capacitance (C) = ?

We know that

$X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$

or $C = \frac{1}{2\pi f X_C}$



HINT: 3

Given,

Voltage amplitude (V_o) = 170 V

f = 60 Hz

Current amplitude (I_o) = 0.85 A

Capacitance (C) = ?

We can write,

$V_o = I_o X_C = I_o \frac{1}{\omega C}$

or $C = \frac{I_o}{2\pi f V_o}$

HINT: 4

Given,

Inductance (L) = 0.450 mH = 0.450×10^{-3} H

Current (I) = 2.60 mA = 2.60×10^{-3} A

Voltage (V) = 12.0 V

Frequency (f) = ?

∵ $V = I X_L$

or $V = I (2\pi f L)$

or $f = \frac{V}{2\pi I L}$

HINT: 5

Given,

R = 250 Ω

C = 4.80 μ F = 4.80×10^{-6} F

$V_C = 7.60 \sin(120\, t)$ . . . (i)

a. Capacitive reactance, X_C = ?

The a.c. voltage is given by

$V_C = V_o \sin \omega t$ . . . (ii)

comparing (i) and (ii), we get

ω = 120 rad/s

Now, we know that

$X_C = \frac{1}{\omega C}$

b. Expression of voltage across resistor, V_R = ?

We know that

$V_R = I R = \frac{V_c}{X_c} R$

HINT: 6

Given,

Resistance (R) = 200 Ω

Inductance (L) = 0.400 H

Capacitance, C = 6 μ F = 6×10^{-6} F

Voltage amplitude (V_o) = 30.0 V

Angular frequency (ω) = 250 rad s^{-1}

a. Impedance of the circuit, Z = ?

For R – L series circuit, impedance is given by

$Z = \sqrt{R^2 + X_L^2}$

$= \sqrt{R^2 + (\omega L)^2}$

b. Current amplitude, I_o = ?

We know that

$I_o = \frac{V_o}{Z}$

c. Voltage across resister, V_R = ?

Voltage across inductor, V_L = ?

∵ $V_R = IR$

Also,

$V_L = I X_L = I \omega$

d. Phase angle, θ = ?

We know that

$\tan \theta = \frac{X_L}{R}$

Since θ is positive so the source voltage leads the current.

HINT: 7

Given,

$L = 20 \text{ mH} = 20 \times 10^{-3} \text{ H}$

$C = 140 \text{ nF} = 140 \times 10^{-9} \text{ F}$

$R = 350\ \Omega$

$V_{rms} = 120 \text{ V}$

$f = 1.25 \text{ k Hz} = 1.25 \times 10^3 \text{ Hz}$

a. Power supplied by generator, $P_{av} = ?$

Average power supplied by source,

$$P_{av} = V_{rms} I_{rms} \cos\theta$$

$$= V_{rms} \times \frac{V_{rms}}{Z} \cos\theta$$

$$= \frac{V_{rms}^2}{Z} \cos\theta \qquad \ldots \text{(i)}$$

$$\because \quad \tan\theta = \frac{X_L - X_c}{R}$$

But $X_L = L\omega = L \times 2\pi f$

Also,

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

So,

$$\tan\theta = \frac{157 - 909}{350}$$

or $\theta = \tan^{-1}(2.15)$

$\therefore \theta = -65.04°$

The impedance of the circuits is given by

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

From (i), we have

$$P_{av} = \frac{V^2}{Z} \cos\theta$$

b. Average power dissipated by the resister, $P_R = ?$

$$\because \quad P_R = I_{rms}^2 R = \left(\frac{V_{rms}}{Z}\right)^2 \times R$$

HINT: 8

Given,

Voltage amplitude (V) = 120 V

Angular frequency (ω) = 50.0 rad s^{-1}

Resistance (R) = 400 Ω

Inductance (L) = 9 H

a. Capacitance (C) = ?

For maximum current, we must have

$X_L = X_C$

$$\text{or} \quad \omega L = \frac{1}{\omega C}$$

$$\text{or} \quad C = \frac{1}{\omega^2 L}$$

b. Voltage amplitude across inductor, $V_L = ?$

We know that

$V_L = I X_L$

$$\therefore \quad V_L = \frac{V}{R} X_L = \frac{V}{R} \omega \times L$$

HINT: 9

Given,

Inductance, $L = 3$ H

Resistance of in coil, $r = 50\ \Omega$

Resistance of Resister, $R = 550\ \Omega$

Voltage of source, $V = 100$ V

$f = 50$ Hz

Total Resistance of the circuit, $R_1 = R + r$

$= 550 + 50 = 600\ \Omega$

Current in the coil, $I = ?$

Voltage across the coil, $V_L = ?$

We know that

$$I = \frac{V}{Z}$$

But total impedance of the circuit is given by

$$Z = \sqrt{R_1^2 + X_L^2} = \sqrt{R_1^2 + \left(\frac{1}{2\pi f L}\right)^2}$$

Hence,

$$I = \frac{V}{Z}$$

Now,

$\therefore \quad V_L = I X_L$

HINT: 10

Given,

$C = 10\ \mu\text{F} = 10 \times 10^{-6} \text{ F} = 10^{-5} \text{ F}$

$R = 1000\Omega$

$V_{rms} = 12$ V

$f = 50$ Hz

$I = ?, \qquad V_C = ?$

Impedance of the circuit

$$Z = \sqrt{X_C^2 + R^2}$$

$$= \sqrt{\left(\frac{1}{2\pi f C}\right)^2 + R^2}$$

Now,

$$\therefore \quad I = \frac{V_{rms}}{Z}$$

Also,

$$V_C = I X_C = I \times \frac{1}{2\pi f C}$$

HINT: 11

Given,

$R = 25\Omega \quad L = 30\text{mH} = 30 \times 10^{-3} \text{ H}$

$C = 10\ \mu\text{F} = 10 \times 10^{-6} \text{ F} \quad V_{rms} = 240$ V

$f = 50$ Hz, $\qquad I = ?$

Voltmeter reading across capacitor (V_C) = ?

Impedance of the circuit, $Z = \sqrt{(X_L - X_C)^2 + R^2}$

But $X_L = 2\pi fL = 9.42\Omega$

and $X_C = \frac{1}{2\pi fC}$

Current in the circuit, $I = \frac{V}{Z}$

Also, $V_C = I X_C$

HINT: 12

Given,

$V = 50$ volt,

$f = 50$ Hz,

$R = 40\ \Omega$, $L = 200$ mH $= 200 \times 10^{-3}$ H

$V_R = 20$ V

Resistance of solenoid, r = ?

If I be the currents in the circuit, we have

$$I = \frac{V_R}{R} = \frac{20}{40}$$

$\therefore\ I = 0.5$ A

Also,

$$I = \frac{V}{\{(R+r)^2 + X_L^2\}^{1/2}}$$

$$\text{or}\quad 0.5 = \frac{50}{\{(40 + r)^2 + (2\pi fL)^2\}^{1/2}}$$

HINT: 13

Given,

$L = 0.20$ H

$R = 50\ \Omega$

$I = 1$ A, $V = 240$ volt, $f = 50$ Hz

Here, the current is in phase with the potential difference and therefore we can have

$X_L = X_C$

Let R_1 be the required resistance while C be the required capacitance in series with inductor. So total resistance,

$R_2 = R + R_1$

Impedance in the circuit,

$Z = \sqrt{(R + R_l)^2 + (X_L - X_C)^2}$

$\therefore\ Z = R + R_1$

Now,

$$Z = \frac{V}{I}$$

$$\text{or}\quad R + R_1 = \frac{240}{1}$$

or $R_1 = 240 - 50$

$\therefore\ R_1 = 190\ \Omega$

Now,

$X_L = X_C$

$$\text{or}\quad 2\pi fL = \frac{1}{2\pi fC}$$

$$\text{or}\quad C = \frac{1}{4\pi^2 F^2 L}$$

HINT: 14

Given,

$C = 33$ pF $= 33 \times 10^{-12}$ F

$f = 810$ kHz $= 810 \times 10^3$ Hz

$$\therefore\ X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$$

$$= \frac{1}{2\pi \times 810 \times 10^3 \times 33 \times 10^{-12}} = 5957.2\ \Omega$$

If L is the required inductance then, we can write

$X_L = 2\pi fL$

$= 2\pi \times 810 \times 10^3$ L

$= 5086800L\ \Omega$

Now,

$X_L = X_C$

❖ ❖ ❖

ELECTRONS

19

CHAPTER

19.1 Introduction

In the quest to study the properties of matter, some of the natural observations led to the conclusion that electric charge is also a fundamental property as the mass. Every material particle that surrounds us has charge on them. But, this property is not visible in the bulk matter usually. This is because every matter consists of equal amount of positive and negative charges. So, they become electrically neutral but they are not chargeless. If we rub a plastic cover of our pen on our hair and take it near a piece of paper, the paper is attracted towards the pen. What makes the plastic to attract the piece of paper? The answer to this question is, the interaction between charges and is a proof for the existence of charge in any matter. There are two types of charges: positive charge and negative charge. Fundamentally, a positive charge is carried by proton and negative charge is carried by electron of an atom. Electron can be transferred from one body to another body easily because of its low binding capacity in an atom. Likewise, electron can be deposited on a matter and conduct through a conductor. Also the beam of electron can be produced experimentally.

19.2 Particle Nature of Electricity

Except hydrogen atom, all atoms contain three subatomic particles: proton, neutron, and electron. Protons and neutrons are relatively heavier than electron and they lie at the nucleus of an atom; the electron revolves around the nucleus. Proton is positive charge particle, neutron is chargeless and electron is negative charge particle. As the electron remains at the orbit, it is easier to remove from the atom. Moreover, in conductors, valence electrons are almost free from the nuclear attraction. If we rub a body, it gets charged. It means, the body either gains electrons or loses electrons. The body which loses the electrons possesses positive potential and the body which gains the electrons possesses the negative potential. If we connect two plates with different potentials using a conducting wire, the electric charge flows from higher potential to lower potential, it means the charge particles (always electrons) flow from negative plate to positive potential plate (conventionally, current flows from positive plate to negative plate). In reality, flow of charge depicts the flow of charge particles. Since the electron are fundamental particles and responsible to carry the charge, so the charge transfer is always quantized, i.e. $q = \pm ne$, where $e = 1.6 \times 10^{-19}$ C and n is positive integer.

Electricity is produced due to the flow of such charged particles. The charge particles possess a fixed value of charge, every electron possess charge of 1.6×10^{-19} C on magnitude. Since the electricity is the flow of such fundamental charged particles, it has a particle nature.

19.3 Millikan's Oil Drop Experiment

After the discovery of electron, investigating its properties became the very important aspect of physicist. One of the prominent physicist of those times, Robert Andrew Millikan conducted series of experiments principally based upon the Stoke's law of viscous force and finally came to a conclusion which provided basics for quantization of charge. This important discovery on this particle nature of electricity awarded him with the prestigious Nobel prize of 1923. In his experiment, a charged non-volatile oil drop was allowed to fall in a viscous medium (air) trapped in between two metal plates kept at a distance sufficient enough to make the oil drop achieve the terminal velocity. The oil drop has to be non-volatile and viscous, so that its geometry is preserved during its motion and the Stoke's law remains valid. This was also ensured by conducting the experiment under isothermal condition. The motion of the oil drop was studied under following two cases:

i. Motion under the effect of gravity alone.

ii. Motion under the combined effect of gravity and the electric field.

The experimental set up for the Millikan's oil drop experiment is as shown in Fig.19.1.

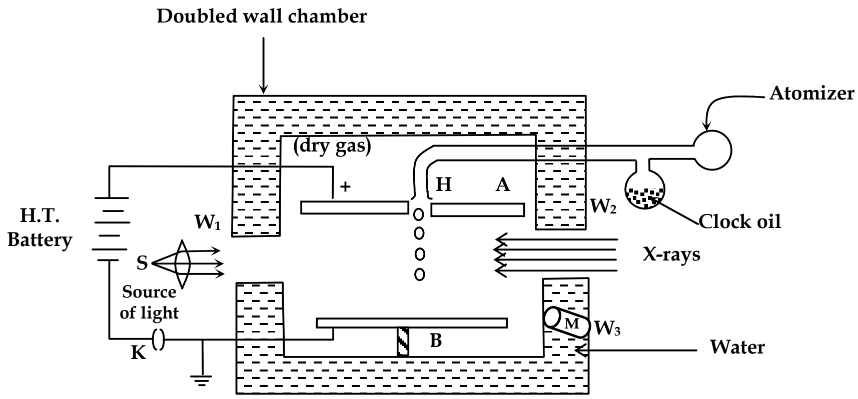


Fig. 19.1: Apparatus arrangement for Millikan's oil drop experiment

In his experiment, he used two metal plates placed parallelly and maintained at a high potential difference of nearly 10,000 V by connecting to high tension battery (HTB), so that the region between the plates is a region of uniform electric field. The upper plate has a hole H at its centre through which small charged oil drops are sprayed in the region between the plates. The oil drops are charged due to friction with air when squeezed through the atomizer. The whole arrangement is kept inside double walled jacket in which cold water keeps on circulating so that the internal temperature remains constant (isothermal condition). This jacket also helps to prevent the zigzag motion of oil drop which would otherwise arise due to the convection current of air set up by the external conditions.

The jacket is also provided with three openings W_1, W_2 and W_3 in the form of windows. One of the window say W_1 has an electric bulb to illuminate the region between the plates. The other window W_2 has a source of X-ray that may be used to charge the oil drop, if the friction with the air is not sufficient enough to charge it. The window W_3 has a travelling microscope fitted to it which would help to calculate the terminal velocity acquired by oil drop by measuring the distance travelled by it.

Experimental procedure and calculations

This experiment is done in two steps:

i. Motion under the effect of gravity alone

First of all, the charged oil drop is allowed to fall under the effect of gravity alone by switching off the electric field and its terminal velocity is calculated. Let ρ be the density of the oil drop and r be its radius. When the oil drop acquires the terminal velocity, the net force acting on it must be zero. The forces acting in the upward direction are the upthrust and viscous force and the force acting on the downward direction is its weight.

For, Net force = 0

Upthrust + viscous force – weight = 0

Upthrust + viscous force = weight . . . (19.1)

Here,

Upthrust (U) = Volume of air displaced × density of air × acceleration due to gravity

= Volume of oil drop × density of air × acceleration due to gravity

$= \frac{4}{3}\pi r^3 \sigma g$

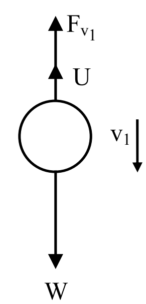
Viscous force (F_v) = $6\pi\eta r v_1$ where v_1 is the terminal velocity of oil drop

Weight (W) = Mass of oil drop × acceleration due to gravity

Here, volume of oil drop is equal to the volume of air displaced. So,

= Volume of oil drop × Density of oil drop × Acceleration due to gravity

$= \frac{4}{3}\pi r^3 \rho g$



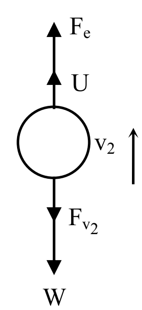
Thus, equation (19.1) can be written as,

$$\frac{4}{3}\pi r^3 \sigma g + 6\pi\eta r v_1 = \frac{4}{3}\pi r^3 \rho g$$

or, $$6\pi\eta r v_1 = \frac{4}{3}\pi r^3(\rho - \sigma)g \qquad \text{. . . (19.2)}$$

or, $$r = \sqrt{\frac{9}{2}\frac{\eta v_1}{(\rho - \sigma)g}} \qquad \text{. . . (19.3)}$$

ii. Motion under the combined effect of gravity and electric field



In the second case, the experiment is conducted by applying the electric field. An identical oil drop which is negatively charged is selected and its motion is studied under the combined effect of gravity and electric field.

Let q be the total charge on the oil drop and E be the strength of the electric field. If the strength of the electric field is strong enough to cause the oil drop to move in upward direction with a terminal velocity v_2,

Again, net force on oil drop = 0 . . . **(19.4)**

The forces in upward directions are upthrust and electric force. The viscous force acts in downward direction as the oil drop is moving in upward direction and the weight also acts in downward direction.

So, equation (19.4) can be written as,

Electrostatic force + upthrust – Viscous force – Weight = 0

$$\text{or,}\quad qE + \frac{4}{3}\pi r^3\sigma g - 6\pi\eta r v_1 - \frac{4}{3}\pi r^3\rho g = 0$$

$$\textbf{or,}\quad \mathbf{qE = \frac{4}{3}\pi r^3(\rho - \sigma)g + 6\pi\eta r v_1} \qquad \text{... (19.5)}$$

Using equation (19.2) in equation (19.5) we get,

$$\mathbf{q = \frac{6\pi\eta r\,(v_1 + v_2)}{E}} \qquad \text{... (19.6)}$$

Using the value of r from equation (19.3) in equation (19.6), we get,

$$\mathbf{q = \frac{6\pi\eta}{E}\sqrt{\frac{9\eta v_1}{2(\rho - \sigma)g}}\,(v_1 + v_2)} \qquad \text{... (19.7)}$$

If the oil drop moves in downward direction even after applying electric field (i.e. electric field is weak), then equation (19.7) becomes,

$$\mathbf{q = \frac{6\pi\eta}{E}\sqrt{\frac{9\eta v_1}{2(\rho - \sigma)g}}\,(v_1 - v_2)} \qquad \text{... (19.8)}$$

Equations (19.7) and (19.8) are the expression for total charge carried by an oil drop. Through the series of experiments conducted for the oil drop of different sizes, it was found that the value of charge q was an integral multiple of some small unit of charge equivalent to charge carried by an electron i.e. (1.6×10^{-19} C). This means, total charge q on the oil drop could be expressed as

$$q = ne \quad \text{where } n = 1, 2, 3, \ldots$$

Thus, Millikan came to a conclusion that, the charge on any object exists as a multiple of small units called quantum of charge. Each quantum of charge is equivalent to 1.6×10^{-19} C. This fact is known as quantization of charge. Before this experiment, the existence of subatomic particles was not universally accepted.

Note

If the oil drop remains stationary in region between the plates, then, viscous force will be zero. At this condition, net force on the drop is,

$$F_{net} = 0$$

$$\text{or,}\quad mg - qE = 0$$

$$\text{or,}\quad mg = qE$$

$$\text{or,}\quad mg = q\left(\frac{V}{d}\right)$$

Where, V is p.d. between two plates and d = separation between them.

19.4 Motion of Electron in Uniform Electric Field

Electrons are negative charge particles. They are deflected by electric field. Since they are negative charge particles, they are attracted towards the positively charged plate. So, we observe them travelling in the opposite direction of electric field. The nature of electrons in an electric field is explained below.

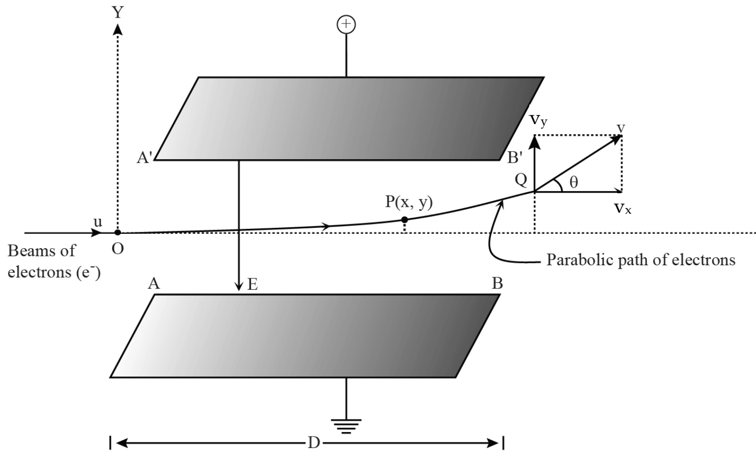


Fig. 19.2: Deflection of electron in electric field

Consider two parallel conducting plates in which upper plate is maintained at positive potential and lower plate is earthed so that a uniform electric field is produced between these plates. Let V be the potential difference between two plates and d be their distance of separation as shown in Fig. 19.2. A horizontal beam of electrons with initial velocity u is allowed to enter into the field, perpendicular with the direction of the field.

In such motion of electron, the velocity along horizontal direction is uniform, i.e., acceleration along x-direction, $a_x = 0$. For the electron of initial velocity u along horizontal direction, the equation of motion is written as,

$$x = ut + \frac{1}{2} \cdot 0 \cdot t^2$$

$$\mathbf{x = ut} \qquad \ldots\mathbf{(19.9)}$$

Where x is the horizontal distance travelled by the electron.

Also, the displacement of electron along y-direction is

$$y = 0 \cdot t + \frac{1}{2} at^2$$

$$\therefore \quad \mathbf{y = \frac{1}{2} at^2} \qquad \ldots\mathbf{(19.10)}$$

In the electric field, the electric field provides the mechanical force to deflect the electrons. So,

$$ma = eE$$

$$\mathbf{a = \frac{eE}{m}} \qquad \ldots\mathbf{(19.11)}$$

Where, a is the acceleration of electron in the field, m is the mass of electron and E is the electric field between two plates. Also,

$$\mathbf{E = \frac{V}{d}} \qquad \ldots\mathbf{(19.12)}$$

Where, V = potential difference between two plates

d = separation of two plates

From equations (19.11) and (19.12), we get,

$$\mathbf{a = \frac{eV}{md}} \quad \text{...(19.13)}$$

Now, using the equations (19.9) and (19.13) in equation (19.10), we get,

$$y = \frac{1}{2}\left(\frac{eV}{md}\right)\left(\frac{x}{u}\right)^2$$

$$\mathbf{y = \left(\frac{eV}{2mdu^2}\right)x^2} \quad \text{...(19.14)}$$

Equation (19.14) is the form of equation of parabola. Hence, we can conclude that, motion of electron in electric field is parabolic in nature.

We can also determine the angle of deflection of electrons in the electric field. To find the angle, components of velocity (v_x and v_y) are to be calculated,

Here, $v_x = u$

and

$$v_y = u_y + at$$

$$v_y = 0 + at$$

$$v_y = \frac{eV}{md} \cdot \frac{x}{u}$$

$$\therefore \quad v_y = \left(\frac{eV}{mdu}\right)x$$

$$\therefore \text{ The deflection, } \tan\theta = \frac{v_y}{v_x} = \left(\frac{eV}{mdu}\right)x\left(\frac{1}{u}\right)$$

$$\therefore \tan\theta = \left(\frac{eV}{mdu^2}\right)x$$

$$\therefore \quad \boldsymbol{\theta = \tan^{-1}\left(\frac{eV}{mdu^2}\right)x} \quad \text{...(19.15)}$$

Total velocity of electron at any point in the electric field,

$$v = \sqrt{v_x^2 + v_y^2}$$

$$\mathbf{= \sqrt{u^2 + \left(\frac{eVx}{mdu^2}\right)^2}} \quad \text{...(19.16)}$$

After crossing the electric field, the electron continues it's motion in a straight line path tangent to the parabola, because at that point the applied electric field terminates and thus, no force acts to change the velocity. Gain in Kinetic Energy is given by,

$$\Delta K.E. = (K.E.)_f - (K.E.)_i$$

$$= \frac{1}{2}m\left[u^2 + \left(\frac{eE\ D}{mu}\right)^2\right] - \frac{1}{2}mu^2$$

$$= \frac{1}{2}m\left(\frac{2E\ D}{mu}\right)^2$$

$$\mathbf{\Delta K.E. = \frac{2\ E^2\ D^2}{mu^2}} \quad \text{...(19.17)}$$

19.5 Motion of Electron in Uniform Magnetic Field

When a charged particle is in motion, magnetic field is induced around it. If electron moves in magnetic field, its induced magnetic field which interacts with the applied magnetic field and the electron deflects from its original path. The magnitude of magnetic force experienced by electron and its path of deflection can be explained at different angles of projection in the field.

i. When electron enters the field parallelly or antiparallelly, i.e. $\theta = 0^o$ or $\theta = 180^o$.

The magnitude of magnetic force is,

$$\mathbf{F = Bev \sin\theta} \quad \text{...(19.18)}$$

For $\theta = 0^o$ or 180^o, $\sin\theta = 0$

$$\therefore F = 0$$

The electron does not experience any magnetic force, if it enters into the field parallelly or anti-parallelly.

ii. When electron enters the field perpendicularly, i.e. $\theta = 90^o$, the magnitude of magnetic force is,

$$F = Bev \sin 90^o$$

$$\mathbf{F = Bev} \quad \text{...(19.19)}$$

The electron experiences maximum force in the magnetic field. Since the force is always perpendicular to velocity vector $\vec{v}$, it obeys the circular path as shown in Fig. 19.3. In this condition, the magnetic force provides the centripetal force to the electron,

$$\text{i.e.} \quad Bev = \frac{mv^2}{r}$$

$$\mathbf{r = \frac{mv}{Be}} \quad \text{...(19.20)}$$

Here, r is the radius of circular path followed by electron in magnetic field.

Also, from equation (19.20), we get,

$$\frac{v}{r} = \frac{Be}{m}$$

$$\omega = \frac{Be}{m}$$

Where, ω is angular velocity of electron.

$$\frac{2\pi}{T} = \frac{Be}{m}$$

$$\mathbf{T = \frac{2\pi m}{Be}} \quad \text{...(19.21)}$$

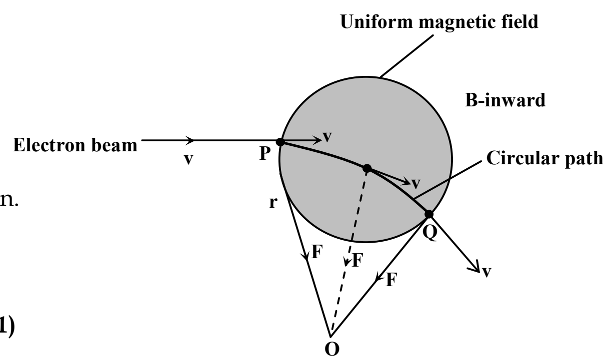


Fig. 19.3 : Circular motion in uniform magnetic field

Here, T is the time period of revolution of electron in the uniform magnetic field. The frequency of revolution of electron is, $f = \frac{1}{T} = \frac{Be}{2\pi m}$.

iii. **When electron enters at any oblique angle θ:** In this condition, the component of velocity which is parallel to field tends to move the electron in linear path, whereas the perpendicular component of velocity tends to move the electron in circular path. Due to the combined effect, electron obeys a helical path (i.e. spiral path) as shown in Fig. 19.4.

The initial velocity of electron in the magnetic field can be resolved into two components along x-axis (i..e v_x) and along y-axis (i.e. v_y). In the given condition, the component v_x (= v cos θ) tends to move the electron along the direction of magnetic field, whereas the component v_y (= v sin θ) perpendicular to the direction of magnetic field tends to move the electron in circular path. The combined effect makes the electrons to move in a helical path as shown in Fig. 19.4. Therefore, the centripetal force provided by the magnetic field perpendicular to the component v_y, we write,

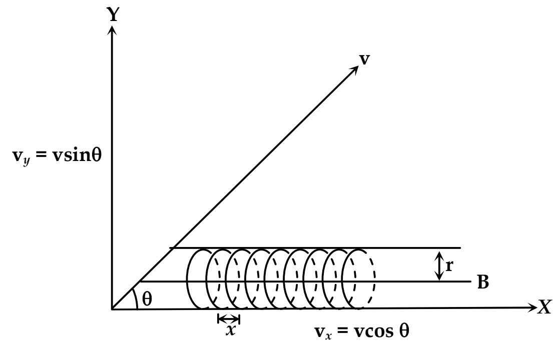


Fig. 19.4: Spiral motion of charged particle in uniform magnetic field

$$\frac{mv_y^2}{r} = Bev_y$$

$$\frac{mv_y}{r} = Be$$

$\therefore$ The radius (r) of the helical path is given by,

$$r = \frac{mv_y}{Be}$$

$$\textbf{or,} \quad \mathbf{r = \frac{mv \sin\theta}{Be}} \qquad \ldots (19.22)$$

Let T be the time period, then,

$$T = \frac{\text{circumference of the circle}}{\text{speed along circle}} = \frac{2\pi r}{v\sin\theta} = \frac{2\pi}{v\sin\theta} \times \frac{mv\sin\theta}{eB}, \text{ (using equation 19.22)}$$

$$\therefore \quad \mathbf{T = \frac{2\pi m}{e B}} \qquad \ldots (19.23)$$

Hence, it is clear that time period of revolution of electron is independent of speed of the particle as well as angle of projection but depends on m, B and e.

The linear distance travelled by the particle during this time period is called pitch of helix. Also, the pitch is defined as linear distance between two consecutive turns of a helical path. So,

Pitch (x) = horizontal velocity × time period (*T*)

$$= v\cos\theta \times \frac{2\pi m}{eB}, \text{ from (19.23)}$$

$$\therefore \mathbf{x = \frac{2\pi m v \cos\theta}{e B}} \qquad \ldots (19.24)$$

This principle is used in focusing a beam of charged particles in electron microscope or TV picture tube.

The above relations are true for any charged particle whether positive or negative. For more general case, if a particle containing charge q is projected at same angle into the uniform magnetic field, the radius of path, time period of revolution and pitch of its path are written as,

$$r = \frac{mv\sin\theta}{Bq}$$

$$T = \frac{2\pi m}{qB}$$

$$x = \frac{2\pi m v\cos\theta}{qB}$$

Cross Fields

A region of uniform electric field and magnetic field applied simultaneously perpendicular to each other such that a charge particle entering normally into this region passes undeviated is called cross-field.

In such fields, the deviation, produced on the charge particle due to magnetic field is equal to that by electric field. This is possible when, electrostatic force = magnetic force

i.e. $qE = Bqv$

or, $$v = \frac{E}{B} \quad \text{...(19.25)}$$

So, on such fields, the velocity of particle is equal to the ratio of electric field to magnetic field.

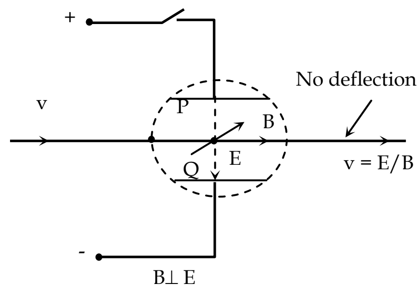


Fig. 19.5: Cross-fields

19.6 Specific Charge of Electron

Electron is a negative charge particle. Its charge is considered as the fundamental charge of nature. The charge of an electron was first measured by R.A. Millikan in 1909 from his famous Millikan's oil drop experiment and he was awarded with Nobel Prize in 1923. After the discovery of charge of electron, many queries about the nature and properties of charge were solved. This experiment proved the concept of quantization of charge. Millikan found the charge of an electron to be -1.6×10^{-19} C. But, the mass of electron could not be measured by any experiment. However, it was possible by employing result of already designed experiment of J.J. Thomson that had determined the charge to mass ratio of electron, which finally paved the way for determining the mass of electron from calculation.

The charge to mass ratio of an electron is called the specific charge of the electron. Similarly, the charge to mass ratio of a proton is called specific charge of the proton. According to J.J. Thomson's experiment, the specific charge of an electron is,

$$\frac{e}{m} = 1.76 \times 10^{11} \text{ C/kg}$$

From Millikan's oil drop experiment,

$$e = 1.6 \times 10^{-19} \text{ C}$$

∴ Solving these, the mass of electron can be determined to be, $m = 9.1 \times 10^{-31}$ kg.

19.7 Determination of Specific Charge (e/m) of an Electron by J.J. Thomson's Experiment

The ratio of charge and mass of a charged particle is called its specific charge. In 1897, Sir J.J. Thomson devised an experiment for measuring the ratio of charge (e) and mass (m) for an electron. This experiment is based on principle of cross fields. The simplified form of the apparatus for determining the specific charge of an electron is as shown in the Fig. 19.6.

The experiment set up consists of an evacuated glass tube provided with a cylindrical anode (A) with a hole (H) at its centre and a cathode (C) in the form of filament which are maintained at a potential difference of V volts. The electrons are emitted from filament cathode when suitable current is passed which then accelerate towards the anode and pass through hole 'H' provided to it. The electron beam then travels straight along the horizontal axis of the, tube and strikes at the middle of the fluorescent screen at the other end of tube at point O (say).

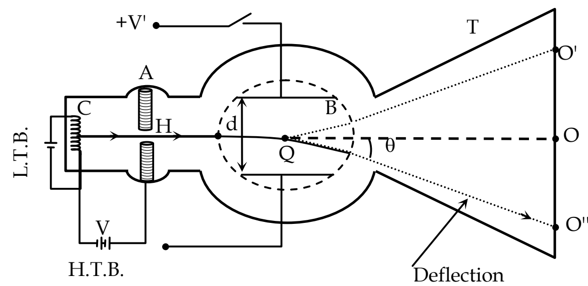


Fig. 19.6: Thomson's apparatus for determination of e/m for electron

A uniform magnetic field is applied to the beam of electrons by using Helmholtz coil as shown in Fig. 19.6 (shown by dotted lines). The magnetic field is into the plane of paper and perpendicular to it. As soon as the electron beam enters the region of uniform magnetic field, a magnetic force (F_m) acts in the downward direction as decided by Fleming's left hand rule. The beam is then deflected downwards and strikes the fluorescent screen at point O' (say). If v be the velocity of electron with which it enters the field region then, the magnetic force experienced by it is given by,

$$\mathbf{F_m = Bev} \qquad \ldots\textbf{(19.26)}$$

The magnetic field is then switched off and a uniform electric field (E) is applied to the electron beam by using two parallel metal plates maintained at a potential difference of V' volts and separated by a distance d. The electric field is parallel to the plane of paper and upward. The beam of electron entering this region of electric field experiences force (F_e) in upward direction and hence is deflected downward. Let the beam strikes the fluorescent screen at O''. The magnitude of electric force is given by,

$$\mathbf{F_e = eE = \frac{eV'}{d}} \qquad \ldots\textbf{(19.27)}$$

Finally, electric and magnetic fields are applied simultaneously perpendicular to each other and the fields are so adjusted that, the electron beam is neither deflected upward nor downward. Rather it passes straight along the axis of tube and strikes at point O again, where it had striked in the absence of both fields. In this condition,

$$F_m = F_e$$

or, $$Bev = \frac{e\,V'}{d} \qquad \text{[Using equations (19.26) and (19.27)]}$$

or, $$\mathbf{v = \frac{V'}{Bd}} \qquad \ldots\textbf{(19.28)}$$

In this way, Thomson made the cross fields arrangements for his experimental study.

When the electron is accelerated between the potential difference of electrodes C and A, it gains kinetic energy given by,

$$K.E = eV$$

or, $$\frac{1}{2}mv^2 = eV$$

$$\mathbf{\frac{e}{m} = \frac{v^2}{2V}} \qquad \ldots\textbf{(19.29)}$$

Further, from equation (19.28), we get,

$$\mathbf{\frac{e}{m} = \frac{V'^2}{2B^2d^2\,V}} \qquad \ldots\textbf{(19.30)}$$

Thus, by knowing the values of V', B, V and d, we can calculate the value of $\frac{e}{m}$. The value of $\frac{e}{m}$ for electron calculated by J. J Thompson was 1.76×10^{11} C/kg.

The ratio $\left(\frac{e}{m}\right)$ is of fundamental importance for the determination of mass of electron as there are no other known experiments devised for accurate measurement of its mass. However, the charge on electron is determined by famous Millikan's oil drop experiment. Moreover, the constant e and m appear combinely in almost all equations and hence for the mathematical simplicity, we use the value combinely as ratio $\left(\frac{e}{m}\right)$.

19.8 Conduction Through Gases

The flow of charged particles in a material medium is known as electric conduction. Atmospheric gas is electrically neutral. Although, the gas contains many positive ions and free electrons, they move randomly and recombine frequently. So, they do not produce any specific pattern of flow of charge particles. If strong electric field is applied using electrodes of high potential difference in presence of low pressure of gas, positive ions are drawn towards the negative terminals and the negative charge particles are drawn towards the positive terminals. This movement of charge particles produces a current through the gas, which is known as ionizing current. *The passage of current so produced in presence of high potential difference and low pressure is called the electric discharge through a gas.* Depending upon several factors, the discharge may radiate visible light.

Electric discharge phenomena can be observed in a special type of device called the discharge tube. Discharge tube consists of a glass tube with two electrodes maintained at high potential difference (10 kV to 15 kV) at two end of the tube of length about 30 cm to 50 cm and diameter 3 cm to 5 cm as shown in *Fig. 19.7*. It is also provided with a vacuum pump to vary the pressure of gas inside it and a pressure gauge to measure the pressure in it. When the pressure of gas is reduced to 10 mm of Hg, discharging appears in the form of light (glow).

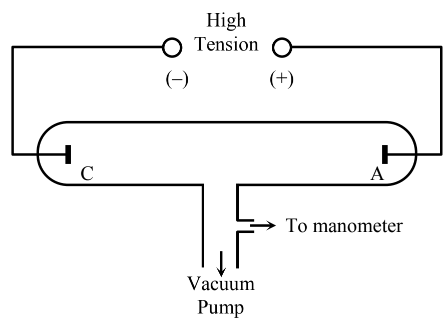
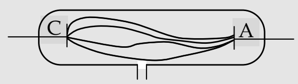
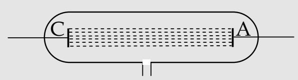
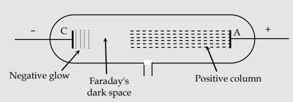
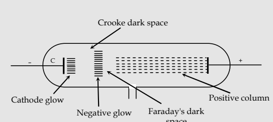
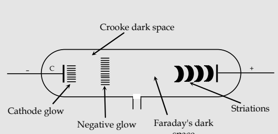
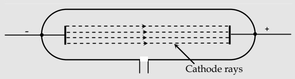


Fig. 19.7: Discharge tube

At low pressure and high voltage, the ions in the gas collide with the other gas molecules in the discharge tube. Due to the collision, the electrons are excited to higher energy state, but the excited electrons de-excite in very short interval of time emitting the visible light.

The patterns of visibility in the tube are different at different pressures. At low pressure, dark spaces may be produced into the tube. They are called Faraday's dark space. Actually, such types of dark spaces are produced when discharging electrons travel through the positive ion clouds. In the positive ion clouds, the speed of moving electrons drastically decrease and can not excite the gas molecules. Hence, the dark spaces are formed at these regions of tube. Furthermore, no discharging phenomenon occurs at very low pressure (10^{-4} mm of Hg), since there is very less number of gas molecules to be excited.

Pressure	Figure	Observations	Conclusion
Above 100 mm of Hg		No discharge takes place	Atoms are not still excited
At 10 mm of Hg		Irregular thin lines travel from cathode plate to anode. These lines are called streamers. Sparks and crackling noise are produced.	Discharging phenomena starts into the tube. The motion of electrons and ions are zig-zag.
At 5 mm of Hg		The discharge fills the tube. Regular lines travel from cathode plate to anode plate. This is called Geissler discharge.	Discharging takes place almost uniformly throughout the tube depends on nature of gas used in the tube. For example air gives red, hydrogen gives blue etc.
At 2 mm of Hg		The regular discharge leaves the cathode plate and a new glow appears on cathode plate. The new glow is called negative glow and linear glow towards anode plate is called positive column. A dark space appears between negative glow and positive column which is called Faraday's dark space.	A cloud of positive ions appears near the cathode plate. The electrons ejected from cathode plate accelerate towards the ion cloud so that gas atoms excite and negative glow appear. In the cloud, ejected electrons suddenly loss kinetic energy due to the attraction to positive ions. So, electrons lose their ionizing capacity. So, Faraday's dark space appears. After crossing the ion cloud, these electrons accelerate towards the anode plate. So, positive column appears.
At 1 mm of Hg		Negative glow shifts slightly away from the surface of cathode plate and a new glow appears on the plate. This new glow is called cathode glow. A dark space appears between cathode glow and negative glow, that is called the Crookes dark space.	A few clouds of positive ions travel towards the cathode plate. The sizes of clouds are different.
At 0.1 mm of Hg		Faraday's dark space becomes larger in size. The positive column is broken into equally spaced alternate bright and dark bands. These bands are called striations.	Hence, the width of Crookes dark space, Faraday's dark space and dark bands in striations are different.

About 10^{-2} mm of Hg		The Crookes dark space fills the tube. Faraday's dark space and all glow disappears. The inner discharge tube around the anode plate glows.	The glow on the wall of tube predicts that there must be the invisible rays travelling from cathode plate to anode plate. These invisible rays are called cathode rays because of their origination from cathode plate.
At 10^{-4} mm of Hg		Discharging phenomenon stops.	No particles are sufficient for discharging.

19.9 Discharging Mechanism

The discharging phenomenon is a continuous conduction of electricity through gas by the formation and movement of charge carriers such as electrons and ions due to an applied electric field. When an orbital electron gains energy by means of radiation or collision by other particles, it jumps to the higher energy states (upper electronic orbits). If the absorbed energy is sufficiently high to remove the electron, the atom gets ionized. The atom which losses electron from its orbit is called positive ion. If the atom gains excess electrons, it becomes negative ion. The detached electron from the atom becomes free to travel in space. This electron is called free electron. If the absorbed energy is not sufficient to ionize the atom, the electron may jump to the higher energy state. This phenomenon is called excitation. The excited electron remains at higher energy state for very short duration, then returns to lower energy state by emitting electromagnetic radiation to the surrounding. This reverse phenomenon of excitation is called de-excitation.

In de-excitation process, an electron (i.e. by the atom) loses energy in the form of electromagnetic radiation of certain wavelength. If the wavelength of radiation so emitted lies within the visible range (i.e. 400 nm to 700 nm), our eyes can detect them. The discharging mechanism in gases relies on both excitation and ionization of gas molecules.

Formation of ions and free electrons

Many gas molecules are originally ionized due to the absorption of cosmic rays and collision with neighbouring molecules. When an electric field is applied, the positive ions move along the direction of electric field and the free electrons move opposite of the electric field. The electrons being light and is swept away very fast by an electric field as compared to the slow moving, heavy positive ions. These moving ions and electrons collide to other molecules of gases and further ionization and excitation takes place. Thus, large number of ions and free electrons are produced in gases.

Liberation of electrons from Cathode plate

The positive ions gain kinetic energy from electric field. Then, they accelerate towards the cathode plate and finally strike on its surface. The collision of positive ions on the cathode plate liberates more electrons from its surface. These liberated electrons are accelerated away from the cathode and ionize the gas further by collision.

Formation of dark space

When the free electrons travel through the positive ions cloud, they get slowed down and hence lose their ionizing capacity. Thus, there is no emission of light for some distance beyond the cathode glow. This region appears dark.

At about 2 mm of Hg pressure of gas in discharge tube, an ion cloud is formed near the cathode plate, so only one dark space is formed. This dark space is called Faraday's dark place. If the pressure of gas is further reduced, many ion clouds are formed into the tube. Free electrons speed up between the ion clouds and slow down at the vicinity of cloud. Therefore, glows appear between the clouds and dark spaces appear in the cloud regions. About 1 mm of Hg, Faraday's dark space shifts away from the cathode plate and a new dark space appears in that place. This newly formed dark space is called Crookes dark space. The successive process of ionizing, losing ionizing power, accelerating for some distance and again ionizing, occurs repeatedly inside the tube till the discharged electrons reach the anode plate. Thus, many alternate dark and bright bands are formed, which are called the striations.

Cathode rays and termination of discharging

As the pressure of gas is gradually lowered from 1 mm of Hg, the mean free path of particles increases and hence the length of Crookes dark space increases. At about 10^{-2} mm of Hg, the mean free path becomes larger than the length of discharge tube. Then, the Crookes dark space fills the entire tube, only the electrons (cathode rays) coming out of the cathode strike the wall of the tube and this impact causes fluorescence. If the pressure is still reduced, the ions are insufficient to emit electrons from cathode plate. So, the current through the gas gradually decreases and finally, the discharging is terminated.

19.10 Cathode Rays and Their Production

The flow of charge particles can be detected experimentally through a special tube, called discharge tube. This tube is an evacuated tube containing positive and negative electrodes at two sides and low pressure of gas is maintained in it. A high tension battery is connected across two electrodes of the tube. At low pressure of gas 10^{-2} mm of Hg and an applied potential difference of 10 kV to 15 kV, charge particles flows from cathode plate to anode plate. This phenomenon of flow of charge particles from cathode plate to anode plate is known as discharging and the rays of particles so produced are known as cathode rays.

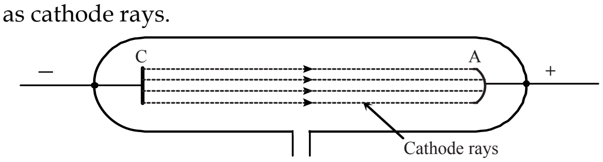


Fig. 19.8: Cathode rays

The invisible rays emitted from the cathode plate of a discharge tube when pressure is maintained about 10^{-2} mm of Hg under the high potential difference 10 kV to 15 kV are called cathode rays. Actually, the cathode streams are the rays of electrons. They are independent of nature of gas filled in the tube.

Properties of Cathode Rays

i. Cathode rays travel in straight lines and cast shadow of obstacles placed in their paths. When an object is placed in the path of cathode rays inside the tube, it casts a shadow on the wall opposite to the cathode.

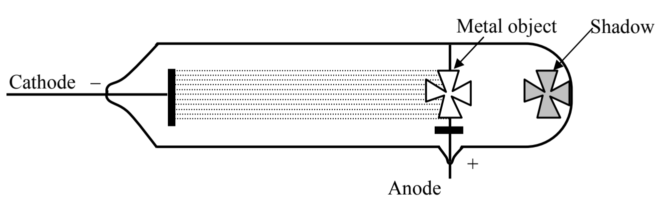


Fig. 19.9: Shadow produced by cathode rays

ii. They produce fluorescence when they strike the glass wall of the discharge tube.

iii. Each particle of cathode rays carries negative charge whose charge is equal to charge of an electron. Hence, they are the stream of electrons.

iv. Cathode rays are deflected by magnetic and electric fields. This property is the good evidence that the cathode is possess mass.

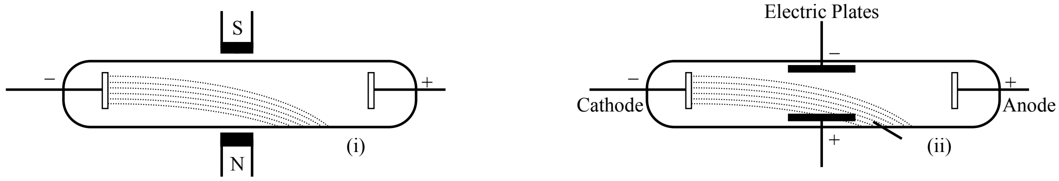


Fig. 19.10: (i) Deflection of cathode rays in magnetic field (ii) Cathode rays deflected towards positive plate

v. They ionize the gas and make them conducting which is the electrical effect of the rays.

vi. They affect the photographic plate.

vii. Cathode rays can travel with a speed nearly equal to the speed of light (about 90% of speed of light).

viii. The energetic cathode ray can produce X-rays when they strike a metallic target like tungsten, platinum etc. in a vacuum.

ix. Their penetrating power is low, they can however penetrate very thin sheets of paper.

x. They produce heat when they fall upon matter. This is because the kinetic energy of cathode rays is converted into heat.

*x*i. They possess mechanical energy. When the cathode rays impinge upon light mica plates fitted upon a wheel, the wheel begins to rotate proving that the rays possess mechanical energy. This also indicates that the cathode rays produces the mechanical pressure.

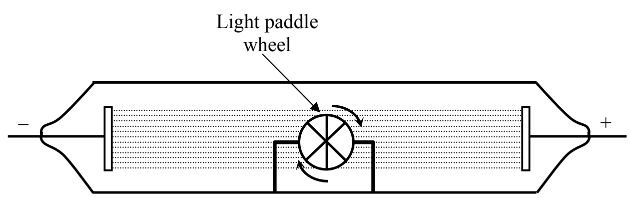


Fig. 19.11: Mechanical pressure exerted by cathode rays

*x*ii. Production of positive rays is always associated with production of cathode rays.

*x*iii. The characteristics of cathode rays do not depend upon the nature of electrodes and the nature of gas present in the cathode rays.

Canal Rays or Positive Rays

At about 1 mm of Hg, many positive ion clouds are formed. If holes are drilled at the cathode plates, the streams of faint luminous glow appear in these holes. This evidence shows that there must be some particles coming out through the holes. Obviously, the particles are attracted towards the cathode plate should have positive charge in them. These streams of positive ions are called canal rays or positive rays. Canal rays are deflected by electric and magnetic field.

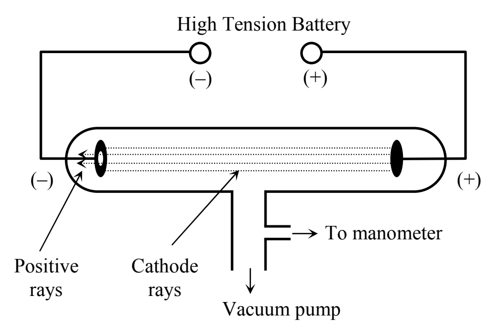


Fig. 19.12: Production of positive rays

Tips for MCQs

1. About Electrons

i. The charge of an electron, $e = 1.6 \times 10^{-19}$ C, negative nature and its mass, $m_e = 9.1 \times 10^{-31}$ kg.

ii. Its specific charge $\left(\frac{e}{m}\right) = 1.76 \times 10^{11}$ C / kg.

iii. It is the fundamental particle in nature and its spin is $\frac{1}{2}$.

iv. It is deflected by electric and magnetic fields.

v. Its rest mass is 0.51 MeV.

vi. It was identified by J.J. Thomson, in 1897.

vii. Its charge was measured by American Physicist Robert Millikan.

2. Electron in uniform electric field

i. It moves against the direction of electric field.

ii. Its path in electric field is parabolic in nature, and the equation of path, $y = \left(\frac{eV}{2mdu^2}\right)x^2$.

iii. Change of kinetic energy of electron, $\Delta E_k = \frac{2E^2D^2}{mu^2}$.

iv. The velocity, momentum and kinetic energy are changed in electric field.

v. Acceleration produced is, $a = \frac{F}{m} = \frac{eE}{m}$.

3. Electron in uniform magnetic field

i. When electron is initially moving parallel to the magnetic field, $\theta = 0$, $F = Bev \sin 0 = 0$, then the velocity, momentum and kinetic energy remains constant.

ii. If an electron incident normally in the magnetic field, $\theta = 90^o$, $F = Bev$. Since the force is always perpendicular to velocity vector $\vec{v}$, it moves in circular path. In such condition the electron experiences maximum force.

a. The radius of circular path, $r = \frac{mv}{Be} = \frac{p}{Be} = \frac{\sqrt{2mE_k}}{Be}$

b. The time period of revolution, $T = \frac{2\pi m}{Be}$

c. The frequency of revolution, $f = \frac{1}{T} = \frac{Be}{2\pi m}$

d. The angular frequency, $\omega = 2\pi f = \frac{Be}{m}$

iii. If an electron enters into the magnetic field, with certain oblique angle, θ the path is helical / spiral.

a. Helical path is the superposition of linear motion and circular motion.

b. The radius of a circle, $r = \frac{mv \sin \theta}{Be}$

c. Time period of revolution, $T = \frac{2\pi m}{Be}$

d. Frequency of revolution, $f = \frac{Be}{2\pi m}$

e. Angular frequency, $\omega = \frac{Be}{m}$

f. The pitch of helical path, $= v\cos\theta \times T = \dfrac{2\pi mv\cos\theta}{Be} = \dfrac{2\pi r}{\tan\theta}$

4. **The speed of electron in cross field, $v = \dfrac{E}{B}$**

5. **Millikan's oil drop experiment**
   i. Proves the quantum nature of charge, $q = ne$
   ii. The charge of electron is determined from, $q = \dfrac{6\pi\eta d}{V}\sqrt{\dfrac{9\eta v_1}{2(\rho - \sigma)g}}(v_1 + v_2)$ and $q = ne$

6. **About Cathode rays**
   i. Beam of electrons, which are deflected with electric and magnetic fields.
   ii. They are discovered by Sir William Crookes.
   iii. Cathode rays are produced from Cathode plate of discharge tube, but positive rays are produced due to the ionization of gas molecules but not from anode plate.
   iv. They can be produced about 0.01 mm of Hg gas pressure in discharge tube.
   v. Discharging phenomena occur in low pressure and high potential difference between electrodes. It is impossible at very high and very low pressure of gas.

Worked Out Problems

1. An electron has velocity of 4×10^6 ms^{-1} and moves in a circular orbit in a magnetic field of flux density 0.4 Tesla. What will be the radius of the orbit? Given $e = 1.6 \times 10^{-19}$ C and $m_e = 9 \times 10^{-31}$ kg.

SOLUTION

Given,

Initial velocity (v) = 4×10^6 ms^{-1}

Magnetic field (B) = 0.4 T

Electronic charge (e) = 1.6×10^{-19} C

Mass of electron (m_e) = 9×10^{-31} kg

Radius of orbit (r) = ?

We know, $r = \dfrac{m_e v}{Be}$

$$= \frac{9 \times 10^{-31} \times 4 \times 10^6}{0.4 \times 1.6 \times 10^{-19}}$$

$$= \frac{3.6 \times 10^{-24}}{0.64 \times 10^{-19}}$$

$$= 5.6 \times 10^{-5}\ \text{m}$$

∴ The radius of orbit is 5.6×10^{-5} m.

2. An electron having 450 eV of energy moves at right angles to a uniform magnetic field of flux density 1.50×10^{-3} T. Find the radius of its circular orbit. Assume that the specific charge, $e/m = 1.76 \times 10^{11}$ Ckg^{-1}.

SOLUTION

Given,

Magnetic field (B) = 1.50×10^{-3} T

Kinetic energy, E_k = 450 eV

$$\frac{1}{2}mv^2 = 450\ \text{eV}$$

$$\frac{1}{2}mv^2 = 450 \times 1.6 \times 10^{-19}\ \text{J}$$

$$v^2 = \frac{2 \times 450\, e}{m} \quad (e = 1.6 \times 10^{-19}\ \text{C})$$

$$= 2 \times 450 \times (e/m)$$

$$= 2 \times 450 \times 1.76 \times 10^{11} = 1.584 \times 10^{14}$$

$$v = 1.26 \times 10^7\ \text{ms}^{-1}$$

Now, the radius of path, $r = \dfrac{mv}{Be}$

$$\text{or, } r = \frac{V}{B(e/m)}$$

$$= \frac{1.26 \times 10^7}{1.50 \times 10^{-3} \times 1.76 \times 10^{11}}$$

$$= \frac{1.26 \times 10^7}{2.64 \times 10^8}$$

$$= 4.77 \times 10^{-2}\ \text{m}$$

∴ The radius of circular orbit is 4.77×10^{-2} m.

3. Two plane metal plates 4 cm long are held horizontally 3 cm apart in a vacuum, one being vertically above the other. The upper plate is at a potential of 300 V and the lower is earthed. Electrons having a velocity of 10^7 m/s are injected horizontally midway between the plates and in a direction parallel to the 4 cm edge. Calculate the vertical deflection of the electron beam as it emerges from the plates.(e/m for electron = 1.8×10^{11} C kg^{-1})

SOLUTION

Given,

Separation between plates (d) = 3 cm = 0.03 m

P.d. between plates (V) = 300 V

Velocity of electron along X axis (v_x) = 10^7 ms^{-1}

Length of plate (D) = 4 cm = 0.04 m

$\frac{e}{m} = 1.8 \times 10^{11}$ C kg^{-1}

Electric field intensity (E) $= \frac{V}{d}$

$= \frac{300}{0.03} = 10^4$ V/m

Vertical deflection of electron (y) = ?

The vertical deflection of the beam of electron is given by,

$y = \frac{1}{2} at^2$

The vertical acceleration of the electron is given by,

$a = \frac{F}{m} = \frac{eE}{m} = \frac{e}{m} \times \frac{V}{d}$

$= 1.8 \times 10^{11} \times 10^4$

$\therefore$ a $= 1.8 \times 10^{15}$ m/s^2

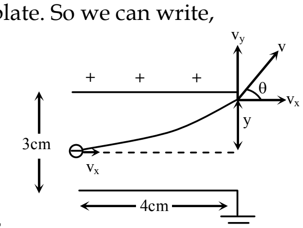
The time taken by an electron for the above deflection is same as the time taken to travel the length of the plate. So we can write,

$v_x = \frac{D}{t}$

$\therefore$ $t = \frac{D}{v_x}$

$= \frac{0.04}{10^7}$

$= 4 \times 10^{-9}$ s



Hence, from (i), we get $y = \frac{1}{2} at^2$

$= \frac{1}{2} \times 1.8 \times 10^{15} \times (4 \times 10^{-9})^2$

$\therefore$ y $= 1.44 \times 10^{-2}$ m

$\therefore$ The vertical deflection of the electron is 1.44×10^{-2} m.

4. An oil drop of mass 3.25×10^{-15} kg falls vertically with uniform velocity, through the air between vertical parallel plates which are 2 cm apart. When a p.d. of 1000 V is applied to the plates, the drop moves to the positively charged plate, being inclined at 45° to the vertical. Calculate the charge on the drop.

SOLUTION

Given,

Mass of oil drop (m) = 3.25×10^{-15} kg

Separation between plates (d) = 2 cm = 2×10^{-2} m

P.d. between plates (V) = 1000 V

Charge on the drop (q) = ?

Angle of inclination (θ) = 45°

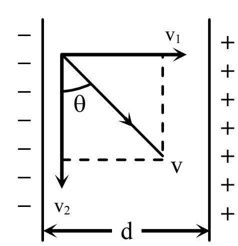
Let v be the terminal velocity of an oil drop moving towards positive plate making angle θ = 45° with the vertical. Let its horizontal component $v_1 = v \sin\theta$ and vertical component $v_2 = v\cos\theta$. Then, we can write,

$qE = 6\pi\eta r v_1$ **. . . (i)**

and $mg = 6\pi\eta r v_2$ **. . . (ii)**

Dividing (i) by (ii), we get,

$\frac{qE}{mg} = \frac{v_1}{v_2} = \frac{v\sin\theta}{v\cos\theta} = \tan\theta$



or $q = \tan\theta \times \frac{mg}{E}$

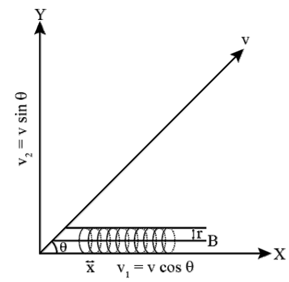
$= \tan 45° \times \frac{mg \times d}{V}$

$$= \frac{1 \times 3.25 \times 10^{-15} \times 9.8 \times 2 \times 10^{-2}}{1000}$$

$\therefore$ q $= 6.37 \times 10^{-19}$ C

5. An electron with a velocity of 10^7 ms^{-1} enters a region of uniform magnetic flux density of 0.10 T, the angle between the direction of the field and the initial path of the electron being 25°. By resolving the velocity of the electron find the axial distance between two turns of the helical path. Assume that the motion occurs in vacuum and illustrate the path with a diagram: (e/m = 1.8×10^{11} C kg^{-1})

SOLUTION



Given,

Velocity of an electron (v) = 10^{-7} m/s

Magnetic flux density (B) = 0.10 T

$\theta = 25°$,

Axial distance between two turns of helical path,

$x = ?$

$\frac{e}{m} = 1.8 \times 10^{11}$ Ckg^{-1}

$v_2 = v \sin\theta$

The electron does not feel any force due to component $v_1 = v \cos\theta$ along B, but it feels perpendicular force due to component $v_2 = v \sin\theta$ and therefore, electron travels in a circular path of radius r, where magnetic force provides necessary centripetal force i.e.,

$$Bev_2 = \frac{mv_2^2}{r}$$

$$\text{or, } Be = \frac{mv_2}{r} = \frac{m\omega r}{r} = m \times \frac{2\pi}{T}$$

$$\therefore \quad T = \frac{2\pi m}{Be}$$

The distance between two turns of helical path (x) is equal to the linear distance travelled in the same time period T. So, $v_1 = \frac{x}{T}$

or, $x = v_1 \times T$

$$= v\cos\theta \times \frac{2\pi m}{Be} = v\cos\theta \times \frac{2\pi}{B \times \frac{e}{m}}$$

$$= 10^7 \times \cos 25° \times \frac{2\pi}{0.1 \times 1.8 \times 10^{11}}$$

$\therefore$ $x = 3.16 \times 10^{-3}$ m

6. An electron having 500 eV energy enters at right angle to a uniform magnetic filed of 10^{-4} T. If its specific charge is 1.75×10^{11} Ckg^{-1}, calculate the radius of its circular orbit.

SOLUTION

Given,

Energy, $\frac{1}{2}mv^2 = eV = 500\text{ eV} = 500 \times 1.6 \times 10^{-19}$ J

or, $v = \sqrt{\frac{2 \times 500 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}} = 1.32 \times 10^7$ m/s

Magnetic field (B) = 10^{-4} T

Specific charge (e/m) = 1.75×10^{11} C kg^{-1}

Radius of orbit = ?

We have,

$$r = \frac{mv}{Be} = \frac{v}{B \cdot e/m}$$

$$= \frac{1.32 \times 10^7}{10^{-4} \times 1.75 \times 10^{11}} = 0.76 \text{ m}$$

7. [NEB 2074] An electron moves in a circular path of radius 20 cm in a uniform magnetic field of 2×10^{-3} T. Find the speed of electron and period of revolution. (Mass of electron = 9.1×10^{-31} kg)

SOLUTION

Given

Radius (r) = 20 cm = 0.20 m

Magnetic field (B) = 2×10^{-3} T

Mass of electron (m) = 9.1×10^{-31} kg

Speed of electron (v) = ?

Period (T) = ?

We have,

$$r = \frac{mv}{Be}$$

$$v = \frac{rBe}{m}$$

$$= \frac{0.20 \times 2 \times 10^{-3} \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}$$

$$v = 7.03 \times 10^{7}\ ms^{-1}$$

Also, period of revolution

$$T = \frac{2\pi m}{Be}$$

$$= \frac{2\pi \times 9.1 \times 10^{-31}}{2 \times 10^{-3} \times 1.6 \times 10^{-19}}$$

$$= 1.78 \times 10^{-8}\ s$$

$\therefore$ The period of revolution = 1.78×10^{-8} s.

8. [HSEB 2070] Calculate the radius of a water drop which would just remain suspended in an electric field of 300 V/cm and charged with one electron.

SOLUTION

Given,

Electric field (E) = 300 V/cm = 30000 V/m

Charge (q)= 1.6×10^{-19} C

Density of water (ρ) = 1000 kgm^{-3}

When the drop suspended in air, the upward electric force must balance the downward weight of the drop,

So,

$$mg = qE$$

$$\frac{4}{3}\pi r^3 \rho g = qE$$

$$r^3 = \frac{3qE}{4\pi\rho g}$$

$$= \frac{3 \times 1.6 \times 10^{-19} \times 30000}{4\pi \times 1000 \times 9.8}$$

$$r^3 = 1.17 \times 10^{-19}$$

$\therefore$ $r = 4.89 \times 10^{-7}$ m

$\therefore$ The radius of water drop is 4.89×10^{-7} m

9. [HSEB 2066] In Millikan-type apparatus, the horizontal plates are 1.5 cm apart. With the electric field switched off an oil drop is observed to fall with the steady velocity 2.5×10^{-2} cm/s. When the electric field is switched on the upper plate being positive, the drop just remains stationary when the p.d. between plates is 1500 V. (a) Calculate the radius of the drop (b) How many electronic charges does it carry? (Given, density of oil = 900 kgm^{-3} and viscosity of air = 1.8×10^{-5} Nsm^{-2}, Neglect air density)

SOLUTION

Given,

Distance (d) = 1.5 cm = 1.5×10^{-2} m

Terminal velocity (v) = 2.5×10^{-2} cm/s = 2.5×10^{-4} m/s

Potential difference (V) = 1500 V

Now, For (a): We have,

$$r = \sqrt{\frac{9}{2}\frac{\eta v}{(\rho - \sigma)g}} = \sqrt{\frac{9}{2} \times \frac{1.8 \times 10^{-5} \times 2.5 \times 10^{-4}}{(900 - 0) \times 9.8}}$$

$$= \sqrt{2.29 \times 10^{-3} \times 10^{-5} \times 10^{-4}} = \sqrt{2.29 \times 10^{-12}}$$

$$= 1.5 \times 10^{-6}\ m$$

Hence, the radius of the drop is 1.5×10^{-6} m.

Again,

For (b): we have,

$$E = \frac{V}{d} = \frac{1500}{1.5 \times 10^{-2}}$$

$$= \frac{1500 \times 10^{2}}{1.5} = 10^{5}\ V/m$$

Again, we have, $qE = mg = \frac{4}{3}\pi r^3 \rho g$

$$q = \frac{\frac{4}{3}\pi r^3 \rho g}{E}$$

$$= \frac{4 \times \pi \times (1.5 \times 10^{-6})^3 \times 900 \times 9.8}{3 \times 10^{5}}$$

$$= 127.23 \times 10^{-20}\ C$$

But, $q = ne$

or, $12.723 \times 10^{-19} = n \times 1.6 \times 10^{-19}$

$\therefore$ n = 8

Hence, the required number of electrons is 8.

10. [NEB 2075] An electron moving with a speed of 10^7 m/s is passed into a magnetic field of intensity 0.1 T normally. What is the radius of the path of the electron inside the field? If the strength of the magnetic field is doubled, what is the radius of the new path? (e/m = 1.8 × 10^{11} C/kg) [4]

SOLUTION

Given,

Speed of electron (v) = 10^7 m/s

Magnetic flux density (B) = 0.1 T

Specific charge of electron $\left(\frac{e}{m}\right)$ = 1.8 × 10^{11} C kg^{-1}

Radius of path (r) = ?

We have,

$$r = \frac{mv}{Be} = \frac{v}{B\,e/m}$$

$$= \frac{10^7}{0.1 \times 1.8 \times 10^{11}}$$

$$= 5.6 \times 10^{-4}\ \text{m}$$

Again on doubling magnetic field,

$$r' = \frac{mv}{B'e} = \frac{v}{B'\,e/m}$$

$$= \frac{10^7}{2 \times 0.1 \times 1.8 \times 10^{11}}$$

$$= 2.78 \times 10^{-4}\ \text{m}$$

Challenging Problems

1. A beam of electron is under potential difference of 1.36 × 10^4 V applied across two parallel plates 4 cm apart and a magnetic field 2 × 10^{-3} T at right to each other. If two fields produce no deflection in the electronic beam, calculate

i. the velocity of electrons

ii. the radius of the orbit in which the beam will move, if the electric field is made zero. [Given, mass of electron = 9.1 × 10^{-31} kg] [HSEB 2067]

Ans: (i) 1.7 × 10^8 m/s (ii) 0.48 m

2. **[ALP]** In the ionosphere, electrons execute 1.4 × 10^6 revolutions in a second. Find the strength of the magnetic flux density in this region. (given: mass of electron = 9.11 × 10^{-31} kg, electronic charge = 1.6 × 10^{-19} C)

Ans: 5 × 10^{-5} T

3. **[ALP]** In a Millikan's oil drop experiment, the horizontal plates are 1.5 cm apart. With the electric field switched off, an oil drop is observed to fall with a steady velocity of 2.5 × 10^{-2} cm/s. When the field is switched on, the upper plate being positive, the drop just remains stationary when the p.d. between the plates is 1500 V. Calculate the radius of drop and the number of electronic charges it carries.

Given, Oil density, ρ = 900 kgm^{-3}, Viscosity of air, η = 1.8 × 10^{-5} Nsm^{-2}, Density of air, σ =1.293 kg/m^3) [HSEB 2067]

Ans: (a) 1.5 × 10^{-6} m (b) 8

4. **[ALP]** An electron beam after being accelerated from rest through a p.d. of 5000 V in a vacuum is allowed to impinge normally on a fixed surface. If the incident current is 50 μA, determine the force exerted on the surface assuming that, it brings the electron to rest.[e = 1.6 × 10^{-19} C, m = 9.11 × 10^{-31} kg]

Ans: 1.2 × 10^{-8} N

5. Electrons are accelerated from rest by a p.d. of 100 V. What is the final velocity? The electron beam now enters normally a uniform electric field of intensity 10^5 Vm^{-1}. Calculate the flux density B of a uniform magnetic field applied perpendicular to the electric field, if the path of the beam is unchanged from its original direction. (Assume e/m_e = 1.8 × 10^{11} Ckg^{-1})

Ans: 6.0 × 10^6 m/s, 1.7 × 10^{-2} T

6. A beam of proton accelerated from rest through a potential difference of 2000 V, enters a region of uniform magnetic field which is perpendicular to the direction of the proton beam. If the flux density is 0.2 T, calculate the radius of the path, which the beam describes. (Proton mass = 1.7×10^{-27} kg. Electronic charge = -1.6×10^{-19} C) **Ans: 0.033 m**

7. In an evacuated tube, electrons are accelerated from rest through a potential difference of 3600 V. These electrons travel as a narrow beam through a field free before entering a uniform magnetic field, the flux lines of which are perpendicular to beam. In the magnetic field, electrons describe a circular arc of radius 0.10 m. Calculate (i) the speed of the electrons entering the magnetic field (ii) the magnitude of the magnetic flux density. If an electron describes a complete revolution in a magnetic field, how much energy will it acquire? ($e/m = 1.8 \times 10^{11}$ Ckg^{-1}) **Ans: (i) 3.6×10^{7} m/s (ii) 2×10^{-3} T**

8. Give an account of a method, by which the charge on an electron is found to be -1.60×10^{-19} C, calculate the potential difference in volt necessary to be maintained between two horizontal conducting plates, one 5 mm above the other, so that a small oil drop, of mass 1.31×10^{-14} kg with two electrons attached to it, remains in equilibrium between them. Which plate would be at the positive potential? (g = 9.8 ms^{-2})

Ans: 2006 V

*[**Note:** Hints to challenging problems are given at the end of this chapter.]*

Conceptual Questions with Answers

1. What is an electron? Write any three properties of electron.

↳ Electron is a fundamental charge particle which possesses the negative charge. It is denoted by e. Its charge is 1.6×10^{-19} C.

Any three properties of an electron are:

i. It is negatively charged particle which contains both charge and mass.

ii. It is deflected by electric and magnetic fields.

iii. It ionizes the gas.

2. What are cathode rays? Do they deflect in the magnetic field?

↳ The invisible rays emitted from the cathode plate of a discharge tube when pressure is maintained at about 10^{-2} mm of Hg under the high potential difference 10 kV to 15 kV are called cathode rays. Actually, the cathode rays are the rays of electrons. Cathode rays are deflected by electric and magnetic fields.

3. Differentiate between cathode rays and alpha rays.

↳ Some important differences between the cathode rays and alpha rays are as follows:

Cathode Rays	**Alpha Rays**
1. Cathode rays are the rays of negatively charged particles, the rays of electrons.	1. Alpha rays are the rays of positively charged particles, the rays of alpha particles.
2. Mass of individual particle in cathode rays is very small.	2. Mass of individual particle in alpha rays is relatively large.
3. These rays deflect towards the positively charged plate.	3. These rays deflect towards the negatively charged plate.
4. These rays have low ionizing power and high penetrating power.	4. These rays have high ionizing and low penetrating power.

4. What is the difference between the deflection of the electron due to electric and magnetic fields?

↳ Electron is a negatively charged fundamental particle. It is influenced by both electric and magnetic fields.

a. If a moving electron is incident into the electric field, it follows the parabolic path and travels against the electric field.

b. If a moving electron is incident into the magnetic field, its path depends upon the angle of incidence into the field.

   i. If the electron is incident parallel or anti-parallel to the field, it is not deflected.

   ii. If the electron is incident perpendicular to the field, it follows circular path.

   iii. If the electron is incident with certain angle θ ($0 < \theta < 90$), it follows helical path.

---

5. What do you mean by cross field?

↳ When uniform electric field and magnetic fields are simultaneously applied perpendicularly in space, a charge particle entering into these regions does not get deflected. This field region is known as cross field. In cross field region,

Force of electric field (F_e) = Force of magnetic field (F_m)

i.e. $eE = Bev$

---

6. Which property of cathode ray shows that it possesses the mass particles?

↳ The cathode rays are deflected by electric and magnetic fields. Electric and magnetic fields deflect only mass particle but not the electromagnetic radiation. This property of cathode ray shows that it possesses the mass particles.

---

7. How can mass of an electron be determined?

↳ Millikan's oil drop experiment determines the charge of an electron, i.e. $e = 1.6 \times 10^{-19}$ C. Also, J. J. Thomson's experiment determines the specific charge of an electron, i.e. $\frac{e}{m} = 1.76 \times 10^{11}$ Ckg^{-1}. By solving above expressions, the mass of electron can be determined.

i.e. $m = 9.1 \times 10^{-31}$ kg.

---

8. Differentiate between electronic charge and specific charge of an electron.

↳ Electronic charge of an electron is the charge of an electron, i.e. $e = 1.6 \times 10^{-19}$ C.

Specific charge of an electron refers to the charge to mass ratio of an electron. i.e, $\frac{e}{m} = 1.76 \times 10^{11}$ Ckg^{-1}.

---

9. What are the properties of cathode rays?

   i. Cathode rays travel in straight lines and cast shadow of obstacles placed in their paths. Whenever an object is placed inside the tube, it casts a shadow on the wall opposite to the cathode. This experiment showed that the cathode rays travel in straight lines. Further, since the shadow falls on the wall opposite to the cathode, it shows that the rays travel from cathode towards the anode.

   ii. They produce fluorescence when they strike the glass wall of the discharge tube.

   iii. They carry negative charge whose charge is equal to charge of an electron. Hence, they are the stream of electrons.

   iv. Cathode rays are deflected by magnetic and electric fields.

   Deflection of cathode rays in an electric field indicates negative charge on its particles

   When the metal plates are given opposite electric charges, the beams of cathode rays are deflected towards the positively charged plate. This shows that the particles in the cathode rays carry negative charge.

   v. They are emitted at right angles from the surface of the cathode and enter at right angles into the surface of anode.

   vi. They ionize the gas and make them conducting which is the electrical effect of the rays.

   vii. They damage the photographic plate.

10. The value of e/m is constant for cathode rays but not for positive rays, why? (NEB 2074)

✎ e/m is the charge to mass ratio of a particle or any ion. Cathode rays are the beam of electrons. The value of e/m of cathode ray is equal to the e/m of an electron, which is constant

(i.e. $\frac{e}{m} = \frac{1.6 \times 10^{-19}}{9.1 \times 10^{-31}} = 1.76 \times 10^{11} \frac{C}{kg}$).

But the positive rays contain many positive ions whose neither charge nor the mass is same. That is why, e/m varies in accordance with their values of charge and mass.

---

11. An electron and a proton move with the same speed in a uniform magnetic field. Compare the radii of their circular paths.

✎ The radius of circular path of charge particle while revolving in a magnetic field is, $\frac{mv}{Be}$

The magnitude of charge of an electron and a proton is the same. For an electron and a proton, when they move with same speed in a uniform magnetic filed, the radii are,

$r_e = \frac{m_e v}{Be}$ and $r_p = \frac{m_p v}{Be}$

Where r_e and r_p are the radii of circular path of electron and proton respectively. Then, $\frac{r_e}{r_p} = \frac{m_e}{m_p}$

This means, $r \propto m$. Since the mass of electron is smaller than the proton, the radius of its circular path is also small.

---

12. Beams of electrons and protons having the same initial kinetic energy enter normally into an electric field, which beam will be more curved, justify. [HSEB 2072]

✎ The transverse deflection of an electron in an electric field is,

$$y = \frac{eV}{2mdu^2} x^2$$

$$y = \frac{eV}{4d\left(\frac{1}{2} mu^2\right)} x^2$$

$$y = \left(\frac{eV}{4d} x^2\right) \frac{1}{E_k}$$

For the identical condition, $y \propto \frac{1}{E_k}$.

If kinetic energy is also equal for both electron and proton, trajectory of particles is also equally curved.

---

13. Explain why electric discharge through a gas takes place in low pressure. (HESB 2071)

✎ Gas is, in general, poor conductor of electricity. At high pressure, free charge particles in gas do not respond to the electric field. There may be some ionization due to cosmic rays etc, but regular recombination of ions of opposite polarity prevents from discharging. As the pressure decreases, the mean free path of charged ions increases. Then, the charge ions can collide on the cathode plate so that electric discharge through gases is possible.

---

14. On what factors does the voltage applied across the discharge tube for electric discharge through a gas depend?

✎ If the potential difference between the electrodes of discharge tube is gradually increased, discharge occurs in the gas at a certain stage. This discharging potential depends on the following factors:

i. nature of gas used

ii. pressure of gas

iii. distance between two electrodes and,

iv. nature of the material of the gas.

15. What is the importance of Millikan's Oil drop experiment? [HSEB 2067]

✎ This experiment has great importance in atomic physics. It determines the fundamental value of charge. It concludes that the charge on any object exists as a multiple of small units called quantum of charge. Each quantum of charge is equivalent to 1.6×10^{-19} C. This fact is known as quantisation of charge.

16. Write down expression for acceleration of a moving charge Q in parallel and perpendicular magnetic field? (HSEB 2069)

i. When electron enters the field parallelly or antiparallely, i.e. $\theta = 0^o$ or $\theta = 180^o$.

The magnitude of magnetic force is,

$\mathbf{F = Bev \sin\theta}$

For $\theta = 0^o$ or 180^o, $\sin\theta = 0$

$\therefore$ F = 0

The electron does not experience any magnetic force, if it enters into the field parallelly or anti-parallelly.

ii. When electron enters the field perpendicularly, i.e. $\theta = 90^o$, the magnitude of magnetic force is,

$F = Bev \sin 90^o$

$\mathbf{F = Bev}$

17. Cathode rays can not be regarded as electromagnetic waves. Why?

✎ Electromagnetic waves are not deflected by electric and magnetic fields. These wave particles have the rest mass zero. On contrary, cathode rays deflect in both electric field and magnetic field. Moreover, cathode rays possess mass, they are the beam of electrons.

18. Water can not be used in place of clock oil in Millikan's oil drop experiment. Why?

✎ Non-volatile and viscous liquid is necessary to perform Millikan's oil drop experiment. This type of liquid has low vapour pressure so that it reduces the problem of evaporation. Water is volatile liquid, that may evaporate along the path and radius of drop may not remain constant. Also, the drop of liquid must be very small in size so that it should acquire terminal velocity to apply stokes law. This is also impossible in water drop. A clock oil is appropriate to perform this experiment.

19. When cathode rays strike a metal, it gets heated up. How?

✎ Cathode rays are beam of electrons, i.e. beam of mass particles. When they collide on a metal, their momentum is changed. On changing momentum, force is exerted onto the metal plate. Consequently, the kinetic energy $E_k = p^2/2m$, is also changed. The loss of kinetic energy of electrons is transformed into heat energy so that the metal plate becomes heated.

20. Why does colour appear in the discharge tube at low pressure?

✎ Cathode rays excite the atoms after collision. In excitation, orbital electron jumps to higher energy state and returns to ground state emitting electromagnetic radiations. As the excitation energy of electron depends on the initial kinetic energy of discharging electrons, the emitted radiations also have the different wave length i.e. different energy), which ultimately contrast in colour in the discharge tube.

21. Which property of cathode rays forces us to believe that cathode rays consists of electrons?

✎ Cathode rays are deflected in electric and magnetic fields. When the specific charge (e/m) of cathode rays are measured, it is found exactly equal to the specific charge of an electron which confirms that the cathode rays are the streams of electrons.

Exercises

Short-Answer Type Questions

1. Does an electron deflect in electric and magnetic field?
2. What is the difference between deflection of the electron due to electric and magnetic fields?

3. What is the charge of an electron?
4. What is the specific charge of an electron?
5. What do you mean by particle nature of electricity?
6. What happens when an electron enters into the electric field?
7. What do you mean by electric discharge?
8. How will you make a gas conducting?
9. Why are cathode rays not regarded as electromagnetic radiation?
10. What is the importance of the ratio e/m?
11. An electron and a proton enter a transverse electric field with the same velocity. Name the particle whose trajectory is more curved?
12. Will it be possible to produce discharge between earth and moon? If not why?
13. "The value of specific charge of cathode rays is constant but it is not constant for positive rays" Why?
14. An electron beam passes through a region of crossed electric and magnetic fields of intensity E and B respectively. For what value of the electron speed, the beam will remain undeflected?
15. What is specific charge?
16. What are positive rays? Why are they called canal rays?
17. What are the main differences between the cathode rays and positive rays?
18. Which type of liquid is used in Milliken's oil drop experiment and why?
19. A charged particle is not deflected in a region. Do you think no field present there?
20. What is the value of specific charge of hydrogen ion?
21. What is the ratio of specific charge of a proton and an α-particle?
22. What are the main differences between the cathode rays and positive rays?

Long-Answer Type Questions

1. Show that, electron motion in magnetic field is circular. Prove that frequency and time period are independent with the velocity of electron.
2. Show that, the path of an electron moving through a transverse uniform electric field is parabola.
3. What are cathode rays? How they are produced? Discuss the important properties of cathode rays. How will you prove that, cathode rays are not electromagnetic waves?
4. Describe the theory of Millikan's oil drop experiment to determine the charge of an electron.
5. Describe an experiment to determine the specific charge of an electron.
6. Describe an experiment to determine the ratio of the charge to mass (e/m) for an electron. Show how the result is derived from the observations.
7. Discuss the trajectory of a charged particle when it is moving in a uniform magnetic field and hence discuss how the specific charge of the particle is obtained.
8. Describe the phenomenon of electrical discharge through gases.
9. Show that electron motion in magnetic field is circular. Prove that frequency and time period are independent with the velocity of electron.

Numerical Problems

1. A cathode ray tube is operating at 10 kV. Calculate the speed of electron.

Ans: 5.9×10^7 ms^{-1}

2. An electron moves at right angle to uniform magnetic field 10^{-3} T. Find the radius of the circular path if the velocity of the electron is 3×10^7ms^{-1}.

Ans: 17.06 cm

3. A beam of electron remains undeflected when passes through a cross field of strength 100 Vm^{-1} and 5 × 10^{-4} T. Find the velocity of electrons.

Ans: 2 × 10^5 ms^{-1}

4. An ion for which the charge per unit mass is 4.40 × 10^7 C/kg, has velocity of 3.52 × 10^5 m/s and moves in a circular orbit in a magnetic flux density 0.4 T. What will be the radius of the orbit?

(Ans: 2 cm)

5. An electron moves in a circular path of radius 20 cm in a magnetic field of 2 × 10^{-3} T.
   i. What is the speed of electron?
   ii. What is the p.d. through which the electron must be accelerated to acquire this speed? (e = 1.6 × 10^{-19} C m= 9.11 × 10^{-31} kg)

(Ans: 7.2 x 10^7m/s, 13642.1V)

6. What is the ratio of the speed of a proton and α-particle when accelerated from rest through same p.d.?

(Ans: 2:1)

7. A stream of electrons moving with a velocity of 10^9 cm/s passes between two parallel plates. The intensity of the electric field between these plates is 300 V/cm. Find the intensity of the magnetic field required so that, there is no deflection of the electrons.

Ans: 3 × 10^{-3} T

8. An electron entering a magnetic field of 10^{-2} T with a velocity of 10^7 m/s describes a circle of radius 6 × 10^{-3} m. Calculate e/m of the electron.

Ans: 1.67 × 10^{11} Ckg^{-1}

9. A stream of electrons moving with a velocity of 6 × 10^6 ms^{-1} passes between two parallel plates. The intensity of magnetic field is 5 × 10^{-4} T. Calculate the strength of the electric field at right angles to the magnetic field required to keep the beam undeflected.

Ans: 3000 V/m

10. An electron beam passes through a magnetic field of 2 × 10^{-3} T and an electronic field of 3.4 × 10^4 V/m both acting simultaneously. If the path of the electron remains undeflected, calculate the speed of the electrons. If the electric field is removed, what will be the radius of the circular path? Mass of an electron is 9.1 × 10^{-31} kg.

Ans: 1.7 × 107 ms^{-1}, 0.0483 m

11. Electrons are accelerated through a potential difference of 3000 V, enter a region of uniform magnetic field, the direction of the field being at right angles to the motion of the electrons. If the flux density is 0.01 T, calculate the radius of the electron path. (e = 1.6 × 10^{-19} C, m = 9.00 × 10^{-31} kg)

Ans: 0.018 m

12. If the specific charge of proton is 9.6 × 10^7 Ckg^{-1}, find the specific charge for an alpha particle.

Ans: 4.8 × 10^7 Ckg^{-1}

13. In a Millikan's oil drop experiment, a drop is observed to fall with a terminal speed 1.4 mm/s in the absence of electric field. When a vertical electric field of 4.9 × 10^5 v/m is applied, the droplet is observed to continue to move downward at a lower terminal speed 1.21 mm/s. Calculate the charge on the drop. (Density of oil = 750 kg/m^3, viscosity of air = 1.81 × 10^{-5} kg/ms, density of air = 1.29 kg/m^3)

Ans: 5.16×10^{-19} C

14. An electron is accelerated through a potential difference of 2000 V and then it enters a uniform magnetic field of 0.02 Tesla in a direction perpendicular to it. Find the radius of the path of the electron in the magnetic field. Mass of an electron is 9.1 × 10^{-31} kg, charge of an electron is 1.6×10^{-19}C.

Ans: 7.5×10^{-4} m

15. An electron is accelerated through a potential difference of 2KV and then it enters a uniform magnetic field of 0.02T, in a direction perpendicular to it. Find the radius of the path of the electron in the magnetic field. (mass of electron = 9.1×10^{-31}kg)

Ans: 7.5×10^{-3}m

16. Two plane metal plates 4 cm long are held horizontally 3cm apart in a vacuum, one being vertically above the other. The copper plate is at a potential of 300V and the lower is earthed. Electrons having velocity of 10^7m/s are injected, horizontally midway between the plates and parallel to the 4cm edge. Calculate the vertical deflection of the electron beam as it emerges from the plates. (e/m for electron = 1.8×10^{11}C kg^{-1}.)

Ans: 1.44×10^{-2}m

17. In a Milliken- type apparatus the horizontal plates are 1.5 cm apart. With the electric field switched off an oil drop is observed to fall with the steady velocity 2.5×10^{-2} $cms.^{-1}$ when the field is switched on the upper plate being positive, the drop just remains stationary when the potential difference between the plates is 1500V. Calculate the radius of the drop and the number of electronic charges. (Given – density of oil = 900 kg m^{-3} and viscosity of air = $1.8\times10^{-5}NSm^2$, Neglect air density)

Ans: 1.5×10^{-6}m, 8

Multiple Choice Questions

1. The ratio of specific charge of a proton to that of an α-particle is:
   a. 4 : 1 b. 1 : 2
   c. 1 : 4 d. 2 : 1
2. An electron enter in magnetic filed with velocity 2×10^6 ms^{-1} perpendicular to the field of 2×10^{-5} T. What is the radius of the path of electron?
   a. 0.57 m b. 7.5 m
   c. 2.4 m d. 0.24 m
3. An electron is moving with a velocity v and enters a uniform electric field perpendicularly. Its trajectory within the filed will be:
   a. Parabolic b. Circular
   c. Hyperbolic d. Elliptic
4. An electron enters in a magnetic filed of 10^{-3} T normally with velocity 10^6 m/s. The radius of path of electron is:
   a. 11.4 cm b. 11.4 mm
   c. 5.7 cm d. 5.7 mm

Answers

1. (d) 2. (a) 3. (a) 4. (d)

Hints to Challenging Problems

HINT: 1

Given,

$V = 1.36 \times 10^4$ V

$d = 4$ cm $= 4 \times 10^{-2}$ m

$B = 2 \times 10^{-3}$ T

$m = 9.1 \times 10^{-31}$ kg

(i) For no deflection, we must have

Magnetic force = Electric force

or Bev = eE

or $v = \frac{E}{B} = \frac{V}{dB}$ $(\because E = \frac{V}{d})$

(ii) If r is the radius of path then we have

$$\frac{mv^2}{r} = Bev$$

$$\therefore\ r = \frac{mv}{Be}$$

HINT: 2

Given,

Number of revolutions in one second,

$f = 1.4 \times 10^6$ rev/s

Magnetic flux density, B = ?

$m = 9.11 \times 10^{-31}$ kg

$e = 1.6 \times 10^{-19}$ C

For an electron to be in circular path, the magnetic force provides necessary centripetal force i.e.,

$$Bev = \frac{mv^2}{r}$$

or $B = \frac{mv}{e \times r}$

$= \frac{m \times \omega r}{e \times r} = \frac{m \times 2\pi f}{e}$

HINT: 3

Given,

$d = 1.5\text{ cm} = 1.5 \times 10^{-2}\text{ m}$

$v = 2.5 \times 10^{-2}\text{ cm/s}$

$= 2.5 \times 10^{-4}\text{ m/s}$

$V = 1500\text{ V}$

density of oil, $\rho = 900\text{ kg/m}^3$, $\eta = 1.8 \times 10^{-5}\text{ Nsm}^{-2}$

density of air, $\sigma = 0$ (neglected)

a. Weight of drop = upward force

or, mg = viscous force + upthrust

Due to negligible density of air, no upthrust acts so we can write

$mg = 6\pi\eta rv$

or $\rho \times \frac{4}{3}\pi r^3 g = 6\pi\eta rv$

or, $r = \sqrt{\frac{9\eta v}{2(\rho - \sigma)g}}$

For, $\sigma = 0$, then,

$r = \sqrt{\frac{9\eta v}{2\rho g}}$

b. In this case, the weight of the drop must be equal to upward electric force i.e.,

$mg = qE$

or $\frac{4}{3}\pi r^3\rho g = ne \times \frac{V}{d}$

or $n = \frac{\frac{4}{3}\pi r^3\rho g \times d}{e \times V}$

HINT: 4

Given,

$V = 5000\text{ V}$

incident current, $I = 50 \times 10^{-6}\text{ A}$

$e = 1.6 \times 10^{-19}\text{ C}$

Force exerted, F = ?

$m_e = 9.1 \times 10^{-31}\text{ kg}$

We have,

$I = \frac{q}{t} = \frac{ne}{t}$

or $\frac{n}{t} = \frac{I}{e}$

Now,

Force on the surface due to electrons = rate of change of momentum of the electrons

or $F = \frac{nm_e v - 0}{t}$ (as final velocity is zero)

$= \frac{nm_e v}{t}$

But, $eV = \frac{1}{2}m_e v^2$

or $v = \sqrt{\frac{2eV}{m_e}}$

So, $F = \frac{nm_e}{t} \times \sqrt{\frac{2eV}{m_e}}$

HINT: 5

Given,

$V = 100\text{ V}$

Final velocity, v = ?

$E = 10^5\text{ Vm}^{-1}$, $\frac{e}{m} = 1.8 \times 10^{11}\text{ Ckg}^{-1}$

i. To find speed of electron, $v = \sqrt{\frac{2eV}{m}}$

ii. Then, B is determined from, $B = \frac{E}{v}$

HINT: 6

Given,

$V = 2000$ volt

Flux density, $B = 2.2\text{ T}$

Radius of the path, r = ?

We have,

$Bev = \frac{mv^2}{r}$

$\therefore \quad r = \frac{mv}{Be}$ . . . (i)

Again,

$eV = \frac{1}{2}mv^2$

$\therefore \quad v = \sqrt{\frac{2eV}{m}}$ . . . (ii)

From (i) and (ii), we get

$r = \frac{m}{Be}\sqrt{\frac{2eV}{m}} = \frac{1}{B}\sqrt{2V\frac{m}{e}}$

HINT: 7

Given,

$V = 3600\text{ V}$

radius of circular path, $r = 0.10\text{ m}$

i. To find speed of electron, $v = \sqrt{\frac{2eV}{m}}$

ii. Then, the magnetic field is determined from,

$B = \frac{mv}{er} = \frac{v}{\frac{e}{m} \times r}$

iii. Electric force is the conservative force so the work done for this force is zero in one complete cycle. Hence, energy acquired by an electron is zero.

HINT: 8

Given,

$e = 1.6 \times 10^{-19}$ C

Separation between the plates, d = 5 mm

$= 5 \times 10^{-3}$ m

Mass of oil drop, $m = 1.31 \times 10^{-14}$ kg

Number of electrons, n = 2

$g = 9.8\ ms^{-2}$

For the oil drop to be in equilibrium, its weight should be equal to upward electric force i.e.,

$mg = qE$

or $mg = ne \times \dfrac{V}{d}$

or $V = \dfrac{mg \times d}{ne}$

❖ ❖ ❖

PHOTONS

20
CHAPTER

20.1 Introduction

Before Planck's discovery of particle nature of light, the light was solely described in terms of a wave. It was believed that the light energy is emitted from a source continuously. But, the concept of continuous emission of light from the source could not explain many observable facts in nature like black body radiation. So, Planck studied the nature of light and discovered that light travels in the form of the tiny discrete packets which were named quantum or photon. On the basis of quantum nature of light, many phenomena like photoelectric effect, Compton effect, pair production, which were unsolved until these date, were solved theoretically and experimentally.

20.2 Quantum Nature of Light

Before 1900, the energy was considered as a continuous spectrum of radiations in which the light wave contains all wavelength from zero to infinity. However, this concept could not verify many experimental facts in nature. Later on, in 1900, **Max Planck** put forward an interesting and revolutionary idea on his quantum theory of radiation. According to this theory, energy is emitted or absorbed in discontinuous (i.e. discrete) units rather in continuous form. In each step of emission of radiation, a tiny packet or bundle of energy is emitted or absorbed. These bundles were called photons or quanta (in singular: quantum). The theory about the quantum nature of energy is called Planck's quantum theory. According to this theory, the energy of every quantum (or photon) is directly proportional to its frequency f,

$$E \propto f$$

$$E = hf, \quad \text{where, } f = \text{frequency of photon}$$

Where, h is proportionality constant and it is called Planck's constant. The experimental value of Planck's constant (h) is 6.62×10^{-34} Js.

For a light photon moving with speed c and wavelength λ is written as,

$$f = \frac{c}{\lambda}$$

So, the energy of photon,

$$\mathbf{E = \frac{hc}{\lambda}} \qquad \textbf{...(20.1)}$$

In 1905, Einstein explained photoelectric effect on the basis of Planck's quantum theory.

Properties of Photon

i. A photon is the building block of all the electromagnetic radiations.

ii. Its speed is equal to the speed of light (3×10^8 m/s) in vacuum but changes when travelling from one place to another in different media and hence, wavelength also changes but frequency remains the same.

iii. Its rest mass (mass in stationary condition) is zero but have dynamic mass (mass in motion). So, its total energy is equal to kinetic energy of photon.

iv. Its charge is zero. So, it is not deflected by magnetic and electric fields.

v. The energy of each photon of frequency f is $E = hf = \frac{hc}{\lambda}$.

vi. It has both particle and wave nature.

vii. It exerts force and pressure when strikes on a surface.

20.3 Photoelectric Effect

Photo refers to light and electric refers to electricity. The term photoelectric refers to the conversion of light energy to electricity. Therefore, the photoelectric effect is defined as *the phenomenon of emission of electrons from a material surface when light of suitable wavelength (or frequency) falls upon it.* Here, the light involves not only visible light but also all types of electromagnetic radiation. The electrons emitted by photoelectric effect are called photoelectrons and the current so produced is called "photoelectric current". This effect is the evidence of particle nature of radiation.

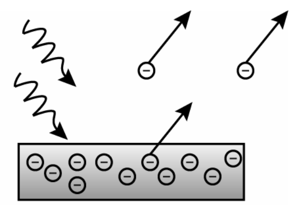


Fig.20.1: Photoelectric effect

i. **Work function:** *The minimum amount of energy required to eject out an electron from the surface of material is known as work function.* It is denoted by ϕ_0. The work functions of alkali metals have small value, so they are used to produce photoelectricity, but the work functions of insulators like wood, plastic are very large, so it is impossible to produce photoelectricity in these materials.

According to Planck's theory, work function

$$\phi_0 = hf_0 \qquad \ldots(20.2)$$

where, f_0 is called threshold frequency

ii. **Threshold frequency:** *The minimum frequency of an incident light which can eject out the electrons from the surface of material is known as threshold frequency.* It is denoted by f_0. The corresponding energy is called threshold energy and is equivalent to work function.

iii. **Threshold wavelength:** *The longest wavelength of incident light which can eject out the electrons from the surface of material is known as threshold wavelength.* It is denoted by λ_0.

The threshold frequency and threshold wavelength are related as,

$$f_0 = \frac{c}{\lambda_0} \qquad \ldots(20.3)$$

Also, work function can be related to threshold wavelength as,

$$\phi_0 = \frac{hc}{\lambda_0} \qquad \ldots(20.4)$$

Photoelectric effect: Evidence of particle nature of light

The phenomena; interference, diffraction, and polarization of light, ensure that light shows the wave picture. According to this picture, light is an electromagnetic wave which consists of electric field and magnetic field propagating mutually perpendicular to each other. If this wave concept is applied to explain the photoelectric phenomenon of light, a contradiction arises. According to wave theory, the free electrons on the surface of metal should continuously absorb the light energy, so greater the intensity of light is exposed; the greater would be the kinetic energy of photoelectron. However, it is experimentally impossible. The experimentally observed fact is that the maximum kinetic energy of photoelectrons is independent of the intensity of light.

Besides the above facts, the photoelectric effect is an instantaneous process. As soon as the light falls on the surface of material, the electrons are emitted readily at an interval of about 10^{-9} s. If light possesses sole wave property, the ejection of electron may take hours long time.

In view of the above contradiction, light must show the particle property. This concludes that the wave theory of light fails to explain the basic features of photoelectric effect.

Demonstration of photoelectric effect

The apparatus arrangement to demonstrate the photoelectric effect is shown in Fig. 20.2 This arrangement consists of an evacuated quartz bulb (Q) with two zinc plates A and B, a microammeter and a d.c. power supply. The plate A is connected to the negative terminal and the plate B is connected to the positive terminal of power supply. As soon as the light of appropriate wavelength is allowed to fall on plate A, deflection in microammeter is observed. If the light is blocked, no current appears in the electric circuit. This evidence shows that electrons are ejected from the metal surface, when light of appropriate wavelength falls upon it. The current so produced is called photoelectric current. The magnitude of current depends on the intensity of beam of incident light and the electric potential provided by power supply.

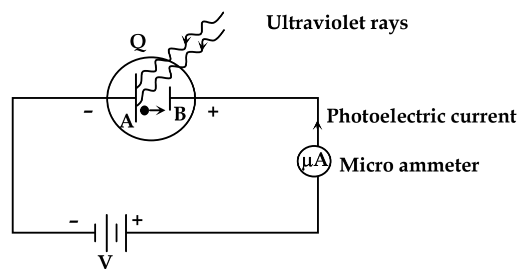


Fig. 20.2: Demonstration of photoelectric effect

20.4 Einstein's Equation of Photoelectric Effect

When light falls on the surface of a material, it interacts with the orbital electrons. If the energy of incident photon is smaller than the work function of the material, the electron is not emitted out. To emit out the electron from the surface of material, the energy of incident photon (E = hf) must be equal to or greater than the work function (ϕ_0). When the energy of incident photon is sufficiently greater than the work function, the photoelectron gains the kinetic energy. This phenomenon of photoelectric effect was firstly described and formulated by **Albert Einstein**. Einstein was awarded with Nobel Prize in 1921, for his discovery of photoelectric effect.

Einstein's formulation on photoelectric effect is based on the principle of conservation of energy. He used the Planck's theory to find the energy of incident photon (E = hf). According to Einstein's theory of photoelectric effect, the incident energy of photon is imparted into two forms (a) some part of energy is used to eject the electron from the orbit of atom (i.e. provide the work function ϕ_0) and (b) remaining part is transferred as kinetic energy of photoelectron $\left(\frac{1}{2}mv_{max}^2\right)$. Therefore,

$$\mathbf{hf = \phi_0 + \frac{1}{2}mv_{max}^2} \qquad \ldots(20.5)$$

where, f = frequency of incident photon

m = mass of photoelectron

v_{max} = maximum velocity of photoelectron

$$\text{or,}\quad \frac{1}{2}mv^2_{max} = hf - \phi_0 \qquad \ldots (20.6)$$

As we have, $\phi_0 = hf_0$, **we get,**

$$\frac{1}{2}mv^2_{max} = hf - hf_0 \qquad \ldots (20.7)$$

This is the famous Einstein's photoelectric equation for which, he was awarded Nobel Prize in 1921. Later on, R.A. Millikan experimentally verified it and was also awarded Nobel Prize in 1923.

If λ and λ_0 are the respective wavelength corresponding to incident frequency and threshold frequency, equation (20.7) becomes,

$$\frac{1}{2}mv^2_{max} = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$$

$$\therefore \quad \frac{1}{2}mv^2_{max} = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right) \qquad \ldots (20.8)$$

Accelerating potential and stopping potential

If light of suitable wavelength is allowed to fall on the cathode plate of quartz bulb as in Fig. 20.3(i) the photoelectrons travel to the anode plate. If the positive potential is gradually increased across two plates keeping intensity and frequency of light fixed, the photoelectric current also increases in the circuit till a stage is reached, when photoelectric current becomes maximum and does not increase further. This positive potential in photoelectric circuit which increases the photoelectric current is called accelerating potential and the maximum possible photoelectric current is called saturation current.

Alternately, if the polarity of power supply is reversed in quartz bulb as in Fig. 20.3(ii), the photoelectron should move towards the cathode plate. If the negative potential is gradually increased in the cathode plate, the electrons are repelled and photoelectric current decreases until it becomes zero. *The value of retarding potential at which the photoelectric current becomes zero is called cut off or stopping potential for a given frequency of incident light.* Stopping potential is denoted by V_s or V_o. At stopping potential, even an electron of maximum kinetic energy fails to reach at cathode plate. It is a kind of critical potential, below which the photoelectric current is possible and above which the current stops completely.

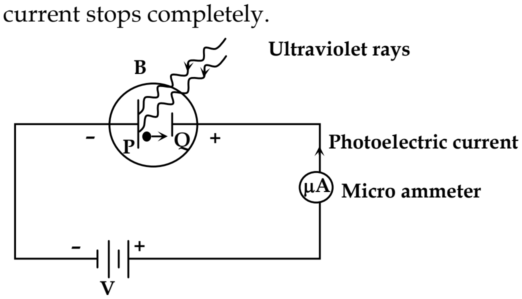


Fig. 20.3(i): Accelerating potential

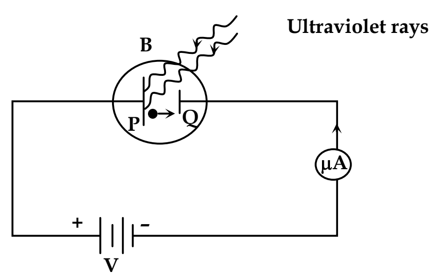


Fig. 20.3 (ii): Stopping potential

For a photoelectron of maximum kinetic energy, the work done by stopping potential must be equal to its kinetic energy.

Hence,

$$\frac{1}{2}mv^2_{max} = eV_s \qquad \ldots (20.9)$$

Where, v_{max} = Velocity of most energetic electron
m = mass of electron
e = charge of electron.

20.5 Laws of Photoelectric Emission

A physicist, Lenard summed up all the experimental observations in the form of laws called Lenard's laws of photoelectric emission which are as follows:

1. *The photoelectric effect is an instantaneous process which occurs almost instantly when the metal surface is exposed to radiation of suitable frequency.* It is found that time lag between the incidence of radiation and emission of electron is less than 10^{-8} s.

2. *The number of photoelectrons emitted and hence the photocurrent depends on the intensity of radiation exposed but is independent of frequency of incident radiation.*

   The intensity of radiation can be varied by changing the distance between source of radiation and cathode plate. The greater the intensity, greater will be the number of photons interacting with the electrons and hence greater will be the number of electrons emitted. This consequently increases the photocurrent. A plot of intensity and photocurrent is as shown in Fig. 20.4.

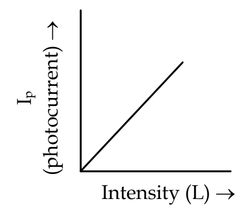


Fig. 20.4: Relation of photocurrent and intensity

   However, for a constant intensity, if we vary the frequency (by using different colours of light radiation), the photocurrent remains constant.

3. **The maximum kinetic energy of photoelectrons emitted is independent of intensity but depends on frequency of light.**

   It is observed that, the photoelectrons are emitted when a light of suitable frequency falls on metal surface. This means, the photon must have energy greater than or equal to the energy with which the electron is bound to metal surface (work function) to eject an electron.

   A part of photon energy equal to work function is used to eject electrons and remaining energy is used to impart kinetic energy to emitted electrons. Since, work function for a material is constant, we can say that,

   kinetic energy of photoelectrons $\propto$ energy of photon

   i.e. $\frac{1}{2}mv^2 \propto hf$

   Thus, we can say that greater the frequency greater will be the kinetic energy of emitted electrons. A plot of kinetic energy and frequency is as shown in Fig. 20.5. The graph shows that emission of electron is impossible when frequency of photon is below f_0. Since intensity is concerned with number of photons incident per unit area per unit time, it has nothing to do with energy of photoelectron.

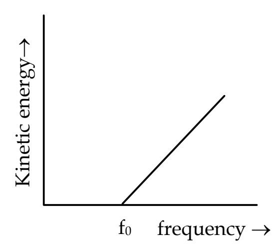


Fig.20.5: Relation of kinetic energy and frequency

4. **The stopping potential is independent of intensity of radiation but depends on the frequency of radiation.**

   As we discussed earlier, the energy of photons and hence the frequency is directly proportional to the kinetic energy of photoelectrons. So, for greater frequency of radiation, the kinetic energy of photoelectrons is greater. The more energetic photoelectrons require larger retarding potential to stop them from reaching the collector. So, we can write,

frequency of photons ∝ kinetic energy of electrons ∝ stopping potential

$$\text{i.e.}\quad f \propto \frac{1}{2} mv^2_{max} \propto e\,V_s.$$

A plot of stopping potential versus frequency is as shown in Fig.20.6.

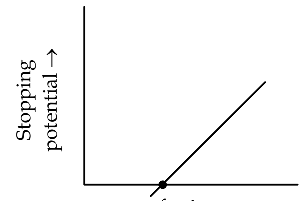


Fig.20.6: Relation of stopping potential and frequency

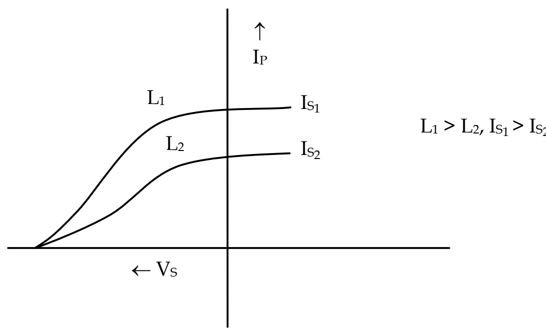


Fig.20.7: Relation of intensity and photo electric current

For a constant frequency but different intensities of radiation, if we gradually increase the retarding potential, the photocurrent gradually decreases and becomes minimum at certain potential called stopping potential. The value of stopping potential does not depend on intensity. A plot of photocurrent versus stopping potential at constant frequency but varying intensity is shown in Fig.20.7.

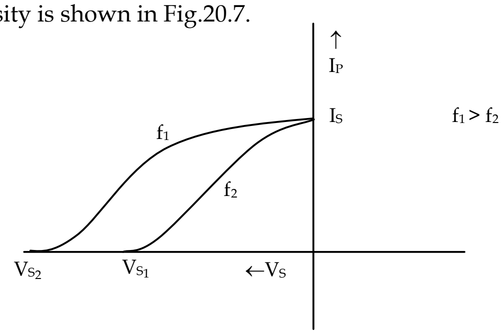


Fig.20.8: Relation of stopping potential and frequency

For constant intensity, but different frequency of incident radiation, a plot of stopping potential versus photocurrent is as shown in Fig.20.8.

Note

i. *Any other electromagnetic waves having wavelength shorter than the ultraviolet rays can produce photo electric effect from any metal.*

ii. *Visible rays can cause the photo electric effect, only from alkali metals such as Na, K, Cs.*

iii. *Infrared rays can cause the effect only from caesium (Cs) which has the lowest value of work function nearly 1.8 eV.*

iv. *In the photoelectric effect, one energetic photon can liberate only one electron. Hence this process is called one to one interaction.*

20.6 Millikan's Verification of Einstein's Equation of Photoelectric Effect

According to Einstein, the maximum K.E. of the emitted photoelectrons depends upon the frequency of the photons.

$$\frac{1}{2}mv^2_{max} \propto hf \qquad \ldots (20.10)$$

Whenever the polarity of the emitter and collector is reversed such that the collector is at negative potential, the emitted photo-electrons have to do work against this negative potential in the expenses of its own kinetic energy in order to reach the collector. If the negative potential of the collector is increased to a value called stopping potential (V_s) such that even the fastest moving electrons are repelled, then in this situation,

$$\frac{1}{2}mv^2_{max} = eV_s \qquad \ldots (20.11)$$

From equation (20.10) and equation (20.11) we can write,

$$eV_s \propto hf$$

$$\text{or,} \quad V_s \propto f \qquad \ldots (20.12)$$

This equation shows that stopping potential is directly proportional to the frequency of photon and hence a plot of stopping potential and frequency of photon must be a straight line.

This fact was experimentally verified by Robert A. Millikan. In this experiment, he took three different photosensitive metals, Na, K, Li mounted on a cylindrical wheel which could be rotated about an axis. This structure was kept inside an evacuated glass tube provided with a strong electromagnet that could be used to turn the wheel. The tube also had a knife N, that could be used to remove the metal oxides from the surface of metal. The monochromatic light was allowed to fall on different metals and the emitted electrons were collected by the collector maintained at negative potential as shown in Fig.20.9. The negative potential of the collector was gradually increased to a maximum magnitude called stopping potential at which even the fastest moving electrons could not reach the collector. Now, the frequency of incident light was changed to a higher value and the value of stopping potential also had to be changed to a greater value. This procedure was repeated by taking different colors (frequency) of light and allowing them to fall on each of the three metals marked on the wheel.

A plot of incident frequency and stopping potential for each three metals were as shown below.

The graph is a straight line with negative y-intercept and thus its equation must be in the form of

$$y = mx + (-c) \qquad \ldots (20.13)$$

Now, according to Einstein's photoelectric equation,

$$\frac{1}{2}mv^2_{max} = hf - hf_0 \qquad \ldots (20.14)$$

From equations (20.11) and (20.14) we get,

$$eV_s = hf - hf_0$$

$$\text{or,} \quad V_s = \frac{h}{e}f + \left(\frac{-h}{e}f_0\right) \qquad \ldots (20.15)$$

This equation is analogous to equation (vi) with $y = V_s$, $m = \frac{h}{e}$, $x = f$ and $c = \left(\frac{-h}{e}f_0\right)$

Thus, his experiment successfully verified Einstein's photoelectric equation.

A closer look an equation (20.15), shows that V_s is actually proportional to $(f - f_0)$ as shown in Fig. 20.10.

Further, the slope of the line, AB is,

$$m = \frac{BC}{AC} = \frac{\Delta V_s}{\Delta f}$$

$$\text{or,} \quad \frac{h}{e} = \frac{\Delta V_s}{\Delta f}$$

$$\textbf{or,} \quad \mathbf{h = \frac{\Delta V_s}{\Delta f} \times e} \qquad \textbf{...(20.16)}$$

Thus, by knowing the slope of line AB, the value of h can be determined experimentally and its value is found to 6.62×10^{-34} Js which is a constant.

Similarly, the slope of this line can be used to calculate the work function ($\phi = hf_0$) of the material.

We have, $\phi = \frac{h}{e} f_0 \times e$

From equation (20.16), we can write,

$$\phi = \frac{h}{e} f_0 \times e = \frac{\Delta V_s}{\Delta f} \cdot f_0 \times e$$

In this way, the value of work function of the material can be determined experimentally.

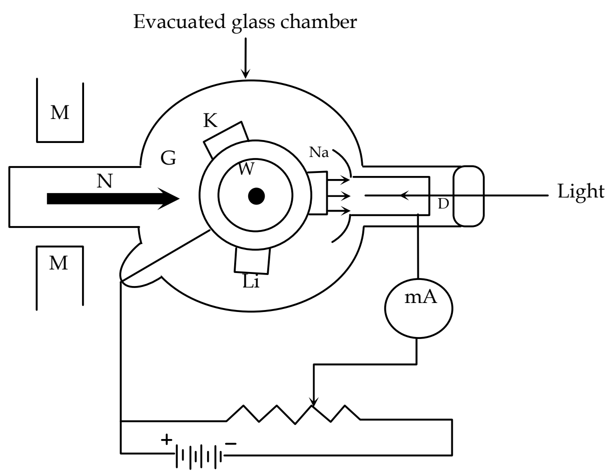


Fig. 20.9: Millikan's apparatus for photoelectric effect

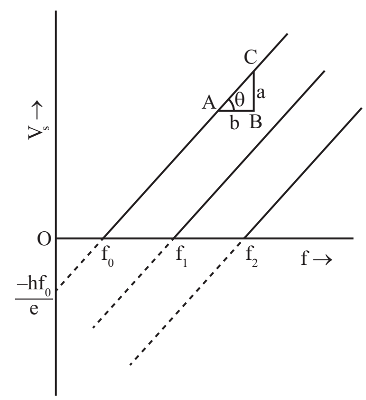


Fig. 20.10: Graph between stopping potential and frequency of incident light

20.7 Photocell

Photocell is an electric energy source that converts light energy into an electric current. It works on the principle of photoelectric effect. The construction and working of photocell is described below.

It is an evacuated glass bulb which contains a cylindrical electrode called cathode C, which is connected to the negative terminal of secondary cell (rechargeable cell) and a thin rod, called anode A, and is connected to positive terminal of the cell as shown in Fig. 20.11. The cathode plate is coated with barium or cesium oxide. The work function of barium or cesium oxide is comparatively very low, so the visible light can eject the electrons from the cathode plate. Anode is allowed to collect the emitted electrons from the cathode. A micro-ammeter is connected to measure current in the circuit. The rechargeable cell connecting across the anode A and cathode C gets charged while the cathode is exposed to light. The charged cell is used as the electric source to drive the electrical appliance. It is an alternative source of electricity. The current and power produced in a photocell depends on (a) light intensity (b) surface area exposed (c) distance from light source.

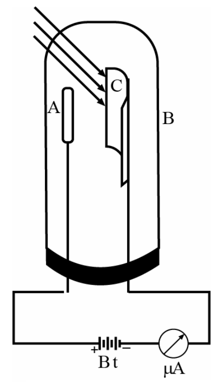


Fig.20.11: Photocell

Tips for MCQs

1. About photons

i. Energy of a photon, $E = hf = \frac{hc}{\lambda}$

ii. It has zero rest mass, $m_o = m\sqrt{1 - \frac{v^2}{c^2}}$, for v = c, $m_o = 0$, but in motion, it has dynamic mass.

iii. It is not deflected by electric and magnetic fields.

iv. From Einstein's mass-energy equivalence, $E = hf = \frac{hc}{\lambda} = mc^2$

So, $m = \frac{hf}{c^2} = \frac{h}{c\lambda}$

v. Momentum of photon, $p = mv = mc = \frac{E}{c} = \frac{h}{\lambda}$

2. Work function and threshold frequency

i. Work function $(\phi_0) = hf_0 = \frac{hc}{\lambda_0}$

ii. Work function is usually expressed in electron volt (eV) unit.

iii. For photoelectric effect, incident energy (hf) must be greater than or equal to work function (ϕ_o) i.e. $hf \geq \phi_0$.

iv. The work function of metal decreases when the temperature of the metal increases.

3. Photoelectric effect

i. One photon ejects one electron, i.e. it is a one to one phenomenon.

ii. Cesium is the best metal for photoelectric effect.

iii. This effect is based on conservation of energy.

iv. This effect establishes the particle nature of light.

v. Photoelectric effect is instantaneous process. The photoelectron is emitted as soon as the light fall upon the material ($\sim 10^{-9}$ s).

vi. Photoelectric current depends on the intensity of incident radiation, but not the frequency of radiation above f_0.

vii. Kinetic energy of photoelectron depends on frequency of incident radiation, but not the intensity.

4. Einstein's photoelectric equation

i. The energy of incident photon is imparted into energy against work function and kinetic energy of photoelectron. i.e. $hf = \phi_0 + E_k$ and $(E_k = \frac{1}{2} mv^2_{max})$

ii. The kinetic energy deduced from Einstein's equation is the maximum value of kinetic energy of photoelectron.

5. **Stopping potential**

i. Stopping potential (V_s) depends on the nature of material.

ii. It depends on frequency of incident radiation but is independent of intensity of radiation.

$$eV_s = hf - \phi_0 = \frac{1}{2}mv^2_{max}$$

6. **Intensity is directly proportional to :** (a) number of photons falling per unit time, (b) number of electrons emitted per unit time, (c) photoelectric current, (d) square of distance between bulb and plate and

i. $I = \frac{Nhf}{At}$, N = total number of photo electron

$$\frac{P}{A} = \frac{Nhf}{At}$$

$\therefore$ Power (P) $= \frac{Nhf}{t} = \left(\frac{N}{t}\right)hf = nhf = \frac{nhc}{\lambda}$

Worked Out Problems

1. **For caesium the value of ϕ_0 is 1.35 electronvolt. (a) What is the longest wavelength that can cause photo-electric emission from a cesium surface? (b) What is the maximum velocity with which photoelectrons will be emitted from a cesium surface illuminated with light of wavelength 4.0×10^{-7} m? [$e = 1.6 \times 10^{-19}$ C, $m = 9 \times 10^{-31}$ kg, $h = 6.6 \times 10^{-34}$ Js]**

SOLUTION

Given,

Work function (ϕ_0) $= 1.35$ eV $= 1.35 \times 1.6 \times 10^{-19}$
$= 2.16 \times 10^{-19}$ J

(a) $\lambda_0 = ?$

We know that,

$\phi_0 = hf_0$

or, $\phi_0 = \frac{hc}{\lambda_0}$ or $\lambda_0 = \frac{hc}{\phi_0}$

$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{2.16 \times 10^{-19}} = 9.17 \times 10^{-7}$ m

(b) $v_{max} = ?$ (if $\lambda = 4 \times 10^{-7}$m)

$m_e = 9 \times 10^{-31}$ kg

From photoelectric equation, we have,

$\frac{1}{2} m_e v^2_{max} = hf - \phi_0$

$\frac{1}{2} m_e v^2_{max} = \frac{hc}{\lambda} - \phi_0$

or, $\frac{1}{2} \times 9 \times 10^{-31} \times v^2_{max}$

$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{4 \times 10^{-7}} - 2.16 \times 10^{-19}$

or, $4.5 \times 10^{-31} \times v^2_{max} = 4.95 \times 10^{-19} - 2.16 \times 10^{-19}$

or, $v_{max} = \sqrt{\frac{2.79 \times 10^{-19}}{4.5 \times 10^{-31}}} = 7.87 \times 10^5$ m/s

2. **When ultraviolet light with a wavelength of 400 nm falls on a certain metal surface, the maximum kinetic energy of the emitted photoelectrons is 1.10 eV. What is the maximum kinetic energy of the photoelectrons when light of wavelength 300 nm falls on the same surface?**

SOLUTION

Given,

For $\lambda_1 = 400$ nm $= 400 \times 10^{-9}$ m,

Maximum K.E. (E_1) $= 1.1$ eV $= 1.1 \times 1.6 \times 10^{-19}$ J

For, $\lambda_2 = 300$ nm $= 300 \times 10^{-9}$ m,

Maximum K.E. (E_2) = ?

From photoelectric equation, we have,

$E_1 = hf_1 - \phi_0$ and, $E_2 = h f_2 - \phi_0$

Subtracting, we get,

$E_2 - E_1 = h (f_2 - f_1)$

or, $E_2 = h\left(\frac{c}{\lambda_2} - \frac{c}{\lambda_1}\right) + E_1$

$= hc\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right) + E_1$

$= 6.62 \times 10^{-34} \times 3 \times 10^8 \left(\frac{1}{300\times10^{-9}} - \frac{1}{400\times10^{-9}}\right) + 1.1 \times 1.6\times10^{-19}$

$= 19.86 \times 10^{-26}\left(\frac{10^7}{3} - \frac{10^7}{4}\right) + 1.76 \times 10^{-19}$

$= 19.86 \times 10^{-19} \times \frac{1}{12} + 1.76 \times 10^{-19}$

$= 1.65 \times 10^{-19} + 1.76 \times 10^{-19}$

$= 3.41 \times 10^{-19}$ J

$= \frac{3.41 \times 10^{-19}}{1.6 \times 10^{-19}}$ eV

$\therefore \quad E_2 = 2.1$ eV

3. A photon has momentum of magnitude 8.24×10^{-28} kgm/s. (a) what is the energy of this photon? Give your answer in joule and electron volt. (b) What is the wavelength of this photon?

SOLUTION

Given,

Momentum (p) = 8.24×10^{-28} kgm/s

(a) Energy of a photon (E) = ?

We know that,

$E = mc^2$

$= mc \times c$

$= p \times c$

$= 8.24 \times 10^{-28} \times 3 \times 10^8$

$\therefore \quad E = 2.47 \times 10^{-19}\ \text{J} = \frac{2.47 \times 10^{-19}}{1.6 \times 10^{-19}}$ eV

$\therefore \quad E = 1.54$ eV

(b) Wavelength of the photon (λ) = ?

From de- Broglie wave equation, we have

$\lambda = \frac{h}{p} = \frac{6.62 \times 10^{-34}}{8.24 \times 10^{-28}}$

$\therefore \quad \lambda = 804 \times 10^{-9}$ m

4. A 75 W light source consumes 75 W of electrical power. Assume all this energy goes into emitted light of wavelength 600 nm. (a) calculate the frequency of the emitted light. (b) How many photons per second does the source emit?

SOLUTION

Given,

Power (P) = 75 W

Wavelength (λ) = 600 nm

$= 600 \times 10^{-9}$ m $= 6 \times 10^{-7}$ m

(a) f = ?

we have

$c = f\lambda$

$\therefore \quad f = \frac{c}{\lambda} = \frac{3 \times 10^8}{6 \times 10^{-7}} = 5 \times 10^{14}$ Hz

(b) Number of photons per second,

$\frac{n}{t} = ?$

we have

$E_n = nhf$

or $\frac{E_n}{t} = \frac{nhf}{t}$

or $P = \left(\frac{n}{t}\right) hf$

or $\left(\frac{n}{t}\right) = \frac{P}{hf}$

$= \frac{75}{6.62 \times 10^{-34} \times 5 \times 10^{14}}$

$= 2.3 \times 10^{24}$ photons/sec.

5. 400 nm wavelength of light falls on a photo sensitive material of work function 2.3 eV. Compute the maximum energy of photoelectrons.

SOLUTION

Given,

Wavelength (λ) = 400 nm = 400×10^{-9} m.

Work function (ϕ_0) = 2.3 eV = $2.3 \times 1.6 \times 10^{-19}$ J

Maximum energy (E_k) = ?

We have,

$hf = \phi_0 + E_k$

$h\frac{c}{\lambda} = \phi_0 + E_k$

$E_k = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{400 \times 10^{-9}} - 2.3 \times 1.6 \times 10^{-19}$

$= 1.285 \times 10^{-19}$ J

$= \frac{1.285 \times 10^{-19}}{1.6 \times 10^{-19}} = 0.803$ eV

6. [NEB 2074] Radiations of wavelength 5400 Å fall on a metal whose work function is 1.9 eV. Find the energy of the photoelectrons emitted and their stopping potential. Planck's constant = 6.62 × 10^{-34} JS.

SOLUTION

Given,

Wavelength (λ) = 5400Å = 5400×10^{-10} m

Work function (ϕ_0) = 1.9 eV

$= 1.9 \times 1.6 \times 10^{-19} = 3.04 \times 10^{-19}$ J

Planck's constant (h) = 6.62×10^{-34} Js

Kinetic energy (E_k) = ? (for photoelectron)

Stopping potential (V_0) = ?

We know,

$$E_k = \frac{hc}{\lambda} - \phi_0$$

$$= \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{5400 \times 10^{-10}} - 3.04 \times 10^{-19}$$

$$= 3.68 \times 10^{-19} - 3.04 \times 10^{-19}$$

$$= 0.64 \times 10^{-19}$$

$\therefore$ Energy of photoelectron (E_k) = 6.4×10^{-18} J

Now,

$eV_s = E_k$

$$V_s = \frac{E_k}{e} = \frac{6.4 \times 10^{-18}}{1.6 \times 10^{-19}} = 0.4 \text{ V}$$

$\therefore$ Stopping potential = 0.4 V.

7. [HSEB 2075] Sodium has a work function of 2 eV. Calculate the maximum energy and speed of the emitted electrons when sodium is illuminated by a radiation of 150 nm. What is the threshold frequency of radiation for which electrons are emitted from sodium surface?

SOLUTION

Given,

Work function (ϕ_0) = 2 eV = $2 \times 1.6 \times 10^{-19}$ J

Maximum Kinetic Energy ($K.E_{max}$) = ?

Speed of electron (v_{max}) = ?

Threshold frequency (f_n) = ?

Wavelength of light (λ) = 150 nm = 150×10^{-9} m

From

Einstein photo-electric equation

$hf = \phi_0 + K.E_{max}$

$$\text{or, } K.E_{max} = h\frac{c}{\lambda} - \phi_0$$

$$= \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{150 \times 10^{-9}} - 2 \times 1.6 \times 10^{-19}$$

$$= 1.004 \times 10^{-18} \text{J}$$

$$v_{max} = \sqrt{\frac{2 \times K.E_{max}}{m}}$$

$$= \sqrt{\frac{2 \times 1.004 \times 10^{-18}}{9.1 \times 10^{-31}}} = 1.483 \times 10^6 \text{ m/s}$$

Again, For threshold frequency

$\phi_0 = hf_0$

$$\text{or, } f_0 = \frac{\phi_0}{h} = \frac{2 \times 1.6 \times 10^{-19}}{6.62 \times 10^{-34}} = 4.8 \times 10^{14} \text{ Hz}$$

Challenging Problems

1. **[UP]** A photon of green light has a wavelength of 520 nm. Find the photon's frequency, magnitude of momentum, and energy. Express the energy both in joules and electron volts.

Ans: 5.76 × 10^{14} Hz, 1.27 × 10^{-27} Ns, energy (3.82 × 10^{-19}J, 2.39 eV)

2. **[UP]** The predominant wavelength emitted by an ultraviolet lamp is 248 nm. If the total power emitted at this wavelength is 12.0 W, how many photons are emitted per second?

Ans: 1.5 × 10^{19} proton/ sec

3. **[UP]** A clean nickel surface is exposed to light of wavelength 235 nm. What is the maximum speed of the photoelectrons emitted from this surface? [ϕ_0 = 5.1 eV]

Ans: 2.5 × 10^5 m/s

4. **[UP]** What would the minimum work function for a metal have to be for visible light (400 nm to 700 nm) to eject photoelectrons?

Ans: 1.77 eV

5. **[UP]** When ultraviolet light with a wavelength of 254 nm falls upon a clean copper surface, the stopping potential necessary to stop emission of photoelectrons is 0.181 V. (a) What is the photoelectric threshold wavelength for this copper surface? (b) What is the work function for this surface?

Ans: (a) 2.64×10^{-7} m (b) 4.70 eV

6. **[UP]** An excited nucleus emits a gamma ray photon with an energy of 2.45 MeV. (a) What is the photon frequency? (b) What is the photon wavelength?

Ans: (a) 5.92×10^{20} Hz (b) 5.06×10^{-13} m

7. **[ALP]** When light of frequency 5.4×10^{14} Hz is shone on to a metal surface the maximum energy of the electrons emitted is 1.2×10^{-19} J. If the same surface is illuminated with light of frequency 6.6×10^{14} Hz, the maximum energy of the electrons emitted is 2.0×10^{-19} J. Use this data to calculate a value for the Planck constant.

Ans: 6.67×10^{-34} Js

8. **[ALP]** The maximum kinetic energy of the electrons emitted from a metallic surface is 1.6×10^{-19} J when the frequency of the incident radiation is 7.5×10^{14} Hz. Calculate the minimum frequency of radiation for which electrons will be emitted. Assume that Planck's constant = 6.6×10^{-34} Js.

Ans: 5.1×10^{14} Hz

9. **[ALP]** Light of frequency 5.0×10^{14} Hz liberates electrons with energy 2.31×10^{-19} J from a certain metallic surface. What is the wavelength of ultra-violet light which liberates electrons of energy 8.93×10^{-19} J from the same surface? (Take the velocity of light to be 3.0×10^{8} ms^{-1} and Planck's constant (h) to be 6.62×10^{-34}Js).

Ans: 0.2×10^{-8} m

10. **[ALP]** The photoelectric work function of potassium is 2 eV and the surface is illuminated with radiation of wavelength 350 nm. What potential difference have to be applied between a potassium surface and the collecting electrode in order just to prevent collection of electrons? What would be the kinetic energy of the electrons? [HSEB 2057]

Ans: 2.47×10^{-19} J

11. **[ALP]** The maximum kinetic energy of the electrons emitted from a metallic surface is 1.6×10^{-19} J when the frequency of the radiation is 7.5×10^{14} Hz. Calculate the minimum frequency of the radiation for which electrons will be emitted. Assume that $h = 6.6 \times 10^{-34}$ Js. [HSEB 2055]

Ans: 5×10^{14} Hz

*[**Note:** Hints to challenging problems are given at the end of this chapter.]*

Conceptual Questions with Answers

1. What is photoelectric effect?

➯ Photoelectric effect is defined as the phenomenon of emission of electrons from a material surface when light of suitable wavelength (or frequency) falls upon it. Here, the light involves not only visible light but all types of electromagnetic radiation. The electrons emitted by photoelectric effect are called photoelectrons and the current so produced is called "photoelectric current". This effect is the evidence of particle nature of radiation.

2. Alkali metals are preferred in photoelectric effect. Why?
Or, Why are metals like Na, Li, and K suited for photoelectric emission.

➯ Alkali metals like Na, K, Li have relatively low work function (ϕ_0). So, visible light can easily eject out the electrons from their surfaces. In photoemitter cells, photo conducting cell, etc,. visible light is used to get the photoelectric current.

3. What is the difference between 'thermionic emission' and ' 'photoelectric emission'?

↳ Some important differences between 'thermionic emission' and 'photoelectric emission' are as follows:

Thermionic Emission	Photoelectric Emission
1. Heat energy is absorbed by the material to eject out the electrons in thermionic emission.	1. Light energy is absorbed by the material to eject out the electrons in photoelectric emission.
2. Thermionic emission is temperature dependent.	2. Photoelectric emission is light frequency and intensity dependent.
3. The emitted electrons are called thermo electrons.	3. The emitted electrons are called photoelectrons.

4. If the intensity of light falling on the emitting substance of a photoelectric cell be increased, what will be the effect on current flowing from the cell?

↳ The intensity of light means the width of light beam (i.e. number of light photons) that comes from the source. High intensity contains large number of light photons and vice-versa. In photoelectric effect, one photon interacts to one electron whatever the energy of photon. Therefore, large intensity of light can produce large number of photo electrons. It means, photoelectric current increases when intensity of light that falls on the electron emitting substances increases.

5. Which is more energetic, a photon of microwave or a photon of ultravoilet?

↳ The energy of photon is inversely proportional to its wavelength i.e.,

$$E = \frac{hc}{\lambda}, \text{ i.e. } E \propto \frac{1}{\lambda}$$

$\lambda_{microwave} > \lambda_{ultraviolet}$

So, $E_{microwave} < E_{ultraviolet}$

It means, the photon of ultraviolet is more energetic than the photon of microwave

6. What do you mean by the work function of cesium is 1.6 eV?

↳ Work function is the minimum energy required to eject out the electron from the surface of a substance. As per the particular example of cesium, at least 1.6 eV light energy is required to eject out electron from the cesium surface. If the energy of light photon is smaller than 1.6 eV, photoelectric effect does not occur in cesium.

7. How does the work function of metal surface affect the kinetic energy of photoelectron?

↳ From Einstein's photoelectric equation,

$$\frac{1}{2} mv^2_{max} = hf - \phi_0$$

where, f = frequency of incident radiation

ϕ_0 = work function of metal

Clearly, if work function of metal increases, the kinetic energy of photoelectron decreases.

8. What is meant by threshold frequency in photoelectric effect? Does it depend upon the intensity of incident light?

↳ The minimum value of frequency of incident radiation for photoelectric effect which is sufficient to eject out the electrons from the surface of a material is called threshold frequency (f_0). The electron ejected out by the light with frequency equal to threshold frequency has zero kinetic energy. Threshold frequency does not depend on the intensity of light.

9. What is meant by momentum of photon?

↳ The momentum of a photon is the product of kinetic (dynamic) mass of the photon and velocity of light. i.e. momentum (p) = $m_{kinetic}$ C

$$= \left(\frac{hf}{c^2}\right) c$$

$$p = \frac{hf}{c}$$

In Einstein's mass-energy relation, $E = mc^2$

$$hf = mc^2$$

$$m = \frac{hf}{c^2}$$

where 'm' is kinetic mass of photon, while rest mass is zero.

---

10. Define thermionic emission. In what sense it is different from photoelectric emission?

✎ When a metal is heated, its free electrons get sufficient thermal energy. Then, these electrons leave the metal surface. This method of removal of electrons is called thermionic emission. In thermionic emission, electrons are emitted due to the absorption of heat energy by metal, however, electrons are ejected due to the exposure of light energy on the metal surface.

---

11. Valence electrons are called free electrons in metal, why don't they emit out spontaneously? Why does external energy (light or heat) require to eject them out?

✎ Free electrons in a metal are free in the sense that they move inside the metal in a constant potential. They are not free to come out of the metal, so additional energy is needed to overcome such potential provided by the nucleus and the surrounding electrons. Hence, heat or light energy must be supplied to eject these electrons.

---

12. Why do different metals have different work functions?

✎ The energy distribution of free electron in different metal is different, i.e. electric potential provided by the nucleus of different metal is different due to their charge distribution. Therefore, the energy required to eject an electron is different in metals. Thus, the average binding energy of electrons is also different in these metals. Hence, different metals have different work functions.

---

13. How does wave theory of light fails to explain the photoelectric effect?

✎ The classical theory (wave theory) of radiation could not explain the main features of photoelectric effect. This theory explains the continuous absorption of energy from radiation, which could not explain: (i) the independence of maximum kinetic energy of photoelectron with intensity of light. (ii) the existence of threshold frequency (iii) the instantaneous nature of phenomena.

---

14. What is a photocell?

✎ Photocell is an electric device which converts light energy into electric energy. It works on the principle of photoelectric effect. When light beam of suitable wave length is exposed on a photosensitive material, photoelectrons are ejected. These photoelectrons are exploited to move in a conducting wire.

---

15. What happens to the wave length of a photon after it collides with an electron?

✎ In photoelectric effect, photon transfers whole energy to eject and to move away the photoelectron from material. So, the incident photon disappears. But in Compton effect, a photon transfers a part of its energy to the colliding electron, so its energy decreases and consequently wave length increases (as $E = \frac{hc}{\lambda}$).

---

16. Ultraviolet light is incident on two photo sensitive materials having work functions W_1 and W_2 ($W_1 > W_2$). In which case, will the kinetic energy of the emitted electrons be greater? Why?

✎ The kinetic energy of photo electron is, $E_k = hf - \phi_0$

where ϕ_0 is work function of a metal.

i. For first metal $E_{k_1} = hf - W_1$, and

ii. For second metal, $E_{k_2} = hf - W_2$

If $W_1 > W_2$, $E_{k_1} < E_{k_2}$

Hence, the kinetic energy the emitted electrons will be greater for the photosensitive material having smaller work function W_2.

---

17. How does 'stopping potential' in photoelectric emission depend upon (i) the intensity of the incident radiation (ii) the frequency of the incident radiation?

(i) Stopping potential does not depend, on the intensity of incident radiation. Intensity of radiation varies the number of emission of photoelectrons but does not change the kinetic energy of photoelectrons. (ii) Stopping potential depends on the frequency of incident radiation, i.e. $eV_s = hf - \phi_0$. ϕ_0 is constant for a material So, $V_s \propto f$.

---

18. What is the effect on the velocity of photoelectrons, if the wavelength of incident light is decreased?

According toe Einstein's Photoelectric equation the kinetic energy of photoelectron is,

$$E_k = hf - \phi_0 = \frac{hc}{\lambda} - \phi_0$$

For constant ϕ_0 ; $E_k \propto \frac{1}{\lambda}$

$$\frac{1}{2} m v^2_{max} \propto \frac{1}{\lambda}$$

$$v^2_{max} \propto \frac{1}{\lambda}$$

$$\therefore \quad v_{max} \propto \frac{1}{\sqrt{\lambda}}$$

As the wavelength of incident light decreases, the velocity of photoelectron increases.

---

19. If ultraviolet rays and x-rays are incident on a metal surface, in which case greater stopping potential is measured?

The stopping potential for a metal is,

$$eV_s = hf - \phi_0 = \frac{hc}{\lambda} - \phi_0$$

work function ϕ_0 is constant for a metal. So,

$$V_s \propto \frac{1}{\lambda}$$

The wavelength of x-rays is shorter than the ultraviolet rays. Hence, greater stopping potential is measured write the metal is exposed with x-rays.

---

20. Can a photon have mass? Explain.

The rest mass of photon is zero. However, the mass and energy are inter convertible quantities. Einstein's mass-energy formula gives relation between them, $E = mc^2$. It means,

$$m = \frac{E}{c^2} = \frac{hf}{c^2}$$

This equivalent mass of photon is called dynamic mass. Therefore, photon has non zero dynamic mass.

---

21. If the frequency of the incident radiation on the cathode of a photocell is doubled, how will the following change.

i. Kinetic energy of the electrons? ii. Photoelectric current? iii. Stopping potential?

(i) Kinetic energy of photoelectron becomes more than double of its original energy. As the work function of the metal is fixed, so incident photon of higher energy will impact more energy to the photoelectron. (ii) Increase in frequency of incident radiation has no effect on photoelectric current. (iii) With the increase in frequency, the kinetic energy of photoelectron increases, so stopping potential also increases.

Exercises

Short-Answer Type Questions

1. What do you mean by quantum nature of radiation?
2. Define (i) photon (ii) work function (iii) threshold frequency
3. What is quantum theory of radiation? How does it explain photoelectric effect?
4. Alkali metals are used as a photoelectric plate, why?
5. What are photoelectrons?
6. What is a photon? Mention its main features.
7. How many photons are required to eject one photoelectron?
8. What is the effect of increase in intensity on photoelectric current?
9. How many electron volt make one joule?
10. Can photoelectric effect be explained on the basis of wave theory of radiation? Explain.
11. What is meant by work function of a metal?
12. Write down the laws of photoelectric emission.
13. Which photon is more energetic blue one or red one?
14. Human skin is relatively insensitive to visible light, but ultraviolet radiation can cause severe burns. Does this have anything to do with photon energies? Explain.
15. What is the rest mass of a photon? What is its significance?
16. What is the relation between momentum and energy of a photon?
17. The work function of silver is 4.73 eV. What does it mean?
18. Define threshold frequency. Is it the same for all metals? Why?
19. "When the intensity of incident light is increased there is no increase in the kinetic energy of the photoelectrons" why?
20. What is a photo cell?

Long-Answer Type Questions

1. What is photoelectric effect? Discuss the Einstein's photoelectric equation.
2. Establish Einstein's photoelectric equation. Use this equation to establish laws of photoelectric of emissions.
3. What is photoelectric effect? Discuss Einstein's photoelectric equation. Does the work function of a metal depend in intensity of light?
4. Discuss photoelectric effect and derive Einstein's photoelectric equation. What is stopping potential?
5. Explain photoelectric effect to write Einstein's photoelectric equation. Describe Millikan's laboratory method to determine Planck's constant.
6. Explain Millikan's experiment for the verification of Einstein's photoelectric equation.
7. What is work function of a metal? Does it depend on the intensity of incident light? Discuss Einstein's photoelectric equation.
8. What is photoelectric effect? Derive Einstien's photoelectric equation. Define various terms used in it.

Numerical Problems

1. What is the energy associated with a photon of wavelength 900 nm?

   Ans: 2.2×10^{-19} J

2. Calculate the wavelength associated with a photon of energy 19.8×10^{-16} J.

   Ans: 1×10^{-10} m

3. The work function of a metal is 2 eV. What is its threshold frequency?

Ans: 4.8×10^{16} Hz

4. Find the threshold wavelength if the work-function of a metal is 1.6 eV.

Ans: 7.76×10^{-7}m

5. What energy is carried by one quantum of sodium light of wavelength 5893 Å ?

Ans: 3.36×10^{-19} J

6. Light of wavelength 4×10^{-7} m falls on a sodium surface. What is the maximum energy of the emitted electron in electron volts? (The work function of sodium = 2.3 eV, h = 6.62×10^{-34} Js)

Ans: 0.3 eV

7. A photon of green light has wavelength of 520 nm. Find the photon's energy. Express the energy both in joules and electron volts.

Ans: 2.38 eV

8. The predominant wavelength emitted by an ultraviolet lamp is 248 nm. If the total power emitted at this wavelength is 120 W, how many photons are emitted per second?

Ans: 149.8 *x* 10^{25}

9. A clean nickel surface is exposed to light of wavelength 235 nm. What is the maximum speed of the photoelectrons emitted from this surface? (ϕ_0 = 5.1 eV)

Ans: 0.25 *x* 10^{-17}J

10. What is the mass, momentum and energy of a photon of wavelength 1 Å ?

Ans: 2.2×10^{-32} kg, 6.6×10^{-24} kgm/s, 19.8×10^{-16} J

11. Calculate the frequency of a photon with energy 7.5 eV.

Ans: 1.8×10^{15} Hz

12. The power output of an FM radio transmitter is 150 kW and it operates at a frequency of 99.7 MHz. How many photons per second does the transmitter emit?

Ans: 2.27×10^{30} photons/s

13. The work function of sodium metal is 2.3 eV. What is the longest wavelength of light that can cause photo-electrons?

Ans: 5397 Å

14. The photoelectric threshold wavelength of a metal is 5000 Å . Find (i) the value of work function in eV, (ii) the kinetic energy of the photoelectrons, in eV ejected by the light of wavelength 4000 Å .

Ans: (i) 2.47 eV, (ii) 0.62 eV

15. The work function of molybdenum is 5 eV. When a light of unknown wavelength falls upon it the maximum velocity of the ejected photoelectron is 1.62×10^{6} m/s. Find the incident wavelength.

Ans: 1000 Å

16. The threshold wavelength for producing photoelectrons from a metal surface is 372 nm. What is the work function of this surface, in eV?

Ans: 3.34 eV

17. The kinetic energy of the most energetic photoelectrons is doubled when the wavelength of the incident radiation is reduced from 400 nm to 310 nm. What is the work function of metal?

Ans: 0.9 eV

18. Sodium has a work function of 2 eV. Calculate the maximum energy and speed of the emitted electrons when sodium is illuminated by radiation of wave length 150 nm. (Given mass of electron = 9.1×10^{-31} kg)

Ans: 1.48×10^{6} m/sec. and 9.96×10^{-19} J

19. Electrons with maximum kinetic energy of 3 eV are ejected from a metal surface by ultra-violet radiation of wavelength 1.5×10^{-7} m. Determine work function, threshold wavelength and the stopping potential for the metal (Planck's constant, h = 6.62×10^{-34} Js)

Ans: 5.275 eV, 2.35×10^{-7} m, 3V

20. When ultraviolet light with a wavelength of 400 nm falls on a certain metal surface, the maximum kinetic energy of the emitted electrons is 1.10 eV. What is the maximum kinetic energy of the photoelectrons when light of wavelength 300 nm falls on the same surface?

Ans: 2.137 eV

Multiple Choice Questions

1. Emission of electrons from the surface of metal by action of light on it is:
   a. Thermionic emission
   b. Photoelectric emission
   c. Electronic emission
   d. Cold emission
2. Emission of electrons due to light is called:
   a. Thermionic emission
   b. Photoelectric emission
   c. Both a and b
   d. none.
3. The wavelength of a particle having mass m and moving with velocity v is given by:
   a. $\frac{h}{mv}$
   b. $\frac{hv}{m}$
   c. $\frac{hm}{v}$
   d. $\frac{mv}{h}$
4. The dimensional formula for Plank's constant is:
   a. $M^1L^2T^{-1}$
   b. $M^1L^2T^2$
   c. MLT^{-2}
   d. $M^2L^2T^{-1}$
5. Photoelectric effect is based on the principle of conservation of:
   a. Energy
   b. Momentum
   c. Angular momentum
   d. Power
6. The UV photon is incident on a metal of photoelectric work function 2 eV and produces a photoelectron of energy 2 eV. The wavelength associated with photon is:
   a. 9300 Å
   b. 6200 Å
   c. 4900 Å
   d. 3100 Å
7. If the λ of electron is 1Å and Planck's constant $h = 6.6 \times 10^{-34}$ Js, then find the momentum of electron:
   a. 3.3×10^{-22} kg ms^{-1}
   b. 6.6×10^{-24} kg ms^{-1}
   c. 6.6×10^{-22} kg ms^{-1}
   d. 6.6×10^{-34} kg ms^{-1}
8. Planck's constant is given as 6.6×10^{-34} Js. The minimum wavelength of x-rays emitted by x-rays tube operating at 30 kilovolt in Å will be nearly:
   a. 0.2
   b. 0.4
   c. 0.6
   d. 0.8
9. Threshold frequency of a metal whose work function is 4.5 eV is:
   a. 1.09×10^{15} Hz
   b. 1.09×10^{12} Hz
   c. 1.09×10^{6} Hz
   d. 0.09×10^{3} Hz
10. What is the wavelength of an electron?
    a. $\frac{h}{2\pi}mv$
    b. $\frac{h}{mv}$
    c. $\frac{h}{2\pi mv}$
    d. $\frac{2\pi}{mv}$
11. Einstein's photoelectric equation states that $E_k = E - W$. In this equation E_k refers to:
    a. K.E. of all the emitted electrons
    b. Mean K.E. of the emitted electrons
    c. Maximum K.E. of the emitted electrons
    d. Minimum K.E. of emitted electrons

12. In an experiment on photoelectric effects, stopping potential is 1.0 V when light of wavelength 6520 Å is incident on the emitting surface. The shopping potential is 2.9 V for light of wavelength 3260 Å. The work function of the metal is

a. 0.9 eV
b. 1.9 eV
c. 5.8 eV
d. Cannot be deduced from the given data.

13. If in a photoelectric experiment, the wavelength of incident radiation is reduced from 6000 Å to 4000 Å, then

a. Stopping potential will decrease.
b. Stopping potential will increase
c. Kinetic energy of emitted electrons will decrease
d. The value of work function will decrease.

14. Graph of maximum kinetic energy of the photoelectrons against v, the frequency of the radiation incident on the metal, is a straight line of slope equal to:

a. work function
b. stopping potential
c. $\frac{h}{e}$
d. h

15. The work functions for metals A, B and C are respectively 1.92 eV, 2.0 eV and 5 eV. According to Einstein's equation, the metals which will emit photoelectrons for a radiation of wavelength 4100 Å is/are

a. none
b. A only
c. A and B only
d. All the three metals

16. The time taken by a photoelectron to come out after the photon strikes is approximately

a. 10^{-16} s
b. 10^{-1} s
c. 10^{-4} s
d. 10^{-10} s

17. The momentum of a photon of energy 1 MeV in kg m s^{-1}, will be

a. 0.33×10^{6}
b. 7×10^{-24}
c. 10^{-22}
d. 5×10^{-22}

18. A non-monochromatic light is used in an experiment on photoelectric effect. The stopping potential is

a. Related to the mean wavelength.
b. Related to the longest wavelength.
c. Related to the shortest wavelength.
d. Not related to any of the wavelength.

19. The work function of a substance is 4.0 eV. The longest wavelength of light that can cause photoelectron emission from this substance is approximately.

a. 540 nm
b. 400 nm
c. 310 nm
d. 220 nm

20. The surface of a metal is illuminated with the light of 400 nm. The kinetic energy of the ejected photoelectrons was found to be 1.68 eV. The work function of the metal is:

(hc = 1240 eV nm)

a. 3.09 eV
b. 1.42 eV
c. 1.51 eV
d. 1.68 eV

21. Maximum velocity of the photoelectrons emitted by a metal surface is 1.2×10^{6} m s^{-1}. Assuming the specific charge of the electron to be 1.8×10^{11} C kg^{-1}, the value of the stopping potential in volt will be

a. 6
b. 4
c. 3
d. 2

22. A monochromatic source of light emits photons of frequency 6×10^{14} Hz. The power emitted by the source is 8×10^{-3} W. Calculate the number of photons emitted per second.
(Take h = 6.63×10^{-34} Js)

a. 6×10^{14} b. 4×10^{15}
c. 2×10^{16} d. 1×10^{17}

23. The energy of a photon of wavelength 390 nm is nearly

a. 6.6 eV b. 3.2 eV
c. 5.5 eV d. 1.2 eV

24. A steel ball of mass m is moving with a kinetic energy K. The de Broglie wavelength associated with the ball is

a. $\frac{h}{2mK}$ b. $\sqrt{\frac{h}{2mK}}$
c. $\frac{h}{\sqrt{2mK}}$ d. meaningless

25. The de Broglie wavelength of an electron moving in the n^{th} Bohr orbit of radius r is given by

a. $\frac{2\pi r}{n}$ b. $n\pi r$
c. $\frac{nr}{2\pi}$ d. $\frac{nr}{\pi}$

Answers

1. (b) 2. (b) 3. (a) 4. (a) 5. (a) 6. (d) 7. (b) 8. (b) 9. (a) 10. (b) 11. (c) 12. (a) 13. (b)
14. (d) 15. (c) 16. (d) 17. (d) 18. (c) 19. (c) 20. (b) 21. (b) 22. (c) 23. (b) 24. (c) 25. (a)

Hints to Challenging Problems

HINT: 1

Given,

λ = 520 nm = 520×10^{-9}m

i. $f = \frac{c}{\lambda} = \frac{3 \times 10^8}{520 \times 10^{-9}}$

ii. Momentum (p) = $\frac{h}{\lambda}$

iii. Energy (E) = hf

HINT: 2

Given,

λ = 248 nm = 248×10^{-9} m

P = 12.0 W

Number of Photons per second $\left(\frac{n}{t}\right)$ = ?

Since energy of each photon is hf, so energy of n - photons estimated is E = nhf

$\therefore\ P = \frac{E}{t}$

or $P = \frac{n\,h\,f}{t}$

or $\frac{n}{t} = \frac{P}{hf}$

$= \frac{P\lambda}{hc}$

HINT: 3

Given,

λ = 235 nm = 235×10^{-9} m, h = 6.62×10^{-34} Js

$\phi_0 = 5.1$ eV = $5.1 \times 1.6 \times 10^{-19}$ J

m = 9.11×10^{-31} kg

v_{max} = ?

From photoelectric equation, we have

$$\frac{1}{2} m_e v^2_{max} = hf - \phi_0$$

$$= \frac{hc}{\lambda} - \phi_0$$

$$\text{or } v_{max} = \sqrt{\frac{2}{m}\left(\frac{hc}{\lambda} - \phi_0\right)}$$

HINT: 4

Given, λ_{max} = 700 nm = 700×10^{-9} m

The minimum work function is associated to the longest wavelength, so

$$\phi_{min} = h\frac{c}{\lambda_{max}}$$

HINT: 5

Given,

λ = 254 nm = 254×10^{-9} m

V_S = 0.181 V

(a) threshold wavelength, λ_0 = ?

From photoelectric equation, we have

$$\frac{1}{2} mv^2_{max} = hf - hf_0$$

or $eV_s = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$

or $\frac{hc}{\lambda_0} = \frac{hc}{\lambda} - eV_s$

or $\lambda_0 = \frac{hc}{\frac{hc}{\lambda} - e V_s}$

(b) Work function, $\phi_0 = ?$

We know that

$$\phi_0 = h f_0 = \frac{hc}{\lambda_0}$$

HINT: 6

Given,

$E = 2.45$ MeV

$= 2.45 \times 1.6 \times 10^{-19} \times 10^6$ J

$= 3.92 \times 10^{-13}$ J

(a) $f = \frac{E}{h}$

(b) $\lambda = \frac{c}{f}$

HINT: 7

Given,

$f_1 = 5.4 \times 10^{14}$ Hz

$E_1 = 1.2 \times 10^{-19}$ J

$f_2 = 6.6 \times 10^{14}$ Hz

$E_2 = 2 \times 10^{-19}$ J

$h = ?$

From Photoelectric equation, we can write

$E_1 = hf_1 - \phi_0$

$E_2 = hf_2 - \phi_0$

Subtracting (i) from (ii), we get

$E_2 - E_1 = h (f_2 - f_1)$

or $h = \frac{E_2 - E_1}{f_2 - f_1}$

HINT: 8

Given,

$E_{max} = 1.6 \times 10^{-19}$ J

$f = 7.5 \times 10^{14}$ Hz

$h = 6.6 \times 10^{-34}$ Js

$f_0 = ?$

From photoelectric equation, we have

$E_{max} = h f - h f_0$

or $h f_0 = h f - E_{max}$

$\therefore f_0 = f - \frac{E_{max}}{h}$

HINT: 9

Given,

$f_1 = 5 \times 10^{14}$ Hz

$E_1 = 2.31 \times 10^{-19}$ J

$E_2 = 8.93 \times 10^{-19}$ J

$\lambda_2 = ?$

From Photoelectric equation, we can write

$E_1 = hf_1 - \phi_0$

$E_2 = hf_2 - \phi_0$

Subtracting (i) from (ii), we get

$E_2 - E_1 = h (f_2 - f_1)$

or $\frac{E_2 - E_1}{h} = \frac{c}{\lambda_2} - f_1$

or $\frac{E_2 - E_1}{h} + f_1 = \frac{c}{\lambda_2}$

HINT: 10

Given,

$\phi_0 = 2 \text{ eV} = 2 \times 1.6 \times 10^{-19} \text{ J} = 3.2 \times 10^{-19}$ J

$\lambda = 350 \text{ nm} = 350 \times 10^{-9}$ m

$e = 1.6 \times 10^{-19}\ J$

$V_s = ?$

Now,

$eV_s = hf - \phi_0$

Find V_S and use it in

or $\frac{hc}{\lambda} = \phi_0 + eV_s$

Maximum K.E. $= eV_s$

HINT: 11

Given,

$E_1 = 1.6 \times 10^{-19}$ J when $f_1 = 7.5 \times 10^{14}$ Hz

$f_0 = ?$

We know,

$hf_1 = hf_0 + E_1$

or $hf_0 = hf_1 - E_1$

❖ ❖ ❖

SEMICONDUCTOR

21
CHAPTER

21.1 Introduction

In the modern world electricity has become a vital part of life. We use electricity as a way of transferring energy from place to place for heating, lighting, moving things from place to place. For this purpose, we use different types of materials depending up on their conducting properties. These conducting properties in turn have their tremendous use in the realm of modern science and technology. It is the study of conducting properties of materials that has helped us to design the instruments such as magnetic levitating trains, magnetic resonance imaging, and many other devices that help us to probe into the world ranging from the atomic to cosmos. In this chapter, we shall discuss the conduction properties of different materials. Especially, the mechanism of conduction in the semiconductors and their use will be dealt in depth here.

21.2 Band Theory of Solids

The electrons revolving round the nucleus in particular orbits carry particular energy of that orbit. So, in an isolated atom, the electrons possess discrete energies as determined by Bohr's theory. But, in a crystal there are many atoms that are arranged in a regular pattern. In a crystal, the inter-atomic spacing decreases and atoms interact with all other neighboring atoms. So, the electrons in different orbit do not possess defined energy. For example, the electrons in valence orbit of different atoms now have different energies that it would have when the atom is isolated. In fact, some of the valence electrons have slightly less energy while others have slightly more energies than in the case of isolated atoms. The valence electrons in different atoms now have different energies that differ by very small amount. The energy difference is so small that, it constitutes a continuous range. In another word, it can be said that energy level of different atoms splits up into many separated levels due to atomic interaction which are so closely spaced that they may be treated as a continuous band of allowed energy states. The energy of electrons can change almost continuously in these bands. However, the electrons in the inner orbits are less affected by this interaction. The plot of inter-atomic spacing with energy is as shown in Fig. 21.1.

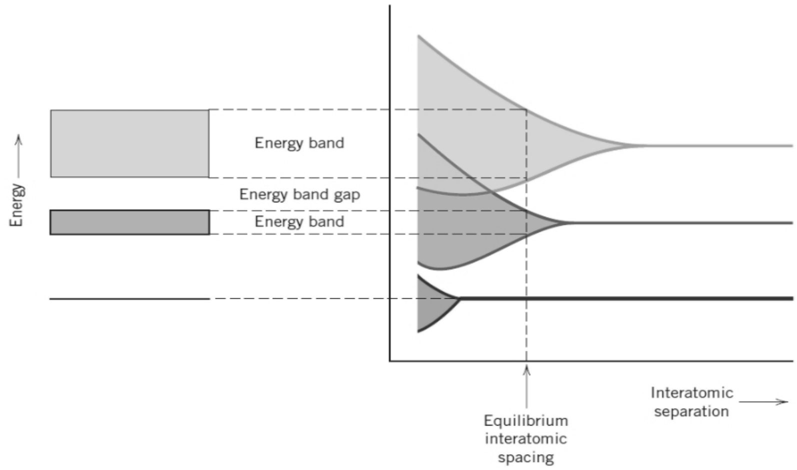


Fig. 21.1: Energy bands in crystalline solid

The range of energy possessed by the electrons in an orbit due to atomic interaction is known as energy bands. There are following important energy bands in solids according to band theory.

Valence band

Valence band is the range of energies possessed by the electrons in the valence orbitals. This is the band that valence electrons actually occupy. The electrons in this band are known as valence electrons. This band is completely or partially filled but is never completely empty.

Conduction band

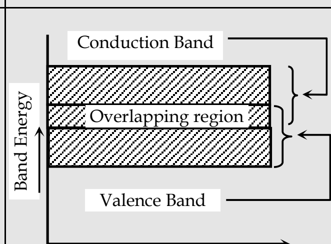
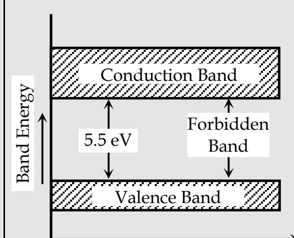
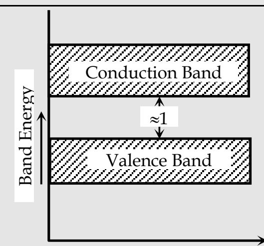
Conduction band is the range of energies possessed by the electrons that have jumped from the valence band when excited. The electrons in these bands have enough energy to move freely in the material and are responsible for the conduction of electricity. So, these electrons are known as conduction electrons. Thus, the range of energy possessed by the conduction electrons is known as conduction band. This band can be empty or partially filled depending upon the material but are never completely filled.

Forbidden band

The energy gap between the valence and conduction band is known as forbidden band. This is also known as band gap and is the characteristic of different materials. This gap corresponds to the energy that must be supplied to excite a valence electron to make it conduction electron. Larger the band gap, the greater is the bond between the valence electrons and the nucleus. And so, greater amount of energy has to be supplied to valence electrons in order to excite them to conduction band. This band is completely empty as there are no allowed energy states. This means, the electrons are forbidden to be in this band and hence the name forbidden band.

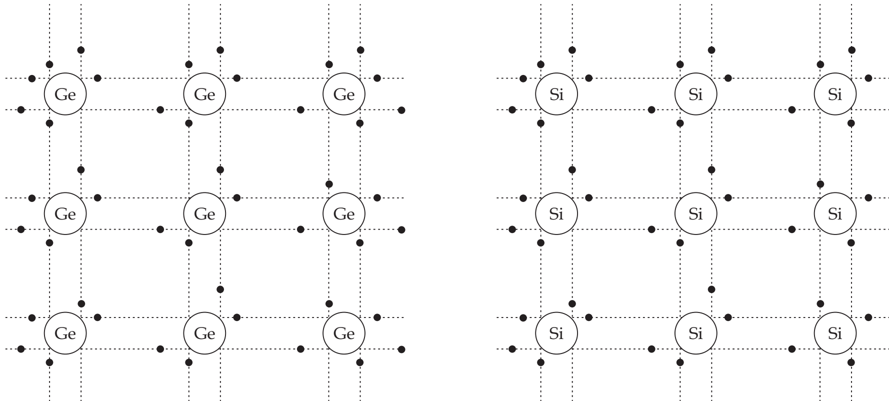
Classification of solids on the basis of band theory:

S. N.	Conductors	Insulators	Semiconductors
1.	In such materials, the valence band and the conduction band overlap each other, i.e. there is no band gap.	There is large bad gap between the valence band and conduction band.	The band gap is very small.

2.	Large number of conduction electrons is available owing to no band gap. So, these materials are good conductor of electricity.	There are no electrons in the conduction band and hence these materials are bad conductors of electricity. Very high electric field has to be supplied to make the valence electrons jump to the conduction band.	Some of the conduction electrons are always available at room temperature. The thermal energy at room temperature is sufficient enough to overcome the band gap in such materials.
3.	Since the valence and conduction band overlap, there are plenty of free electrons available in the conduction band.	These have completely filled valence band and completely empty conduction band.	The conduction band of such materials is completely empty at absolute zero and hence serves as perfect insulator at this temperature. However, at room temperature, both valence band and conduction band are partially filled.
4.	Examples:metals like copper, sodium , silver, etc.	Examples: Nonmetals like diamond, paper, glass, air etc.,	Examples: Germanium, Silicon, graphite etc.
			

21.3 Semiconductors

Semiconductors are those substances whose electrical resistivity is intermediate between those of good conductors and good insulator. These substances usually form the Group IV - elements of the periodic table. Semiconductor in a bulk is regular crystal of these elements which bind each other with covalent bond. Out of many semiconductor elements, Silicon and Germanium are the most studied in terms of their use in modern electronics. Both of these elements have four electrons in their outermost orbit each of which are involved in the covalent bonding with other atoms to form a crystalline structure as shown in Fig. 21.2 (i) and (ii). The figure shows, the covalent bonding between the electrons in Ge and Si.



(i) (ii)

Fig. 21.2: (i) Ge-atoms in a crystal (ii) Si-atoms in a crystal

Practically, there are no free electrons (charge carriers) in them. This is the case when the temperature is at absolute zero. And hence, such materials behave as perfect insulator at absolute zero of temperature. In terms of band theory, the conduction band of such materials is completely empty at absolute zero. The band gap between conduction band and valence band is however very small for such materials (1.12 eV for Si a 0.67 eV for Ge). When the temperature is slightly raised, even at room temperature; the covalent bonds are broken and the electron acquire enough energy to jump to conduction band owing to small band gap. The electrons in the conduction band are those dissociated from their parent atoms which are free to move about the crystal. This means, the semiconductor now becomes conducting. The number of these electrons increases rapidly with temperature. But, only the electrons in the conduction band don't tell about the electrical conduction in the semiconductors which shall be discussed in this chapter.

21.4 Charge Carriers in Semiconductor

As discussed above, when the electrons from valence band jump to the conduction band, these leave behind a vacancy for an electron. This vacancy of electron in the valence band is called hole. This vacancy can be occupied by an electron from neighbouring atom, leaving it again with a vacancy. This vacancy has an effective positive electronic charge and behaves as an apparent free particle with a charge +e. So, this vacancy can travel through the material and serve as an additional current carrier. This means there are two types of charge carrier in a semiconductor; one the electrons in the conduction band and the other holes in the valence band.

21.5 Types of Semiconductor

The electrical sensitivity of semiconductor tremendously varies when small concentration of suitable impurities is added. The process of adding suitable impurities to pure semiconductor is known doping. These semiconductors added with impurities are called extrinsic semiconductors where as the pure form of semiconductors are called intrinsic semiconductors.

Semiconductor

Intrinsic Semiconductor
(Pure form of semiconductor)

Extrinsic Semiconductor
(Impurity added semiconductor)

Intrinsic Semiconductor

A semiconductor in its pure form and free from all kind of impurities is called intrinsic semiconductor. That means, pure Silicon (Si) and pure Germanium (Ge) are intrinsic semiconductor.

The Silicon and Germanium each has 4 electrons in its outermost orbit. Each of these valence electrons forms a covalent bond with neighbouring atoms in a crystalline structure and form a perfect diamond like structure. That is, all the electrons of Si or Ge are bonded and hence are not available for conduction. However, even at room temperature, these electrons get enough thermal energy to excite them to the conduction band thereby breaking covalent bond. These electrons in conduction band account for the electrical conductivity. If the temperature is increased, more electrons break the covalent bonds and become available for conduction. This means, conductivity of semiconductor increases with temperature i.e. resistivity decreases with temperature. So, semiconductors are said to have negative temperature coefficient of resistance.

After the electrons jump to the conduction band, empty spaces are left behind in the valence band. These empty spaces are called holes. So, in intrinsic semiconductor number of holes in valence band is equal to number of electrons in conduction band.

Extrinsic Semiconductor

These are the semiconductors obtained by adding suitable impurities to pure form of semiconductor. This process of adding impurities to pure semiconductor is called doping. Such process also can lead to increased carrier concentration. Depending upon the impurities used for doping, extrinsic semiconductors can be classified into following two categories.

i. p-type ii. n-type

P-type semiconductor

These are the semiconductors obtained by doping trivalent impurities such as indium, aluminium, Gallium etc. to pure form of Silicon or Germanium. When a trivalent impurity say indium (In) is added to a Silicon crystal three of its valence electrons share covalent bonding with three neighbouring Silicon (host) atoms but the fourth bond with the Silicon is incomplete. So, the indium atom robs an electron from neighbouring covalent bond and possesses eight electrons in its valence shell as shown in Fig. 21.3. Meanwhile, a vacancy of electron called hole is created in the covalent bond from where electron has been robbed. Thus, for every trivalent impurity added to the Silicon crystal a hole will be created and hence accepts electron from Silicon crystal. So, it is also called acceptor atom. The number of acceptor atoms is equal to number of holes. At higher temperatures, the electrons can get knocked out of the bond and rise to conduction band. And again, this process also creates a hole for each transition. Hence, holes are greater in number than conduction electrons in such materials and are called majority charge carriers. Conduction electrons are called minority charge carriers in p-type semiconductor. Since the majority charge carriers are the positive holes, it is called P-type (or p-type) semiconductor. This can be remembered as (**P**-for **p**ositive). Though we call it positive type, the material as a whole is electrically neutral. However, the concentration of the positive holes is more.

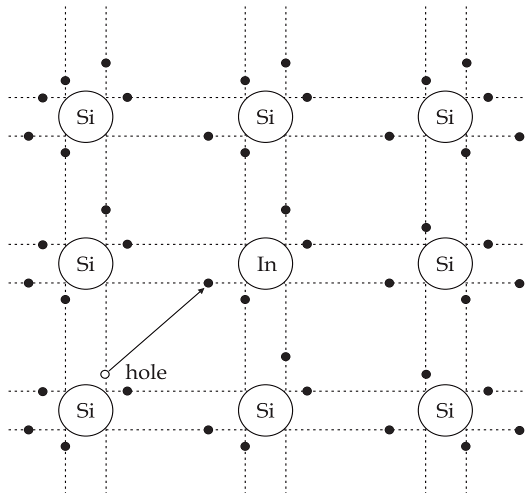


Fig. 21.3: p-type crystal formed by doping Indium (In) in Silicon (Si) crystal

N-type semiconductor

When a pure form of semiconductor is doped with pentavalent impurities such as arsenic (As), antimony, phosphorus etc, the resulting semiconductor is N-type (or n-type) semiconductor. Such impurities have five valence electrons in their outer most orbits, four of which are engaged in the covalent bonding with the Silicon atoms. The fifth electron is unattended as shown in Fig. 21.4 and hence is available for conduction. As such impurities provide free electrons for conduction, they are called donor impurities. Again, due to temperature effects, some of the covalent bonds are broken and electrons free from such bonds jump to conduction band leaving behind corresponding hole in the valence bond. However, the number of conduction electrons in conduction band are greater and are known as majority charge carriers. And holes in valence band are called minority charge carriers. Since majority charge carriers are the negatively charged electrons, these are called **n**-type semiconductor. It can be remembered as **n** for negative, however the material as a whole is electrically neutral.

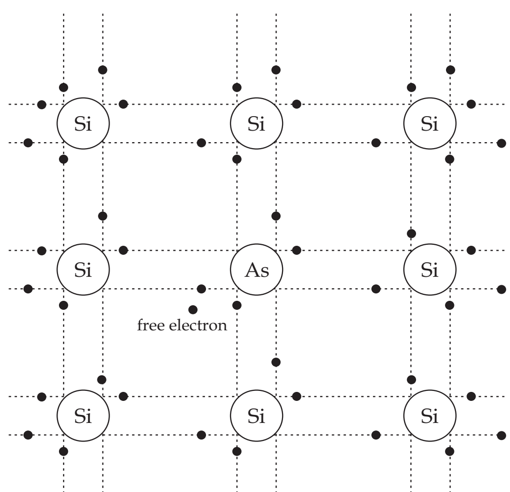


Fig. 21.4: N-type crystal formed by doping arsenic (As) in the Silicon (Si) crystal

21.6 P-N Junction Diode (Semiconductor diode)

When a pure form of a semiconductor is doped with p-type material at one end and n-type of material at another extreme end the resulting semiconductor material is called P-N junction (or p-n junction) diode. A p-n junction diode can also be formed by fusing a p-type semiconductor with n-type semiconductor forming a continuous structure. The surface of contact between two types of material is called a p-n junction. The block diagram for p-n junction diode is as shown in Fig. 21.5 (i) and its circuit symbol is as shown in Fig. 21.5 (ii).

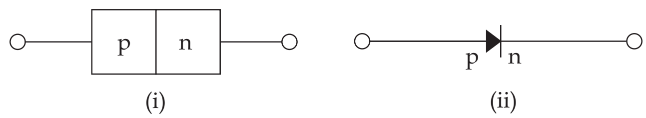


Fig. 21.5: (i) Block diagram for p-n junction diode (ii) Symbolic representation

A junction diode is called so, because it consists of two electrodes. P-side of the semiconductor acts as anode and n-side acts as cathode. In Fig. 21.5 (ii), the arrow head represents p-side (anode) and bar represents n-side (cathode). The conventional current flows in the direction of arrow head.

The p-side of diode has higher concentration of positive charges (holes) and n-side of diode has higher concentration of negative charges (free electrons). These are known as majority charge carriers. However small concentration of negative charge carriers (electrons) exists in p-type and that of positive charge carriers (holes) exists in n-type which are the minority charge carriers in them respectively.

As we know, the charges always flow from higher concentration region to lower concentration region; as soon as the junction diode is formed; positive charge (holes) from p-side start to diffuse towards n-side and negative charge (free electrons) diffuse towards p-side from n-side.

Actually, the free-electrons is n-side occupy conduction band but holes in p-side occupy valence band.

When the free electrons from conduction band of n-side diffuse towards p-side, the n-side near to junction leaves behind a positive immobile ion where as the p-side near to junction becomes negative after accepting the diffused electron.

Similarly, when electrons from valence band of n-side move towards p-side, it recombine with a hole resulting a negative immobile ion near the junction of p-side. And the n-side near the junction becomes positive. It seems as if, the hole of the p-side has diffused to n-side to recombine with the electron so as to leave behind a negative immobile ion.

The result is that, the p-side of the semiconductor near the junction is left with negative immobile ions and the n-side of the semiconductor near the junction is left with positive immobile ions. This process continues till equilibrium is reached. Due to the formation of these immobile ions, the electrons in the n-region are pushed away from the junction and holes in the p-side are pushed away from the junction due to electrostatic repulsive force.

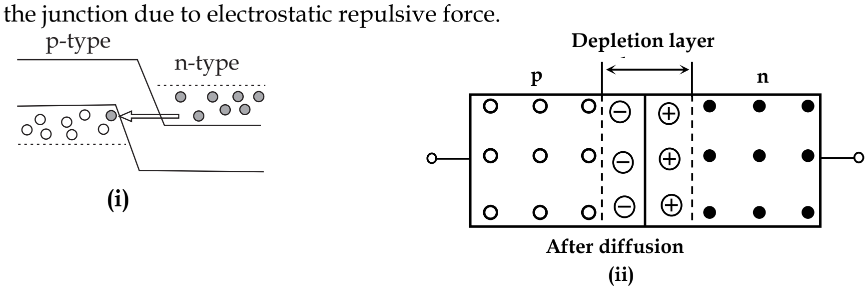


Fig. 21.6: (i) Diffusion of charge particles in a diode (ii) Formation of depletion layer

Thus, near the junction on its either side, a region is formed which is practically devoid or depleted of free charge carriers. This region is known as depletion layer or region as shown in Fig.21.6(ii). In this region an electric field is set up due to the difference in potential that is directed from n to p region which is responsible to stop further diffusion of charge particles. This potential difference which acts as a barrier for the further diffusion of charge carrier is known as potential barrier.

The barrier potential is found to depend upon the concentration of free charge carriers. Smaller the concentration, greater the distance that charge particles have to travel to suffer collision with holes or electrons where they get annihilated or recombined. And the potential barrier will be weak. If the concentration is greater then, the charge particles will have to travel small distances and the barrier potential will be stronger. This barrier potential also depends on the nature of crystal and temperature as the charge concentration up turn depends on the temperature.

21.7 Working of a P-N Junction Diode

A junction diode should be connected to suitable external voltage for its operation. The process of applying external voltage to the junction diode is known as biasing of junction diode. Biasing can be done in following two ways.

i. Forward biasing ii. Reverse biasing

Forward biasing

A p-n junction diode is said to be forward biased if its p-side is connected to positive terminal and n-side is connected to negative terminal of the external voltage source as shown in Fig.21.7.

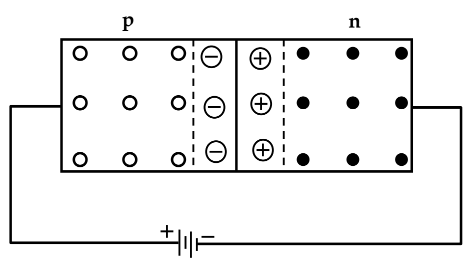


Fig. 21.7: Forward biasing in semiconductor diode

During forward bias condition, the free electrons that are abundant in the n-region experience electrostatic repulsive force of the applied field and hence move towards the junction region. In this process they leave behind positive fixed ions on the right side of the junction in the n-region. These immobile positive ions attract electrons from the negative terminal of the battery and a flow of charge is maintained.

Similarly, the free holes are pushed toward the junction due to repulsive force of the field which leaves behind the negative immobile ions of left hand side of junction in the p-region. The positive terminal of battery attracts electrons from these negative ions thus creating holes again in the p-region. And the same cycle is true for the holes in p-region and electrons in n-region. In this way, a continuous current is set up in the circuit. Thus, a diode conducts in forward biased condition. In this condition, the current inside the semiconductor is due to the recombination of holes and electrons where as in the external circuit, the current is due to electrons only. The width of the depletion layer decreases in forward biased condition.

Reverse biasing

A p-n junction diode is said to be reversed biased, if its p-side is connected to negative terminal and n-side is connected to positive terminal of the external voltage source as shown in Fig.21.8.

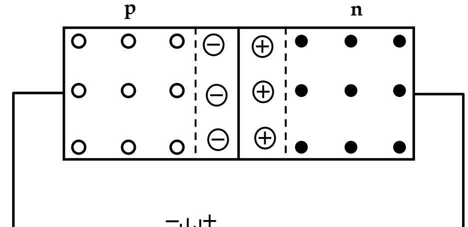


Fig. 21.8 Reverse biasing in semiconductor diode

Due to the applied reverse voltage, the free conduction electrons abundant in n-region are pulled toward the positive terminal and the holes abundant in p-region are pulled towards the negative terminal of the battery. The result is that, the junction region towards p becomes more negative and that toward n become more positive. This is to say, the width of the depletion layer continuously increases till the barrier potential equals the applied potential. At this condition, the applied potential and barrier potential are in the same direction and are thus added which prevents the recombination of holes and electrons. This means no charge particle can cross the barrier and hence the current in the circuit is zero. So, a p-n junction diode does not conduct in reverse bias condition.

However, a semiconductor diode has minority charge carriers on the either side of the junction even at ordinary temperature, i.e. at higher (or room) temperature, p-side always has small concentration of free electrons and n-side has small concentration of holes. So, a small current is always present in the circuit due to flow of these minority charge carriers even in the reverse bias condition.

21.8 Diode Characteristics and Its Study

Diode characteristics refer to the current and voltage relationship especially the variation in current due to the external voltage. A plot of current as a function of voltage is known as I-V characteristics of diode.

The circuit arrangement to study the I-V characteristics both for forward and reverse biased condition are shown in Fig. 21.9 (i) and 21.9 (ii) respectively.

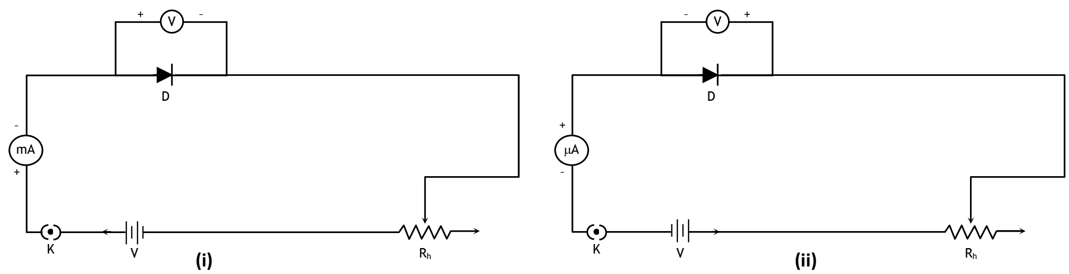


Fig. 21.9: (i) Forward biased diode (ii) Reverse biased diode

During the forward biased condition, when the applied voltage is gradually increased from zero, current in the circuit also increases from zero. This current can be recorded by a milliameter (mA) connected to the circuit as shown in Fig.21.9 (i). The current slowly increases upto a certain value of applied voltage which is 0.7 V for Silicon and 0.3 V for Germanium diode. After the applied voltage reaches this value there is abrupt rise in current and this particular value of voltage is called knee voltage. The I-V characteristics for forward biased condition is shown by right part of the graph in Fig. 21.10 (i).

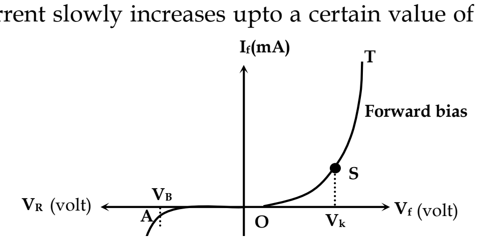


Fig. 21.10(i): I-V characteristics of diode

In reversed biased condition, the reverse current due to majority is zero (however small current always exists due to minority charge carriers) for zero of the applied voltage.

When the applied reverse voltage is slightly increased, there is small current in the circuit due to minority charge carriers which is almost constant for wide range of voltage applied. This current is known as saturation current. This current is in the range of nanoampere for Silicon diode and in the range of microampere for Germanium diode. However, when the reverse voltage is applied beyond a particular value, the current rises sharply due to breakdown of diode. There are two types of breakdown occurring in a junction diode.

i. Avalanche break down   ii. Zener breakdown

i. Avalanche breakdown

Under reverse biased condition, the conduction in diode takes place due to minority charge carriers. In moderately doped semiconductor diodes when the reverse bias voltage is increased, the minority charge carriers tend to accelerate and their kinetic energy increases. These energetic minority carriers collide with stationary atoms and impart some energy to valence electrons present in the covalent bonds. These electrons after acquiring energy break their covalent bonds and jump to conduction band to become free conduction electrons. And these electrons further knock some of the valence electrons out. In this process, holes are also created due to vacancy left behind in valence shell. And in this way, charge carrier multiplication takes place and hence huge current is observed. This is known as avalanche breakdown.

ii. Zener breakdown

Zener breakdown occurs in junction diodes which are heavily doped. In such diodes, the depletion layer is very thin. So the electrons (minority) in valance band of p-type material tunnel to the conduction band of n-type material giving rise to a reverse current. This is known as the zener effect and is the quantum tunneling effect.

A typical I-V characteristic curve showing forward current and reverse current in zener and avalanche breakdown is as shown in Fig.21.10 (ii).

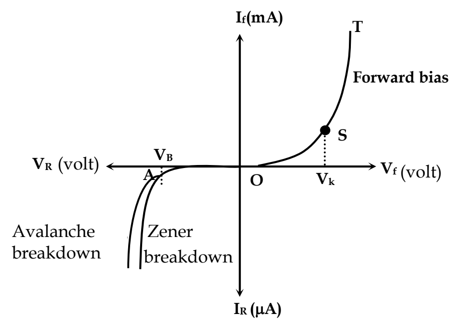


Fig. 21.10 (ii): I-V Curve

21.9 Semiconductor Diode as Rectifier

A rectifier is an electric circuit that converts alternating signals to unidirectional signals. A rectifier circuit can be constructed using one or more diode so as to convert half or both the cycles of a.c. if half wave of a.c. is converted to d.c., then the circuit is called half wave rectifier and if both the cycles of a.c. are converted to d.c. then it is called full wave rectifier.

a. Half wave rectifier

The half-wave rectifier circuit using a p-n junction diode (D) with a load resistor (R_L) is as shown in Fig. 21.11 (i). The diode is connected in series with the secondary of a transformer and the load resistor R_L. The primary of the transformer is fed with the a.c. supply.

The a.c. voltage across the secondary windings changes polarity after every half cycle of the input a.c. wave. During the positive half cycles of input a.c. voltage, the upper end of secondary (S_1) is positive with respect to the lower end (S_2) and hence the diode D is forward biased condition. Therefore, diode conducts current. If the forward resistance of the diode is assumed to be zero, the input voltage during positive half cycles is directly applied to the load resistor R_L and the waveforms of output voltage and output current are of same shape as that of input a.c. voltage. A typical graph for input a.c. voltage and rectified output d.c. is as shown in Fig.21.11 (ii).

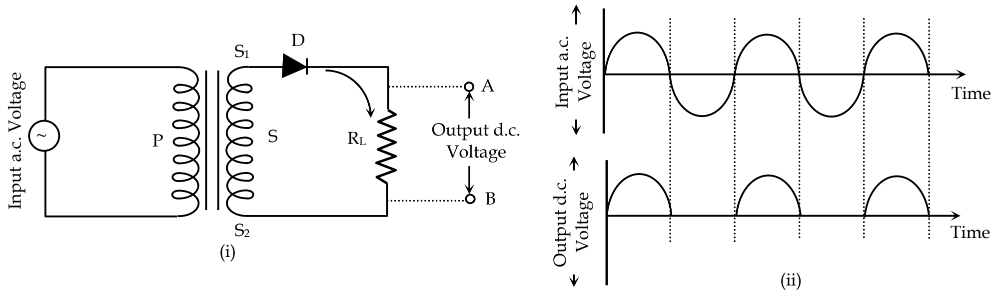


Fig. 21.11 (i): Circuit diagram for half wave rectifier (ii) nature of input and output signal

During the negative half cycles of the input a.c., the upper end of secondary is negative with respect to its lower end. So the diode is reverse biased and does not conduct.

Thus, in a half wave rectifier, the output voltage developed across a load resistor is a series of positive half cycles of alternating voltage.

b. Full wave rectifier

A full wave rectifier is a circuit that rectifies both the cycles of the input a.c. voltage. The full wave rectifier circuit using two diodes (D_1 and D_2) and a load resistor (R_L) is as shown in Fig. 21.12(i).

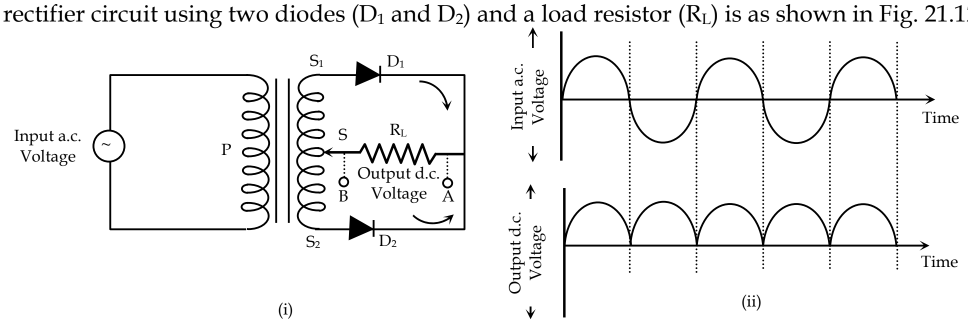


Fig. 21.12 (i): Circuit diagram for full wave rectifier (ii) nature of input and output signal

The primary (P) of transformer is feed with input a.c. signal. The diodes D_1 and D_2 are connected across the secondary of a transformer which is central tapped. When an additional wire is connected across the exact middle of the secondary winding of a transformer, it is called a central tapped transformer. The wire used is known as central tap. (The central tapped transformer divides the a.c. signal into two parts. The upper part of the secondary winding produces a positive voltage say V_1 and the lower part of the secondary winding produces a negative voltage V_2)

During the positive half cycle, the upper end of the secondary winding is positive while the lower end is at negative potential. So, the diode D_1 is forward biased and becomes conducting. But, D_2 is reversed biased and does not conduct. Thus, during the positive half cycle, there is current in the circuit in the direction shown by arrow along the load resistor as shown in Fig. 21.12(i) and hence output voltage appears across it.

During the negative half cycle, upper end of the winding of secondary is negative while its lower end is at positive potential. So, D_1 is reversed biased and does not conduct, while D_2 is forward biased and hence conducts. During this cycle also, there is current along the load R_L in the direction shown by dotted arrow as in Fig. 21.12(i) and is along the same direction as is during the positive half cycle.

Thus, in both the cycles of input a.c. signal there is current in the same direction (i.e. unidirectional) along the load which is d.c. In this way a full wave rectifier converts both the cycles of a.c. into d.c. and hence known as full wave rectifier. The input and output wave form for full wave rectification is as shown in Fig. 21.12(ii).

21.10 Filter Circuits

A diode conducts in a forward bias condition and does not conduct at reverse bias condition. However, due to the presence of minority charge carriers in both the p and n side of diode, a small current always persists in the circuit. So, the output of the rectifier is not completely free from the a.c. signals. Such output d.c. signals containing a.c. component due to minority carriers are called pulsating d.c. Such pulsating d.c. are not desired and hence the a.c. components should be filtered from pulsating d.c., so as to obtain perfect d.c. Such circuits used to remove a.c. components from pulsating d.c. are called filter circuits. There are many types of filter circuits, among which L-C filter is discussed below.

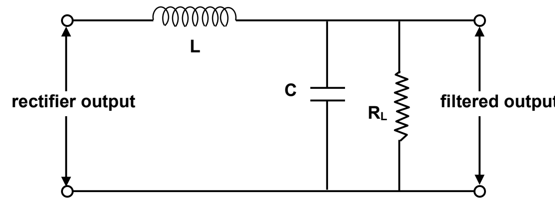


Fig. 21.13: Rectified signal through filters

L-C filter: In this type of filter, an inductor is connected in series to the output of rectifier and a capacitor is connected in parallel to the load R_L as shown in Fig. 21.13.

We know,

impedance of inductor = $X_L = \omega L = 2\pi fL$

So, the inductor offers high resistance to a.c. but provides easy access to d.c.

Similarly, impedance of capacitor = $X_C = \frac{1}{\omega L}$. So it will offer low resistance to a.c. But ω for d.c. is 0.

Therefore, X_C for d.c. = ∞. This means, capacitor provide infinite resistance to d.c.

Thus, the rectifier output containing pulsating d.c. first passes from inductor where a.c. is blocked. Secondly, the d.c. if still contains a.c., will bypass through capacitor C and hence rectifier output has perfect d.c.

21.11 Zener Diode

A zener diode is a heavily doped p-n junction diode preferably a Silicon diode that operates in reverse bias condition in the reverse breakdown voltage region without being damaged, whereas a normal p-n junction diode operates in forward biased condition only. The circuit symbol for zener diode is as shown in Fig. 21.14 (i) and the name zener is given in honour of the inventor Clarence Melvin Zener, an American scientist. Silicon is preferred to Germanium for the manufacture of such diodes because of their capability of operating at high temperature and currents.

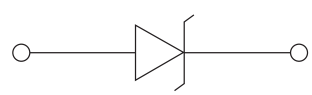


Fig.: 21.14 (i): Symbolic representation of Zener Diode

Zener diode as a voltage regulator

The process of maintaining a constant output voltage across the load is known as voltage regulation or voltage stabilization. In the case where the applied voltage from the power supply keeps on fluctuating, it is desired to regulate or keep it steady for the proper working and durability of the electrical appliances. For this purpose, a zener diode can be suitably connected to a circuit, so that a steady output is maintained.

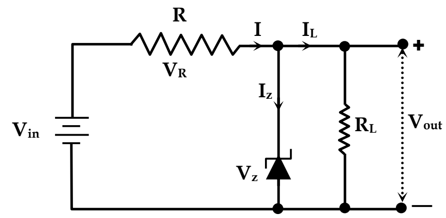


Fig. 21.14 (ii): Circuit diagram of zener diode as a voltage regulator

The circuit arrangement in order to understand the action of zener diode as voltage regulator is as shown in Fig. 21.14 (ii). It consists of a zener diode of zener voltage (V_z) arranged parallel to a load resistor R_L across which the constant output is desired. The fluctuating supply from any source is passed through a series resistor R and across the zener diode as shown in Fig. 21.14 (ii). This series resistor R tends to absorb the voltage fluctuation so as to maintain constant voltage across the load. The zener diode is so connected that it is in reverse biased condition and hence comes to it full operation.

Let V_{in} be the supplied input voltage and I be the total current through the circuit. If I_z and I_L be the currents through diode and load respectively, then $I = I_z + I_L$.

If V_{in} is greater than zener voltage (V_z) then, the voltage equal to V_z appears at the zener diode and the excess voltage appears across the series resistor R. Actually, if the voltage across the zener diode tends to increase beyond zener voltage, the resistance of the zener diode decreases significantly as a result there is sharp rise in current which helps to maintain a constant voltage across it. The excess voltage is dropped across the series resistor R. As a result, the ouput voltage lowers back to normal value i.e. V_{out} can't exceed V_z rather it is always equal to V_z.

Here, $I = I_z + I_L$

The load current $I_L = \frac{V_o}{R_L} = \frac{V_z}{R_L}$

Since V_z and R_L are constant, I_L is also constant. This means, if I increases, there is proportional increase in I_z only.

Now if the voltage across the diode tends to decrease, the current through the diode also decreases. So that the voltage drop across the series resistor is very less. Thus, the output voltage is again raised to normal. But, when the supply voltage decreases below certain value, the circuit will not work.

21.12 Transistor

A transistor is a three terminal semiconducting device essentially consisting of two p-n junctions in a single material. This type of transistor also known as bipolar transistor has a n or a p type material sandwiched between two p or n type materials as shown in Fig. 21.15(i) and 21.15(ii).

If the n-type material lies between two p-type materials, the transistor is called as p-n-p transistor and if a p-type material lies between two n-type materials, the transistor is known as n-p-n transistor.

The middle region of the transistor is usually thin as compared to its adjacent regions and is known as the base of the transistor. Among the two regions, one is slightly wider than the other. The former wider part is called collector and the later is called emitter. So, in terms of width; collector region > emitter region > base region.

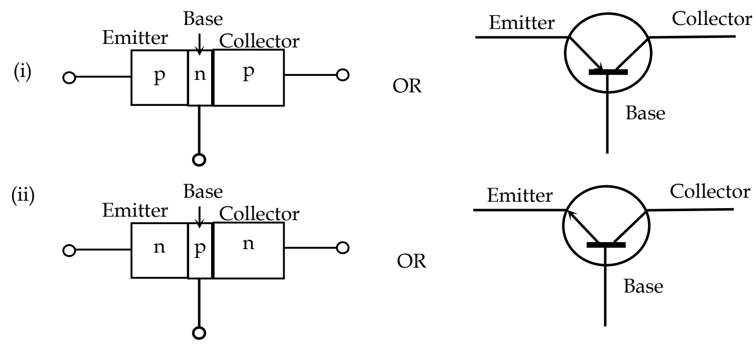


Fig. 21.15: (i) Block diagram and Circuit symbol of p-n-p transistor
(ii) Block diagram and Circuit symbol of n-p-n transistor

What differentiates these regions is the level of doping done to them. The emitter is heavily doped and hence supplies (emits) charge carriers to base, so the name emitter. The base is very lightly doped. The collector is doped in such a way that, its doping level lies between emitter and base.

As the name suggests, its major function is to collect the charge carriers from the base.

The circuit symbols n-p-n and p-n-p transistor are as shown in Fig. 21.15 (i) and 21.15 (ii).

21.13 Working of a Transistor

The main function of transistor is amplification and switching. Amplification means the magnification of an input signal by transferring energy to it from an external source, where as switching refers to the controlling of a relatively large current between or voltage across two terminals by means of small control current or voltage applied at a third terminal. So, for proper functioning of a transistor, it is necessary to maintain proper polarity between the junctions. This process is known as biasing of the transistor. *The biasing of transistor is done in such a way that, the emitter base junction is always forward biased and collector base junction is always reverse biased.* The emitter current (I_E) is always equal to sum of collector current (I_c) and base current (I_B).

i.e. $I_E = I_C + I_B$

i. n-p-n transistor

In this type of transistor a p-type material sandwiched between two n-type materials serves as the base, whereas the n-type materials act as emitter and collector. For its proper functioning n-p junction is forward biased and p-n junction is reversed biased. i.e. in n-p junction n and p are connected respectively to negative and positive terminal of the external voltage source where as in p-n junction p and n are connected respectively to negative and positive terminal of the voltage source as shown in Fig. 21.16.

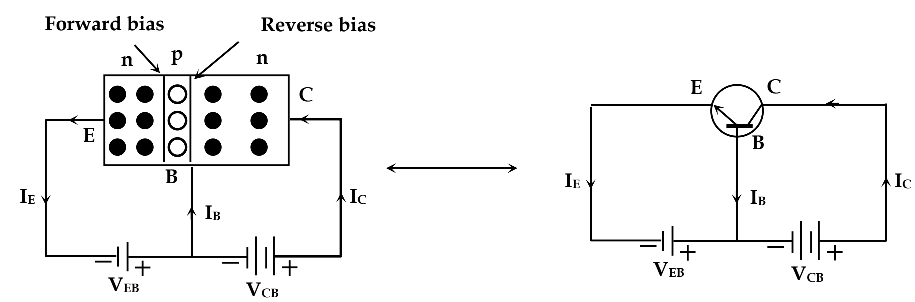


Fig. 21.16: Working of n-p-n transistor

Symbolically, the circuit diagram can be represented as shown in Fig. 21.16. Usually, an arrow is shown in the emitter that indicates the conventional direction of current in the n-p-n transistor, the direction of current is emitter can be remembered as (**n**ot **p**ointed i**n**)

N-region usually abundant with free electrons (majority charge carriers) pushes the electron towards base due to the forward biasing field which constitutes emitter current I_E. The electrons after reaching the base, recombine with the holes present in it. Since the base is lightly doped, only few electrons (nearly 5%) recombine with hole and constitute base current (I_B). This current is very small owing to the fact stated above. The remaining electrons (nearly 95%) cross into collector region and constitute collector current (I_C). In this way, almost all of the emitter current flows through the collector circuit. Thus, it is always true that, $I_E = I_B + I_C$. It is to be noted that, the charge carriers are the electrons both within the transistor and external circuit. The electrons entering collector region are attracted by the positive potential of V_{CB} where they gain energy to enter the emitter region again.

ii. p-n-p transistor

In this type of transistor a n-type material sandwiched between two p-type materials serves as the base where as the p-type materials act as emitter and collector. For its proper functioning p-n junction is forward biased and n-p junction is reversed biased, i.e. in p-n junction, p and n are respectively connected to positive and negative terminal of the voltage source where as in n-p junction n and p are connected respectively to positive and negative terminal of the voltage source as shown in Fig. 21.17.

Symbolically, the circuit representation is as shown in Fig. 21.17. The direction of current is shown by an arrow in the emitter and the direction can be remembered as (**p**ointed i**n**).

P-region usually abundant with holes pushes the holes towards base due to the external bias voltage which constitutes emitter current. Since the n-type base is lightly doped, only small number of holes recombines with electrons to constitute base current. The majority holes then cross to the collector region to constitute collector current (I_C). In this case also, almost emitter current flows through the collector circuit and $I_E = I_B + I_C$. Thus, within the p-n-p transistor, current is due to the holes, however in the external circuit, current is due to electrons. When the holes from p-region are forced to base, negative ions are left in this region. So, the electrons from p-region are attracted to the positive V_{EB}. Similarly, when the holes reach to collector region (p-region), it becomes more positive and hence electrons are attracted from negative terminal of V_{CB}. Thus, current in external circuit is due to electrons only.

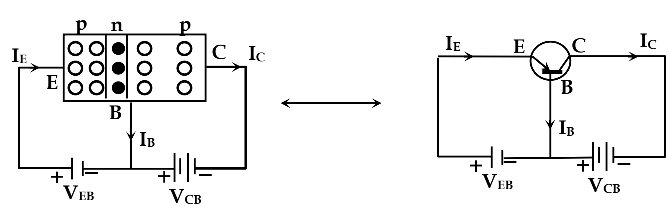


Fig. 21.17: Circuit diagram for working of p-n-p transistor

21.14 Transistor Configuration

A transistor is a three terminal device, so there are basically three possible ways to connect it within an electric circuit with one terminal being common to both input and output. Each method of connection responds differently to its input signal within a circuit. This method of arrangement of a transistor in a electric circuit is known as transistor configuration. The three possible ways for transistor configuration are as follows.

i. Common base configuration.

ii. Common collector configuration

iii. Common emitter configuration

Out of these configurations, only common emitter configuration will be discussed here in detail.

Common emitter (C-E) configuration

This is the circuit arrangement in which the emitter is common to both the input and output. The input is applied between the base and emitter and output is taken from the collector and emitter. The circuit arrangement for common emitter configuration for n-p-n transistors is as shown in Fig. 21.18.

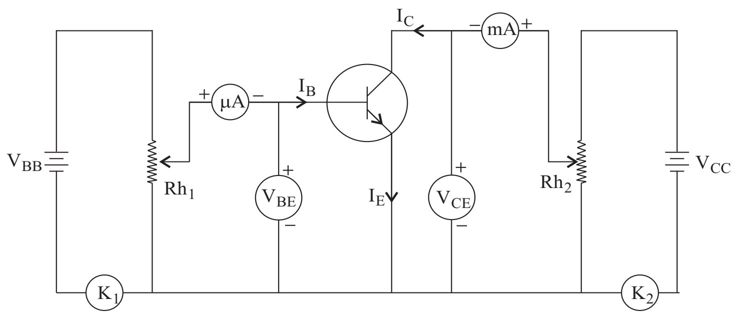


Fig. 21.18: Circuit diagram for transistor characteristics

Characteristics for CE configuration

Characteristics refer to different set of curves plotted between the different set of voltages and their corresponding values for currents in a transistor. To state in a simple way, characteristic curves are the curves of current plotted against the voltages. Basically, there are three types of characteristics curves which are discussed below.

i. Input characteristics

It is the curve obtained between base current (I_B) and base emitter voltage (V_{BE}) when emitter-collector voltage (V_{CE}) is kept constant. To study this characteristic, the potentiometer Rh_2 is adjusted for a suitable voltage V_{CE}. Then, by sliding the potentiometer Rh_1, the base current (I_B) is measured as a function of base-emitter voltage (V_{BC}). For different V_{BE}, corresponding base currents (I_B) are noted. The variation of I_B with V_{BE} for constant V_{CE} is as shown in Fig. 21.19.

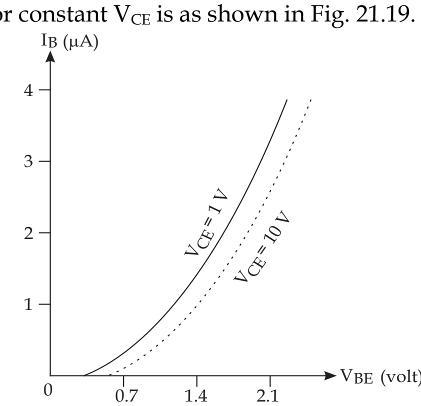


Fig. 21.19: Input characteristics

The ratio of change in base voltage (ΔV_{BE}) to change in base current (ΔI_B) which is actually the slope of (I_B Vs V_{BE}) curve defines the resistance offered by the input part. Since the emitter base junction is always forward biased, the resistance offered is very low.

ii. Output characteristics

It is the curve obtained between the collector current (I_C) and collector-emitter voltage (V_{CE}) when the base current is constant. To study this characteristic, the potentiometer Rh_1 is adjusted to obtain the suitable constant value I_B. Then, by sliding the potentiometer Rh_2, collector current (I_C) is measured as a function of V_{CE}. For different collector-emitter voltage (V_{CE}), corresponding collector current I_C are measured. The variations of I_C with V_{CE} for different constant values of I_B is as shown in Fig. 21.20.

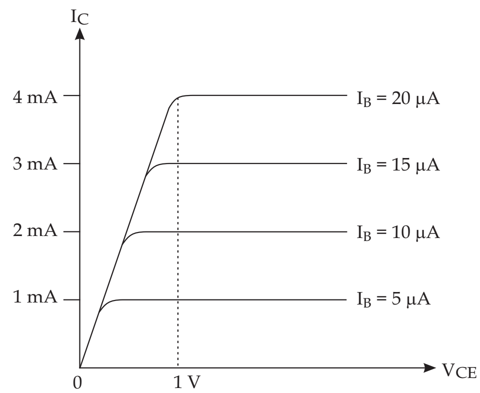


Fig. 21.20: Output characteristics

The graph shows that for each constant values of I_B, the collector current varies with V_{CE} only between 0 V to 1 V. After than the current (I_C) becomes more or less constant.

iii. Transfer characteristics

A curve obtained for collector current (I_C) against base current (I_B) when the collector-emitter voltage is constant is called transfer characteristic. The variation I_C with I_B at constant V_{CE} is as shown in Fig. 21.21. It is a straight line passing through origin showing linear relationship between I_C and I_B.

The slope of this line is $\frac{\Delta I_C}{\Delta I_B}$ gives the current gain which is also known as transfer ratio or current amplification factor (β) in CE configuration.

i.e. $\beta = \frac{\Delta I_C}{\Delta I_B}$

Similarly, for common-base configuration current amplification factor is denoted by α and is defined as the ratio of change in collector current to the change in emitter current i.e. $\alpha = \frac{\Delta I_C}{\Delta I_E}$.

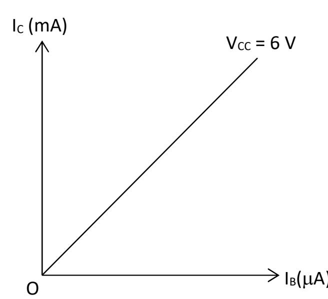


Fig. 21.21: Transfer characteristics

21.15 Transistor as an Amplifier

An amplifier is an electronic circuit that is used to get a large output signal from a small input signal. The amplifier circuit using n-p-n transistor in CE configuration is as shown in Fig. 21.22.

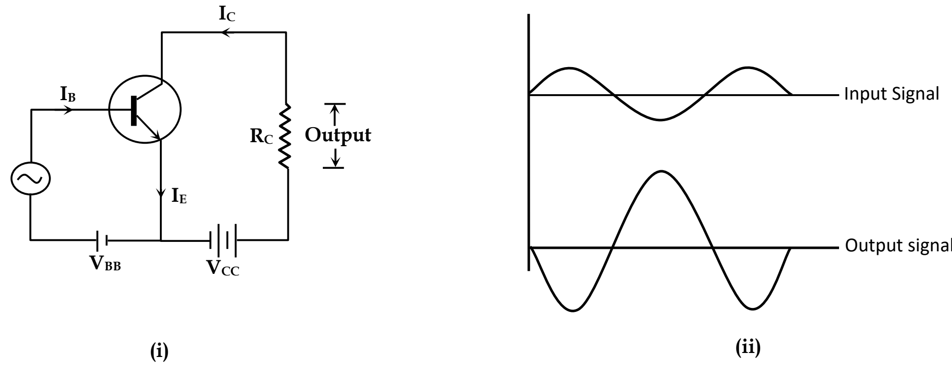


Fig. 21.22: (i) Circuit diagram of a transistor as an amplifier (ii) nature of input and output signal through a transistor

A d.c. source V_{BB} connected to the input of the circuit which maintains a forward bias between emitter base junction in n-p-n transistor at all circumstances. An input a.c. signal is fed to the input part of the circuit, whose output is taken across the collector load resistance R_C as shown in Fig. 21.22 (i).

During positive half-cycle of a.c., the forward bias voltage increases, as a result more electrons flow from emitter region to the base and then to collector. This results in increased collector current and hence greater voltage is dropped across R_C.

During the negative half-cycle of a.c., the forward bias voltage of input part decreases. As a result, few electrons flow from emitter to collector via base. This results in smaller collector current (I_C) and smaller voltage drop across the load resistor R_C. Smaller change in current in input, β times larger change in output. Hence, the voltage is amplied.

But over a complete cycle of a.c. input, the change in collector current (ΔI_C) is much greater than the input base current and hence an amplified output is always obtained across the load.

Relation between α and β

We know, for any transistor,

$$I_E = I_B + I_C$$

$$\text{So, } \Delta I_E = \Delta I_B + \Delta I_C$$

Dividing both sides by ΔI_c, we get,

$$\frac{\Delta I_E}{\Delta I_C} = \frac{\Delta I_B}{\Delta I_C} + \frac{\Delta I_C}{\Delta I_C}$$

But, $\frac{\Delta I_C}{\Delta I_E} = \alpha$ is the current amplification factor for common base configuration.

and $\frac{\Delta I_C}{\Delta I_B} = \beta$ is the current amplification factor for common emitter configuration.

$$\therefore \quad \frac{1}{\alpha} = \frac{1}{\beta} + 1$$

$$\text{or,} \quad \frac{1}{\alpha} = \frac{1+\beta}{\beta}$$

$$\text{or,} \quad \alpha = \frac{\beta}{1+\beta}$$

$$\text{Also,} \quad \frac{1}{\alpha} - 1 = \frac{1}{\beta}$$

$$\text{or,} \quad \frac{1-\alpha}{\alpha} = \frac{1}{\beta}$$

$$\therefore \quad \beta = \frac{\alpha}{1-\alpha}$$

21.16 Logic Gate

In electronics, we may use a set of rules to have different function of a device. Such set of rules are derived from a special mathematics called Boolean Algebra (Coined after Mathematician Bool).

Basically, a logic gate is an idealized or physical device implementing a Boolean function. A logic gate performs a logical operation on one or more binary inputs and produces a single binary output.

Logic gates are built using diodes or transistors acting as electronic switches. The other like components may be vacuum tubes, electromagnetic relays, fluidic logic, pneumatic logic, optics molecules or mechanical elements.

Logic gates are part of complex circuit called logic circuits. Some of the devices using logic circuits are, multiplexer, de-multiplexer, registers, ALU, computer memory, microprocessors etc.

Boolean algebra

Boolean data: A set of data with only two possible values is called Boolean data. Boolean data type is mostly used in computer programming. The two possible values or states may be true or false; 0 or 1; on or off; high or low; as per the requirement.

Boolean Algebra is a logical formulation of truth values or set correlation.

Boolean expression is an expression in a programming language that produces a Boolean value when evaluated.

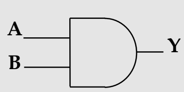
Truth table

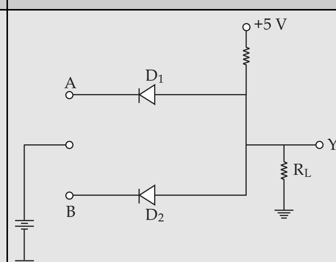
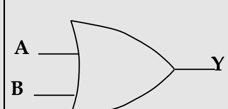
A truth table is a mathematical table used in logical analysis which sets out the operational function of a device using functional values and logical expressions. A truth table has one or more columns for each input variable and one final column showing all possible results based on the relation.

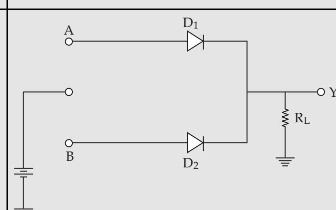
The logic which is always true regardless of input is called logical true.

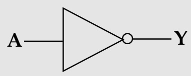
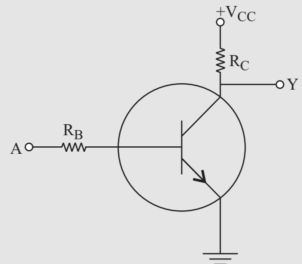
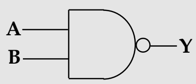
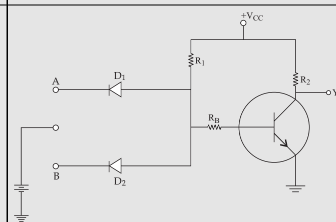
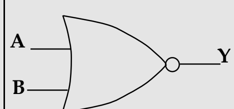
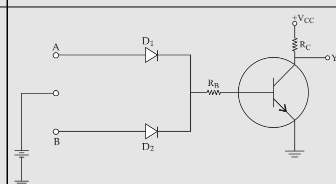
The output value is never true, that is always false regardless of the input value is called logical false.

The various types of logic gates along with their symbol Boolean algebra and truth table are tabulated below.

<table>
<tr><th>Type</th><th>Symbols</th><th>Boolean algebra</th><th>Circuit diagram</th><th>Truth table</th></tr>
<tr><td>AND</td><td>

Output is ON, if all inputs are ON</td><td>Y =A . B</td><td>
</td><td><table><tr><th colspan="2">INPUT</th><th>OUTPUT (Y)</th></tr><tr><th>A</th><th>B</th><th>A AND B</th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table></td></tr>
<tr><td>OR</td><td>

Output is ON, if either or all inputs are ON</td><td>Y =A + B</td><td>
</td><td><table><tr><th colspan="2">INPUT</th><th>OUTPUT (Y)</th></tr><tr><th>A</th><th>B</th><th>A OR B</th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table></td></tr>
</table>

<table>
<tr>
<td>NOT</td>
<td>Output is OFF, if input is ON and vice-versa</td>
<td>$Y = \overline{A}$</td>
<td></td>
<td><table><tr><th>INPUT</th><th>OUTPUT (Y)</th></tr><tr><td>A</td><td>NOT A</td></tr><tr><td>0</td><td>1</td></tr><tr><td>1</td><td>0</td></tr></table></td>
</tr>
<tr>
<td>NAND</td>
<td>Reverse result of AND gate</td>
<td>$Y = \overline{A . B}$</td>
<td></td>
<td><table><tr><th colspan="2">INPUT</th><th>OUTPUT (Y)</th></tr><tr><td>A</td><td>B</td><td>A NAND B</td></tr><tr><td>0</td><td>0</td><td>1</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>0</td></tr></table></td>
</tr>
<tr>
<td>NOR</td>
<td>Reverse result of OR gate</td>
<td>$Y = \overline{A + B}$</td>
<td></td>
<td><table><tr><th colspan="2">INPUT</th><th>OUTPUT (Y)</th></tr><tr><td>A</td><td>B</td><td>A NOR B</td></tr><tr><td>0</td><td>0</td><td>1</td></tr><tr><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td><td>0</td></tr></table></td>
</tr>
</table>

NAND Gate as universal gate

NAND gate is a combination of NOT gate and AND gate in which the output of AND gate is connected to the input of NOT gate. When suitably combined with other types of gates, NAND get can be used to design all three basic gates. So, this gate is called universal gate.

i. NOT gate from NAND gate

A NAND gate serves as a NOT gate, when its input are simply joined together as shown in Fig. 21.23.

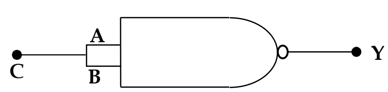


Fig. 21.23 NOT gate by NAND gate

Since, both the input of AND gate are joined together, there is only one input. The truth table for such case is as follows.

Input (A = B = C)	Output Y
0	1
1	0

The truth table justifies that, the gate combination is a NOT gate.

ii. OR gate from NAND gate

OR gate can be obtained from NAND gate when output of two NOT gates formed from two NAND gates is connected to two inputs of a NAND gate as shown in Fig. 21.24.

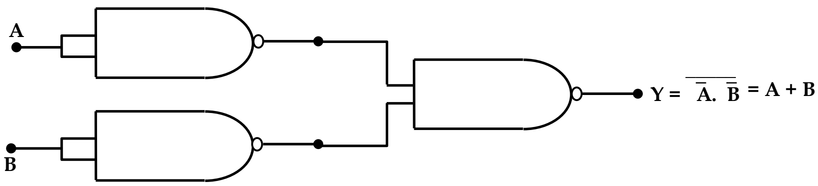


Fig. 21.24: OR gate by NAND gate

The truth table of obtaining an OR gate from a NAND gate is given below:

A	B	$\overline{A}$	$\overline{B}$	$Y = \overline{\overline{A}\,\overline{B}} = A + B$
0	0	1	1	0
0	1	1	0	1
1	0	0	1	1
1	1	0	0	1

From the truth table, it is clear that the output of the combination is high (1) if one or all of inputs are high (1) which is the condition of OR gate. So, the above arrangement behaves as an OR gate which is obtained from the NAND gate.

iii. AND gate from NAND gate

When the output of a NAND gate is connected to input of a NOT gate, formed from NAND gate, we get AND gate as shown in Fig. 21.25.

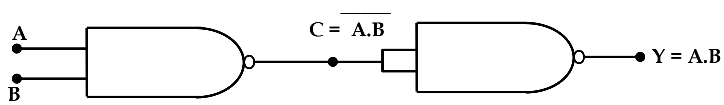


Fig. 21.25: AND gate by NAND gate

The truth table of obtaining an AND gate from a NAND gate is given below:

A	B	C	Y
0	0	1	0
1	0	1	0
0	1	1	0
1	1	0	1

From this truth table, it is clear that output is high (1) only when all the input are high (1) which is the condition of AND gate. So, the above arrangement behaves as an AND gate which is obtained from the NAND gate.

21.17 Nanotechnology

Nanotechnology refers to the study of materials, their structures and control of the phenomena that help for the design and production of the devices with enhanced properties. The word 'nano' is not a mere indication of the measurement scale of the order of nanometer (10^{-9}m) but it simply refers to the size which is very small particularly below 100 nm in a measurement scale. The size is often found to be compared with a strand of human hair, which is about 80,000 nm wide.

The term nanotechnology was coin ed in 147 AD by Norio Tanguichi of Tokyo Science University to describe semiconductor process such as thin film deposition that deal with control on the order of nanometers. According to Norio, "Nanotechnology is the science mainly consisting of the processing of separation, consolidation and deformation of materials by one atom or one molecule." And this definition still stands as the basic statement today. Nanotechnology basically deals with the understanding and control of matter at dimension between 1 and 100 nm approximately, where unique phenomena enable to devise novel applications. This field of science involves imaging, measuring, and modelling and manipulating matter at this length scale. At nanoscale, unusual physical, chemical and biological properties can emerge in materials which differ significantly from the properties of the bulk material. These peculiar properties form the basis in the design of devices with superior characteristics.

This multidisciplinary scientific education which encompasses physics, chemistry, material science, biochemistry, biotechnology and many more, is the forefront of modern research. Some of the basic applications of this field of study are discussed below.

i. **Medicine:** Researchers are developing customized nanoparticles the size of molecules that can deliver drugs directly to diseased cells in our body.
ii. **Electronics:** It is used to increase the capabilities of electronics devices while we reduce their weight and power consumption.
iii. **Food:** Nanotechnology is having an impact on several aspects of food science, from how food is grown to how it is packaged.
iv. **Fuel Cells:** Nanotechnology is being used to reduce the cost of catalysts used in fuel cells to produce hydrogen ions from fuel such as methanol and to improve the efficiency of membranes used in fuel cells to separate hydrogen ions from other gases such as oxygen.
v. **Solar Cells:** Companies have developed nanotech solar cells that can be manufactured at significantly lower cost than conventional solar cells.
vi. **Space:** Nanotechnology may hold the key to making space-flight more practical. Advancements in nanomaterials make lightweight spacecraft and a cable for the space elevator possible.
vii. **Fuels:** Nanotechnology can address the shortage of fossil fuels such as diesel and gasoline by making the production of fuels from low grade raw materials economical, increasing the mileage of engines, and making the production of fuels from normal raw materials more efficient.
viii. **Better Air Quality:** Nanotechnology can improve the performance of catalysts used to transform vapors escaping from cars or industrial plants into harmless gasses.
ix. **Cleaner Water:** Nanotechnology is being used to develop solutions to three very different problems in water quality.
x. **Chemical Sensors:** Nanotechnology can enable sensors to detect very small amounts of chemical vapors.
xi. **Fabric:** Making composite fabric with nano-sized particles or fibers allows improvement of fabric properties without a significant increase in weight, thickness, or stiffness

Tips for MCQs

1. **About band theory:**
   i. There are two types of energy bands in solids: Valence energy band and conduction energy band. The separation of these energy bands is called forbidden gap.

ii. The conduction band always lies above the forbidden gap and valence band lies below the forbidden gap.
iii. The maximum energy possessed by an electron in the energy band at 0 K temperature is called Fermi energy and the corresponding energy level is called Fermi level.
iv. At 0 K, conduction energy band is complete empty.

2. **Characteristics of semiconductor:**
   i. Its resistivity is greater than conductor and smaller than insulator. Likewise, its conductivity is greater than insulator and smaller than conductor.
   ii. Its forbidden energy gap is quite small about 1 eV, (however it varies over a certain range).
   iii. It behaves a perfect insulator at 0K.
   iv. The conductivity of semiconductor can be increased by (i) raising temperature (ii) by doping trivalent or pentavalent impurities.
   v. Holes and free electrons are the charge carriers in a semiconductor.

3. **About holes and free electrons in semiconductor:**
   i. Holes are vacancy in valence band, which behaves as positive charge particles and the magnitude of charge is equal to that of an electron.
   ii. Holes acts as a virtual charge because of no physical charge in it.
   iii. Free electrons are the mobile electrons in conduction band.
   iv. Electron are real charge particles.
   v. The mobility of electrons is greater than that of holes.
   vi. Holes act as positive charge carriers and free electrons act as negative charge carriers.
   vii. The effective current in the semiconductor is the sum of hole current and electron current.

4. **p-type and n-type semiconductor:**
   i. p-type semiconductor:
   a. It is made by doping trivalent impurities like Indium, Boron, Aluminium, etc. in semiconductor.
   b. Holes are majority charge carriers and electrons are minority charge carriers.
   c. It is called acceptor type, because it accepts the electrons for conduction.

   ii. n-type semiconductors:
   a. It is made by doping pentavalent impurities like Arsenic, Antimony, Phosphorus, etc, in semiconductor.
   b. Free electrons are the majority charge carriers and holes are minority chare carriers.
   c. It is called donor type because the dopped impurity atom donates the electron.
   d. Both p-type and n-type semiconductors are electrically neutral.
   e. The electrical conductivity in semiconductors is written, $\sigma = \sigma_e + \sigma_n$.

5. **p-n junction diode:**
   i. p-n junction diode is the suitably joined p-type and n-type semiconductor.
   ii. It can be compared with capacitor in which the depletion region acts as diectric between two capacitor plates.
   iii. Two important process occur during formation of p-n junction: diffusion and drifting.
   iv. Electric field is set up across the depletion layer due to the accumulation of charge forming two different polarities.
   v. The value of barrier potential is about 0.7 V in Silicon and 0.3 V in Germanium.
   vi. In forward biasing, conduction takes place in low voltage. However, the conduction doesnot occur in low voltage in reverse biasing. At large voltage, conduction occurs in reverse biasing is due to minority change carrier. Conduction occurs due to majority charge carriers in forward biasing.

6. **Half wave rectifier and full wave rectifier:**
   i. Rectifiers convert a.c. into d.c., but the magnitude of voltage or current is not steady.
   ii. Half wave rectifier converts half cycle of a.c. into d.c. signal, whereas the full wave rectifier converts full cycle of a.c. signal into d.c. signal.
   iii. The efficiency of half wave rectifier is about 40.6% and efficiency of full wave rectifier is about 81.2%.
7. **Zener diode:**
   i. It is the special heavily doped diode which works in reverse biased condition.
   ii. It is used in voltage regulation.
   iii. A junction diode can be used as a rectifier, capacitor and off switch, whereas Zener diode can be used as stabilizer.
8. **p-n-p transistor and n-p-n transistor**
   i. It is used as an amplifier and an oscillator.
   ii. Transistor has three parts, emitter (E), base (B), and collector (C).
   iii. Emitter-base junction are always forward biased and collector base junction is reverse biased.
   iv. The current relation is $I_E = I_B + I_C$.
   v. There are three types of configuration in transistor: Common base (CB), Common emitter (CE) and Common collector (CC).

Conceptual Questions with Answers

1. What is a semiconductor?

↳ Semiconductor is a solid substance that has a conductivity between conductor and insulator, either due to the addition of an impurity or because of temperature effect. The conductivity of semiconductor is smaller than conductors like metals and greater than insulators like plastics. The semiconductor materials contain four electrons in their valence orbit, so they form covalent bonding in crystal formation.

2. What are the energy bands in solids?

↳ Solids have two energy bands: Valence energy band and conduction energy band. The valence energy band is the outermost electron orbital of an atom of any specific material that electrons actually occupy. Conduction energy band is a delocalized band of energy levels in a crystalline solid which is partially filled with electrons. These electrons have great mobility and are responsible for electrical conductivity.

3. What forbidden energy gap?

↳ The energy difference between the top of the valence band and the bottom of the conduction band is called forbidden energy gap. Electrons are not found in this gap, hence the name forbidden. Such type of gap is found in semiconductors and insulators. Forbidden gap separates the conduction energy band and valence energy band in solids.

4. Write any three characteristics of semiconductors.

↳ There are several characteristics of semiconductor. Three of them are written below.
   i. Its conductivity is smaller than conductors and greater than insulators. Also, its resistivity is greater than conductors and smaller than insulators.
   ii. It behaves perfect insulator of zero Kelvin. Also, the conductivity increases as the temperature increases.
   iii. Its forbidden energy gap is quite small about 1 eV.

5. What is charge carrier? What type of charge carries are found in semiconductors?

↳ A charge carrier is a particle free to move carrying electric charge. In semiconductors, holes and free electrons are charge carriers. Holes are positive charge carriers and free electrons are negative charge carriers.

6. What are holes in semiconductors?

↳ Holes are the vacancy created in the valence band of semiconductor when an electron on acquiring energy jumps from valence energy band to conduction energy band. Electrons comes to fulfill the holes in valence band, hence it is considered positive charge carriers. It acts as the virtual charge because there is no physical charge on it.

7. Distinguish between hole current and electron current.

↳ When an electron from a nearby covalent bond jumps to fill vacancy, the vacancy shifts in a direction opposite to that in which the electron jumps. This gives rise to hole current. On the other hand, electron current is constituted by the drift of free electrons.

8. What is doping in semiconductor?

↳ The process of addition of a desirable impurity atoms deliberately to a pure semiconductor to modify its properties in a controlled manner is called doping in semiconductor. The doping of a semiconductor increases its conductivity to a great extent. Trivalent or pentavalent impurities can be dopped in pure semiconductors to increase holes or free electrons.

9. What are dopants? Give examples.

↳ The impurities which are added to increase the conductivity of semiconductors are called dopants. There are two types of dopants used in tetravalent Silicon or Germanium.

i. Trivalent dopants: They increases the number of holes when appropriately dopped in semi-conductors. Examples; Indium (In), Boron (B), Aluminium (Al) etc.

ii. Pentavalent dopants: They increases the number of free electrons when appropriately dopped in semiconductors. Examples: Arsenic (As), Anti-mony (Sb) and Phosphorus (P), etc.

10. Does p-type or n-type semiconductor crystal is electrically charged? Explain.

↳ No. In these semiconductors, the charge carriers can be increased by dopping trivalent or pentavalent impurities. In p-type semiconductor, trivalent impurities are dopped to enhance the number of holes. In n-type semiconductor, pentavalent impurities are dopped to increase the concentration of free electrons. However, the semiconductor materials (Silicon and Germanium) and added material both are chargeless. When they mix, net charge is still zero, only the conductivity will be changed.

11. Why does a pure semiconductor behave like an insulator at absolute zero temperature?

↳ For a pure semiconductor at a temperature of absolute zero (-273.15^{o} C)the valence band is usually full and there are no electrons in the conduction band. It is difficult to provide additional energy required for lifting electron from valence band to conduction band by applying electric field. Hence, the conductivity of a pure semiconductor at absolute zero temperature is zero and it behaves like an insulator.

12. What happens to the conductivity of semiconductor with the rise in temperature? Compare with the conductivity of metals.

↳ When semiconductors are heated, valence electrons jump to the conduction band creating more holes in valence band and consequently more free electrons in conduction band. With the increase in temperature, the concentration of charge carriers increases resulting in increase in conductivity of semiconductors. However, the conductivity of metal decreases with the increase in temperature.

13. Why temperature coefficient of resistance of a semiconductor is negative?

↳ With the increase in temperature, the concentration of charge carriers (electrons and holes) increases. As more charge carriers are made available, the conductivity of a pure semiconductor increases i.e. resistivity of a pure semiconductor decreases with the rise in temperature. Thus, semiconductors are said to have negative temperature coefficient of resistance.

14. What do you mean by donor and acceptor impurities?

✎ Donor impurities (such as arsenic, antimony, bismuth or phosphorous) when added to a pure semiconductor lattice, form n-type extrinsic semiconductor. The pentavalent impurities are called donor impurities as such impurities donate electrons to the lattice. Acceptor impurities (such as boron, gallium, indium or aluminium) when added to a semiconductor lattice form p-type extrinsic semiconductor. The trivalent impurities are called acceptor impurities because such impurities accept electrons from the lattice.

15. What are the differences between intrinsic and extrinsic semiconductor?

✎ Differences between intrinsic and extrinsic semiconductors are as follows:

Intrinsic semiconductor	**Extrinsic semiconductor**
1. Impurity are not added in intrinsic semiconductor.	1. A small amount of impurity is doped in a pure semiconductor for preparing extrinsic semiconductor.
2. The number of free electrons in the conduction band is equal to the number of holes in the valence band.	2. The number of electrons and holes are not equal.
3. Electrical conductivity is low.	3. Electrical conductivity is high.
4. Electrical conductivity is a function of temperature alone.	4. Electrical conductivity depends on temperature as well as on the amount of impurity doped in the pure semiconductor.

16. What are the differences between half wave rectifier and full wave rectifier?

✎ Differences between half wave rectifier and full wave rectifier

Half wave rectifier	**Full wave rectifier**
1. These type of rectifier converts only one half of a.c. into d.c. signal, either positive or negative cycle	1. It converts entire a.c. into fluctuating d.c. that is it covert both half cycle to d.c.
2. As it rectifies a.c. partially its efficiency is also less. Maximum efficiency is 40.6%	2. Its efficiency is almost double of half wave rectifier. Efficiency is 81.2 %.
3. Half wave rectifier needs only single diode for rectification.	3. It consists of more than one diode
4. During reverse bias condition (i.e. negative half of input cycle), diode D is reverse biased. This results into no current in the circuit	4. Positive half cycle is converted by one diode and negative half cycle is converted by other diode

17. What is depletion region?

✎ Generally, depletion refers to reduction or decrease in quantity of something. However, in semiconductor physics, the depletion region refers to a region where flow of charge carriers are decreased over a given time and finally results in empty mobile charge carriers or full of immobile charge carriers

18. What are the differences between p-type and n-type semiconductor

p-type semiconductor	**n-type semiconductor**
1. In p type semiconductor, trivalent impurity like Al, Ga, In etc. are added.	1. In n type semiconductor, Pentavalent impurity like P, As, Sb, Bi etc. are added.
2. Holes are majority carriers and electrons are minority carriers	2. Electrons are majority carriers and holes are minority carriers

3. Impurity added creates a vacancy of electrons called holes and is known as acceptor atom.	3. Impurity added creates electrons called free electrons and is known as donor atom.
4. The electron density is much greater than the hole density. (i.e. holes are the majority charge carriers.)	4. Electrons are majority charge carriers.

19. What is p-n junction diode? How do free electrons and holes flow in p-n junction diode?

✍ If p-type semiconductor is joined with n-type semiconductor, a p-n junction diode is formed. The region in which the p-type and n-type semiconductors are joined is called p-n junction. This p-n junction separates n-type semiconductor from p-type semiconductor.

In n-type semiconductors, large number of free electrons is present. Near the junction free electrons and holes are close to each other. Hence, the free electrons from n-side are attracted towards the holes at p-side. Thus, the free electrons move from n-side to p-side. Similarly, holes move from p-side to n-side.

20. Explain the mechanism of formation of positive and negative ions at p-n junction.

✍ The free electrons that are crossing the junction from n-side provide extra electrons to the atoms on the p-side by filling holes in the p-side atoms. The atom that gains extra electron at p-side has more number of electrons than protons. We know that, when the atom gains an extra electron from the outside atom it will become a negative ion.Thus, each free electron that is crossing the junction from n-side to fill the hole in p-side atom creates a negative ion at p-side. Similarly, each free electron that left the parent atom at n-side to fill the hole in p-side atom creates a positive ion at n-side.

21. What is Zener diode?

✍ A zener diode is a p-n junction semiconductor device designed to operate in the reverse breakdown region. The breakdown voltage of a zener diode is carefully set by controlling the doping level during manufacture.Zener diodes are the basic building blocks of electronic circuits. They are widely used in all kinds of electronic equipments. Zener diodes are mainly used to protect electronic circuits from over voltage.

22. What are the applications of Zener diode?

✍ Some important applications of zener diodeis as follows:

i. They are normally used as voltage reference

ii. They are used in voltage stabilizers or shunt regulators.

iii. They are used in switching operations

iv. They are used in clipping and clamping circuits.

iv. They are used in various protection circuits

23. Draw a circuit diagram for a p-n junction diode in forward bias. Sketch the voltage-current graph for the same.

✍

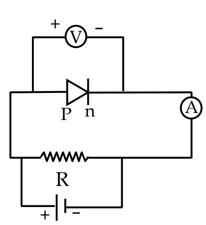


Fig: Forward biasing of p-n junction

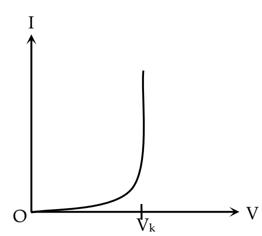
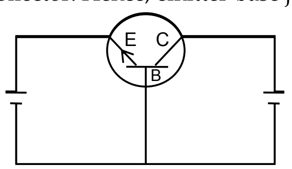
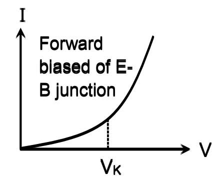


Fig: I-V curve in forward biasing

24. In a transistor, emitter-base junction is always forward biased. Why?

✍ The proper application of d.c. voltage across three terminals of a transistor (Emitter, Base and Collector) is called biasing of transistor. The voltage is applied either in emitter-base junction or in collector-base junction in such a manner that emitter-base junction is forward biased and collector-base junction is reverse biased. When emitter-base junction is made forward biased its resistance decreases. The current flows through the junction beyond the knee voltage, i.e. charge from emitter cross base to reach collector. Hence, emitter-base junction is always forward biased in transistor.





25. Give the circuit symbol and truth table of NAND gate.

↳ NAND gate is the combination of AND and NOT gate in such a way that the output of AND gate is connected to the input of a NOT gate. The symbolic representation of the NAND gate is shown in figure below. This gate produces high output if any one of the input is low.

If A and B represent the inputs and Y represents the output of NAND gate, then Y = $\overline{A}.\overline{B}$. The truth table for NAND gate is given by:

Inputs		A·B	Output
A	B		Y = $\overline{A.\ B}$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

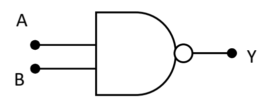


Fig. Symbol of NAND gate

26. What are logic gates? Give truth table for a two-input AND gate.

↳ The logic gates are the electronic circuits which give the logic decisions. AND gate is an electronic circuit which gives high output when all of the inputs are high. The symbol of two inputs AND gate is given below which consists of two inputs named A and B and one output say Y.

The truth table for two-inputs AND gate is given below:

Inputs		Output
A	B	Y = A·B
0	0	0
0	1	0
1	0	0
1	1	1

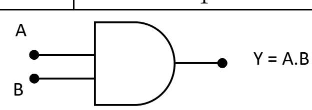


Fig. Symbol of AND gate

27. What is the biasing of transistor? Write its proper biasing.

↳ Transistor Biasing is the process of setting a transistors d.c. operating voltage or current conditions to the correct level so that any a.c. input signal can be amplified correctly by the transistor. In transistor biasing, emitter base junction is forward biasing and base collector junction is reverse biasing.

28. What is nanotechnology?

↳ Nanotchnology is the branch of technology that deals with dimensions and tolerances of less than 100 nanometres, especially the manipulation of individual atoms and molecules. Nanotechnology is helping to considerably improve, even revolutionize, many technology and industry sectors:

information technology, homeland security, medicine, transportation, energy, food safety, and environmental science, and among many others.

Exercises

Short-Answer Type Questions

1. What are the basic properties of semiconductor?
2. Why do Ge and Si behave like semiconductor?
3. What do you mean by doping? Why is it done?
4. Why are n-type semiconductors so called?
5. Why are p-type semiconductors so called?
6. How does the conductivity of a semiconductor change with rise in its temperature?
7. What happens if both, the emitter and collector of a transistor are forward biased?
8. What are valance band, conduction band and forbidden energy gap?
9. What do you mean by biasing of a junction diode?
10. What is the advantage of a semiconductor over a metal?
11. Distinguish between intrinsic and extrinsic semiconductors.
12. Distinguish between an n-type and p-type semiconductors.
13. Electrons and holes in a semiconductor move in opposite direction when a cell is connected. Why don't they recombine resulting zero current?
14. Is there any hole in an n-type semiconductor?
15. What are merits and demerits of half wave rectifier?
16. What are merits and demerits of full wave rectifier?
17. What is a zener diode?
18. What is a p-n junction? Give its circuit symbol.
18. What is depletion layer in the p-n junction?
20. Can there be any current flow across a reverse biased p-n junction? Explain.
21. Why is a transistor so named?
22. What will happen if emitter and collector terminals of a transistor are interchanged?
23. What is avalanche breakdown?
24. What happens if the base of a transistor is doped heavily?
25. What happens if the emitter of a transistor is doped lightly thick?
26. Can you exchange the emitter and collector of a transistor? Explain.
27. Why are most of the transistors n-p-n type and not p-n-p type?
28. What are the types of a logic gates.
29. What is truth table?
30. Under what conditions, the output of an OR gate is high?
31. Under what conditions, the output of an AND gate is high?

Long-Answer Type Questions

1. What is band theory of metals? What are the differences among conductor, semiconductor and bad conductor in terms of energy band?
2. What are intrinsic and extrinsic semiconductors? What is doping? Explain the process of doping in semiconductor.
3. Discuss the working of p-n junction diode when it is (i) forward biased and (ii) reversed biased.
4. What is a junction diode? Explain full wave rectifier produced by a filter circuit. (HSEB 2054)

5. Explain the characteristics of a diode and discuss its application as a full wave rectifier. (HSEB 2055)
6. What are n-type and p-type semiconductors? Describe with a neat diagram the working mechanism of a full wave rectifier for a junction diode. (HSEB 2067)
7. What is a junction diode? Explain its working as a half wave rectifier. (HSEB 2061)
8. What is an extrinsic semiconductor? Explain the formation of potential barrier and depletion region in p-n junction. [HSEB 2072]
9. What do you mean by avalanche effect and zener effect? Describe with necessary circuit diagram the use of zener diode as a voltage regulator. (HSEB 2065)
10. What is a transistor? How is it formed?
11. Describe the working of n-p-n transistor with proper diagram.
12. Describe the working of p-n-p transistor with proper diagram.
13. How is an n-p-n transistor formed? Discuss the input and output characteristics of the transistor in CE configuration. (HSEB 2060)
14. Define α and β parameter of a transistor. Establish the relation between them.
15. Explain a n-p-n transistor in CE configuration as an amplifier.
16. Explain how a transistor can be used as a switch.
17. Name the three basic logic gates. Draw an equivalent circuit for each gate and prepare the truth table for each gate.
18. Give the logic symbol, truth tale and Boolean expression for NOT gate.
19. Explain the working of a NAND gate with its truth table.
20. Explain the working of a NOR gate with its truth table.
21. Explain how the NAND gate can be used to obtain all three basic gates.
22. Write a note on Nanotechnology.

Numerical Problems

1. Find the value of α if $\beta = 200$.

   Ans: 0.99
2. The emitter current in a common base transistor is 10 mA and $\alpha = 0.95$. Find the base current and common collector current.

   Ans: 0.5 mA, 9.5 mA
3. The base current of a transistor is 105 μA. Find the values of β, I_E and α.

   Ans: 19.5, 2.155 μA, 0.95
4. In a n-p-n transistor, 10^{10} electrons enter the emitter in 10^{-6} s. 2% of the electrons are lost in the base, calculate the values of α and β.

   Ans. 0.98, 49
5. The input voltage of a transistor is 1mV and its output is 150mV. Calculate its voltage gain.

   Ans: 150

Multiple Choice Questions

1. The depletion layer in a p-n junction is caused by
   a. drift of holes. b. diffusion of charge carriers.
   c. migration of impurity atoms. d. drift of electrons.
2. During reverse biasing, thickness of depletion layer
   a. decreases. b. increases.
   c. may increases an decreases. d. remains constant.
3. Which get is an inverter?
   a. NOT gate b. OR gate
   c. NAND gate d. AND gate

4. A transistor is essentially a
   a. voltage operated device.
   b. current operated device .
   c. power driven device.
   d. resistance operated device.
5. Holes are majority charge carriers in
   a. n-type semiconductors.
   b. ionic solids.
   c. p-type semiconductors.
   d. metals.
6. When a semiconductor is doped with a donor impurity, then
   a. the hole concentration increases.
   b. the hole concentration decreases.
   c. the electron concentration increases.
   d. the electron concentration decreases.
7. The avalanche breakdown in p-n junction is due to:
   a. shift of Fermi level.
   b. cumulative effect of conduction band electron collision.
   c. widening of forbidden gap.
   d. high impurity concentration.
8. In the depletion region of unbiased p-n junction diode there are only
   a. electrons.
   b. holes.
   c. both electrons and holes.
   d. only fixed ions.
9. The energy gap between the conduction band and the valence band of certain material is 0.7 eV. The material is
   a. an insulator
   b. a conductor
   c. semi conductor
   d. semimetal
10. In a diode when current flows from p to n side it is called
    a. forward biased
    b. reverse biased
    c. biasing opposite
    d. negative biased
11. The impurity atom with which pure Silicon is doped to make p-type semiconductor is
    a. Indium
    b. Phosphorus
    c. Antimony
    d. Arsenic
12. In a transistor circuit, the emitter-base circuit of a n-p-n transistor is always
    a. reverse biased
    b. neutral biased
    c. forward biased
    d. not biased

Answers

1. (b) 2. (b) 3. (a) 4. (b) 5. (c) 6. (c) 7. (b) 8. (d) 9. (c) 10. (a) 11. (a) 12. (c)

❖ ❖ ❖

ATOMIC MODELS

22

CHAPTER

22.1 Introduction

Every matter is composed of discrete units, called atoms. The word atom comes from ancient Greek objective 'atomos', its meaning is indivisible. Before nineteenth century, this was the part of Philosophy, then it entered into the scientific mainstream and is explained in subject of atomic theory. In the beginning days of atomic theory, it was considered that atom is the most fundamental unit of matters, so it was named atomos. After the discovery of three different sub-atomic particles; electron, proton and neutron, the concept of 'uncuttable atom' has been changed. On the basis of electromagnetic theory and radioactivity, physicists discovered that the so called "indivisible" atom actually is composed of subatomic particles.

The different theories regarding the divisibility of an atom and its constituent particles have been presented by different physicist at different times which are atomic models. The validity of these theories has been tested through different experiments. But all the models are not successful enough to describe the experimentally observed facts. So, different models of atoms have been given modifying the pre-existing models and overcoming their shortcomings or drawbacks. The following are the important atomic models.

Thomson’s atomic model

Rutherford’s atomic model

Bohr’s atomic model

Sommerfeld relativistic atomic model

Vector atomic model

In this chapter, we will focus on Rutherford's and Bohr's atomic model.

22.2 Rutherford's Atomic Model

A British Physicist, Ernest Rutherford proposed a model of the atomic structure known as Rutherford's model of atoms. He conducted an experiment where he bombarded α-particles in a thin sheet of gold (will be described briefly in chapter 24). In this experiment, he studied the trajectory of the α-particles after interaction with a thin sheet of gold. From the experiment, he proposed a model, which his somehow similar to what we accept today; the nucleus (a dense matter) at the center and electrons revolving around it. Although it could not explain many phenomena regarding the stability of atom, it is the basis of the quantum mechanics and helped the future development of quantum mechanics.

Rutherford's experiment was unable to explain the following phenomena:

i. Rutherford's model was unable to explain the stability of an atom. In his model, electrons revolve at a very high speed around a nucleus of an atom in a fixed orbit. According to classical theory, an accelerated charge particle must continuously lose energy in the form of electromagnetic radiation. So, the orbiting electrons must finally collapse into the nucleus thereby losing all of its energy at some instant of time. And, stable atoms must not exist. However, the electrons revolving in fixed orbits do not emit electromagnetic radiation. And the stable atoms too exist. This fact could not be explained by Rutherford's atomic model.

ii. This model did not mention anything about the arrangement of electrons in the orbit.

22.3 Bohr's Atomic Model

Although the Rutherford atomic model had given the basic idea about the structure of atom, it could not explain the stability of atom and the atomic spectra that are produced in electron transition from one atomic orbit to another. Afterward Neil Bohr, in 1913, modified the classical concept given by Rutherford by adding the early quantum concepts. To explain the newly combined theory regarding the classical and quantum concepts, Bohr proposed the following three basic postulates.

1. An electron in an atom revolves in certain stable orbit without radiation of energy. It means an atom has certain specific energy state and they have definite total energy. These are called the stationary states of the atom.

2. The electron revolves around the nucleus only in those orbits for which the angular momentum of that electron is some integral multiple of $\frac{h}{2\pi}$. It means the angular momentum of the orbiting electron is quantized.

   i.e. angular momentum, $L = \frac{nh}{2\pi}$ where, n = 1, 2, 3, . . .

   and h = Planck's constant

   If m be the mass and v be the speed of an electron in an atom, that revolves in a specified orbit of radius r, the angular momentum of that electron is,

   $$mvr = \frac{nh}{2\pi}$$

   This is known as Bohr's quantization condition.

3. Electron might make a transition from one energy state to another energy state by absorbing or radiating energy. When electron transits from higher energy state to lower energy state, it radiates that energy in the form of photon and absorbs the energy in reverse transition. The emitted or absorbed photon has the energy equal to the energy difference between the initial and final energy states.

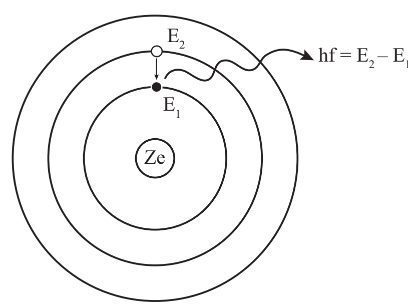


Fig. 22.1: Electron transition in an atom

   If E_i and E_f be the energies of the initial and final energy states, then the energy of emitted photon is.

   $hf = E_i - E_f$, where, f = frequency of emitted radiation

   This is known as the Bohr's frequency condition.

Bohr's theory of hydrogen atom

Bohr's theory can be applied to hydrogen atom, which is a one electron system and also to hydrogen like ions- He^{+}, Li^{++}, Be^{+++}. Bohr postulated that an electron revolves around the nucleus in a specified orbit. In this condition, the electrostatic force between the nucleus and electron in orbit provides the centripetal force required to revolve around nucleus as shown in Fig. 22.2. The gravitational attraction between electron and nucleus is negligible.

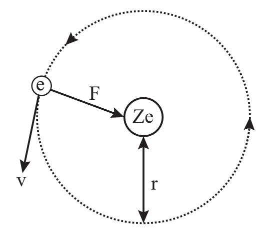


Fig. 22.2: Revolving of an electron around nucleus

Let Ze be the charge of nucleus and e be the charge of an electron that revolves around the nucleus through an specified orbit of radius r. The electrostatic force between the nucleus and the electron when separated with distance r (i.e. radius of orbit) is,

$$F_e = \frac{1}{4\pi\epsilon_0}\frac{Ze.\,e}{r^2} \qquad \ldots (22.1)$$

Where Z = atomic number of an atom and ϵ_0 = permitivity of free space

Then, the centripetal force for electron to revolve it around the nucleus is,

$$F_c = \frac{mv^2}{r} \qquad \ldots (22.2)$$

where

m = mass of an electron

v = speed of electron

r = radius of orbit

Since the centripetal force required by the electron is provided by the electrostatic force of attraction between the nucleus and the electron. So, we write,

$$\frac{mv^2}{r} = \frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r^2} \qquad \ldots (22.3)$$

i. Radii of Bohr's stationary orbit

From the Bohr's second postulate,

$$mvr = n\frac{h}{2\pi}$$

$$v = \frac{nh}{2\pi mr} \qquad \ldots (22.4)$$

Substituting the value of v from equation (22.4) in equation (22.3), we get,

$$\frac{m}{r}\left(\frac{nh}{2\pi mr}\right)^2 = \frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r^2}$$

$$\frac{m}{r}\cdot\frac{n^2h^2}{4\pi^2m^2r^2} = \frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r^2}$$

$$\frac{n^2h^2}{4\pi^2mr^3} = \frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r^2}$$

$$\frac{n^2h^2}{\pi mr} = \frac{Ze^2}{\epsilon_0}$$

$$\therefore \quad \mathbf{r = \frac{\epsilon_0 n^2 h^2}{\pi m Z e^2}} \qquad \text{. . . (22.5)}$$

In general, for n^{th} orbit,

$$\mathbf{r_n = \frac{\epsilon_0 n^2 h^2}{\pi m Z e^2}} \qquad \text{. . . (22.6)}$$

This equation gives the radii of permitted orbits. The radii of permitted orbit are proportional to the square of principal quantum number (i.e. $r_n \propto n^2$), provided other parameters are constant.

The radii of stationary orbits are in the order of square of natural numbers, i.e. $r_1: r_2: r_3: . . . : r_n = 1^2: 2^2: 3^2: . . . : n^2$. Thus, the stationary orbits are not equally spaced. Consecutive orbits nearer the nucleus are closer than the farther orbits.

The radius of hydrogen atom is.

$$r_n = \frac{\epsilon_0 n^2 h^2}{\pi m e^2} \text{ (Here, Z = 1 in hydrogen atom)}$$

Using,

$\epsilon_0 = 8.85 \times 10^{-12}$ Fm^{-1}

$h = 6.62 \times 10^{-34}$ Js

$m = 9.1 \times 10^{-31}$ kg

$e = 1.6 \times 10^{-19}$ C

We get,

$$r_n = \frac{8.85 \times 10^{-12} \times n^2 \times (6.62 \times 10^{-34})^2}{\pi \times 9.1 \times 10^{-31} \times (1.6 \times 10^{-19})^2}$$

$$r_n = 0.53 \times 10^{-10}\, n^2$$

$$r_n = 0.53\, n^2 \text{ Å}$$

For first orbit, n = 1,

$$r_1 = 0.53 \text{Å}$$

This is the radius of lowest orbit of hydrogen atom and is known as Bohr radius.

Velocity of electron

From Bohr's second postulate,

$$mvr = n\frac{h}{2\pi}$$

$$v = \frac{nh}{2\pi m r}$$

In general, the speed of electron in n^{th} orbit is,

$$\mathbf{v_n = \frac{nh}{2\pi m r_n}} \qquad \text{...(22.7)}$$

Also, the radius of the n^{th} orbit,

$$\mathbf{r_n = \frac{\epsilon_0 n^2 h^2}{\pi m Z e^2}} \qquad \text{. . . (22.8)}$$

Substituting the value of r_n in equation (22.8) in equation (22.7), we get,

$$v_n = \frac{nh}{2\pi m} \times \frac{\pi m Z e^2}{\epsilon_0 n^2 h^2}$$

$$v_n = \frac{Ze^2}{2\epsilon_0 nh} \quad \text{. . . (22.9)}$$

This equation gives the speed of electron in n^{th} orbit of an atom. Providing other parameters constant, $v_n \propto \frac{1}{n}$. i.e. the speed of electron decreases as it goes to higher energy states. The speed is maximum for the innermost orbit. The speed of electron in hydrogen atom is,

$$v_n = \frac{e^2}{2\epsilon_0 nh} \quad \text{. . . (22.10)}$$

Using the values of e, ϵ_0 and h in equation (22.10), we get,

$$v_n = \frac{(1.6 \times 10^{-19})^2}{2 \times 8.85 \times n \times 6.62 \times 10^{-34}}$$

$$= \frac{2.19 \times 10^6}{n}$$

$$v_n = \left(\frac{1}{137}c\right)\frac{1}{n}$$

Where c = speed of light in vacuum.

So, speed of an electron in the innermost orbit of hydrogen atom (i.e. n = 1) is $\frac{1}{137}$ of the speed of light in vacuum.

Frequency of revolution of electron

The frequency of revolution of an electron around the nucleus is the number of revolution completed by the electron in one second.

The speed of electron (v) can be expressed in terms of angular velocity (ω) and radius (r) of the orbit.

i.e. $v = r\omega$

$= r.\ 2\pi f$, where, f = frequency of revolution

$$\therefore f = \frac{v}{2\pi r} \quad \text{. . . (22.11)}$$

For innermost orbit of hydrogen atom,

$v = 2.19 \times 10^6\ ms^{-1}$

$r = 0.53\ Å = 0.53 \times 10^{-10}$ m

So, $f = \frac{2.19 \times 10^6}{2\pi \times 0.53 \times 10^{-10}}$

$f = 6.56 \times 10^{15}$ Hz

22.4 Energy of Electron

An electron revolving around the nucleus possesses both kinetic energy and potential energy. The kinetic energy is due to the motion of electron in any orbit, whereas the potential energy is due to its position in the field of nucleus. So, the total energy is the sum of kinetic energy and potential energy.

i.e. Total energy of the electron $E = E_k + E_P$

We have, kinetic energy of n^{th} orbit,

$$E_k = \frac{1}{2} mv_n^2 \quad \text{. . . (22.12)}$$

Using the value of v_n from equation (22.9) in equation (22.12), we get,

$$E_k = \frac{1}{2}m\left(\frac{Ze^2}{2\epsilon_0 nh}\right)^2$$

$$\mathbf{E_k = \frac{mZ^2e^4}{8\epsilon_0^2 n^2h^2}} \qquad \text{. . . (22.13)}$$

The potential energy E_p of electron in the electric field of nucleus is written as,

$$\mathbf{E_p = -e\left(\frac{1}{4\pi\epsilon_0}\frac{Ze}{r_n}\right)} \qquad \text{. . . (22.14)}$$

$\because$ Electric potential energy = qV

Here, q = e and V is provided by nucleus. The negative sign indicates that the electron is bound to the nucleus. Using the value of r_n from equation (22.6) in equation (22.14), we get.

$$E_p = -\frac{1}{4\pi\epsilon_0}\frac{Ze^2}{\left(\frac{\epsilon_0 n^2h^2}{\pi mZe^2}\right)}$$

$$\mathbf{= -\frac{mZ^2e^4}{4\epsilon_0^2 n^2h^2}} \qquad \text{. . . (22.15)}$$

Now, total energy, $$E = \frac{mZ^2e^4}{8\epsilon_0^2 n^2h^2} - \frac{mZ^2e^4}{4\epsilon_0^2 n^2h^2}$$

$$E = -\frac{mZ^2e^4}{8\epsilon_0^2 n^2h^2}$$

The general form of total energy of an electron in an atom is,

$$\therefore \quad \mathbf{E_n = -\frac{mZ^2e^4}{8\epsilon_0^2 n^2h^2}} \qquad \text{. . . (22.16)}$$

Total energy of an electron is indicated by negative sign. It shows that, the electron is bound to the nucleus and some work should be done to separate it from the nucleus. Here, kinetic energy of nucleus has been neglected based on assumption that nucleus is heavy compared to electron.

For a hydrogen atom, Z = 1

So, total energy of electron in n^{th} orbit of hydrogen atom is,

$$\mathbf{E_n = -\frac{me^4}{8\epsilon_0^2 n^2h^2}} \qquad \text{. . . (22.17)}$$

Using,

$m = 9.1 \times 10^{-31}$ kg

$e = 1.6 \times 10^{-6}$ C

$\epsilon_0 = 8.85 \times 10^{-12}$ Fm^{-1}

$h = 6.62 \times 10^{-34}$ Js

Then,

$$E_n = -\frac{9.1 \times 10^{-31} \times (1.6 \times 10^{-19})^4}{8 \times (8.85 \times 10^{-12})^2 \times n^2 \times (6.62 \times 10^{-34})^2}$$

$$E_n = \frac{-13.6 \times 1.6 \times 10^{-19}}{n^2} \text{ joule}$$

or, $E_n = \frac{-13.6}{n^2}$ eV [$\because$ 1 eV = 1.6×10^{-19} J]

i.e. $E_n \propto -\frac{1}{n^2}$

As n increases, E_n becomes less negative. It means the energy of electron possesses greater energy in higher energy state.

At n = ∞, E_n = 0, which shows that the maximum energy of electron is zero. It means electron gets free from the nucleus.

Numbers of orbit n	Total energy $E_n = -\frac{13.6}{n^2}$ eV
n = 1	E_1 = – 13.60 eV
n = 2	E_2 = – 3.40 eV
n = 3	E_3 = – 1.51 eV
n = 4	E_4 = – 0.85 eV
n = 5	E_5 = – 0.54 eV
. . .	. . .
n = ∞	E_∞ = 0

The energy of various energy states of hydrogen atom can be visualized by energy diagram as shown in Fig (22.3). The lowermost energy state, n = 1, is called ground state (ground level) of a hydrogen atom. All other energy states above the ground state are called excited states. The energy state of n = 2 is called first excited state, n = 3 is called second energy state and so on. The difference of energy that possesses by the electron in different energy states are indicated by the spacing of energy levels. Larger the spacing between the energy states greater the energy required for the transition of electrons in these orbits. The space between n = 1 and n = 2 is the largest. It means maximum energy is required for the transition of electron from 1 → 2 energy state than required for any other two consecutive energy states.

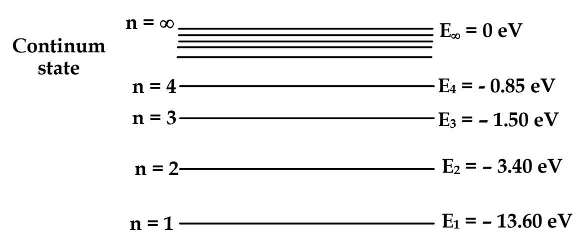


Fig. 22.3: Energy level diagram

Energy emitted during electron Transition

Let ΔE be the energy emitted in moving an electron from a higher energy level (n_2) to lower energy level (n_1). Then,

$$\Delta E = E_{n_2} - E_{n_1} \qquad \text{. . . (22.18)}$$

Where E_{n_1} and E_{n_2} are the energies corresponding to principal quantum numbers n_1 and n_2 respectively.

The energy of energy states n_1 and n_2 are,

$$E_{n_1} = -\frac{mZ^2e^4}{8\epsilon_0^2 n_1^2 h^2}$$

and $E_{n_2} = -\frac{mZ^2e^4}{8\epsilon_0^2 n_2^2 h^2}$

Substituting the values of E_{n_2} and E_{n_1} in equation (22.18), we get,

$$\Delta E = -\frac{mZ^2e^4}{8\epsilon_0^2 n_2^2 h^2} - \left(-\frac{mZ^2e^4}{8\epsilon_0^2 n_1^2 h^2}\right)$$

$$= \frac{mZ^2e^4}{8\epsilon_0^2 n_1^2 h^2} - \frac{mZ^2e^4}{8\epsilon_0^2 n_2^2 h^2}$$

$$= \frac{mZ^2e^4}{8\epsilon_0^2 h^2}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\Delta E = 13.6Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \text{eV}$$

For hydrogen atom, Z = 1

$$\therefore \mathbf{\Delta E = 13.6\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) eV} \qquad \text{. . . (22.19)}$$

Frequency and Wavelength of Emitted Radiation

When an electron jumps from higher energy state to lower energy state, energy is radiated to the surrounding. The emitted radiation is quantum of energy and such quantized energy packet is called photon. The frequency and wavelength of radiation so emitted can be determined using the Bohr's third postulate of atomic model. According to this postulate,

$$hf = E_{n_2} - E_{n_1}$$

Where f = frequency of emitted radiation

$$hf = -\frac{mZ^2e^4}{8\epsilon_0^2 n_2^2 h^2} - \left(-\frac{mZ^2e^4}{8\epsilon_0^2 n_1^2 h^2}\right)$$

$$\text{or,} \quad hf = \frac{mZ^2e^4}{8\epsilon_0^2 n_1^2 h^2} - \frac{mZ^2e^4}{8\epsilon_0^2 n_2^2 h^2}$$

$$\textbf{or,} \quad \mathbf{f = \frac{mZ^2e^4}{8\epsilon_0^2 h^3}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)} \qquad \text{. . . (22.20)}$$

This is the required expression for frequency for emitted radiation.

$$\text{Also, } f = \frac{c}{\lambda}$$

Where, c = Speed of light

λ = Wavelength of radiation

Now, equation (22.20) becomes,

$$\frac{c}{\lambda} = \frac{mZ^2e^4}{8\epsilon_0^2 h^3}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\text{or,} \quad \frac{1}{\lambda} = \frac{mZ^2e^4}{8\epsilon_0^2 ch^3}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

or, $$\frac{1}{\lambda} = \left(\frac{me^4}{8\epsilon_0^2 ch^3}\right) Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\therefore \quad \frac{1}{\lambda} = RZ^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Where, R is called Rydberg constant after the name of the Swedish spectroscopist J. Rydberg.

$$R = \frac{me^4}{8\epsilon_0^2 ch^3}$$

or, $$R = \frac{9.1 \times 10^{-31} \times (1.6 \times 10^{-19})^4}{8 \times (8.85 \times 10^{-12})^2 \times 3 \times 10^8 \times (6.62 \times 10^{-34})^3}$$

$$R = 1.097 \times 10^7 \text{ m}^{-1}$$

For hydrogen atom, Z = 1

$$\mathbf{\frac{1}{\lambda} = R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)} \qquad \textbf{. . . (22.21)}$$

The term $\frac{1}{\lambda}$ is denoted by $\bar{\nu}$ and it is called wave number. Wave number ($\bar{\nu}$) is defined as the number of waves formed in unit distance.

$$\therefore \quad \mathbf{\bar{\nu} = R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)} \qquad \textbf{. . . (22.22)}$$

This relation is known as Rydberg formula for the spectrum of the hydrogen atom. It is very useful in the study of origin of spectral lines.

22.5 Spectral Series of Hydrogen Atom

When hydrogen atom absorbs energy from its surrounding, the electron in its lower energy state jumps to higher energy states. The electron that jumps to higher energy states is called excited electron. The excited electron stays at the higher energy for very short period (~ 10^{-8} s). Then, it returns to lower energy state immediately. The electron may return directly from excited state to ground state or through other lower energy. In the de-excitation, it loses the energy to the surroundings. The amount of energy radiated in de-excitation is equal to the difference of energy in corresponding energy states. In each transition of electron, electromagnetic radiation of definite wavelength is emitted, which is known as spectral line. These lines may or may not be visible.

The wavelength of emitted radiation depends on the difference of energy between the energy level through which the electron jumps, greater the energy difference of energy states, shorter the wavelength of emitted radiation (i.e. $\Delta E = \frac{hc}{\lambda}$). The spectral lines of different wavelengths are known as spectral series. Hydrogen spectrum was studied systematically by many scientists in different periods. These series are named after the name of scientists who studied the corresponding series. The spectral series for hydrogen atom is shown in Fig. 22.4. The energy level diagram including the spectral series is also known as Kossel diagram.

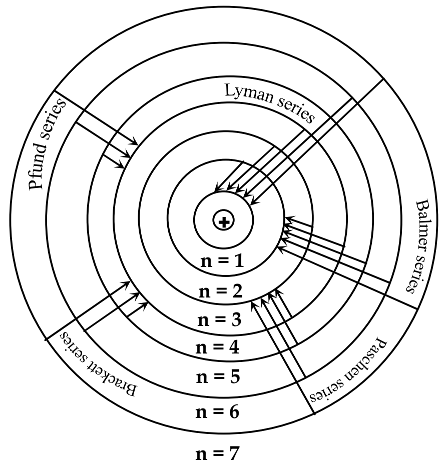


Fig. 22.4: Spectral series in hydrogen atom

The wavelength of radiation emitted when electron jumps in different energy states are explained clearly by using the following relation.

$$\bar{\upsilon} = \frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \qquad \ldots (22.23)$$

The relation in equation (22.23) explains the complete spectrum of hydrogen atom. The spectral series is a group of different spectral lines. They are different due to difference in energy or wavelength or frequency.

i. **Lyman series:** The spectral series of hydrogen atom when an electron jumps from higher energy states to ground state is called Lyman series. It lies in the ultraviolet region of electromagnetic spectrum. So, for Lyman series $n_1 = 1$ and $n_2 = 2, 3, 4, \ldots \infty$. This series was discovered by Lyman in 1915. The wavelengths of spectral lines of Lyman series are,

$$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$$

a. **The longest wavelength:** The wavelength of the first member of Lyman series, $n_1 = 1$ and $n_2 = 2$

If λ_ℓ is the longest wavelength, we have,

$$\frac{1}{\lambda_\ell} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$

$$\therefore \quad \lambda_\ell = \frac{4}{3R} = \frac{4}{3 \times 1.097 \times 10^7} = 1.216 \times 10^{-7}\ \text{m} \approx 1216\ \text{Å}$$

b. **The shortest wavelength (λ_s):** The wavelength of the last member of this series, $n_1 = 1$ and $n_2 = \infty$

$$\therefore \quad \frac{1}{\lambda_s} = R\left(\frac{1}{1^2} - \frac{1}{\infty}\right) = R\,(1 - 0)$$

$$\text{or} \quad \lambda_s = \frac{1}{R} = \frac{1}{1.097 \times 10^{-7}} = 0.9115 \times 10^{-7}\ \text{m} = 911.6\ \text{Å}$$

Therefore, the ratio of shortest and longest wavelength of Lyman series

$$= \frac{\lambda_s}{\lambda_\ell} = \frac{\frac{1}{R}}{\frac{4}{3R}} = \frac{3}{4}$$

In this way, other wavelengths between $n_1 = 1$ to $n_2 = \infty$ can be found out.

ii. Balmer series: The spectral series of hydrogen atom when an electron jumps from higher energy states to first excited state is called Balmer series. It lies in the visible region of electromagnetic spectrum. So, for Balmer series, $n_1 = 2$ and $n_2 = 3, 4 \ldots \infty$. This series was discovered by Balmer in 1885. The wavelength of spectral lines in Balmer series is given by

$$\bar{\upsilon} = \frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n_2^2}\right)$$

For first member of Balmer series, $n_1 = 2$ and $n_2 = 3$

$$\therefore \quad \frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{5R}{36}$$

$$\text{or,} \quad \lambda = \frac{36}{5R}$$

$$\text{or,} \quad \lambda = \frac{36}{5 \times 1.097 \times 10^7} \approx 6563 \times 10^{-10}\ \text{m} = 6563\ \text{Å}.$$

The longest wavelength or wavelength of first member of Balmer series. This line is also called **H_α–line**.

$$\lambda_\alpha = \lambda = \frac{36}{5R} = 6563\ \text{Å}$$

For **H_β – line**, we have,

$n_1 = 2$ and $n_2 = 4$

$$\therefore \quad \lambda_\beta = \frac{64}{12R} = \frac{16}{3R}$$

$$\therefore \quad \frac{\lambda_\alpha}{\lambda_\beta} = \frac{\frac{36}{5R}}{\frac{16}{3R}} = \frac{108}{80} = \frac{27}{20}$$

In the same way, we can find out for H_γ and H_δ when $n_2 = 4$ and $n_2 = 5$ respectively.

Similarly, for $n_1 = 2$ and $n_2 = \infty$

$$\lambda_s = \frac{4}{R}$$

This is the shortest wavelength of a line in the Balmer series.

iii. Paschen series: The spectral series of hydrogen atom when an electron jumps from higher energy states to second excited state is called Paschen series. It lies in the infrared region of electromagnetic spectrum. So, for Paschen series, $n_1 = 3$ and $n_2 = 4, 5, \ldots \infty$. This series was discovered by Paschen in 1896. The wavelength of the spectral lines in Paschen series is given by,

$$\bar{\upsilon} = \frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n_2^2}\right) \qquad \text{where } n_2 = 4, 5, \ldots \infty$$

iv. Brackett series: The spectral series of hydrogen atom when an electron jumps from higher energy states to third excited state is called Brackett series. It lies in the infrared region of electromagnetic spectrum. For Brackett series, $n_1 = 4$ and $n_2 = 5, 6, 7, \ldots \infty$. This series was discovered by Brackett in 1922.The wavelength of spectral lines of this series are,

$$\bar{\upsilon} = \frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n_2^2}\right) \qquad \text{where } n_2 = 5, 6, 7, \ldots \infty$$

v. Pfund series: The spectral series of hydrogen atom when an electron jumps from higher energy states to fourth excited state is called Pfund series. It lies in the far infrared region of electromagnetic spectrum.

So, for pfund series, $n_1 = 5$ and $n_2 = 6, 7, 8, \ldots$ This series was discovered by Pfund in 1925. The wavelength of spectral lines of pfund series is given by

$$\bar{\upsilon} = \frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n_2^2}\right), \text{ where, } n_2 = 6, 7, 8, \ldots$$

With each series, the spectral lines get closer together with increasing frequency. No two elements have the same atomic emission spectrum.

Energy-level diagram

We just explained, the different spectral lines on the basis of geometrical concept of orbits. A much more important concept than above is in terms of energy level diagram which is as shown in Fig. 22.5

A diagram showing the total energies of an electron in different stationary orbits of an atom is called energy level diagram. The various possible energy values are shown by parallel horizontal lines while transitions of electron among stationary orbits are shown by parallel vertical lines.

The total energy of an electron in n^{th} orbit of H-atom is given by

$$E_n = -\frac{13.6}{n^2} \text{ eV}.$$

For, $n = 1, 2, 3, \ldots$, we get the energies of electron in different orbits as,

n = 1,	$E_1 = -13.6$ eV	This is the ground state energy
n = 2,	$E_2 = -3.4$ eV	This is the 1st excited state
n = 3,	$E_3 = -1.51$ eV	This is the 2nd excited state
. . ..	. . ..	. . ..
. . ..	. . ..	. . ..
$n = \infty$,	$E_\infty = 0$	(The atom is said to be ionized in this case.)

The total energy of an electron in an atom is found negative value. It means larger the magnitude gives lower energy. The negative value of energy of the electron indicates that electron is bound to the nucleus and some work should be done to separate it from the atomic orbit.

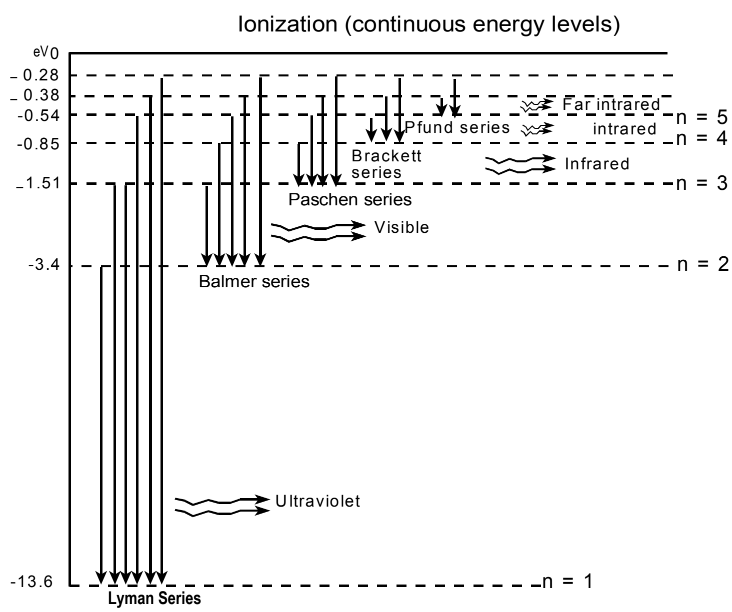


Fig. 22.5: Spectra of H-atom

From this diagram, it is clear that, atom emits line spectra i.e. the atom emits radiation of particular wave length only, which is due to transition of electron between two fixed stationary orbits.

22.6 Heinsenberg Uncertainty Principle

Quantum physics, mostly studies the behaviour of microscopic objects, specifically, the atomic and sub-atomic particles. Classical physics deals about the properties of mediocre objects, mostly the objects which we entertain in our daily life. Theory of relativity mostly studies about the planetary and celestial objects. Study of the behaviour of classical objects is relatively easier than very small and very large objects. The precise measurement of position, velocity, momentum, etc. of classical objects is possible. However, it is impossible to measure such variables for microscopic objects like electrons, protons. Out of many physical variables, some of them form a pair which cannot be measured simultaneously in nature. These pairs are called canonically conjugate variables. Position-momentum pair, energy-time pair and angular momentum-angular displacement pair are canonically conjugate variables.

On the regards of such canonically conjugate variables, German physicist Werner Heinsberg, in 1927 proposed a principle called Heinsenberg Uncertainty principle. This principle states that "it is impossible to measure the conjugate variables of an object with unlimited precision."

i. **Position-momentum uncertainty:** In accordance with uncertainty principle, the position and momentum uncertainty principle can be stated as "it is impossible to measure both the position and momentum of a subatomic particle at a time accurately." The product of the uncertainty in position (Δx) and the uncertainty in momentum (Δp) is greater than or equal to $\hbar\left(=\frac{h}{2\pi}\right)$, i.e.

$$\Delta x \times \Delta p \geq \hbar$$

ii. **Energy-time uncertainty:** With the similar fashion as position-momentum uncertainty, the uncertainty principle in energy and time is stated as "it is not possible to measure both energy and time of a subatomic particle at a time accurately." The product of uncertainty in energy (ΔE) and the uncertainty in time (Δt) is never less than $\hbar\left(=\frac{h}{2\pi}\right)$. i.e.

$$\Delta E \times \Delta t \geq \hbar$$

It is to be noted that the value of Planck's constant ($h = 6.62 \times 10^{-34}$ Js) is so small with respect to the uncertainty in macroscopic object that the study of uncertainty in these objects is irrelevant.

Application of Uncertainty Principle

The principle of uncertainty explains a large number of facts which could not be explained by classical ideas.

Non-existence of electron in the nucleus: The radius of the nucleus of any atom is of the order of 10^{-14} m so that, if an electron is confined within nucleus, the uncertainty in its position must not be greater than 10^{-14} m.

$$\therefore \quad \Delta x \times \Delta p \geq \hbar$$

$$\text{or,} \quad \Delta p \geq \frac{\hbar}{\Delta x}$$

$$\text{or,} \quad \Delta p \geq \frac{1.054 \times 10^{-34}}{10^{-14}}$$

$$\text{or,} \quad \Delta p \geq 1.054 \times 10^{-20} \text{ kg m/s}$$

If this is the uncertainty in a nuclear electron's momentum, the momentum p itself must be at least comparable with this magnitude i.e.

$$p \geq 1.054 \times 10^{-20}$$

i.e. $$mv \geq 1.05 \times 10^{-20}$$

$$v \geq \frac{1.054 \times 10^{-20}}{m}$$

$$v \geq \frac{1.054 \times 10^{-20}}{9.1 \times 10^{-31}}$$

$$v \geq \frac{10.54 \times 10^{-21}}{9.1 \times 10^{-31}}$$

$$v \geq 1.15 \times 10^{10}\ ms^{-1}$$

It means that, the electron exists into the nucleus only when its velocity exceeds $1.15 \times 10^{10}\ ms^{-1}$. However, it is a well known fact that the velocity of any body cannot be greater than $3 \times 10^{8}\ ms^{-1}$ (velocity of light in vacuum). It concludes that, the electron does not exist inside the nucleus.

Wave Particle Duality

In 1924, a French physicist Louis Victor de-Broglie developed a new concept on property of matters. He put forward the concept that matters show wave nature too. All particles electrons, protons, atoms, marbles and even humans-have a wavelength that is related to the momentum of the particles by wavelength $= \frac{h}{\text{momentum}}$. The wavelength of a particle is called de-Broglie wavelength. This concept of de-Broglie is called de-Broglie hypothesis and the double characters of matter-wave and particle nature is called the wave particle duality.

A particle of large mass and ordinary speed has too small wavelength to be detected. However, a subatomic particle such as an electron moving at typical speed has a detectable wavelength.

For a one kg mass object when travelling with speed 100 ms^{-1} speed, the de-Broglie wavelength is

$$\lambda = \frac{h}{mv} = \frac{6.62 \times 10^{-34}}{1 \times 100} = 6.62 \times 10^{-36}\ m$$

But, an electron when travelling with speed $10^{6}\ ms^{-1}$

$$\lambda = \frac{h}{mv} = \frac{6.62 \times 10^{-34}}{9.1 \times 10^{-31} \times 10^{6}} = 7.27 \times 10^{-11}\ m$$

This value of wavelength of electron is smaller than visible light but large enough for noticeable diffraction. Similarly, the radiations also have the dual nature. The wave theory of light can satisfactorily describe the wave phenomena: interference, diffraction and polarization. On the other hand effects like photoelectric effect, Compton effect and pair production are effectively explained by quantum (particle) theory of light. Thus, the radiations behaves both as wave and particle, which ultimately supports the de-Broglie hypothesis.

de-Broglie Wave Equation

From Einstein's mass-energy equivalence relation, we have,

$$\mathbf{E = mc^2} \qquad \ldots (22.24)$$

Where m = mass of particle and c = speed of light in vacuum.

Again, according to quantum theory, we have,

$$\mathbf{E = hf} \qquad \ldots (22.25)$$

Combining equations (22.24) and (22.25), we get,

$$hf = mc^2$$

But $c = f\lambda$ for a wave. So, we can write,

$$\therefore \quad h\frac{c}{\lambda} = mc^2$$

$$\text{or,} \quad \lambda = \frac{h}{mc}$$

$$\textbf{or,} \quad \lambda = \frac{h}{p} \quad \text{(where p = mc is the momentum of the photon.)} \qquad \ldots (22.26)$$

When de-Broglie computed the relation between wavelength and momentum of photon, he also proposed this theory for material particles such that the motion of any material particle also possesses wave. The momentum of a material particle, viz., electron, proton, neutron or a ball of mass m and velocity v is $p = mv$ and its de-Broglie wave-length is,

$$\therefore \quad \lambda = \frac{h}{p} = \frac{h}{mv} \qquad \ldots (22.27)$$

Equation (22.27) shows that, greater the particle's momentum, the shorter its wavelength. Equation (22.27) is known as *de-Broglie wave equation* and the wavelength given by this equation is called de-Broglie wavelength. Equation (22.27) couples intimately the dual behaviour of a material particle because λ has a meaning only for a wave and momentum p has a meaning only for a particle.

If E is the kinetic energy of the material particle, then,

$$E = \frac{1}{2}mv^2 = \frac{1}{2}\frac{m^2v^2}{m} = \frac{p^2}{2m} \qquad (\because p = mv)$$

$$\therefore \quad p = \sqrt{2mE} \qquad \ldots (22.28)$$

So, from equations (22.27) and (22.28), de-Broglie wavelength is also given by,

$$\lambda = \frac{h}{\sqrt{2\,mE}} \qquad \ldots (22.29)$$

Also, if $v = 0$ then $\lambda = \infty$ and if $v = \infty$ then $\lambda = 0$. (from equation 22.27)

It means, the waves are associated with a matter only when it is in motion. The matter (particles) may be charged or uncharged but must be in motion. The waves associated with it are independent of charge. It means, de-Broglie wave cannot be electromagnetic wave because electromagnetic wave is produced by only accelerated charged particles. The existence of matter waves was first experimentally verified by Davisson and Germer in 1927.

Matter waves

The waves associated with the matters when they are in motion are called matter waves. Matter waves can be detected in subatomic particles like electrons, protons, atoms and molecules. The concept of matter waves was firstly introduced by de-Broglie. So, these waves are also named de-Broglie waves. He proposed this concept while he was student and later on he was awarded with Nobel Prize. The wavelength of matter wave is,

$$\lambda = \frac{h}{mv}$$

Some important properties of matter waves are as below:

i. Smaller is the velocity of the particle, greater the de-Broglie wavelength.
ii. Matter waves are generated from the motion of particle. If the particle at rest, wavelength become infinity.
iii. The velocity of matter wave is greater than the velocity of light.
iv. Matter waves are neither transverse nor longitudinal

22.7 Excitation Energy and Excitation Potential

Excitation Energy

The energy required to raise an electron from ground state to any one of the higher energy state is known as excitation energy.

Let E_1 be the energy of ground state and E_i be the energy of any excited state of an atom. Then, the excitation energy for the atom is,

$$\Delta E = E_i - E_1$$

If the electron jumps to first excited state from ground state in hydrogen atom then

$$E_2 = -3.4 \text{ eV and } E_1 = -13.6 \text{ eV}$$

Excitation energy for the given condition is,

$$\begin{aligned} \Delta E_1 &= E_2 - E_1 \\ &= -3.4 - (-13.6) = 10.2 \text{ eV} \end{aligned}$$

The second excitation energy for hydrogen atoms is,

$$\begin{aligned} \Delta E_2 &= E_3 - E_1 \\ &= -1.51 - (-13.6) = 12.09 \text{ eV} \end{aligned}$$

Excitation Potential

The accelerating potential which gives sufficient energy for a bombarding electron to excite the ground state electron to any one of the excited state is called excitation potential.

The first excitation energy of hydrogen atom is,

$$\Delta E_1 = E_2 - E_1$$

or, $e\Delta V_1 = -3.4 - (-13.6)$

or, $e\Delta V_1 = 10.2 \text{ eV}$

or, $\Delta V_1 = \dfrac{10.2 \text{ eV}}{e}$

$\therefore$ $\Delta V_1 = 10.2 \text{ V}$

$\therefore$ The first excitation potential of hydrogen atom, $\Delta V_1 = 10.2$ V.

Similarly, the second excitation energy of hydrogen atom is,

$$\Delta E_2 = 12.09 \text{ eV}$$

or, $e\Delta V_2 = 12.09 \text{ eV}$

or, $\Delta V_2 = \dfrac{12.09 \text{ eV}}{e}$

$\therefore$ $\Delta V_2 = 12.09 \text{ V}$

Above result shows that numerical value of excitation energy and excitation potential are equal.

22.8 Ionization Energy and Ionization Potential

Ionization Energy

The energy required to knock out an electron from an atom is known as ionization energy. To knock out an electron, it should be sent to infinity orbit (i.e. $n = \infty$). After removal of an electron from an atom, the atom becomes ionized, which is called positive ion.

In hydrogen atom, energy of electron at ground state (i.e. E_1) = –13.6 eV and the energy of atom at infinity, $E_\infty = 0$

So, ionization energy of hydrogen atom is,

$$\Delta E_i = E_\infty - E_1$$
$$= 0 - (-13.6) = 13.6 \text{ eV}$$

Ionization Potential

The accelerating potential which gives sufficient energy for a bombarding electron to ionize the target atom by knocking one of its electrons completely out of the atom is known as ionization potential. The ionization energy of hydrogen atom is,

or, $\Delta E_i = 13.6$ eV

or, $e\Delta V_i = 13.6$ eV

or, $\Delta V_i = \dfrac{13.6 \text{ eV}}{e}$

$\therefore$ $\Delta V_i = 13.6$ V

Limitations of Bohr's Theory

Although the Bohr's theory could account for the stability of atom and somehow describe the spectrum of hydrogen atoms, it has many limitations. Some limitations are as follows:

i. This theory is appropriate only for hydrogen and hydrogen like atoms, but can not explain the atoms with two or more electrons.
ii. This theory can explain only the circular orbits of the atom but unable to explain elliptical orbits, which are possible in many atoms.
iii. Electron shows both particle and wave properties. However, this theory could not say anything about the wave nature of electrons.
iv. Bohr's theory determines only the frequency of spectral lines of hydrogen like atom, but unable to estimate the intensity of spectral lines.
v. This theory could not explain the effect of electric field and magnetic field in spectral lines.
vi. This theory does not tell anything about how much time an excited electron remains in higher energy state.
vii. This theory could not explain why the atomic orbits are quantized.

22.9 Emission and Absorption Spectra

In light waves, a spectrum refers the arrangement of waves in accordance with wavelength or frequency. The light spectrum is measured with a special device called the spectrometer. Various colours in the light spectrum can be detected by spectrometer. The spectra in hydrogen atom are both visible and invisible.

Emission Spectra

The spectra originated due to the de-excitation of subatomic particles in atom are known as emission spectra. These spectra are emitted out from the atom; hence it is named 'emission'. On the basis of the character of the source, emission spectra are of three types:

i. Line spectra
ii. Continuous spectra
iii. Band spectra

i. **Line spectra:** The spectral lines consisting of discrete lines of definite wavelength are called line spectra. Each spectral line is distinguished with another line by dark spaces. So, that the lines are seen clearly. Hydrogen spectrum, sodium spectrum, mercury spectrum, etc. are some examples of line spectra.

If an atom acquires energy from the surrounding, it excites to the higher energy states. However, the life time of atom in higher energy state is very short (~ 10^{-8} s), so it returns to the lower energy state. During this process, energy is emitted in the form of photon. When many atoms of an element are in same energy states, they always emit particular colour sets in the spectrum. The skematic diagram of line spectrum is shown in Fig. 22.6.

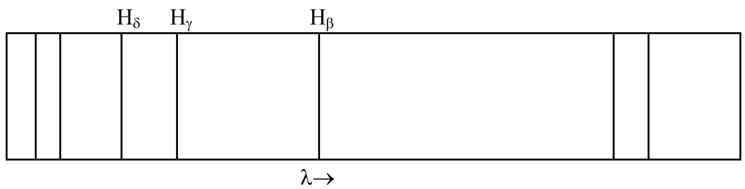


Fig. 22.6: Line spectra

ii. **Continuous spectra:** The spectra which cover a wide range of wavelength with negligible frequency gap are known as continuous spectra. In such spectrum, no spectral line is separately distinguished as shown in Fig. 22.7. These types of spectra are originated by hot solids, liquids and high density gases. The spectra originated from gases in the sun are continuous. This form of spectrum depends on the temperature and surface condition of body.

Fig. 22.7: Continuous emission spectrum

iii. **Band spectra:** The spectra which consist of separate group of lines are known as band spectra. Each band has one sharp edge and is distinctly separated from another band. Many gases produce band spectra O_2, CO_2, NH_3, N_2 can produce band spectra. The molecules which contain two or more atoms can originate band spectra. Each atom of a molecule can produce its own line spectra. Due to the overlapping of the line spectrum of different atoms, group of such lines (bands) are obtained.

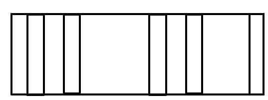


Fig. 22.8: Band spectra

Absorption Spectra

The spectra of electromagnetic radiation transmitted through a substance, showing dark lines or bands due to absorption at specific wavelengths are called absorption spectra. When a white beam of light transmitted through a sample material (for example: gas), the light photons that match the energy gap of the molecules present in that sample are absorbed in order to excite the molecule. Other photons transmit unaffected. If the transmitted beam is observed with a spectrometer, some dark lines are seen in place of absorbed spectra. Such spectra are called absorption spectra. The dark lines in the absorption occur exactly in place of bright lines in emission spectra of that sample.

Types of absorption spectra are similar to emission spectra: line spectra, continuous spectra and band spectra. The line spectra are produced by atoms of gas molecules; continuous spectra are produced by solids and liquids. Likewise band absorption spectra are produced by gaseous state of matter.

22.10 Laser

A laser is a device that emits light through a process of optical amplification based on the stimulated emission of electromagnetic radiation. The full form of LASER is "Light Amplification by Stimulated Emission of Radiation." First working laser was operated by Thedore Maiman at the Hughs Research Laboratories in 1960. Actually, Albert Einstein had given the idea of stimulated emission that could produce a laser. The light produced from laser is very intense, monochromatic, coherent and highly unidirectional. It is a rare process in nature.

Stable state

As explained in Bohr's atom model, the atomic orbitals can either be in ground state or in excited states. Electrons remain in ground state most of the time. If it excites to upper state (i.e. excited state), it returns to the ground energy state after small interval of time ($\sim 10^{-8}$ s). Therefore, ground state is also termed as stable state.

Metastable state

Metastable state is a particular excited state of an atom in which the electron remains relatively longer time than the ordinary excited states. Metastable state has great importance in producing the laser, since electrons stay in this state for relatively longer time. A metastable state may be considered as a kind of temporary energy trap. 2S state of helium, 5S state of neon are the examples of metastable states. Electrons of an atom in the metastable state remains excited for a considerable time in the order of 10^{-6} s to 10^{-3} s.

Some terms Related to Laser Production

i. Spontaneous emission

Spontaneous emission is the process of emission of electromagnetic radiation in an atom or molecule, when the electron transition occurs from an excited state to lower energy state, usually ground state. In this process, a photon of energy equal to the energy gap of two energy states is emitted. Spontaneous emission occurs not only in orbital electrons, but also in the nuclear excitation. This type of emission is responsible for most of the light we see all around us.

If an electron jumps spontaneously from higher energy state E_2 to lower energy state E_1, then the emitted energy is, $E_2 - E_1 = hf$ as shown in Fig. 22.9.

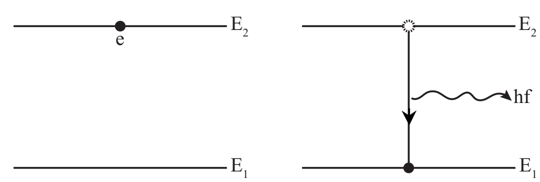


Fig. 22.9: Spontaneous emission of radiation

ii. Induced or stimulated absorption

When a photon of light having energy equal to the energy gap between two energy levels of an atom, is absorbed by ground state electron, it jumps to the higher energy state. This process is called induced or stimulated absorption. This is called so because the incident photon has stimulated the atom to absorb the energy. For an electron at lower energy state E_1 to get raised to higher energy state E_2, the electron in the ground

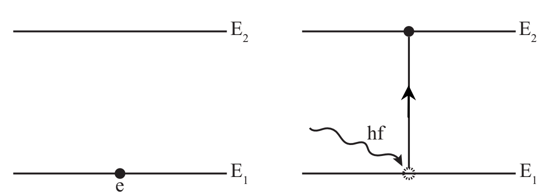


Fig. 22.10: Induced absorption

state must absorb photon of energy, $E_2 - E_1 = hf$, where, hf is the energy of absorbed photon by the electron in lower energy state as shown in Fig. 22.10.

iii. Stimulated emission

The process by which an incoming photon of a specific frequency can interact with an electron at excited state, causing it to drop to a lower energy state is known as stimulated emission. If an electron of an atom in the excited state E_2 interacts with an incident photon of energy exactly equal to $E_2 - E_1 = hf$, then it stimulates the electron to come down to the lower energy state E_1. The emitted photon has the frequency exactly equal to the incident photon. Thus, a pair of identical photons is emitted in stimulated emission. The stimulated emission is shown in Fig. 22.11.

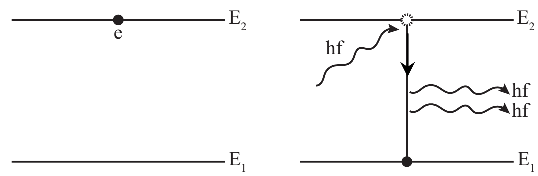


Fig. 22.11: Stimulated emission

Differences between spontaneous emission and stimulated emission

Spontaneous emission	Stimulated emission
1. It is a natural transition in which an atom is de-excited after the end of its life time in the higher energy level.	1. It is an artificial transition which occurs due to de-excitation of an atom before the end of its life time in the higher energy state.
2. The photon emitted due to spontaneous emission can move in any direction.	2. The photon emitted due to stimulated emission can move only in the direction of the incident photon.
3. The probability of spontaneous emission depends only on the properties of the two energy levels between which the transition occurs.	3. The probability of stimulated emission depends on the properties of two energy levels involved in the transition as well as on the energy density of incident radiation.

Population inversion

It is the condition in atoms in which the numbers of electrons stay more in higher energy state than in lower energy state. In ordinary condition, the number of electron remains more in ground state than the excited state. If we reverse the situation by any means, population inversion takes place. To produce the laser, population inversion is an essential condition, which is required for continuous emission of radiation by stimulated process. The Boltzmann distribution explains the ratio of the number of atoms in each state by using a factor called Boltzmann factor,

$$\text{Since, } N_2 \propto e^{-\frac{E_2}{k_B T}}$$

$$N_1 \propto e^{-\frac{E_1}{k_B T}}$$

$$\therefore \quad \frac{N_2}{N_1} = e^{-\frac{(E_2 - E_1)}{k_B T}} < 1 \text{ (in normal condition)}$$

So, for population inversion, we need to produce a condition, $\frac{N_2}{N_1} > 1$.

Where, N_2 = number of electrons in higher energy state E_2

N_1 = number of electrons in lower energy state E_1

k_B = Boltzmann constant

T = absolute temperature

If $\frac{N_2}{N_1} > 1$, population inversion occurs.

Optical pumping

The process in which light is used to raise the electrons from a lower energy level in an atom to a higher energy level is known as optical pumping. It is commonly used in laser production so as to achieve population inversion. This technique was developed by Alfred Kastler in 1966.

This process pumps the electrons to well defined higher energy state, mostly in metastable state in laser production.

Principle of laser

The working principle of laser basically depends on the stimulated emission. It also depends on two features (i) population inversion and (ii) optical pumping. If the electron in excited state absorbs photon with equal energy as that of energy gap between it and lower energy state, it releases a second photon of same frequency, in phase with the first photon. In this situation, the electron de-excites into the lower energy state. This causes the stimulated emission. The emitted photon is identical to the stimulating photon with the same frequency, polarization and direction of propagation. The photons, as a result, are totally coherent. This is the critical property that allows optical amplification to take place.

Helium – Neon laser

The essential components of He-Ne laser are shown in Fig. 22.12. Its usual operation wavelength is 632.8 nm, in the red portion of the visible spectrum.

It consists of a discharge tube of nearly 0.5 m length and diameter about 5 mm whose two ends are cut at Brewster's angle. The tube is filled with the mixture of helium and neon in the ratio 5:1 at a total pressure of about 1 torr. The tube is also provided with two parallel mirrors at the ends, one of which is 100% reflecting and while the other is partially transparent. The distance between the mirror is half integer multiple of the wavelength of laser light.

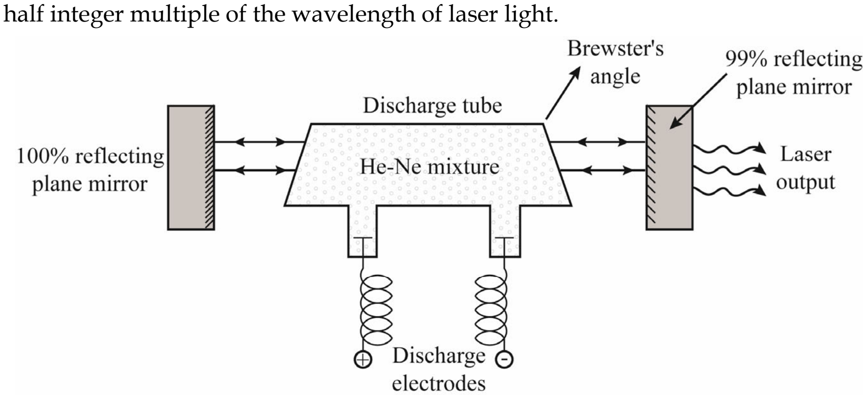


Fig. 22.12: He-Ne Laser

Initially, some of the atoms in the mixture are ionized by applying high electric field and the electrons thus created are accelerated due to this field. These electrons then acquire sufficient kinetic energy to excite the neutral atoms by collision.

Thus, the helium atoms are excited by the electron impact to 2S state which is a relatively long lived state and has energy of 20.61 eV. While the neon atoms are much less excited by the electron impact. This energy of He-atom in 2S state is nearly equal to the energy of the 5S level of Ne-atom which is 20.66 eV. So, during collision, some of the He-atoms transfer their energy to ground state Ne-atom with 0.05 eV of extra energy provided by the kinetic energy of the atoms. The Ne-atom is then excited to 5S state. Thus, He-atom helps to achieve population inversion of the Ne-atom which is necessary for the laser action.

The Ne-atoms in the 5S state are stimulated to jump to 3P state there by emitting a photon whose wavelength corresponds to wavelength of laser light.

The wavelength of laser light is given by,

$$\lambda = \frac{hc}{E_{5S} - E_{3P}}$$

$$= \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{(20.66 - 18.70) \times 1.6 \times 10^{-19}} \approx 634 \text{ nm}$$

The photons emitted during the process are reflected back and forth between the parallel mirrors and hence number of photons is amplified. Thus, an avalanche of photons is created which come out from the partially transparent mirror as an intense unidirectional beam known as laser.

Further, the Ne-atom goes a spontaneous emission from 3P to 3S state and the transition from 3S state to 2P state is a non radiative transfer.

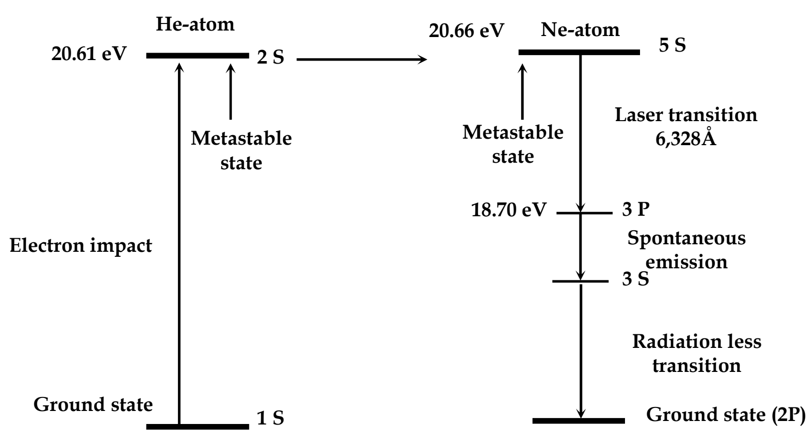


Fig. 22.13: Energy level diagram of helium-neon laser

Applications of Laser

1. Laser technology finds its extensive use in manufacturing industries. For example, for cutting, drilling, welding cladding, soldering, hardening, engraving, etc.
2. Lasers especially semi-conductor lasers are widely used in communication system. These are used in information handling systems; for example, they read digitally coded music on CDs and retrieve data from computer disc. They are also used extensively particularly for long-distance optical data transmission.

3. Laser therapy has become more common in medical field especially in performing bloodless surgery.
4. Lasers are used for the precision measurement of length surveying for example in the construction of tunnels, measuring the distance between earth & moon (lunar ranging).
5. Laser is most efficiently used in defence mechanism such as detecting and destroying missiles.
6. Laser beam is used in the production of true three-dimensional pictures in space without use of lens. The record of this three dimensional image of the object on a film is called hologram and the phenomenon is holography.

Properties of Laser

i. Laser can travel over long distance without much loss of energy (or intensity).
ii.. It is highly monochromatic.
iii. It is highly unidirectional beam of light.
iv. It is coherent i.e. all atoms emit radiations simultaneously.
v. It is very intense.
vi. Its wavelength is very, very short.
vii. It is highly energetic light.
viii. It stays on at a single frequency.
ix. It is well collimated i.e. all rays are perfectly parallel to each other. Hence, a laser beam is very narrow and can travel to long distance without spreading. It can be brought to an extremely sharp focus.
x. It can vaporize even the hardest metal because of its high energy density and directional property. A laser beam can produce temperature of order of 10^4 °C at a focused point.

Comparison of Ordinary Light and Laser Light

S.N.	Ordinary light	Laser
1.	It is less intense than laser.	It is a light of larger intensity.
2.	It is chromatic.	It is monochromatic.
3.	It is multidirectional.	It is purely unidirectional.
4.	It is incoherent source.	This is highly coherent.
5.	Wavelength is longer than laser.	Wavelength is very short of the order 10^{-11} m.
6.	Less energetic than laser.	Highly energetic.
7.	It consists of many wavelength and frequency.	It has only a particular frequency.
8.	It can produce ordinary temperature when strikes on an object.	But, it produces high temperature in order of 10^4°C.

Tips for MCQs

1. Bohr's theory
   i. Mathematical form of basic postulates
      a. $\frac{mv^2}{r} = \frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r^2}$ b. $mvr = \frac{nh}{2\pi}$ c. $\Delta E = hf = E_i - E_f$

ii. Bohr's theory explains

a. Stability of atom b. spectrum of hydrogen c. energy of electron in n^{th} orbit

iii. Radius of orbit of hydrogen like atom, $r_n = \frac{\epsilon_0 n^2 h^2}{\pi m Z e^2}$, $\therefore r_n \propto n^2$

iv. Velocity of electron in an orbit, $v_n = \frac{Ze^2}{2\epsilon_0 nh}$, $v_n \propto \frac{1}{n}$

As, $r_n \propto n^2$, $v_n \propto \frac{1}{\sqrt{r_n}}$

v. Energy of electron in an orbit of hydrogen like atom,

$E = \frac{me^4 Z^2}{8\epsilon_0^2 n^2 h^2} = -13.6\frac{Z^2}{n^2}$ eV

vi. Rydberg constant, $R = \frac{me^4}{8\epsilon_0^2 n^2 ch^3} = 1.097 \times 10^7 m^{-1}$.

vii. Total energy of electron is more in outer orbit than inner orbit. The energy of free electron is zero.

viii. Total energy of electron in a stationary orbit is negative, which means the electron is bound to the nucleus.

ix. The wavelength of radiation that is emitted in electron transition, $\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

x. The wavelength range for:

a. Lyman series is 121.6 nm to 91.2 nm, lies in UV range.

b. Balmer series is 656.3 nm to 365 nm, lies in visible range.

c. Paschen series is 1875 nm to 822 nm, lies in infrared range.

d. Bracket series is 4051 nm to 1458 nm, lies in infrared range.

e. Pfund series is 7460 nm to 2279 nm, lies in far infrared range.

2. Dual nature of radiation

i. Both radiation and matter show dual nature: Wave nature and particle nature.

ii. Wave length of particle wave is also called de Broglie wavelength,

$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mE_k}} = \frac{h}{\sqrt{3mk_BT}}$

iii. For a particle at rest, $v = 0$, $\lambda = \infty$ and for a particle in motion, $v \neq 0$, $\lambda \neq 0$. It means wave is associated only with the particle in motion.

iv. Wavelength of particle is independent with its charge, but depends on accelerating potential.

a. Wavelength of electron, $\lambda_e = \frac{12.27}{\sqrt{V}}$ Å. b. Wavelength of proton, $\lambda_p = \frac{0.287}{\sqrt{V}}$ Å.

Therefore for constant potential $\lambda_e > \lambda_p$.

3. Heisenberg's uncertainty principle

i. Position momentum uncertainty, $\Delta x \times \Delta p \geq \frac{h}{2\pi}$

ii. Energy time uncertainty, $\Delta E \times \Delta t \geq \frac{h}{2\pi}$

iii. Angular momentum and angular displacement uncertainly, $\Delta L \times \Delta\theta \geq \frac{h}{2\pi}$

iv. This theory successfully explains the non-existence of electrons in the nucleus.

4. Generally, ground state consists of a large number of electrons.

Physical quantity	Symbols	Dimensions	units	Remarks
Wavelength	λ	[L]	m	
Rydberg constant	R	[L^{-1}]	m^{-1}	
Bohr radius	a_0	[L]	m	Radius of the first Bohr orbit in a H-atom.
Atomic number	Z			Z = Number of electrons = Number of Protons

Worked Out Problems

1. [NEB 2074] Calculate de Broglie wavelength of an electron which has been accelerated through a potential difference of 200 V. Given mass of electron = 9.1 × 10^{-31} kg and Planck's constant, h = 6.6 × 10^{-34} Js.

SOLUTION

Given,

Potential difference (V) = 200 V

Mass of electron (m) = 9.1 × 10^{-31} kg

Planck's constant (h) = 6.6 × 10^{-34} Js

de Broglie wavelength (λ) = ?

We know,

$$\lambda = \frac{h}{mv}$$

The velocity of electron is determined from,

$$\frac{1}{2}mv^2 = eV$$

$$v^2 = \frac{2eV}{m}$$

$$v = \sqrt{\frac{2eV}{m}}$$

$$= \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 200}{9.1 \times 10^{-31}}}$$

$$= 8.4 \times 10^6 \text{ ms}^{-1}$$

Now, wavelength (λ) $= \dfrac{6.6 \times 10^{-34}}{9.1 \times 10^{-31} \times 8.4 \times 10^6}$

$$= 8.6 \times 10^{-11} \text{ m}$$

$\therefore$ The de Broglie wavelength is 8.6 × 10^{-11} m.

2. [HSEB 2072] Calculate the wavelength of electromagnetic radiation emitted by a hydrogen atom which undergoes a transition between energy levels of –1.36 × 10^{-19} J and –5.45 × 10^{-19} J. Given Planck's constant = 6.6 × 10^{-34} Js.

SOLUTION

Upper energy level (E_2) = –1.36 × 10^{-19} J

Lower energy level (E_1) = –5.45 × 10^{-19} J

Planck's constant (h) = 6.6 × 10^{-34} Js

From Bohr's postulate

$$\frac{hc}{\lambda} = E_2 - E_1$$

$$\therefore \lambda = \frac{hc}{E_2 - E_1}$$

$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{-1.36 \times 10^{-19} - (-5.45 \times 10^{-19})}$$

$$= \frac{19.8 \times 10^{-26}}{4.09 \times 10^{-19}} = 4.84 \times 10^{-7} \text{ m}$$

3. [HSEB 2070] A cricket ball is moving with a speed of 120 km/h. What would be its de Broglie wavelength if its mass is 400 g.

SOLUTION

Speed (v) = 120 km/h = 33.33 ms^{-1}

Mass of ball (m) = 400 g = 0.4 kg

de Broglie wavelength (λ) = ?

We know,

$$\lambda = \frac{h}{mv}$$

$$= \frac{6.6 \times 10^{-34}}{0.4 \times 33.33}$$

$$= 4.95 \times 10^{-35} \text{ m}$$

$\therefore$ The de Broglie wavelength is 4.95 × 10^{-35} m.

4. [HSEB 2067] Find the wavelength of the radiation emitted from a hydrogen atom, when an electron jumps from third orbit to second orbit. (Give $\in_0$ = 8.854 × 10^{-12} $C^2N^{-1}m^2$, h = 6.62 × 10^{-34} Js, m_e = 9.1 × 10^{-31} kg).

SOLUTION

Permitivity ($\in_0$) = $8.854 \times 10^{-12}\ C^2N^{-1}m^{-2}$

Planck's constant (h) = 6.62×10^{-34} Js

Mass of electron (m_e) = 9.1×10^{-31} kg

$n_1 = 2,\ n_2 = 3$

Wavelength (λ) = 2

We have,

$$\frac{1}{\lambda} = \frac{me^4}{8\in_0^2 ch^3}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = \frac{9.1 \times 10^{-31} \times (1.6 \times 10^{-19})^4}{8 \times (8.854 \times 10^{-12})^2 \times 3 \times 10^8 \times (6.62 \times 10^{-34})^3}\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$= 1.097 \times 10^7\left(\frac{1}{4} - \frac{1}{9}\right) = 1.097 \times 10^7\left(\frac{5}{36}\right)$$

$$\lambda = 6.56 \times 10^{-7}\ m$$

Therefore, the required wavelength is 6.56×10^{-7} m.

5. If an electron position can be measured to an accuracy of 10^{-9} m. How accurately can its velocity be measured? (m_e = 9.1 × 10^{-31} kg)

SOLUTION

Given,

Position uncertainty (Δx) = 10^{-9} m

Mass (m_e) = 9.1×10^{-31} kg

Velocity uncertainty (Δv) = ?

From Heisenberg uncertainty principle,

$$\Delta x\ .\ \Delta p \approx \frac{h}{2\pi}$$

$$\Delta x\ .\ m_e\ \Delta v \approx \frac{h}{2\pi}$$

$$\Delta v \approx \frac{h}{2\pi\ \Delta x\ m_e}$$

$$\approx \frac{6.62 \times 10^{-34}}{2\pi \times 10^{-9} \times 9.1 \times 10^{-31}}$$

$$\approx 1.16 \times 10^5\ ms^{-1}$$

6. Calculate the wave length of the first and last members of the Blamer series for H-atom.(R = 10^7m^{-1}).

SOLUTION

Given,

$\lambda_1 = ?$, $\lambda_\infty = ?$

For Balmer series, $n_1 = 2$

and n_2 = 3, 4, ...

We know that,

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

For the first member, $n_2 = 3$

$$\therefore \quad \frac{1}{\lambda_1} = 10^7\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 10^7 \times \frac{5}{36}$$

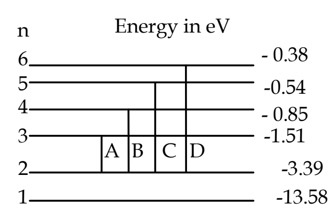
$$\therefore \quad \lambda_1 = \frac{36}{5} \times 10^{-7} = 7.2 \times 10^{-7}\ m$$

For the last member, $n_2 = \infty$

$$\therefore \quad \frac{1}{\lambda_\infty} = 10^7\left(\frac{1}{2^2} - \frac{1}{\infty}\right) = \frac{10^7}{4}$$

$$\therefore \quad \lambda_\infty = \frac{4}{10^7} = 4 \times 10^{-7}\ m$$

7. Figure, which represents the lowest energy levels of the electrons in the hydrogen atom, specifies the values of the principal quantum number n associated with each state and the corresponding value of the energy level, measured in electron volts. Work out the wavelength of the lines associated with the transitions A, B, C, D marked in the figure. (Take 1 eV to be 1.6 × 10^{-19} J; Planck constant h to be 6.5 × 10^{-34} Js; and c, the velocity of light in vacuum, to be 3 × 10^8 ms^{-1})



SOLUTION

For transition A, we can write,

$$E_3 - E_2 = \frac{hc}{\lambda_A}$$

or, $\lambda_A = \dfrac{hc}{E_3 - E_2} = \dfrac{6.65 \times 10^{-34} \times 3 \times 10^8}{\{-1.51 - (-3.39)\} \times 1.6 \times 10^{-19}} = \dfrac{19.5 \times 10^{-7}}{1.88 \times 1.6 \times 10^{-19}}$

$\therefore \quad \lambda_A = 6.5 \times 10^{-7}$ m.

Similarly, we can have,

$\lambda_B = \dfrac{hc}{E_4 - E_2} = \dfrac{19.5 \times 10^{-7}}{(-0.85 + 3.39) \times 10^{-19}}$

$\therefore \quad \lambda_B = 4.8 \times 10^{-7}$m

$\lambda_C = \dfrac{hc}{E_5 - E_2} = \dfrac{19.5 \times 10^{-7}}{(-0.54 + 3.39) \times 1.6 \times 10^{-19}}$

$\therefore \quad \lambda_C = 4.3 \times 10^{-7}$ m

$\lambda_D = \dfrac{hc}{E_6 - E_2} = \dfrac{19.5 \times 10^{-7}}{(-0.38 + 3.39) \times 1.6 \times 10^{-19}}$

$\therefore \quad \lambda_D = 4 \times 10^{-7}$ m

8. a. An atom initially in an energy level with E = –6.52 eV absorb a photon that has wavelength 860 nm. What is the internal energy of the atom after it absorbs the photon?

b. An atom initially in an energy level with E = – 2.68 eV emits a photon that has wavelength 420 nm. What is the internal energy of the atom after it emits the photon?

SOLUTION

(a) Given,

$E_1 = -6.25 \text{ eV} = -6.52 \times 1.6 \times 10^{-19}$ J
$= -10.43 \times 10^{-19}$ J

$\lambda = 860 \text{ nm} = 860 \times 10^{-9}$ m

$h = 6.625 \times 10^{-34}$ Js

Internal energy of the atom after absorbing photon (E_2) = ?

When a photon is absorbed, we can write,

$\therefore \quad E_2 - E_1 = hf$

or, $E_2 = hf + E_1$

$= \dfrac{hc}{\lambda} + E_1$

$= \dfrac{6.625 \times 10^{-34} \times 3 \times 10^8}{860 \times 10^{-9}} - 10.43 \times 10^{-19}$

$= 0.0231 \times 10^{-17} - 0.10 \times 10^{-17}$

$= -0.0769 \times 10^{-17} \text{ J} = -\dfrac{0.077 \times 10^{-17}}{1.6 \times 10^{-19}}$ eV

$\therefore \quad E_2 = -4.80$ eV

(b) Given,

$E_1 = -2.68 \text{ eV} = -2.68 \times 1.6 \times 10^{-19}$ J

$\lambda = 420 \text{ nm} = 420 \times 10^{-9}$ m

Internal energy of an atom after emitting photon (E_2) = ?

When a photon is emitted, we can write,

$\therefore \quad E_1 - E_2 = hf$

or, $E_2 = E_1 - hf = E_1 - \dfrac{hc}{\lambda}$

$= -4.288 \times 10^{-19} - \dfrac{6.625 \times 10^{-34} \times 3 \times 10^8}{420 \times 10^{-9}}$

$= -4.288 \times 10^{-19} - 4.73 \times 10^{-19}$

$= -9.018 \times 10^{-19}$ J

$= \dfrac{9.018 \times 10^{-19}}{1.6 \times 10^{-19}}$ eV

$\therefore \quad E_2 = -5.64$ eV

9. How many photons per second are emitted by a 7.50 mW CO_2 laser that has a wave length of 10.6 μm?

SOLUTION

Given,

$P = 7.50 \text{ mW} = 7.5 \times 10^{-3}$ W

$\lambda = 10.6 \ \mu\text{m} = 10.6 \times 10^{-6}$ m.

Number of photons per second, $\dfrac{n}{t}$ = ?

We know that

$\text{Power} = \dfrac{\text{Energy}}{\text{Time}}$

or, $p = \dfrac{nhf}{t}$

or, $\dfrac{n}{t} = \dfrac{P}{hf} = \dfrac{P \times \lambda}{hc} \quad (\because c = f\lambda)$

$= \dfrac{7.5 \times 10^{-3} \times 10.6 \times 10^{-6}}{6.62 \times 10^{-34} \times 3 \times 10^8}$

$\therefore \quad \dfrac{n}{t} = 4 \times 10^{17}$ photons/sec

10. An electron is confined to a box of size 100 Å. Calculate the uncertainty introduced in the velocity of an electron.

SOLUTION

Given,

$m_e = 9.1 \times 10^{-31}$ kg

$h = 6.62 \times 10^{-34}$ Js

$\Delta x = 100$ Å $= 100 \times 10^{-10}$m $= 10^{-8}$ m

$\Delta v = ?$

From Heisenberg uncertainty principle, we have,

$$\Delta x \times \Delta p = h/2\pi$$

or, $\Delta x \times (m_e \times \Delta v) = h/2\pi$

or, $\Delta v = \dfrac{h}{2\pi\,\Delta x \times m_e}$

$$= \frac{6.62 \times 10^{-34}}{2\pi \times 10^{-8} \times 9.1 \times 10^{-31}}$$

$\therefore$ $\Delta v = 1.15 \times 10^{4}$ m/s^{-1}

11. [HSEB 2073] Calculate the de Broglie wavelength of electron having kinetic energy of 400 eV.

SOLUTION

Given,

De Broglie wavelength (λ) = ?

Kinetic energy (E_k) = 400 eV = $400 \times 1.6 \times 10^{-19}$ J

We have,

$$\lambda = \frac{h}{\sqrt{2mE_k}} = \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 400 \times 1.6 \times 10^{-19}}} = 6.13 \times 10^{-11} \text{ m}$$

12. [NEB 2075] A hydrogen atom is in ground state. What is the quantum number to which it will be excited absorbing a photon of energy 12.75 eV?

SOLUTION

Given,

Energy of absorption (ΔE) = 12.75 eV

Quantum number (n) = ?

We know,

The energy of hydrogen atom in n^{th} state,

$$E_n = \frac{-13.6 \text{ eV}}{n^2} \quad \ldots \text{(i)}$$

The energy gap between n^{th} state and the ground state of hydrogen atom is,

$\Delta E = E_n - E_1$

Here,

$\Delta E = 12.75$ eV and

$E_1 = -13.6$ eV

Then,

$12.75 = E_n - (-13.6)$

or, $E_n = 12.75 - 13.6 = -0.85$ eV

$\therefore$ $E_n = -0.85$ eV

Also, from (i),

$$E_n = -\frac{13.6}{n^2} \text{ eV}$$

or, $-0.85 = -\dfrac{13.6}{n^2}$

or, $n^2 = \dfrac{13.6}{0.85} = 16$

$\therefore$ $n = 4$

Hence, the quantum number is 4.

Challenging Problems

1. An excited hydrogen atom has energy – 3.4 eV. Find the angular momentum of the electron.

Ans: 2.1×10^{-34} Js

2. **[ALP]** The ground state of the electron in the hydrogen atom may be represented by the energy – 13.6 eV and the first two excited states by – 3.4 eV and – 1.5 eV respectively. On a scale in which an electron completely free of the atom is at zero energy. Use this date to calculate the ionization potential of the hydrogen atom and the wavelengths of these lines in the emission spectrum of hydrogen.

Ans: 13.6 V, 1.3×10^{-7} m, 1.02×10^{-7} m, 6.54×10^{-7} m

3. **[ALP]** An electron of energy 20 eV comes into collision with a hydrogen atom in its ground state. The atom is excited into a state of higher internal energy and the electron is scattered with reduced velocity. The atom subsequently returns to its ground state with the emission of a photon of wavelength 1.216×10^{-7} m. Determine the velocity of the scattered electron. (mass of electron = 9.1×10^{-31} kg, e = 1.6×10^{-19} C, c = 3×10^{8} m/s, h = 6.62×10^{-34} Js). [HSEB 2055]

Ans: 1.86×10^{6} m/s

4. **[UP]** A hydrogen atom initially in the ground level absorbs a photon, which excites it to the $n = 4$ level. Determine the wavelength and frequency of the photon.

Ans: 97.4×10^{-9} m, 3.08×10^{15} Hz

5. **[UP]** A hydrogen atom is in a state with energy – 1.51 eV. In the Bohr model, what is the angular momentum of the electron in the atom, with respect to an axis at the nucleus?

Ans: 3.16×10^{-34} kg m^2/s

6. **[UP]** A hydrogen atom undergoes a transition from the n = 5 to the n = 2 state. (a) What are the energy and wavelength of the photon that is emitted? (b) If the angular momentum is conserved and if the Bohr model is used to describe the atom, what must the angular momentum be of the photon that is emitted?

Ans: (a) 4.56×10^{-19}J, 4.36×10^{-7} m (b) 3.16×10^{-34} Js

7. **[UP]**
   (a) Using the Bohr model, calculate the speed of the electron in hydrogen atom in the n = 1, 2 and 3 levels.
   (b) Calculate the orbital period in each of these levels.
   (c) The average life time of the first excited level of hydrogen atom is 1.0×10^{-8} s. In the Bohr model how many orbits does an electron in the $n = 2$ level complete before returning to the ground level?

Ans: (a) 2.18×10^{6} m/s (b) 1.52×10^{-16} sec (c) 8.2×10^{6}

8. **[UP]** Calculate the wavelength of the first line of the Balmer series if the wavelength of the second line of this series is 4.86×10^{-7} m. [HSEB 2054]

Ans: 6.56×10^{-7} m

9. **[UP]** Determine the wavelength, frequency and hence energy of the photon emitted for H_γ line in Balmer series. (Rydberg constant = 1.07×10^{7} m^{-1})

Ans: 434.1×10^{-9} m, 6.90×10^{14} Hz, 4.576×10^{-19} J

10. **[UP]** Calculate the de Broglie wavelength of an electron having Kinetic energy 0.4 KeV.

Ans: 0.61Å

*[**Note:** Hints to challenging problems are given at the end of this chapter.]*

Conceptual Questions with Answers

1. A proton and an electron have the same kinetic energy. Which has longer de-Broglie wavelength? [NEB 2074]

➯ The de-Broglie wavelength for a particle of kinetic energy E is,

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$$

The wavelength of electron wave, $\lambda_e = \dfrac{h}{\sqrt{2m_eE}}$

The wavelength of proton wave, $\lambda_p = \dfrac{h}{\sqrt{2m_pE}}$

For equal kinetic energy E in both electron and proton,

$$\frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_p}{m_e}}$$

$$\lambda \propto \frac{1}{\sqrt{m}}$$

This relation shows that greater the mass, smaller the wavelength. It concludes that electron has longer de-Broglie wavelength than proton.

---

2. The accelerating voltage of a proton is increased to twice. How will its de-Broglie wavelength change? Explain.

➪ The de-Broglie wavelength for a particle is,

$$\lambda = \frac{h}{2mE}$$

Where, E is the kinetic energy of a particle (say proton). We have, E = eV. So,

$$\lambda = \frac{h}{\sqrt{2meV}}$$

If accelerating potential is increased by twice, V' = 2V, then new wavelength,

$$\lambda' = \frac{h}{\sqrt{2me\,(2V)}}$$

$$= \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2meV}}\right) = \frac{1}{\sqrt{2}}\lambda$$

∴ Wavelength is decreased by $\sqrt{2}$ times.

---

3. "The total energy of an electron of an atom in an orbit is negative". What does this negative energy indicate? [HSEB 2072]

➪ The total energy of an electron in an atom is found negative value. It means larger the magnitude gives lower energy. The negative value of energy of the electron indicates that electron is bound to the nucleus and some work should be done to separate it from the atomic orbit.

---

4. Why is gravitational force not taken into consideration while evaluating the energy of an electron in an atom? [HSEB 2070]

➪ The electric force between an electron in ground state and the nucleus hydrogen atom is,

$$f_e = \frac{1}{4\pi\epsilon_0}\frac{e \cdot Ze}{r^2}$$

$$= 9 \times 10^9 \frac{1.6 \times 10^{-19} \times 1.6 \times 10^{-19}}{(0.53 \times 10^{-10})^2} \quad \text{(for example, hydrogen Z = 1)}$$

$$= 8.2 \times 10^{-8}\ \text{N}$$

Also, the gravitational force between an electron and nucleus (i.e. proton) of a hydrogen atom is,

$$f_g = G\frac{m_e\, m_p}{r^2}$$

$$= 6.67 \times 10^{-11} \frac{9.1 \times 10^{-31} \times 1.67 \times 10^{-27}}{(0.53 \times 10^{-10})^2}$$

$$= 3.61 \times 10^{-47}\ \text{N}$$

This shows that the gravitational force between electron and proton is about 10^{39} times weaker than the electric force. Hence, gravitational force is not taken into account in evaluating the energy of an atom.

---

5. The wave nature of particles is not observable in daily life. Why? [HSEB 2070]

➪ A particle of large mass and ordinary speed has too small a wavelength to be detected. However, a tiny particle such as an electron moving at typical speed has a detectable wavelength.

For a one kg mass object when travelling with speed 100 ms^{-1} speed, the de-Broglie wavelength is

$$\lambda = \frac{h}{mv} = \frac{6.62 \times 10^{-34}}{1 \times 100} = 6.62 \times 10^{-36} \text{ m}$$

But, an electron when travelling with speed 10^6 ms^{-1}

$$\lambda = \frac{h}{mv} = \frac{6.62 \times 10^{-34}}{9.1 \times 10^{-31} \times 10^6} = 7.27 \times 10^{-11} \text{ m}$$

This value of wavelength of electron is smaller than visible light but large enough for noticeable diffraction.

---

6. A stone is dropped from the top of a building. How does its de-Broglie wavelength change? [HSEB 2067]

➪ The de-Broglie wavelength of a body (say a stone) is,

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

While the stone is dropped from a top, its velocity increases continuously. It means the wavelength gradually decreases.

---

7. What are the differences between matter wave and electromagnetic wave? [HSEB 2067]

➪ The important difference between electromagnetic waves and matter waves are given below:

Electromagnetic waves	Matter waves
1. Electromagnetic waves are associated with electric and magnetic fields perpendicular to each other and to the direction of propagation of radiation.	1. Matter waves may not be associated with electric and magnetic fields.
2. Electromagnetic waves can be emitted or radiated into space.	2. Matter waves are neither radiated into space nor emitted by the particles. These are simply associated with the particles.
3. All electromagnetic waves travel with the same velocity in vacuum.	3. Matter waves travel with different velocities in different direction in a homogeneous media.
4. The wavelengths of electromagnetic radiations are much large and are given by the relation, $\lambda = \frac{c}{f}$ where f is the frequency of wave.	4. The matter waves have shorter wavelengths given by de-Broglie equation, $\lambda = \frac{h}{mv}$ where, mv is the momentum of the particle.
5. They are not the mechanical wave because these waves are not due to the cause of the vibration of medium particles.	5. They are neither electromagnetic nor mechanical wave because they travel in vacuum and material medium also.

---

8. Distinguish between stimulated emission and spontaneous emission.

➪ The important difference between spontaneous emission and stimulated emission are given below:

Spontaneous emission	Stimulated emission
1. It is a natural transition in which an atom is de-excited after the end of its life time in the higher energy level.	1. It is an artificial transition which occurs due to de-excitation of an atom before the end of its life time in the higher energy state.
2. The photon emitted due to spontaneous emission can move in any direction.	2. The photon emitted due to stimulated emission can move only in the direction of the incident photon.
3. The probability of spontaneous emission depends only on the properties of the two energy levels between which the transition occurs.	3. The probability of stimulated emission depends on the properties of two energy levels involved in the transition as well as on the energy density of incident radiation.

9. What do you mean by uncertainty principle?

➯ Heinsenberg's uncertainty principle states that "it is impossible to measure the conjugate variables of an object with unlimited precision." For example,

Position-momentum uncertainty: In accordance with uncertainty principle, the position and momentum uncertainty principle can be stated as "it is impossible to measure both the position and momentum of a subatomic particle at a time accurately." The product of the uncertainty in position (Δx) and the uncertainty in momentum (Δp) is greater than or equal to $\hbar \left(= \frac{h}{2\pi}\right)$, i.e.

$$\Delta x \times \Delta p \geq \hbar$$

10. What are the importance of de-Broglie wave? [HSEB 2055]

➯ de-Broglie wave has great importance in atomic and quantum physics. Some importance are written below:

i. It is the firm evidence of particle nature of light, which made it possible to explain the quantization of energy.

ii. It shows the wave nature of particle. Each material particle exhibit the wave nature on its motion i.e., $\lambda = \frac{h}{mv}$

iii. Many important devices, like electron microscope, work in the principle of de-Broglie hypothesis.

11. Can any type of wave move faster than light? Explain.

➯ Matter wave can move faster than light. Its wave length is, $\lambda = \frac{h}{mv}$ and frequency, $f = \frac{mc^2}{h}$

The speed of matter wave, $v_d = f\lambda$

$$v_d = \frac{mc^2}{h} \times \frac{h}{mv}$$

$$v_d = \frac{c^2}{v} = \left(\frac{c}{v}\right) c$$

We know, no material moves faster than light i.e. $c > v$.

Hence, $\frac{c}{v} > 1$

So, $v_d > c$.

Thus, de-Broglie wave velocity must be greater than the speed of light.

12. Why are Bohr's orbits are called stationary orbits?

➯ The electron in an orbit of an atom does not radiate energy, while revolving around the nucleus. It can neither radiate energy nor absorb energy in that specific orbit. So, the Bohr's orbit is called stationary in the sense of constant energy carried by electron in that orbit.

13. A hydrogen atom contains one electron. But the spectrum of hydrogen atom has many lines. Why?

➯ A source of hydrogen spectrum consists of millions of hydrogen atoms. Also, each atom contains infinitely large number stationary orbits. As the electron transits from lower energy level to any higher energy level, it returns to the lower energy level in very short time ($\sim 10^{-8}$ s). Therefore, all possible transitions can occur from any higher level to any lower energy level. This gives rise to a large number of spectral lines.

14. Define ionization energy. What is its value for a hydrogen atom?

➯ The energy required to knock out an electron from an atom is known as ionization energy. To knock out an electron, it should be sent to infinity orbit (i.e. $n = \infty$). After removal of an electron from an atom, the atom becomes ionized, which is called positive ion.

In hydrogen atom, energy of electron at ground state (i.e. E_1) = –13.6 eV and the energy of atom at infinity, $E_\infty = 0$

So, ionization energy of hydrogen atom

$$\Delta E_i = E_\infty - E_1$$
$$= 0 - (-13.6) = 13.6 \text{ eV}$$

---

15. Define excitation energy. What is its value when electron jumps from ground state to first excited state?

✎ The energy required to raise an electron from ground state to any one of the higher energy state is known as excitation energy.

Let E_1 be the energy of ground state and E_i be the energy of any excited state of an atom. Then, the excitation energy for the atom is,

$$\Delta E = E_i - E_1$$

If the electron jumps to first excited state from ground state in hydrogen atom then

$$E_2 = -3.4 \text{ eV and } E_1 = -13.6 \text{ eV}$$

Excitation energy for the given condition is,

$$\Delta E_1 = E_2 - E_1$$
$$= -3.4 - (-13.6) = 10.2 \text{ eV}$$

---

16. What is Lyman series? What are the shortest and longest wavelength of Lyman series?

✎ **Lyman series:** The spectral series of hydrogen atom when an electron jumps from higher energy states to ground state is called Lyman series. It lies in the ultraviolet region of electromagnetic radiation. So, for Lyman series $n_1 = 1$ and $n_2 = 2, 3, 4, \ldots \infty$. This series was discovered by Lyman in 1915. The wavelengths of spectral lines of Lyman series are,

$$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$$

The longest wavelength in Lyman series is 1216 Å and the shortest wavelength in Lyman series is 912 Å.

---

17. Which spectral series of hydrogen atom lie within visible range? What are the shortest and longest wavelength of that spectral series?

✎ The spectral series of hydrogen atom when an electron jumps from higher energy states to first excited state is called Balmer series. It lies in the visible region of electromagnetic radiation.. So, for Balmer series, $n_1 = 2$ and $n_2 = 3, 4 \ldots \infty$. This series was discovered by Balmer in 1885. The wavelength of spectral lines in Balmer series is given by

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n_2^2}\right)$$

The shortest wavelength is 365 nm and longest wavelength is 365.3 nm of spectral series.

---

18. What are stationary orbits?

✎ To explain the atomic model of hydrogen like atom, Bohr has postulated that electrons can revolve around the nucleus in a certain discrete, non-radiating orbits in which the angular momentum of an electron is an integral multiple of $\frac{h}{2\pi}$. Such orbits are called stationary orbits.

---

19. What is the principle of laser?

✎ The full form of laser is "light amplification by stimulated emission of radiation". In the production of laser, the intensity of light is amplified, proceeding the stimulated emission. So, the laser production depends on the principle of stimulated emission of light.

---

20. What do you mean by stimulated emission?

✎ The process by which an incoming photon of a specific frequency can interact with an electron at excited state, causing it to drop to a lower energy state is known as stimulated emission. If an electron of an atom in the excited state E_2 interacts with an incident photon of energy exactly equal to $E_2 - E_1 = hf$, then it stimulates the electron to come down to the lower energy state E_1. The emitted photon has the frequency exactly equal to the incident photon.

21. What is population inversion in laser?

↬ It is the condition in atoms in which the numbers of electrons stay more in higher state than lower energy. In ordinary condition, the number of electron remains more in ground state than the excited state. If we reverse the situation by any means, population inversion takes place. To produce the laser, population inversion is an essential condition, which is required for continuous emission of radiation by stimulated process.

22. What is optical pumping in the production of laser? [HSEB 2072]

↬ The process in which light is used to raise the electrons from a lower energy level in an atom to a higher energy level is known as optical pumping. It is commonly used in laser production so as to achieve population inversion.

23. Why are the amount of helium taken more than the Neon in Ne-Ne laser, although the laser is produced by Neon?

↬ In He-Ne laser, excited helium works to carry the electrons of neon from ground state to meta-stable state. In the de-excitation of electron from 5s state to 3s state, laser is produced. To do so, population inversion is needed in neon which can be done only if there are large number of excited helium atoms.

24. What are the properties of laser?

↬ The major properties of laser are:

i. Laser can travel over long distance without much loss of energy (or intensity).

ii.. It is highly monochromatic.

iii. It is highly unidirectional beam of light.

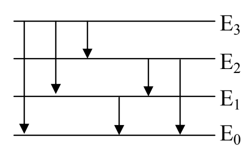
iv. It is coherent i.e. all atoms emit radiations simultaneously.

v. Its wavelength is very, very short.

vi. It is highly energetic light.

25. An electron is in the third excited state. How many different photon wavelengths are possible? (HSEB 2053)

↬ There are six possible ways of transition of electron if it is in the third excited state as shown in figure.



Corresponding to these six transitions, there are six wavelengths.

The above transition can also be shown by the formula,

Total number of spectral lines = $\frac{n(n-1)}{2}$, where n is the number of principal quantum number.

In the question, n = 4 so the total spectral lines will six.

26. How is Brackett series originated in hydrogen spectra? (HSEB 2052)

↬ When an electron jumps from higher energy levels to the fourth energy level, the Brackett series is obtained**.** In this series, the wavelength of spectra produced in Bracket series is,

$$\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n^2}\right), \text{ where, } n = 5, 6, 7, \ldots$$

Exercises

Short-Answer Type Questions

1. What are the main differences between Rutherford model and Bohr model?
2. The energies of the hydrogen atom orbits are negative. Is this true?
3. What are the evidences for dual nature of light?

4. What are the drawbacks of Rutherford's atomic model?
5. Write down the formula for the energy of an electron in the n^{th} Bohr orbit of a hydrogen atom. State the significance of the sign associated with it.
6. Explain what is meant by ionization energy of an atom. What is ionization energy for a hydrogen atom?
7. What do you mean by excitation potential?
8. Explain what is meant by ionization potential. What is the value of ionization potential for hydrogen atom?
9. Given $R_n = 1.097 \times 10^7 \text{ m}^{-1}$. Calculate the longest and the shortest wavelength in Balmer series of hydrogen spectra.
10. An electron is in the third excited state. How many different photon wavelength are possible?
11. How are different series in hydrogen spectra originated?
12. Why is an atom in its ground state called stable?
13. State and explain de Broglie's hypothesis.
14. Differentiate between matter waves and electromagnetic waves.
15. Why is the wave nature of particles not observable in daily life?
16. An electron and a proton have the same kinetic energy. Which one of them has the longer wavelength?
17. Point out the importance of de Broglie wave.
18. Show that de Broglie hypothesis of matter wave is in agreement with Bohr's theory.
19. What is metastable state?
20. Explain (a) spontaneous emission (b) stimulated or induced absorption (c) stimulated or induced emission (d) optical pumping and (e) population inversion.
21. Mention some uses of laser.
22. What is the principle of laser?
23. An electron is in Bohr's orbit n = 2 around a nucleus. What is the frequency of revolution of electron ? (Ans:- 819×10^{12} HZ).
24. Mention the differences between proton and photon.
25. The de Broglie wavelength of the particle of K.E. is λ. What would be the wavelength of the particle if it's K.E. were E/4? (2λ).
26. If the ionization energy of H-atom is 13.6 ev, what is the ionization energy of He^+? What are the differences between ordinary light and laser light?
27. Name the series obtained in H-spectrum and their spectroscopic range.
28. Differentiate between ordinary light and laser light.
29. Two electrons revolve around in the second and the third orbit respectively. Which of them posses more energy?
30. How can you say that energy is quantised?

Long-Answer Type Questions

1. Explain how Bohr modified the Rutherford model of an atom to explain the emission of radiation from atoms. (HSEB 2062)
2. What are Bohr's postulates of hydrogen atom? Derive an expression for the radius of Bohr's orbit. (HSEB 2053, 2059)
3. What are Bohr's postulates? Derive expression for the total energy of electron in n^{th} orbit of hydrogen atom. (HSEB 2057, 2063, 2064, 2065, 2066, 2067)

4. Derive expression for the speed of electron in n^{th} orbit of hydrogen atom. What are the limitations of Bohr's atomic model?
5. State Bohr's postulate of the stationary orbit for the theory of the hydrogen atom. Express this postulate in terms of the de Broglie wavelength of the electron.
6. Explain the formation of various series of spectral lines in the hydrogen spectrum. Draw the energy level diagram. On the diagram show the various transitions leading to all the series.
7. Show that the speed of an electron in the inner most orbit of H-atom is 1/137 times the speed of light in the vacuum.
8. Distinguish between ionization potential and excitation potential. How do you arrive at the conclusion that the electrons in an atom are arranged in various closed orbits?
9. Explain what is meant by wave particle duality. Derive an expression for de Broglie wavelength. An electron is accelerated under a potential difference of V volt. Obtain an expression for the de Broglie wavelength of electron.
10. How did de Broglie arrive at the existence of matter waves? Mention a practical application of de Broglie waves.
11. What is laser? Describe the construction and working principle of He-Ne laser. (HSEB 2067)

Numerical Problems

1. What is the angular momentum of electron in third excited state of hydrogen atom?

   Ans: 4.24×10^{-34} kgm^2/s

2. An electron has a de Broglie wavelength of 2.80×10^{-10} m. Determine (a) the magnitude of its momentum; (b) its kinetic energy (in joules and in electron volts).

   Ans: 2.36×10^{-24} kgms^{-1}, 19.17 eV

3. (a) A non-relativistic free particle with mass m has kinetic energy K. Derive an expression for the de Broglie wavelength of the particle in terms of m and K. (b) what is the de Broglie wavelength of an 800 eV electron? What is the de Broglie wavelength of an 800 eV electrons?

   Ans: 4.335×10^{-11} m

4. Calculate the de Broglie wavelength of a 5 g bullet that is moving at 340 m/s. Will the bullet exhibit wavelike properties?

   Ans: 6.2×10^{-9} m, 2.7×10^{-10} m

5. When electron and photon have the same de-Broglie wavelength. Find the ratio of their kinetic energy?

   Ans: 5.46×10^4

6. Obtain the de-Broglie wavelength of neutron of K.E. 150 eV.(mass of neutron=1.67×10^{-27} kg)

   (Ans: 2.3×10^{-12} m)

7. Calculate the wavelength of the de-Broglie wave associated with thermal neutrons at temperature 27°C.

   Ans: 1.45 A°

8. The de-Broglie wavelength of a particle at 27°C is 1.5Å. What is the value when the temperature falls to 17°C?

   Ans: 1.53 A°

9. In the Bohr model of the hydrogen atom, what is the de-Broglie wavelength of electron when it is in n= 4 level?

   Ans: 1.33×10^{-9} m

10. The de-Broglie wavelength of a particle at 300 K is 4×10^{-7}m. What is the de-Broglie wavelength when temperature rises to 400 K?

    Ans: 3.4×10^{-7} m

11. The electron in the hydrogen atom jumps from the fourth orbit to second. Find the frequency of the spectral line emitted. Given that the Rydberg constant is 1.097×10^7 m^{-1}.

    Ans: 6.17×10^{14} Hz

12. Calculate the de Broglie wavelength of a particle of mass 2 kg moving with a velocity 5 m/s? (h = 6.62×10^{-34} Js)

Ans: 6.62×10^{-35} m

13. Calculate the de Broglie wave length of an electron moving with a velocity of 1/100th of the velocity of light? (m = 9.1×10^{-31} kg)

Ans: 2.42×10^{-10} m

14. Calculate the K.E. of a neutron in electron volt if it has a de Broglie wavelength of 0.04 $\AA$. (m = 9.1×10^{-27} kg).

Ans: 51.25 eV

15. Through what voltage an electron must be accelerated such that it has a de Broglie wavelength of 0.386 $\AA$? (m = 9.1×10^{-31} kg, e = 1.6×10^{-19} C).

Ans: 1010 V

16. An electron and photon have the same de Broglie wavelength of 10^{-10} m. Which of the two has greater kinetic energy?

Ans: Kinetic energy of photon is greater

17. Compare the de Broglie wavelength of an electron of kinetic energy 1 eV and neutron of kinetic energy 1 eV.

Ans: 43 : 1

18. Estimate the wavelength of an electron accelerated through a p.d. of 3600 V(e = 1.6×10^{-19} C, m = 9.1×10^{-31} kg, h = 6.6×10^{-34} Js).

Ans: $2\text{x}10^{-11}$ m

19. The ionization potential of the hydrogen atom is 13.6 V. Calculate:
   (a) The speed of an electron which could just ionize the hydrogen atom.
   (b) The minimum wavelength which the hydrogen atom can emit. (charge on electron = -1.6×10^{-19}C, mass of electron = 9.11×10^{-31} kg, plank constant = 6.63×10^{-34} Js, speed of light = 3.00×10^{8} ms^{-1})

Ans: (a) 2.19 × 106 ms^{-1}, (b) 0.91×10^{-7} m

20. The wavelength of yellow light in air is 6.0×10^{-7}m. Calculate its wavelength in water of refractive index $\frac{4}{3}$.

Ans: 4.5×10^{-7} m

Multiple Choice Questions

1. The spectral region for the Lyman series of hydrogen spectrum is in the:
   a. Ultra-violet region   b. Infra-red region
   c. Visible region   d. Far Infra-red region
2. In an electronic transition, atom cannot emit:
   a. UV radiation   b. IR radiation
   c. Visible light   d. Gamma rays
3. Ratio of wavelength of first line of Lyman series to first line of Balmer
   a. 1 : 4   b. 4 : 1
   c. 2   d. 5 : 27
4. The series of visible spectral lines of hydrogen atom is known as:
   a. Balmer series   b. Lyman series
   c. P fund series   d. Paschen series
5. Line spectrum is given by:
   a. Electron   b. Atom
   c. Molecule   d. All

6. Radiations coming from Lyman series fall in the range:
   a. Visible
   b. Infrared
   c. Ultraviolet
   d. Far infrared
7. Radio of wavelength of 1st member of Balmer series to 1st member of Lyman series is:
   a. 32 : 27
   b. 27 : 5
   c. 32 : 5
   d, 32 : 7
8. The full form of LASER is:
   a. Light amplified by strong emission of radiation
   b. Light amplification by stimulated emission of radiation
   c. Light amplified by stimulated emission of radiation
   d. Light amplification by string emission of radiation
9. The Bohr model of atoms
   a. assumes that the angular momentum of electrons is quantised.
   b. uses Einstein's photoelectric equation
   c. predicts continuous emission spectra for atoms
   d. predicts the same emission spectra for all types of atoms.
10. Which state of triply ionised Beryllium (Be^{+++}) has the same orbital radius as that of the ground state of hydrogen?
    a. n = 3
    b. n = 4
    c. n = 1
    d. n = 2
11. If the energy of a hydrogen atom in nth orbit is E_n, then energy in the nth orbit of a singly ionised helium atom will be
    a. $4E_n$
    b. $E_n/4$
    c. $2E_n$
    d. $E_n/2$
12. Which of the following transitions in hydrogen atoms emit photons of highest frequency?
    a. n = 1 to n = 2
    b. n = 6 to n = 2
    c. n = 2 to n = 6
    d. n = 2 to n = 1
13. The ratio of minimum to maximum wavelength in Balmer series is
    a. 5 : 9
    b. 5 : 36
    c. 1 : 4
    d. 3 : 4
14. The ionisation energy of Li^{2+} is equal to
    a. 9 hcR
    b. 6 hcR
    c. 2 hcR
    d. hcR
15. The transition in He^+ ion that will give rise to a spectral line having the same wavelength as that of some spectral line in hydrogen atom is
    a. n = 3 to n = 1
    b. n = 3 to n = 2
    c. n = 4 to n = 2
    d. n = 4 to n = 3
16. If λ_1 and λ_2 are the wavelengths of the first members of the Lyman series and Paschen series respectively, then $\lambda_1 : \lambda_2$ is
    a. 1 : 3
    b. 1 : 30
    c. 7 : 50
    d. 7 : 108
17. The ground state energy of hydrogen atom is –13.6 eV. When its electron is in the first excited state, its excitation energy is:
    a. 0
    b. 3.4 eV
    c. 6.8 eV
    d. 10.2 eV
18. If an electron jumps from 1st orbit to 3rd orbit, then it will
    a. absorb energy
    b. release energy
    c. no sign of energy
    d. none of these

19. Hydrogen atom does not emit X-rays because
    a. It contains only a single electron
    b. Energy levels in it are far apart
    c. Its size is very small
    d. Energy levels in it are very close to each other.
20. The ratio of minimum wavelengths of Lyman series and Balmer series will be
    a. 1.25 b. 0.25
    c. 5 d. 10
21. The de Broglie wavelength of the electron in the ground state of the hydrogen atom is ....
    (radius of the first orbit of hydrogen atom = 0.53 Å).
    a. 1.67 Å b. 3.33 Å
    c. 1.06 Å d. 0.53 Å
22. Hydrogen atom is excited from ground state to another state with principal quantum number equal to 4. Then the number of spectral lines in the emission spectra will be:
    a. 3 b. 5
    c. 6 d. 2
23. The transition from the state n = 3 to n = 1 in a hydrogen like atom results in ultraviolet radiation. Infrared radiation will be obtained in the transition from:
    a. $2 \rightarrow 1$ b. $3 \rightarrow 2$
    c. $4 \rightarrow 2$ d. $4 \rightarrow 3$
24. Calculate the highest frequency of the emitted photon in the Paschen series of spectral lines of the Hydrogen atom.
    a. 3.7×10^{14} Hz b. 9.1×10^{15} Hz
    c. 10.23×10^{14} Hz d. 29.7×10^{15}
25. The energy state of doubly ionised lithium having the same energy as that of the first excited state of hydrogen is
    a. 4 b. 6
    c. 3 d. 2

Answers

1. (a) 2. (d) 3. (d) 4. (a) 5. (b) 6. (c) 7. (b) 8. (b) 9. (a) 10. (d) 11. (a) 12. (d) 13. (a) 14. (a) 15. (c) 16. (d) 17. (d) 18. (a) 19. (d) 20. (b) 21. (b) 22. (c) 23. (d) 24. (a) 25. (b)

Hints to Challenging Problems

HINT: 1

Given,

$E = -3.4$ eV, angular momentum (L) = ?

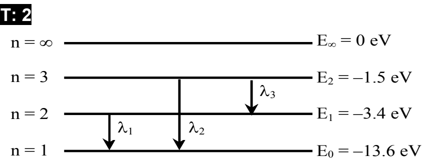
Firstly, find n from,

$$E = \frac{-13.6}{n^2} \text{ eV}$$

Then apply n in,

$$L = \frac{nh}{2\pi}$$

HINT: 2



Given,

$E_1 = -13.6$ eV

$E_2 = -3.4$ eV

$E_3 = -1.5$ eV

The ionisation potential,

$eV = E_\infty - E_1$

$= 0 - (-13.6) = 13.6$ eV

$\therefore$ V = 13.6 V.

i. For transition of election from E_2 to E_1,

$$E_2 - E_1 = h f_1 = \frac{hc}{\lambda_1}$$

or, $$\lambda_1 = \frac{hc}{E_2 - E_1}$$

ii. For transition of electron from E_3 to E_1, we can write

$$\lambda_2 = \frac{hc}{E_3 - E_1}$$

iii. Similarly transition of electron from E_3 to E_2, we can write,

$$\lambda_3 = \frac{hc}{E_3 - E_2}$$

HINT: 3

Here

$\lambda = 1.216 \times 10^{-7}$ m

Initial energy of electron (E) = 20 eV

$= 20 \times 1.6 \times 10^{-19}$ J $= 32 \times 10^{-19}$ J

Let v be the velocity of scattered electron.

Here, initial energy of electron = Energy of photon + energy of scattered electron

or, $E = hf + \frac{1}{2} m_e v^2$

or, $\frac{1}{2} m_e v^2 = E - \frac{hc}{\lambda}$

HINT: 4

Given,

Energy in ground state,

$E_1 = -13.6$ eV

$= -13.6 \times 1.6 \times 10^{-19}$ J $= -21.76 \times 10^{-19}$ J

For n = 4, $E_4 = \frac{-13.6}{n^2}$ eV

i. Frequency (f) $= \frac{E_4 - E_1}{h}$

ii. Wavelength $(\lambda) = \frac{c}{f}$

HINT: 5

Given,

E = –1.51 eV

To find angular momentum of an electron, firstly find n, from

$$E = \frac{-13.6}{n^2} \text{ eV}$$

Then, $L = \frac{nh}{2\pi}$

HINT: 6

Given,

Transition of a atom is from n = 5 to n = 2 state.

a. We have,

$$E_n = \frac{-13.6}{n^2} \text{ eV}$$

$\therefore$ $E_2 = \frac{-13.6}{(2)^2} = -34$ eV

$E_5 = \frac{-13.6}{(5)^2} = -0.544$ eV

From Bohr's postulate,

$E = E_5 - E_2$

Now,

$E = hf = \frac{hc}{\lambda}$

or, $\lambda = \frac{hc}{E}$

b. According to the law of conservation of angular momentum,

Decrease in angular momentum of electron = angular momentum of photon

or, $\left(5\frac{h}{2\pi} - \frac{2h}{2\pi}\right) = L \quad (\because L = \frac{nh}{2\pi})$

HINT: 7

(a) Given,

Speed of electron in n = 1 is v_1 and v_1 = ?

We know that,

$$v_n = \frac{e^2}{2 \in_o nh}$$

For n = 1, $v_1 = \frac{e^2}{2\in_0 n \times 1}$

For n = 2, we have,

$$v_2 = \frac{e^2}{2 \in_0 h \times 2}$$

Similarly, for n = 3,

$$v_3 = \frac{e^2}{2 \in_0 h \times 3}$$

(b) Orbital period in each level = ?

For n = 1, we have,

$$v_1 = \frac{2\pi r_1}{T_1}$$

or, $T_1 = \frac{2\pi r_1}{v_1}$

For n = 2,

$$T_2 = \frac{2\pi r_2}{v_2}$$

Similarly, for n = 3

$$T_3 = \frac{2\pi r_3}{v_3}$$

(c) Here, $v_2 = 1.09 \times 10^6$ m/s

$T_2 = 1.22 \times 10^{-15}$ s

$\because$ $v_2 = \frac{\text{Circumference of second orbit } (C_2)}{T_2}$

Now,

$\because$ In 1.22×10^{-15} sec, 1.329×10^{-9} m distance covered by electron

$\therefore$ In 10^{-8} sec, $\dfrac{1.329 \times 10^{-9}}{1.22 \times 10^{-15}} \times 10^{-8}$ m

$= 1.089 \times 10^{-2}$ m

So, Number of orbits completed

$$= \frac{\text{Total distance covered}}{\text{Circumference of second orbit}}$$

HINT: 8

Given,

Wavelength of first line, $\lambda_1 = ?$

Wavelength of second line, $\lambda_2 = 4.86 \times 10^{-7}$ m

i. For first line of Balmer series,

$n_1 = 2$ and $n_2 = 3$

$$\frac{1}{\lambda_1} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\therefore \quad \lambda_1 = \frac{36}{5R} \qquad \text{. . . (i)}$$

ii. For the second line of Balmer series,

$n_1 = 2$ and $n_2 = 4$.

$$\therefore \quad \frac{1}{\lambda_2} = R\left(\frac{1}{2^2} - \frac{1}{4^2}\right)$$

$$\therefore \quad \lambda_2 = \frac{16}{3R} \qquad \text{. . . (ii)}$$

Dividing (ii) by (i), we get,

$$\text{or,} \quad \lambda_1 = \frac{20}{27}\lambda_2$$

HINT: 9

Given,

$\lambda = ?$

$f = ?$

$E = ?$

For H_r line in Balmer series,

$n_1 = 2$ and $n_2 = 5$

To find λ_1

$$\frac{1}{\lambda_1} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

To find f_1,

$$\therefore \quad f_1 = \frac{c}{\lambda_1}$$

Again, Energy of photon emitted,

$E_1 = hf_1$

HINT: 10

Given,

$E = 0.4$ keV $= 0.64 \times 10^{-19}$ J

$\lambda = ?$

From de- Braglie wave equation, we have,

$$\lambda = \frac{h}{\sqrt{2mE}}$$

❖ ❖ ❖

X-RAYS

23

CHAPTER

23.1 Introduction

All of us are familiar with the X-ray ward in hospital and you might also have been referred to such ward by your doctor to take the X-ray image of different parts of your body. What actually is X-ray and how does it produce image? We will try to answer these questions and other related topics in this chapter. In X-ray ward, energetic rays are produced which can penetrate human flesh but not the bones. This property of the X-rays is used to take the image of internal body parts so that we can detect the fracture of bones and diagnose the diseases. These highly penetrating rays are called x-rays. Actually, X-rays are electromagnetic radiations that can travel in vacuum as well as in medium.

Fig. 23.1: x-ray machine

X-rays were discovered by **Wilhelm Conrad Roentgen** in 1895 while he was working to study the cathode rays from discharge tube and was awarded Nobel Prize for this accidental discovery (the first Nobel Prize in its history). Nowadays, X-rays are widely used in hospitals, research centres, engineering fields and detective departments.

The wavelength of X-rays is very short (ranging from 10^{-9} m to 10^{-12} m). Since the wavelength is very short, X-ray photons are highly energetic $\left(\text{i.e. } E = \dfrac{hc}{\lambda}\right)$. They can easily penetrate through low atomic number materials such as carbon, hydrogen, oxygen, which are the constituents of human flesh.

23.2 Production of X-Rays

The modern method for the production of X-rays consists of a discharge tube which was designed by W.D. Coolidge in 1916 A.D. and is commonly known as modern Coolidge tube. It consists of highly evacuated glass tube as shown in Fig. 23.2, the pressure inside which is maintained in the order of 10^{-5} mm of Hg. The tube is provided with a filament cathode and an anode between which a very high potential difference of the order of kV is applied. The thermo-electrons are produced by heating the filament cathode (F) by passing small d.c. current through it from low tension (L.T.)

supply. The cathode filament is kept inside a metal cup (M) in order to focus the electrons in the target. The current passing through the filament can be controlled with the help of rheostat (Rh) as shown in Fig 23.2. The anode (A) is usually made of a heavy (high atomic number) element like molybdenum or tungsten which have very high melting point and is cut at 45^{o} angle with the direction of incident electrons.

When the electrons emitted from cathode are accelerated through high potential difference applied through high tension (H.T.) supply between the electrodes they acquire very high kinetic energy. When these energetic electrons suddenly encounter the hard target, they are heavily retarded and these retarded electrons emit energy in the form of radiation which is called X–radiation (X-rays). The efficiency of X-ray tube is very low. It converts only about 1% of energy of incident electrons into X-radiation, remaining part will be convert into heat energy.

During this process, high heat is generated in the target material due to continuous hitting by the electrons. Due to this the target may melt. In order to prevent it from melting it is kept inside a jacket made of copper filled with cold water or oil. Copper is good conductor of heat, so heat easily transfers to the water through it.

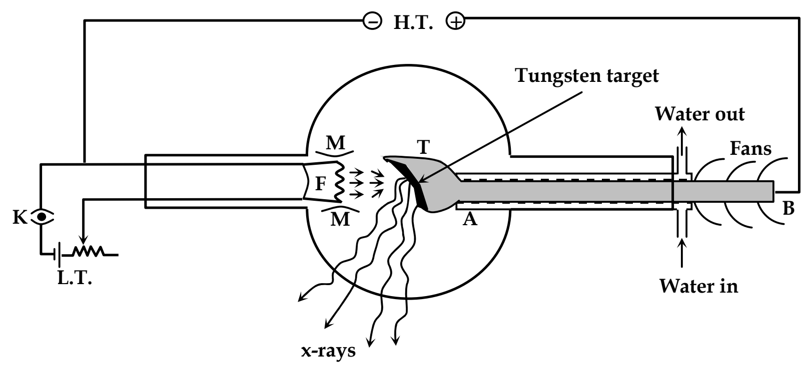


Fig. 23.2: Production of x-rays in Coolidge tube

This method of production of X-rays is called the modern method because the quality and intensity of the X-rays thus produced can be controlled.

Intensity control: The intensity of X-rays can be controlled by controlling the number of incident electrons striking the target which in turn can be controlled by controlling the current flowing through the filament. Filament current regulation controls the no. of thermal electrons emitted from the cathode. This current can be easily changed with the help of rheostat (Rh).

Quality control: The quality refers to the energy of the X-rays. The energy of the X-rays depends upon the kinetic energy of the electrons which in turn depend upon the potential difference across the electrodes used to accelerate the electrons. So, quality of X-rays can be controlled by controlling the p.d. across the electrodes.

X-rays of higher frequencies are highly penetrating and are called hard X-rays. Those with low frequency are less penetrating and are known as soft x-rays.

If all of the kinetic energy of the electrons is converted in the form of x-radiation, then

$$\text{Energy of radiation (hf)} = \frac{1}{2} mv^2_{max}$$

$$\text{Also, } \frac{1}{2} mv^2_{max} = eV$$

$$\text{Therefore, } hf_{max} = eV$$

or, $f_{max} = \dfrac{eV}{h}$

or, $\dfrac{c}{\lambda_{min}} = \dfrac{eV}{h}$

$$\therefore \quad \boldsymbol{\lambda_{min} = \frac{hc}{eV}} \qquad \textbf{... (23.1)}$$

Thus, we see that frequency of X –radiation is proportional to the potential difference across the electrodes.

Properties of X-Rays

1. They are electrically neutral rays. They contain photons which travel with speed of light in vacuum but their wavelength is very short (0.1 Å to 100 Å or 0.01 nm to 10 nm).
2. They are not deflected by electric and magnetic fields.
3. They ionize the gases through which they pass and make them more conducting.
4. They affect photographic plate, similar to light and so this fact is exploited in X-ray photography.
5. The penetrating power of X-rays is very high so they can pass through many substances such as paper, flesh, cardboard, wood, thin, concrete walls etc.
6. They exhibit phenomenon of reflection, refraction, interference, diffraction and polarization similar to that of light, but not as easily as in case of ordinary light wave.
7. They can cause the photoelectric effect on any metal.
8. They are scattered, and after scattering their wavelength may remain constant or increase but do not decrease.
9. X-rays cannot pass through lead, iron, bones etc. and this property is used in radiography. X-rays are strongly absorbed by lead, bones etc. Absorption of X-rays increases with increase of thickness and increase in atomic number of atoms in the medium.
10. They produce fluorescence and phosphorescence effect in some substances, such as zinc sulphide, barium platinocyanide, etc.
11. They have property of destroying some living cells. So, exposure to these rays must be avoided, but this character is benefited in radiotherapy.

Uses of X-Rays

X-rays are used in many fields. Some important applications of X-rays are explained below:

1. Medical uses: X-rays are used in diagnosis of fractures of bones and some diseases, and also in the therapy.

a. Diagnosis: X-rays are highly penetrating radiations. They can penetrate through human flesh easily but not through the bones. So, the X-rays exposed to a part of our body provide the contrast between the bones, muscles and fractured part. Also, the image produced from X-ray radiograph is also used to diagnose the tuberculosis in lungs unusual growth and presence

Fig. 23.3: X-ray image of fractured bone

of bullet into the body. In CT scan, a three-dimensional image is obtained using X-rays from different angles.

b. **Therapy:** Highly energetic X-rays (hard x-rays) are allowed to fall on the cancerous tissue so that these tissues are destroyed. When proper amount of radiation dose is exposed to infected tissue, they die and infection can be cured. But, it has very high side effect, since it damages the nearby fresh tissue.

2. **Engineering uses:** X-rays are used to check the quality of goods and machines.
3. **Industry:** X-rays are used to detect the cracks in casting and welding in bridges, metal pipes and other metallic devices.
4. **Detective departments:** X-rays are used to detect the contraband goods like gold, sealed parcels, and illegal drugs.
5. **Scientific research:** X-rays are used to study the crystal structure of solid state materials, atomic structures of proteins and nucleic acids, a technique called X-ray diffraction (XRD). X-rays can be used in chemical analysis using Moseley's law.

23.3 X-ray Spectra

X-ray photon produced even from source does not contain equal energy. So, they do not have the same wavelength, varying between a minimum and maximum values. The intensity of X-rays at different wavelengths is also obtained different. Thus, the intensity distribution of x-radiations among the wavelengths varies. This variation in intensity produces a band of X-rays which is called X-ray spectrum. In accordance with its origin, X-ray spectra are categorized into two: continuous spectra and characteristics spectra. The graph between X-ray intensity and wavelength at different applied voltage are shown in Fig 23.4.

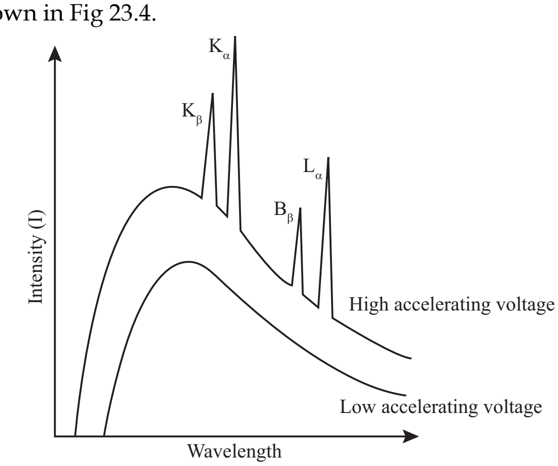


Fig. 23.4: Continuous X-ray spectrum

Continuous Spectra

The nucleus of an atom contains protons and neutrons. Protons are positive charge particles and neutrons are chargeless. So, the net charge of nucleus is positive. When an electron (negative charge particle) travels close to the nucleus, it slows down due to the attraction of nucleus. As a result, the electron deviates from its original path and then losses kinetic energy. Thus, the electromagnetic radiation is emitted in the expense of loss of kinetic energy of the decelerating electron. This process of loss of energy by a charge particle is called Bremsstrahlung. It has a maximum frequency (or minimum wavelength λ_{min}) beyond which the intensity is zero. In high atomic number metals, the wavelength of emitted radiation lies within the wavelength range (10^{-9} m to 10^{-12} m) of x-radiation. Such type of x-radiation is called continuous X-rays and the corresponding X-ray spectrum is called continuous spectrum. Continuous spectrum consists of all possible wavelength within a range upto a minimum wavelength limit.

If total kinetic energy of moving electron is converted into x-radiation, the wavelength is shortest, otherwise contains longer wavelength that of the minimum limit.

The X-rays of shortest wavelength are possible when total kinetic energy of an electron is converted into x-rays. If V is the applied p.d. between the filament and target and λ_{min} is the shortest wavelength, then,

Maximum K.E. of incident electrons

= Maximum energy of X-rays [X-rays of shortest wavelength (λ_{min})]

$$\therefore \quad eV = hf_{max} = \frac{hc}{\lambda_{min}}$$

$$\therefore \quad \lambda_{min} = \frac{hc}{eV}$$

The values of h, c and e are constant, the minimum value of wavelength (λ_{min}) of X-ray is,

$$\therefore \quad \lambda_{min} \propto \frac{1}{V}$$

The energy of photon increases on decreasing the wavelength and the wavelength of X-ray decreases on increasing supplied voltage. Energy (penetrating power or quality) can be increased by increasing the p.d.

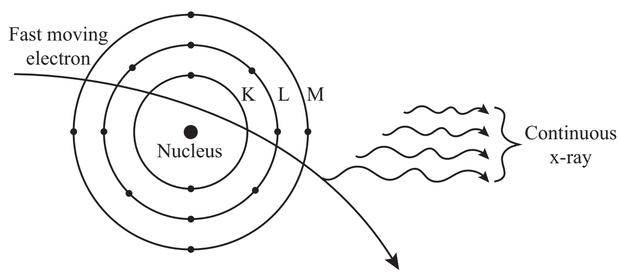


Fig. 23.5: X-rays produced due to losing of energy of electron

Energy of continuous X-ray depends on the location of electron to nucleus and degree of its deceleration. The closer the electron gets to the nucleus, the more it slows down, changes direction and the greater the energy of resultant x-ray. Experimentally, continuous spectra are obtained in various values of potential differences provided by the high tension battery. In tungsten target, continuous spectra are obtained in various values of applied voltages: 40 kV, 50 kV, 60 kV, 70 kV, etc, as shown in Fig. 23.5

Characteristics Spectra

When an energetic electron strikes a bound electron in an atom, (the electron in target atom) the bound electron is ejected from the inner orbit, it may be kicked out or excited to upper orbit. After the ejection of electron, the atom is left with a vacant energy level. Then, the electron in the upper orbit falls into the lower level, emitting quantized photons with an energy level (equivalent to the energy difference between the higher and lower states). The x-ray, thus produced is called characteristics x-ray. Each element has a unique set of energy levels and thus the transition from higher to lower energy levels produces X-rays with different X-rays spectra produced due to the electron transition in target atoms are known as characteristics spectra.

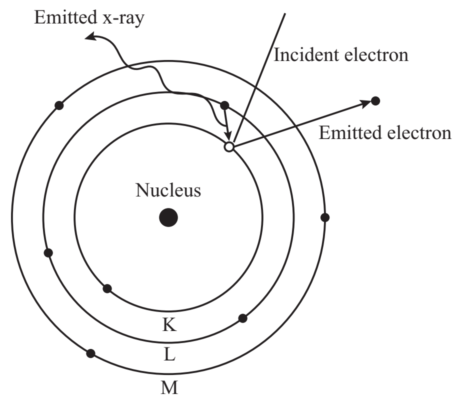


Fig. 23.6: Characteristics X-rays

Characteristics spectrum consists of discrete spectral lines in the form of small groups superimposed on the continuous spectrum. When an electron falls from L-orbit to K-orbit, the X-ray spectrum is called K_α spectrum. Similarly, when an electron falls from the M orbit to K orbit, the X-ray emitted is called a K_β x-ray. Many other characteristics X-rays are produced like as in Fig. 23.6.

Differences between continuous spectra and characteristics spectra

Continuous spectra	Characteristics spectra
1. Continuous spectra consist of radiations of all possible wavelengths within a range upto with a minimum wavelength limit.	1. The characteristics spectra have a few sharp peaks at certain wavelengths. These peaks appearing in continuous spectrum form line spectrum.
2. The fast moving electrons during decelleration radiates energy in the form of continuous x-rays.	2. The fast moving electrons excite the inner electrons to the higher energy state and make transition to the lower energy state. The x-rays, thus produced is called characteristics x-rays.
3. This type of spectrum consists of radiation with λ_{min} depending upon the potential applied.	3. The characteristics spectrum depends on the atom through which X-rays are emitted.
4. In tungsten target, continuous X-rays are obtained in many values of applied voltage, for examples 40 kV, 50 kV, 60 kV, 70 kV.	4. In tungsten target, characteristics X-rays are obtained in specific value of applied voltage, for example 70 kV.

23.4 X-rays Diffraction

Diffraction is the phenomenon of spreading of wave through a small aperture or from an obstacle. All types of wave exhibit diffraction property. X-ray is an electromagnetic wave, so it shows the diffraction phenomenon when passes through tiny aperture or obstacle. They can be diffracted through the atomic spacing of crystals and liquids. X-ray diffraction (XRD) is one of the most important tools to analyze all kinds of matters ranging from fluids, to powder and crystals. This technique is widely used to obtain the information about the structure of crystalline materials and to identify the molecular structure of biomolecules like protein, carbohydrates, DNA, RNA, etc.

The diffraction phenomenon of X-ray was firstly experimented by Von Laue and his coworkers in April 1912. They studied about the X-ray diffraction by passing it through ZnS crystal. Their experiment not only discussed the wave nature of X-ray but also the atomic arrangement in the crystals. The results of the experiment confirmed that X-rays were electromagnetic radiation of short wavelength.

Von Laue Experiment

The apparatus arrangement in Von-Laue experiment is shown in Fig. 23.7. The apparatus consists of a X-ray source which emits the x-rays. The emitted X-rays are collimated by two parallel slits S_1 and S_2. Then, the collimated beam is passed through the zinc sulphide (ZnS) crystal. The transmitted beam of X-rays from ZnS are observed by exposing it on the photographic plate. After appropriate exposure on the plate, it is developed. Upon close observation in the developed film, a definite pattern of sports is seen. There is a central spot where X-ray beam falls directly. Moreover, there are many small spots around it in a regular pattern corresponding to the crystal structures as shown in Fig.23.7. These regular patterns on the film are called Laue's pattern. The diffraction spots that surrounded the central spot of the primary beam could be explained by Laue as interference pattern due to the diffraction through crystal's space lattice. The diffraction pattern depends upon the lattice constant and the wavelength of x-rays.

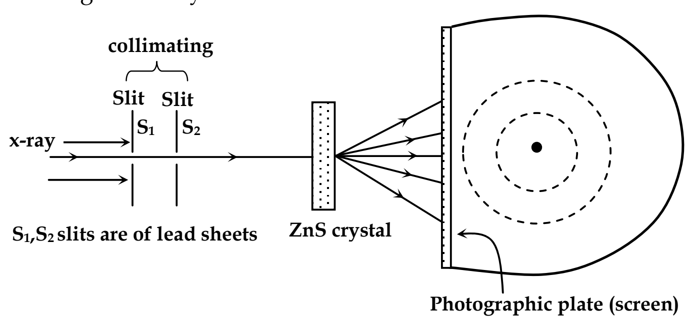


Fig. 23.7: Diffraction of x-ray by a crystal of ZnS

Major findings of Von Laue XRD experiment

i. The atoms (ions) of a crystal are arranged in a regular three dimensional lattice.

ii. X-rays are electromagnetic waves.

23.5 Bragg's Law

When X-ray wave travels through a very small aperture, it bends from its original path, this phenomenon is called diffraction of x-rays. X-rays can be diffracted passing through some crystals like zinc sulphide because its wavelength is comparable to the space between the atoms in crystal (lattice spacing). Bragg studied the diffraction of X-rays through crystals and established a mathematical relation between the lattice spacing and wavelength of x-rays. This mathematical equation is written as,

$$2d \sin \theta = n\lambda$$

and is known as Bragg's law, where d symbolizes the lattice spacing and λ is the wavelength of x-rays.

Proof

Let us consider three crystal planes L_1, L_2, and L_3 of a crystal containing an array of atoms as shown in Fig. 23.8. A beam of X-ray is incident on the first crystal plane L_1. When the X-ray beam is incident on this plane, a part of it is reflected and remaining part diffract through spacing between the atoms. Also, a part of the diffracted beam is reflected from the second plane. Similar phenomenon can be observed in the third plane.

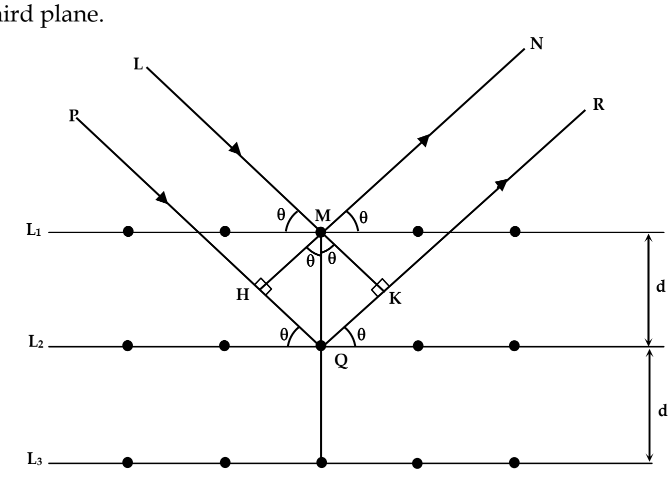


Fig. 23.8: Diffraction of x -ray

Let LM and PQ be two incident rays on first and second crystal planes making glancing angle θ. When the rays fall on atoms, they reflect back through the path MN and QR respectively. Let d be the distance between two successive planes. Two perpendicular line MH and MK are drawn on PQ and QR respectively as shown in Fig. 23.8.

The path difference of these rays is HQ + QK. In this experiment, the interference phenomenon takes place when reflected rays of certain path difference superimpose. For a bright pattern, the path difference of two rays must be integral multiple of wavelength (λ). Therefore,

$$HQ + QK = n\lambda, \quad \text{where n is the order of diffraction, } n = 1, 2, 3, \ldots$$

In ΔMHQ,

$$\sin \theta = \frac{HQ}{MQ} = \frac{HQ}{d}$$

$$\therefore \quad HQ = d \sin \theta$$

Similarly, in ΔMKQ,

$$QK = d \sin \theta$$

So, for constructive interference,

$$d \sin \theta + d \sin \theta = n\lambda$$

$$\mathbf{2d \sin \theta = n\lambda} \qquad \ldots (23.2)$$

This is the mathematical form of Bragg's law. We can calculate the wavelength λ of X-rays when lattice spacing d is known. The value of glancing angle (θ) and order of brightness (n) can be determined from this experiment. If the wavelength of X-rays is known, the lattice spacing d can be calculated.

Tips for MCQs

1. X-rays were discovered by Roentgen. He is the first Nobel Laurent in the history of Nobel Prize.
2. X-rays wavelength range lies between 1 nm to 10^{-3} nm.
3. The minimum value of wavelength of X-rays is, $\lambda = \frac{hc}{eV}$, i.e. $\lambda \propto \frac{1}{V}$
4. When electron are accelerated by potential difference V, the energy acquired by electron is $\frac{1}{2} mv^2_{max}$ = eV.
5. Depending upon the penetrating capacity, X-rays are categorized into two:
a. Soft x-rays, wavelength ranging approximately 10 Å – 100 Å,
b. Hard x-rays, wavelength ranging approximately 0.1 Å– 10 Å.
It is to be noted that, there is no special demarcation value to distinguish between soft X-rays and hard x-rays.
6. a. The intensity X-rays is controlled by current from low tension battery (LTB).
b. The quality of X-rays is controlled by potential difference provided high tension battery (HTB).
7. X-rays spectrum is classified into two categories:
a. Continuous X-ray and b. Characteristics x-rays
8. Bragg's law of X-rays difference is, $2d \sin\theta = n\lambda$, where n = 1, 2, 3, ……..
9. The intensity of X-ray (I) that emerges from a material is, $I = I_0 e^{-\mu x}$
Where, I_o = incident intensity of x-ray
μ = absorption coefficient of material (SI unit is m^{-1})
x = thickness of material
10. The quantity of radiative energy absorbed per unit mass by a substance is termed as dose of radiation, $D = \frac{Q}{m}$. Its unit is Gray (Gy), 1 Gy = 1 Jkg^{-1}.
   1. The energy of X-ray photon, $E = \frac{hc}{\lambda}$.
   2. The minimum wavelength of X-ray is determined from, $\lambda_{min} = \frac{hc}{eV}$.
   3. The kinetic energy of thermo electrons due to the voltage supplied by high tension battery, $\frac{1}{2} mv^2_{max} = eV$.
   4. Bragg's law for diffraction of X-rays is
   $2d \sin\theta = n\lambda$, where, n = 1, 2, 3, …

Worked Out Problems

1. Calculate the energy in electron volt of a quantum of x-radiation of wavelength 0.15 nm. ($h = 6.6 \times 10^{-34}$ Js)

SOLUTION

Given,

Wavelength (λ) = 0.15 nm = 0.15×10^{-9} m
Planck's constant (h) = 6.6×10^{-34} Js
Energy (E) = ?

We have, velocity of light (c) = 3×10^8 ms^{-1}

We know,

$$E = \frac{hc}{\lambda}$$

$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{0.15 \times 10^{-9}}$$

$$= \frac{1.98 \times 10^{-25}}{0.15 \times 10^{-9}} = 1.32 \times 10^{-15} \text{ J}$$

We know,

1 eV = 1.6×10^{-19} J

So, $E = \left(\frac{1.32 \times 10^{-15}}{1.6 \times 10^{-19}}\right) \text{eV} = 8250 \text{ eV}$

2. An X-ray tube operates at 18 kV. Calculate the maximum speed of electron striking the target. ($m_e = 9 \times 10^{-31}$ kg, e = 1.6×10^{-19} C)

SOLUTION

Given,

Operating voltage (V) = 18 kV = 18×10^3 V

Mass of electron (m_e) = 9×10^{-31} kg

Charge of electron (e) = 1.6×10^{-19} C

If the electrons strike the target with maximum speed (v_{max}) then we must have,

$$eV = \frac{1}{2} m_e v^2_{max}$$

$$\text{or, } v_{max} = \sqrt{\frac{2\,eV}{m_e}} = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 18 \times 10^3}{9 \times 10^{-31}}}$$

$\therefore$ $v_{max} = 8 \times 10^7$ m/s

3. Electrons are accelerated form rest through a potential difference of 10000 V in an X-ray tube. Calculate (i) the resultant energy of the electrons in eV; (ii) wave length of the associated electron waves; (iii) the maximum energy and the minimum wavelength of the x-radiation generated, (Charges of electron = 1.6×10^{-19} C, mass of the electron = 9.11×10^{-31} kg. Planck constant = 6.62×10^{-34} Js. Speed of electromagnetic radiation in vacuum = 3.0×10^8 ms^{-1}.)

SOLUTION

Given,

Potential difference (V) = 10000 volt = 10^4 V

(i) Energy of electrons (E) = ?

we know that,

$$eV = \frac{1}{2} mv^2$$

or, eV = E

or, E = $1.6 \times 10^{-19} \times 10^4$ J

$$= \frac{1.6 \times 10^{-19} \times 10^4}{1.6 \times 10^{-19}} \text{ eV} = 10^4 \text{ eV}$$

(ii) Wave length of the associated electron waves (λ) = ?

The wave associated with an electron is the matter wave whose wavelength is given by,

$$\lambda = \frac{h}{m_e v} \text{ (de – Broglie wave equations)}$$

But, $E = \frac{1}{2} m_e v^2$

$$\therefore v = \sqrt{\frac{2E}{m_e}}$$

$$\text{So, } \lambda = \frac{h}{m_e \times \sqrt{\frac{2E}{m_e}}} = \frac{h}{\sqrt{2m_eE}}$$

$$= \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 9.11 \times 10^{-31} \times 1.6 \times 10^{-15}}}$$

$$= \frac{6.62 \times 10^{-34}}{\sqrt{29.152 \times 10^{-46}}}$$

$$= \frac{6.62}{5.4} \times 10^{-11}$$

$$= 1.23 \times 10^{-11} \text{ m}$$

(iii) Maximum energy of X-ray (E_{max}) = ?

Minimum wavelength of X-ray generated (λ_{min}) = ?

For Maximum energy of x-ray, the total energy of an incident electron on the target is absorbed. So, we can write,

$E_{max} = eV = 1.6 \times 10^{-19} \times 10^4 = 1.6 \times 10^{-15}$ J

For, λ_{min}, the total energy must be converted into x-ray, i.e.,

$eV = hf_{max}$ (since each photon of X-ray is hf)

$$\text{or, } eV = \frac{hc}{\lambda_{min}}$$

$$\text{or, } \lambda_{min} = \frac{hc}{eV} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19} \times 10^4}$$

$\therefore$ $\lambda_{min} = 1.24 \times 10^{-10}$ m

4. The spacing of atomic planes in crystal is 1.1×10^{-10} m and when incident on them at glancing angle of 5°, a first order image is produced. Calculate the wavelength. What is the glancing angle for the second order image?

SOLUTION

Given,

Atomic spacing (d) = 1.1×10^{-10} m

Glancing angle (θ) = 5°

order of diffraction (n) = 1
Wavelength (λ) = ?
For the second order (n = 2), θ_2 = ?
From the Bragg's law, we have
$2d \sin\theta_n = n\lambda$
For first order image (n) = 1
or, $2 \times 1.1 \times 10^{-10} \times \sin 5° = 1 \times \lambda$
or, $\lambda = 1.914 \times 10^{-11}$ m
For second order image (n) = 2
$2d \sin \theta_2 = n\lambda$
or, $2 \times 1.1 \times 10^{-10} \times \sin\theta_2 = 2 \times 1.91 \times 10^{-11}$
or, $\text{Sin}\theta_2 = \dfrac{1.91 \times 10^{-11}}{1.1 \times 10^{-10}} = 0.17$
or, $\theta_2 = \sin^{-1}(0.17) = 9.78°$

5. **[HSEB 2072] An X-ray spectrometer has a crystal of rock salt for which atomic spacing is 2.82 Å set at an angle of 14° to the beam coming from a tube operated at a constantly increasing voltage. An intense first line appears when the voltage across the tube is 9045 V. Calculate the value of Planck's constant.**

SOLUTION

Given,
Spacing (d) = 2.82 Å = 2.82×10^{-10} m
Angle (θ) = 14°
Order (n) = 1
Voltage (V) = 9045 V
Planck's constant (h) = ?
We have,
From Bragg's law,
$2d \sin \theta = n\lambda$
$$\lambda = \frac{2d \sin\theta}{n}$$
$$= \frac{2 \times 2.82 \times 10^{-10} \times \sin 14}{1}$$
$= 1.36 \times 10^{-10}$ m
Now, $\lambda = \dfrac{hc}{eV}$
$$\therefore h = \frac{\lambda \times eV}{c}$$
$$= \frac{1.36 \times 10^{-10} \times 1.6 \times 10^{-19} \times 9045}{3 \times 10^8}$$
$= 6.56 \times 10^{-34}$ Js
$\therefore$ The value of Planck's constant is 6.56×10^{-34} Js.

6. **[HSEB 2052] X-ray beam of wavelength 2.9 Å is diffracted from the plane of cubic crystal. The first order diffraction is obtained at angle 35°. Calculate the spacing between the planes.**

SOLUTION

Given
Wavelength (λ) = 2.9 Å = 2.9×10^{-10} m
Order (n) = 1
Angle (θ) = 35°
Spacing (d) = ?
We have,
From Bragg's law
$2d \sin \theta = n\lambda$
$$d = \frac{n\lambda}{2 \sin\theta}$$
$$= \frac{1 \times 2.9 \times 10^{-10}}{2 \times \sin 35°}$$
$= 2.5 \times 10^{-10}$ m

7. **[HSEB 2065] X-rays of wavelength 0.36 Å are diffracted by a Bragg's crystal spectograph at a glancing angle of 4.8°. Find the spacing of the atomic planes in the crystal.**

SOLUTION

Given,
Wavelength of X-ray (λ) = 0.36 Å
= 0.36×10^{-10} m
Glancing angle (θ) = 4.8°
Spacing of atomic planes (d) = ?
Now,
From Bragg's law, we have
$2d\sin \theta = n\lambda \qquad n = 1, 2, 3, \ldots$
for n = 1; we have
$2d\sin\theta = \lambda$
or, $d = \dfrac{\lambda}{2\sin\theta} = \dfrac{0.36 \times 10^{-10}}{2 \times \sin 4.8}$
$\therefore d = 2.15 \times 10^{-10}$ m
Hence, the required spacing is 2.15×10^{-10} m

Challenging Problems

1. An X-ray tube works at a d.c. potential difference of 50 kV. Only 0.4% of the energy of the cathode rays is converted into x-radiation and heat is generated in the target at the rate of 600 W. Estimate (i) the current passed into the tube; (ii) the velocity of the electrons striking the target. (electron mass = 9.00×10^{-31} kg, Electron charge = -1.6×10^{-19} C)

Ans: (i) 0.012 A (ii) 1.3×10^8 m/s

2. An X-ray tube is operated with an anode potential of 10 kV and anode current of 15 A (i) Estimate the number of electrons hitting the anode per second. (ii) Calculate the rate of production of heat at the anode, stating any assumptions made.

Ans: (i) 9.37×10^{16} electron/sec (ii) 150 W

3. Determine the ratio of the energy of a photon of x-radiation of wavelength 0.1 nm to that of a photon of visible radiation of wavelength 500 nm.

Ans: 5000 : 1

4. X-ray beam of wavelength 2.9 Å is diffracted from the plane of cubic crystal. The first order diffraction is obtained at an angle 35°. Calculate the spacing between the planes.

Ans: 2.52×10^{-10} m

5. (a) What is the minimum p.d. between the filament and the target of an X-ray tube if the tube is to produce X-rays with a wavelength of 0.150 nm ? (b) what is the shortest wavelength produced in an X-ray tube operated at 30 kV?

Ans: (a) 8281.3 V (b) 0.414×10^{-9} m

*[**Note:** Hints to challenging problems are given at the end of this chapter.]*

Conceptual Questions with Answers

1. Can X-ray diffraction experiment be performed by an ordinary grating? Why? [HSEB 2073]

↳ No. It is impossible to perform XRD experiment by an ordinary grating. Wave diffraction is possible only when the wavelength of wave and width of aperture are comparable. The width of ordinary grating is about 10^{-6} m, whereas the wavelength of X-rays ranges from 10^{-9} m to 10^{-12} m. So, these values are not comparable to the spacing between ordinary gratings. The spacing of ordinary grating is at least one thousand times larger than the longest wavelength of x-rays.

2. Can X-rays be produced from gases? [HSEB 2070]

↳ X-rays can not be produced from gases. The spectrum produced by gases lies in ultraviolet, visible and infrared range, but not in the X-ray region. X-rays are produced only in the Bremsstralung effect or the electron transition in high atomic number metals like tungsten.

3. Can Bragg's law of X-ray diffraction be verified with yellow light of wavelength 600 nm? Explain. [HSEB 2069]

↳ Bragg's law depends on the X-ray diffraction experiment. For the diffraction of any wave, its wavelength should be comparable to the width of opening. However, the spacing between the atoms in crystal is about 10^{-10} m, which is about 10^4 times narrower than the wavelength of yellow light of wavelength 600 nm (= 6×10^{-6} m). So, Bragg's law can not be verified by using yellow light.

4. What are the differences between X-rays and ordinary ray of light?

↳ Although X-rays and ordinary light are electromagnetic radiations, they have some basic differences:

X-rays	ordinary light
1. X-rays are high energy electromagnetic waves.	1. Ordinary light is medium energy electromagnetic waves

2. They have invisible spectrum.	2. They have visible spectrum.
3. X-rays can penetrate through the soft tissue of human body.	3. They can not penetrate even through these tissues.
4. X-ray spectrum has wide range of spectrum of wavelength range 10^{-9} m to 10^{-12} m.	4. Ordinary light has narrow range of spectrum of wavelength 4×10^{-7} m to 7×10^{-7} m.

5. What are soft X-rays and hard x-rays?

✎ X-rays of low penetrating power are called soft x-rays. They possesses low energy photons. In contrast, hard X-rays are more energetic and have high penetrating power. They possesses high energy photons. The distinction between hard and soft X-rays is not well defined. Hard X-rays are typically those with energies greater than around 10 keV than soft x-rays.

6. Where are X-rays on the electromagnetic spectrum?

✎ X-rays are energetic than the ultraviolet rays and weaker than γ-rays. Energetic in the sense that X-rays photons have wavelength shorter than ultraviolet photons and weaker in the sense that X-rays photons have wavelength longer then γ-rays photons. Therefore, X-ray falls between ultraviolet and γ-ray region in electromagnetic radiation.

7. What are the differences between cathode rays and x-rays?

✎ Some fundamental differences between cathode rays and X-rays are as follows:

Cathode rays	X-rays
1. They are the rays of charged particles i.e. rays of electrons	1. They are the rays of photons. They are electromagnetic radiations.
2. They are deviated by electric and magnetic fields.	2. They are not deviated by electric and magnetic field.
3. The speed of cathode rays is less than speed of visible light.	3. The speed of X-rays is equal to the speed of visible light in vacuum.
4. The particles of cathode rays have non-zero rest mass.	4. The particle of X-rays have zero rest mass.

8. When X-rays are produced, only about 1% of the initial input energy appears as X-ray energy. What has happened to the other 99% of the energy? Explain.

✎ The efficiency of X-ray is very low. About 99% of incident energy is converted into other types of energy and only about 1% of energy is useful in the production of x-rays. This majority part of unused energy in X-ray is converted into the heat energy which heats the target material. Only a small fraction of that energy is converted into sound energy.

9. How do X-rays image the internal parts of our body?

✎ X-rays are highly penetrating rays. They penetrate through the soft tissue, but not through the bones. The contrast between bones and muscles is obtained on the photographic film in accordance with the intensity of penetrating x-radiations. Not only bone and muscle, different tissues have different absorption capacity, which makes the contrasts in the photographic film of these tissues. Thus, the X-rays takes image of the internal body parts.

10. Why X-rays are called harmful radiation?

✎ X-rays are electromagnetic radiation. X-ray photon carries energy. The photons transfer their energy to the material so that the molecules ionize in that medium. Ionization causes the defects seriously in biomolecules like protein and nucleic acids. This eventually may cause cancer and other chronic diseases. Since, they harm the genetic and sensory molecules, they are called harmful radiations.

11. Why characteristic X-rays are named so?

✎ Characteristics X-rays are produced due to the excitation or ionization of atoms in the target material. The excitation and ionization energy of different materials are different. Therefore, the X-rays of different spectrum can be produced by using different materials. It means, characteristics X-rays are the properties of materials. So, they are named so.

12. Write the important properties of x-rays?

a. They are electrically neutral rays.
b. They are not deflected by electric and magnetic fields.
c. They ionize the gases through which they pass and make them more conducting.
d. They affect photographic plate, similar to light and so this fact is exploited in X-ray photography.
e. The penetrating power of X-rays is very high. So, they can pass through many substances such as paper, flesh, cardboard, wood, thin concrete walls etc.
f. They can cause the photoelectric effect on any metal.
g. They are scattered, and after scattering their wavelength may remain constant or increase but not decrease.

13. Discuss the use of X-rays in medicine.

➪ **Medical uses:** X-rays are used in diagnosis of fractures in bones and some diseases, and also in the therapy.

a. Diagnosis: X-rays are highly penetrating radiations. They can penetrate through human flesh but stop passing through the bones. So, the X-rays exposed to a part of our body provides the contrast between the bones, muscles and fractured part. Also, the image produced from X-ray is also used to diagnose the tuberculosis in lungs and presence of bullet into the body.

b. Therapy: Highly energetic X-rays (hard x-rays) are allowed to fall on the cancerous tissues so that these tissues are destroyed. When proper amount of radiation dose is exposed to infected tissue, they die and infection can be cured. But, it has very high side effect, since it damages the near by fresh tissue.

14. A patient is suggested to put off the gold ornaments before entering into X-ray room. Explain why.

➪ X-rays are highly penetrating rays. They can easily penetrate from the muscles and fracture part of our body, but can not penetrate through the bones and high atomic number metals. If X-ray imaging of a person is taken with gold ornaments, bone shaped contrast image is produced in the photographic plates due to the interaction of X-rays with gold. This makes the difficulty in identifying the abnormality of the body.

Also, the gold is high atomic number metal, so it may produce the penetrating radiations when X-ray falls upon it.

15. Photoelectric effect and X-ray production are the reverse effect. Justify?

➪ Photoelectric effect and X-ray production are the reverse effects. In photoelectric effect, radiation is incident on a metal surface to eject out the electrons, whereas in X-ray production, electrons are impinged on the metal surface to eject out the radiation.

16. How can you control the intensity of incident x-ray?

➪ The width (intensity) of X-ray beam in Coolidge tube, depends on the number of energetic electrons that strikes on the target materials per second. Larger the number of electron striking the target material, wider the X-ray beam. The production of electrons depends on the temperature of filament. This temperature can be varied by the current passing through the low tension battery. Therefore, current produced by low tension battery controls the intensity of x-rays.

17. How can you control the quality of x-ray?

➪ Quality of X-ray refers to the energy of X-ray beam. The beam of higher quality can have the greater penetrating power (greater energy) and vice-versa. The energy of X-ray photon is equal to the kinetic energy of most energetic electron, i.e. (If there is no loss of energy)

$$\frac{1}{2} mv^2_{max} = h\nu_{max} = \text{Energy of X-ray photon}$$

And the potential difference provided by the high tension battery is responsible to provide the kinetic energy of electrons that strike on the target materials,

i.e. $\frac{1}{2}mv^2_{max} = eV$

$\therefore \quad hv_{max} = eV$

It means, the energy of X-ray photon increases when potential difference provided by high tension battery across the filament and target material increases. Therefore by controlling the p.d. across the electrode the quality of X-ray can be controlled.

---

18. What are the advantages of Bragg's law?

↳ Bragg's law have two main advantages:

i. Wavelength of X-ray can be determined, if lattice spacing of crystal used is known.

ii. Lattice spacing of a crystal can be determined if wavelength of X-rays is known.

---

19. Why is the target of a Coolidge tube made of tungsten, why not of aluminium or steel?

↳ The material of high melting point and high atomic weight should be used as the target material to produce the x-rays. The target should be of high melting point so that it does not melt by heat developed due to collision of electrons. Also, it should be of high atomic weight because only then X-rays of high energy will be emitted. However, aluminium and steel do not fulfil these requirements to be the target, but the tungsten does.

Exercises

Short-Answer Type Questions

1. Production of X-rays is the reverse of photoelectric effect. Explain.
2. How do X-rays produce?
3. X-rays are harmful for our body. Explain
4. Which can penetrated more, X-ray or γ-ray?
5. Why visible light cannot take the image of internal body parts?
6. How is the intensity of X-rays controlled?
7. What is the difference between hard X-rays and soft x-rays?
8. Write down the properties of x-rays.
9. What are the important uses of x-rays?
10. A patient is suggested to drink barium sulphate solution before taking the X-ray photograph of internal organs, why?
11. What is the difference between X-rays and cathode rays?
12. How are electrons produced in X-ray tube?

Long-Answer Type Questions

1. What are x-rays? Give some important properties and applications.
2. What are x-rays? How are they produced by Coolidge tube method? Write down their some properties.
3. Derive Bragg's law and explain how is this law used to determine the crystal plane spacing.
4. Describe Coolidge tube method for the production of x-rays. How would you control (i) the intensity and (ii) the penetrating power of the emitted x-rays.

Numerical Problems

1. An X-rays tube operated at 30 kV emits X-rays with a short wavelength limit of 0.4 Å. Calculate Planck's constant.

Ans: 6.4×10^{-34} Js

2. An X-ray tube operates at 20 kV. A particular electron loses 5% of its kinetic energy to emit an X-ray photon at the first collision. Find the wavelength corresponding to this photon.

Ans: 1.24×10^{-9} m

3. A monochromatic beam of ray incident on a crystal at a glancing angle of 5° produces a first order image. What is the glancing angle for a second order image?

Ans: 10^{o}

4. (a) What is the minimum potential difference between the filament and the target of an x ray tube is to produce x rays with a wavelength of 0.150 nm? (b) What is the shortest wavelength produced by an x ray tube operated at 30.0 kV?

Ans: 8281.3 V, 0.414 nm

5. X-rays from a tube undergo first order reflection at a glancing angle of 12^{o} from the face of a calcite crystal is 3.04×10^{-8} cm. Calculate the wavelength of x rays. At what angle will the third order reflection take place from the crystal?

(Ans: 1.3×10^{-10} m, 39.3^{o})

6. An X-ray tube operated at 30 kV emits x rays with a short wavelength limit of 0.4 A^{o}. Calculate the Planck's constant.

Ans: 6.4×10^{-34} JS

7. Calculate the minimum applied potential required to produce X-rays of 1 Å wavelength.

Ans: 12,400 V

8. The potential difference across an X-ray tube is 10^{5} V and a current 5 mA flows through it. Find the maximum speed of the electrons striking the target. If only 0.15 percent of the incident energy is converted into X-radiations, find the rate of production of heat.

Ans: 1.88×10^{8} m/s, 499.25 J/sec.

9. If the potential difference across the tube is 1.5×10^{3} V, and the current 1.0×10^{-3} A, find (a) the number of electrons crossing the tube per second and (b) the kinetic energy gained by an electron traversing the tube without collision.

Ans: (a) 6.3×10^{15} electrons per sec. (b) 2.4×10^{-16}J

10. Calculate the energy in electron volt and velocity of electron beam giving rise to X-ray of wavelength 1 Å?

Ans: 12.4×10^{3} eV, v = 6.6×10^{7} m/s

11. X-rays of wavelength 0.0850 nm are scattered from the atoms of crystal. The second order maximum in the Bragg reflection occurs when the angle θ is 21. 50, what is the spacing between the adjacent atomic planes in the crystal?

Ans: 0.232 nm

12. An X-ray tube operated at a d.c. p.d. of 40 kV produces heat at the target at the rate of 720 W. Assuming 0.5 % of the energy of the incident electrons is converted into x-radiation. Calculate (i) the number of electrons per second striking the target (ii) velocity of incident electrons. $\left(\text{Given } \frac{e}{m} = 1.8 \times 10^{11}\ \text{Ckg}^{-1}\right)$

Ans. 1.1×10^{17} electrons /s, 1.2×10^{8} m/s

Hints to Challenging Problems

HINT: 1

Given,

V = 50 kV = 50×10^{3} V, P = 600 W

(i) Current passed into tube, I = ?

By question,

0.4 % energy of cathode rays (qV) is converted into x-ray and remaining (100 – .04)% = 99.6% into heat so,

Heat energy generated = 99.6 % of qV

or Rate of heat energy generated = $\frac{99.6}{100} \times \frac{qV}{t}$

or $600 = 99.6 \times 10^{-2} \times I \times V$ $\left(\because I = \frac{q}{t}\right)$

(ii) Velocity of electron striking the target, v = ?

We can write

$$\frac{1}{2} m_e v^2 = eV$$

or $v = \sqrt{\frac{2eV}{m_e}}$

HINT: 2

Given,

$V = 10\text{ kV} = 10 \times 10^3\text{ V} = 10^4\text{ V}$

$I = 15\text{ mA} = 15 \times 10^{-3}\text{A}$

(i) Number of electrons per sec hitting the anode $\left(\frac{N}{t}\right) = ?$

We know that

$$I = \frac{q}{t} = \frac{Ne}{t} \quad (\because q = Ne)$$

or $\frac{N}{t} = \frac{I}{e}$

(ii) Rate of production of heat at the anode

P = VI

HINT: 3

Given,

$\lambda_1 = 0.1\text{ nm} = 0.1 \times 10^{-9}\text{m}$

$\therefore$ $E_1 = hf_1 = \frac{hc}{\lambda_1}$

$\lambda_2 = 500\text{ nm} = 500 \times 10^{-9}\text{ m}$

$\therefore$ $E_2 = hf_2 = \frac{hc}{\lambda_2}$

$\frac{E_1}{E_2} = ?$

Now,

$$\frac{E_1}{E_2} = \frac{\frac{hc}{\lambda_1}}{\frac{hc}{\lambda_2}} = \frac{\lambda_2}{\lambda_1}$$

HINT: 4

Given,

$\lambda = 2.9\text{ Å} = 2.9 \times 10^{-10}\text{ m}$ ($\because$ 1 Å $= 10^{-10}$ m)

Number of order, n = 1, $\theta = 35°$

Spacing between the planes, d = ?

From Bragg's law, we know that

$$2 d \sin\theta = n\lambda$$

HINT: 5

Given,

$\lambda = 0.150\text{ nm} = 0.15 \times 10^{-9}\text{ m}$

(a) Minimum potential difference (v_{min}) =?

We can write

$eV_{min} = hf$

or $V_{min} = \frac{hf}{e} = \frac{hc}{\lambda e}$

(b) For $V = 30\text{ kV} = 30 \times 10^3\text{ V}$, $\lambda_{min} = ?$

$\because eV = hf_{max} = \frac{hc}{\lambda_{min}}$

or $\lambda_{min} = \frac{hc}{eV}$

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NUCLEAR PHYSICS

24

CHAPTER

24.1 Introduction

In the early years of 20th century, much less was known about the structure of atoms beside the fact that they contain electrons. J.J. Thompson discovered electrons in 1897, but its mass was still unknown. So, it was not possible even to say how many electrons were contained in an atom. Since the atoms are electrically neutral, scientist reasoned that an atom must also contain positive charge in order to compensate the negative charge. But, nobody knew in what form did this positive charge exist. Different models were proposed to describe the structure of an atom, but none of them had convincing explanation of experimentally observed facts. It was Earnest Rutherford, who proposed a satisfactory model to explain the existence of positive charge and its position in an atom based on his α-scattering experiment by gold foil. According to him, the positive charge of the atom is densely concentrated at the centre of atom forming its nucleus. Many other properties of nucleus could then be known. The branch of physics that deals with the study of properties of nucleus and the nuclear phenomena in terms of its constituents, interaction of nuclei, nuclear transmutation and their application is called nuclear physics. This field of physics finds broad application in high energy physics, medicine, material engineering, archaeology, etc. The most commonly known applications of nuclear physics are nuclear power plants and nuclear weapons.

24.2 Nucleus of an Atom

As mentioned earlier, the discovery of nucleus of an atom was made by Rutherford α-scattering experiment. In his experiment, α particles emitted from a source were made to interact with gold foil, and their corresponding deflections were studied. Based on the experimental observations, following conclusions were made.

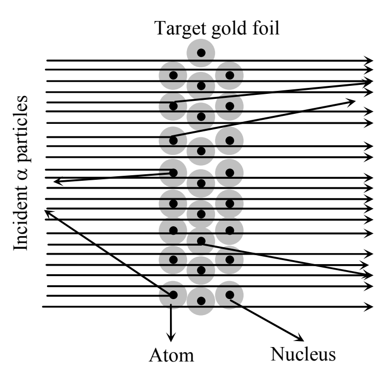

Fig. 24.1 Deflection of α particles by gold

1. Most of the α particles passed undeviated from gold foil which indicates that an atom is mostly an empty space.
2. Some α particles were scattered through large angles which is due to the interaction (Coulomb repulsion) with a massive dense core probably located at the centre. This central core of an atom was named nucleus.

3. A very few α particles nearly 1 in 8,000 traced back their original path which is due to head on collision with the nucleus. This suggests that the nucleus is impenetrable. These facts are shown in the Fig. 24.1.

Thus, according to Rutherford, nucleus is a positively charged dense sphere located at the centre of an atom. The experimental measurement of its diameter is about 10^{-15} m (1 fm). So, the size of the nucleus is significantly small as compared to that of atom (~ 10^{-10} m). Nucleus is about 10^5 times smaller than atom.)

24.3 Constituents of a Nucleus

Every atom consists of a nucleus and electrons. Electrons revolve around the nucleus through a specified orbit. The nucleus is dense part of an atom. The existence of nucleus was first modelled by the Rutherford's α-scattering experiment.

Nucleus contains two types of particles proton and neutrons which are of nearly equal masses. The particles in the nucleus are collectively called as nucleons. Hydrogen nucleus contains only one proton, but the nucleus of atoms of all other elements contains both protons and neutrons. Protons are positively charged particles and neutrons are chargeless. Therefore, the nucleus is positively charged. The mass of neutron is slightly greater than that of the proton. Total mass of an atom is supposed to be concentrated at the nucleus because electron is around 1836 times lighter than the proton or neutron

Important Facts about Nucleus

i. A nucleus is regarded as a positively charged sphere, which is impenetrable.

ii. The charge of nucleus is provided by the charge of proton. Let Z be the number of protons in a nucleus, then the charge of nucleus of an atom is written Ze, where e is the charge of an electron (equivalently, the charge of a proton).

iii. Nucleus is the central (or core) part of an atom.

iv. The radius of nucleus is about fermi unit (i.e. 10^{-15} m). The radius of a nucleus depends on the mass number of an atom. It is calculated from,

$$\mathbf{R = R_0 A^{1/3}} \qquad \ldots \textbf{(24.1)}$$

where, $R_0 = 1.2 \times 10^{-15}$m, is a constant quantity and A is the atomic mass of an atom.

v. The nuclear density has extremely large value. Its density is in the order of 10^{17} kg/m^3. It is independent of atomic number.

24.4 Nuclear Density

The mass per unit volume of the nucleus is called nuclear density. Consider a nucleus of mass number A and radius R.

Mass of nucleus (m) = A amu = A × 1.66 × 10^{-27} kg

also, the volume of nucleus (V) = $\frac{4}{3}\pi R_0^3 = \frac{4}{3}\pi\,(1.2 \times 10^{-15})^3\,A = 7.24 \times 10^{-45}\,A\ m^3$

Now, the density of nucleus (ρ) = $\frac{m}{V} = \frac{A \times 1.66 \times 10^{-27}}{7.24 \times 10^{-45} \times A} = 2.29 \times 10^{17}\ kgm^{-3}$

This shows that, the density of nucleus has extremely large value. It does not depend on mass number of an atom. All nuclei possess nearly the same value. It should be clear that, nuclear density is not uniform through out the nucleus. It has maximum density at the centre and decreases gradually towards the surface.

24.5 Atomic Number and Atomic Mass

The physical and chemical property of an element depend on the number of nucleons of an atom. So, it is very important to know the number of protons and neutrons in the nucleus. Moreover, a nucleus provides individuality of an atom.

Atomic Number

The number of protons in the nucleus of an atom is called its atomic number. Atomic number is denoted by Z. For example,

i. The atomic number of hydrogen is 1, i.e. Z = 1 (since there is only one proton in the nucleus of hydrogen atom).
ii. The atomic number of helium is 2, i.e. Z = 2 (since the helium nucleus contains two protons).
Similarly, for lithium, Z = 3; for calcium, Z = 20; for uranium, Z = 92, and so on.

Atomic Mass Number

Total number of neutrons and protons in the nucleus of an atom is called its atomic mass number. It is denoted by A. Therefore,

$$A = Z + N$$

where, N = number of neutrons.

For example, hydrogen has only one proton in its nucleus, so it has A = 1. Also, helium has two protons and two neutrons in its nucleus, so it has A = 4.

24.6 Representation of a Nucleus of an Atom

The nucleus of an atom is represented symbolically as $_ZX^A$,

where, Z = atomic number
X = name of element
A = atomic mass number

For example,

i. Hydrogen nucleus, $_1H^1$
ii. Helium nucleus, $_2He^4$
iii. Oxygen nucleus, $_8O^{16}$
iv. Uranium nucleus, $_{92}U^{235}$ (for an isotope)

24.7 Isotopes, Isobars, Isotones

Isotopes

Two or more nuclei having same atomic number but different atomic mass numbers are called isotopes of an element. $_6C^{12}$, $_6C^{13}$, $_6C^{14}$ are the isotopes of carbon. Similarly, $_{19}K^{40}$, $_{19}K^{42}$ are the isotopes of potassium.

Isobars

Two or more nuclei having same atomic mass number but different atomic numbers are called isobars. Isobars are formed from two or more elements. For example, C^{14}, $_7N^{14}$ are the isobars of carbon and nitrogen. Similarly, $_{19}K^{40}$ and $_{20}Ca^{40}$ are the isobars of potassium and calcium. The chemical properties of isobars are different.

Isotones

Two or more nuclei having equal number of neutrons are called isotones. For examples, $_{17}Cl^{37}$ and $_{19}K^{39}$ are isotones of chlorine and potassium. Similarly, $_{12}Mg^{24}$ and $_{11}Na^{23}$ are the isotones of magnesium and sodium.

24.8 Einstein's Mass-Energy Relation

At the beginning, the terms mass and energy emerged as two entirely different concepts independent to each other. Specifically, during the initial development of science of chemistry, it was assumed that in chemical reactions energy and mass are conserved separately. It was Einstein in 1905, as a consequence of his special theory of relativity, who showed that mass and energy are inter related to each other. In fact, these two quantities are equivalent and can be converted into one another i.e. mass can be converted to energy and vice versa.

In any chemical reaction, the amount of mass that is converted into other forms of energy is very tiny fraction compared to total mass involved. So, there is no hope of measuring the mass change even with the best laboratory balance and hence, mass and energy truly seem to be conserved separately. However, in a nuclear reaction, the energy released is tremendous (about a million times greater than in a chemical reaction) and the change in mass can easily be measured. So, mass and energy are conserved combinedly in such reaction. Thus, the conservation of energy is really the law of conservation of mass and energy.

According to Einstein, "*in an isolated system when the sum of rest masses changes, there is always a change in* $\frac{1}{c^2}$ *times the total energy other than rest mass energy*". This means if m is the change in rest mass of an isolated system and E is the corresponding change in the rest mass energy, then,

$$m = \frac{1}{c^2} E$$

$$\mathbf{E = mc^2} \qquad \ldots (24.2)$$

This change is equal in magnitude but opposite in sign to the change in sum of the rest masses. For example, when a uranium nucleus undergoes fission in a nuclear reactor, the sum of the rest masses of the resulting fragments is less than the rest mass of the parent nucleus. This decrease in mass when multiplied by c^2 (a conversion factor) equals the energy that is released in the process.

Mass – energy equivalence is the concept that asserts mass of a body as a measure of its energy content. In this concept, the total internal energy of a body at rest is equal to the product of its rest mass (m_0) and a suitable conversion factor c^2, to transform from units of mass to units of energy i.e., $E_0 = m_0c^2$, where c is the speed of light in vacuum and m_0 is the rest mass of the body.

In general, if a body is moving with velocity v, then the total energy of the body according to Einstein, given by $E = mc^2$ is the sum of rest mass energy and the kinetic energy of the body.

Thus, if the object is moving with speed v, its total energy is given by,

$$\mathbf{E = mc^2} \qquad \ldots (24.3)$$

From mass velocity relation, we know that,

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

Therefore, equation (24.3) can be written as,

$$E = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$= m_0c^2\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}}$$

Using binomial expansion $(1 + x)^n = 1 + nx + \ldots$, we can write,

$E = m_0c^2\left(1+\frac{1}{2}\frac{v^2}{c^2}\right)$, Neglecting the terms containing higher powers of $\left(\frac{v}{c}\right)$.

Therefore,

$$\mathbf{E = m_0c^2 + \frac{1}{2}m_0v^2} \qquad \ldots (24.4)$$

= rest mass energy + kinetic energy of object

For v = 0 (mass at rest), we get ,

$$\mathbf{E = m_0c^2} \qquad \ldots (24.5)$$

Equation (24.5) shows that the energy of stationary particle is not zero, rather it has energy in the form of mass, which we call rest mass energy. This discovery is popularly known as Einstein's mass energy relation.

24.9 Units of Energy

The SI unit of energy is joule (J). When we measure in the atomic level, the quantity of energy is relatively small. Therefore, we use different unit of energy in nuclear level. Nuclear energy is generally measured in electron volt (eV). It is defined as the amount of energy gained by an electron when accelerated through a potential difference of 1 volt.

$$\therefore\ 1\text{ eV} = 1.6 \times 10^{-19}\text{ C} \times 1\text{V}$$

$$= 1.6 \times 10^{-19}\text{ J}$$

The megaelectron volt (MeV) is a large energy and has the relation,

$$10^6\text{ eV} = 1\text{ MeV}$$

$$\text{So,}\quad 1\text{ MeV} = 1.6 \times 10^{-13}\text{ J}$$

24.10 Atomic Mass Unit

The measurement of mass in the nuclear level is not scientific when we measure in kilogram unit. So, another unit is used to measure the mass of nucleus. It is named atomic mass unit (amu). The atomic mass unit (amu) of nucleus is determined by comparing it with highly stable nucleus of carbon (C^{12}).

One atomic mass unit (1 amu) is defined as the $\frac{1}{12}^{\text{th}}$ of the mass of carbon atom $_6C^{12}$.

One mole of carbon has mass 12 g. It means 6.023×10^{23} atoms of carbon has the mass of 12 g. Therefore,

$$\text{Mass of 1 atom of carbon} = \frac{12}{6.023 \times 10^{23}}\ \text{g}$$

$$\therefore \quad 1\ \text{amu} = \frac{1}{12} \times \frac{12}{6.023 \times 10^{23}}\ \text{g} = 1.66 \times 10^{-24}\ \text{g}$$

$$\therefore \quad 1\ \text{amu} = 1.66 \times 10^{-27}\ \text{kg}$$

The energy equivalence of 1 amu mass is

$$E = mc^2$$

$$E = 1\ \text{amu} \times c^2$$

$$= 1.66 \times 10^{-27} \times (3 \times 10^8)^2$$

$$= 931 \times 1.6 \times 10^{-13}\ \text{J (approximately)}$$

$$\therefore \quad 1\ \text{amu} = 931\ \text{MeV (approx)}$$

An electron mass, $m_e = 9.1 \times 10^{-31}$ kg = 0.000568 amu

The energy equivalence of this mass of electron is 0.51 MeV.

Similarly, A proton mass, $m_p = 1.007276$ amu

A neutron mass, $m_n = 1.008665$ amu

Nuclear Forces

The nucleons (protons and neutrons) are bound tightly within a very small dimension with high density of the order of ~ 10^{17} kgm^{-3}. The gravitational attraction among the nucleons is of the order of 10^{-34} N. If we calculate the electrostatic repulsive force among the protons in nucleus, it is of the order of 10^{-2} N. So, the repulsive force between the nucleons is 10^{36} times greater than the gravitational force. This shows that the nucleus would not be stable. But in reality, it is not so. Nucleus is stable in many atoms. So, there must be another force which must dominate the electrostatic repulsion among the nucleons. This third force is called nuclear force or strong force. Nuclear force binds the nucleons in a small volume and provides the stability of nucleus. Some important properties of nuclear force are mentioned below:

i. Nuclear forces are attractive in nature.
ii. Nuclear force are charge independent.
iii. They are short range forces. Nuclear forces vanish beyond 10 fm.
iv. They are spin dependent. The force between two nucleons having parallel spins is stronger than the anti parallel spins.
v. Nuclear forces are non-central forces.
vi. They show saturation effect i.e. a nucleon interacts only with its neighbouring particle.

24.11 Mass Defect

The observable fact shows that, the mass of a composite nucleus of an atom is smaller than that of sum of individual masses of nucleons. For example, the sum of mass of 6 protons and 6 neutrons is found greater than the composite mass of carbon nucleus ($_6C^{12}$). Similar property can be observed in all other nuclei. This difference of mass in such condition is termed as mass defect. *The difference between the rest mass of the nucleus and the sum of the masses of the nucleons constituting a nucleus is known as mass defect.* It is denoted by Δm.

Let M be the composite mass of a nucleus of an atom having atomic mass numbers A and atomic number Z. Also, m_p and m_n be the mass of a proton and a neutron respectively. Then, the total mass of constituent nucleus is,

$$Zm_p + (A - Z)\, m_n$$

Then,

$$\textbf{the mass defect } (\Delta m) = [Zm_p + (A - Z)\, m_n] - M \qquad \text{...(24.6)}$$

24.12 Packing Fraction

Packing fraction of a nucleus in an atom is defined as the mass defect per nucleon of that nucleus. It is also called atomic packing fraction.

$$\text{Packing fraction (f)} = \frac{\text{mass defect } (\Delta m)}{\text{atomic mass number (A)}}$$

$$\therefore \quad f = \frac{\Delta m}{A}$$

24.13 Binding Energy

Initially, when mass defect was observed in the nucleus, it seemed to be very curious matter. Later on, it was disclosed that this reduced mass of nucleus is actually converted into energy that binds the nucleons in a nucleus. This energy was termed as binding energy. This means, binding energy is the energy equivalence of mass defect. Conversely, *the binding energy is the amount of energy required to break up a nucleus into its constituent parts and place them at an infinite distance from one another.*

Therefore, the binding energy of a nucleus is written as,

$$\text{Binding energy} = \Delta m\, c^2$$

where, c is the velocity of light.

In terms of MeV

$$\textbf{Binding energy (BE)} = \Delta m \times 931 \text{ MeV}$$

$$= [\{Zm_p + (A - Z)\, m_n\} - M] \times 931 \text{ MeV} \qquad \text{... (24.7)}$$

Binding energy per nucleon is calculated by dividing the binding energy of a nucleus with atomic mass number of corresponding nucleus. Therefore,

$$\textbf{Binding energy per nucleon} = \frac{\textbf{Binding energy}}{\textbf{A}} \qquad \text{... (24.8)}$$

The binding energy per nucleon is very important to study the stability of nucleus. The nucleus having greater binding energy per nucleon has greater stability. Thus, this quantity gives a better information about the stability of nucleus. Fig. 24.2, shows the plot of average binding energy per nucleon versus atomic mass number for naturally occurring nuclei.

Important Features of Binding Energy per Nucleon

i. The maximum binding energy per nucleon occurs at around mass number A = 60 and corresponds to the most stable nuclei. An isotope of nickel Ni^{62} has the maximum binding energy per nucleon, then Fe^{58}, Fe^{56}.

ii. Nuclei with very low or very high mass number have lesser binding energy per nucleon and are less stable.

iii. The smaller the binding energy per nucleon, the easier it is to disrupt the nucleus into its constituent nucleons.

iv. Nuclei with low mass number may undergo nuclear fusion, where light nuclei are joined together under certain conditions so that the final product may have a greater binding energy per nucleon and become stable.

v. Nuclei with high mass numbers may undergo nuclear fission, where the nucleus splits to give two daughter nuclei with the release of neutrons. The daughter nuclei will possess greater binding energy per nucleon.

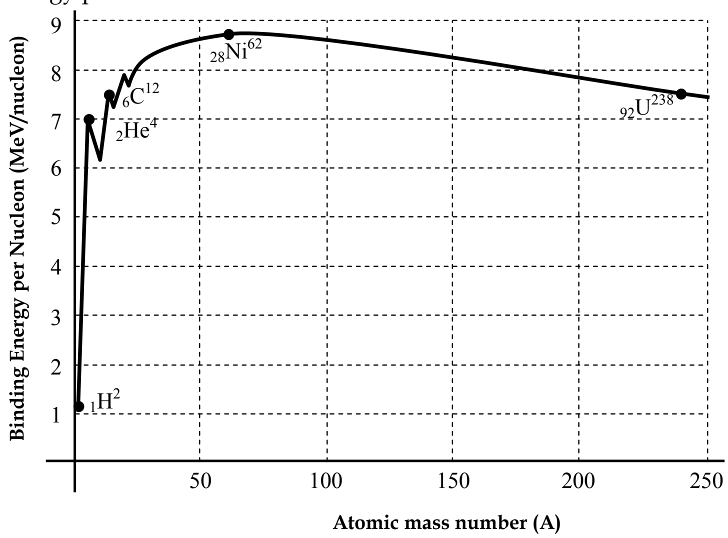

Fig. 24.2: Binding energy per nucleon versus atomic mass number

24.14 Nuclear Reaction

Nuclear reaction is defined as the change in composition of a nucleus when it is bombarded with nucleons or other sub-atomic particles. When a light particle collides with the nucleus, the interaction between the nuclear particles and the light particles takes place and nucleus transforms into new nucleus with different mass and energy.

Nuclear reactions are basically two types; nuclear decay reaction and nuclear transmutation reaction. Nuclear decay reaction is also called radioactivity. Radioactivity will be studied in next chapter.

In contrast, the nuclear transmutation reaction refers to the interaction of a nucleus with other nucleus. These reactions occur at very special conditions. For example, the fusion reaction (in which two nuclei combine to form a single nucleus) can take place whenever two nuclei come in the nuclear range, so that nuclear force becomes effective. For this, one of the nuclei should be accelerated to very high energy and made to collide with the other. Similarly, in fission reaction (in which a single nucleus splits into two or more lighter nuclei) very energetic charged particles such as protons, α-particles, etc. are used to bombard the nucleus or neutral particles such as thermal neutrons.

The particles or nucleus which are used to initiate nuclear reaction are called projectile particles. The projectile particles may be α-particles, protons, ions of element, electrons, neutrons, etc. The kinetic energy of these projectiles are very high extending from megaelectron volts to a few giga electron volts. The nucleus that undergoes transmutation is called target nucleus.

The nucleus which is bombarded with a light particle is known as mother nucleus and the bombarding particles are known as projectile. This pair of mother nucleus and the projectile is called parent pair. The new nucleus which is formed after transformation is known as daughter nucleus and the ejected particle is known as emitted particle. This pair is called final pair. The nuclear reaction can be represented as the following nuclear equation.

$$\mathbf{A + a = B + b + Q} \qquad \ldots(24.9)$$

Where, A = mother nucleus

a = projectile

B = daughter nucleus

b = emitted particle

Q = radiated energy

The value of Q may be positive or negative. If the energy is evolved in the nuclear reaction, Q is greater than zero, $Q > 0$ (i.e. positive value of Q). This type of reaction is called exoergic or exothermic reaction. If the energy is absorbed in the nuclear reaction this is called endoergic or endothermic reaction ($Q < 0$, negative value of Q). If the energy is neither evolved nor emitted, $Q = 0$. It should be noted that mass-energy conservation is strictly obeyed in nuclear reaction.

Important nuclear reactions are:

(a) Particle disintegration (b) Photo disintegration (c) Radioactive capture

Two types of basic nuclear reactions are explained below:

Nuclear Fission

The nuclear reaction in which a heavy nucleus disintegrates into two nuclei of nearly comparable mass along with emission of some particles and liberation of energy is known as nuclear fission. In nuclear fission, heavy nucleus is made to collide with a light particle in order to disintegrate it.

In 1939, (in the beginning days of world war second), a German scientist **Otto Hahn** and **Strassmann** studied the nuclear fission reaction in uranium nucleus and discovered that when a uranium nucleus ($_{92}U^{235}$) is bombarded with a neutron, it explodes into two nearly equal fragments, barium ($_{56}Ba^{141}$) and krypton ($_{36}Kr^{92}$) along with the emission of three neutrons ($_0n^1$), releasing some energy (Q) in the form of γ-rays. This fission reaction is represented by the following nuclear equation,

$$_0n^1 + {_{92}U^{235}} \longrightarrow {_{92}U^{236}} \longrightarrow {_{56}Ba^{141}} + {_{36}Kr^{92}} + 3\ {_0n^1} + Q$$

It is noted that barium and krypton are not produced in all fission reaction, the fragments may be other nuclei.

Energy released in fission reaction

In the nuclear reaction, the mass-energy conservation must be strictly followed. In the above nuclear reaction,

Before reaction

Mass of $_{92}U^{235}$ = 235.0439 amu

Mass of $_0n^1$ = 1.0087 amu

Total mass of parent pair = 236.0526 amu

After reaction

Mass of $_{56}Ba^{141}$ = 140.9129 amu

Mass of $_{36}Kr^{92}$ = 91.8973 amu

Mass of three $_0n^1$ = 3.0261 amu

Total mass of final pair = 235.8363 amu

Now,

Mass defect (Δm) = mass of parent pair – Mass of final pair = 236.0526 – 235.8363 = 0.2163 amu.

This lost mass equivalently appears in the form of energy so that total mass energy is conserved.

$$1 \text{ amu} = 931 \text{ MeV}$$

$$\text{Energy released} = 931 \times 0.2163 = 201.37 \text{ MeV} \approx 200 \text{ MeV}$$

Thus, large amount of energy is released which is mainly in the form of lights γ-rays and K.E. of the fission products.

Fission chain reaction

When a $_{92}U^{235}$ nucleus is bombarded with a slow neutron, two almost equal mass nuclei ($_{56}Ba^{141}$ and $_{36}K^{92}$) are produced, which are called fission pair (F.P.) and three neutrons are released simultaneously. These released neutrons are absorbed in the body of the source (and some may be lost). If the number of absorbed neutrons is greater than the lost neutrons, the reaction continues to the further steps. Suppose two neutrons are absorbed and one is lost in every reaction, the reaction takes place in faster rate and the whole process proceeds in a geometric progression. Thus, the reaction once started continues until whole source disintegrates, which is known as chain reaction. A chain reaction is a self propagating nuclear reaction process in which number of product neutrons is more than the number of neutrons required to initiate the reaction so that the reaction proceeds as a chain. Enormous energy is released from the Uranium source in chain reaction, which once started becomes uncontrolled. The chain reaction of $_{92}U^{235}$ is shown in Fig. 24.3.

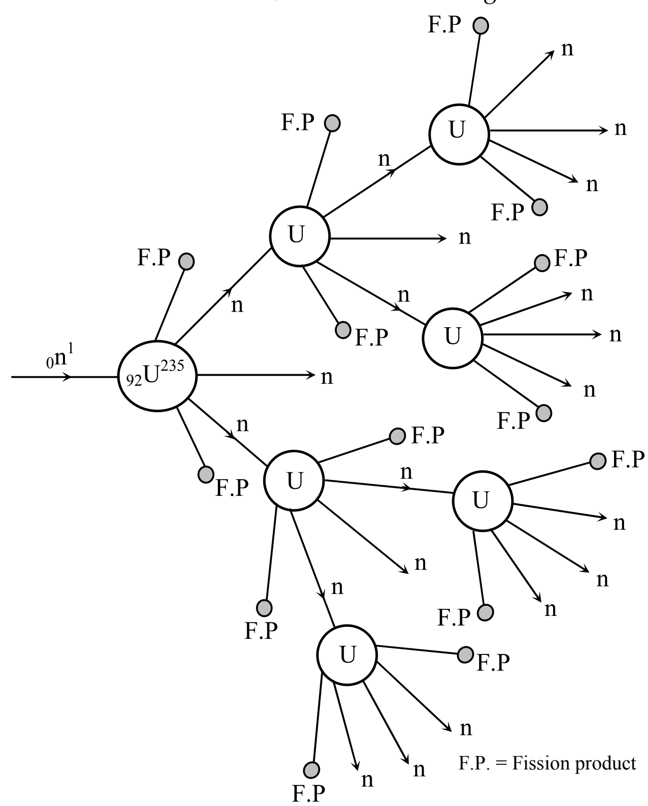

Fig. 24.3: Uncontrolled chain reaction

Uncontrolled chain reaction

When nucleus of $_{92}U^{235}$ is bombarded by a slow neutron, it undergoes fission by capturing neutron and split into two fragments $_{56}Ba^{141}$ and $_{36}Kr^{92}$ together with three neutrons. As explained above, three neutrons are emitted in every reaction of uranium fission. These emitted neutrons are employed to combine other nearby uranium nuclei to continue the sustainable chain reaction. But all the neutrons so produced are not used for the further nuclear reaction. They may interact with air molecules or escape out from the source. If the multiplication factor of chain reaction is greater than 1, the fission rate is multiplied rapidly so that whole source would explode radiating enormous energy and hence becomes uncontrolled. This type of nuclear reaction is called uncontrolled chain reaction. In such reaction, the large amount of thermal energy is produced in a very short time. Thus, nuclear disaster occurs. Atom bomb works on the principle of uncontrolled chain reaction.

Controlled chain reaction

Nuclear chain reaction is not always devastating. If the reaction is preceded in a controlled way, the energy so produced can be used in electricity generation, propulsion of ships and submarines. This type of nuclear reaction in which the rate of reaction can be varied at our will is known as controlled chain reaction. Simply, number of neutron and energy can be controlled in required level. Controlled reaction can be performed by making the absorption of excess neutrons in the nuclear source as shown in Fig. 24.4. A special type of material is used for the absorption of excess neutrons which is called moderator. The moderator is arranged so suitably that the multiplication factor can be 1. In general graphite, heavy water (D_2O), beryllium, etc. are used as the moderators. Mostly, moderators are made with elements of low atomic number.

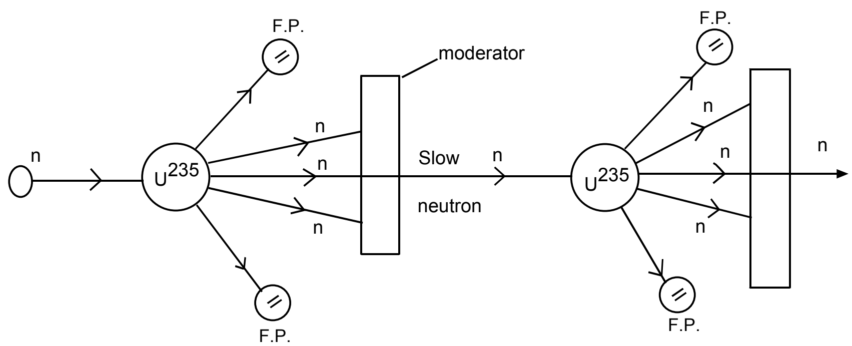

Fig. 24.4: Controlled chain reaction

Multiplication factor

When fission chain reaction is started, it may or may not be sustained until all the nuclei undergo fission. To examine whether the chain reaction increases, decreases or remains steady, a parameter is to be defined, which is called multiplication factor.

The multiplication factor of a fissionable mass is defined as the ratio of number of neutrons present at the beginning of particular generation to the number of neutrons present at the beginning of the previous generation. It is denoted by k.

$$\therefore \quad k = \frac{\text{Number of neutrons present at the beginning of one generation}}{\text{Number of neutrons present at the beginning of previous generation}}$$

The physical meaning of multiplication factor for its different values are as follows:

i. If k > 1, the fission chain reaction grows. It is also called uncontrolled chain reaction. If the chain reaction is started for k > 1, whole the source is exploded within a few second. Explosion of atom bomb is an example of uncontrolled chain reaction.

ii. If k = 1, the chain reaction remains steady. This type of chain reaction is controlled by means of machinery. It is also called controlled chain reaction. This principle is used in nuclear power generation from power plants.

iii. If k < 1, the chain reaction gradually dies out. Due to the lack of necessary number of neutrons for nuclear fission, the rate of fission decreases and is terminated.

Critical size and critical mass

In nuclear fission reaction, slow neutron bombards the uranium atom to break into daughter nuclei. The emitted neutron after fission reaction, travels a certain average distance through the material before it encounters another uranium nucleus and triggers another fission event. If the size of uranium source is too small, a neutron is likely to escape through the surface before it finds another nucleus. Therefore, for the sustained chain reaction, the size and mass of uranium source must have at least a critical value. If the size and mass of the source is smaller than critical value, the nuclear fission reaction decreases and dies out.

The amount of mass in fission source for which each fission event produces one additional fission event is called critical mass and the corresponding size of source is known as critical size. The multiplication factor k,

i. If k = 1, the neutron population and critical mass remains stationary and the nuclear reaction proceeds steadily.

ii. If k > 1, the neutron population increases rapidly in the source and chain reaction proceeds very fast. The mass for such condition is supercritical mass and the size is called supercritical size.

iii. If k < 1, the neutron population decreases rapidly and the chain reaction ceases. This mass in the source is called sub-critical mass and corresponding size is called sub-critical size.

24.15 Nuclear Fusion Reaction

The nuclear reaction in which two or more lighter nuclei merge into a single nucleus releasing some energy is known as nuclear fusion reaction. In nuclear fission reaction, one heavy nucleus splits into lighter nuclei, in contrast, two or more lighter nuclei fuse together, in order fusion.

When two deuterium nuclei $_1H^2$ are fused together, a single helium nucleus is formed with the release of energy about 24 MeV. This fusion reaction is written in the following nuclear equation.

$$_1H^2 + {_1H^2} \longrightarrow {_2He^4} + Q$$

To find the released energy, we have,

mass of a deuteron, $_1H^2$ = 2.01471 amu

$\therefore$ mass of two deuteron = 4.02942 amu

mass of a helium nucleus= 4.00388 amu

Now, the mass defect (Δm) = 4.02942 – 4.00388

= 0.02554 amu

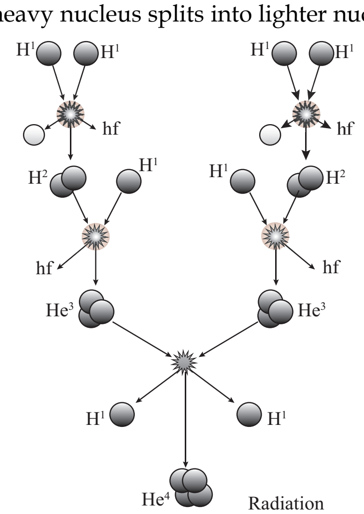

Fig. 24.5: Nuclear fusion chain

Also, 1 amu = 931 MeV

The energy liberated, Q= 0.02554 × 931

= 23.71 MeV

≈ 24 MeV

The energy released during fusion is much less than that in fission. However, energy released per nucleon during fusion is much greater than that liberated during fission.

The energy radiated from the sun and stars is considered due to the nuclear fusion reaction on its surface. For the nuclear fusion, large temperature and pressure is required, which is possible only on the sun and stars.

The hydrogen bomb is an example of the uncontrolled nuclear fusion reaction in which tremendous amount of energy is released.

Difference between Nuclear Fission and Nuclear Fusion

The major differences between nuclear fission reaction and nuclear fusion reaction are mentioned below.

Nuclear Fission	Nuclear Fusion
1. Nuclear fission is a nuclear reaction in which a heavy nucleus is bombarded with a light particle such that two fragments of roughly equal masses are formed along with emission of energetic neutrons and energy.	1. Nuclear fusion is a nuclear reaction in which two or more lighter nuclei are fused together to form a single nucleus releasing some energy
2. It can occur in room temperature and normal pressure.	2. Very high temperature and pressure are required for this reaction.
3. The energy released from a nucleus is relatively high (≈ 200 MeV). But the energy released per nucleon is about 0.85 MeV.	3. The energy released from a nucleus is relatively low but the energy released per nucleon is about 6.75 MeV.
4. This reaction takes place in high atomic number nuclei. For example: Uranium, Plutonium and Thorium.	4. This reaction takes place in low atomic number, nuclei. For example: Hydrogen, Deuterons, Tritium.
5. The radiations produced in nuclear fission are harmful.	5. The produced rays are relatively less harmful.
6. Radiation pollution is created.	6. Thermal pollution is created.
7. Atom bomb is based on this principle.	7. Hydrogen bomb is based on this principle.
8. It completes in single stage.	8. It is multistage reaction.

Tips for MCQs

1. Subatomic particles are: electron, proton and neutron. Proton and neutron lie in the nucleus and electron revolves around them.
2. Except hydrogen, the nucleus of each atom contains proton and neutron. Hydrogen atom contains only one proton.
3. The density of nucleus is very high ~ 2.29×10^{17} kgm^{-3}. Density of nucleus does not depend on atomic mass number.
4. The radius (R) and volume (V) of nucleus are determined from the formula,

$R = R_0 A^{\frac{1}{3}}$, where $R_0 = 1.2 \times 10^{-15}$ m

$V = \frac{4}{3}\pi R_0^3 A$

5. The symbol of nucleus, $_ZX^A$ (A = Atomic mass number, Z = atomic number, X is name of element)
6. Isotopes of an element consist of same atomic number but different atomic mass number. (i.e. $_ZX^A$, $_ZX^{A'}$) Where $A \neq A'$.
7. Isobars of different elements consists of different atomic number but same atomic mass number.
8. Mass of atom and subatomic particles is measured in atomic mass unit (amu)
   1 amu = 1.66×10^{-27} kg
9. Nuclear force is the strongest force in nature, it is also termed as strong force. This force holds the nucleons in a very small volume.
10. The size of nucleus is in the order of 10^{-15} m and the size of atom is in the order of 10^{-10} m.
11. Einstein's mass energy relation, $m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$
12. The nuclei containing even number of protons and even number of neutrons are relatively more stable.
13. **Mass defeat (Δm)**
    i. The mass of the nucleus (M) formed is less than the sum of the masses of the individual nucleons $(Zm_p + (A - Z)m_n)$. This difference is called mass defect (Δm).
    ii. $\Delta m = [Zm_p + (A - Z)m_n] - M$.
    iii. Packing fraction = $\frac{\Delta m}{A}$
14. **Binding energy**
    i. The mass defect (Δm) is converted into the binding energy to hold the nucleons in a small dimension.
    ii. Binding energy (BE) = $\Delta m \times 931$ MeV
    or, B.E. = $\Delta m\, c^2$ (joule)
    iii. Binding energy is usually expressed in MeV.
    1 amu = 931 MeV
    iv. Binding energy per nucleon = $\frac{\text{B.E.}}{A}$

$$\therefore \quad \frac{\text{B.E.}}{\text{nucleon}} = \frac{\Delta m \times 931}{A}\left(\frac{\text{MeV}}{\text{nucleon}}\right)$$

$$= \frac{\Delta mc^2}{A}\left(\frac{\text{joule}}{\text{nucleon}}\right)$$

15. **Nuclear reaction**
    i. Equation for nuclear reaction is, $_ZX^A + a = {}_{Z'}Y^{A'} + b + Q$
    It is also represented on, $_ZX^A$ (a, b) $_{Z'}Y^{A'}$
    ii. In nuclear reaction
        a. number of nucleons is conserved
        b. total charge is conserved
        c. linear momentum is conserved
        d. total energy is conserved

16. **Nuclear fission**
    i. Discovered by Otto Hann and Strassmann
    ii. Example of nuclear fission
    $_{92}U^{235} + {}_{0}n^{1} \rightarrow {}_{92}U^{236} \rightarrow {}_{56}B^{141} + {}_{36}Kr^{92} + 3\,{}_{0}n^{1} + Q$
    Here, Mass defect = 0.2153 amu
    Exothermic energy ≈ 200 MeV
    iii. Chain reaction:
        i. In uncontrolled chain reaction:
            a. multiplication factor (K) is greater than 1.
            b. mass is greater than 1
            c. principle of atom bomb.
        ii. In controlled chain reaction:
            a. multiplication factor (K) is equal to 1.
            b. mass is equal to 1
            c. principle of nuclear power production.
17. **Nuclear fusion:**
    i. Example: $_{1}H^{2} + {}_{1}H^{2} \rightarrow {}_{2}He^{4}$ + 24 MeV
    ii. Energy released per unit atom is greater in nuclear fission, but the energy released per unit mass is greater in nuclear fusion.
    iii. It occurs in large temperature and pressure, so it is also called thermonuclear reaction.
    iv. The source of solar energy and stellar energy are considered due to the effect of nuclear fusion reaction.

Worked Out Problems

1. How much energy will be created if a man of mass 50 kg is destroyed completely?

SOLUTION

Given,
Mass (m) = 50 kg
Speed of light (c) = 3.0×10^{8} ms^{-1}
Now, total energy released (E) = ?
From Einstein's mass-energy relation,

$E = mc^2$
$= 50 \times (3.0 \times 10^{8})^2$
$E = 4.5 \times 10^{18}$ J

∴ 4.5×10^{18} J energy is released when a man of mass 50 kg is destroyed completely.

2. A neutron is absorbed by a $_{3}Li^{6}$ nucleus with subsequent emission of an α-particle. Write the corresponding nuclear reaction and calculate the energy released in the reaction.

SOLUTION

Given,
Mass of neutron = 1.008665 amu
Mass of $_{3}Li^{6}$ = 6.015126 amu
Mass of $_{2}He^{4}$ = 4.002603 amu
Mass of $_{1}H^{3}$ = 3.0164049 amu
1 amu = 931 MeV
The appropriate nuclear reaction to satisfy the given condition,
$_{3}Li^{6} + {}_{0}n^{1} \longrightarrow {}_{1}H^{3} + {}_{2}He^{4}$ (α- particle)
Mass of constituents before reaction
$m_1 = {}_{3}Li^{6} + {}_{0}n^{1}$
$= 6.015126 + 1.008665 = 7.023791$ amu
Mass of constituents after reaction
$m_2 = {}_{1}H^{3} + {}_{2}He^{4}$
$= 3.016049 + 4.002603$ amu
$= 7.018652$ amu
The mass loss, $\Delta m = m_1 - m_2$
$= (7.023791 - 7.018652)$ amu
$= 0.005139$ amu
Now, equivalent energy is,
$\Delta E = \Delta m \times 931$ MeV
$= 4.784$ MeV

3. Calculate the mass defect, binding energy and binding energy per nucleon of helium ($_2He^4$) nucleus.

SOLUTION

Given,

Mass of proton (m_p) = 1.007276 amu

Mass of neutron (m_n) = 1.008665 amu

Mass of $_2He^4$ (M) = 4.001506 amu

Mass defect (Δm) = (Zm_p = Nm_n) – M

Here, Z = 2, A = 4 $\quad \therefore$ A – Z = – 2 = 2

Now,

Δm = (2 × 1.007276 + 2 × 1.008665) – 4.001506
= 0.030376 amu

ii. Binding energy (BE) = Δm × 931 MeV
= 0.030376 × 931 MeV
= 28.28 MeV

iii. Binding energy per nucleon = $\frac{BE}{A}$
= $\frac{28.28}{4}$
= 7.07 MeV

4. Using the values given below, calculate binding energy value for $^{238}_{92}U$.
($^{238}_{92}U$ = 238.0508 amu, $^{1}_{0}n$ = 1.008665 amu, $^{1}_{1}p$ = 1.007825 amu, 1 amu = 931 MeV.)

SOLUTION

The nucleus $_{92}U^{238}$ has 92 protons and (238 – 92) = 146 neutrons.

Therefore, mass defect (Δm)

= Mass of (92 protons and 146 neutrons) – mass of $_{92}U^{238}$
= (92 × 1.007825 + 146 × 1.008665) – 238.0508
= (92.7199 + 147.26509) – 238.0508
= 239.98499 – 238.0508

$\therefore$ Δ m = 1.93419 amu

Binding energy = Δm × 931 MeV
= 1.93419 × 931 MeV = 1800.730 MeV

5. [NEB 2074] The mass of $_{17}Cl^{35}$ is 34.9800 amu. Calculate its binding energy and binding energy per nucleon. Mass of one proton = 1.007825 amu and mass of one neutron = 1.00865 amu.

SOLUTION

Given,

Mass of $_{17}Cl^{35}$ (M) = 34.9800 amu

Mass of proton (m_p) = 1.007825 amu

Mass of neutron (m_n) = 1.00865 amu

Binding energy (BE) = ?

Binding energy per nucleon = ?

We have,

Mass defect (Δm) = Zm_p + (A – Z)m_n – M
= 17 × 1.007825 + (25 – 17) × 1.00865 – 34.9800 = 0.308725 amu

We have,

1 amu = 931 MeV

So, BE = Δm × 931 = 287.42 MeV

Also, binding energy per nucleon = $\frac{BE}{A} = \frac{287.42}{35}$ = 8.21 MeV.

6. [HSEB 2073] The energy librated in the fission of single uranium – 235 atom is 3.2 × 10^{-11} J. Calculate the power production corresponding to the fission of 1 g of uranium per day. Assume Avogadro constant as 6.02 × 10^{23} mol^{-1}.

SOLUTION

Given,

Mass number of Uranium (A) = 235

Energy (E) = 3.2 × 10^{-11} J per atom

Mass (m) = 1 g

Avogadro constant (N_A) = 6.02×10^{23} mol^{-1}

Total number of uranium disintegration per day

$$N = nN_A$$
$$= \left(\frac{m}{A}\right) N_A \quad m = \text{total mass and } A = \text{molar mass}$$
$$= \left(\frac{1 \times 10^{-3}}{235 \times 10^{-3}}\right) \times 6.02 \times 10^{23}$$
$$= 2.56 \times 10^{21}$$

Total energy production per day,

$$E_t = N \times E$$
$$= 2.56 \times 10^{21} \times 3.2 \times 10^{-11}$$
$$= 8.19 \times 10^{10} \text{ J}$$

Now, power production (P) $= \dfrac{E_t}{\text{time}} = \dfrac{8.19 \times 10^{10}}{60 \times 60 \times 24} = 9.48 \times 10^5$ watt

7. Assuming that about 200 MeV energy is released per fission of $_{92}U^{235}$ nuclei, what would be the mass of U^{235}, consumed per day in the fission reactor of power 1 MW approximately? [HSEB 2068]

SOLUTION

Given,

Energy per atom (E) = 200 MeV

$$= 200 \times 10^6 \times 1.6 \times 10^{-19}$$
$$= 3.2 \times 10^{-11} \text{ J}$$

Molar mass (A) = 235×10^{-3} kg

Power (P) = 1 MW = 10^6 W

Now, total energy released per day

$$E_{total} = P \times \text{time}$$
$$= 10^6 \times 24 \times 3600$$
$$= 8.64 \times 10^{10} \text{ J}$$

Now, total number of atoms disintegrated

$$N = \frac{E_{total}}{E} = \frac{8.64 \times 10^{10}}{3.2 \times 10^{-11}}$$

$$N = 2.7 \times 10^{21}$$

Also,

We have,

$$N = nN_A$$
$$= \left(\frac{m}{A}\right) N_A$$
$$m = \frac{N \times A}{N_A}$$
$$= \frac{2.7 \times 10^{21} \times 235 \times 10^{-3}}{6.023 \times 10^{23}}$$
$$= 1.05 \times 10^{-3} \text{ kg}$$

Mass consumed = 1.05 g

8. [NEB 2075] A city requires 10^7 watts of electrical power on the average. If this is to be supplied by a nuclear reactor of efficiency 20%. Using $^{92}U_{235}$ as the fuel source, calculate the amount of fuel required per day (Energy released per fission $^{92}U_{235}$ = 200 MeV).

SOLUTION

Given,

Output power (P_{out}) = 10^7 watts

Efficiency (η) = 20%

Input power (P_{in}) = ?

Mass required = ?

Time (t) = 1 day = 24 × 60 × 60 = 86400 sec

Energy, released per fission of $^{235}_{92}U$ = 200 MeV

Mass of $^{235}_{92}U$ = ?

We have,

$$\eta = \frac{P_{out}}{P_{in}}$$

or, $0.20 = \frac{10^7}{P_{in}}$

or, $P_{in} = \frac{10^7}{0.20} = 5 \times 10^7$ watt

Energy $E = P_{in} \times t = 5 \times 10^7 \times 86400 = 4.32 \times 10^{12}$ J

Again,

235 amu $= 235 \times 1.66 \times 10^{-27}$ kg $= 3.9 \times 10^{-25}$ kg

Energy released per fission of $^{235}_{92}U$ = 200MeV

$= 200 \times 10^6 \times 1.6 \times 10^{-19} = 3.2 \times 10^{-11}$J

$\therefore$ 3.2×10^{-11} J energy is released by 3.9×10^{-25} kg of uranium.

or, 4.32×10^{12} J energy is released by $\frac{3.9 \times 10^{-25}}{3.2 \times 10^{-11}} \times 4.32 \times 10^{12}$ kg of uranium.

$= 0.0527$ kg

$\therefore$ Mass of $^{235}_{92}U$ required = 0.0527 kg

Challenging Problems

1. The nuclear radius of $_8O^{16}$ is 3×10^{-15} m. Calculate the nuclear radius of $_{82}Pb^{205}$.

Ans: 7.02 fermi

2. Calculate in MeV the energy liberated when a helium nucleus ($_2He^4$) is produced (a) by fusing two neutrons and two protons, and (b) by fusing two deuterium nuclei ($_1H^2$). Mass of neutron = 1.00898 amu, Mass of proton = 1.00759 amu, mass of helium = 4.00277 amu, mass of deuterium = 2.01419 amu,1 amu = 931 MeV.]

Ans: (a) 28.27 MeV (b) 23.84 MeV

3. If 10 g of a matter is completely annihilated, find the quantity of energy produced.

Ans: 9×10^{14} J

4. The energy liberated in the fission of a single uranium- 235 atom is 3.2×10^{-11} J. Calculate the power production corresponding to the fission of 1 g of uranium per day. Assume, Avogadro constant = 6.0 $\times 10^{23}$ mol $^{-1}$.

Ans: 0.946 MW

5. A nucleus of uranium 238 can disintegrate with the emission of an alpha particle according to the reaction.

$$^{238}_{92}U \rightarrow ^{234}_{90}Th + ^{4}_{2}He.$$

Calculate the total energy related in the disintegration [Mass of ^{238}U = 238.12492 amu. Mass of ^{234}Th = 234.11650 amu. Mass of ^4He = 4.00387 amu . 1 amu is equivalent to 930 MeV]

Ans: (a) 4.23 MeV (b) 4.16 MeV

*[**Note:** Hints to challenging problems are given at the end of this chapter.]*

Conceptual Questions with Answers

1. Why is neutron considered the most effective bombarding particle in a nuclear reaction?

[NEB 2074]

➪ Neutron is a charge less subatomic particle, it does not interact electrically with electrons and protons. As it passes into an atom, it is not deflected by orbital electrons, and also by the proton in the nucleus. Hence, it can combine into the nucleus easily. Further, when it enters into the nucleus, it increases the neutron to proton ratio. This makes the nucleus unstable. Thus, the nuclear reaction takes place.

---

2. According to properties of charges, like charges repel each other. Then, how do the protons in a nucleus stay together? [NEB 2074]

✎ Nucleons are bound together not by the electrical force but by another nature of attractive nuclear force, called strong force, but is very strong in magnitude within the nuclear range (~ 10^{-15} m). Electric force between the charge particles decreases as the inverse square law $\left(F \propto \frac{1}{r^2}\right)$. In nucleus, neutrons occupy the space between protons, so the electrical influence between protons is dominated by strong force. Hence, protons in a nucleus stay together.

---

3. Why is the mass of a nucleus slightly less than the mass of constituent particles? [HSEB 2073]

✎ In nucleus, nucleons bind tightly with a nuclear force, called the strong force. This force provides the binding energy for these particles. This binding energy is produced in the expense of some fraction of mass of nucleons, which makes the ultimate reduction of mass in the nucleus as compared with the sum of mass of constituent particles.

---

4. Diameter of Al^{27} nucleus is D_{Al}. How can one express the diameter of Cu^{64} in terms of D_{Al}? Explain.

✎ Let D_{Al} and D_{Cu} be the diameter of alumium nucleus and copper nucleus respectively. We have,

$$D_{Al} = 2\left(R_0\, A_{Al}^{1/3}\right) \quad \ldots\text{(i) and}$$

$$D_{Cu} = 2\left(R_0\, A_{Cu}^{1/3}\right) \quad \ldots\text{(ii)}$$

From equation (i),

$$R_o = \frac{D_{Al}}{2\,A_{Al}^{1/3}} \quad \ldots\text{(iii)}$$

Using R_0 in equation (ii), we get

$$D_{Cu} = 2\left(\frac{D_{Al}}{2\,A_{Al}^{1/3}}\right)\cdot A_{Cu}^{1/3}$$

$$= D_{Al}\left(\frac{A_{Cu}}{A_{Al}}\right)^{\frac{1}{3}}$$

$$= D_{Al}\left(\frac{64}{27}\right)^{\frac{1}{3}} = \left(\frac{4}{3}\right) D_{Al}$$

$$\therefore \quad D_{Cu} = \left(\frac{4}{3}\right) D_{Al}$$

---

5. By what factor, must the mass number of a nucleus increase to double its volume? Explain. [HSEB 2072]

✎ The volume of nucleus, $V = \frac{4}{3}\pi R_0^3 A$.

The factor $\frac{4}{3}\pi R_0^3 A$ is constant. So, to double the volume, the atomic mass number also should be doubled.

---

6. All the nuclei have nearly the same density. Justify. [HSEB 2072]

✎ The average mass of a nucleon is 1.66×10^{-27} kg. For any nucleus, its total mass can be $A \times 1.66 \times 10^{-27}$ kg.

Also, the volume of nucleus, $V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi (R_0 A^{1/3})^3 = \frac{4}{3}\pi R_0^3 A$

Now, the density, $\rho = \frac{m}{V} = \frac{A \times 1.66 \times 10^{-27}}{\frac{4}{3}\pi (1.2 \times 10^{-15})^3 A} = 2.29 \times 10^{17}$ kgm^{-3}

This shows that density of nucleus does not depend on atomic mass number. Hence, the density of any nucleus is almost constant.

---

7. Which is more stable, $_3Li^7$ or $_3Li^4$?

↳ The nucleus having greater number of neutrons has less mutual electrostatic force between the protons. $_3Li^7$ contains more number of neutrons than $_3Li^4$. Due to greater number of neutrons in $_3Li^7$, it is more stable than $_3Li^4$.

---

8. What are the number of protons and the number of neutrons in a nucleus of $_{82}Pb^{206}$?

↳ In $_{82}Pb^{206}$,

Atomic number (Z) = 82 and atomic mass number (A) = 206

Therefore, number of neutrons, N = A – Z = 206 – 82 = 124

$\therefore$ There are 82 protons and 124 neutrons in the nucleus of $_{82}Pb^{206}$.

---

9. Give the mass number and atomic number of elements on the right hand side of the decay process, $_{86}Rn^{220} \longrightarrow Po + {_2He^4}$

↳ In nuclear reaction, atomic number and atomic mass number are always conserved. Therefore, the atomic number of Po is, 86 – 2 = 84 and the atomic mass number is 220 – 4 = 216.

Therefore, the decay equation is written as,

$_{86}Rn^{220} \longrightarrow {_{84}Po^{216}} + {_2He^4}$

---

10. Why should the emitted neutrons be slowed down in sustainable chain reaction?

↳ The emitted neutrons in the nuclear fission reaction should combine to other uranium nuclei to proceed the reaction continuously. If the fission neutrons were produced instantaneously and move swiftly, there would be no time for the neutron capture into the nucleus that ultimately ceases the reaction.

---

11. Why is the nuclear fusion not possible in laboratory?

↳ Nuclear fusion takes place at a very high temperature and pressure. These conditions can not be realized in laboratory. Nuclear fusion reaction takes place in the sun and other celestial bodies.

---

12. The sun is constantly losing mass due to thermo nuclear reaction. Comment.

↳ Nuclear fusion reaction takes place in the sun so that enormous heat and light are produced. In each fusion reaction, a small fraction of mass of atom is converted into thermal energy. Since, the mass is reduced continuous to convert into energy, it is constantly loosing the mass.

---

13. A fusion reaction is more energetic than a fission. Explain.

↳ The energy released per unit mass in fusion reaction is more than that of fission reaction. Nuclear fission reaction occurs in heavy nuclei like U^{235}, but the nuclear fusion reaction occurs in light like H^1, H^2, etc. Although, energy released per unit atom is larger in fission reaction energy, released per unit mass is much greater in fusion reaction.

---

14. Why are fusion reactions also known as thermo nuclear reaction?

↳ In nuclear fusion reaction, two or more nuclei has to be combined to form a heavy nuclei. In such nuclear combination, large thermal energy is required to work against the electrostatic repulsion between these nuclei. That is why, nuclear fusion reactions are also known as thermo-nuclear reaction.

---

15. Why do lighter nuclei tend to fuse together?

↳ The binding energy per unit nucleon is relatively greater in middle class nucleus. For instance, the binding energy per nucleon of helium is greater than that of hydrogen. The nuclei having greater binding energy per nucleon are relatively more stable. Hence, to be stable nuclei in nature, lighter nuclei tend to fuse together.

16. Define mass defect and binding energy.

✎ **Mass defect:** The difference between the rest mass of the nucleus and the sum of the masses of the nucleons composing a nucleus is known as mass defect. It is denoted by Δm.

Let M be the composite mass of a nucleus of an atom having atomic mass number A and atomic number Z. Also, m_p and m_n be the mass of a proton and a neutron respectively. Then, the total mass of constituent nucleus is,

$$Zm_p + (A - Z)\, m_n$$

the mass defect $(\Delta m) = [Am_p + (A - Z)\, m_n] - M$

Binding energy: The binding energy is the amount of energy required to break up a nucleus into its constituent parts and place them at an infinite distance from one another.

Therefore, the binding energy of a nucleus is written as,

$$\text{Binding energy} = \Delta m\, c^2$$

17. The binding energy per nucleons of Fe^{56} is 8.8 MeV. What does this mean?

✎ This means, the minimum energy of 8.8 MeV is required to eject a nuclear particle (one proton or one neutron) from the nucleus of an iron nucleus. More clearly, total energy $56 \times 8.8 = 492.8$ MeV is required to separate every nucleon to infinite distance apart from the iron nucleus.

18. How mass defect is related to the binding energy of nucleons?

✎ Mass defect is the loss of mass in the nucleus, when nucleons are composed unitely at a position. Nucleons in the nucleus are bound strongly which can not be removed with small amount of energy, rather it requires enormous energy. The nucleons gain binding energy by the conversion of mass defect into the form of energy.

So, binding energy(BE) = Mass defect $(\Delta m) \times c^2$.

19. In heavy nuclei, the numbers of neutrons are much greater than number of protons. Why?

✎ Nucleus contains protons and neutrons. Protons are positive charge particles, they repel to each other. So, they always tend to move away from each other due to the charge of similar nature. For the stability of nucleus, the repulsive force between these particles should be minimized. That can be done by separating them placing far to each other, which is naturally possible only when number of neutrons (neutral particles) are much greater than the number of protons.

20. What is atomic mass unit (amu)? Why this unit is necessary?

✎ 1 amu (1 atomic mass unit) is defined as the one-twelfth the mass of one ${}_6C^{12}$ atom, which is the most abundant naturally occuring isotope of carbon.

1 amu $= 1.66 \times 10^{-27}$ kg (in mass)

and 1 amu = 931 MeV (equivalent energy)

In the study of atomic mass, the unit 'kilogram (kg)' seems unscientific because of relatively high unit in the measurement of atomic mass. So, for the efficient comparison and calculation of mass in atomic level, amu is appropriate.

21. What does high binding energy per nucleon mean?

✎ Binding energy per nucleon means the average energy required to remove a nucleon from the nucleus. In order to compare the stability of different nucleons, we require to find the binding energy per nucleon of that nucleus. Greater the binding energy per nucleus of a nucleus, greater the stability of nucleus.

22. What is nuclear fission? Give one example.

✎ Nuclear fission is a nuclear reaction in which a heavy nucleus is bombarded with a light particles such that two fragments of roughly equal masses are formed along with emission of energetic neutrons and energy. When a nucleus of ${}_{92}U^{235}$ is bombarded with a slow neutron, two fragments of ${}_{56}Ba^{235}$ and ${}_{36}Kr^{92}$ are produced accompanied with three neutrons and about 200 MeV energy, i.e.

$${}_{92}U^{235} + {}_0n^1 \longrightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3\,{}_0n^1 + Q\ (\approx 200\text{ MeV})$$

23. What is nuclear fusion? Write an example.

↳ Nuclear fusion is a nuclear reaction in which two or more light nuclei are fused together to form a single nucleus releasing some energy. In general, the mass of the composite nucleus is less than the sum of the masses of the nuclei which are fused. The lost mass is converted into energy which is released in fusion process. For example, when two deuterons are fused, a triton is formed accompanied with a proton and some energy, i.e.

$$_1H^2 + {_1H^2} \longrightarrow {_1H^3} + {_1H^1} + 4.0 \text{ MeV}$$

24. Which principles are applied in the production of atom bomb and hydrogen bomb?

↳ Atom bomb is produced from the nuclear fission reaction and hydrogen bomb is produced from nuclear fusion reaction.

25. Complete the nuclear fusion equation.

$$_1H^3 + {_1H^2} \longrightarrow {_2He^4} + \ldots\ldots\ldots\ldots + 17.6 \text{ MeV}$$

$$_1H^3 + {_1H^2} \longrightarrow {_2He^4} + {_0n^1} + 17.6 \text{ MeV}$$

26. Prove 1 amu = 931 MeV.

↳

We have,

1 amu = 1.66×10^{-27} kg

We know,

$c = 3 \times 10^8 \text{ ms}^{-1}$

Now,

From Einstein's mass-energy relation,

$$E = mc^2$$
$$= 1.66 \times 10^{-27} \times (3 \times 10^8)^2 = 14.94 \times 10^{-11} \text{ J}$$

We know,

1 eV $= 1.6 \times 10^{-19}$ J

so, $$E = \left(\frac{14.94 \times 10^{-11}}{1.6 \times 10^{-19}}\right) \text{eV}$$
$$\approx 931 \times 10^6 \text{ eV}$$
$$E = 931 \text{ MeV}$$

$\therefore$ 1 amu = 931 MeV proved.

Exercises

Short-Answer Type Questions

1. What are the constituents of a nucleus?
2. Define (a) atomic mass unit (b) mass defect (c) binding energy (d) binding energy per nucleons.
3. Explain Einstein's mass-energy relationship theory.
4. How is energy released from the decay of radioactive isotopes?
5. Differentiate between nuclear fission and nuclear fusion.
6. What do you mean by nuclear reaction?
7. Discuss health hazards and safety related to radiation.
8. Intermediate mass elements are more stable than light and heavy elements. Explain.
9. Explain the terms (i) nuclear binding energy, (ii) nucleon and (iii) nuclide.
10. Define binding energy and binding energy per nucleon.
11. Is it possible that, the mass defect of an atom is negative?
12. What is binding energy per nucleon? What is its maximum value?
13. "Heavy nuclei split into lighter nuclei, by a process called fission." Why?
14. Lighter nuclei fuse together under suitable conditions." Why?
15. Define nuclear fission. Why it is called so?
16. What are thermonuclear reactions? Why are they called so?
17. Light energy emitted by the sun and stars comes from the fusion process. What conditions in the interior of star makes this possible?

18. A chain reaction dies out sometimes, why?
19. What is the difference between nuclear fission and radioactivity?
20. Why do nuclear reactions not occur just like chemical reactions?
21. Why is the neutron so effective as a bombarding particle?
22. Distinguish between nuclear fission and fusion.
23. Distinguish between chemical and nuclear reactions.

Long-Answer Type Questions

1. State and explain Einstein's mass energy relation with example.
2. Define binding energy. Draw a graph showing the relation between the binding energy per nucleon and atomic number.
3. Define the terms, binding energy and mass defect. Establish the relation between them.
4. What is nuclear fission? Give an example of nuclear reaction.
5. Distinguish between nuclear fusion and fission with examples.
6. What do you mean by fission? How energy is released in fission of uranium nucleus?
7. What is nuclear fission? How energy is released in nuclear fission reaction?

Numerical Problems

1. Find (i) mass defect (ii) binding energy (iii) binding energy per nucleon and (iv) packing fraction for the Helium atom ($_2He^4$). (mass of $_2He^4$ = 4.001509 amu, mass of $_1H^1$ = 1.007277 amu, mass of neutron = 1.0086666 amu)

   Ans: 0.030377 amu, 28.3 MeV, 7.07 MeV, 7.59 x 10^{-3} amu

2. The mass of the nucleus of the isotope $_3Li^7$ is 7.0143514. Find it's binding energy and binding energy per nucleon. (Mass of proton=1.0072754 amu, Mass of neutron =1.0086654 amu) (1 amu = 931 MeV)

   Ans: 39.2 MeV, 5.6 MeV

3. Calculate the Q-value of the nuclear reaction represented by $_7N^{14}$ (α,p) $_8O^{17}$ Relevant masses in amu are ($_7N^{14}$ =14.007514 m_α= 4.003837 amu $_8O^{17}$ = 17.004533 amu m_p = 1.008142 amu)

   Ans: -1.233 MeV

4. How much energy will be liberated if 1.0 g of matter is destroyed completely?

   Ans: a. 9.0 × 10^{13} J

5. Calculate the energy released in the nuclear reaction:

   $_1H^2 + {}_1H^2 \longrightarrow {}_2He^4$ + Q (energy)

   (Given, mass of $_1H^2$ = 2.014102 amu, mass of $_2He^4$ = 4.002604 amu, 1 amu = 931 MeV).

   Ans: b. 23.83 MeV

6. Calculate the mass defect, binding energy and binding energy per nucleon of $_{26}Fe^{56}$.

   (Given, mass of proton = 1.007276 amu, mass of neutron = 1.008665 amu, mass of $_{26}Fe^{56}$ = 55.934939 amu).

   Ans: 0.514 amu, 478.7 MeV, 8.55 MeV

7. Calculate the energy released in fission of a uranium$_{92}U^{235}$ atom in the following nuclear reaction,

   $_{92}U^{235} + {}_0n^1 = {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3{}_0n^1$ + Q,

   (mass of $_{92}U^{235}$ = 235.045933 amu, mass of $_0n^1$ = 1.008665 amu, $_{56}Ba^{141}$ = 140.9177 amu,

   $_{36}Kr^{92}$ = 91.8854 amu, 1 amu = 931 MeV)

   Ans: 209.8 MeV

8. Find the energy equivalent to 1 gm in kWh.

   Ans: 2.5 × 10^7 kWh

9. The mass of $_8O^{16}$ is 15.9949 amu. Calculate its binding energy. What is the binding energy per nucleon? (Given m_n = 1.008665 amu, m_p = 1.007825 amu).

   Ans: 127.54 MeV, 7.97 MeV / nucleon

10. Calculate (i) the mass defect (ii) binding energy (iii) binding energy per nucleon for $_6C^{12}$ nucleus. Atomic mass of $_6C^{12}$= 12 amu, mass of proton = 1.007825 amu and mass of neutron = 1.008665 amu.

Ans: (i) 0.09894 amu (ii) 92.113 MeV (iii) 7.676 MeV per nucleon

11. When $_3Li^6$ is bombarded by neutron so that, $_1H^3$ and $_2H^4$ are released. Write the reaction and find the reaction energy

Mass of $_3Li^6$ = 6.015126 amu, Mass of $_2He^4$ = 4.002604 amu, Mass of $_1H^3$ = 3.016049 amu
Mass of $_0n^1$ = 1.008665 u

Ans: 4.78 MeV

12. Calculate in MeV the energy liberated when a helium nucleus ($_2He^4$) is produced by fusing two deuterium nuclei.

Mass of deuterium = 2.01419 amu, Mass of helium = 4.00277 amu, 1 amu = 931 MeV

Ans: 23.8 MeV

13. Calculate the total amount of energy released if 25 g of matter is completely annihilated.

Ans: 2.25 × 10^{15} J

Multiple Choice Questions

1. The percentage of mass which changes into energy during fission is in the order of:
   a. 10% b. 1%
   c. 0.4% d. 0.1%
2. For an isobaric family members of nuclei, which of the following condition is true?
   a. Neutron number remains same
   b. Atomic number remains same
   c. Both neutron and proton numbers remain same
   d. Mass number remain same
3. The diameter of an atom is of the order of:
   a. 10^{-8} cm b. 10^{-9} cm
   c. 10^{-10} cm d. 10^{-12} cm
4. In a nuclear reaction, a deuteron particle is bombarded with a target nucleus, then energy is released along with a neutron and a product. The new product has the atomic mass:
   a. Smaller than a parent nucleus b. Greater than the parent nucleus
   c. Equal to the parent nucleus d. Can't be concluded
5. Energy equivalent of 1 gm of U^{235} is nearly:
   a. 3×10^{16} J b. 3×10^{23} J
   c. 9×10^{19} J d. 9×10^{13} J
6. The radius of gold nucleus is approximately:
   a. 4.29×10^{-14} m b. 1.5×10^{-10} m
   c. 2.5×10^{-8}m d. 6.0×10^{-24} m
7. A nucleus $_ZX^A$ decays to $_{Z+1}Y^A$ plus an additional nuclear particle. The resulting particle may be:
   a. Positron b. Alpha
   c. Beta d. Gamma
8. What are the appropriate conditions for a fusion reaction to occur?
   a. High temperature and low pressure b. Low temperature and high pressure
   c. High temperature and high pressure d. Low temperature and low pressure
9. The ratio of the mass defect of the nucleus to its mass number is maximum among following nuclei in
   a. ^{14}N b. ^{28}Si
   c. ^{56}Fe d. ^{238}U

10. A nuclear transformation is denoted by X(n, α) $^{7}_{3}Li$. Which of the following is the nucleus of element X?

a. $^{9}_{5}B$ b. $^{11}_{4}Be$
c. $^{12}_{6}C$ d. $^{10}_{5}B$

11. What is the size of gold nuclei?

a. $3\,R_0$ b. $4\,R_0$
c. $5\,R_0$ d. $5.8\,R_0$

12. On bombarding ^{235}U by slow neutron, 200 MeV energy is released. If the power output of atomic reactor is 1.6 MW, then the rate of fission will be

a. $5 \times 10^{22}\ s^{-1}$ b. $5 \times 10^{16}\ s^{-1}$
c. $8 \times 10^{16}\ s^{-1}$ d. $20 \times 10^{16}\ s^{-1}$

13. If the radius of a nucleus of ^{256}X is 8 fermi, then the radius of ^{4}He nucleus will be

a. 16 fermi b. 2 fermi
c. 32 fermi d. 4 fermi

14. The density of a nucleus of mass number A is proportional to

a. A^3 b. $A^{1/3}$
c. A^1 d. A^0

15. The energy equivalent of neutron-proton mass differences is 1.3 MeV and the rest mass energy of electron is 0.51 MeV. What is the maximum kinetic energy of electron emitted in neutron decay?

a. 1.81 MeV b. 1.3 MeV
c. 0.79 MeV d. 0.905 MeV

16. The ratio between the radii of nuclei with mass number 27 and 125 is

a. 5 : 3 b. 3 : 5
c. 27 : 125 d. 125 : 27

17. Four atoms of hydrogen combine to form an $^{4}_{2}He$ atom with a release of energy of

a. 26.7 MeV b. 216 MeV
c. 3.27 MeV d. 1 MeV

Answers

1. (d) 2. (d) 3. (a) 4. (b) 5. (d) 6. (a) 7. (c) 8. (c) 9. (c) 10. (d) 11. (d) 12. (b) 13. (b) 14.(d) 15. (c) 16. (b) 17. (a)

Hints to Challenging Problems

HINT: 1

Given,

Mass number of $_8O^{16}$ nucleus, $A_1 = 16$
Nuclear radius of $_8O^{16}$ nucleus, $R_1 = 3 \times 10^{-15}m$
Mass number of $_{82}Pb^{205}$ nuclear, $A_2 = 205$
Nuclear radius of $_{82}Pb^{205}$, R_2 = ?
We know that
$R \propto A^{1/3}$

$$\frac{R_1}{R_2} = \frac{(A_1)^{1/3}}{(A_2)^{1/3}}$$

HINT: 2

a. Given,

$2\,_1H^1 + 2\,_0n^1 \longrightarrow {}_2He^4$

Mass defect (Δm)
$= (2 \times 1.00759 + 2 \times 1.00898) - 4.00277$
$= 4.03314 - 4.00277$

∴ $\Delta m = 0.03027$ u

∴ Energy liberated = $\Delta m \times 931$ MeV

b. Given,

$_1H^2 + {}_1H^2 \longrightarrow {}_2He^4$

Mass defect (Δm) $= 2 \times 2.01419 - 4.00277$

∴ Energy liberated $= \Delta m \times 931$ MeV
$= 23.84$ MeV

HINT:3

Given,

$M = 10\ g = 10 \times 10^{-3}\ kg = 10^{-2}\ kg$
Speed of light, $c = 3 \times 10^8\ m/s$

Einstein's mass energy formula,

$E = mc^2$

HINT: 4

Given,

Energy liberated by a single

$E({}_{92}U^{235}) = 3.2 \times 10^{-11}$ J

Power production due to 1 g of ${}_{92}U^{235}$ per day, p = ?

Avogadro's constant, $N_A = 6 \times 10^{23}$ mol^{-1}

$\because$ 235 g contains 6×10^{23} atoms of ${}_{92}U^{235}$

$\therefore$ 1 g contains $= \frac{6 \times 10^{23}}{235}$ atoms

So, $N = 2.56 \times 10^{21}$ atoms

Hence, total energy produced by 1 g (N atoms)

$= N \times E({}_{92}U^{235})$

Now, Power production, $P = \frac{E}{t}$

HINT: 5

Given,

${}_{92}U^{238} \longrightarrow {}_{90}Th^{234} + {}_{2}He^{4}$ + energy (E)

Total energy released, E = ?

mass defect (Δm)

= mass of ${}_{92}U^{238}$ – mass of (${}_{90}Th^{234}$ and ${}_{2}He^{4}$)

Then,

$\therefore$ Energy released = $\Delta m \times 931$ MeV

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RADIOACTIVITY

25 CHAPTER

25.1 Introduction

In 1896, **Henry Becquerel** discovered an interesting phenomenon of heavy elements like Uranium, Thorium, etc. that emit certain invisible radiations which affect the photographic plate. Later on, many other lighter nuclei were also discovered which show the similar properties. Piere Curie and Madam Curie discovered a new element, Polonium, which shows the exactly similar property as the Uranium. A common property was observed among these elements which was the emission of the radiations spontaneously. *This phenomenon of emission of radiations spontaneously from the nucleus of some isotopes of elements is known as radioactivity.* The emission of nuclear radiation is a purely random event. The elements that show the radioactive property are known as radioactive elements.

Radiations emitted by radioactive nuclei are of three distinct types. They are named as α-rays, β-rays and γ-rays. α rays and β-rays are the rays of particles, hence they are particulate radiations. But, γ-rays are electromagnetic radiation. Radioactive elements can produce all three types of radiations but they are not emitted simultaneously. These radiations are harmful for human tissues, however they are used in diagnosis and treatment of some diseases.

Radioactivity can be of two types:

i. Natural radioactivity ii. Artificial radioactivity

If the radiations are emitted spontaneously from naturally occuring isotopes then, it is termed as natural radioactivity. However, the radiations emitted from artificially created radioactive elements is termed as artificial radioactivity.

Three types of radiations emitted from the radioactive source can be distinguished by enclosing the beam into the strong electric field. The experimental set up is shown in Fig. 25.1. A radioactive source is kept into a very thick lead block. The upper face of block contains a window which allows the radiation to emit out when it opens. Two oppositely charged parallel plates are placed at two sides just above the lead block. When the window of the block is open, the deflection of radiation can be observed. Some of the particles are found deflecting towards the negative plate, which are relatively heavier than others. They are named α-particles or α-rays. Some of the lighter particles are observed deflecting towards the positive plate, which are named β-particles or β-rays. A ray is observed passing straight up without deflecting towards any plates, they are named γ-rays. This experiment confirms three types of radiation emitted in radioactivity and nature of charge possessed by corresponding rays. The deflection of these rays can also be studied in magnetic field.

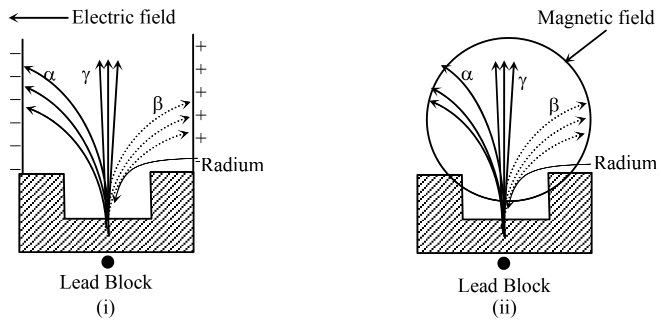

Fig. 25.1: Experimental set up to demonstrate the three types of radioactivity

25.2 Radioactive Decay

The rules of radioactivity inside atomic nuclei are governed by mass-energy equivalence. Particles decay only when the parent nucleus has greater mass than its products. For example, when a neutron decays to a proton and an electron (with an antineutrino), the mass of neutron is greater than the sum of masses of proton and the electron. This process of decaying of a particle is spontaneous. However, the decay is not spontaneous if the products have greater mass than the mass of parent nucleus. This induced decay needs external energy input.

25.3 Stability of Nucleus and Radioactive Isotopes

An atom is composed of three subatomic particles: neutrons, protons and electrons. The number of protons in an atomic nucleus is equal to the number of electrons in electronic orbits for a neutral atom. If the electron is removed or added, the atom becomes electrically charged. This charged atom is called an ion. An ionized atom contains unequal number of protons and electrons. But the neutrons do not have any direct relation to the electronic configuration of an atom. However, in atomic physics, the number of neutrons plays vital role to stabilize the nucleus. The principal role of neutrons is to act as a sort of nuclear cement to hold the nucleus intact.

One may surprise, how an electrically neutral particle play a crucial role to bind the nucleons together! Actually, nucleons are bound together not by the electrical force but by another nature of attractive nuclear force appropriately called strong force. Strong force is a very short range force which is effective only within the nuclear dimension, whereas electrical force between charges decreases as the inverse square of the distance $\left(F \propto \frac{1}{r^2}\right)$. It means, the nuclear force decreases far more rapidly, even it tends to zero at a few nucleons diameters apart. Within nuclear dimension, although the electrostatic force is also large, the nuclear force is dominant. The presence of neutrons enhances to the nuclear attraction and holds the protons from flying away.

Heavy nucleus contains large number of protons, so more neutrons are needed to hold them together. In light elements, the nuclei can be stable, though protons and neutrons are approximately equal in number. However, extra neutrons are required for heavy elements to hold the nucleons together. The most common form of lead contains 82 protons and 126 neutrons. i.e. about one and a half times as many neutrons as protons. For elements with more than 83 protons ($Z > 83$), even the addition of extra neutrons cannot stabilize the nucleus. The nuclei of these elements are unstable and finally decay radiating special rays called radioactive rays: α-rays, β-rays and γ-rays, until they become stable.

The unstable nuclei which decay spontaneously overtime are called radioactive nuclei. There are many isotopes which possess the decaying characters, called radioactive isotopes. All isotopes of elements heavier than Bismuth (Z = 83) are radioactive in nature. Many light elements are also radioactive. C^{14}, K^{40}, Na^{24}, Co^{60}, I^{131} are some examples of radioactive isotopes.

25.4 Nature of Radioactivity

Radioactive source shows that, the emission or radiation is both spontaneous and random. It is a spontaneous process because it is not affected by any external factors such as variation of temperature, pressure and electric and magnetic fields. Decay is random in the sense that, it is not possible to predict which nucleus in a sample will decay next. There is however, a constant probability that a nucleus will decay, within any fixed period of time.

Radioactive Rays

Radioactive atoms emit three distinct types of rays. The rays are named: alpha, beta and gamma. These three rays can be separated by putting magnetic field or electric field across their path as shown in Fig. 25.1.

25.5 Alpha Rays (α-rays)

An alpha ray is a stream of particles that are made of two protons and two neutrons and are identical to the nuclei of helium atoms. They are particles and so called as alpha particles. The phenomenon of emission of an α-particles from a radioactive nucleus is called alpha decay. When a nucleus undergoes alpha decay, its atomic number and atomic mass number are reduced by 2 and 4 respectively.

The transformation of ${}_{Z}X^{A}$ nucleus into daughter nucleus ${}_{Z-2}Y^{A-4}$ nucleus by an alpha decay is expressed by the following equation.

$$ {}_{Z}X^{A} \longrightarrow {}_{Z-2}Y^{A-4} + {}_{2}He^{4} \ (\alpha\text{-particle}) $$

The energy released in this process can be obtained from Einstein's mass-energy relation,

$$ Q = (m_x - m_y - m_\alpha)\, c^2 $$

This energy (Q) is shared both by daughter nucleus Y and alpha particle (α).

Properties of α-Rays

i. α-rays are positively charged particles and charge on an α-particle is $+ 3.2 \times 10^{-19}$ C (+ 2e).

ii. They have mass and charge equal to that of He-nucleus. Thus, α-particles are the doubly ionized helium atoms (He^{++}).

iii. They have very little penetrating power because they are massive particle and so, they can be stopped by a sheet of paper, 0.01 mm thick aluminium foil or even thin layer of air extending upto a few cm.

iv. They have got high ionizing power, greater than both β-rays and γ-rays.

v. They are deflected by both electric and magnetic fields.

vi. They affect a photographic plate, i.e. a chemical change takes place due to the exposure of α-rays.

vii. They produce brilliant scintillation (tiny flashes of light) on a fluorescent screen such as zinc sulphide, barium platinocynide, etc.

viii. The speed of α-particles ranges 5 to 7 percent of speed of light in vacuum depending on the radioactive material emitting them.
ix. They produce heating effect when they are stopped.
x. They can burn the muscles of the body and hence they are ruinous.
xi. Their energy lies between 4 to 10 MeV. Due to its high energy, they are used as bullets in bombarding atomic nuclei.
xii. Its specific charge (q/m) is 4.815×10^7 C kg^{-1}.

25.6 Beta Rays (β-rays)

Beta ray is simply a stream of electrons. An electron (β-particle) is ejected from the nucleus when a neutron is transformed into a proton, i.e. ${}_0n^1 \longrightarrow {}_1p^1 + {}_{-1}e^0$. It may seem that, electron exists into the neutron, but this is not so. The electrons are ejected from nucleus immediately after its creation. β-decay occurs for the conservation of mass and charge in radioactive disintegration.

β-decay is generally expressed in the following ways.

$$ {}_ZX^A \longrightarrow {}_{Z+1}Y^A + {}_{-1}e^0\ (\beta^-) + (\bar{\nu}) $$

Here, $\bar{\nu}$ is antineutrino.

The energy released in this process is,

$$ Q = (m_x - m_y - m_\beta)\ c^2 $$

This released energy is shared by daughter nucleus, β-particle and antineutrino.

Properties of β-Rays

i. β-particles (β-rays) are negatively charged particles and charge on a β-particle is equal to an electronic charge -1.6×10^{-19} C.
ii. They have the same mass as that of electrons. Thus, β-particles are identical with electrons.
iii. They have greater penetrating power than that of α-rays, having a range of several metres in air, but lower power of penetration than that of γ-rays.
iv. They have got less ionizing power than that of α-particles.
v. They are deflected by electric and magnetic fields more than α-particles owing to their smaller mass but the deflection is in opposite direction.
vi. They affect a photographic plate. i.e. they also produce chemical change.
vii. They can produce scintillation on a fluorescent screen.
viii. Their emission speed is in the range of 33% to 99% of the speed of light in vacuum.
xi. Its specific charge (e/m) is 1.76×10^{11} C kg^{-1}.
xiii. They are only emitted from certain elements.

Existence of neutrino

In β-decay condition, it is seen there is the clear violation of three important conservation laws: law of conservation of energy, law of conservation of linear momentum and the conservation of angular momentum. This was the unrealistic evidence in the study of nature and natural laws. To resolve the problem, Wolfgang Pauli, in 1933, purposed a new particle having three especial properties: zero rest mass, zero charge and spin half particle. Later on, Enrico Fermi, developed a theory regarding the newly purposed particle by Pauli, and named it as "neutrino". Then, its antiparticle was also discovered, so a neutrino and an antineutrino, accompanied with a beta-minus (β^-) and beta–plus (β^+)

decays respectively. In neutron to proton transformation, an electron (beta – minus) and an antineutrino are emitted.

$$n \longrightarrow p + \beta^- + \bar{\nu} \qquad (\bar{\nu} \text{ is antineutrino})$$

In proton to neutron transformation, a positron (beta – plus) and a neutrino are emitted.

$$n \longrightarrow p + \beta^+ + \nu \qquad (\nu \text{ is neutrino})$$

Mystery of neutrino:

Neutrinos are extremely swift ($v \sim c$). Whether they have mass is still questionable. If they do, it is thousand times less than the mass of an electron.

25.7 Gamma rays (γ-rays)

Gamma rays are massless energy (i.e. rest mass is zero). Like visible light, gamma rays are also the photons of electromagnetic radiation, but of much higher energy. Visible light is emitted when electron jumps in an atomic orbit from higher energy state to lower energy state, however gamma rays are emitted when same event occurs in nucleons. In most of the cases, when an excited nucleon return to the ground state, γ-rays are emitted. Sometimes, after the emission of an alpha, beta or positron particle, the nucleus is still in an exited state, called a meta-stable state. In order to get to a lower energy state, it emits a quantum of energy in the form of a gamma ray.

Properties of γ-Rays

i. γ-rays are stream of electrically neutral particles called photons.
ii. The rest mass of photons is zero.
iii. They are electromagnetic waves of small wavelength so, they are not called radioactive particle.
iv. They have the greatest penetrating power among all nuclear radiations. It penetrates very deeply into matter before, its energy has been used up. It can pass through even 5 cm thick sheet of lead or 30 cm thickness of iron.
v. Their ionizing power is the least.
vi. They are not deflected by electric or magnetic fields.
vii. Their emission speed is exactly the same as the speed of light in vacuum.
viii. They affect photographic plate very strongly.
ix. When it is exposed on human body or living thing, it affects strongly.
x. They are used in radio therapy to destroy cancerous cells.
xi. They can produce nuclear reaction.
xii. They can produce heating effect on the exposed surface.

Penetrating power of Radiation

The radioactive rays, α-rays, β-rays, and γ-rays have different penetrating capacity when they fall on a material. Alpha rays can be stopped by papers. β-rays cannot be stopped by paper, but by the combination of several aluminium plates. Gamma rays are highly penetrating rays. Thick lead plates are required to stop the γ-rays. They can penetrate even a thick concrete wall. The penetrating scheme of α-, β- and γ-rays is shown in Fig. 25.2.

An alpha particle is easy to stop because it moves relatively slow and its double positive charge interacts with the molecules it encounters along its path. Because of its strong positive charge, the particles of the medium in its path get ionized. When it receives two electrons from the matter, it becomes a chargeless and harmless helium atom.

Beta particles move faster than the alpha particle. It possesses the charge equal to the charge of an electron. Its electrical interaction with other atoms is relatively weaker than the alpha particles. So, it can travel much farther than the alpha particles in same medium. Consequently, the ionization power in a medium is relatively low.

Since gamma rays have no charge, they do not stop until these hit directly on the electron or the nucleus. They can travel long distance in any medium, only the dense material like lead can stop the gamma rays. Therefore, they have the least ionizing power and the most penetrating power out of three radiations.

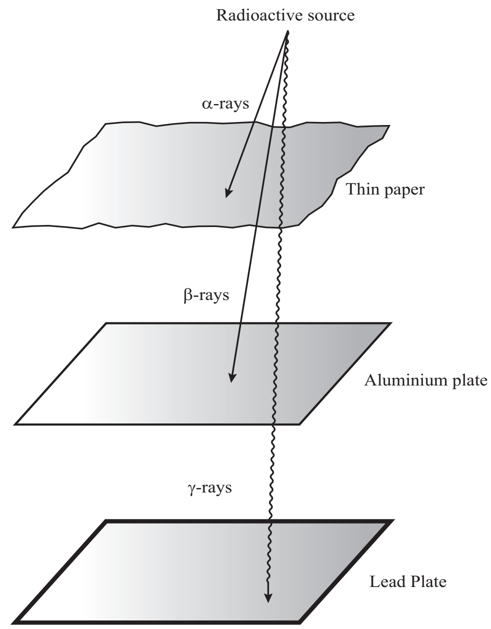

Fig. 25.2: Penetrating power of radioactive rays

25.8 Laws of Radioactive Transformation

There are many laws of radioactive transformation. Some important laws are listed below:

i. When a particle ejects an α-particle, the mass is reduced by 4 units and charge decreases by 2 units. The radioactive transformation in α-particle emission is represented by the following equation:

$$_{Z}X^{A} \longrightarrow {}_{Z-2}Y^{A-4} + \alpha\ (He^{++}) \qquad \ldots(25.1)$$

ii. When a radioactive nucleus emits a β-particle, the mass number remains unchanged but the charge increases by 1 unit. The radioactive transformation in β-particle emission is represented by the following equation:

$$_{Z}X^{A} \longrightarrow {}_{Z+1}Y^{A} + \beta\ ({}_{-1}e^{0}) \qquad \ldots(25.2)$$

iii. When a radioactive nucleus emits γ-rays, the mass and the charge remains unchanged. Only some energy is radiated and the original nucleus shifts from higher energy level to lower energy level. The radioactive transformation in γ-rays emission is represented by the following equation:

$$\left({}_{Z}X^{A}\right)^{*} \longrightarrow {}_{Z}X^{A} + \gamma \qquad \ldots(25.3)$$

iv. The decay of radioactive materials is purely a random process.

v. The rate of decay is completely independent of the physical condition and chemical composition of the material. It is not affected by temperature and pressure.

vi. The rate of decay of radioactive nuclei is directly proportional to the quantity of material actually present at that instant.

25.9 Radioactive Decay Law

This law states that "*the rate of decay (disintegration) of radioactive nuclei of atoms at any instant is directly proportional to the number of atoms present at that instant*".

Let N be the number of radioactive nuclei present in a radioactive substance at any instant of time t. Let dN be the number of such nuclei that disintegrates in a short interval of time dt. Then, the rate of disintegration $-\frac{dN}{dt}$ is directly proportional to N.

$$\text{i.e.,}\quad -\frac{dN}{dt} \propto N$$

$$-\frac{dN}{dt} = \lambda N$$

$$\textbf{or,}\quad \mathbf{\frac{dN}{dt} = -\lambda N} \qquad \textbf{...(25.4)}$$

Where, λ is a proportionality constant called decay constant or disintegration constant or transformation constant. Its value is different for different radioactive substances. And negative sign indicates that, number of atoms decreases with time. Its value depends on nature of radioactive element only and not on other factors like temperature, pressure, amount of element, etc.

Equation (25.4) can be written as,

$$\mathbf{\frac{dN}{N} = -\lambda dt} \qquad \textbf{... (25.5)}$$

Let N_0 be the number of radioactive atoms present at a time $t = 0$ and N be the number of atoms left at time t.

Integrating equation (25.5), we get,

$$\int_{N_0}^{N} \frac{dN}{N} = \int_{0}^{t} -\lambda dt$$

$$\text{or,}\quad [lnN]_{N_0}^{N} = -\lambda \int_{0}^{t} dt \qquad (\text{here, } ln = \log_e)$$

$$\text{or,}\quad lnN - lnN_0 = -\lambda[t]_0^t$$

$$\text{or,}\quad ln\frac{N}{N_0} = -\lambda[t-0] \qquad (\because ln\frac{x}{y} = ln\,x - ln\,y)$$

$$\text{or,}\quad ln\frac{N}{N_0} = -\lambda t$$

Taking antilog on both sides, we get,

$$\frac{N}{N_0} = e^{-\lambda t}$$

$$\therefore\quad \mathbf{N = N_0 e^{-\lambda t}} \qquad \textbf{... (25.6)}$$

Equation (25.6) concludes that, the number of atoms of a given radioactive substance decreases exponentially with time i.e. in beginning the decay occurs rapidly and then becomes more and more slow. N becomes zero only when t approaches infinity. Therefore, a radioactive substance will never disintegrate completely. This fact is clear from the graph between time (t) along x-axis and number of atoms (N) present at time t as shown in Fig. 25.3.

Half Life Period

Radioactive isotopes decay at different rates. The decay rate of radio isotopes is measured in terms of characteristics of time, the half life. The half life of radioactive material is the time needed for half of the radioactive atoms to decay. It is denoted by T_h or $T_{1/2}$. For example, radium 226 has a half life of 1620 years. This means, half of any given specimen of Ra-226 will have undergone radioactive decay by the end of 1620 years. In the next 1620 years, half of the remaining Radium will decay, leaving only one fourth the original numbers of Radium atoms. Likewise, similar process proceeds for the remaining atoms.

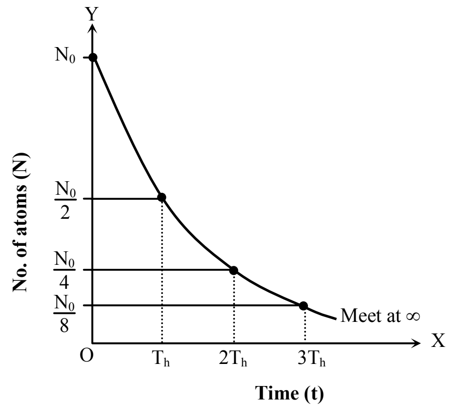

Fig. 25.3: Radioactive decay with time

In half life period, $N = \frac{N_0}{2}$ at $t = T_h$.

From equation (25.6), we get,

$$\frac{N_0}{2} = N_0 e^{-\lambda T_h}$$

$$\frac{1}{2} = e^{-\lambda T_h}$$

Taking antilog,

$$ln\frac{1}{2} = -\lambda T_h$$

$$\lambda T_h = ln\,2$$

$$T_h = \frac{ln\,2}{\lambda}$$

$$\mathbf{T_h = \frac{0.693}{\lambda}} \quad \textbf{...(25.7)}$$

This is the very important relation to study the radioactive phenomena.

Mean Life

The average time for which an atom of a radioactive substance exists is called average life or mean life of the radioactive element. It is denoted by τ.

Let dN number of atoms disintegrated at time dt. Also, there are N number of atoms that survive for a time t. So, the combined age of dN atoms is tdN. Therefore, the life span of all atom that survive from time 0 to ∞ is $\int_0^\infty t\,dN$.

Now, the average life, $\tau = \frac{\text{Combined life}}{\text{Total atoms}}$

$$= \frac{1}{N_0}\int_0^{\infty} t dN = \frac{1}{N_0}\int_0^{\infty} t[d(N_0 e^{-\lambda t})]$$

Differentiating the term in the bracket with respect to time, we get,

$$= \frac{1}{N_0}\int_0^{\infty} t(-\lambda N_0 e^{-\lambda t} dt) = -\lambda\int_0^{\infty} te^{-\lambda t} dt$$

The – ve sign indicates that the number of atoms decreases as time progresses. So, we can ignore negative sign.

$$\therefore\ \boldsymbol{\tau = \lambda\int_0^{\infty} te^{-\lambda t} dt} \qquad \textbf{...(25.8)}$$

Integrating by parts,

$$\tau = \lambda\left[\left|\frac{te^{-\lambda t}}{-\lambda}\right|_0^{\infty} - \int_0^{\infty}\frac{e^{-\lambda t}}{-\lambda}dt\right] = \left|\frac{e^{-\lambda t}}{-\lambda}\right|_0^{\infty}$$

$$\boldsymbol{\tau = \frac{1}{\lambda}} \qquad \textbf{...(25.9)}$$

$$T_{mean} = \left(\frac{1}{\frac{0.693}{T_h}}\right) = \frac{T_h}{0.693}$$

∴ So, the average life of a radioactive element is the reciprocal of its decay constant.

Alternative form of decay law: Activity of decay

In many cases, we are interested in decay rate $R\left(= -\frac{dN}{dt}\right)$ rather the calculation of number of radioactive isotopes N itself. Decay rate gives the number of nuclei decaying per unit time. It is represented by R.

From radioactive decay law,

$$R = -\frac{dN}{dt} = \lambda N$$

$$\therefore \quad R = \lambda N_0 e^{-\lambda t}$$

$$\boldsymbol{R = R_0\, e^{-\lambda t}} \qquad \textbf{...(25.10)}$$

Where R_0 is the radioactive decay rate at time t = 0 and R is the rate at any subsequent time t. i.e. $R = \lambda N$. The total decay rate R of a sample of radio nuclides is called the activity of that sample.

25.10 Number of Atoms Left After n^{th} Half Lives

Let N_0 be the number of radioactive nuclei in a sample specimen in the beginning of radioactivity.

After time T_h, the number of atoms left will be = $\frac{N_0}{2}$

After time $2T_h$, the number of atoms left will be = $\frac{1}{2} \times \left(\frac{N_0}{2}\right) = \left(\frac{1}{2}\right)^2 N_0$

After time $3T_h$, the number of atoms left will be $= \frac{1}{2}$ of $\left(\frac{1}{2}\right)^2 N_0$

$$= \frac{1}{2} \text{ of } \left(\frac{1}{2}\right)^2 N_0 = \left(\frac{1}{2}\right)^3 N_0$$

Similarly,

After time nT_h, the number of atoms left will be = $\left(\frac{1}{2}\right)^n N_0$

Therefore, number of radioactive atoms left after n^{th} half life is,

$$\mathbf{N = \left(\frac{1}{2}\right)^n N_0} \qquad \text{...(25.11)}$$

25.11 Kinetic Energy of Emitted α-particle from nucleus

In nuclear reaction, when a parent nucleus is splitted into a daughter nucleus and a light particle (here, alpha particle), the nuclear reaction can be written as,

$X \rightarrow Y + \alpha$

X = parent nucleus

Y = daughter nucleus

α = alpha particle

Let M, m and m_α be the masses of parent nucleus, daughter nucleus and alpha particle respectively.

For the condition, $M > (m + m_\alpha)$, the difference of mass is converted into the kinetic energy of daughter nucleus and alpha particle.

Before reaction, total momentum is zero, i.e., $p_1 = 0$

After reaction, both products gain kinetic energy and travel with certain speed. Let v and v_α be the speeds of daughter nucleus and α–particle respectively. In this condition, total momentum,

$p_2 = mv + m_\alpha v_\alpha$

From the conservation of momentum, $p_1 = p_2$

$mv + m_\alpha v_\alpha = 0$

$$\textbf{or,} \quad \mathbf{v = -\frac{m_\alpha v_\alpha}{m}} \qquad \text{... (25.12)}$$

The negative sign shows that daughter nucleus and alpha particle move in opposite direction. In terms of magnitude,

$$\mathbf{v = \frac{m_\alpha v_\alpha}{m}} \qquad \text{... (25.13)}$$

Let Q be the mass equivalent energy released in the nuclear reaction, i.e.,

$$Q = \frac{1}{2}mv^2 + \frac{1}{2}m_\alpha v_\alpha^2$$

From equation (25.13), we get,

$$Q = \frac{1}{2}m\left(\frac{m_\alpha v_\alpha}{m}\right)^2 + \frac{1}{2}m_\alpha v_\alpha^2 = \frac{1}{2}\frac{m_\alpha^2 v_\alpha^2}{m} + \frac{1}{2}m_\alpha v_\alpha^2$$

$$= \frac{1}{2}m_\alpha v_\alpha^2\left(\frac{m_\alpha + m}{m}\right) = (E_k)_\alpha\left(\frac{m_\alpha + m}{m}\right)$$

Where $(E_k)_\alpha$ is the kinetic energy of α-particle.

$$\textbf{So, } (E_k)_\alpha = \left(\frac{m}{m_\alpha + m}\right)Q \qquad \text{... (26.14)}$$

25.12 Uses of Radioactive Nuclei

Radioactive nuclei are used for many purposes like medical use, finding the age of fossils and artifacts, detection of elements, etc. Medical uses of radioactive isotopes and carbon dating are explained below.

Medical uses

As explained earlier in this chapter, many isotopes of elements are radioactive. Co^{60}, Na^{24}, K^{40}, C^{14}, etc., are some examples of radioactive isotopes. The radiation emitted from the radioactive isotopes are always damaging in nature. When these radiations are incident on the materials, the atoms of the materials get ionised. The cell may be damaged or badly harmed in case of living cells. Their physical and chemical properties may alter and harm them.

The damaging nature of radiations can be utilized in the diagnosis and therapy of many diseases like cancer. Some of the medical uses of radiation are listed below:

Radioactive isotopes are used for the diagnosis and treatment of many diseases:

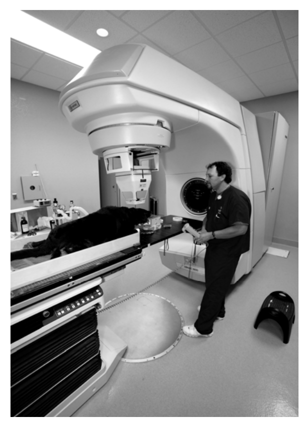

Fig. 25.4: Co^{60} Radiotherapy

a. **Radiodiagnosis:** The detection of causes of diseases using radioactive isotopes is known as radiodiagnosis. Some applications of radioactive isotopes in radiodiagnosis are as follows:
   i. Radioactive mercury (Hg-203) is used to detect kidney and liver functions.
   ii. Radioactive iodine (I–131) is used to study the thyroid functions.
   iii. To detect the haemorrhage location in human body, radio chromium (Cr - 51) is used.
b. **Radiotherapy:** The treatment of diseases using radioactive isotopes is known as radiotherapy. Some applications of radio isotopes in radiotherapy are as follows:
   i. Co^{60} isotopes are used to destroy the cancerous tissues.
   ii. I^{131} isotopes are used to destroy the overactive thyroid gland.
   iii. Radiophosphorous, radiogold are used to cure leukemia.

Carbon dating

Radio carbon dating is the technique of determining the age of archaeological specimens from the examination of radioactive carbon isotopes. A carbon isotope C^{14} is the major source of carbon dating. The rate of disintegration of such radioactive carbon isotopes in the fossils, woods, rocks or parts of meteorite are observed to find their ages.

Carbon is an essential component of organisms. Every organic compound contains carbon. Animals and plants receive carbon from food, air, water, etc. In the living state, they contain certain proportion of C^{14} and C^{12} (in almost constant proportion). But, after death, radioactive C^{14} decays gradually into stable carbon C^{12}. It takes a very long time (half life - 5730 years) for the decay process. Analyzing the proportion of C^{14} and C^{12}, the age of archaeological specimen can be determined.

Let N_0 be the number of radioactive C^{14} in an organism at the time of death, t = 0. At time *t* after its death, the organism contains N_{14} and N_{12} number of atoms of ${}_6C^{14}$ and ${}_6C^{12}$ respectively. Obviously, the present number of ${}_6C^{12}$ atoms and the number of ${}_6C^{14}$ atoms must be equal to N_0, the number of ${}_6C^{14}$ atoms present at t = 0 i.e.

$$\mathbf{N_0 = N_{12} + N_{14}} \quad \ldots (25.15)$$

According to radioactive decay, we have,

$$\mathbf{N_{14} = N_0 e^{-\lambda t}} \quad \ldots (25.16)$$

From equations (25.15) and (25.16) we get,

$$N_{14} = (N_{12} + N_{14})\, e^{-\lambda t}$$

$$\text{or} \quad e^{\lambda t} = \frac{N_{12}}{N_{14}} + 1$$

$$\text{or} \quad ln\, e^{\lambda t} = ln\left(\frac{N_{12}}{N_{14}} + 1\right)$$

$$\text{or} \quad \lambda t = ln\left(\frac{N_{12}}{N_{14}} + 1\right)$$

$$\text{or} \quad t = \frac{1}{\lambda}\, ln\left(\frac{N_{12}}{N_{14}} + 1\right)$$

$$\textbf{or} \quad \mathbf{t = \frac{2.303}{\lambda}\, \log_{10}\left(\frac{N_{12}}{N_{14}} + 1\right)} \quad \ldots (25.17)$$

$$\text{where, } \lambda = \frac{0.693}{T_h} \text{ and } T_h = 5730 \text{ yr}$$

Measuring (N_{12}/N_{14}) and decay constant λ, age of the archaeological specimen is determined.

Measurement of radiation dose

Chemical and biological changes in tissue exposed to ionizing radiation depends upon the energy absorbed in the tissue from the radiation, rather than the amount of ionization that the radiation produces in air kept inside the GM tube. Therefore, in the study of biological effect of radiation, new physical parameters are defined. Absorbed dose is a physical quantity which is used in radiodiagnosis and radiotherapy units to measure the biological damage of radiation. Absorbed dose is defined as the quantity of energy absorbed per unit mass by a substance. It is denoted by D.

$$\therefore \quad \text{Absorbed dose (D)} = \frac{\text{Energy absorbed (E)}}{\text{mass (m)}}$$

Its unit is Gray (Gy). One gray (Gy) is defined as the absorption of one joule of energy per kilogram of absorbing material.

i.e. 1 Gy = 1 Jkg^{-1}

Another unit of absorbed does is rad.

1 Gy = 100 rad

Units of radioactivity

In the study of radioactivity, different types of units are used. Some important units that are used to study the various phenomena regarding the radioactivity are given below:

i. Becquerel (Bq) : 1 Bq = 1 disintegration/second

ii. Rutherford (R) : 1 R = 10^6 disintegration/second

iii. Curie (Cu) : 1 Cu = 3.7×10^{10} disintegration/second

25.13 Geiger Muller Counter: A Radiation Detector

Geiger Muller Counter (GM Counter) is an instrument used for detection and measurement of all types of nuclear radiations: α-rays, β-rays and γ-rays. It was invented by German physicist Hans Geiger and Walter Muller. Hans Geiger invented its principle in 1908 and his collaborator Walter Muller developed a technique to produce a physical tube in 1928. Thus, the experimental device was invented from the combined effort of Geiger and Muller, so it was named Geiger Muller tube (GM tube) or GM Counter in the sense that it not only detects the radiation but also counts the number of radiations. This instrument works in the principle of ionizing effect of radiation.

Construction

This instrument consists of two main parts: a tube (GM tube) and a radiation counter (GM counter). GM tube is usually cylindrical in shape with a wire down the centre. The tube is filled with a mixture of inert gases, argon, helium and neon at low pressure about 10 cm of Hg. The centre wire is connected to positive terminal and metallic wall is connected to the negative terminal of a power supply. It is operated at high voltage typically, 400 V – 900 V. If the voltage is too low, the electric field in the tube is too weak to cause a current pulse. If the voltage is too high, the tube will go continuous discharge and the tube can be damaged. Before it is used for the radiation detection, its operating voltage has to be determined. Another part, the counter receives the electric pulse feeds from the tube. It magnifies the pulse and count every pulse sent from the tube. Finally numerical value is displayed in counter.

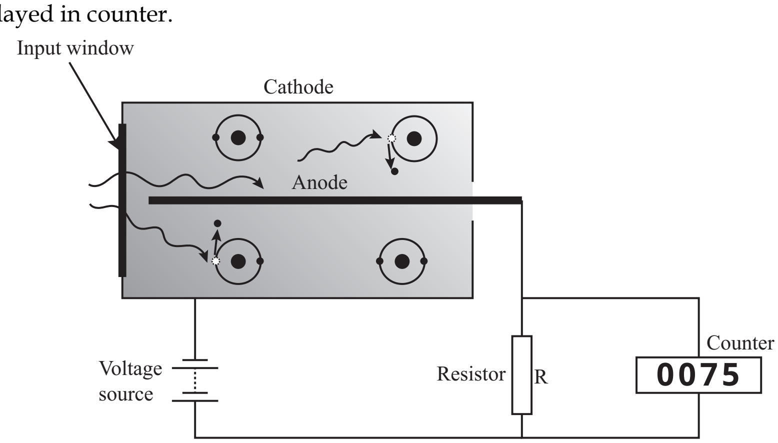

Fig. 25.5: GM counter

Working

When ionizing radiation such as alpha particle, beta particle or gamma photon enters into the tube, it ionizes the gas. From the ionized atom, an electron is knocked out and the remaining part of atom is positively charged ion. High operating voltage in the tube produces electric field into it. The electrons which were knocked out from the atoms are attracted to the positive electrode, and the positively charged ions are attracted to the negative electrode. This produces an electric pulse in the wire connecting the electrodes, and this pulse is counted in the counter section. Ionization of only one or a few atoms cannot produce such significant pulse to be detected. Actually, the incident radiation is captured by an atom of gas and made it ionized. After ionization, negative particle (i.e. electron) travels towards the central wire with high speed. Similarly, the positive ion moves towards the positive terminal. In their movement, they collide with other atoms in their path, which produces the avalanche effect into the tube so that many atoms ionize in very short time. This phenomenon is responsible to produce the electric pulse.

After the pulse is counted, the charged ions become neutralized and GM Counter again ready to record the next count. Thus, the GM Counter detects and measures the intensity of nuclear radiations.

Measurement of operating voltage

Very small voltage across two terminals of the counter produces too small electric field into the tube. In this condition, the ionized atoms again attract the detached electrons and recombine. So, no electric pulse is generated. When the voltage is gradually increased, some of the radiations are counted, but still many ionized atoms recombine, which voltage is still not appropriate to operate the tube. After a certain value of applied voltage, the count rate remains almost constant. This is the voltage for correct count. If the applied voltage is further increased, the counter shows the regular counting whether the radiations enter or not. This happens due to the continuous discharge into the tube. Therefore, for the correct measurement of radiation, the voltage across the tube should be set within the constant count region. In graph, it is called plateau region. The correct operating voltage is estimated from the following formula,

$$V = V_1 + \frac{1}{3}(V_2 - V_1) \qquad \text{... (25.18)}$$

Where, V = operating voltage

V_1 = Voltage of initial point of plateau region

V_2 = Voltage of final point of plateau region

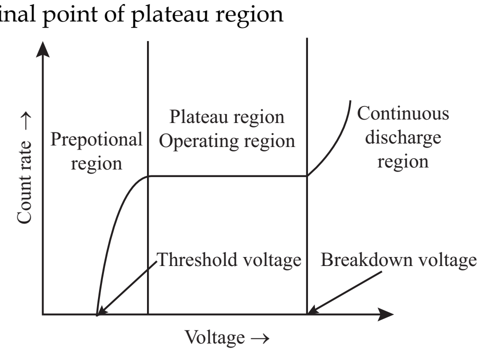

Fig. 25.6: Voltage characteristic of a GM tube

Limitations

There are many limitations of G.M. Counter. Some major limitations are listed below:

i. It counts the same magnitude regardless of the energy of the incident radiation. It means it can not distinguish the radiations of different energy.
ii. It is not able to measure high radiation rate. If two or more radiations enter the tube at a time or within the relaxation time, they are counted only one.
iii. It usually needs high voltage for the operation. However, very high operation voltage also give wrong counting.

25.14 Radiation Hazard

Exposure of high level of radiation causes severe problems in the human health and produces wide range of symptoms. Nausea and vomiting are the initial effects of the radiation exposure. Long term exposure may result in fatal damage of internal organs. Similarly, numerous cases of skin related illness can be observed which leads to skin cancer. Radioactive materials decays producing radiation and breaking the chemical bond that makes up our tissues which damages the DNA. Ultimately damaging cells and in most cases causing deformity. In addition, one of most infamous accident, Chernoby disaster, which occurred due to radiation and other debris, led many people and their next generation causing mutation in gene.

Radio isotopes are produced by radiation and its subsequent absorption, that includes nuclear reaction. Some important radio isotopes are radio cobalt (Co^{60}), radio sulphur (S^{35}) etc.

Fig. 25.7: Sources of Radiation Hazard

Causes of radiation hazard

The pollution caused by radioactive materials/waste emitted from industries and into the atmosphere as a by-product is as radioactive pollution. Radioactive waste includes gamma radiation, burnt fuels, oils, cosmic rays, radioactive minerals. Some causes of radioactive pollution is highly hazardous causing severe damage to the atmosphere and human health as well. Radiation causes the gene to mutate by breaking DNA molecules. Secondly, radioactive pollution causes skin burns which may lead to skin cancer. Moreover, birth defects and cognitive disabilities in children are likely to occur. These radioactive materials aggregate in some animals and are transferred via food chain. So, basically speaking, radioactive pollution affects all the organisms.

Remedial measures of radiation hazard

Safety measures from radiation hazard are briefly explained below:

1. First and foremost precaution one can take is by keeping a radiation source at distant place where people are less exposed.
2. Shielding the radioactive waste by constructing a barrier via concrete lead container which will somehow reduce radiation intensity.
3. The nuclear explosion and other activities which create radioactive waste should be carried out far away from public area.
4. People working around radioactive isotopes should be asked to wear aprons and handle with the help of remote control device.

Tips for MCQs

1. The activity dN/dt of a source is related to the number N of undecayed nuclei by the equation: $dN/dt = -\lambda N$.
2. The number N of undecayed nuclei in a radioactive sample at time t is given by the equation: $N = N_0 e^{-\lambda t}$, Where N_0 is the number of undecayed nuclei at time t = 0.
3. The half-life $T_{1/2}$ and the decay constant λ are related by the equation: $T_{1/2} = 0.693/\lambda$.
4. The relation of mean life and decay constant is $T_m = \frac{1}{\lambda}$

1. Radioactivity is the spontaneous and random process.
2. All the elements with atomic number greater than 82 are naturally radioactive.
3. Properties of α, β and γ-rays:

Rays / Properties	**α-rays**	**β-rays**	**γ-rays**
Nature	Helium nucleus	Fast moving electrons	Electromagnetic wave
Nature of charge	Positive	Negative	Chargeless
Magnitude of charge	3.2×10^{-19} C	1.6×10^{-19} C	Zero
Mass	6.68×10^{-27} kg	9.1×10^{-31} kg	Rest mass is zero
Velocity	$\sim \frac{1}{10}$ of c	1% to 99% of c	c (= 3×10^8 ms^{-1})
Effect of electric and magnetic fields	Deflected	Deflected	Not deflected
Ionizing power	Maximum	Intermediate	Minimum
Penetrating power	Minimum	Intermediate	Maximum

Worked Out Problems

1. A radioactive source decayed to $\frac{1}{128}^{th}$ of its initial activity after 50 days. What is its half life?

SOLUTION

Given,

$\frac{N}{N_0} = \frac{1}{128}$

Time of activity (t) = 50 days

Half life (T_h) = ?

We have,

$N = N_o e^{-\lambda t}$

or, $\frac{N}{N_0} = e^{-\lambda t}$

or, $ln\left(\frac{N}{N_0}\right) = -\lambda t$

or, $\lambda = -\frac{ln\left(\frac{N}{N_0}\right)}{t} = -\frac{ln\left(\frac{1}{128}\right)}{50}$

$\lambda = 0.097\ day^{-1}$

Now,

$T_n = \frac{0.693}{\lambda} = \frac{0.693}{0.097} = 7.14$ days

The half life of given source is 7.14 days.

2. A certain radioactive substance has its half life 10 h. If its initial number is 6×10^{20}. Calculate the decay constant and the number of atoms after 30 h.

SOLUTION

Given,

Half life (T_h) = 10 h

Initial number (N_0) = 6×10^{20}

Total time (t) = 30 h

Decay constant (λ) = ?

Number of atoms after 30 h (N) = ?

We have,

$\lambda = \frac{0.693}{T_h} = \frac{0.693}{10} = 0.0693\ h^{-1}$

Given,

Now, total number of radioactive atoms remaining,

$N = N_o e^{-\lambda t}$

$= 6 \times 10^{20} \times e^{-(0.0693 \times 30)} = 0.75 \times 10^{20}$

Therefore, 0.75×10^{20} radioactive atoms remains after 30 h.

3. How long will it take a sample of radioactive substance to decrease to 20%, if the half life is 4 days?

SOLUTION

Given,

$\frac{N}{N_0} = \frac{20}{100} = \frac{1}{5}$

Half life (T_h) = 4 days

Now, decay constant (λ) $= \frac{0.693}{T_h}$

$= \frac{0.693}{4} = 0.173\ day^{-1}$

Now,

$N = N_o e^{-\lambda t}$

or, $\frac{N}{N_0} = e^{-\lambda t}$

Taking log,

$ln\left(\frac{N}{N_0}\right) = -\lambda t$

$t = -\frac{ln\left(\frac{N}{N_0}\right)}{\lambda} = -\frac{ln\left(\frac{1}{5}\right)}{0.173}$

t = 9.3 days

∴ 9.3 days is required to decrease the sample to 20%.

4. The half life of radium is 1600 years. How long will it take for $\frac{7}{8}$ of a given sample of radium to decay?

SOLUTION

Given,

Half life (T_h) = 1600 years

number of atoms decayed = $\frac{7}{8}$ of N_0

So, number of atoms remaining (N) is N

$$= N_0 - \frac{7}{8}N_0 = \frac{1}{8}N_0$$

$$\frac{N}{N_0} = \frac{1}{8} \quad \dots(i)$$

Now,

$$\text{Decay constant } (\lambda) = \frac{0.693}{T_h}$$

$$= \frac{0.693}{1600} = 4.33 \times 10^{-4} \text{ year}^{-1}$$

Now,

$$N = N_0 e^{-\lambda t}$$

$$\frac{N}{N_0} = e^{-\lambda t}$$

$$ln\left(\frac{N}{N_0}\right) = -\lambda t$$

$$t = -\frac{ln\left(\frac{N}{N_0}\right)}{\lambda} = -\frac{ln\left(\frac{1}{8}\right)}{4.33 \times 10^{-4}}$$

$$= 4802.4 \text{ years}$$

$\therefore$ 4802.4 years is required to decay $\frac{7}{8}$ of a given sample of radium.

5. The half life of a radioactive substance is 1.192×10^7 s against α-decay. Calculate decay rate for 2.0×10^{20} atoms.

SOLUTION

Given,

Half life (T_h) = 1.192×10^7 s

Number of atoms (N) = 2.0×10^{20} atoms

Now,

$$\text{Decay rate}\left(\frac{dN}{dt}\right) = ?$$

We know, $\frac{dN}{dt} = \lambda N$

and $$\lambda = \frac{0.693}{T_h} = \frac{0.693}{1.192 \times 10^7} = 5.81 \times 10^{-8} \text{ s}^{-1}$$

Now,

$$\frac{dN}{dt} = 5.8 \times 10^{-8} \times 2.0 \times 10^{20}$$

$$= 1.162 \times 10^{13}/\text{second.}$$

6. The half life of a radioactive materials is 10 years. How many radioactive atoms will remain after 15 years, if the initial number of atoms are 1.2×10^{21}.

SOLUTION

Given,

Half life (T_h) = 10 years

Time (t) = 15 years

Initial number (N_0) = 1.2×10^{21}

Remaining number (N) = ?

We have, Decay constant $(\lambda) = \frac{0.693}{T_h} = \frac{0.693}{10}$

$$= 0.0693 \text{ year}^{-1}$$

Now, $$N = N_0 e^{-\lambda t}$$

$$= 1.2 \times 10^{21} \times e^{-0.0693 \times 15}$$

$$= 4.24 \times 10^{20} \text{ atoms}$$

7. The half life of strontium 90 is 30 years. The initial number 6.0×10^{12} of radioactive strontium will be disintegrated after 20 years. If the energy emitted per disintegration of nucleus is 1.2×10^{-13} J, calculate the total energy released in 20 years.

SOLUTION

Given,

Half life (T_h) = 30 years

Initial number (N_0) = 6.0×10^{12}

Time (t) = 20 years

Decay number = $N_0 - N$ = ?

Energy released per decay (E_p) = 1.2×10^{-13} J

Total energy released (E) = ?

We know,

$$\text{Decay constant } (\lambda) = \frac{0.693}{T_h} = \frac{0.693}{30} = 0.0231 \text{ year}^{-1}$$

Now,

$$N = N_0 e^{-\lambda t}$$
$$= 6.0 \times 10^{12} \times e^{-(0.0231 \times 20)}$$
$$= 3.78 \times 10^{12}$$

Now, number of nuclei decayed

$$= N_0 - N = 6.0 \times 10^{12} - 3.78 \times 10^{12}$$
$$= 2.22 \times 10^{12}$$

Now, total energy released,

$$E = (N_0 - N)\, E_p = 2.22 \times 10^{12} \times 1.2 \times 10^{-13}$$
$$E = 0.2664 \text{ J}$$

8. The half life of radium is 1620 years. After how many years 25% of a radium block remains un-decayed?

SOLUTION

Given,

Half life (T_h) = 1620 years

N = 25% of N_0

or, $\frac{N}{N_0} = 25\%$

or, $\frac{N}{N_0} = \frac{25}{100}$

We have,

$$\therefore \quad \lambda = \frac{0.693}{T_h} = \frac{0.693}{1620} = 4.28 \times 10^{-4} \text{ year}^{-1}$$

Now,

$$N = N_0 e^{-\lambda t}$$

or, $\frac{N}{N_0} = e^{-\lambda t}$

Taking logarithm on both sides,

or, $ln\left(\frac{N}{N_0}\right) = -\lambda t$

or, $t = -\frac{ln\left(\frac{N}{N_0}\right)}{\lambda}$

or, $t = -\frac{ln\left(\frac{1}{4}\right)}{4.28 \times 10^{-4}}$

$\therefore$ t = 3239 years

9. [HSEB 2063] A sample of Ra-226 has half life of 1620 years. What is the mass of the sample which undergoes 20000 disintegrations per second?

SOLUTION

Given,

Half life ($T_{1/2}$) = 1620 yrs = (1620 × 12 × 30 × 24 × 60 × 60) s = 5.04×10^{10} s

No. of disintegrations per second $\left(\frac{dN}{dt}\right)$ = 20000 dis/s

Avogadro's number (N_A) = 6.02×10^{23} mol^{-1}

we have,

$$\frac{dN}{dt} = \lambda N$$

or, $\frac{dN}{dt} = \frac{0.693}{T_h} \times N$

or, $20000 = \frac{0.693}{5.04 \times 10^{10}} \times N$

$\therefore$ N = 1.45×10^{15} atoms

Again, we have

6.02×10^{23} number of atoms of Ra are present in 226 g of it.

$\therefore$ 1.45×10^{15} number of atoms of Ra are present in $\frac{226}{6.02 \times 10^{23}} \times 1.45 \times 10^{15}$ g

$$= 5.44 \times 10^{-7} \text{ g} = 5.44 \times 10^{-10} \text{ kg}$$

Hence, the required mass of sample is 5.44×10^{-10} kg.

Challenging Problems

1. **[UP]** The isotope $_{38}Sr^{90}$ undergoes β^- decay with a half-life of 28 years. (a) What nucleus is produced by this decay? (b) If a nuclear power plant is contaminated with ^{90}Sr, how long will it take for the radiation level to decrease to 1.0% of its initial value?

   Ans: (a) $_{38}Sr^{90} \longrightarrow {}_{39}Y^{90} + {}_{-1}\beta^0$ (a) 186 years

2. **[UP]** Tritium (H^3) undergoes decay β^- with a half life of 12.3 years. If some tritium gas is released into the atmosphere in a nuclear power plant accident, how long will it take for 90% of the tritium to become non-radioactive?

   Ans: 41.2 yrs

3. **[UP]** The isotope ^{226}Ra undergoes α decay with a half-life of 1620 years. What is the activity of 1.00 g of ^{226}Ra? Express your answer in Bq and in Ci.

   Ans: 3.6×10^{10} Bq, 0.98 *Ci*

4. **[UP]** The wood of a living tree has $_6C^{14}$ activity of 12 disintegrations per minute per gram. An ancient piece of wood of mass 36g shows $_6C^{14}$ activity of 240 disintegration. Estimate the age of the ancident wood. Half life of $_6C^{14}$ = 5730 years.

   Ans: 4860 years

5. If the half life period of a radioactive substance is 2 days, after how many days will $\frac{1}{64}^{\text{th}}$ part of the substance be left behind? [HSEB 2067]

   Ans: 12 days

6. **[ALP]** A radioactive source has decayed to $\frac{1}{128}$ th of its initial activity after 50 days. What is its half life?

   Ans: 7.12 days

7. **[ALP]** The isotope $_{19}K^{40}$ with a half of 1.37×10^9 years, decays to $_{18}Ar^{40}$ which is stable. Moon rocks from the sea of tranquility show that the ratio of these potassium atoms to argon atoms is $\frac{1}{7}$. Estimate the age of the rock.

   Ans: 4.11×10^9 years

8. **[ALP]** A source, of which the half-life is 130 days, contains initially 1.0×10^{20} radioactive atoms, and the energy released per disintegration is 8×10^{-13} J. Calculate (a) the activity of the source after 260 days have elapsed and (b) the total energy released during this period.

   Ans: (a) 1.54×10^{12} dis/sec (b) 6×10^7 J

9. **[ALP]** A small volume of a solution which contained a radioactive isotope of sodium had an activity of 12000 disintegration per minute. When it was injected into the blood stream of a patient. After 30 hours the activity of 1 cm^3 of the blood was found to be 0.5 disintegration per minute. If the half life of the sodium isotope is taken as 15 hours, estimate the volume of blood in the patient.

   Ans: 6000 cm^3

10. **[ALP]** The half life period of the Po-210 is about 140 days. During this period, the average number of α-emission per day from a mass of polonium (Po) initially equal to 1 microgram is about 12×10^{12}. Assuming that one emission takes place per atom and that the approximate density of polonium is 10 gcm^{-3}, estimate the number of atoms in 1 cm^3 of polonium.

    Ans: 3.36×10^{22} atoms

*[**Note:** Hints to challenging problems are given at the end of this chapter.]*

Conceptual Questions with Answers

1. A nucleus does not contain electrons, yet it is ejected them. How?

↳ A neutron in a nucleus can decay into a proton, an electron and antineutrino. This phenomenon takes place for the charge and mass conservation. The electron so emitted is called beta particle. Actually, electron does not remain in nucleus, but it is produced in nuclear decay as shown in the reaction below.

$_0n^1 \longrightarrow {_1p^1} + {_{-1}e^0} + \bar{\nu}$

2. Can radioactivity be controlled?

↳ No, radioactivity is a spontaneous process. It is not affected by external physical conditions like the variation of pressure and temperature. The rate of radioactive decay depends on the number of radioactive nuclei present in a source.

3. Comment on the statement "a nucleus contains no electrons but still eject them".

↳ Heisenberg uncertainty principle confirms that electron does not exist in nucleus. However, in β-decay process, a neutron breaks up into a proton, electron and anti neutron as,

$_0n^1 \longrightarrow {_1H^1} + {_{-1}e^0} + \bar{\nu}$

4. What are the significances of half life?

↳ The value of half life of radioactive isotopes gives an idea of the relative stability of that isotope. An isotope of longer half life is more stable than the isotope with shorter half life.

5. What are the important features of exponential curve of radioactive element?

↳ There are many significances of exponentially decay curve of radioactive element. Some of them are as follows:

i. The number of radioactive nuclei in a radioactive sample decreases exponentially with time. The disintegration is fast in the beginning but becomes slower and slower as the time elapses.

ii. Irrespective of its nature, a radioactive sample will take infinitely long time to disintegrate completely.

iii. Larger the value of decay constant λ, the higher is the rate of disintegration.

6. What is α-decay? Write the α-decay equation?

↳ α-decay is a nuclear process in which an unstable nucleus transforms itself into a nucleus by emitting an α-particle. The α-decay equation is expressed as,

$_ZX^A \longrightarrow {_{Z-2}Y^{A-4}} + \alpha(= {_2He^4}) + Q$

Where, Q is the energy released in radioactive process.

7. Natural radioactive nuclei are the nuclei of high mass number. Why?

↳ Free neutrons are unstable particles. They decay spontaneously with a mean life of about 1000 s. Heavy nuclei contains more neutrons than protons. These neutrons behave as if they are free in some extent. This process involve emission of radiation.

8. Why do all β-particles emitted during beta decay not have the same energy?

↳ In β-decay process, particles like antineutrinos are also emitted along with β-particle itself. The available energy in this process is shared by β-particle and antineutrino in all possible proportions. The energy of β-particle is no longer fixed, it depends on the energy of antineutrino.

9. Why are α-particles emitted rather than protons in radioactivity?

↳ The α-particles have very high binding energy. With the emission of α-particles, the binding energy per nucleus of residual nucleus increases appreciably. However, the emission of proton may not be energetically favourable.

10. Why is it not possible to define total life of a radioactive substance?

↳ Radioactivity is a spontaneous and random process. So, the nucleus can have any value of total life between zero and infinity. So, it is not possible to define the total life of a radioactive substance.

11. What is mean life of radioactive isotopes?

➯ The mean life of radioactive isotope is defined as the ratio of the combined age of all the atoms to the total number of atoms present in the given sample.

$$T_{mean} = \frac{1}{\lambda} = \frac{T_h}{0.693} = 1.44\ T_h$$

12. How does Co^{60} cure the cancer disease?

➯ Co^{60} emits the energetic γ-rays in its disintegration. These γ-rays are exposed to cancerous cells to destroy them. As the infected cells are damaged, the disease is cured.

13. Why are γ-rays also called electromagnetic waves?

➯ γ-rays consist of short wavelength which travel with the speed of electromagnetic waves and show properties similar to electromagnetic waves. So, γ-rays are called electromagnetic waves.

14. Write four properties of γ(gamma) rays?

➯ The main properties of γ-rays are:

i. γ-rays are stream of electrically neutral particles called photons.

ii. The rest mass of photons is zero.

iii. They are electromagnetic waves of small wavelength so, they are not called radioactive particle.

iv. They have the greatest penetrating power among all the nuclear radiations. They can pass even through 5 cm thick sheet of lead or 30 cm of iron.

15. A certain radioactive substance has a half life period of 30 days. What is the disintegration constant?

➯ The half life of radioactive substance, T_h = 30 days

Disintegration constant (λ) = $\frac{0.693}{T_h} = \frac{0.693}{30}$ = 0.023 day^{-1}.

16. Define 'disintegration constant' and 'half life' of a radioactive substance.

➯ Reciprocal of the time interval during which the number of radioactive nuclei in a given sample reduces to 36.8% of its initial value is known as disintegration constant. It is denoted by λ. It's unit is per second (s^{-1}).

The time interval during which half of the initial number of radioactive nuclei disintegrates from its initial value is known as half life. It is denoted by T_h. The relation between half life and disintegration constant of a radioactive sample is,

$$T_h = \frac{0.693}{\lambda}$$

17. Why α-particles have a high ionizing power?

➯ α-particles are massive particles that are produced during radioactive disintegration. When these massive particles are incident on any material, they imparts large amount of energy to the material medium. So, the particles in the medium are highly ionized, when α-particle travels.

18. Which are more penetrating α-rays, β-rays or γ-rays?

➯ γ-rays are the most penetrating rays among α-rays, β-rays and γ-rays. γ-rays are electromagnetic radiation with very short wavelength, so that these rays rarely interact with particles in their path. Since these rays rarely interact with the particles in a medium, they can penetrate easily. The rays which have low ionizing power have high penetrating power.

19. How do the atomic number and atomic mass number change when an α-particle is emitted out from a radioactive material.

➯ During a emission of α-particle, atomic number is reduced by number 2 and atomic mass number is reduced by number 4. In terms of nuclear equation,

$${}_{Z}X^{A} \longrightarrow {}_{Z-2}Y^{A-4} + {}_{2}He^{4}\ (\alpha\text{-particle})$$

20. What is the difference between natural radioactivity and artificial radioactivity?

➥ The radioactivity occurring in natural materials is known as natural radioactivity. For example, the carbon dating in a fossil is an example of natural radioactivity.

The radioactivity occurring in the sources which are formed artificially is known as artificial radioactivity. The radioactive isotope Co^{60} can be formed artificially for the cancer treatment.

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21. What are the medical use of radioactive isotopes?

➥ Radioactive isotopes are used to diagnose and treat many diseases:

a. **Radiodiagnosis:** The detection of causes of diseases using radioactive isotopes is known as radiodiagnosis. Some applications of radioactive isotopes in radiodiagnosis are as follows:

i. Radioactive mercury (Hg-203) is used to detect kidney and liver functions.
ii. Radioactive iodine (I–131) is used to study the thyroid functions.
iii. To detect the haemorrhage location in human body, radio chromium (Cr - 51) is used.

b. **Radiotherapy:** The treatment of diseases using radioactive isotopes is known as radiotherapy. Some applications of radio isotopes in radiotherapy are as follows:

i. Co^{60} isotopes are used to destroy the cancerous tissues.
ii. I^{131} isotopes are used to destroy the overactive thyroid gland.
iii. Radiophosphorous, radiogold are used to cure the leukemia.

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22. What is radio carbon dating?

➥ Radio carbon dating is the technique of aging archaeological specimens from the examination of radioactive carbon isotopes. A carbon isotopes C^{14} is the major source of carbon dating. The rate of disintegration of such radioactive carbon isotopes in the fossils, woods, rocks or parts of meteorite are observed to find their age.

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23. What is the difference between an electron and a beta particle?

➥ An electron is identical to the beta particle. They have equal mass, charge and some properties. But they have different origin. Beta particle is emitted from the nucleus in nuclear reaction or in radioactivity, but the electron are the extra-nuclear particles.

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24. Write units regarding radioactivity.

➥ Becquerel, Rutherford and Curie are commonly used units in radioactivity.

i. Bacquerel (Bq) : 1 Bq = 1 disintegration/second
ii. Rutherford (R) : 1R = 10^6 disintegration/second
iii. Curie (Cu) : 1 Cu = 3.7×10^{10} disintegration/second

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25. The half life of radium is 1600 years. Calculate its disintegration constant.

➥ Given,

Half life (T_h) = 1600 years

Decay constant (λ) = ?

We know,

$$\lambda = \frac{0.693}{1600} = 4.33 \times 10^{-4} \text{ year}^{-1}$$

Exercises

Short-Answer Type Questions

1. What is meant by radioactivity?
2. Differentiate between natural and artificial radioactivity?
3. What are the laws of radioactive disintegration?
4. What do you mean by half life of radioactive source? Relate it with of decay constant.
5. What are the uses of radiation and radioactive isotopes?
6. Explain briefly carbon dating.

7. Discuss briefly about the harmful aspects of radiation.
8. What is meant by radioactive isotope?
9. Differentiate between natural and artificial radioactivity?
10. What are the properties of α, β and Υ rays?
11. What are the laws of radioactive disintegration?
12. What do you mean by half life of radioactive source? Relate it with of decay constant.
13. What are the uses of radiation and radioactive isotopes.
14. What is natural radioactivity?
15. Does a nucleus emit α, β and γ–ray at the same time? Explain.
16. What are alpha rays? Write down their some properties.
17. Why does a nucleus emit a gamma ray photon?
18. What is the difference between a gamma ray photon due to jumping of electron from one orbit to another and a gamma ray photon due to radioactive decay?
19. Explain why the β-decay of a free proton is not possible but that a proton bound in the nucleus is possible?
20. What happens to the atomic number and mass number of an element when it emits an α–particle?
21. What is β – decay? What happens to the atomic number and mass number of an element when it emits a β– particle?
22. Define decay constant or disintegration constant.
23. Define activity. Give its unit.
24. On what factor does activity of a radioactive substance depends?
25. What are radioisotopes?
26. What are the uses of radioisotopes?

Long-Answer Type Questions

1. What is radioactivity? Obtain $N = N_0e^{-\lambda t}$ in radioactive decay law. Describe the significance of decay curve.
2. State the law of radioactive disintegration. Derive a relation between half life and decay constant. [NEB 2075]
3. Explain what is meant by radioactivity and half-life. How are the atomic number and mass number of a radioactive nucleus change by the emission of (i) alpha particle (ii) beta particle and (iii) a gamma ray?
4. What is artificial radioactivity? What are radioisotopes? How are they produced?
5. What is radiocarbon dating? How would you estimate the age of an ancient object?
6. Describe the construction and working principle of GM counter.
7. What is meant by radiation pollution? Describe its remedies.

Numerical Problems

1. A radioactive substance is disintegrated to $\left(\frac{1}{128}\right)^{th}$ of its initial value in 30 h. Find its half life.

   Ans: 4.28 h

2. The decay rate of a radioactive materials is 4.88×10^{-18} s^{-1}. Calculate its half life.

   Ans: 1.42×10^{17} s

3. A radioactive source has half life 10 days. Calculate the disintegration rate of 3.7×10^{16} atoms of the materials.

Ans: 2.564×10^{15}/day

4. 75% of a radioactive element disintegrates in 24 years. Calculate the half life of the element.

Ans: 12 years

5. The activity of radium decreases about 1% every 25 years, compute the half life.

Ans:3.76 years

6. The isotope ^{226}Ra undergoes α-decay with a half-life 0f 1620 years. What is the activity of 1.00 g of ^{226}Ra? Express your answer in Bq and in Ci.

Ans: 0.98Ci

7. At certain instant a piece of radioactive element contains 10^{12} atoms. The half life of the material is 15 days. Calculate the rate of decay after 30 days have elapsed.

Ans: 4.15×10^{11} decay/day

8. The half-life of thorium is 1.4×10^{10} years. Find the time required for 15% of a sample of thorium to disintegrate.

Ans: 3.28×10^{9} years

9. Find the half-life and average life of a radioactive sample whose disintegration constant is 25.72×10^{-3} per day.

Ans: 121.15 days, 174.82 days

10. A radioactive material of mass 10 mg with a half-life period of two years is kept in store for six years. How much of the material remains unchanged?

Ans: 1.25 mg

11. The half-life of radium is 1620 years. After how many years 25% of radium block remains undecayed?

Ans: 3240 years

12. Determine the half-life of a radioactive material if its activity falls to $(1/16)^{th}$ of its initial value in 30 years.

Ans: 7.5 years

13. The half-life of radon is 3.8 days. After how many days will only one – twentieth of a radon sample be left over?

Ans: 16.42 days

14. The half-life of U^{238} against alpha decay is 1.42×10^{17} s. How many disintegrations per second occur in 1 g of U^{238}. [Avogadro's number = 6.02×10^{23} mol^{-1}]

Ans: 1.23×10^{4} dis per second

15. At a certain instant, a piece of radioactive material contains 10^{12} atoms. The half life of the material is 30 days. (i) Calculate the number of disintegration in the first second. (ii) How long will elapse before 10^{4} atoms remain? (iii) What is the count rate at this time?

Ans: (i) 2.7×10^{5}, (ii) 797 days (approx.), (iii) 9.6/h

16. A radioactive material has a half life of T_h years. After how much time is its activity reduced to 1% of original value?

Ans: 6.65 years

17. The activity of 1g of $_{92}U^{235}$ is 1 curie. What is its half life?

Ans: 4.8 × 108 sec

Multiple Choice Questions

1. The time taken to reduce a substance to $(1/8)^{th}$ of the original value is 6 days. Its half life is:
   a. 2 days
   b. 4 days
   c. 12 days
   d. 3/4 days
2. The mass of a radioactive salt of half life 2 days is 10 g. What amount of the salt will be left after 10 days?
   a. 2.54 g
   b. 5 g
   c. 6.24 g
   d. 0.31 g
3. The half-life of a radioactive substance is 2 months, then the amount of substance left after 1 year is:

a. $\frac{M}{64}$ b. $\frac{M}{32}$

c. $\frac{M}{16}$ d. $\frac{M}{8}$

4. The radioactive substance after 90 days reduces to 12.5%, then find the decay constant of substance

a. 0.3845/day b. 0.0234/day

c. 0.467/day d. 0.0467/day

5. Which of the following rays has the highest ionising power?

a. α-rays b. β-rays

c. γ-rays d. Visible light

6. Which of the following is α particle?

a. $_1H^2$ b. $_2He^3$

c. $_2He^4$ d. $^3Li^{-6}$

7. Initial mass of a radioactive sample of half-life 6 hours is 0.8 kg. The amount of the sample left after 1 day (24 hours) is:

a. 0 b. 50 g

c. 100 g d. 200 g

8. The half-life of a certain radioactive element is such that 7/8 of a given quantity decays in 12 days. What fraction remains undecayed after 24 days?

a. 0 b. $\frac{1}{128}$

c. $\frac{1}{64}$ d. $\frac{1}{32}$

9. The half-life of ^{215}At is 100 μs. The time taken for the radioactivity of a sample of ^{215}At to decay to 1/16th of its initial value is

a. 400 μs b. 6.3 μs

c. 40 μs d. 300 μs

10. A radioactive material has a half-life of 10 days. What fraction of the material would remain after 30 days?

a. 0.5 b. 0.25

c. 0.125 d. 0.33

11. A radioactive isotope has a half-life of 2 yr. How long will it take the activity to reduce to 3% of its original value?

a. 4.8 yr b. 7 yr

c. 10 yr d. 9.6 yr

12. The fraction of the radioactive sample that will remain undecayed after 4 half-life periods is

a. $\frac{1}{2}$ b. $\frac{3}{4}$

c. $\frac{15}{16}$ d. $\frac{1}{16}$

Answers

1. (a) 2. d 3. (a) 4. (b) 5. (a) 6. (c) 7. (b) 8. (c) 9. (a) 10. (c) 11. (c) 12. (d)

Hints to Challenging Problems

HINT: 1

a. In β decay,

$$_{38}Sr^{90} \longrightarrow {}_{39}Y^{90} + {}_{-1}\beta^{0}$$

b. Half life $T_{1/2}$ = 28 yrs.

t = ?

By question,

$$\frac{N}{N_o} = 1\% = \frac{1}{100}$$

$\therefore$ $N_o = 100 \times N$

We know that

$$N = N_o e^{-\lambda t}$$

or $N = 100 \times N e^{-\lambda t}$

or $t = \dfrac{ln\, 100}{\lambda}$

HINT: 2

Half life $(T_{1/2})$ = 12.3 yrs

Since, 90% of tritium is non-radioactive so 10% is radioactive.

$\therefore$ $\dfrac{N}{N_o} = 10\% = \dfrac{10}{100} = \dfrac{1}{10}$

$\therefore$ $N_o = 10 \times N$

We know that

$$N = N_o e^{-\lambda t}$$

or $N = 10 \times N e^{\frac{-0.693}{T_{1/2}} \times t}$

or $\dfrac{1}{10} = e^{\frac{-0.693}{12.3} \times t}$

Then, find t.

HINT: 3

Given,

$T_{1/2}$ = 1620 years

226 g of Ra contains = 6.0×10^{23} atoms

$\therefore$ 1 g of Ra contains = $\dfrac{6.02 \times 10^{23}}{226}$ atoms

i.e. $N = \dfrac{6.02 \times 10^{23}}{226}$ atoms

The activity is,

$$\frac{dN}{dt} = \lambda N$$

HINT: 4

Given,

$\dfrac{dN_0}{dt}$ = 12 dis. per min per gram. . . . (i)

$\dfrac{dN}{dt} = \dfrac{240}{36} = \dfrac{20}{3}$ dis. per min per gram. . . . (ii)

Age of ancient wood, t = ?, $T_{1/2}$ = 5730 yrs

From (i) and (ii), we have

$$\therefore \quad \frac{\frac{d.N_0}{dt}}{\frac{dN}{dt}} = \frac{12 \times 3}{20} = 1.8$$

or $\dfrac{-N_0\lambda}{-N\lambda} = 1.8$

or $\dfrac{N_0}{N_0 e^{-\lambda t}} = 1.8$

HINT: 5

Given,

Half life, $T_{1/2}$ = 2 days

Required time, t = ? when $\dfrac{N}{N_0} = \dfrac{1}{64}$

Now,

$$\frac{N}{N_0} = \frac{1}{64}$$

or $\dfrac{N_0 e^{-\lambda t}}{N_0} = \dfrac{1}{64}$

HINT: 6

According to questions,

$$\frac{dN}{dt} = \frac{1}{128} \times \frac{dN_o}{dt}$$

or $-N\lambda = \dfrac{1}{128} \times -\lambda N_o$

or $\dfrac{N}{N_o} = \dfrac{1}{128}$

But, $N = N_o e^{-\lambda t}$ so we can write

$$\frac{N_0 e^{-\lambda t}}{N_0} = \frac{1}{128}$$

HINT: 7

Given,

$T_{1/2} = 1.37 \times 10^9$ years

$\frac{N}{N_o} = \frac{1}{8}$ [since out of 8 parts (1 + 7), 1 part is Ar and 7 parts is K, so the fraction remain is $\frac{1}{8}$]

Now, $\frac{N}{N_0} = \frac{1}{8}$

or $\frac{N_0 e^{-\lambda t}}{N_0} = \frac{1}{8}$

HINT: 8

Given,

$T_{1/2} = 130$ days

$N_o = 1.0 \times 10^{20}$

$E = 8 \times 10^{-13}$ J

a. $\frac{dN}{dt} = ?$

$t = 260$ days

We know that

$\frac{dN}{dt} = \lambda N = \lambda N_0 e^{-\lambda t}$

b. Number of atoms decayed = $N_0 - N$

Hence, total energy released

$= (N_0 - N) \times 8 \times 10^{15}$

HINT: 9

Given,

$t = 30$ h

$T_{1/2} = 15$ h

Let V cm^3 be the volume of blood.

$\frac{dN_o}{dt}$ for volume Vcc = 12000 dis/min

$= \frac{12000}{60}$ dis/s

$= 200$dis/s

$\therefore \frac{dN_o}{dt}$ for 1 $cm^3 = \frac{200}{V}$ dis/s . . . (i)

Also,

After 30 hours,

$\frac{dN}{dt}$ for 1 cm^3 = 0.5 dis/min $= \frac{0.5}{60}$ dis/s

$\therefore \frac{dN}{dt} = \frac{1}{120}$ dis/s . . . (ii)

From (i) and (ii), we get

$$\frac{\frac{dN_o}{dt}}{\frac{dN}{dt}} = \frac{200/V}{1/120}$$

or $\frac{-\lambda N_o}{-\lambda N} = \frac{200 \times 120}{V}$

or $\frac{N_o}{N} = \frac{24000}{V}$

or $\frac{N_0}{N_0 e^{-\lambda t}} = \frac{24000}{V}$ $(\because N = N_0 e^{-\lambda t})$

$e^{\lambda t} = \frac{24000}{V}$

And find volume (V) of the blood.

HINT: 10

Given,

$T_{1/2} = 140$ days

mass, $m = 10^{-6}$ g

density, $\rho = 10$ g/cm^3

$\therefore V = \frac{m}{\rho} = \frac{10^{-6}\ g}{10\ g/cm^3} = 10^{-7}\ cm^3$

During half life average number of α - emission per day = 12×10^{12} atoms so,

Initially the number of atoms = $140 \times 2 \times 12 \times 10^{12}$ atoms

i.e., 10^{-7} cm^3 contains $140 \times 2 \times 12 \times 10^{12}$ atoms

$\therefore$ 1 cm^3 contains $140 \times 2 \times 12 \times 10^{12} / 10^{-7}$

$= 3.36 \times 10^{22}$ atoms

❖ ❖ ❖

NUCLEAR ENERGY AND OTHER SOURCES OF ENERGY

26
CHAPTER

26.1 Introduction

The advancement of science and technology has direct impact on the society. The contribution of every disciplines of science on the society can not be ignored. Out of many branches of science, physics contributes the major role in the society and the advancement of the world as a whole. Most of the developments made in the field of physics have a direct impact on the society. Some important impacts on society are: exploration of new source of energy, computational technology, means of transportation, development of radio, television, telephone and satellites etc. These inventions have made the world a very narrow and a comfortable place for the human habitation. However, there are equally and in-ignorable dark sides that at any time could lead to a devastating effect to the inhabitants of this earth. The environment of the world has been deteriorating day by day due to the consumption of fossil fuels, destruction of natural resources and development of nuclear weapons. This chapter focuses particularly the various types of energy, their impact on the society and the negative impacts created on the natural environment and the society by the experimentation and inventions done on the field of physics.

26.2 Energy and Energy Sources

Energy is usually defined as the ability to do work. More generally, energy is a fundamental entity whose availability and flow are required for all phenomena, natural or artificial. Energy exists in various forms; mechanical, heat, light, sound, chemical, electric, magnetic and atomic energy. It is a conserved quantity. All the energy in nature cannot be used for useful work. To exploit more energy from nature, more advance technologies are be required.

There are many sources of energy. The sun is the major and common source of energy. The sun provides us the heat and light energy. Heat and light received from the sun transforms to other forms of energy like chemical energy, electric energy, magnetic energy, etc. There are some other sources of energy. They are:

(a) wind (b) moving water (c) fuels (wood, coal, oil, natural gas) (d) nuclear fuels (e) geothermal energy (f) biomass, etc.

26.3 Conservation of Energy and Degradation of Energy

Conservation of energy

The conservation of energy refers that the total energy of an isolated system remains constant. Total energy is conserved over time. More general definition of this law is "energy can neither be created nor be destroyed; rather it can only be transformed from one form to another." When we use energy, we do not use it completely. We just change its forms. We can transform one form of energy to another more useful form. For example, the kinetic energy of flowing water is no more useful as it is, but when this mechanical form of energy is changed into electric energy, it will be useful for many purposes. Similarly, a car engine burns gasoline, converting the chemical energy into mechanical energy that makes the car move. Also, solar cells change radiant energy into electric energy. In all above examples, energy can change forms, but the total energy in the universe remains the same.

It is well known fact that, energy cannot be destroyed. Someone may think, why we worry in energy crisis if energy is conserved forever. But the main challenge is, how it can be converted into workable form. The conservation phenomenon of energy is not sufficient to prevent against energy crisis, rather it is necessary to enhance the efficiency of engines to perform mechanical work.

Degradation of energy

The available energy of a system decreases as its temperature or pressure decreases and approaches equilibrium to the surroundings. When the heat is transferred from a system, its temperature decreases and hence the quality of its energy deteriorates. While the first law states that energy is always conserved quantity wise, the second law emphasizes that energy always degrades quality wise.

In nature, all physical operations are irreversible. In every physical process, a certain quantity of energy is wasted in nature in terms of friction, thermal conduction and radiation. In this way, all the energies existing in different forms will be gradually converted into heat energy and it is impossible to convert into mechanical work. Therefore, useful energy of the universe will tend to be zero. This makes the thermal equilibrium in all bodies in the universe so that no heat flows would be possible. Hence no heat engine works. In this condition, although total energy of the universe remains conserved, the useful energy will be vanished. This is called degradation of energy or heat death.

Transformation of energy

Energy transformation is the process of changing energy from one of its forms into another. Energy transformations occur everywhere every second at the day. A ball held at high point at rest has only potential energy. When the ball is released from the height, then the potential energy of the ball is gradually transformed into kinetic energy.

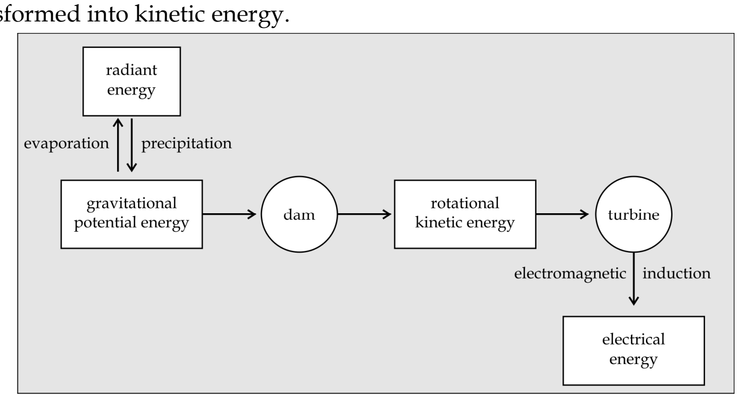


Fig. 26.1: Transformation of energy

26.4 Global Energy Consumption Pattern

Global energy consumption is the total energy used by the entire human civilization. Human beings consume energy from both renewable sources and non-renewable sources. Fossil fuels, coal, natural gases and nuclear power are some examples of non-renewable sources of energy. The energy sources of non-renewable energy are depleting day by day. Hydro-power, wind energy, solar energy and biomass energy are some renewable sources of energy. Global community usually chooses the efficient source of energy whether it be renewable or non-renewable, but the time has been alarming us to reduce the consumption of non-renewable energy source and to develop the technology for the consumption of renewable sources. There is a strong relationship between energy consumption and economic growth.

Energy consumptions by different countries reflect their income level and climate. Large differences exist in terms of energy consumption between some of the most developed countries and underdeveloped countries. The extent of energy consumption by a country is measured in terms of energy per capita. The energy per capital of United State is greater than the countries like Germany and Japan. China is currently the largest primary energy consumer in the world, however per capita energy consumption in China is less than USA. Due to the advancement in new technology and population growth, the energy consumption scenario has been observed increasing, however the global needs rise more slowly than in the past. But it has been speculated that the consumption still expands by 30% between today and 2040. A global economy growing at an average rate of 3.4% per year, a population that expands from 7.4 billion today to more than 9 billion in 2040, and a process of urbanization that adds city are the key factors which enhances the energy consumption in the world.

World Economic Outlook (WEO) has reported some important facts analyzing the energy consumption trend of the world.

1. Global primary energy consumption increased by 1% in 2016, following growth of 0.9% in 2015 and 1% in 2014. This compares with the 10-year average of 1.8% a year.
2. As was the case in 2015, growth was below average in all regions except Europe and Eurasia. All fuels except oil and nuclear power grew at below average rate.
3. Energy consumption in China grew by 1.3% in 2016.
4. Global Oil consumption growth averaged 1.6 million barrels per day or above in its 10 years average.
5. World's natural gas consumption grew by 63 billion cubic metre or 1.5% slower than 10 years average of 2.3%.
6. Global coal consumption fell by 53 million tonnes, which is just 1.7 percent of the equivalent oil consumption annually.
7. Renewable power (excluding hydro) grew by 14.1% in 2016, below the 10-year average.
8. In renewable power supply, hydroelectricity is in the first, wind power in second and solar power in the third.
9. Global nuclear power generation was increased by 1.3% in 2016, comparing the previous 10 years average.
10. Hydroelectric power generation rose by 2.8% in 2016.

26.5 Energy use in Nepal

Nepal is a underdeveloped country. It consumes a small fraction of total energy consumption in global scenario. It has no known reserves of gas, coal and oil. Although its most significant energy resources is water, less than one percent of the potential 84,000 MW of hydropower is currently harnessed. However, it does not yet have a strategy for sustainable efficient energy use for either the electricity sector, like hydro-electricity, wind energy and solar energy, or its main primary energy source, bio-mass.

Biomass and biogas are the most important primary energy source in Nepal. Biomass comprises forest, wood, agriculture resides and dung. More than 95% of biomass is used for cooking and heating purposes in households. About 1.2 households owe cattle and buffaloes. The technical biogas potential is therefore very high. The majority of households in remote villages use biogas plants for cooking and electricity.

International Hydropower Association has recently reported that Nepal's theoretical hydropower potential has been estimated to be around 84,000 MW of which 43,000 MW has been identified as economically viable. Currently Nepal's installed hydropower capacity is about 100 MW, coming from 88 hydropower plants across the country. 441 MW has been produced by 60 hydropower plants owned by independent power producers. The latest report of March 2018, 113 hydropower plants are under construction. These plants will have a combined capacity of 3090 MW once completed.

In many rural area, solar energy production is one of the efficient method of power production in Nepal. Most of the part in Nepal is occupied by hilly region, where the efficiency of solar radiation is very high. Private installations of solar panels are more frequent in Nepal. Recently, the solar panels are installed and whole energy supply is fulfilled by solar energy in office in Singh Durbar.

26.6 Nuclear Energy

The energy that is sourced from the atomic nucleus is called the nuclear energy. Protons and neutrons are tightly bound in a nucleus. If the nucleus is spitted up into two or more nuclei or light nuclei are fused together, energy is released in the form of heat energy. Sometimes, the nuclear reaction occurs in a heavy nucleus when bombarded with a light particle, it splits into two or more lighter nuclei. This process is called nuclear fission. The energy released from nuclear fission can be used to drive a turbine and generate electricity. In other situation, two or more light nuclei fuse together to form a relatively heavy nucleus. This is the process of nuclear fusion, nuclear fusion reaction takes place in the sun and other stars, releasing enormous energy. However, artificial energy production by nuclear fusion has not succeeded yet.

All the nuclear power stations currently operating in the world use the process of nuclear fission, and most use uranium as their main fuel. U-235 source is used in nuclear power plants. When the nucleus of a U-235 atom is hit by a slow neutron, it splits up into two almost identical and lighter nuclei and in the process releases a large amount of energy and more neutrons. The energy released in this process is absorbed as heat by coolant, and then produces steam that drives a turbine in electric generator. Some of the neutrons hit more U-235 nuclei and so keep the fission process going; the others are absorbed by the controlled rods.

The total mass of the products of the reaction (fission products and neutrons) is minutely less than the original mass of the original nucleus and impacting neutron. The difference of mass is converted into energy according to Einstein's mass-energy equivalence relation, $E = mc^2$. Most of the energy is carried by fission products while they collide with nearby atoms and quickly loss most of their kinetic energy into heat energy. In nuclear power plants, this energy is used to generate electricity.

A nuclear power plant comprises a number of systems and components, including the reaction for itself, that together are designed to harness and control the energy of nuclear fission, and to turn it into electricity. Nuclear power plants use a nuclear fission. Nuclear fusion has the potential to be safer but has not yet been developed. Though there are many types of nuclear reactors, they have several components; fuel, moderator, coolant and control rods. The schematic diagram is shown in Fig. 26.2.

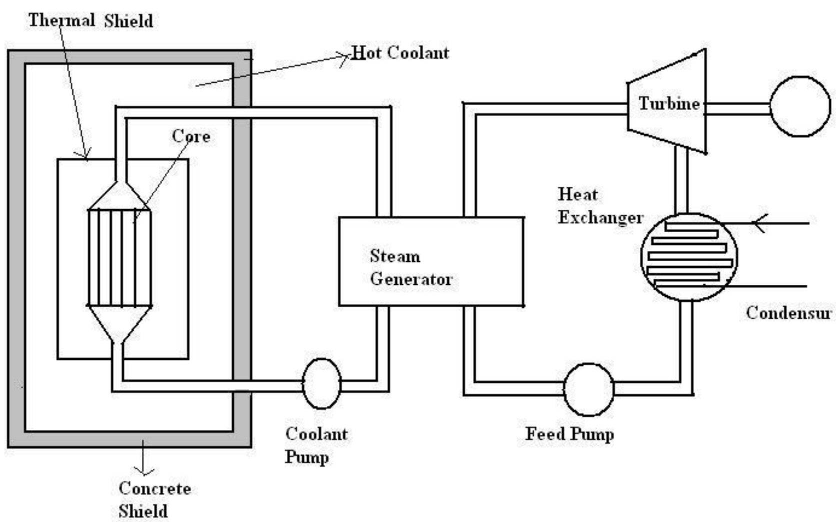


Fig. 26.2: Nuclear reactor

Advantages of nuclear energy source

1. A nuclear power station uses a steam turbine and generator to produce electricity in exactly the same way as any other thermal power station.
2. It does not produce carbon dioxide and other pollutants that are formed when things are burnt.
3. It is more clean and more environment friendly than coal, oil or gas fire power stations.
4. It is compact, competitive and practically inexhaustible.
5. Its efficiency is relatively high and a best alternative source of energy at the location where hydropower plant is impossible.

Disadvantages of nuclear energy source

1. The nuclear fission creates materials that are still radioactive and harmful to human beings and to the environment for thousands or even millions of years.
2. Another problem is regarding the safe way to store the waste.
3. Accidents at nuclear power stations are very rare, but when they occur, they can be catastrophic.
4. Steam coming out of the turbine is also still hot, and so adds heat to the environment.

26.7 Renewable Energy and Nonrenewable Energy

Renewable Energy

The energy which is generated from those energy sources that the continuously replenished by nature is called renewable energy. The major sources of renewable energy sources are the sun, the wind, the water, the earth's heat, and plants. Renewable energy technologies are used to produce usable energy from these sources. Hydropower is the most developed and largest source of renewable energy. Solar energy, wind energy, geothermal energy, biomass energy are other alternative sources of renewable energy.

Advantages

a. The sun, wind, geothermal, ocean energy are available in the abundant quantity and free to use.
b. Renewable sources have low carbon emissions, therefore they are considered as green and environment friendly.
c. Renewable helps in stimulating the economy and creating job opportunities. The money that is used to build these plants can provide jobs to thousands to millions of people.
d. We don't have to rely on any third country for the supply of renewable sources as in case of non-renewable sources.
e. Renewable sources can cost less than consuming the local electrical supply.
f. Solar energy is renewable, non-polluting and relatively maintenance free.
g. Wave and tide is a non-polluting source of energy. Wave turbines are relatively quiet to operate and do not affect wildlife.

Challenges

a. It is not easy to set up a plant as the initial costs are quite steep.
b. Solar energy can be used during the day time and not during night or rainy season.
c. Geothermal energy which can be used to generate electricity has side effects too. It can bring toxic chemicals beneath the earth surface onto the top and can create environmental changes.
d. Hydroelectric energy provides pure form of energy but building dams across the river which is quite expensive can affect natural flow and affect wildlife.
e. To use wind energy, we have to rely on strong winds, therefore we need to choose suitable site to operate them.

Hydro Electricity

Hydropower plants are constructed to generate the hydroelectricity. The power plants convert the energy in flowing water into electricity. The most common form of hydropower uses a dam on a river to retain a large reservoir of water.

Energy from the sun evaporates water in the water resources like Earth's ocean, rivers, lake, etc and draws it upward as water vapour. When water vapour rises up at the cooler air, mostly the mountainous region, it condenses and forms clouds. The moisture eventually falls to the earth as rain and snow, replenishing the water in the oceans and rivers. Gravity drives the moving water, transporting it from high ground to low ground. The force of moving water can be extremely powerful. The fall and movement of water is part of a continuous natural cycle called the water cycle. Thus, hydropower is called a renewable energy source.

Nepal is rich in water resources. Till date, Kali Gandaki A hydropower project is the largest, which produces 144 MW power. Besides this, Middle Marsyangdi 70 MW, Marsyandi 69 MW, Kulekhani I 60 MW, Upper Mersyandi A 650 MW are the major hydropower in Nepal. Upper Tamakoshi project is the largest hydropower project which is under construction, its capacity is 456 MW. Hydropower supplies 19% of all electricity in the world.

Solar Energy

Human beings and all living beings have been using heat and light energy from sun, since the origin of life in the world. So, we use solar energy directly or indirectly in every moment of our life. But, the solar energy which we are discussing here is quite different that how solar technology taps the infinite power of the sun to convert it into electricity. The electricity produced from solar energy can, then, be used in the form of heat, light etc.

Solar panels are made up of photovoltaic (PV) cells, when more solar energy is generated it can be stored in battery as DC electricity. Which convert sunlight into direct current (DC) electricity. Then, the inverter converts the DC electricity generated by solar panels into the alternating currents (AC) electricity. Thus, the electric panels send power to our light bulbs and electric appliances. It is the clean sources of energy. It is the free source of energy that is sustainable and totally inexhaustible, unlike fossil fuels that are finite. It does not emit any greenhouse gases when producing electricity.

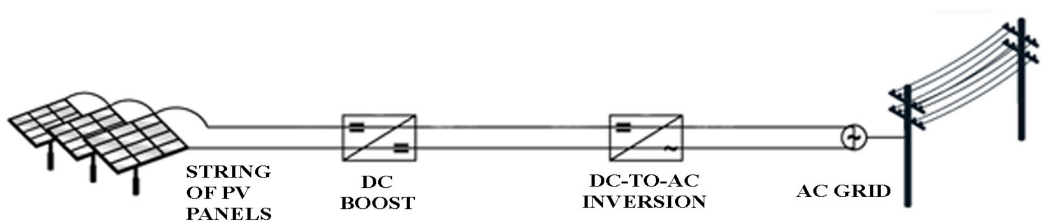


Fig. 26.3: Solar device

The most advantage using solar energy is that, it is distributed over a wide geographical area, ensuring that developing countries like Nepal have access to electricity generation at a stable cost for the long term future. Solar energy production is beneficial in many places of Nepal. Since the device is portable to carry in hilly regions. This technology has been popular in remote villages of Nepal. Nowadays, it is also popular in urban areas. Recently, the solar power has been installed in every office into Singh Durbar premises. First time, the Nepal Electricity Authority (NEA) has been installing solar energy in the national grid line.

Wind Energy

Moving air is called wind. When air is heated by solar heat, it rises up in the atmosphere. The place from where the air rises becomes partially vacant hence the air pressure decreases. Then, the air nearby the partially vacant region blows towards this region, resulting winds of various speeds. More precisely, during the day, the air above the land heats up more quickly than the air over water. The warm air over the land expands and rises, and the heavier, cooler air rushes in to take its place, creating wind. At night, the winds are reversed because the air cools more rapidly over land than over water.

The moving air molecules possess kinetic energy. The combined kinetic energy of all air molecules is exploited to rotate the wind turbines. In this process, kinetic energy is converted into the mechanical work to rotate turbines, powering a rotor inside the generator and producing electricity.

Wind energy is a renewable energy, so it will never run out. While rotating turbines, wind losses the kinetic energy but the air molecules are not destroyed. The air once used, can be used again and again. It does not generate green house gases or heat emission and other pollutants. Once a wind turbine has been constructed, the land can still be used for other purposes, such as farming or agriculture. Besides these advantages, it has some disadvantages. Wind forms produce the noise, sometimes intolerable to its surroundings. Also, it covers large area and highly visible. So, many people dislike it. Wind energy currently generates only 1% of all electricity on global scale. In Nepal, it is not efficient as solar energy and hydroelectric energy.

Biomass Energy

Biomass energy is the conversion of biomass/organic material and collection and storage of the sun's energy (through photosynthesis) into useful forms of energy such as heat, electricity and liquid fuels. Various processes used for conversion of biomass into energy are:

1. Direct combustion
2. Thermo-chemical conversion
3. Bio-chemical conversion

Biomass energy is another form of renewable energy. Biomass energy source is derived from organic matters such as wood, crop waste or garbage. Biomass is a renewable energy source because its supplies are not limited. Trees and crops can always be grown, and waste will always be existed.

Biomass receives its energy from the sun. In the process of photosynthesis, chlorophyll in plants captures the sun's energy by converting carbondioxide from the air and water from the ground into carbohydrates. When these carbohydrates are burned, they turn back into carbondioxide and water and release the sun's energy they contain. In this way, biomass function as a sort of natural battery for storing solar energy. The exploitation of energy from biomass has played a key role in the evolution of mankind. It is still, the main source of energy for more than half the world's population for domestic energy needs.

Benefits of biomass energy

The advantages of using biomass as a source of energy are illustrated below.

i. Biomass energy is an abundant, secure environment friendly and renewable source of energy.

ii. It can be used to generate electricity with the same equipment or in the same power plants that are now burning fossil fuels.

iii. Biomass energy is non pollutant of a atmosphere and is not associated with environmental impacts such as acid rain, radioactive waste disposal or the damming of river.

iv. Biomass energy is sustainable.

v. Biomass is easily available and can be grown with relative ease in all parts of the world.

Non-Renewable Energy

The energy which is generated from those energy sources that are not continuously replenished by nature is called non-renewable energy. The major sources of non-renewable energy are the fossil fuels, nuclear fuels, coal, natural gases. Fossil fuel is the most developed and largest source of non-renewable energy.

Advantages

a. Non-renewable sources are cheap and easy to use. We can easily fill up our car tank and power our motor vehicle.
b. We can use small amount of nuclear energy to produce large amount of power.
c. Non-renewable energy has little competition. For example: if we are driving a battery driven car and our battery gets discharged then we won't be able to charge it in the middle of the road, rather it is easy to find a gas pumping station.
d. They are considered as cheap when converting from one type of energy to another.

Challenges

a. Non-renewable sources will expire some day and we have to use our endangered resources to create more non-renewable sources of energy.
b. The speed at which such resources are being utilized can have serious environmental changes.
c. Non-renewable sources release toxic gases in the air when burnt which are the major cause for global warming.
d. Since these sources are going to expire soon, prices of these sources are soaring day by day

Differences between Renewable Energy and Non-renewable Energy

Renewable Energy	Non-renewable energy
1. The Renewable resources are present in the atmosphere of the earth.	1. The Non-Renewable resources are typically found in the underground layers of the earth.
2. The Renewable resources are replaced by nature itself in a very short period.	2. The Non-Renewable resources cannot be replaced by nature during the time of human life span.
3. The Renewable energy resources are plently available and abundant in nature.	3. The Non-Renewable resources are scarce resources and not available in an abundant manner in nature.
4. The Renewable resources are obtained free of cost or at very less cost in nature.	4. The Non-Renewable resources are very costly and not easily available.
5. The Renewable resources do not affect the environment of the earth and don't cause any climate changes in the atmosphere.	5. The Non-Renewable resources seriously affect the environment and cause climate changes in the environment.
6. The Renewable resources are called as 'Clean and Green' energy sources because they don't produce harm to the environment.	6. The Non-Renewable resources release 'Green House' gasses into the atmosphere which leads to global warming.

26.8 Pollution

Any unwanted product that causes adverse change in environment and human health is considered as pollution. There are various factors that cause pollution. Introduction of different kinds of toxic chemicals and poor disposal of waste are regarded as the major cause of pollution. In addition, by-products released from numerous industries and human activities contribute towards the pollution. For instance, toxic waste released from industries gets mixed in the soil and air which ultimately results in degradation of quality of soil and air. Our surrounding is being polluted due to the burning of fossil fuels, nuclear accidents. Some of the pollutions are naturally occurring too. For example, volcanic eruption: releasing toxic gas in the environment. Eutrophication which occurs generally in the soil and water when rain carries and deposits nitrogen in river and soil which results in algal growth in water bodies making condition worse for other living organism.

26.9 Air Pollution

Any unwanted or foreign substance present in atmosphere that reduces the quality of air and causes adverse effect on human health is considered as air pollution. It occurs when dust particles, biological molecules harmful gas molecules, and smokes are introduced into the atmosphere. It may causes diseases, allergies and even death of the human. Mostly, many diseases communicate through polluted air. Besides the harmful effects on human health, it has the variety of environment effect like acid rain.

Major air pollutants

- Carbon monoxide (10)
- Ozone (O_3)
- Nitrogen dioxide (NO_2)
- Sulfur oxides (SO_2)
- Carbon dioxide (CO_2)
- Lead (pb)

Fig. 26.4: Sources of air pollution

Causes of air pollution

The primary cause of air pollution is described below:

1. Exhaust emission from vehicles.

2. Industrial emission.
3. Burning of fuel like wood, cow dung, coal, kerosene, etc.
4. Use of fertilizer and pesticides, mining activities which releases particulate matter in air.
5. Nuclear power plant pollute air by releasing radioactive rays. Similarly, use of CFCs in refrigerator, fire extinguisher, sprayers, etc. pollutes air by depleting ozone layer.
6. War, site destruction, different construction activities also contribute to the pollution

Remedial measures of air pollution

The various methods of controlling air pollution are as follows:

1. Encouraging people to use public transport, walk rather than using private vehicles.
2. Plantation of trees along the busy streets as they remove particulates and absorb pollutants.
3. Industries and waste disposal sites should be built farther away from human residence.
4. Avoid using adulterated fuel.
5. For sustainable healthy environment, implementation of Environment Impact Assessment (EIA) in development projects.
6. If possible apply and process the control equipment for monitoring and controlling the air pollution.
7. Fuel substitution is another way of controlling air pollution. For instance, use of electric transportation, bio-fuel.

26.10 Water Pollution

In simple words, water pollution is the contamination of water bodies like: lake, river, aquifers, ocean, etc. Sewage and waste water is responsible for almost half of all ocean pollution and billions of tons of industrial toxic waste are dumped into source of water untreated. Two general categories of cause of water pollution are direct and indirect contaminant sources. Direct contaminant includes effluent and waste from factories, refineries, treatment plants, whereas indirect contaminant are those that enter water supply from atmosphere via rain water, and human agricultural practices as well.

Fig. 26.5: Sources of water pollution

Remedial Measures for Water Pollution

The following are the general possible ways of minimizing water pollution.

1. Proper disposal of household products like paints, used oil, organic waste, detergents used in cleaning of clothes will surely help reduce the water pollution in a effective manner.
2. Secondly, waste water treatment or sewage treatment can be applied which removes pollutants by means of physical, chemical and biological treatment methods.
3. Using water hyacinth in order to prevent the pollution of water sources which is extremely efficient in absorbing and concentrating dissolved nutrients from water.
4. Recycling, renovation, recharge and reuse (4R concept) of waste water.

26.11 Ozone Layer

Ozone exists in earth's stratosphere and is responsible for protecting humans from harmful ultraviolet (uv) rays. Ozone hole refers to the severe depletion of ozone in the region of ozone layer, particularly region high above the earth in the stratosphere. Ozone hole isn't actually a hole in a technical sense but rather more of a thinning of layer of ozone.

Major ozone layer depleting substances

- Chlorofluoron carbons (used in freezers)
- Methyl chroroform
- Hydrochlorofluro carbon (HFC)
- Carbon Tetrachloride

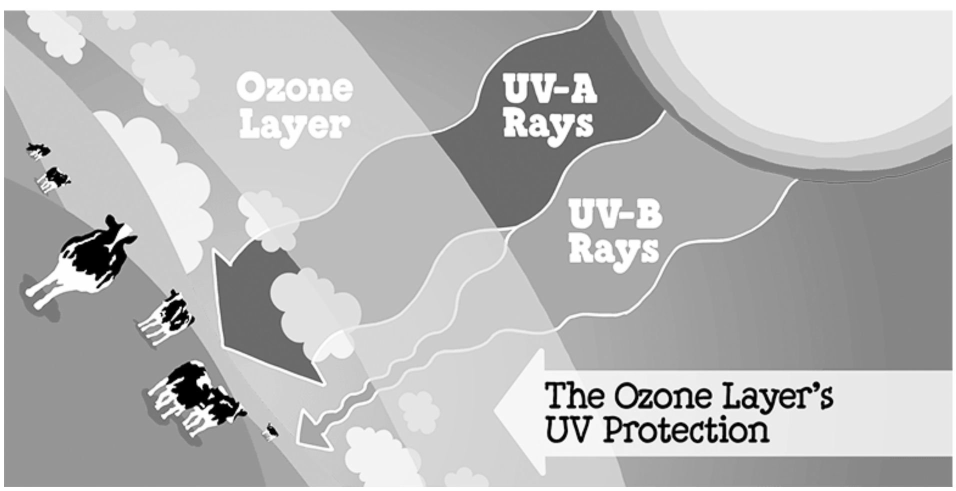


Fig. 26.6: Ozone layer's depletion

The effects of ozone hole are briefly explained in following points.

1. If ozone layer depletes exposure of radiation eventually results in serious impacts on human health, plants and animals due to more UV rays entering the earth. For instance, sun burns, skin cancers, premature aging of skin and damage in genetic materials in the cells is likely to happen.
2. In addition to human health, depletion of ozone also causes overall cooling trend on Antarctica continent leading to climatic effects causing storms and winds in high frequency and strength.

3. Reduction in phytoplankton in ocean that form the basic cycle of food chain in marine ecosystem, gradually making harsh living condition for aquatic plants and animals.
4. Finally, the effect of ozone hole and the damage done is not still comprehended by many scientists but is believed to cause a fatal damage in DNA which can be catastrophic.

26.12 Green House Effect

Green house effect is a natural phenomenon in which the atmosphere of a planet traps radiation emitted by sun caused by gases such as CO_2 (carbon dioxide), water vapour, nitrous oxide (NO_2) and methane that allows the incoming light to pass but retains the heat that radiates back from planet's surface. This process maintains the earth's temperature at around 33°C, allowing life on earth to exist without green house gases the average temperature of earth's surface would be about –18°C (0°F). Basically, green house effects are caused by natural and man-made activities. Volcanic eruption, thermal pollution helps to produce green house gases (CO_2, methane) and increased use of vehicles and chemicals that emits green house gases. Green house gases are helpful to maintain atmospheric temperature but an increased amount over optimum has a huge impact on earth's temperature causing global warming and many other consequences related to climate change. Some of the effects are:

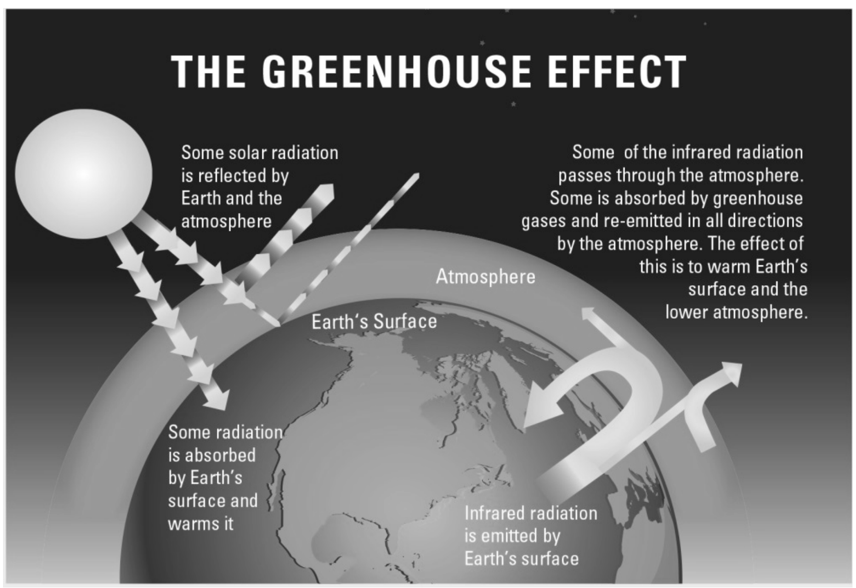


Fig. 26.7: Greenhouse effect

1. Ocean acidification is caused by increase in CO_2 level in atmosphere. The ocean serves as a sink for gas and absorbs most of CO_2 emission.
2. Over the last century, ozone concentration has become much larger at ground level that is a major component of smog which is dangerous for both humans and plants.
3. NO_2, a green house gas which damages the ozone layer and is the most important ozone depleting substances.
4. One of the major aftermaths of greenhouse effect is global warming since the staring days of industrial revolution and human interference in nature.

5. Sea level rises because of melting ice and snow due to thermal expansion of sea. Areas that are just above sea level now, may become submerge.
6. Rising sea level extreme weather conditions.

26.13 Acid Rain

Acid rain is the precipitation of acidic components on the atmosphere which fall to the ground in the form of rain. Sulphuric acid (H_2SO_4) and nitric acid (HNO_3) are the acidic components that are formed on the atmosphere and fall on the earth's surface. Acid rain is caused by the chemical reaction of air with compounds like sulphur dioxide and nitrogen oxides. These substances can rise very high in the atmosphere, where they react with water, oxygen and other chemicals to form more acidic pollutants, which come down in the form of rain.

The gas sulphur dioxide (SO_2) when oxidized in the reaction with hydroxyl radical (OH) via an intermolecular reaction, $HOSO_2$ is formed.

i.e. $SO_2 + OH \longrightarrow HOSO_2$

Then, the chemical reaction of $HOSO_2$ with oxygen yields HO_2 and SO_3, as,

$$HOSO_2 + O_2 \longrightarrow HO_2 + SO_3$$

When the sulphur trioxide reacts with water vapour in the atmosphere, sulphuric acid is formed.

$$SO_3 + H_2O \longrightarrow H_2SO_4$$

Also, the nitrogen dioxide when reacts with hydroxyl radical (OH), nitric acid is formed.

i.e. $NO_2 + OH \longrightarrow HNO_3$

Finally, sulphuric acid and nitric acid falls to the earth's surface, which is popularly called acid rain.

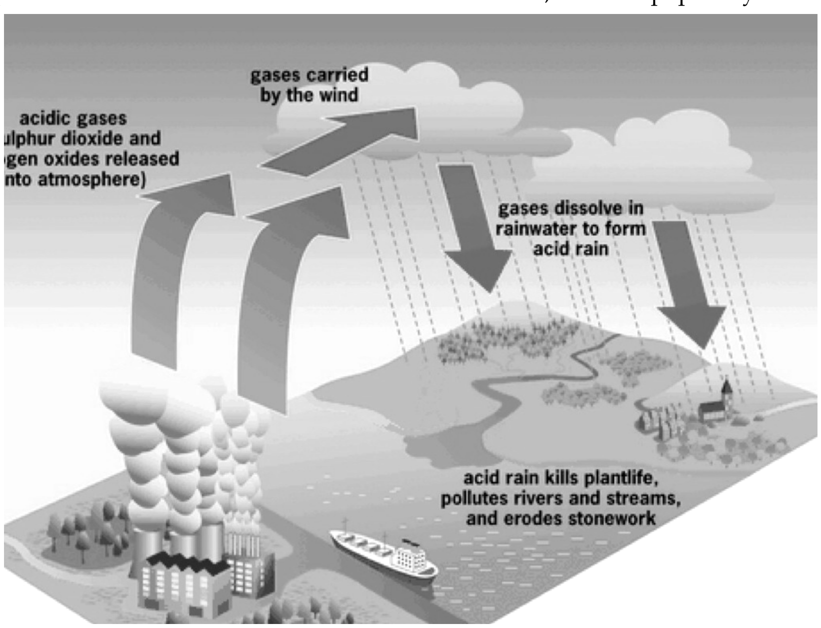


Fig. 26.8: Acid rain

Acid rain has many ecological effects. It has many adverse effects to aquatic as well as terrestrial animals and plants. Acid rain kills trees and harms animals like fish, other wild life and even human civilization.

Conceptual Questions with Answers

1. Write four possible ways of minimizing water pollution.

➯ The following are the general possible ways of minimizing water pollution.

i. Proper disposal of household products like paints, used oil, organic waste, detergents used in cleaning of clothes will surely help reduce the water pollution in a effective manner.

ii. Secondly, waste water treatment or sewage treatment can be applied which removes pollutants by means of physical, chemical and biological treatment methods.

iii. Using water hyacinth in order to prevent the pollution of water sources which is extremely efficient in absorbing and concentrating dissolved nutrients from water.

iv. Recycling, renovation, recharge and reuse (4R concept) of waste water.

2. What are the health hazards due to radiation?

➯ Long term exposure may result in fatal damage of internal organs. Numerous case of skin related illness can be observed which leads to skin cancer. Radioactive materials decays producing radiation and breaking the chemical bond that makes up our tissues which damages the DNA. Ultimately damaging cells and in most cases causing deformity. In addition, one of most infamous accident, Chernobyl, disaster which occurred due to radiation and other debris that led many people and their next generation causing mutation in gene.

3. What is ozone hole? Write its effects.

➯ Ozone hole refers to the severe depletion of ozone in the region of ozone layer, particularly region high above the earth in the stratosphere. Ozone hole isn't actually a hole in a technical sense but rather more of a thinning of layer of ozone. The effects of ozone hole are briefly explained in following points.

i. If ozone layer depletes exposure of radiation eventually results in serious impacts on human health, plants and animals due to more UV rays entering the earth. For instance, sun burns, skin cancers, premature aging of skin is likely to happen.

ii. In addition to human health, depletion of ozone also causes overall cooling trend on Antarctica continent leading to climatic effects causing storms and winds in high frequency and strength.

iii. Reduction in phytoplankton in ocean that form the basic cycle of food chain in marine ecosystem, gradually making harsh living condition for aquatic plants and animals.

iv. Finally, the effect of ozone hole and the damage done is not still comprehended by many scientists but is believed to cause a fatal damage in DNA which can be catastrophic.

4. Explain greenhouse effect.

➯ Green house effect is a natural phenomenon in which the atmosphere of a planet traps radiation emitted by sun caused by gases such as CO_2 (carbon dioxide), water vapour, nitrous oxide (NO_2) and methane that allows the incoming light to pass but retains the heat that radiates back from planet's surface. Basically, green house effects are caused by natural and man-made activities.

5. Write down the radioactive pollution?

➯ The radioactive pollution can be defined as the release of radioactive substances of high energy particles in the air, water or soil as a result of human activity. Radioactive pollution directly or indirectly harms the human health. Leakage of radiation by nuclear accidents or by design is the cause of radioactive pollution.

6. What are the renewable sources of energy?

➯ Renewable energy sources are energy sources that are always being replenished. They can never be depleted. Examples;

a. Hydropower b. Geothermal c. Wind d. Solar.

e. Biomass—includes: Wood and wood waste, solid waste, Landfill gas and biogas, ethanol, biodiesel.

7. What is energy degradation?

➯ Energy degradation is associated with movement towards equilibrium in a quantity potentially associated with work (such as temperature, pressure, or concentration).

8. What is energy transformation? Give one example.

➯ The process of changing one form of energy into another, such as nuclear energy into heat or solar energy into electrical energy is known as energy transformation.

9. What are the main advantages of nuclear energy?

➯ The main advantages of nuclear energy are as follows:

i. A nuclear power station uses a steam turbine and generator to produce electricity in exactly the same way as any other thermal power station.

ii. It does not produce carbon dioxide and other pollutants that are formed when things are burnt.

iii. It is more clean and more environment friendly than coal, oil or gas fire power stations.

iv. It is compact, competitive and practically inexhaustible.

v. Its efficiency is relatively high and a best alternative source of energy at the location where hydropower plant is impossible.

10. What are the challenges of nuclear power?

➯ The challenges of nuclear power are as follows:

i. The nuclear fission creates materials that are still radioactive and harmful to human beings and to the environment for thousands or even millions of years.

ii. Another problem is regarding the safe way to store the waste.

iii. Accidents at nuclear power stations are very rare, but when they occur, they can be catastrophic.

iv. Steam coming out of the turbine is also still hot, and so adds heat to the environment.

11. What is solar energy? Why are its advantages?

➯ Solar energy is radiant light and heat from the Sun that is harnessed using a range of ever-evolving technologies such as solar heating, photovoltaics, solar thermal energy, solar architecture, molten salt power plants and artificial photosynthesis. Solar energy is an alternative for fossil fuels as it is non-polluting, clean, reliable and renewable source of energy. Solar energy also does not require any fuel to produce electricity and thus avoids the problem of transportation of fuel or storage of radioactive waste

12. What is the possibility of wind energy in Nepal?

➯ A study by the Alternative Energy Promotion Center (AEPC) in coordination with the UN Environment Program (UNEP) and other institutions has showed that at least 30,000 MW of wind energy can be generated in Nepal due to adequate hilly and riparian corridors where wind blows regularly.

13. Why biomass energy is important in Nepal?

➯ Looking at the deforestation caused by the excessive use of firewood, the National Energy Strategy of Nepal under draft version has aimed for focusing on biomass energy to fulfill the energy needs on short and medium term and on the longer term energy needs to be met by electricity reducing the consumption of biomass energy. It is also necessary to keep in mind that the reduction of import of fossil fuel will enforce the increased dependency on firewood collected from forest which may cause the environmental and economic losses. Though the electricity for longer term has been strategically considered as the main-energy, there will be a consumption of biomass energy in some form and quantity which need to be managed sustainably and environment-friendly manner. Biomass energy is an important part of our living. Proper consideration for public health and socioeconomic life, through promotion of technology and positive attitude towards the effective management, sustainable and efficient use of biomass energy seems necessary

14. What is non-renewable energy?

➩ A non-renewable resource is a resource of economic value that cannot be readily replaced by natural means on a level equal to its consumption. Most fossil fuels, such as oil, natural gas and coal are considered non-renewable resources in that their use is not sustainable because their formation takes billions of years. Non-renewable resources are those found inside the earth, and they took millions of years to form. These include the fossil fuels, oil, natural gas, and coal and nuclear energy

15. Why is it necessary to develop the technology to use the renewable energy?

➩ There is a limited supply of Non-Renewable resources is on the Earth. We're using them much more rapidly than they are being created. Eventually, they will run out and our future generations are left with no crude oil and nuclear resources. We have a responsibility to transfer the resource to our future generations, for that we have to use the non-renewable and renewable resources in a balanced way and promote sustainability of resources.

16. Do biofuels have any social impact?

➩ Yes. First they can take land for food crops out of food production, which can increase food cost, cause disease and social unrest. There are some arguments that biofuels need to be based on wastes and not occupy food land or newly cleared forests. Second, in cities, smoke from wood burning can increase pollution, which leads to respiratory disease and can worsen asthma.

17. Is using nuclear power really the answer to clean, environmentally friendly energy?

➩ No. There is nothing environmentally friendly about nuclear power. It only creates different environmental problems than fossil fuel energy sources. But neither fossil fuels nor nuclear power are safe, sustainable, or healthy for humans and the environment.

Exercises

Short-Answer Type Questions

1. How is development of physical science related to human kinds?
2. Why do we worry about the energy crisis, although energy doesnot destroy in universe?
3. How energy is released in nuclear fusion?
4. What is source of energy in sun and stars?
5. What are the major hydropower projects in Nepal?
6. What are the benefits and drawbacks of wind energy?
7. Why solar power is called alternative energy source?
8. What is the major energy source in Nepal?
9. Why biomass is very important for Nepal?
10. How geothermal energy is produced?
11. What is acid rain? How it is formed?
12. Describe briefly noise pollution, air pollution and water pollution.
13. Discuss ozone depletion, greenhouse effect, acid rain.
14. Discuss strategies to reduce pollution at local and national levels.
15. Discuss the wide spectrum of electromagnetic radiation from radiowave to cosmic rays.
16. What are the major contents of green house gases?

Long-Answer Type Questions

1. What is greenhouse effect? What are its causes? How can depletion of ozone layer be protected?
2. Describe the causes and remedies of air pollution and water pollution.
3. What are radiation hazards? Explain its harmful effects.
4. How the energy sources are associated with human civilization?
5. What are the major energy sources in Nepal?
6. Why do we face energy crisis? How can we solve the problem of energy crisis?
7. Differentiate between renewable and nonrenewable sources of energy.
9. Describe the renewable sources of energy in Nepal.
10. Describe the global scenario of energy consumption.

❖ ❖ ❖

PARTICLE PHYSICS

27

CHAPTER

27.1 Introduction

Elementary particles are those particles whose internal structures are unknown. The internal structure is said "unknown" in the sense that no simpler particles have been detected other than these particles. So, they are considered as the most fundamental particles in nature. Before the discovery of electron by J.J. Thomson, atoms were considered as the fundamental particles. In Greek language, 'atomos' means 'indivisible' it means atom can not be broken into more fundamental particles. J.J. Thomson purposed that an atom is composed of electrons and nucleus. After the discovery of neutron, in 1932, by Chadwick, it was considered that an atom is composed of three subatomic particles: electron, proton and neuron. After the development of quantum mechanical theory, it was established that some of the subatomic particles like proton and neutron have also internal structure, they are composed of quarks. Likewise, many other elementary particles have also been identified. Till date, Leptons, quarks and mediator particles are considered as the elementary particles.

27.2 History of Elementary Particles

- In 1897, J.J.Thomson discovered negatively charged particle, electron and another positively charged particle; proton.
- In 1911, Rutherford discovered positively charged central core of atom known as nucleus.
- Bohr in 1932 purposed that the nucleus consists of proton and electron is revolving around the nucleus. Electron and proton are basic units of charge.
- Chadwick in 1932 discovered existence of chargeless particle in the nucleus called neutron.
- In 1924 de-Broglie suggested that the photon behaves as particle. (de-Broglie hypothesis)
- In 1928 Dirac-predicated the existence of positron (e^+), antiparticle of electron, having the same mass and the positive charge.
- Anderson in 1932 discovered the positron.
- In 1955, the existence of antiproton was discovered by Serge, Chamberlain and their collaborators.
- In 1956, similarly, the existence of antineutron, the antiparticle of neutron, was discovered by Cook, Chamberton and Wenzel.

- In 1928, Pauli discovered neutrino to account the missing energy of particle along with antineutrino.
- The discovery of antiparticle of a particle finally leads to the existence of antimatter of the matter and antiworld of world.
- In 1995, Japanese scientists Yukawa gave the idea of a particle of mass intermediate between that of an electron and proton, i.e. meson which makes the proton and neutron glued together in nucleus.
- In 1937, the μ-meson was discovered by Anderson and Neddermeyer in the cosmic ray researches. Their discovery was confirmed in 1940 by Leprince-Ringuet and found that the μ-mesons are 207 times heavier than electorn.
- In 1947, Powell group in England discovered π-meson where π^+ and π^- meson have rest masses of 273 m_e and π^0-meson is slightly less than 264m_e.
- Similarly, around or more than 200 particles have been discovered up to now.
- By the end of 1977 five flavours of quark (u,d,s,c,b) were known to exist together with six flavours of lepton (e, μ, τ, ν_e, ν_μ, ν_τ). Assuming that quarks and leptons are the fundamental constituents of matter, many of the strong and weak interactions of hadrons and the weak interactions of leptons are explained. However anticipating a symmetry in nature's building blocks, it was expected that a sixth quark would eventually reveal itself. This quark, labeled top (t), would be 2/3 electronic charge partner to the (bottom) quark. In 1998 the top quark was found at CERN in Geneva and the symmetry of six quarks with six leptons was finally verified.
- In 1978 the standard model was proposed as the definitive theory of the fundamental constituents of matter. In the current view, all matter consists of three kinds of particles : leptons, quarks and mediators. Mediators are the particles by which the four fundamental interactions are mediated.

27.3 Particles and antiparticles

A particle is said to be antiparticle of a particle if that has equal mass and magnitude of electric charge, but nature of charge is opposite. The antiparticle of an electron is positron. Positron has exactly equal mass of electron (= 9.1×10^{-31} kg) and magnitude of charge 1.6×10^{-19} C, but the nature of its charge is positive. Dirac purposed that every particle in nature must have its antiparticle. Another important method of identifying particle-antiparticle pair is the property of annihilation. In particle-antiparticle annihilation, they combine to form energy, usually, a pair of γ-rays satisfying the conservation of energy and momentum. Some examples of particle-antiparticle pair are explained below.

i. **Electron and Positron:** The positron is exactly the counterpart of an electron having equal mass and one unit of positive charge (i.e. 1.6×10^{-19} C). It was discovered by Anderson in 1932. Its mean life is approximately 10^{-10} s. When an electron combines with positron, they disappear and form two quanta of γ-rays.

ii. **Proton and antiproton:** Antiproton is the antiparticle of a proton. The existence of antiparticle was discovered by Serge, chamberlin and their coworkers in 1955.

iii. **Neutron and antineutron:** Antineutron is the anti-particle of a neutron. Antineutron was discovered by Cork, Lamberton and Wenzel in 1950. Although neutron and anti-neutron have zero electric charge, they are supposed to have a certain internal charge distribution.

iv. **Neutrino and anti-neutrino:** Pauli purposed the existence of neutrino in β-decay process. He purposed, in 1931, that β-decay is always accompanied by another particle of almost zero rest mass and zero-charge, called neutrino. Antineutrino is the antiparticle of neutrino.

v. **Matter and antimatter**: A matter is composed of electron, and nucleus. If we take a consideration of hydrogen, it is composed of an electron and proton. According to particle-antiparticle concept, when a positron and an antiproton combine, an antihydrogen is formed. Likewise, anti-elements are formed from positron, antiproton and antineutron. Antielements are responsible to form antimatter. If matters are annihilated with anti-matters, possibly tremendous energy will be released.

27.4 Annihilation

When a particle interacts with its antiparticle, whole masses of both particle and antiparticle are completely converted into energy (photons), usually, a pair of γ-rays (sometimes x-rays). This process of conversion of matter into energy is called annihilation. Most common annihilation on Earth occurs between an electron and its antiparticle, positron.

A particle and an antiparticle can not annihilate into a single photon, they have to annihilate into at least two photons to conserve energy and momentum.

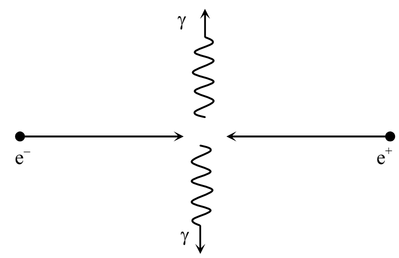


Fig. 27.1: Electron Position annihilation

27.5 Pair Production

An x-ray or γ-ray, may interact with the matter while traversing nearer from the nucleus. When a photon of x-ray or γ-ray passes through the nuclear field, a electron-positron pair, one negative and one positive, appears in place of the photon. This materialization process of energy is known as pair production. Since the energy equivalent to the mass of an electron is 0.51 MeV, the creation of electron-positron pair requires, 2 × 0.51 = 1.02 MeV. Consequently, photons with energy less than 1.02 MeV do not interact by pair production. During pair production, energy in excess of 1.02 MeV is released as kinetic energy of the pair particles.

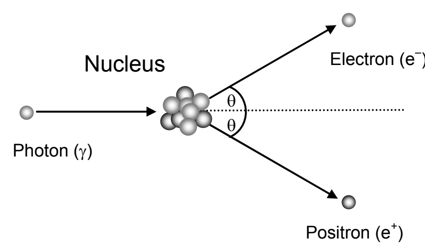


Fig. 27.2: Pair Production

27.6 Concept of Spin

In classical physics, spin is simply the rotation of an object along its axis, but it is quite different in the context of elementary particles. It is somehow difficult to have insight of the concept of spin by using classical physics. It is quantity that requires quantum mechanical explanation to be described at its full. However, the basic concept regarding the spin can be visualized considering the following Fig. 27.3.

Every particle is assigned with certain value of spin in the form of numbers. Each number has the special information regarding rotation of the particle.

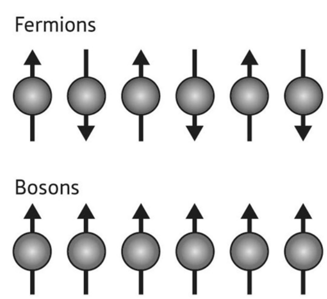


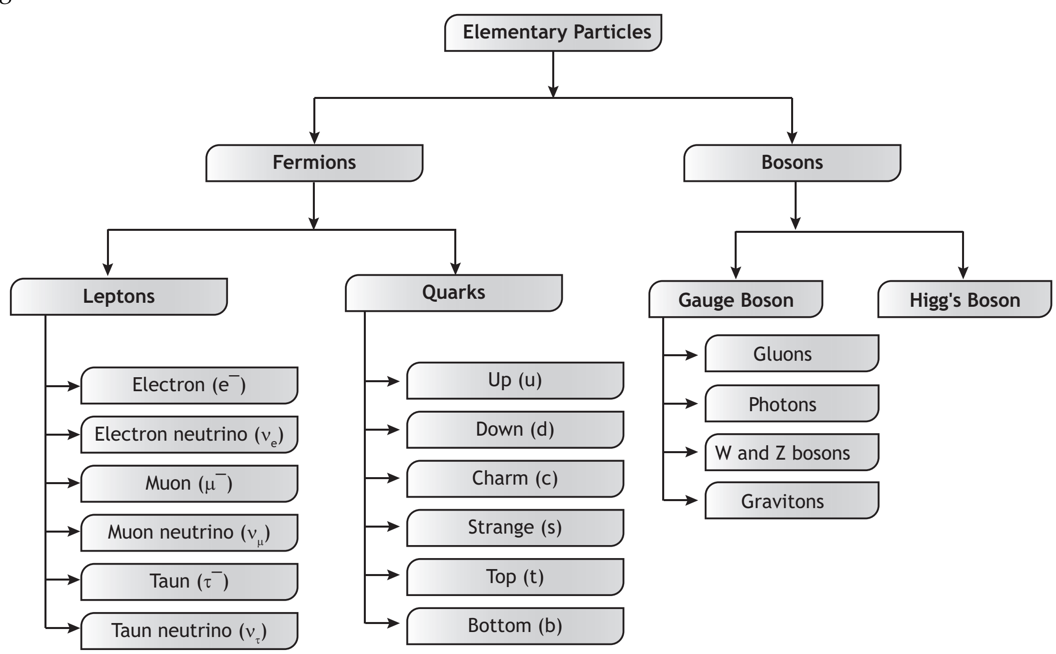
Fig. 27.3: Spin of particles

i. **Half spin:** The particles having half spin, reverse their direction in 2π rotation (i.e. upside turns down) and after another 2π rotation, come to their original position. That is to say, such particles can recover their original position after 4π rotation. Fermions are half-spin particles.

ii. **Integer spin:** The particles having integer spin, recover their original position after 2π rotation. Bosons are integer spin particles.

27.7 Classification of Elementary Particles

Elementary particles are considered as the structureless and are not regarded as made up of any other particles. They can be detected in the matter of the earth and cosmic rays. Elementary particles are broadly classified into two types: fermions and Bosons. Their antiparticles are also the elementary particles. The classification of elementary particles in accordance with standard model is given below.



27.8 Fermions

The elementary particles with half-integer spins are called fermions. Half integer can be the odd multiples of $\frac{1}{2}$. i.e. $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}$. Leptons, quarks and composite particles made up of quarks belong to this family. These particles obey Pauli Exclusion Principle. According to Pauli Exclusion Principle, the particles cannot occupy same quantum state simultaneously.

27.9 Leptons

Light elementary particles are incorporated in this class. In Greek language, 'Lepton' refers the 'light particles'. In this class, all particles are stable except muon and taun. Taun is heavier than many mesons (lie in the class of heavy particles), but it has no internal structure and have no measurable size. Its electric charge is similar to electron. Therefore, taun belongs to lepton family. There are six leptons. Some important information of lepton's are tabulated below.

Particle	Symbol	Charge	Rest Mass $\left(\frac{MeV}{c^2}\right)$
Electron	e^-	-e	0.51
Muon	μ^-	-e	106
Taun	τ^-	-e	1784
e-neutrino	ν_e	0	0
μ-neutrino	ν_μ	0	0
τ-neutrino	ν_τ	0	0

(All antiparticles have charge just opposite to that of particles)

27.10 Quarks

Quarks are elementary particles which are the fundamental constituents of matter. They are fermions. They combine to form composite particles called hadrons. Protons and neutrons are the most stable composite particles, they are the components of nucleus. Quark model was independently purposed by Murray Gell-Mann and George Zweig in 1964. We cannot see quark separately, since the strong force between them increases as we try to separate them. Although M. Gell-Mann contributed prime role in the discovery of quark, he gave the name "quark", when he found the word quark in James Joyce's book "Finnegan's wake."

Initially, M.Gell Mann, and George Zweig purposed only three quarks, up, down and strange, and their antiquarks. Then, other three more quarks namely charm, bottom, and top quarks were discovered in Fermi lab. Each quark has baryon number $\frac{1}{3}$. Quarks have fractional electric charge value $+\frac{2}{3}$ e or $-\frac{1}{3}$ e. (Where e = 1.6×10^{-19} C). The short description of quarks are tabulated below.

Types of quarks	Symbol	Charge	Baryon number	Antiquarks
Up	u	$+\frac{2}{3}e$	$\frac{1}{3}$	$\bar{u}$
Down	d	$-\frac{1}{3}e$	$\frac{1}{3}$	$\bar{d}$
Charm	c	$+\frac{2}{3}e$	$\frac{1}{3}$	$\bar{c}$
Strange	s	$-\frac{1}{3}e$	$\frac{1}{3}$	$\bar{s}$
Top	t	$+\frac{2}{3}e$	$\frac{1}{3}$	$\bar{t}$
Bottom	b	$-\frac{1}{3}e$	$\frac{1}{3}$	$\bar{b}$

27.11 Bosons

The elementary particles with zero or integer spins (0, 1, 2,.......) are bosons. Gauge bosons and Higgs bosons are the examples of bosons. These particles do not obey Pauli Exclusion Principle. Bosons which are responsible for the four fundamental forces are called gauge bosons. Strong interaction is mediated by gluons, electromagnetic interaction is mediated by photons, weak interaction is mediated by W and Z bosons, and the gravitational interaction is mediated by gravitons. Higgs bosons are supposed to explain the origin of particles mass. The properties of four fundamental forces are tabulated below.

Four fundamental forces

Types of force	Nature
Strong force	This force holds the nucleons together in nucleus. It squeezes the protons and neutrons into volume that is about 10^{-15} m. It is the strongest among all four forces.
Electromagnetic force	This force acts between electrically charged particles. It includes the electrostatic force acting between charged particles at rest and combined effect of electric and magnetic forces acting between charged particles moving relative to each other.
Weak force	This force is responsible for radioactive decay, specially, beta decay where a neutron within nucleus changes into a proton and an electron, also acts in nuclear fusion in stars. It is weaker than electromagnetic force and stronger than gravitational force.
Gravitational force	It is the weakest force among all four forces. This force is relevant for large celestial bodies such as planets, stars and galaxies and attraction between them.

The brief description of mediators is tabulated below.

Mediator Particles

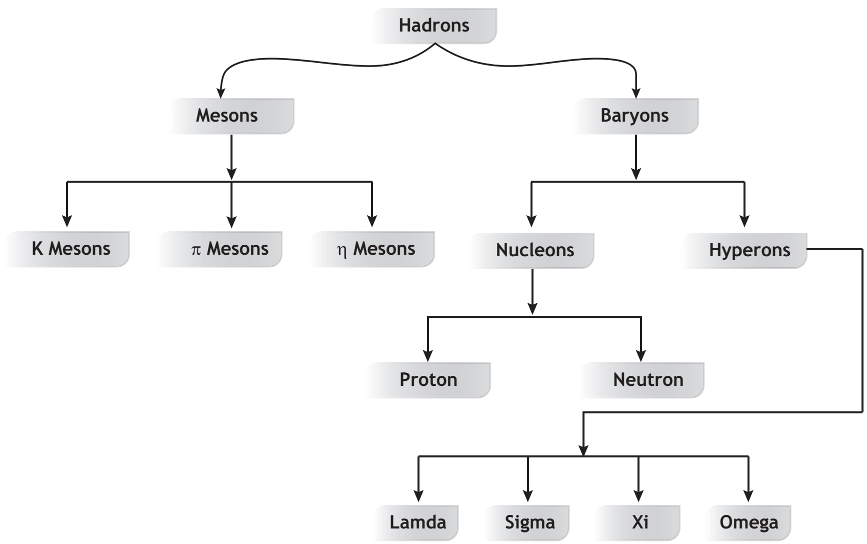
All particles in the mediator group mediate in the interaction for the four kinds of forces: strong nuclear forces, electromagnetic force, weak force and gravitational force.

Particle	Symbol	Force	Charge
Gluon	G	Strong	0
Photon	ν	Electromagnetic	0
Plus W	W+	Weak	+e
Minus W	W-	Weak	-e
Zero Z	Z^0	Weak	0
Graviton	G	Gravitational	0

27.12 Hadrons

Hadrons are strongly interacting heavy composite particles. They are composed of quarks. Hadrons are basically two types: Mesons and Baryons. Mesons contain one quark and an antiquark. Baryons contain three quarks. Quarks held together to form a hadrons by the strong force. Hadrons containing more than three quarks are also discovered in recent years. A tetraquark state was discovered in 2007 and two pentaquark states were discovered in 2015. Tetraquark and pentaquark states will not be described below.

The hadrons are classified as below.



27.13 Mesons

Ordinary mesons are made up of a quark and an antiquark. Since they are composed from a quark and an antiquark, they have the spin 0 or 1. They possess a single unit of charge (except zero meson) and mass intermediate between electron and proton. The rest mass of these particles varies from 250 m_e to 1000 m_e. Short information for some mesons is tabulated below.

Particle	Symbol	Charge	Rest mass $\left(\frac{\text{MeV}}{c^2}\right)$	Mean life, sec
Pion-zero	π^0	0	135	8.3×10^{-3}
Pion plus/minus	$\pi^{\pm}$	$\pm$ e	140	2.6×10^{-8}
Kaon-zero	K^0	0	498	9×10^{-11}
Kaon plus/minus	$K^{\pm}$	$\pm$ e	494	1.2×10^{-8}
Eta meson	η^0	0	549	7×10^{-19}

Quark and Mesons

Each meson is a combination of a quark and antiquark. The baryon number of each meson is zero. Mesons are unstable and they decay into lighter mesons or leptons. The charge and baryon number of π^+ meson is taken as an example below.

Pion π^+ meson

$\pi^+ = u\ \bar{d}$

$Q = (+2/3\ \ +1/3)\ e = +e$

Baryon number (B) = $(1/3)+(-1/3) = 0$

Quark Structure of Source Mesons

Mesons	Quark combination	Charge
π^+	$u\bar{d}$	+e
π^-	$\bar{u}d$	-e
K^o	$d\bar{s}$	0
K^+	$u\bar{s}$	+e
K^-	$\bar{u}s$	-e

27.14 Baryons

Baryons are composite particles including nucleons. They have equal or greater mass than the mass of a proton. They have half integer spins. Baryons heavier than nucleons are hyperons. Baryons are basically divided into two classes: nucleons and hyperons. Every baryon has an antiparticle.

Nucleons

These are the lightest baryons. This group includes protons, neutrons and their antiparticles. A nucleus of an atom is composed up of nucleons.

Hyperons

These are the special baryons having the mass value intermediate between those of neutron and deuteron. Lamda (λ), Sigma (Σ), Xi (Ξ) and omega (Ω) are examples of hyperons. Actually, hyperons contain strange quarks. Their decay time is very much greater than the time of formation.

The short description of nucleons and hyperons are presented below.

Particle	**Symbol**	**Charge**	**Energy equivalence of Rest mass $\left(\frac{\text{MeV}}{c^2}\right)$**
Proton	p	+ e	938.3
Neutron	n	0	939.6
Lambda	λ^0	0	1116
Sigma	Σ^+	+ e	1189
	Σ^0	0	1192
	Σ^-	– e	1197
Xi(cascade)	Ξ^0	0	1315
	Ξ^-	– e	1321
Omega	Ω^-	– e	1672

Quarks and baryon

Each baryon is combination of three quarks and baryon number is 1 for each baryon.

i. **Proton:** It consists of three quarks uud, i.e. two u quarks and one d quarks.
p = uud
Total charge, Q = (+2/3 +2/3 -1/3) e = +e
Also baryon number, B = 1/3 + 1/3 + 1/3 = 1

Similarly, Antiproton contains, $\bar{p} = \bar{u}\ \bar{u}\ \bar{d}$

ii. **Neutron:** It consists of one up quark and two down quarks.
n = udd, charge Q = (+2/3 - 1/3 - 1/3) e = 0
Also baryon number (B) = +1/3 + 1/3 +1/3 = 1
Similarly, Antineutron, $\bar{n} = \bar{u}\ \bar{d}\ \bar{d}$

iii. **Sigma:** It is a hyperon. The Σ^+ quark is made up of two up quarks and a strange quark.
Σ^+ = uus, Charge no. Q = (+2/3 +2/3 – 1/3) e = +e
Similarly, Σ^0 = uds
Σ^- = dds

Quark structure of baryons

Baryon	Q-combination	Charge
p	uud	+e
n	udd	0
Λ^0	uds	0
Σ^+	uus	+e
Σ^0	uds	0
Σ^-	dds	–e
Ξ^0	uss	0
Ξ^-	dss	–e
Ω^-	sss	–e

27.15 Three Generations of Quarks and Leptons

The generations of elementary particles are the divisions of particles in accordance with flavour, quantum number and mass. Each generation is divided into two types of leptons and two types of quarks. Two leptons are classified into one with one electric charge -1 (electron-like) and one neutral (neutrino); two quarks may be classified into one with $+\frac{2}{3}$ e and another with $-\frac{1}{3}$ e.

The first generations contains two leptons, the electron and the electron neutrino, and two quarks, up and down. All the properties of ordinary matter can be understood on the basis of these particles. The second generation includes the muon and muon-neutrino and the charm and strange quarks. These particles are responsible for most of the unstable particles and resonances created in high energy collisions. The third generations includes the tau and the tau-neutrino and the top and bottom quarks.

Generations of matter			
Type	**First**	**Second**	**Third**
Quarks			
up-type	up (u)	charm (c)	top (t)
down-type	down (d)	strange (s)	bottom (b)
Leptons			
charged	electron (e)	muon (μ)	taun (τ)
neutral	electron neutrino (ν_e)	muon neutrino (ν_μ)	taun neutrino (ν_τ)

Tips for MCQs

1. Elementary particles are structureless. They are not composed of any other fundamental particles.
2. Every elementary particle has its anti-particle.
3. Fermions (spin half particle) and bosons (zero or integer spin particles) are two main categories of elementary particles.
4. Pair production is the materialization process and annihilation is the mass to energy conversion process.
5. There are six leptons, six quarks and four types of mediator particles.
6. Hadrons and Mesons are composite particles Hadrons are composed up of three quarks and mesons are composed up of a quark and an anti-quark.
7. Hyperons and nucleons are Baryons. Hyperons are heavier than nucleons.
8. The basic forces in nature are strong, electromagnetic, weak and gravitational.

Conceptual Question Answer

1. What are elementary particles?

↳ Elementary particles are those particles whose internal structure is unknown. The internal structure is said to be "unknown" in the sense that no simpler particles has been detected than these particles. So, they are considered as the most fundamental particles in nature.

2. Define antiparticle of a particle. Give examples.

↳ A particle is said to be an antiparticle of a particle if it has equal mass and magnitude of electric charge, but nature of charge is opposite. Every particle has its antiparticle. For example: (i) antiparticle of electron is positron (ii) antiparticle of proton is antiproton. (iii) antiparticle of neutron is antineutron.

3. Define fermions. Give two examples.

↳ The elementary particles with half-integer spins are called fermions. Half integer can be the odd multiples of $\frac{1}{2}$. i.e. $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}$. Leptons, quarks and composite particles made up of quarks belong to this family. These particles obey Pauli exclusion principle. According to Pauli exclusion principle, the particles cannot occupy same quantum state simultaneously.

4. Define bosons.

↳ The elementary particles with zero or integer spins (0, 1, 2,.......) are bosons. Gauge bosons and Higg's bosons are the examples of bosons. These particles do not obey Pauli exclusion principle. Bosons which are responsible for the four fundamental forces are called gauge bosons.

5. Write the quark combination of proton and neutron.

↳ The quark combination of proton is uud. It means a proton is composed with two up quarks and one down quark.

The charge combination is $\frac{2}{3}e + \frac{2}{3}e - \frac{1}{3}e = e$

The quark combination of neutron is, udd,

The charge combination is, $\frac{2}{3}e - \frac{1}{3}e - \frac{1}{3}e = 0$

6. Write the quark combination of antiproton and antineutron.

➪ The quark combination of antiproton is $\bar{u}\bar{u}\bar{d}$. It means a antiproton is composed with two antiup quarks and one antidown quark.

The charge combination is $-\frac{2}{3} -e - \frac{2}{3} e + \frac{1}{3} e = -e$

The quark combination of antineutron is, $\bar{u}\bar{d}\bar{d}$,

The charge combination is, $-\frac{2}{3} e + \frac{1}{3} e + \frac{1}{3} e = 0$

---

7. Hadrons are not truly fundamental particles, why?

➪ Hadrons are composite particles. They are composed of quarks. Their internal structure is known. Hadrons are of basically two types: mesons and baryons. To be fundamental particles, internal structure should be unknown.

---

8. What are mediator particles?

➪ All particles in the mediator group mediate in the interaction for the four kinds of forces: strong nuclear forces, electromagnetic force, weak force and gravitational force. Strong interaction is mediated by gluons, electromagnetic interaction is mediated by photons, weak interaction is mediated by W and Z bosons, and the gravitational interaction is mediated by gravitons.

---

9. What are mesons? Write the name of three mesons.

➪ Ordinary mesons are made up of a quark and an antiquark. Hence, they have the spin 0 or 1. They possess a single unit of charge (except zero meson) and mass varies between electron and proton. Pion, Kaon, Eta, etc. are some examples of mesons.

---

10. What are quarks? Do they exist separately?

➪ Quarks are elementary particles which are the fundamental constituents of matters. They are fermions. They combine to form composite particles called hadrons. Protons and neutrons are the most stable composite particles, they are the components of nucleus. No, they do not exist separately.

---

11. What are the fundamental forces in nature?

➪ Strong force, electromagnetic force, weak force and gravitational force are the fundamental forces in nature. The elementary particles, gluons mediate for strong interaction, photons mediate for electromagnetic interaction, W and Z bosons mediate for weak interaction and gravitons mediate gravitational interaction.

---

12. Give quark combination of K^+, K^-, K^0

➪ Kaon (K-meson):

i. K^+ meson: It made of one up quark and another antistrange quark. i.e. $K^+ = u\,\bar{s}$,
Total charge, Q = (+ 2/3 + 1/3) e = +e
Total baryon number (B) = 1/3 – 1/3 = 0

ii. K^- meson: It is made of one antiup and a strange quark. i.e. $\bar{u}\,s$
Q = (– 2/3 – 1/3) e = –e
B = – 1/3 + 1/3 = 0

iii. K^0 meson: It is made of a down quark and an antistrange quark. . i.e. $d\,\bar{s}$
Q for $d\,\bar{s}$ = (– 1/3 + 1/3) e = 0
B = 1/3 – 1/3 = 0

---

13. What is meant by annihilation of particle-antiparticle pair?

➪ When a particle interacts with its antiparticle, whole mass of both particle is completely converted into energy photons, usually, a pair of γ-rays (sometimes x-rays). This process of conversion of matter into energy is called annihilation.

---

14. What is meant by pair production?

➪ Pair production is the direct conversion of radiant energy to matter. It is the materialization of particle antiparticle pair, when electromagnetic rays pass in the vicinity of an electronic nucleus. The law of conservation of energy is obeyed in pair production, so the photon energy must exceed a certain threshold value, for example to produce electron - positron pair, the incident energy must be greater than or equal to 1.02 MeV.

Exercises

Short-Answer Type Questions

1. What are the similarities and differences between a neutrino and a photon?
2. How many types of quark you know? Name them with their electronic charges.
3. Show that proton contains three quarks: up, up and down.
4. What are quarks? Write their names with charge.
5. What are the similarities and differences between quarks and leptons?
6. Show that a proton contains three quarks: up, up and down (uud)
7. Which particle does the $\bar{u}\ \bar{u}\ \bar{d}$ combinations produce?
8. What are mesons? Write the names of two mesons.
9. Give two examples of the pairs of particle-antiparticle system.

Long-Answer Type Questions

1. Name the quarks you know. Also present the quark combinations of baryon and meson groups of particles.
2. Give an account of simple classification of elementary particles with examples.
3. Give brief history of elementary particles.
4. What are mediator particles? To which interactions they are associated?
5. Write brief notes about fermions and bosons.
6. What are generations of elementary particles? Describe their significances.

Multiple Choice Questions

1. Which is the particle-antiparticle pair?
   a. electron and proton b. electron and positron
   c. proton and neutron d. neutron and electron
2. Which of the following particle is considered as responsibility of mass giving?
   a. proton b. neutron
   c. Higgs boson d. Graviton
3. The quark combination of antineutron is,
   a. uud b. udd
   c. $\bar{u}\bar{u}\bar{d}$ d. $\bar{u}\bar{d}\bar{d}$
4. Which is the weakest fundamental force?
   a. Gravitational b. Electromagnetic
   c. Weak d. Strong
5. Mesons are made up of,
   a. One quark and an antiquark b. two quarks
   c. two antiquarks d. three quarks
6. Which is not the fundamental particle?
   a. up b. down
   c. electron d. proton

Answers

1. (b) 2. (c) 3. (d) 4. (a) 5. (a) 6. (d)

❖ ❖ ❖

COSMOLOGY

28

CHAPTER

28.1 Introduction

The branch of science, which deals with the study of the origin, evolution and nature of the universe, is called cosmology. Cosmology includes the study of the nature of the universe on its very large scales: planets orbit stars, stars are controlled into galaxies, galaxies are gravitationally bound into clusters and even clusters of galaxies are found with in larger super clusters.

In the earliest form, the study of cosmology was considered as the study of heavens, now it is known as celestial mechanics. In the beginning of study on it, Greek philosophers Aristotle and Ptolemy proposed different cosmological theories to explain the mystery of universe. Later on, Newton disclosed many unsolved problems associated with the universe after his universal law of Gravitation. Modern scientific cosmology is considered to have begun in 1917 with Albert Einstein's publication on his final modification of "General Theory of Relatively (GTR)".

28.2 The Universe

The universe is all around us, in our vision and beyond our vision. It is all of space and time and their contents, including solar system, other stars and planets, galaxies, and all other forms of matter and energy. The size of universe is still unknown. Many matters and energy of the universe have not been measured yet. Many of its constituents are invisible and are called dark matter and dark energy. The aggregation of matters and energy that are in measured form is known as observable universe.

Solar system

Solar system is the collection of the sun, eight planets and their moons in orbit round the sun, together with smaller bodies in the form of asteroids, meteoroids and comets. The sun is the center of our solar system. It is the largest body of the system. Eight planets revolve around the sun in elliptical path and orbits of these planets lie roughly in the same plane called elliptic plane. These planets are Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus and Neptune.

Moons, asteroids, comets, and meteoroids are also part of solar system. Moons orbit the planets. Asteroids, comets and Meteoroids orbit around the sun. Giant dust storms freezing temperatures, colourful clouds and beautiful rings can be found throughout the solar system.

Asteroids orbit the sun between the orbits of Mars and Jupiter. This region is called the asteroid belt. They are made of rock and iron. They are also called minor planets; scientists believe that early in the history of the solar system asteroid - like bodies colliding with each other grew to form the planets. They have very irregular in shape.

Meteoroids are basically small asteroids. There is no exact diameter that distinguishes an asteroid from a meteoroid. The vast majority of all meteoroids are just a few millimeters and less in size.

Comets are icy bodies in space that release gas or dust. They contain dust, ice, carbon dioxide, ammonia, methane and more. They appear fuzzy and has a tail, and sometimes bright. Since it contains water, ice and other frozen volatiles, its tail sublimate into gases and appears when gets closer to the sun. Comets usually have highly eccentric orbits, and they have a wide range of orbital periods ranging from several years to several millions of years. Comets are distinguished from asteroids by the presence of an extended, gravitationally unbound atmosphere surrounding their central nucleus.

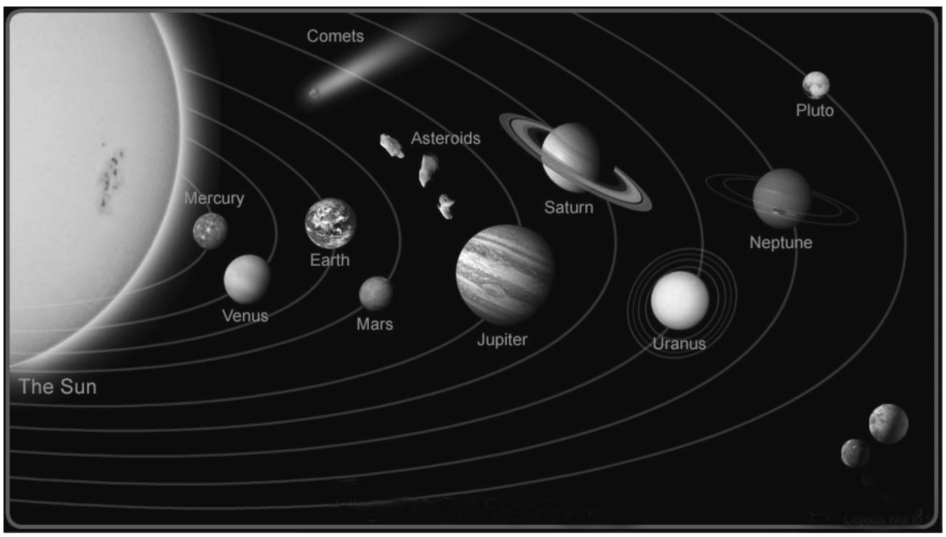


Fig. 28.1: Solar System

Differences between comets and asteroids

Comets	Asteroids
1. They reside mostly in Kuiper belt beyond the orbit of Neptune.	1. Most of them reside in asteroid belt between orbits of Mars and Jupiter.
2. Their diameter ranges from 6-25 miles.	2. Their diameter ranges from the size of small rocks to more than 600 miles.
3. They contain a lot of ice, along with rock and hydrocarbon.	3. They are composed of rocks and metals.
4. Their surface is very unstable and changeable, as ice boils off when comet approaches near to the sun.	4. Their surface is solid and stable, showing crates.

Stars

Stars are the astronomical objects consisting of fusion gases like hydrogen. They are the ball of hydrogen and helium with enough masses. The nearest star to the earth is the sun. Many other stars are visible in the sky with naked eyes. They are the most fundamental building blocks of galaxies. The study of the birth, life and death of stars is central to the field of astronomy.

Hydrogen fusion reaction is the dominant process of energy generation in the core of stars. This process is called hydrogen burning. Hydrogen burning is not the destruction; it is actually the fusion process. In hydrogen fusion in stars; four hydrogen nuclei ($4\ {}_1H^1$) fuse together to form a helium nucleus along with two positrons and release energy about 26 MeV. The nuclear fusion reaction is explained in chapter 24. The colour of stars depends on the temperature of star on its surface.

Constellation

A constellation is the group of stars that are considered to form meaningful patterns on the celestial sphere. The shape of a constellation looks like animals, mythological people or gods and manufactured devices. In the sky, there are 88 constellations which have been observed in the sky. The sun, its planets, and all other solar system objects move across the constellation of the Zodiac: Aries, Taurus, Gemini, Cancer, Leo, Virgo, Libra, Scorpios, Sagittarius, Capricorn, Aquarius, Pisces and Ophiuchus.

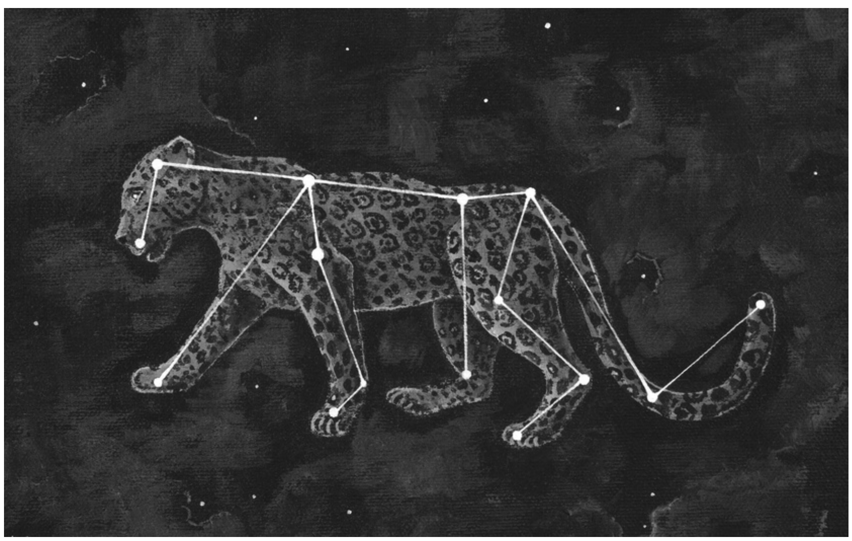


Fig. 28.2 A constellation : Leo

Galaxy

A galaxy is a gravitational bound system of stars, stellar remnants, interstellar gas, dust and dark matter. Hubble speculated that there are 100 billion galaxies in the universe. However, the number of galaxies has been observed many times greater than the Hubble's estimation. After the advancement of sophisticated telescope technology, it has been estimated that there are about 200 billions galaxies in the universe. Galaxies are classified into three main types: Spiral galaxies, elliptical galaxies and irregular galaxies. The majority of galaxies are gravitationally organized into groups, clusters and superclusters.

There are also some other types of galaxies which emit enormous radiation than normal galaxies. They are known as radio galaxy. Quasars and pulsars are the examples of radio galaxy.

Milky way galaxy

Our solar system lies within the Milky Way galaxy. Milky Way galaxy is a spiral galaxy. Hubble confirmed that the Milky Way galaxy is just one of many galaxies. It is visible from the earth as hazy bands of white light some 30 degrees wide arching across the sky. The visible region of Milky Way when viewed from the earth occupies an area of the sky that includes 30 constellations.

Milky way galaxy contains about 10^{11} stars. Its side to side length is about 105 light years. It bulges at the middle about 3000 light years wide. We are 30,000 light years far from the galactic center point.

Fig. 28.3: Milky way

28.3 Evolution of Star

The description of the way that stars change with time is called the evolution of stars. In close observation with sophisticated telescopes, many new stars are found created and existing stars are found dying in the universe. The time span between the birth and death of stars is called their lifetime. Their lifetime ranges from a few millions of years for highly massive to trillions of years for the least massive stars. The lifetime of some stars in accordance to their mass are tabulated below.

Mass (Solar masses)	Time (Years)
60	3 million
30	11 million
10	32 million
3.0	370 million
1.5	3 billion
1.0	10 billion
0.1	1000 billion

Birth of stars

Stars are heavy masses light emitting bodies. They are formed when atoms are of squeezed under enough pressure for their nuclei to undergo fusion. Initially, the clouds of gas and dust particles in the space are pulled together. A few grains of dust collect a few more, and then form a large ball. When this ball pulls more clouds and dust, it becomes a giant ball. The materials inside the giant ball are compressed together so that the temperature reaches 15 million degrees and so, the pressure at the centre of the giant ball becomes 1 billion atmospheres. Then, the nuclear fusion reaction begins in presence of high temperature and pressure. Eventually, the ball of gas and dust starts to glow Thus, a new star begins its life in the universe.

Death of star

Nuclear fuels are the resources of a star. By the steadily burning of nuclear fuel in its deep interior, a star fills up the heat that radiates from its hot surface into the cold depths of interstellar space. Each day, a star burns millions of tons of fuel at its center. Unfortunately, for any star, its fuel supply is limited. Therefore, the thermal and visible energy of the star not more exist forever. Finally, the star exhausts its nuclear fuel and dies.

The fate of a star is determined largely by its mass when it exhausts its nuclear fuel. Smaller stars die gently, by gradually cooling. But the larger stars die violently. They contract slowly at first and then collapse catastrophically. Sudden release of energy may convert this collapse. In the beginning of death of star, it turns into red giant.

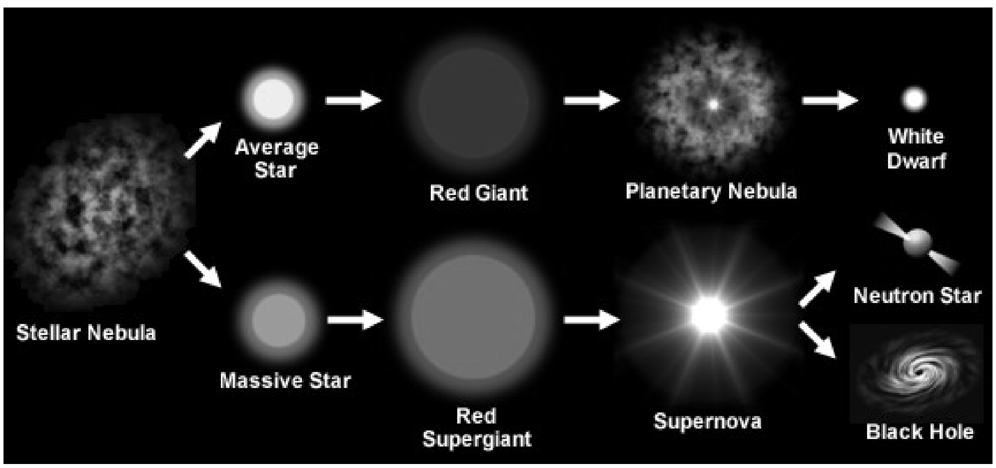


Fig. 28.4: Life Cycle of Star

Red giant

A red giant star is a dying star in the last stages of stellar evolution. A red giant star is formed when a star like our sun or one larger, runs out of its hydrogen fuel.

The fuel of star is the hydrogen fusion to helium. As the time passes, the hydrogen fuel will be depleted. Thus, the stars tend to shrink due to the losing of source. This is a dying process of stars. However, dying star continues radiating becoming hotter than before. Then its heat pressure weakens and gravity pulls it inward. The gravitational compression reheats the interior. About half of the heat generated transfer outward to its surface and radiates away. Remaining half part is trapped in the interior. The continuous cooling the exterior and heating at the inner core generates the huge temperature gradient. Then the star expands and grows to more than 10 times its original size. When the temperature inside the star reaches about 10^6 K, the helium is formed after the hydrogen fusion starts fusion again to form the higher elements like, beryllium, carbon oxygen, eventually turning to red colour. This is called red giant. The mass of red giant when exceeds three times the mass of sun, they are called super red giant.

White dwarfs

After depletion of nuclear fuel in the star, the surface cools and tends to shrink. But, the temperature rises at the inner part due to the gravitational compression. Gravitational compression and heating cannot continue indefinitely. When after millions of years, the star has diminished upto several thousand miles in diameter. Then, its central temperature rises to billions of degrees. The density of matter at its center has risen from several pounds to several tons per cubic inch. Then, the force of gravity within the star increases significantly. Despite the growth of gravity, the compression of small stars (M_s = mass of the sun) gradually stops due to fermi pressure (i.e. no two fermions lie in same quantum state). These stars are called white dwarfs. No write dwarfs is observed larger the 1.4 times the mass of sun.

Neutron stars

A neutron star is a collapsed core of large star which before collapse had a mass of larger than 3 times the mass of sun. It is the stellar remnant of a super giant. If the mass of red giant is greater than $3M_S$, the collapsing core raises its temperature and density so large that the nuclear fusion further begins from the carbon stage. If the mass is sufficiently large, the carbon fuses to neon at 6×10^7 K. Further at temperature 10^9 K, neon to oxygen. Likewise, further fusion reaction ends up to be iron, which is highly stable. Then, no energy is released by fusion reaction. The fusion reaction rate at its terminating stage is very fast. Then, a violent collapse of the core occurs. Eventually, outer layer of the star is thrown off, which is called supernova explosion.

In other cases, the nuclear explosion may be too weak to eject the outer shell. The entire star may continue to collapse, with rising densities and temperature, until its core becomes as dense as an atomic nucleus. During the Core's collapse, which lasts only a few seconds, the electrons in the atoms, unable to resist, are squeezed into the atomic nuclei, transmuting protons into neutrons. The collapse quickly packs 10^{57} neutrons side by side, as in a gigantic atomic nucleus, forming a neutron star.

Black hole

There are two main processes going on continuously in stars. One process is gravitation, which tends to crunch all solar material towards the centre. The other is thermo nuclear fusion consisting of reactions similar to those in a hydrogen bomb when the processes of gravitation and thermonuclear fusion balance each other, the result is the stars, they are existing now.

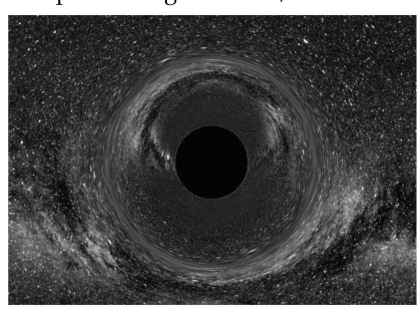


Fig. 28.5: Black hole

For a heavy star, one that is at least three times the mass of sun, once the flame of thermonuclear fusion is extinguished, gravitational collapse take over. The collapse does not stop and the density becomes literally infinite. Gravitation near it is so enormous that nothing can get back out. Even light can not escape. They have crushed themselves out of visible existence. They are called black holes. Although black holes can't be seen, their effect can be measured.

The velocity of escape from the surface of a spherical mass M with radius R is given by

$$v = \sqrt{\frac{2GM}{R}} \quad \ldots(28.1)$$

If ρ be the average density of the body, V be its volume then,

$$M = \rho V = \frac{4}{3}\pi R^3 \rho$$

Using this value in equation (28.1) we get,

$$\mathbf{v = \sqrt{\frac{8\,\pi G\rho}{3}}\,R} \qquad \text{...(28.2)}$$

This equation shows that, for a given value of density ρ, the escape velocity v is directly proportional to radius R. In 1783, John Mitchell noted that for a body with same average density as the sun and radius 500 times the radius of sun, the magnitude of escape velocity would be greater than velocity of light c. So, all the light emitted from such bodies would return toward it i.e., no light (radiation) can escape from the field of such bodies. Such bodies are called black hole. This a black hole is a region of space time exhibiting such strong gravitational effects that nothing -not even particles and electromagnetic radiation such as light can escape from it.

Again from equation (28.1), the radius R can be expressed in terms of escape speed as,

$$\mathbf{R = \frac{2\,GM}{v^2}} \qquad \text{... (28.3)}$$

Thus, a body of mass M will act as a black hole if its radius R is less than or equal to certain critical radius (R_s). In 1961, Karl Schwarzchild used Einstein's special theory of relativity to derive an expression for the critical radius, which is so called as Schwarzchild radius (R_s). The expression for R_s is obtained by setting v = c in equation (28.3) as,

$$\mathbf{R_s = \frac{2\,GM}{c^2}} \qquad \text{... (28.4)}$$

which is required expression for Schwarzchild's radius.

Thus, *if a spherical non-rotating body with mass M has a radius less than R_s, nothing, not even light can escape from the surface of the body, such body is called black hole.* Any other body within a distance of R_s from the center of black hole is attracted by the immense gravitational pull of black hole and hence can't escape from it. The surface of the sphere with radius R_s surrounding a black hole is called event horizon, and we can't see events occurring inside. All that can be known about black hole from outside the event horizon is its mass (due to gravitational effect on other bodies), its electric charge (from the electric forces it exerts on other charged bodies) and its angular momentum (because a rotating black hole tends to drag space and everything in that space-around within it). All other information of the body is lost when it collapses inside it (event horizon). At points far from a black hole, its gravitational effects are the same as those of any normal body with the same mass. So, if somehow sun collapsed to form a black hole, the orbits of the planets would be unaffected provided, the planets revolve far from the event horizon of sun.

28.4 Big Bang

It is the big curiosity of scientists that how our universe was created and what is its age. Many scientists put their views regarding the origin and evolution of the universe, however this question is still debatable. The broadly accepted theory on this issue is big bang model. This model states that the universe began as an incredibly hot, dense point roughly 14 billion years ago. At this time all matter was compacted into a very small ball with finite density and intense heat called a singularity. Suddenly, the singularity began expanding, and the universe as we know it began. Between 10^{-36} seconds to 10^{-33} seconds after Big Bang the universe expanded as fast as speed of light. The fundamental particles formed in the first three minutes after Big Bang. The first particles to form were the quarries and as the universe expanded and cooled, they jointed to gather to make protons, neutrons and other particles. The solar system formed about 9 billion years after Big bang. The universe was subjected to mere activity during Big bang than in all the billions of years since.

Big bang theory is the most widely accepted and popular theory. It explains not only the origin of all known matter, the laws of physics and the large scale structure of the universe, it also accounts for the expansion of universe and broad range of other phenomena. Besides big bang model, there are other theories on this regard: Steady state theory and the oscillating universe theory. The steady state theory purposed that the overall mass and size of universe remain constant, where, as the pulsating theory assumes that the universe is expanding and contacting periodically in billions of years.

Cosmic rays

Cosmic rays are the highly energetic atomic nucleus or other particles travelling through space at a speed approaching that of light. They are mainly originated outside the solar system and even from distant galaxies. Upon impact with Earth's atmosphere, cosmic rays can produce showers of secondary particles that sometimes reach the surface.

It is still impossible to trace where they come from. It is because their path has been changed as they travelled through multiple magnetic fields. Scientists are trying to trace back cosmic ray origins by looking at what the cosmic rays are made up of. They have been trying to figure out the origin from spectroscopic signature each nucleus gives off in radiation.

Red shift

The shifting of colour of light coming from a distant object into red, when the objects are going away from an observer is known as red shift. The red shift that can be observed in light from distant galaxies suggests that the universe is expanding, and thus supports the Big Bang theory. According to Doppler's effect, when a source of wave recedes from the observer, the wavelength of wave is observed larger than actual value. This effect is the basic concept of explanation of red shift. In visible spectrum, red colour has the longest wavelength. The colour of light as observed from earth, is gradually shifting to red colour. This means, the source of light might traversing away from us.

If there is blue shift, the object would come towards us. The red shift of a distant galaxies or quasars can be easily measured by comparing its spectrum with a reference laboratory spectrum. Atomic emission and absorption line occur at well known wavelengths. By measuring the location of these lines in astronomical spectra, astronomers can determine the red shift of the receding source.

28.5 Expanding of Universe

From the early era of human civilization, the curiosity regarding the origin, size and age of the universe began to hunt their mind. The questions such as how large the universe is? Does it have edge? From how long has this universe been existing and for how long will it exist?, remained unanswered. There were many myths regarding these questions but were not acceptable in scientific community as there were no strong proof behind them. In 1929, Edwin Hubble an astronomer at Carnegie observatories, made a critical discovery that the universe is expanding, which led to a interpretation consistent with Big bang theory. If the universe is expanding today, it was smaller and denser in the past.

In 1929 Edwin Hubble, working at the Carnegie observatories in Pasadena, California, measured the redshifts of a number of distant galaxies. He observed that the colour of light coming from distant galaxies is shifting to red. It shows that the wavelength of light is increasing as explained by

Doppler's effect. When a source of light is moving away from us, its wavelength is observed increasing. This phenomenon had disclosed two important consequences: One the universe is not static and the another it is expanding, rather contraction.

28.6 Hubble's Law

Edwin Hubble, an astronomer, measured the relative distant galaxies by measuring the apparent brightness of a class of variable stars called Cepheid's in each galaxy. When he plotted red-shifts against relative distance, he found that the red shift of distant galaxies increased as a linear function of their distance. Then, he formulated what he observed regarding the expanding of universe and is then called Hubble's law.

Hubble's law states that the speed of recession of a galaxy is directly proportional to the distance from the earth.

Let v be the speed of recession of a galaxy at distance r from the earth, then the law is expressed mathematically as,

$$v \propto r$$

$$\mathbf{v = H_0 r} \qquad \text{... (28.5)}$$

Where, H_0 is proportionality constant and is called Hubble constant.

The dimension of H_0: $H_0 = \frac{v}{r} = \frac{[LT^{-1}]}{[L]} = [T^{-1}]$.

$$H_0 = \frac{71 \text{ km/s}}{\text{mpc}} = 2.3 \times 10^{-18} \text{ s}^{-1}$$

The exact value of the Hubble constant is still somewhat uncertain, but is generally believed to the around 65 kilometers per second for every mega parsec in distance. (A mega parsec is given by 1 Mpc = 3×10^6 light years). This means that a galaxy 1 mega parsec away will be moving away from us at a speed of 65 km/s, while another galaxy 100 mega parsecs away will be receding at 100 times this speed. Thus, the recession velocities of distant galaxies are known from the red shift. Hubble's constant reflects the rate at which the universe is expanding.

Hubble discovered that the light coming from the distant galaxies are all red shifted. More the distance from galaxies, higher the red shift. Then, he plotted the recession velocity as a function of distance from the earth the graph was found linear as shown in Fig. (28.6).

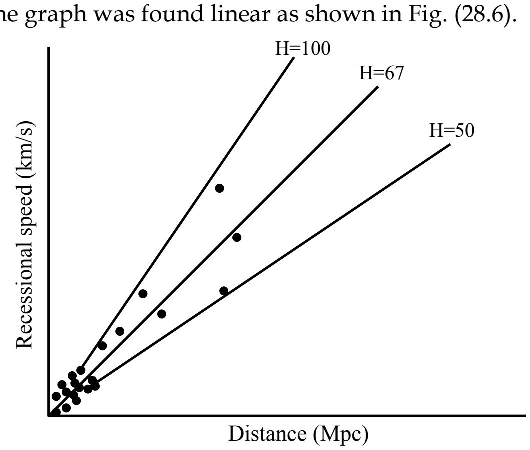


Fig. 28.6: Interpretation of Hubble's law

Consequence of Hubble's law and age of universe

Once scientist understood that the universe was expanding, they immediately realized that it would have been smaller in the past. At some point in the past, the entire universe would have been a single point. This point, later called the big bang, was the beginning of the universe as we understood it today.

Although the value of Hubble's constant is still debated issue, the present estimation is

$$H_0 = 50\,\frac{\text{kms}^{-1}}{\text{Mpc}} \approx 1.6 \times 10^{-18}\ \text{s}^{-1} = 5 \times 10^{-11}\ \text{year}^{-1}$$

Considering the recession velocity is the speed of light, the Hubble's radius is,

$$R_H = \frac{c}{H_0} = \frac{3 \times 10^5\ \text{km/s}}{50\,\dfrac{\text{kms}^{-1}}{\text{Mpc}}} = 6000\ \text{Mpc}$$

The time taken by light to travel about 6000 Mpc is called Hubble's time, and is taken to estimate the age of universe,

$$\begin{aligned} \text{The age of universe } (\tau) &= \frac{1}{H_0} \\ &= \frac{1}{50\,\dfrac{\text{kms}^{-1}}{\text{Mpc}}} \\ &= 19.3 \times 10^9\ \text{years} \end{aligned}$$

28.7 Critical Density

The universe includes planets, stars, galaxies dust clouds, light and even time. It contains billions of galaxies, each containing millions or billions of stars. The space between the stars and galaxies is largely empty. In accordance with Hubble's law, the universe is expanding. It means the density of universe is changing. The challenging question is that the expansion is continued forever or stopped somewhere. To solve this query, the concept of critical density is essential.

The critical density of the universe is the average density of matter required for the universe to just stop its expansion. This condition may come after an infinite time. After the density becomes critical, the universe will begin to contract and it will eventually become closed and will ultimately end up collapsing in on itself.

Let R be the radius of universe considering earth as the center and ρ be the average density of the universe. Then, the total mass of the universe,

$$\begin{aligned} M &= \text{Volume} \times \text{density} \\ &= \frac{4}{3}\pi R^3 \times \rho \end{aligned}$$

The expansion of universe continues until the critical density will be reached. At this condition, the gravitational potential energy of the universe is equal to the kinetic energy of recessive galaxies, so,

$$\frac{GmM}{R} = \frac{1}{2}\,mv^2 \qquad \text{... (28.6)}$$

Where m is the mass of the galaxy and v is its recession velocity.

When the critical density is reached, $M = \frac{4}{3}\pi R^3\rho_c$, ρ_c is the critical density of universe. Also, $v = H_0R$, then,

$$\frac{G}{R}\left(\frac{4}{3}\pi R^3\rho_c\right) = \frac{1}{2}(H_0R)^2$$

$$\frac{4}{3}\pi R^2 G\rho_c = \frac{1}{2}H_0^2 R^2$$

$$\rho_c = \frac{3}{8}\frac{H_0^2}{\pi G} \qquad \dots (28.7)$$

For $H_0 = 50$ kms^{-1}/Mpc and $G = 6.67 \times 10^{-11}$ Nm^{2}kg^{-2}

$\rho_c = 5.8 \times 10^{-27}$ kgm^{-3}.

It shows that the universe continues expanding until the density drops to 5.8×10^{-27} kgm^{-3} then halts the expansion.

28.8 Dark Matter and Dark Energy

Dark matter

The undetectable form of mass in the universe that emits little or no light but its existence we infer from its gravitational influence is known as dark matters. It neither emits or absorbs light or other electromagnetic radiation at any significant level. It is believed that the dark matter may account for approximately 80% of the mass energy of matter in the observable universe. Dark matter has not been directly observed, but its presence is implied in a variety of astrophysical measurements, notably when observing gravitational effects that cannot be explained by visible matter alone.

Different astronomical observations show that the average density of the matter in the universe is 27% of the critical density, but the average density of the luminous matter is only about 4% of the critical density. It means, most of the matter in the universe is not visible, it does not emit electromagnetic radiation of any kind such type of invisible matters are, now, termed as dark matters.

Dark energy

Gravitational force is always attractive in nature. In our general sense, the expansion of universe would be slowed down due to the gravitational attraction between matters in different parts of the universe. But in reality, it has been observed that the expansion of universe is speeding up rather than slowing down. This can be confirmed observing the red shift in extremely distant galaxies. Very distant galaxies actually have smaller red shifts than predicted by Hubble law, which would be the evidence of accelerating universe. This evidence convinced the astronomers and physicists that the space must contain a kind of energy that has no gravitational effect and emits no electromagnetic radiation, but rather acts as a kind of antigravity that produces a universal repulsion. This invisible form of energy which can be the source of a repulsive force causing the expansion of the universe to accelerate is known as dark energy.

Tips for MCQs

1. Modern scientific cosmology is consider to have begun in 1917 with Albert Einstein's Publication on GTR.
2. Universe consists of solar system, other stars and planets, galaxies and all other forms of matter and energy.
3. The death of stars can be white dwarf, neutron stars and black holes.
4. Before death of star, they become red giants.
5. According to Hubble's law, recession speed of galaxy is directly proportional to the distance (r) from the earth, i.e.

   $v \propto r$

   so, $v = H_0 r$

   Where H_0 is Hubble's constant. The reciprocal of Hubble's constant gives the age of universe ($\sim 19.3 \times 10^9$ years)
6. The critical density of universe is about 5.8×10^{-27} kgm^{-3}.
7. The universe is not only expanding, but also accelerating. It may be the expense of dark energy.

Worked Out Problems

1. Find the distance of the galaxy moving with speed 1.55×10^7m/s from the earth, according to the Hubble law. ($H_0 = 17 \times 10^{-3}$ ms^{-1}/ly).

Solution

Given,

Speed (v) = 1.55×10^7 m/s

$H_0 = 17 \times 10^{-3}$ ms^{-1}/ly (ly = light year)

distance (r) = ?

From Hubble's law, we have

$v = H_0 r$

$$\therefore \quad r = \frac{v}{H_0} = \frac{1.55 \times 10^7}{17 \times 10^{-3}} \, ly$$

$$= 9.2 \times 10^8 \, ly$$

$$= 9.2 \times 10^8 \times 9.46 \times 10^{15} \text{ m}$$

$$\therefore \quad r = 87.032 \times 10^{23} \text{ m}.$$

2. If the galaxy moving with the speed 6480 km/s is at a distance of 430 million light years from us, determine Hubble's constant H and the corresponding age of the universe.

Solution

Given,

Speed (v) = 6480 km/s

= 6.48×10^6 m/s

Distance (r) = 430 million light year

= $430 \times 10^6 \times 9.46 \times 10^{15}$ m ($\because$ 1 million = 10^6 and 1 light year = 9.46×10^{15} m)

H_0 = ?

$$\therefore \quad H_0 = \frac{v}{r} = \frac{6.48 \times 10^6}{430 \times 10^6 \times 9.46 \times 10^{15}}$$

$$= \frac{6.48}{4067.8} \times 10^{-15} = 1.59 \times 10^{-18} \, s^{-1}$$

The reciprocal of Hubble's constant (H) gives the age of the universe. So,

$$t = \frac{1}{H_0}$$

$$= \frac{1}{1.59 \times 10^{-18}}$$

$$= 0.629 \times 10^{18} \text{ s}$$

$$= \frac{0.629 \times 10^{18}}{3.15 \times 10^7} \quad (\because 1 \text{ year} = 3.15 \times 10^7 \text{ s})$$

$$= 0.199 \times 10^{11} \text{ year}$$

$$= 1.99 \times 10^{10} \text{ years}$$

Conceptual Questions with Answers

1. What is cosmology?

➯ Cosmology is the study of the origin, evolution, and eventual fate of the universe. Physical cosmology is the scientific study of the universe's origin, its large-scale structures and dynamics, and its ultimate fate, as well as the scientific laws that govern these areas. Modern cosmology is dominated by the Big Bang theory, which brings together observational astronomy and particle physics.

2. What are astrology and astrophysics?

➯ Astrology is the study of the movements and relative positions of celestial bodies interpreted as having an influence on human affairs and the natural world. Astrophysics is the branch of astronomy concerned with the physical nature of stars and other celestial bodies, and the application of the laws and theories of physics to the interpretation of astronomical observations.

3. What are the constituents of universe?

➯ The Universe is all of space and time and their contents, including planets, stars, galaxies, and all other forms of matter and energy. While the spatial size of the entire Universe is still unknown, it is possible to measure the observable universe.

4. Does the universe have edge?

➯ There is no evidence that the universe has an edge. The part of the universe we can observe from the earth is filled more or less uniformly with galaxies extending in every direction as far as we see. We know that the galaxies must extend further than we can see, but we do not know whether the universe is infinite or not.

5. What is solar system? What are its constituents?

➯ The Solar System is the gravitationally bound system comprising the Sun and the objects that orbit it, either directly or indirectly. Solar system consists of the Sun, the planets: Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune. It includes: the satellites of the planets; numerous comets, asteroids, and meteoroids; and the interplanetary medium.

6. What is the significance of red shift?

➯ The changing of colour of light sent out by an celestial object into red that is moving away from an observer is called red shift The red shift that can be observed in light from distant galaxies suggests that the universe is expanding, and thus supports the Big Bang theory.

7. What is the evidence of expanding of universe?

➯ The event of red shift in the distant stars gives the strong evidence for the expanding of universe. Hubble's telescope has confirmed that the speed of distant stars is directly proportional with the distance from the earth. So, it has been speculated that the universe is not only expanding but also accelerating outwards.

8. What is Hubble’s constant? Does Hubble’s constant is universal constant?

➯ The Hubble’s constant is the unit of measurement used to describe the expansion of the universe. It is Hubble’s constant gives the expansion rate of the universe and the universe is accelerating. So, It is not universal constant.

9. It is believed that the universe is not only expanding, but speeding up also. Is there an antigravity?

➯ Current studies of distant exploding stars have led astronomers to conclude that the universe is not only expanding, the expansion may be accelerating with time. There is not due to antigravity force, but may be by the dark energy.

10. Will the Universe expand forever or recollapse?

➯ This depends on the ratio of the density of the Universe to the critical density. If the density is higher than the critical density the Universe will recollapse in a Big Crunch. But current data suggest that the density is less than or equal to the critical density so the Universe will expand forever.

11. What is the evidence for the Big Bang?

➯ The evidence for the Big Bang comes from many pieces of observational data that are consistent with the Big Bang. None of these *prove* the Big Bang, since scientific theories are not proven. Many of these facts are consistent with the Big Bang and some other cosmological models, but taken together these observations show that the Big Bang is the best current model for the Universe. These observations include:

i. The darkness of the night sky.

ii. The Hubble Law - the linear distance versus redshift law.

iii. Fair data showing that our location in the Universe is not special.

iv. Very strong data showing that the sky looks the same in all directions.

v. Time dilation in supernova light curves.

12. Why big bang is most accepted theory?

➯ Astronomers Edwin Hubble and Milton Humason in the early 20th century discovered that galaxies are moving away from the milkyway. Every galaxy is moving away from every other galaxy on average, which means the whole universe is expanding. In the past, then, the whole cosmos must have been much smaller, hotter and denser.

13. What are cosmic rays?

➯ Cosmic rays are the highly energetic atomic nucleus or other particle travelling through space at a speed approaching that of light. They are mainly originating outside the solar system and even from distant galaxies. Upon impact with Earth's atmosphere, cosmic rays can produce showers of secondary particles that sometimes reach the surface.

Exercises

Short-Answer Type Questions

1. Which is the modern theory of study in cosmology?
2. What is visible universe?
3. What are the major constituents of solar system?
4. What is solar system?
5. Define the following terms: Comet, Asteroid, Meteor, Meteorite, Constellation, Galaxy.
6. What is Milky Way? Why is it called so?
7. What is stellar evolution?
8. What are meteorites?
9. Why were atoms unable to exist until hundreds of thousands of years after the Big-Bang?
10. Under what circumstance would the universe eventually collapse into itself?
11. Where are cosmic rays come from?
12. What will determine whether the universe continues to expand forever or eventually slows down and re-collapses?
13. Why will the sun stop fusing hydrogen long before all its hydrogen has been converted to helium?
14. What are galaxies? How many galaxies approximately are there?
15. Why has a comet a tail?
16. What is difference between a planet and a star?
17. What is the difference between a neutron star and a black hole?
18. State Hubble's law and give its significance.
19. What is red giant?
20. Write down the expression of critical density of the universe and its significance.
21. Why dark energy and dark matter are named so?

Long-Answer Type Questions

1. What is universe? Explain the constituents of the universe.
2. Explain how universe expands. Explain the Hubble's law. (HSEB 2062)
3. Describe the birth of a star.
4. Describe the death of a star. (HSEB 2066)
5. What is the critical density of the universe? Derive its expression.
6. State Hubble's law. How does the Hubble's constant help to estimate the age of the universe?
7. State Hubble's law. How this law can be used to explain that universe is expanding?
8. What is red shift? How does it support the expansion of universe?
9. State briefly Big Bang theory and mention observational evidence that supports this theory.
10. Discuss the future of the universe on the basis of critical density.
11. Describe the evidence of dark matter and dark energy?

Numerical Problems

1. Estimate the temperature of the sun from the following data: Average radius of the sun = 7.0×10^5 km; solar constant = 1400 Wm^{-2}. Average radius of the earth's orbit = 1.5×10^8 km.

   Ans: 5802.7 K
2. If a galaxy is at a distance of 500 million light years from us and is receding with a speed of 8×10^6 m/s, find the value of Hubble's constant and the corresponding age of the universe.

   Ans: 1.6×10^{-5} km s^{-1} light $year^{-1}$; 1.875×10^{10} years

Multiple Choice Questions

1. From Hubble's law, it has been estimated that the age of this universe is,
   a. 1.93×10^9 years b. 19.3×10^9 years
   c. 1930 years d. Will be destroyed in a few years.
2. 1 Mega parsec is equal to
   a. One hour b. 1 light year
   c. 3×10^6 light year d. Infinity
3. The critical density of universe is,
   a. 5.8×10^{-27} kgm^{-3} b. 5.8×10^{27} kgm^{-3}
   c. 8.5×10^{-27} kgm^{-3} c. 8.5×10^{27} kgm^{-3}
4. Which of the following theories is the most satisfactory about the origin of the universe?
   a. Big-Bang theory. b. Pulsating theory.
   c. Steady state theory. d. None of above.
5. One main characteristics of black hole is that, it
   a. emits a photon. b. absorbs a photon.
   c. changes photon into mass. d. charges all colours into black one.
6. Hubble's law is based on
   a. Stefans' law. b. Wien's law.
   c. Doppler's effect. d. Newton's law of gravitation.

Answers

1. (b) 2. (c) 3. (a) 4. (a) 5. (b) 6. (c)

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Model Questions

Time 3 hours | Full Marks: 75

(All answers of numerical problems should be expressed in S.I. system) | Pass Marks: 27

Group A

1. Attempt any FOUR questions: [2×4 = 8]

a. Two wires, one of copper and other of iron, have he same diameter and carry the same current. In which wire will the drift velocity of electrons be more?

b. Differentiate between fuse wire and a heating wire.

c. Why are the pole-pieces of magnets cut into cylindrical form in a galvanometer?

d. Hall voltage is much more measurable in semi-conductor than in metals. Why?

e. Explain why two parallel wires carrying current in the opposite direction repel each other?

f. 220V a.c. is more dangerous than 220V d.c., why?

2. Attempt any FOUR questions [2 × 4 = 8]

a. If the discharge tube is filled up with various gases in turn, will the discharge in all gases take place at the same electrode potential?

b. A photon and an electron have got the same de-Broglie wave length. Explain which has greater total energy.

c. How is NOT gate realised?

d. It is said that a very powerful crane is required to lift a nuclear mass of microscopic size. Comment on this.

e. Comment on the statement ''A nucleus contains no electrons and yet can eject them."

f. What are the effects of pollution on living organisms?

3. Attempt any ONE question (1 x 2 = 2)

a. How can bats fly around without colliding with objects that come in their way?

b. Longitudinal waves cannot he polarized. Why?

4. Attempt any ONE question (1 x 2=2)

a. Differentiate between wave-front and wavelet?

b. What is the difference between Fresnel and Fraunhofer diffraction?

Group B

5. Attempt any THREE question (4 x 3 = 12)

a. State Biot and Savart law and use it to obtain an expression for the magnetic field at the centre of the circular coil.

b. What are the categories in which magnetic materials are classified? Explain their differences.

c. State Faraday's laws of electrolysis. How will you verify Faraday's second law experimentally?

d. Show that Lenz's law is an example of conservation of energy.

6. Attempt any THREE question (4 x 3 = 12)

a. Show, in Bohr's model, that radii of electronic orbits increase an n^2, where n is the quantum number of the orbit.

b. Define decay constant of a radioactive element. How is it related to half-life'?

c. Discuss a zener diode and its use as voltage stabilizer.

d. Describe a theory which accounts for the origin and evolution of the universe.

7. Attempt any ONE question (4 x 1 = 4)

a. Show that both harmonics, odd and even, can he produced in an organ pipe open at both ends.

b. What is Doppler's effect? Obtain an expression for the apparent pitch when a source moves towards a stationary observer.

8. Attempt any ONE question (4 x I = 4)

a. Show that in Young's double slits experiment widths of dark and bright fringes are equal.

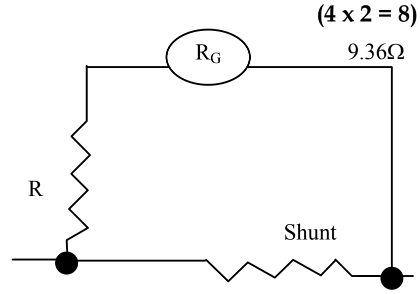
b. Describe Focault's method of determining the speed of light.

Group C

9. Attempt any TWO questions **(4 x 2 = 8)**

a. The resistance of the coil of a pivoted-coil galvanometer coil is and a current of 0.0224 A causes it to deflect full scale. We want to convert this galvanometer to an ammeter reading 20.0 A full-scale. The only shunt available has a resistance of 0.025 Ω. What resistance R must be connected in series with the coil?

Ans: 12.94Ω

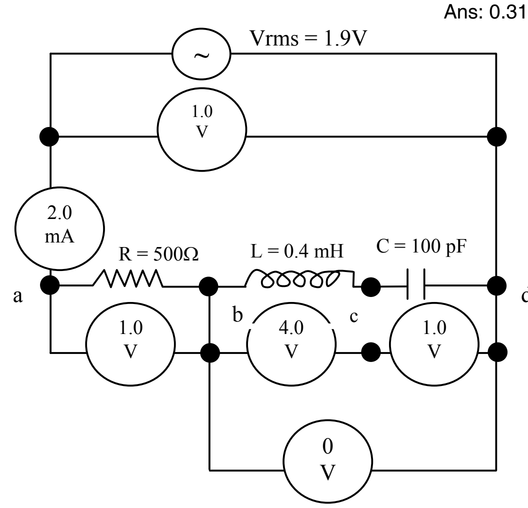


b. A standard cell of 1.0185 V, when used in a one meter long slide wire potentiometer balances at 60 cm. Calculate the percentage error in a voltmeter which balances at 65 cm when reading is 1.1 volt.

Ans: 0.31%

c. The series circuit in figure is a similar to arrangements that are sometimes used in radio tuning circuits. The circuit is connected to the terminals of on a.c. source with a constant r.m.s. terminal voltage of 1.9 V and a variable frequency. Find (i) the resonance frequency (ii) the inductive reactance and the impenance at the resonance frequency (iii) the r.m.s. current at the resonance and (iv) the r.m.s. voltage across each circuit element at resonance.

Ans: (i) $W_0 = 5.0 \times 10^6$ rad s^{-1} (ii) $X_L = 2000\Omega$; $X_C = 2000\Omega$ (iii) $I_{rms} = 2.0$ mA (iv) $V_{R-rms} = 1V$, $V_{L-rms} = 4V$; $V_{C-rms} = 4V$



10. Attempt any TWO questions **(4×2=8)**

a. A city requires 10^8 watts of electrical power on the average. If this is to be supplied by a nuclear reactor of efficiency 20% using $^{235}_{92}U$ as the fuel. Calculate the amount of fuel required for one day's operation. (Given: energy released per fission of $^{235}_{92}U$ 200 MeV).

Ans: 0.527 kg

b. A clean nickel surface of work function 5.1 eV is exposed to light of wavelength 235 nm. What is the maximum speed of the photoelectrons emitted from their surface?

Ans: 2.52×10^5 ms^{-1}

c. An electron moving with a speed of 10^7 m/s is passed into a magnetic field of intensity 0.1×10^{-2}T normally. What is the radius of the path of the electron inside the field? If the strength of the magnetic field is doubled, what is the new radius of the path? (e/m = 1.8×10^{11} $C.kg^{-1}$)

Ans: –5.55cm

11. What is the difference between the speed of longitudinal waves in air at 27°C and their speed at -13°C? What is the speed at 0°C? (4)

Ans: 23.84ms^{-1}; V_0= 332.16 ms^{-1}

12. Light travelling in water strikes a glass plate at all angle of incidence of 53°, part of the beam is refracted and part is reflected. If the refracted and reflected portions make an angle of 90° with each other, what is the index of refraction of glass? (3)

Ans: $\mu_g = 1.76$ assuming $\mu_w = 1.33$

2074 Set A

Group 'A'

1. Answer, in brief, any four questions: [4×2=8]

a. You are given n wires, each of resistance R. What is the ratio of maximum to minimum resistance obtainable from these wires? [2]

b. Why do we prefer a potentiometer to measure emf of a cell rather than a voltmeter? [2]

c. What is angle of dip? How is it related with components of earth's magnetic field? [2]

d. Why is soft iron used to make core of a transformer? [2]

e. If the number of turns of a solenoid is doubled, keeping the other factors constant, how does the self inductance of the solenoid change? [2]

f. The emf of an ac source is given by the expression, E=300 sin 314 t volts. Write the values of peak voltage and frequency of source. [2]

2. Answer, in brief, any four questions. [4×2=8]

a. Why is neutron considered the most effective bombarding particle in a nuclear reaction? [2]

b. The value of e/m is constant for cathode rays but not for positive rays, why? [2]

c. The output of two-input AND gate is fed to a NOT gate. Give its logic symbol and write down its truth table. Identify the new logic gate formed. [2]

d. How does a daughter nucleus differ from its parent nucleus when it emits i) an α-particle and ii) a β- particle? [2]

e. State Hubble's law and write the significance of Hubble's constant. [2]

f. What is energy crisis? Explain. [2]

3. Answer, in brief, any one question. [2]

a. Longitudinal waves are called pressure waves. Why? [2]

b. What is the threshold of hearing? Define one bel. [2]

4. Answer, in brief, any one question. [2]

a. Explain with proper sketch, the differences between wavefronts and wavelets. [2]

b. What is polarizing angles? Does it depend on wavelength of light used? [2]

Group 'B'

5. Answer any three questions. [3×4=12]

a. Describe the mechanism of current flow in a conductor and derive a relation between current density and drift velocity of electrons. [4]

b. What is Seebeck effect? Explain the variation of thermo emf with gradual increase in the temperature of hot junction, keeping the cold junction at 0^0c. [4]

c. State Biot-Savart law. Use this law, to find the magnetic field due to a current carrying circular coil at any point on the axis of the coil. [4]

d. State and explain Faraday's law of electromagnetic induction. Obtain an expression for the emf induced in the rectangular coil rotating in a uniform magnetic field. [4]

6. Answer any three questions. [3×4=12]

a. What is zener diode? Explain its use as a voltage regulator. [4]

b. Discuss photoelectric effect and derive Einstein's photoelectric equation. What is stopping potential? [4]

c. Define mass defect and binding energy of a nucleus. Draw a graph showing the variation of binding energy per nucleon and atomic number of the elements. Also, interpret the graph. [4]

d. Explain renewable and non-renewable source of energy with examples. Give an account of the energy consumption scenario in Nepal. [4]

7. Answer any one question. **[4]**

a. Describe Newton's formula for the velocity of sound in air. Explain why and how this formula is modified by Laplace. [4]

b. Describe an experiment with the necessary theory by which the speed of sound in air is determined by using resonance tube method. [4]

8. Answer any one question. **[4]**

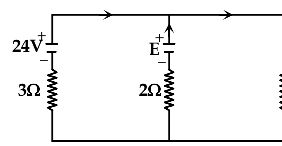
a. Define coherent sources of light. Prove that the dark and bright fringes are equally spaced in Young's double slit experiment. [4]

b. What is diffraction grating? Discuss the formation of diffraction pattern due to a diffraction grating. [4]

Group 'C'

9. Answer any two questions. **[2×4=8]**

a. What must be the emf E in the circuit so that the current flowing through the 7Ω resistor is 1.80A? Each emf source has negligible internal resistance. [4]



Ans: 8.6 V

b. A straight horizontal rod of length 20 cm and mass 30 gm is placed in a uniform horizontal magnetic field perpendicular to the rod. If a current of 2A through the rod makes it self supporting in the magnetic field, calculate the magnetic field. [4]

Ans: 0.75 T

c. A coil of inductance 0.1H and negligible resistance is in series with a resistance 40Ω. A supply voltage of 50v (rms) is connected to them. If the voltage across L is equal to that across R, calculate the voltage across the inductor and frequency of the supply. [4]

Ans: 63.7 Hz, 28.3 V

10. Answer any two questions. **[2×4=8]**

a. An electron moves in a circular path of radius 20 cm in a uniform magnetic field of 2×10^{-3}T. Find the speed of the electron and period of revolution. Mass of electron $=9.1\times10^{-31}$kg. [4]

Ans: 7.02×10^7 m/sec and 5.6×10^7 rev/sec

b. Calculate de Broglie wavelength of an electron which has been accelerated through a potential difference of 200V. Given-mass of electron $=9.1\times10^{-31}$kg and Planck's constant $h=6.6\times10^{-34}$JS. [4]

Ans: 8.7×10^{-11} m

c. The isotope Ra-226 undergoes α decay with a half life of 1620 years. What is the activity of 1 g of Ra-226? Avogadro number = 6.023×10^{23}/mole. [4]

Ans: 3.47×10^{10} dis/sec

11. A car is approaching towards a cliff at a speed of 20m/s. The driver sounds a whistle of frequency 800 Hz. What will be the frequency of the echo as heard by the car driver? Velocity of sound in air = 350m/s. [4]

Ans: 896.96 Hz

12. A plane mirror is placed at the centre of a concave mirror having radius of curvature 40 m. The plane mirror rotates at the speed of 2600 revolutions per second. Calculate the angle between ray incidents on the plane mirror and then reflected from it after the light has travelled to the concave mirror and back to the plane mirror. Given -speed of light is 3×10^8 m/s. [3]

Ans: 0.5°

2074 Set B

Group 'A'

1. Answer, in brief, any four questions. **[4×2=8]**

a. Resistors R_1 and R_2 are connected in parallel to an emf source that has negligible internal resistance. What happens to the current through R_1 when R_2 is removed from the circuit? [2]

b. Why do we prefer potentiometer of longer length for accurate measurement? [2]

c. What is temperature of inversion? How does it change, if temperature of cold junction decreases? [2]

d. What are eddy currents? How can these be reduced in a transformer? [2]

e. Define rms value of ac. How is it related with the peak value of ac? [2]

f. The conductivity of an electrolyte is very low as compared to a metal at room temperature, why? [2]

2. Answer, in brief,any four questions. **[4×2=8]**

a. Draw a circuit diagram for p-n junction diode in forward bias. Sketch the voltage versus current graph for it. [2]

b. A proton and an electron have the same kinetic energy. Which has longer de Broglie wavelength? [2]

c. According to properties of charges, like charges repel each other. Then, how do the protons in a nucleus stay together? [2]

d. If a radioactive nucleus has a half life of one year, will it be completely decayed at the end of two year? Explain. [2]

e. State Hubble's law. What do you mean by dark matter? [2]

f. What is energy crisis? Explain. [2]

3. Answer, in brief, any one question. **[2]**

a. Velocity of sound in solids is more than that in liquids, why? [2]

b. The frequency of a fundamental note of a closed organ pipe and that of an open organ pipe are the same. What is the ratio of their lengths? v

4. Answer, in brief, any one question. **[2]**

a. What are coherent sources of light? Can two different bulbs, similar in all respects, act as coherent sources? [2]

b. Can Sound waves be polarized? Explain. [2]

Group 'B'

5. Answer any three questions. **[3×4=12]**

a. State and explain Kirchhoff's laws of current and voltage. Explain how these laws are used to obtain balance condition of Wheatstone's bridge. [4]

b. Derive an expression for the force per unit length between two infinitely long parallel straight wires carrying current in the same direction. Hence define one ampere. [4]

c. Define permeability and susceptibility of magnetic materials. Derive a relation between them. [4]

d. Derive an expression for the impedance of an ac circuit with an inductor L, a capacitor C and a resistor R in series. Draw the phase diagram if the voltage across the inductor is greater than that across the capacitor. [4]

6. Answer any three questions. **[3×4=12]**

a. Describe the theory of Millikan's oil drop experiment to determine the charge of an electron. [4]

b. What is rectification? With the help of a circuit diagram, explain full wave rectification by using junction diodes. [4]

c. State Bohr's postulates. Using these postulates obtain an expression for the total energy of an electron in the n_{th} orbit of hydrogen atom. [4]

d. What is green house effect? Discuss its effects, sources and the controlling measures. [4]

7. Answer any one question. [4]

a. How is a progressive wave different from a stationary wave? Derive an equation for a progressive wave. [4]

b. What is Doppler's effect in sound? Obtain an expression for the apparent frequency when both source of sound and observer move towards each other. [4]

8. Answer any one question. [4]

a. What is a wavefront? Using Huygen's principle proves that for a parallel beam of light incident on a reflecting surface, the angle of incidence is equal to angle of reflection. [4]

b. Describe Fraunhofer's diffraction at a single slit. [4]

Group 'C'

9. Answer any two numerical questions. [2×4=8]

a. A copper wire has a diameter of 1.02 mm and carries a constant current of 1.67A. If the density of free electrons in copper is $8.5\times10^{28}/m^3$, calculate the current density and the drift velocity of the electrons. [4]

Ans: 1.5×10^{-4} m/sec and 2.05×10^{6} A/m^2

b. A coil consisting of 100 circular loops with radius 60 cm carries a current of 5A. Find the magnetic field at a point along the axis of the coil, 80 cm from the centre. ($\mu_0=4\pi\times10^{-7}Tm/A$) [4]

Ans: 1.13×10^{-4} T

c. An aircraft with a wingspan of 40 m files with a speed of 1080 km hr^{-1} in the eastward direction at a constant altitude in the northern hemisphere. Where the vertical component of earth's magnetic filed is 1.75×10^{-5} T. Find the emf that develops between the tips of the wings. [4]

Ans: 0.21 V

10. Answer any two numerical questions. [2×4=8]

a. Radiations of wavelength 5400 Å fall on a metal whose work function is 1.9 eV. Find the energy of the photoelectrons emitted and their stopping potential. Planck's constant = 6.62×10^{-34} JS. [4]

Ans: 0.58 V

b. The mass of $_{17}CL^{35}$ is 34.9800 amu. Calculate its binding energy and binding energy per nucleon. Mass of one proton = 1.007825 amu and mass of one neutron =1.00865 amu. [4]

Ans: 287.66 MeV and 8.21 MeV

c. Calculate the mass in grams of a radioactive sample Pb-214 having an activity of 3.7×10^4 decays / s and a half life of 26.8 minutes. Avogadro number = 6.02×10^{23} /mole. [4]

Ans: 3.05×10^{-17} kg

11. A steel wire of length 20 cm and mass 5 gram is under the tension of 500N and is tied down at both ends. Calculate the frequency of fundamental mode of vibration. [4]

Ans: 353.55 Hz

12. In a Young's double slit experiment, the separation of four bright fringes is 2.5 mm. The wavelength of light used is 6.2×10^{-5} cm and the distance from the slits to the screen is 80 cm. Calculate the separation of slits. [3]

Ans: 7.9×10^{-4} m

2075 Set A

Group 'A'

1. Answer, in brief, any four questions. [4×2=8]

a. A wire is stretched to double its length. What will happen to its resistivity and resistance?[2]

b. Differentiate between a fuse wire and a heating wire. [2]

c. Does the thermoelectric effect obey the law of conservation of energy? Justify? [2]

d. A solenoid tends to contract when a current passes through it. Why? [2]

e. A bar magnet falls through copper ring. Will its acceleration be equal to 'g'? Justify. [2]

f. Why is choke coil preferred over a resistance in a.c.? [2]

2. Answer, in brief, any four questions. [4×2=8]

a. Gases are insulators at ordinary pressure and start conducting at low pressure. Why? [2]

b. A photon and an electron have got the same de-Broglie wavelength. Which one has greater total energy? Explain. [2]

c. What happens to the kinetic energy of photo electrons when intensity of light is doubled? [2]

d. A semiconductor has electrons and holes as charge carriers. Do conductors also have the holes as charge carriers? Justify. [2]

e. A nucleus consists of positively charged protons and electrically neutral netutrons in a small volume. How can this be possible as the like charges repel each other? [2]

f. If energy is conserved, why is there an energy crisis? [2]

3. Answer, in brief, any one question. [2]

a. We can't hear echo in a small room. Why? [2]

b. Justify the proverb "An empty vessel makes much noise". [2]

4. Answer, in brief, any one question. [2]

a. Differentiate between wavelets and wavefront. [2]

b. Does the polarizing angle for a transparent medium depend upon the wavelength of the light? [2]

Group 'B'

5. Answer any three questions. [3×4=12]

a. State Biot's and Savart's law and used it to obtain an expression for the magnetic field at the center of a circular coil. [4]

b. State the principle of potentiometer. Discuss the application of potentiometer to determine the internal resistance of a cell. [4]

c. State Faraday's laws of electrolysis. How will you verify Faraday's second law experimentally? [4]

d. What is an LCR circuit? Derive the condition for resonant frequency for an LCR series circuit with an a.c. supply. [4]

6. Answer any three questions. [3×4=12]

a. Stating the Bohr's postulates, deduce an expression for the total energy of an electron in nth orbit of hydrogen atom. [4]

b. Discuss J.J Thomson's experiment to determine the specific charge of an electron. [4]

c. What is the difference between a zener diode and a common diode? Discuss the function of Zener diode as a voltage regulator. [4]

d. Differentiate between nuclear fission and fusion. Explain the production of energy in the Sun. [4]

7. Answer any one question. [4]

a. What do you understand by harmonics and overtones in the case of organ pipes? Prove that only odd harmonies are produced in closed organ pipes. [4]

b. What is Doppler's effect? Obtain an expression for the apparent pitch when a source moves away from a stationary observer. [4]

8. Answer any one question. [4]

a. Discuss the Young's double slit experiment and show that the width of bright and dark fringes are equal. [4]

b. Describe Foucault's method of determining the speed of light. [4]

Group 'C'

9. Answer any two numerical questions. [2×4=8]

a. A straight conductor of length 25 cm is moving perpendicular to its length with a uniform speed of 10 m/s making an angle of 45^o with a uniform magnetic field of 10 T. Calculate the emf induced across its length. [4]

Ans: 0.35 V

b. Two lamps rated 25 W – 220 V and 100 W – 220 V are connected to 220 V supply. Calculate the powers consumed by the lamps. [4]

Ans: 16W, 4W

c. A bar magnet, 10 cm in length, has pole strength of 10 AM. Determine the magnetic field at a point on its axis at a distance of 15 cm from the center of the magnet. (μ0 = 4π × 10–7 H/m) [4]

Ans: 7.5×10^{-5}T

10. Answer any two numerical questions. [2×4=8]

a. A city requires 107 watts of electrical power on the average. If this is to be supplied by a nuclear reactor of efficiency 20%. Using 92U235 as the fuel source, calculate the amount of fuel required per day (Energy released per fission 92U235 = 200 MeV). [4]

Ans: 0.527 kg

b. A clean nickel surface of work function 5.1eV is exposed to light of wavelength 235 nm. What is the maximum speed of the photoelectrons emitted from its surface? [4]

Ans: 2.91×10^{-20}J, 2.52×10^5 m/sec

c. An electron moving with a speed of 107 m/s is passed into a magnetic field of intensity 0.1 T normally. What is the radius of the path of the electron inside the field? If the strength of the magnetic field is doubled, what is the radius of the new path? (e/m = 1.8 × 1011C/kg) [4]

Ans: 0.056m, 0.0278m

11. What is the difference between the speed of longitudinal waves in air at 27^o and at – 13^oC? What is the speed of 0^oC? [4]

Ans: 23.96m /sec, 331.1 m/sec

12. How wide is the central diffraction peak on a screen 3.5 m behind a 0.01 mm slit illuminated by 500 nm light source? [3]

Ans: 0.35 m

2075 Set B

Group 'A'

1. **Answer, in brief, any four questions.** **[4×2=8]**
   a. Why do we prefer a potentiometer with longer wire? [2]
   b. Why is Lead (Po) used as a standard reference metal in thermo-electricity? [2]
   c. Why is the conductivity of an electrolyte low in comparison to that of metal? [2]
   d. A solenoid tends to contract when a current flows through it. Why? [2]
   e. What is the significance of the area of a hysteresis loop? [2]
   f. Birds sitting on a high tension line wire fly off when current is switched on. Why? [2]
2. **Answer, in brief, any four questions.** **[4×2=8]**
   a. Why is a magnetic field used to deflect electron beam but not an electric field in a T.V. picture tube? [2]
   b. In a transistor, emitter-base junction is always forward biased. Why? [2]
   c. Define acid rain and write it's adverse effects. [2]
   d. All nuclei have nearly the same density. Why? [2]
   e. Distinguish between leptons and quarks. [2]
   f. Give two evidences to show that the universe is expanding. [2]
3. **Answer, in brief, any one question.** **[2]**
   a. Frequency is the most fundamental property of a wave. Why? [2]
   b. Why is the voice of a woman more intelligible than that of a man? [2]
4. **Answer, in brief, any one question.** **[2]**
   a. Distinguish between wavelet and wavefront. [2]
   b. Can ultrasonic waves be polarized? [2]

Group 'B'

5. **Answer any three questions.** **[3×4=12]**
   a. State and explain Joule's law of heating. Deduce an expression for heat developed in a conductor due to the passage of an electric current. [4]
   b. State Kirchhoff's laws and use them to derive Wheat Stone's bridge principle. [4]
   c. State Ampere's law and use it to find magnetic field due to a long straight current carrying conductor and toroid. [4]
   d. Derive an expression for the impedance of an a.c. circuit containing a resistor an inductor and a capacitor. Hence derive resonance frequency. Also, draw the phase diagram. [4]
6. **Answer any three questions.** **[3×4=12]**
   a. Describe the theory of Milikan's oil drop experiment to determine the charge of an electron. [4]
   b. What are avalanche effect and Zener effect? How can a Zener diode be used as a voltage regulator? [4]
   c. Describe coolidge tube for the production of X-rays. How do you control (i) the intensity (ii) the penetrating power of the emitted X-rays? [4]
   d. State the laws of radioactive disintegration. Derive the relation between half life and decay constant. [4]

7. **Answer any one question.** **[4]**
   a. Describe Michelson's method for determining the speed of light. [4]
   b. Explain Fraunhoffer diffraction at a single slit. [4]

8. **Answer any one question.** **[4]**
   a What are harmonics? Explain the formation of overtones in an open and a closed organ pipe. [4]
   b. What is Doppler effect? Obtain an expression for the apparent frequency heard by a listener due to a source when both are approaching each other. [4]

Group 'C'

9. **Answer any two numerical questions.** **[2×4=8]**
   a. Two resistance of 1000 Ω and 2000 Ω are placed in series with 50 V mains supply. What will be the reading on a voltmeter of internal resistance 2000 Ω when placed across the 1000 Ω resistor? What fractional change in voltage occurs when voltmeter is connected? [4]

   Ans: 12 V, 25%

   b. A horizontal straight wire 5 cm long weighing 1.2 gm^{-1} is placed perpendicular to a uniform horizontal magnetic field of flux density of 0.6 T. If the resistance per unit length of the wire is 3.8 Ω m^{-1}, calculate the p.d. that has to be applied between the ends of the wire to make it just self-supporting. [4]

   Ans: 3.7×10^{-3}V

   c. A coil of 100 turns, each of area 2×10^{-3} m^2 has a resistance of 12 Ω. It lies in a horizontal plane in a vertical magnetic flux density of 3×10^{-3} Wbm^{-2}. What charge circulates through the coil if its ends are short-circuited and the coil is rotated through 180^o about a diametrical axis? [4]

   Ans: 10^{-4} C

10. **Answer any two numerical questions.** **[2×4=8]**
   a. Sodium has a work function of 2 eV. Calculate the maximum energy and speed of the emitted electrons when sodium is illuminated by a radiation of 150 nm. What is the threshold frequency of radiation for which electrons are emitted from sodium surface? [4]

   Ans: 1.004×10^{-18}J, 1.483×10^6 m/sec, 4.8×10^{14}HZ

   b. A hydrogen atom is in ground state. What is the quantum number to which it will be excited absorbing a photon of energy 12.75 eV? [4]

   Ans: 4

   c. A nucleus of ${}_{92}U^{238}$ disintegrates according to [4]

   ${}_{92}U^{238} \rightarrow {}_{90}Th^{234} + 2He^4$.

   Calculate:

   i. the total energy released in the disintegration process.

   ii. the k.e. of the α particle, the nucleus at rest before disintegration.

   [Mass of ${}_{92}U^{238}$ = 3.859×10^{-25} kg

   Mass of ${}_{90}Th^{234}$ = 3.787×10^{-25} kg

   Mass of ${}_{2}He^4$ = 6.648×10^{-27} kg]

   Ans: 4.236 MevV, 4.16 MeV

11. At what temperature, the velocity of sound in air is increased by 50% to that at 27^oC? [4]

12. In a Newton' rings experiment, the diameter of 15^{th} ring was found as 0.590 cm and that of 5^{th} ring was 0.336 cm. Calculate the radius of curvature of the plano-convex lens if the wavelength of light used is 5880A. [3]

    Ans: 100cm

2076 Set B

Group 'A'

1. Answer, in brief, any four questions. [4×2=8]

a. State the principle of potentiometer and write down its one application. [2]

b. What is thermoelectric effect? [2]

c. Distinguish between ionic and electronic conduction. [2]

d. An electron beam and a proton beam are moving parallel to each other in the beginning. Do they always maintain this status? Justify your answer. [2]

e. Define one ampere current in terms of force. [2]

f. 220 V A.C. is more danger than 220 V D.C., why? [2]

2. Answer in brief, any four questions. [4×2= 8]

a. Why discharge does not take place at very low pressure? [2]

b. What do you mean by hole in a semiconductor? [2]

c. Which has more energy- a proton in the infrared or photon in the ultraviolet? Given reasons. [2]

d. All the radioactive series terminate at lead as their final product. Why? [2]

e. What do you mean by greenhouse effect? Write its effects. [2]

f. Does the universe have a centre? Explain. [2]

3. Answer in brief**, any one question. [2]**

a. Can longitudinal wave be polarized? Explain. [2]

b. An empty vessel sounds more than a filled one when it is struck. Why? [2]

4. Answer in brief, any one question. [2]

a. State Huygen's principle. Does it apply to sound wave in air? [2]

b. Differentiate unpolarized and polarized light. [2]

Group 'B'

5. Answer any three questions. [3×4=12]

a. What do you mean by shunt? Describe its use in converting a galvanometer into an ammeter. [4]

b. State Joule's law of heating and verify it experimentally. [4]

c. State Biot and Savart law. Derive an expression for the magnetic field at a point due to a long straight conductor carrying current. [4]

d. An alternating current passes through a circuit containing an inductor and a resistor in series. Derive expressions for the current flowing and phase relation between the current and the voltage. [4]

6. Answer any three questions. [3×4=12]

a. What is quantization of charge? Describe the theory of Millikan's oil drop experiment to determine the number of charges on an oil drop. [4]

b. What is P-N junction diode? Discuss its applications as full wave rectifier. [4]

c. List out the laws of radioactive disintegration. Deduce the expression N = Noe–μ where symbols have their usual meaning. [4]

d. What are sources of energy? Discuss global energy consumption pattern and demands. [4]

7. Answer any one question. [4]

a. Does the propagation of sound wave cause change in thermodynamic condition of medium? Derive Laplace formula of velocity of sound in air. [4]

b. What is Doppler's effect? Derive an expression for the apparent frequency received by a stationary observer when a source of sound is moving away from the observer. [4]

8. Answer any one question. [4]

a. Describe Newton's ring experiment and derive expression for wavelength of light. [4]

b. Describe Foucault's method of determining the speed of light. [4]

Group 'C'

9. Answer any two numerical questions. [2×4=8]

a. Two resistors of resistance 1000Ω and 2000Ω are joined in series with a 100 V supply. A voltmeter of internal resistance 4000Ω is connected to measure the potential difference across 1000Ω resistor. Calculate the reading shown by the voltmeter. [4]

Ans: 28.57V

b. Two galvanometers, which are otherwise identical, are fitted with different coils. One has a coil of 50 turns and resistance 10Ω while the other has 500 turns and a resistance of 600Ω. What is the ratio of the deflection when each is connected in turns to a cell of e.m.f. 25 V and internal resistance 50Ω? [4]

Ans: 13:12

c. The magnetic flux passing perpendicular to the plane of coil is given by $\phi = 4t^2 + 5t + 2$ where ϕ is in weber and t is in second. Calculate the magnitude of instantaneous emf induced in the coil when t = 2 sec. [4]

Ans: 21V

10. Answer any two numerical questions. [2×4=8]

a. An ion for which the charge per unit mass is 4.40×10^7 c/kg has velocity of 3.52×10^5 m/s and moves in a circular orbit in a magnetic field of flux density 0.4T. What will be the radius of this orbit? [4]

Ans: 0.02m

b. Obtain the de Broglie wavelength of neutron of kinetic energy 150 eV. (mass of neutron = 1.675×10^{-27} kg. Planck's constant = 6.6×10^{-34} Js. 1eV = 1.6×10^{-19} J.) [4]

Ans: 2.33×10^{-12} m

c. Calculate the binding energy per nucleon of $_{26}Fe^{56}$. Atomic mass of $_{26}Fe^{56}$ is 55.9349u and that of $_1H^1$ is 1.00783u. Mass of $_0n^1$ = 1.00867u and 1u = 931 MeV. [4]

Ans: 8.78 MeV/ nuclear

11. A wire whose mass per unit length is 10^{-3} kg/m is stretched by a load of 4 kg over the two bridges of a sonometer wire 1 m apart. It is struck at its middle point, what would be the wavelength and frequency of its fundamental vibration? [4]

Ans: 100 Hz

12. How wide is the central diffraction peak on a screen 5 m behind a 0.01 mm slit illuminated by 500 nm light source? [3]

Ans: 0.5m

2076 Set C

Group 'A'

1. **Answer, in brief, any four questions.** **[4×2=8]**
   a. The conductivity of an electrolyte is low as compared to that of metal at room temperature. Why? [2]
   b. If the temperature of cold junction of a thermocouple is lowered, what will be the effect on neutral temperature and the temperature of inversion? [2]
   c. How will the magnetic field intensity at the centre of a circular coil carrying current change, if the current through the coil is doubled and the radius of the coil is halved? [2]
   d. Can a charged particle move through a magnetic field without experiencing any force? Explain. [2]
   e. A copper ring is suspended by a thread in a vertical plane. One end of a magnet is brought horizontally towards the ring. How will the position of the ring be affected? [2]
   f. A choke coil is preferable to a resistor in an ac circuit. Why? [2]
2. **Answer in brief, any four questions.** **[4×2=8]**
   a. An electron and a proton have the same kinetic energy. Which one of them has the longer wavelength? [2]
   b. Why is the emitter region of a transistor doped heavily? [2]
   c. Neutron is considered the most effective bombarding particle in a nuclear reaction. Why? [2]
   d. How does a daughter nucleus differ from its parent nucleus when it emits an α-particle? [2]
   e. What is acid rain? Write its any two effects. [2]
   f. Write the quark composition of proton and neutron. [2]
3. **Answer in brief, any one question.** **[2]**
   a. How are stationary waves formed? [2]
   b. Sound waves are called pressure waves. Why? [2]
4. **Answer in brief, any one question.** **[2]**
   a. Differentiate wave front and wavelet. [2]
   b. Can sound waves be polarized? Explain. [2]

Group 'B'

5. **Answer any three questions.** **[3×4=12]**
   a. Discuss how the current is established in a conductor when it is connected across a source of e.m.f. Derive the relation $J = nev$, where the symbols have their usual meanings. [4]
   b. State and explain Kirchhoff's laws and use these laws to find the balance condition in a wheatstone bridge circuit. [4]
   c. Define angle of dip. If δ is the true dip at a place, δ_1 and δ_2 are the apparent dips observed in two vertical planes at right angles to each other at that place, then prove the relation, $\cot^2\delta = \cot^2\delta_1 + \cot^2\delta_2$. [4]
   d. State and explain Faraday's laws of electromagnetic induction and derive an expression for the emf induced in a rectangular coil rotating in a uniform magnetic field. [4]
6. **Answer any three questions.** **[3×4=12]**
   a. What are cathode rays? How are they produced? Mention the properties of cathode rays. [4]
   b. Distinguish between intrinsic and extrinsic semiconductors. Explain the formation of potential barrier and depletion region in a PN junction. [4]
   c. What is photoelectric effect? Discuss Einstein's photoelectric equation. Write some applications of photoelectric effect. [4]

d. What are the major energy sources? Discuss the global energy consumption pattern and demands. [4]

7. Answer any one question. [4]

a. Define end correction of a pipe. Prove that both odd and even types of harmonics can be obtained from an organ pipe open at both ends. [4]

b. What is Doppler's effect? Derive an expression for the apparent frequency when a source of sound and the observer are moving towards each other. [4]

8. Answer any one question. [4]

a. State and explain Huygen's principle and use it to verify laws of reflection on the basis of wave theory. [4]

b. Discuss Fraunhofer diffraction at a single slit. [4]

Group 'C'

9. Answer any two numerical questions. [2×4=8]

a. The resistance of the coil of a galvanometer is 9.36Ω and a current of 0.0224 A causes it to deflect full scale. The only shunt available has a resistance 0.025Ω. What resistance must be connected in series with the coil to make it an ammeter of range 0 - 20A? [4]

Ans: 12.94 Ω

b. A flat silver strip of width 1.5cm and thickness 1.5mm carries a current of 150A. A magnetic field of 2T is applied perpendicular to the flat face of the strip. The emf developed across the width of the strip is measured to be 17.9μV. Calculate the free electron density in the silver. [4]

Ans: 6.98×10^{27} m^{-3}

c. A circuit consists of a capacitor of 2μF and a resistor of 1000Ω. An alternating emf of 12V and frequency 50Hz is applied. Find the voltage across the capacitor and the phase angle between the applied emf and the current. [4]

Ans: 6.37×10^{-3}A, 10.2V, 57.9°

10. Answer any two numerical questions. [2×4=8]

a. An x-ray tube, operated at a dc potential difference of 10kv, produces heat at the target at the rate of 720 watt. Assuming 0.5% of the incident electrons is converted into x-radiation, calculate the number of electrons striking per second at the target and velocity of the incident electrons. (given, $e/m = 1.8\times10^{11}$ Ckg^{-1}) [4]

Ans: 0.072 A, 6×10^7 m/sec

b. Calculate the binding energy per nucleon of calcium nucleus ($_{20}Ca^{40}$). [4]

Given:

mass of $_{20}Ca^{40}$ = 39.962589 u

mass of neutron, m_n = 1.008665 u

mass of proton, m_p = 1.007825 u

1u = 931 Me V

Ans: 8.54 MeV/ nuclear

c. Find the half life of U^{238}, if 1 gm of it emits 1.24×10^4 α-particles per-second Avogadro's number = 6.025×10^{23}. [4]

Ans: 4.5×10^9 years

11. A source of sound of frequency 550Hz emits waves of wavelength 60 cm in air at 20°C. What would be the wavelength of sound from the source in air at 0°C? [4]

Ans: 0.58 m

12. In a Young's double slit experiment, the separation of four bright fringes is 2.5 mm. The wavelength of light used is 6.2×10^{-7} m. If the distance from the slits to the screen is 80 cm, calculate the separation of two slits. [3]

Appendix A: The International System of Units (SI)

The SI Base Unit

Quality	Name	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	Kelvin	K
Amount of substance	mole	mol
Luminous intensity	candela	cd

Some SI Derived Units

Quantity	Name of Unit	Symbol	Equivalent
Area	square meter	m^2	
Volume	cubic meter	m^3	
Frequency	hertz	Hz	s^{-1}
Mass density (density)	kilogram per cubic meter	kg/m^3	
Speed, velocity	meter per second	m/s	
Angular velocity	radian per second	rad/s	
Acceleration	meter per second squared	m/s^2	
Angular acceleration	radian per second squared	rad/s^2	
Force	newton	N	$kg.m/s^2$
Pressure	pascal	Pa	N/m^2
Work, energy, quantity of heat	joule	J	N.m
Power	watt	W	J/s
Quantity of electricity	coulomb	C	A.s
Potential difference, electromotive force	volt	V	N.m/C
Electric field	volt per meter	V/m	N/C
Electric resistance	ohm	Ω	V/A
Capacitance	farad	F	A.s/V
Magnetic flux	weber	Wb	V.s
Inductance	henry	H	V.s/A
Magnetic field	tesla	T	Wb/m^2, N/A.m
Entropy	joule per Kelvin	J/K	
Specific heat capacity	joule per kilogram Kelvin	J/(kg.K)	
Thermal conductivity	watt per meter Kelvin	W/(m.K)	
Radiant intensity	watt per steradian	W/sr	

The SI Supplementary Units

Quality	Name	Symbol
Plane angle	radian	rad
Solid angle	steradian	sr

Appendix B: Fundamental Physical Constants

Quality	Symbol	Computational Value
Speed of light in a vacuum	c	3.00×10^{8}m/s
Elementary charge	e	1.60×10^{-19} C
Electric permittivity	ε_0	8.85×10^{-12} F/m
Magnetic permeability	μ_0	$4\pi \times 10^{-7}$ H/m
Electron mass	m_e	9.11×10^{-31} kg
Electron mass	m_e	5.49×10^{-4} u
Proton mass	m_p	1.67×10^{-27} kg
Proton mass	m_p	1.0073 u
Neutron mass	m_n	1.67×10^{-27} kg
Neutron mass	m_n	1.0087 u
Electron charge-to-mass ratio	e/m_e	1.76×10^{11} C/kg
Proton-to-electron mass ratio	m_p/m_e	1840
Planck constant	h	6.63×10^{-34} J.s
Molar gas constant	R	8.31 J/mol.K
Avogadro constant	N_A	6.02×10^{23} mol^{-1}
Boltzmann constant	k	1.38×10^{-23} J/K
Molar volume of ideal gas at STP	V_m	2.24×10^{-2} m^3/mol
Faraday constant	F	9.65×10^{4} C/mol
Stefan-Boltzmann constant	σ	5.67×10^{-8} $W/m^2.K^4$
Rydberg constant	R_∞	1.10×10^{7} m^{-1}
Gravitation constant	G	6.67×10^{-11} $m^3/s^2.kg$
Bohr radius (radius of first orbit in H^- atom)	r_1	5.29×10^{-11} m

Appendix C: Astronomical Data

Property	Sun[a]	Earth	Moon
Mass (kg)	1.99×10^{30}	5.98×10^{24}	7.36×10^{22}
Mean radius (m)	6.96×10^{8}	6.37×10^{6}	1.74×10^{6}
Mean density (kg/m^3)	1410	5520	3340
Surface gravity (m/s^2)	274	9.81	1.67
Escape velocity (km/s)	618	11.2	2.38
Period of rotation[c] (d)	26–37[b]	0.997	27.3
Mean orbital radius (km)	2.6×10^{17}[d]	1.50×10^{8}[e]	3.82×10^{5}[f]
Orbital periods	2.4×10^{8} y[d]	1.00 y[e]	27.3 d[f]

a *The Sun radiates energy at the rate of 3.90×10^{26}W; just outside the Earth's atmosphere solar energy is received, assuming normal incidence, at the rate of 1380 W/m^2*

b *The Sun– a ball of gas – does not rotate as a rigid body. Its rotational period varies between 26 d at the equator and 37 d at the poles.*

c *Measured with respect to the distance stars.*

d *About the galactic centre.*

e *About the Sun.*

f *About the Earth.*

Some Properties of the Planets

	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune	Pluto
Mean distance from Sun (10^6 km)	57.9	108	150	228	778	1,430	2,870	4,500	5,900
Period of revolution (y)	0.241	0.615	1.00	1.88	11.9	29.5	84.0	165	248
Period of rotation (d)	58.7	243°	0.997	1.03	0.409	0.426	0.451[b]	0.658	6.39
Orbital speed (km/s)	47.9	35.0	29.8	24.1	13.1	9.46	6.81	5.43	4.74
Inclination of axis to orbit	<28°	≈ 3°	23.4°	25.0°	3.08°	26.7°	97.9°	29.6°	57.5°
Inclination of orbit to Earth's orbit	7.00°	3.39°	–	1.85°	1.30°	2.49°	0.77°	1.77°	17.2°
Equatorial diameter (km)	4,880	12,100	12,800	6,790	143,000	120,000	51,800	49,500	2,300
Mass of Earth = 1	0.0558	0.815	1.000	0.107	318	95.1	14.5	17.2	0.002
Average density (g/cm^3)	5.60	5.20	5.52	3.95	1.31	0.704	1.21	1.67	2.03
Escape speed (km/s)	4.3	10.3	11.2	5.0	59.5	35.6	21.2	23.6	1.3

Appendix D: Elementary Particles and Force Carriers

1. The Fundamental Particles

Leptons

Particle	Symbol	Anti-particle	Charge (e)	Spin (h/2π)	Rest Energy (MeV)	Mean Life (s)
Electron	e^-	e^+	–1	1/2	0.511	∞
Electron neutrino	ν_e	$\bar{\nu}_e$	0	1/2	<0.000015	∞
Muon	μ^-	μ^+	–1	1/2	105.7	2.2×10^{-6}
Muon neutrino	ν_μ	$\bar{\nu}_\mu$	0	1/2	<0.19	∞
Tau	τ^-	τ^+	–1	1/2	1777	2.9×10^{-13}
Tau neutrino	ν_τ	$\bar{\nu}_\tau$	0	1/2	<18	∞

Quarks

Particle	Symbol	Anti-particle	Charge (e)	Spin (h/2π)	Rest Energy (MeV)
Up	u	$\bar{u}$	+ 2/3	1/2	3
Down	d	$\bar{d}$	– 1/3	1/2	6
Charm	c	$\bar{c}$	+ 2/3	1/2	1300
Strange	s	$\bar{s}$	– 1/3	1/2	120
Top	t	$\bar{t}$	+ 2/3	1/2	174,000
Bottom	b	$\bar{b}$	– 1/3	1/2	4300

2. Field Particles (Mediator Particles)

Particle	Symbol	Interaction	Charge (e)	Spin (h/2π)	Rest Energy (GeV)
Gravition[b]		Gravity	0	2	0
Weak boson	W^+, W^-	Weak	± 1	1	80.4
Weak boson	Z°	Weak	0	1	91.2
Photon	γ	Electromagnetic	0	1	0
Gluon	g	Strong (color)	0	1	0

2. Some Composite Particles

Baryons

Particle	Symbol	Quark Content	Anti-particle	Charge (e)	Spin (h/2 π)	Rest Energy (MeV)
Proton	p	uud	$\bar{p}$	+1	1/2	938
Neutron	n	udd	$\bar{n}$	0	1/2	940
Lambda	Λ°	uds	$\bar{\Lambda}^{\circ}$	0	1/2	1116
Omega	Ω^{-}	sss	$\bar{\Omega}$-	–1	1/2	1672
Delta	Δ^{++}	uuu	$\bar{\Delta}^{++}$	+2	1/2	1232
Charmed Lambda	Λ_c^{+}	udc	$\bar{\Lambda}^{+}{}_{c}$	+1	1/2	2285

Mesons

Particle	Symbol	Quark Content	Anti-particle	Charge (e)	Spin (h/2 π)	Rest Energy (MeV)
Pion	π^{+}	$u\bar{d}$	π^{-}	+1	0	140
Pion	π^{0}	$u\bar{u} + d\bar{d}$	π^{0}	0	0	135
Kaon	K^{+}	$u\bar{s}$	K^{-}	+1	0	494
Kaon	K^{0}	$d\bar{s}$	$\bar{K}^{0}$	0	0	498
Rho	ρ^{+}	$u\bar{d}$	ρ^{-}	+1	1	770
D-meson	D^{+}	$c\bar{d}$	D^{-}	+1	0	1869
Psi	ψ	$c\bar{c}$	ψ	0	1	3069
B-meson	B^{+}	$u\bar{b}$	B^{-}	+1	0	5279
Upsilon	Y	$b\bar{b}$	Y	0	1	9460

a *The rest energies listed for the quarks are not those associated with free quarks; since no free quarks have yet been observed, measuring their rest energies in the free state has not yet bee possible. The tabulated values are effective rest energies corresponding to quarks bond in composite particles.*

b *Particles expected to exist but not yet observed.*

Appendix E: Periodic Table of the Elements

GROUP IUPAC 13, GROUP CAS IIIA, ATOMIC NUMBER 5, RELATIVE ATOMIC MASS (1) 10.811, SYMBOL B, ELEMENT NAME BORON

- Metal
- Semimetal
- Nonmetal
- Alkali metal
- Alkaline earth metal
- Transition metals
- Lanthanide
- Actinide
- Chalcogens element
- Halogens element
- Noble gas

STANDARD STATE (25 °C; 101 kPa)
Ne - gas **Fe** - solid
Hg - liquid **Tc** - synthetic

PERIOD \ GROUP	1 IA	2 IIA	3 IIIB	4 IVB	5 VB	6 VIB	7 VIIB	8 VIIIB	9 VIIIB	10 VIIIB	11 IB	12 IIB	13 IIIA	14 IVA	15 VA	16 VIA	17 VIIA	18 VIIIA
1	1 H 1.0079 HYDROGEN																	2 He 4.0026 HELIUM
2	3 Li 6.941 LITHIUM	4 Be 9.0122 BERYLLIUM											5 B 10.811 BORON	6 C 12.011 CARBON	7 N 14.007 NITROGEN	8 O 15.999 OXYGEN	9 F 18.998 FLUORINE	10 Ne 20.180 NEON
3	11 Na 22.990 SODIUM	12 Mg 24.305 MAGNESIUM											13 Al 26.982 ALUMINIUM	14 Si 28.086 SILICON	15 P 30.974 PHOSPHORUS	16 S 32.065 SULPHUR	17 Cl 35.453 CHLORINE	18 Ar 39.948 ARGON
4	19 K 39.098 POTASSIUM	20 Ca 40.078 CALCIUM	21 Sc 44.956 SCANDIUM	22 Ti 47.867 TITANIUM	23 V 50.942 VANADIUM	24 Cr 51.996 CHROMIUM	25 Mn 54.938 MANGANESE	26 Fe 55.845 IRON	27 Co 58.933 COBALT	28 Ni 58.693 NICKEL	29 Cu 63.546 COPPER	30 Zn 65.38 ZINC	31 Ga 69.723 GALLIUM	32 Ge 72.64 GERMANIUM	33 As 74.922 ARSENIC	34 Se 78.96 SELENIUM	35 Br 79.904 BROMINE	36 Kr 83.798 KRYPTON
5	37 Rb 85.468 RUBIDIUM	38 Sr 87.62 STRONTIUM	39 Y 88.906 YTTRIUM	40 Zr 91.224 ZIRCONIUM	41 Nb 92.906 NIOBIUM	42 Mo 95.96 MOLYBDENUM	43 Tc (98) TECHNETIUM	44 Ru 101.07 RUTHENIUM	45 Rh 102.91 RHODIUM	46 Pd 106.42 PALLADIUM	47 Ag 107.87 SILVER	48 Cd 112.41 CADMIUM	49 In 114.82 INDIUM	50 Sn 118.71 TIN	51 Sb 121.76 ANTIMONY	52 Te 127.60 TELLURIUM	53 I 126.90 IODINE	54 Xe 131.29 XENON
6	55 Cs 132.91 CAESIUM	56 Ba 137.33 BARIUM	57-71 La-Lu Lanthanide	72 Hf 178.49 HAFNIUM	73 Ta 180.95 TANTALUM	74 W 183.84 TUNGSTEN	75 Re 186.21 RHENIUM	76 Os 190.23 OSMIUM	77 Ir 192.22 IRIDIUM	78 Pt 195.08 PLATINUM	79 Au 196.97 GOLD	80 Hg 200.59 MERCURY	81 Tl 204.38 THALLIUM	82 Pb 207.2 LEAD	83 Bi 208.98 BISMUTH	84 Po (209) POLONIUM	85 At (210) ASTATINE	86 Rn (222) RADON
7	87 Fr (223) FRANCIUM	88 Ra (226) RADIUM	89-103 Ac-Lr Actinide	104 Rf (267) RUTHERFORDIUM	105 Db (268) DUBNIUM	106 Sg (271) SEABORGIUM	107 Bh (272) BOHRIUM	108 Hs (277) HASSIUM	109 Mt (276) MEITNERIUM	110 Ds (281) DARMSTADTIUM	111 Rg (280) ROENTGENIUM	112 Cn (285) COPERNICIUM	113 Uut (. . .) UNUNTRIUM	114 Fl (287) FLEROVIUM	115 Uup (. . .) UNUNPENTIUM	116 Lv (291) LIVERMORIUM	117 Uus (. . .) UNUNSEPTIUM	118 Uuo (. . .) UNUNOCTIUM

LANTHANIDE

57 La 138.91 LANTHANUM	58 Ce 140.12 CERIUM	59 Pr 140.91 PRASEODYMIUM	60 Nd 144.24 NEODYMIUM	61 Pm (145) PROMETHIUM	62 Sm 150.36 SAMARIUM	63 Eu 151.96 EUROPIUM	64 Gd 157.25 GADOLINIUM	65 Tb 158.93 TERBIUM	66 Dy 162.50 DYSPROSIUM	67 Ho 164.93 HOLMIUM	68 Er 167.26 ERBIUM	69 Tm 168.93 THULIUM	70 Yb 173.05 YTTERBIUM	71 Lu 174.97 LUTETIUM

ACTINIDE

89 Ac (227) ACTINIUM	90 Th 232.04 THORIUM	91 Pa 231.04 PROTACTINIUM	92 U 238.03 URANIUM	93 Np (237) NEPTUNIUM	94 Pu (244) PLUTONIUM	95 Am (243) AMERICIUM	96 Cm (247) CURIUM	97 Bk (247) BERKELIUM	98 Cf (251) CALIFORNIUM	99 Es (252) EINSTEINIUM	100 Fm (257) FERMIUM	101 Md (258) MENDELEVIUM	102 No (259) NOBELIUM	103 Lr (262) LAWRENCIUM

(2009)
Relative atomic masses are expressed with five significant figures. For elements that have no stable nuclides, the value enclosed in brackets indicates the mass number of the longest-lived isotope of the element. However three such elements (Th, Pa and U) do have a characteristic terrestrial isotopic composition, and for these an atomic weight is tabulated.

Appendix F: Conversion Factors

Plane Angle

	Degree (°)	Minute (')	Second (")	Radian (rad)	rev
1 degree =	1	60	3600	1.745×10^{-2}	2.778×10^{-3}
1 minute =	1.667×10^{-2}	1	60	2.909×10^{-4}	4.630×10^{-5}
1 second =	2.778×10^{-4}	1.66×10^{-2}	1	4.848×10^{-6}	7.716×10^{-7}
1 radian =	57.30	3438	2.063×10^{5}	1	0.1592
1 revolution =	360	2.16×10^{4}	1.296×10^{6}	6.283	1

Sold Angle

1 sphere = 4π steradians = 12.57 steradians

Length

	cm	m	km	in.	ft	mi
1 centimeter =	1	10^{-2}	10^{-5}	0.3937	3.281×10^{-2}	6.214×10^{-6}
1 meter =	100	1	10^{-3}	39.37	3.281	6.214×10^{-4}
1 kilometer =	10^{5}	1000	1	3.937×10^{4}	3281	0.6214
1 inch =	2.540	2.540×10^{-2}	2.540×10^{-5}	1	8.333×10^{-2}	1.578×10^{-5}
1 foot =	30.48	0.3048	3.048×10^{-4}	12	1	1.894×10^{-4}
1 mile =	1.609×10^{5}	1609	1609	6.33×10^{4}	5280	1

1 angstrom = 10^{-10}m
1 nautical mile = 18452 m
= 1.151 miles = 6076 ft
1 fermi = 10^{-15} m

1 light-year = 9.460×10^{12} km
1 parsec = 3.084×10^{13} km
1 fathom = 6 ft
1 Bohr radius = 5.292×10^{-11} m

1 yard = 3 ft
1 rod = 16.5 ft
1 mil = 10^{-3} in.
1 nm = 10^{-9}m

Area

	m^2	cm^2	ft^2	$in.^2$
1 Square meter =	1	10^{4}	10.76	1550
1 square centimeter =	10^{-4}	1	1.076×10^{-3}	0.1550
1 square foot =	9.290×10^{-2}	929.0	1	144
1 square inch =	6.452×10^{-4}	6.542	6.944×10^{-3}	1

1 square mile = 2.788×10^{7} ft ^{2} = 640 acres
1 acre = 43.560 ft^2
1 barn = 10^{-28} m^2
1 hectare = $10^4 m^2$ = 2.471 acre

Volume

	meter³	cm³	L	ft³	in³
1 cubic meter =	1	10^6	1000	35.31	6.102×10^4
1 cubic centimeter=	10^{-6}	1	1.000×10^{-3}	3.531×10^{-5}	6.102×10^{-2}
1 liter =	1.000×10^{-3}	1000	1	3.531×10^{-2}	61.02
1 cubic foot =	2.832×10^{-2}	2.832×10^4	28.32	1	1728
1 cubic inch =	1.639×10^{-5}	16.39	1.639×10^{-2}	5.787×10^{-4}	1

1 U.S. fluid gallon = 4 U.S. fluid quarts = 8 U.S. pints = 128 U.S. fluid ounces = 231 in.^{3}
1 British imperial gallon = 227.4 in.^{3} = 1.201 U.S. fluid gallons

Mass

	g	kg	slug	u	oz	lb	ton
1 gram =	1	0.001	6.852×10^{-5}	6.022×10^{23}	3.527×10^{-2}	2.205×10^{-3}	1.102×10^{-6}
1 kg =	1000	1	6.852×10^{-2}	6.022×10^{26}	35.27	2.205	1.102×10^{-3}
1 slug =	1.459×10^4	14.59	1	8.786×10^{27}	514.8	32.17	1.609×10^{-2}
1 u =	1.661×10^{-24}	1.661×10^{-27}	1.138×10^{-28}	1	5.857×10^{-26}	3.662×10^{-27}	1.830×10^{-30}
1 ounce =	28.35	2.835×10^{-2}	1.943×10^{-3}	1.718×10^{25}	1	6.250×10^{-2}	3.125×10^{-5}
1 pound =	453.6	0.4536	3.108×10^{-2}	2.732×10^{26}	16	1	0.0005
1 ton =	9.072×10^5	907.2	62.16	5.463×10^{29}	3.2×10^4	2000	1

1 metric ton = 1000 kg

Time

	y	d	h	min	second
1 year =	1	365. 25	8.766×10^3	5.259×10^5	3.156×10^7
1 day =	2.738×10^{-3}	1	24	1440	8.640×10^4
1 hour =	1.141×10^{-4}	4.167×10^{-2}	1	60	3600
1 minute =	1.901×10^{-6}	6.944×10^{-4}	1.667×10^{-2}	1	60
1 second =	3.169×10^{-8}	1.157×10^{-5}	2.778×10^{-4}	1.667×10^{-2}	1

Force

	dyne	newton	lb	pdl	gf	kgf
1 dyne =	1	10^{-5}	2.248×10^{-6}	7.233×10^{-5}	1.020×10^{-3}	1.020×10^{-6}
1 newton =	10^5	1	0.2248	7.233	102.0	0.1020
1 pound =	4.448×10^5	4.488	1	32.17	453.6	0.4536
1 poundal =	1.383×10^4	0.1383	3.108×10^{-2}	1	14.10	1.410×10^{-2}
1 gram force =	980.7	9.807×10^{-3}	2.205×10^{-3}	7.093×10^{-2}	1	0.001
1 kilogram force =	9.807×10^5	9.807	2.205	70.93	1000	1

Energy, Work, Heat

	Btu	erg	ft. lb	hp. h	joule	cal	kWh	eV	MeV	kg	u
1 British thermal unit =	1	1.055×10^{10}	777.9	3.929×10^{-4}	1055	252.0	2.930×10^{-4}	6.585×10^{21}	6.585×10^{15}	1.174×10^{-14}	7.070×10^{12}
1 erg	9.481×10^{-11}	1	7.376×10^{-8}	3.725×10^{-14}	10^{-7}	2.389×10^{-8}	2.778×10^{-14}	6.242×10^{11}	6.242×10^{5}	1.113×10^{-24}	670.2
1 foot-pound =	1.285×10^{-3}	1.356×10^{7}	1	5.051×10^{-7}	1.356	0.3238	3.766×10^{-7}	8.464×10^{18}	8.464×10^{12}	1.509×10^{-17}	9.037×10^{9}
1 horsepower-hour =	2545	2.685×10^{13}	1.980×10^{6}	1	2.685×10^{6}	6.413×10^{5}	0.7457	1.676×10^{25}	1.676×10^{19}	2.988×10^{-11}	1.799×10^{16}
1 joule =	9.481×10^{-4}	10^{7}	0.7376	3.725×10^{-7}	1	0.2389	2.778×10^{-7}	6.242×10^{18}	6.242×10^{12}	1.113×10^{-17}	6.702×10^{9}
1 calorie =	3.969×10^{-3}	4.186×10^{7}	3.088	1.560×10^{6}	4.186	1	1.163×10^{-6}	2.613×10^{19}	2.613×10^{13}	4.660×10^{-17}	2.806×10^{10}
1 kilowatt-hour =	3413	3.6×10^{13}	2.655×10^{6}	1.314	3.6×10^{6}	8.600×10^{5}	1	2.247×10^{25}	2.247×10^{19}	4.007×10^{-11}	2.413×10^{16}
1 electron volt =	1.519×10^{-22}	1.602×10^{-12}	1.182×10^{-19}	5.967×10^{-26}	1.602×10^{-19}	3.827×10^{-20}	4.450×10^{-26}	1	10^{-6}	1.783×10^{-36}	1.074×10^{-9}
1 kilogram =	8.521×10^{13}	8.987×10^{23}	6.629×10^{16}	3.348×10^{10}	8.987×10^{16}	2.146×10^{16}	2.497×10^{-10}	5.610×10^{35}	5.610×10^{29}	1	6.022×10^{26}
1 unified atomic mass unit =	1.415×10^{-13}	1.492×10^{-3}	1.101×10^{-10}	5.559×10^{-17}	1.492×10^{-10}	3.564×10^{-11}	4.146×10^{-17}	9.32×10^{8}	932.0	1.661×10^{-27}	1

Quantities in the coloured areas are not properly energy units but are included for convenience. They arise from relativistic mass – energy equivalence formula $E = mc^2$ and represent the energy equivalent of a mass of one kilogram or one unified atomic mass unit (u)

Pressure

	atm	dyne/cm²	inch of water	cm Hg	pascal	lb/in.²	lb/ft²
1 atmosphere =	1	1.013×10^{6}	406.8	76	\`1.013×10^{5}	14.70	2116
1 dyne per cm² =	9.869×10^{-7}	1	4.015×10^{-4}	7.501×10^{-5}	0.1	1.405×10^{-5}	2.089×10^{-3}
1 centimeter of mercury[a] at 0°C	1.316×10^{-2}	1.333×10^{4}	5.353	1	1333	0.1934	27.85
1 pascal =	9.869×10^{-6}	10	4.015×10^{-3}	7.501×10^{-4}	1	1.450×10^{-4}	2.089×10^{-2}
1 pound per in.² =	6.805×10^{-2}	6.895×10^{4}	27.68	5.171	6.895×10^{3}	1	144
1 pound per ft²=	4.725×10^{-4}	478.8	0.1922	3.591×10^{-2}	47.88	6.944×10^{-3}	1

[a] Where the acceleration of gravity has the standard value 9.80665 m/s².

1 bar = 10^6 dyne/cm² = 0.1 MPa   1 millibar = 10^3 dyne/cm² = 10^2 Pa

1 torr = 1 millimeter of mercury

Power

	Btu/h	ft.lb/s	hp	cal/s	kW	watt
1 British thermal unit per hour =	1	0.2161	3.929×10^{-4}	6.998×10^{-2}	2.930×10^{-4}	0.2930
1 foot-pound per second =	4.628	1	1.818×10^{-3}	0.3239	1.356×10^{-3}	1.356
1 horsepower	2545	550	1	178.1	0.7457	745.7
1 calorie per second =	14.29	3.088	5.615×10^{-3}	1	4.186×10^{-3}	4.186
1 kilowatt =	3413	737.6	1.341	238.9	1	1000
1 watt	3.413	0.7376	1.341×10^{-3}	0.2389	0.001	1

Magnetic Flux

	maxwell	weber
1 maxwell =	1	10^{-8}
1 weber =	10^{8}	1

Magnetic Field

	gauss	tesla	milligauss
1 gauss =	1	10^{-4}	1000
1 tesla=	10^{4}	1	10^{7}
1 milligauss =	0.001	10^{-7}	1

1 tesla = 1 weber/m^2