Inscribed angles of circles

In this lesson we'll look at inscribed angles of circles and how they're related to arcs, called intercepted arcs.

Chord of a circle

The **chord** of a circle is a line segment that has both of its endpoints on the circle. A diameter of a circle is a special type of chord that passes through the circle's center. The yellow line is an example of a chord.

Inscribed angle

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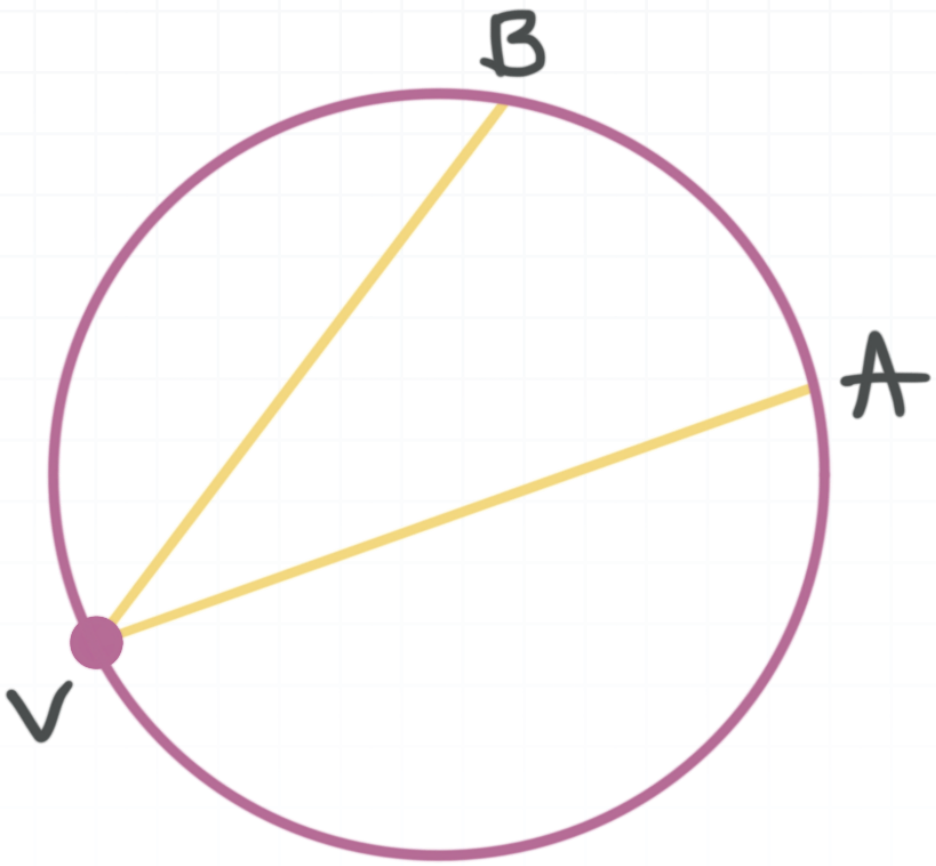
An inscribed angle is formed by two chords that have one endpoint in common, which is the vertex of the angle. The points at which the other endpoint of each chord intersects the circle are the endpoints of an arc which is called the **intercepted arc**.



Angle AVB is an **inscribed angle**, and arc AB is the intercepted arc.

Inscribed and central angles, and intercepted arcs

The measure of an intercepted arc is equal to the measure of the central angle that corresponds to it.

The measure of an inscribed angle is equal to half the measure of the central angle that corresponds to the intercepted arc.

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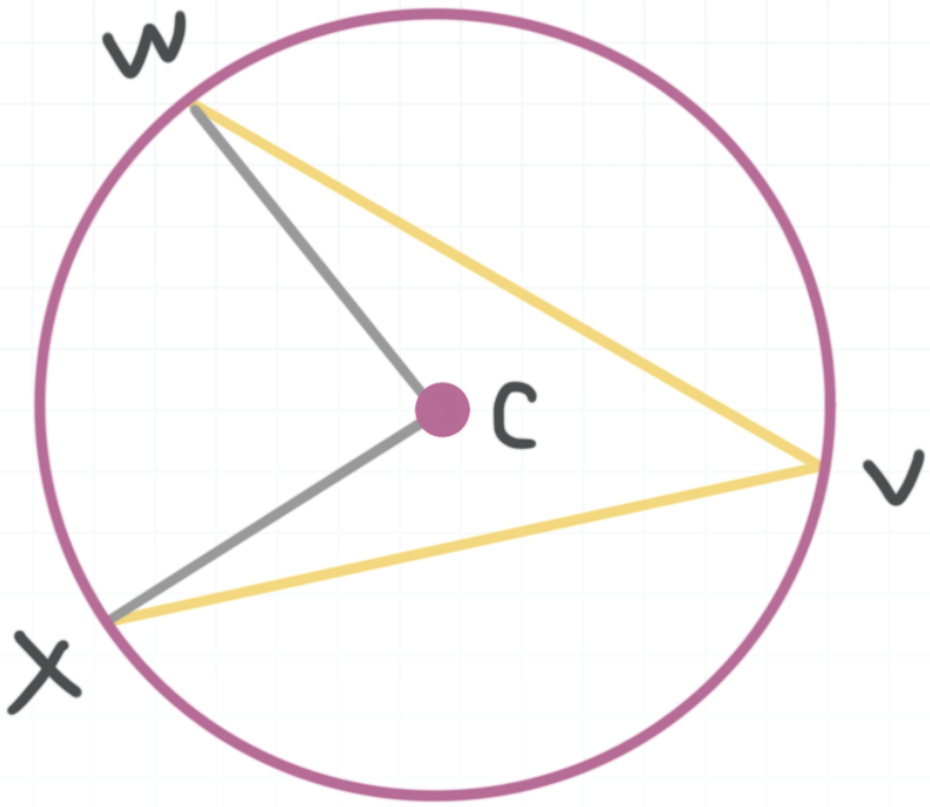
The measure of an inscribed angle is equal to half the measure of its intercepted arc.



$$m\overset{\frown}{WX} = m\angle WCX$$

$$\frac{1}{2}m\angle WCX = m\angle WVX$$

$$\frac{1}{2}m\overset{\frown}{WX} = m\angle WVX$$

We can do some algebra to show that the following is also true.

$$m\overset{\frown}{WX} = m\angle WCX = 2m\angle WVX$$

Let's do a couple of example problems.

Example

Find the measure of the inscribed angle WVX if $m\angle WCX = 88°$.

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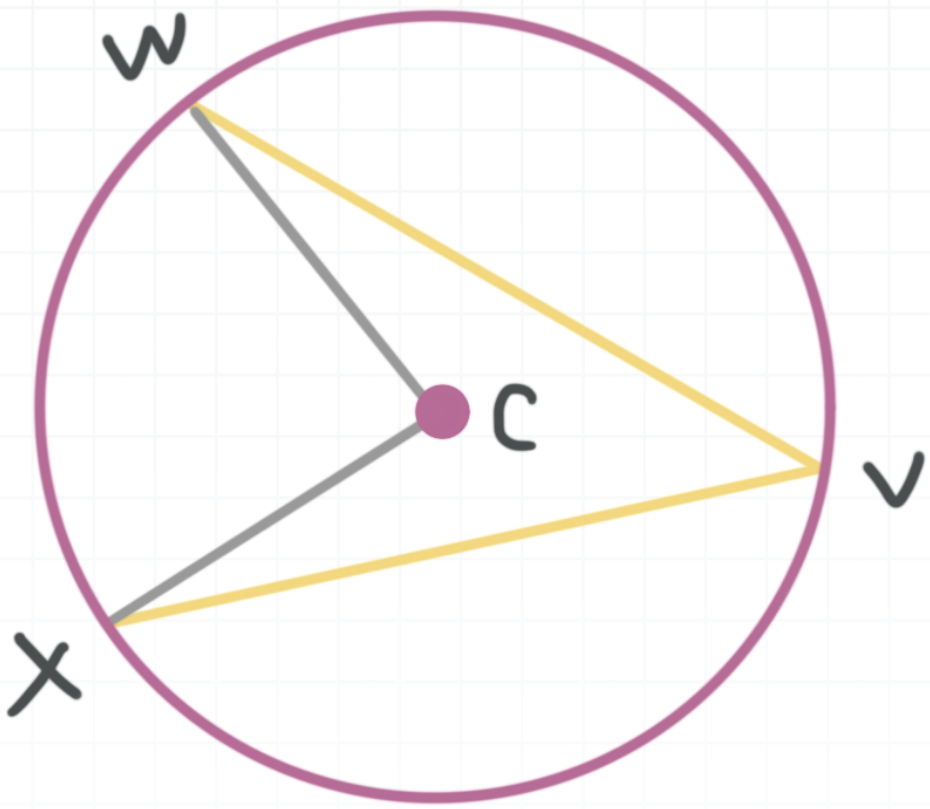

We can see from the figure that $\angle WCX$ is the central angle that corresponds to the intercepted arc $\overset{\frown}{WX}$. Which means that the measure of the inscribed angle ($\angle WVX$) is half that of $\angle WCX$. We know

$$\frac{1}{2}m\angle WCX = m\angle WVX$$

and

$$m\angle WCX = 88^\circ$$

So

$$\frac{1}{2}(88^\circ) = m\angle WVX$$

$$44^\circ = m\angle WVX$$

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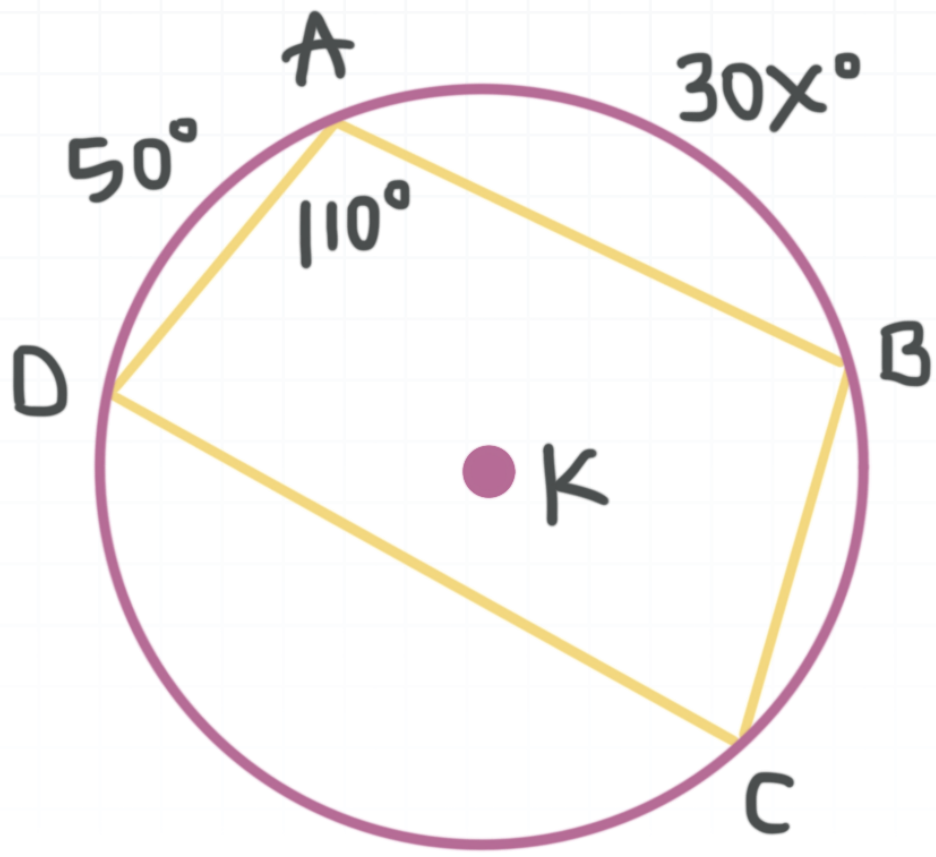
Let's do a problem with a few more steps.

Example

Find the value of x.



The arc that consists of the complete circle has measure $360°$. If we can find the measure of $\overset{\frown}{DCB}$ (remember that this is the arc we'd trace out by starting at point D and going counterclockwise, through C, around the circle to point B), we can set up an equation to solve for x, because $m\overset{\frown}{DCB} + m\overset{\frown}{BA} + m\overset{\frown}{AD} = 360°$, and we know that $m\overset{\frown}{BA} = 30x°$ and $m\overset{\frown}{AD} = 50°$.

From the diagram, we can see that $m\angle DAB = 110°$. The arc intercepted by this inscribed angle is $\overset{\frown}{DCB}$. The intercepted arc has measure twice that of the inscribed angle.

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$$m\overset{\frown}{DCB} = 2m\angle DAB = 2(110^\circ)$$

$$m\overset{\frown}{DCB} = 220^\circ$$

Now we can use the equation we wrote earlier to solve for x.

$$m\overset{\frown}{DCB} + m\overset{\frown}{BA} + m\overset{\frown}{AD} = 360^\circ$$

$$220^\circ + 30x^\circ + 50^\circ = 360^\circ$$

$$270^\circ + 30x^\circ = 360^\circ$$

$$30x^\circ = 90^\circ$$

$$x = 3$$

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