Geometry Workbook Solutions

Congruence

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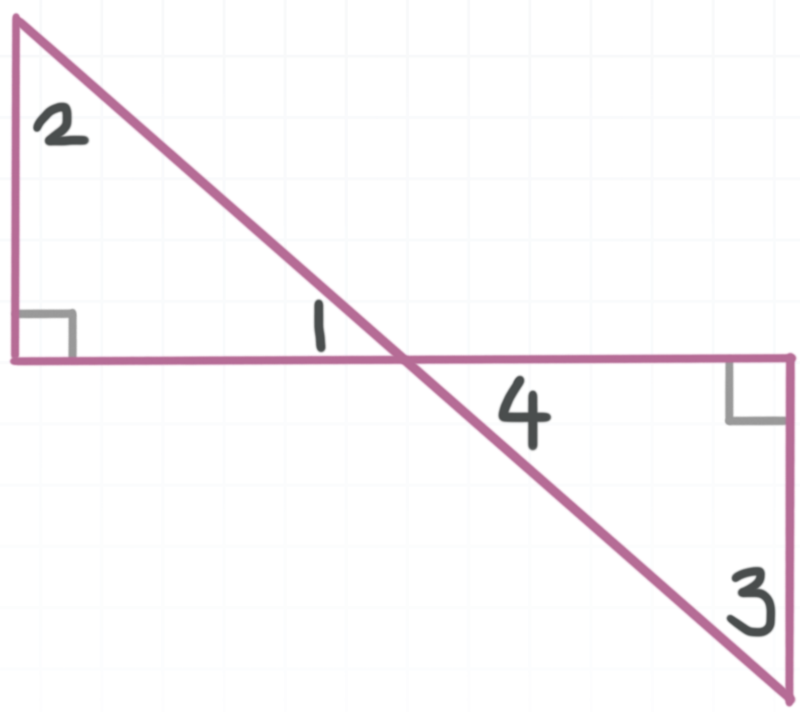
CONGRUENT ANGLES

■ 1. $m\angle 3 = 4x - 11$ and $m\angle 1 = 5x + 2$. Find $m\angle 2$.



Solution:

$m\angle 2 = 33^\circ$. $\angle 1 \cong \angle 4$ because they are vertical angles. And because the three interior angles of a triangle always sum to 180°,

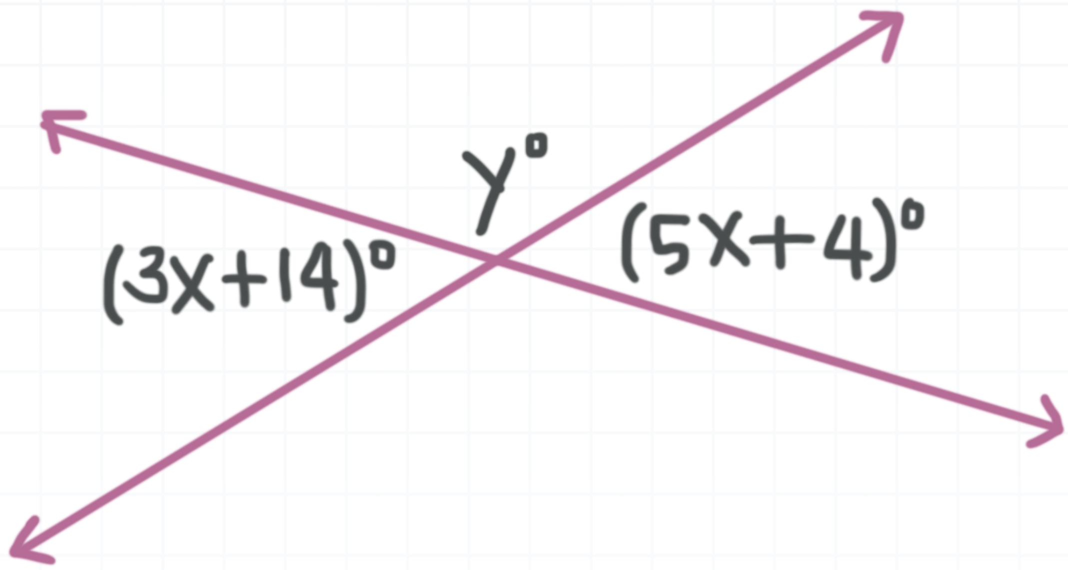
$$m\angle 3 + m\angle 4 + 90 = 180$$

$$4x - 11 + 5x + 2 + 90 = 180$$

$$x = 11$$

Then $m\angle 2 = m\angle 3 = 4(11) - 11 = 33^\circ$.

2. Find the values of x and y.



Solution:

$x = 5$ and $y = 151$. Because they are vertical angles, we know that

$$3x + 14 = 5x + 4$$

$$10 = 2x$$

$$x = 5$$

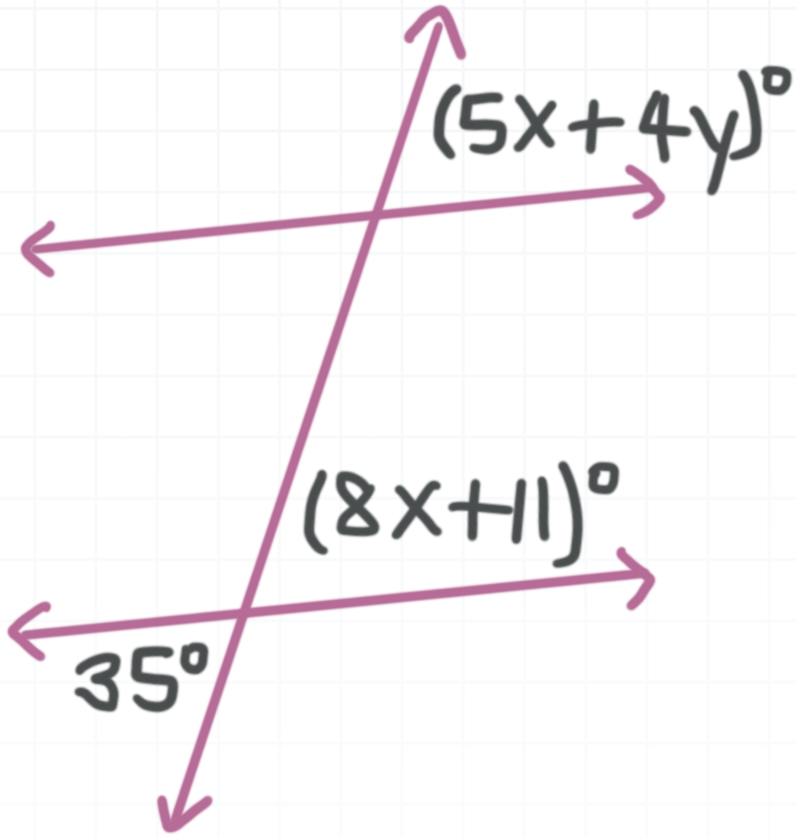
And because $(3x + 14)°$ and $y°$ are supplementary angles, we can say

$$3x + 14 + y = 180$$

$$3(5) + 14 + y = 180$$

$$y = 151$$

3. Find the value of x and y.



Solution:

$x = 3$ and $y = 5$. Because they are vertical angles, we know that

$$35 = 8x + 11$$

$$24 = 8x$$

$$x = 3$$

And because they are alternate exterior angles, we can say

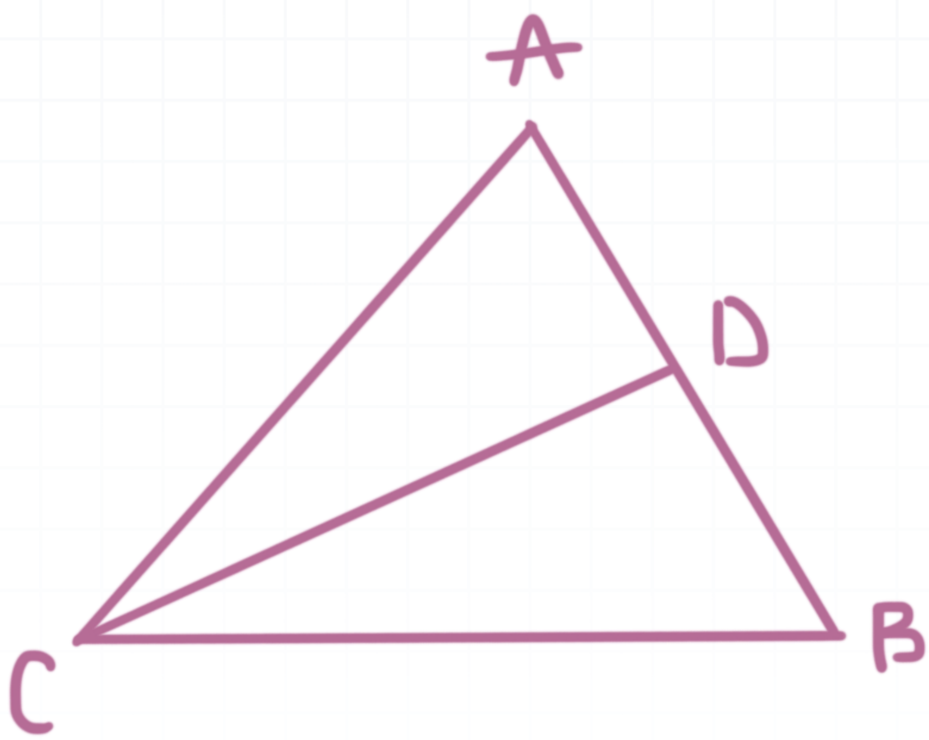
$$5x + 4y = 35$$

$$5(3) + 4y = 35$$

$$4y = 20$$

$$y = 5$$

■ 4. $\overline{CD}$ is an angle bisector of the triangle and $\overline{CD} \perp \overline{AB}$. $m\angle CAD = 5x - 10$ and $m\angle BCD = 25$. Find x.



Solution:

$x = 15$. We know the interior angles of a triangle sum to 180°, so

$$m\angle DBC = 180^\circ - 90^\circ - 25^\circ = 65^\circ$$

And because $m\angle DBC = m\angle CAD$, we can say

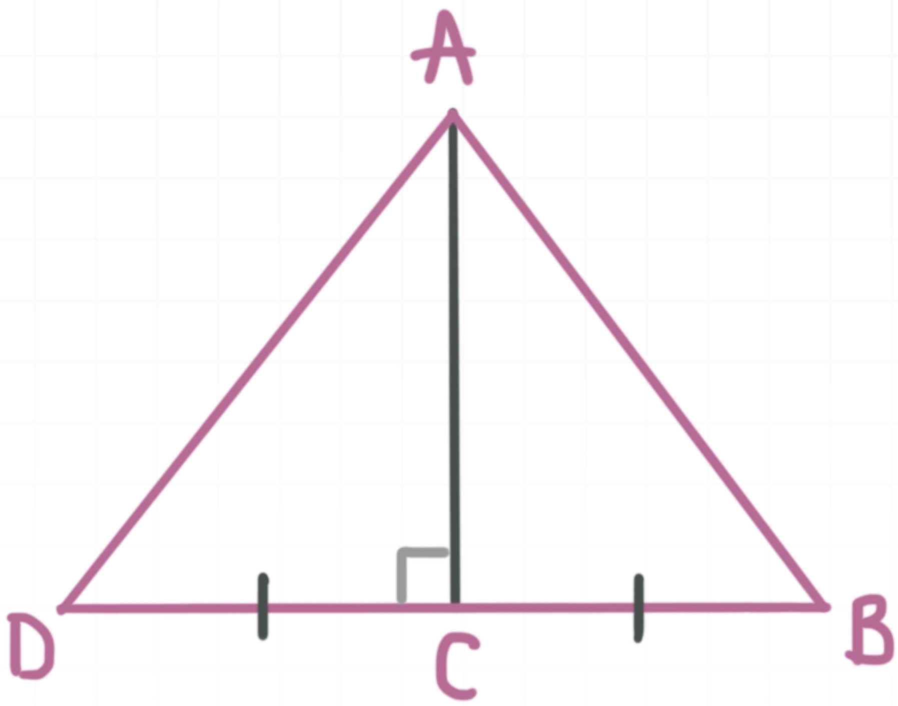
$$5x - 10 = 65$$

$$5x = 75$$

$$x = 15$$

TRIANGLE CONGRUENCE WITH SSS, ASA, SAS

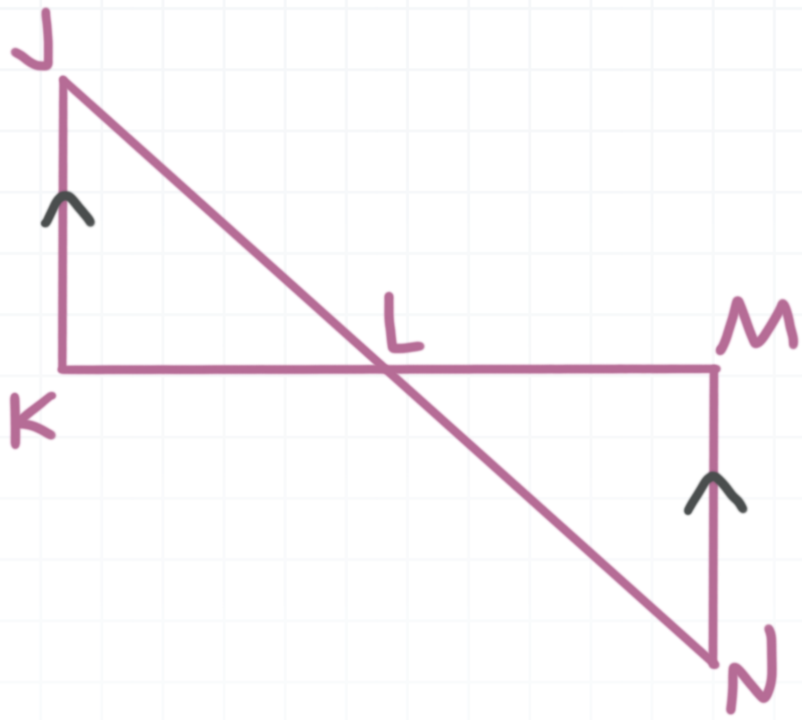
■ 1. Fill in the blank. $\triangle ABC \cong \triangle ADC$ by the ______________ Theorem.



Solution:

SAS (Side-Angle-Side) Theorem. We know $\overline{AC} \cong \overline{AC}$ by the Reflexive Property of Congruence. We know $\angle ACD \cong \angle ACB$ because they are both right angles. And we know $\overline{DC} \cong \overline{BC}$ because of the markings shown on the diagram.

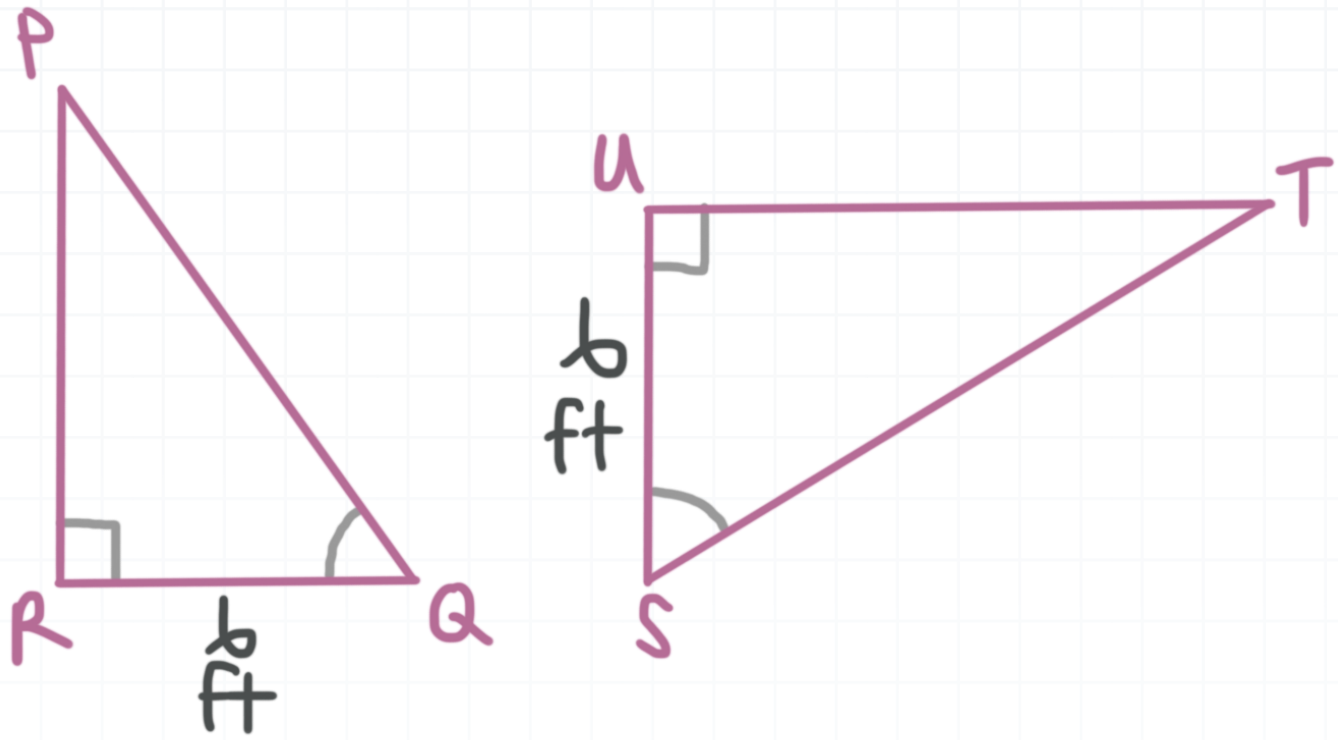
■ 2. Fill in the blank. L is a midpoint of $\overline{JN}$. $\triangle JKL \cong \triangle NML$ by the ______________ Theorem.



Solution:

ASA (Angle-Side-Angle) Theorem. $\angle JLK \cong \angle NLM$ because they are vertical angles. $\angle J \cong \angle N$ because they are alternate interior angles. $JL = NL$ because L is a midpoint of $\overline{JN}$.

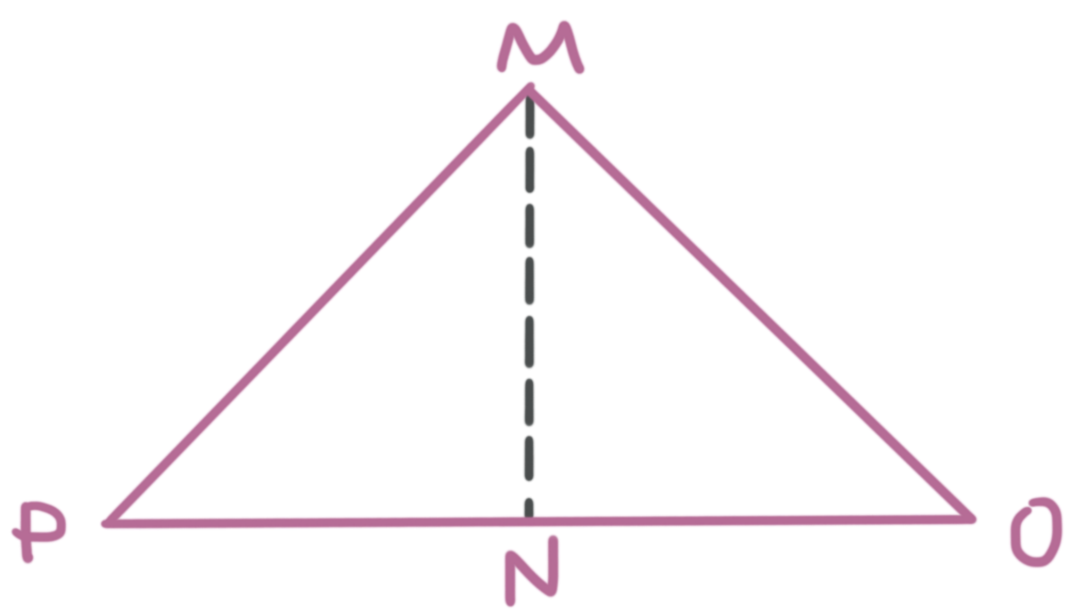
■ 3. $\triangle PRQ \cong \triangle$ ________________ by the ________________ Theorem.



Solution:

$\triangle TUS$ by the ASA (Angle-Side-Angle) Theorem. In the diagram, $\angle Q \cong \angle W$, $\angle R \cong \angle U$, and $RQ = 6 = US$.

■ 4. $\triangle PMO$ is an isosceles triangle with vertex angle at M. N is a midpoint of $\overline{PO}$. $\triangle PMN \cong \triangle OMN$ by the ________________ Theorem.

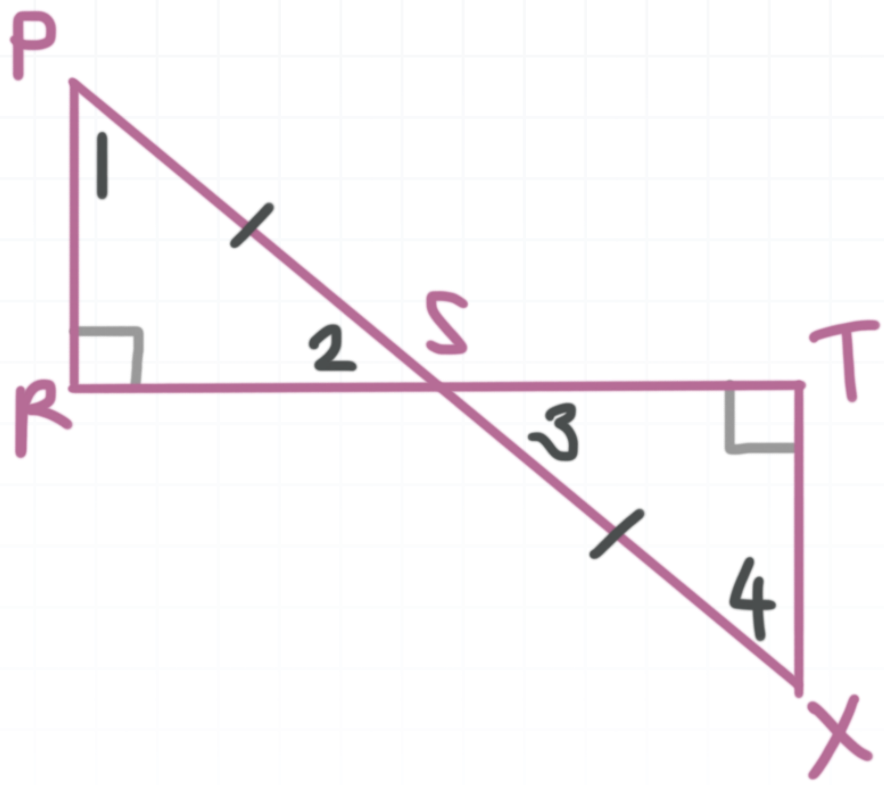


Solution:

SSS (Side-Side-Side) Theorem. We know that $\overline{MN} \cong \overline{MN}$ by the Reflexive Property of Congruence. $\overline{PM} \cong \overline{OM}$ because $\triangle PMO$ is isosceles. And $\overline{PN} \cong \overline{ON}$ because N is a midpoint.

TRIANGLE CONGRUENCE WITH AAS, HL

■ 1. Which theorem could be used to prove $\triangle PRS \cong \triangle XTS$?

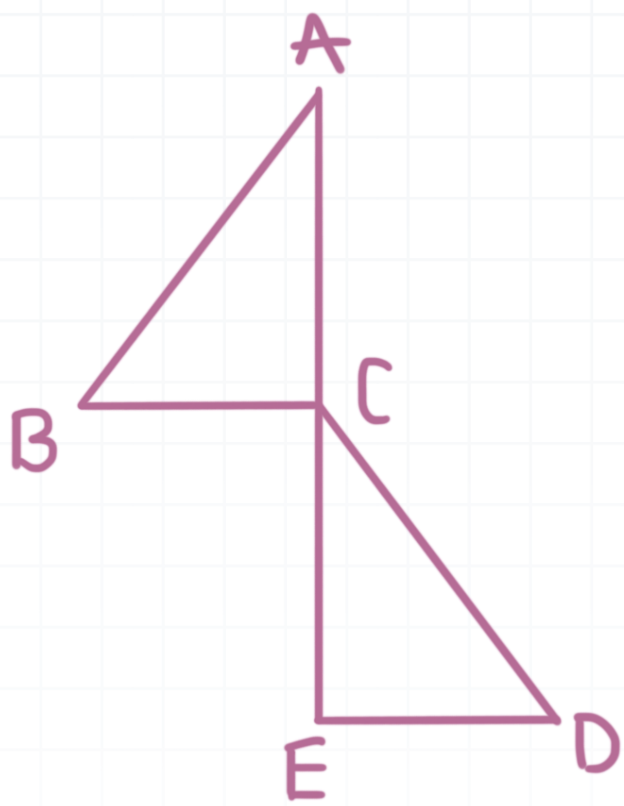


Solution:

AAS (Angle-Angle-Side) Theorem. We know that $\angle 2 \cong \angle 3$ because they are vertical angles, and that $\angle R \cong \angle T$ because they are both right angles, and the diagram shows $\overline{PS} \cong \overline{XS}$.

■ 2. Which theorem could be used to prove $\triangle ACB \cong \triangle CED$? The following facts are given about the triangles.

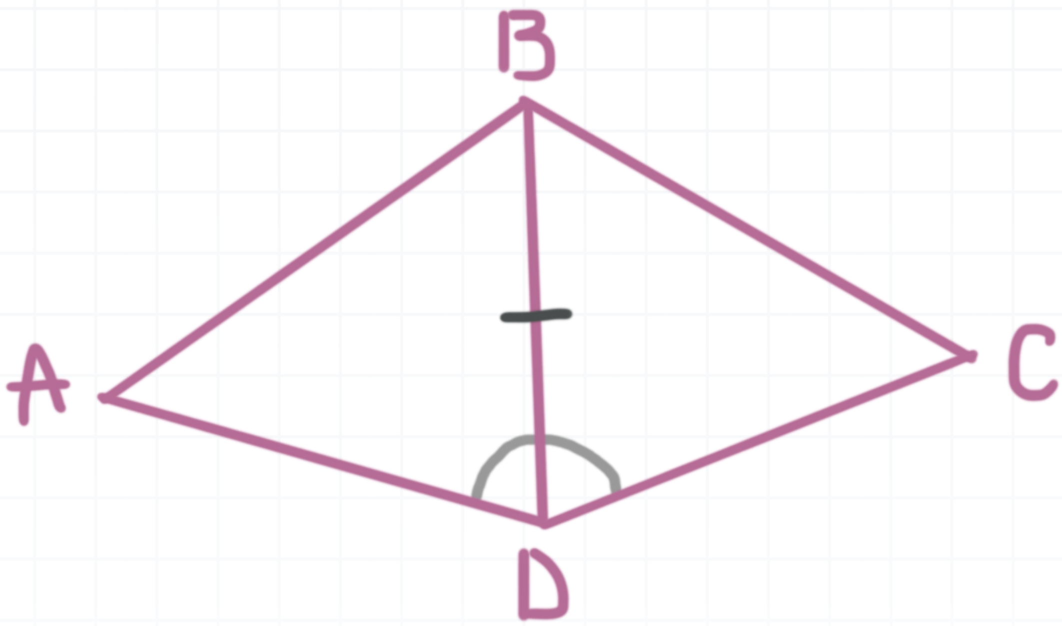
$\overline{AE} \perp \overline{BC}$, $BC||DE$, $\overline{AB} \cong \overline{DC}$, and C is a midpoint of $\overline{AE}$



Solution:

HL. We're told that the hypotenuses are congruent. We also know that $AC = EC$, because C is a midpoint. This makes a set of legs congruent.

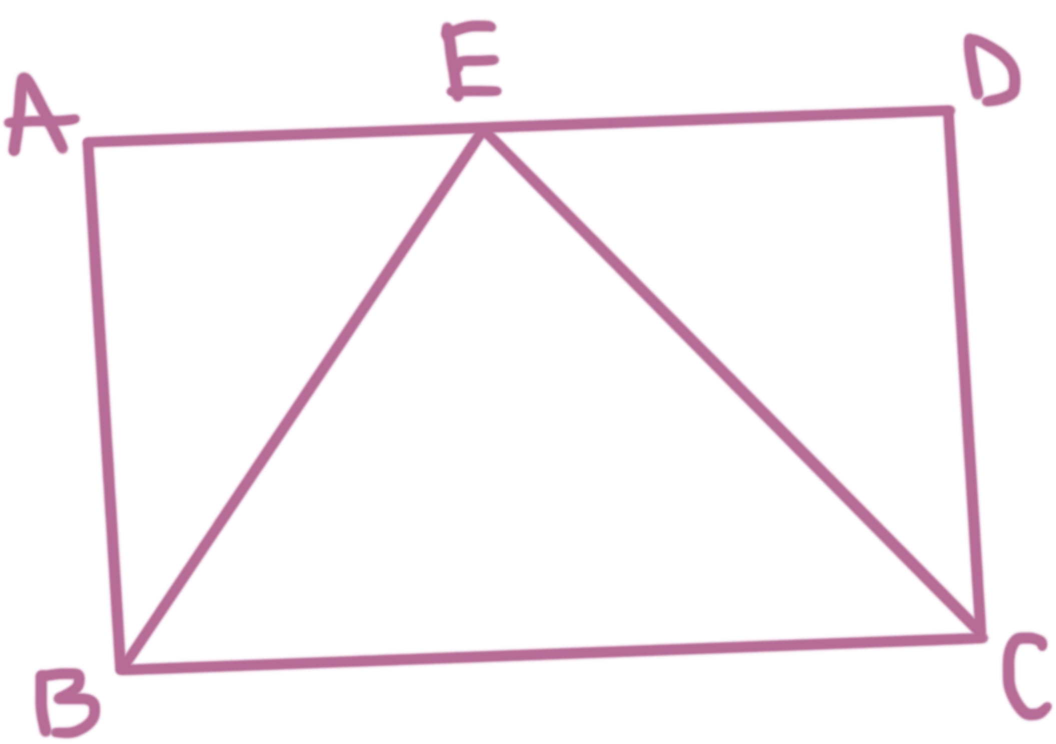
■ 3. What additional information would we need to prove these triangles are congruent using AAS Theorem?



Solution:

$\angle A \cong \angle C$

■ 4. $ABCD$ is a rectangle. BEC is an isosceles triangle with vertex angle at E. Write a proof to verify that $\triangle BAE \cong \triangle CDE$ by the HL Theorem.

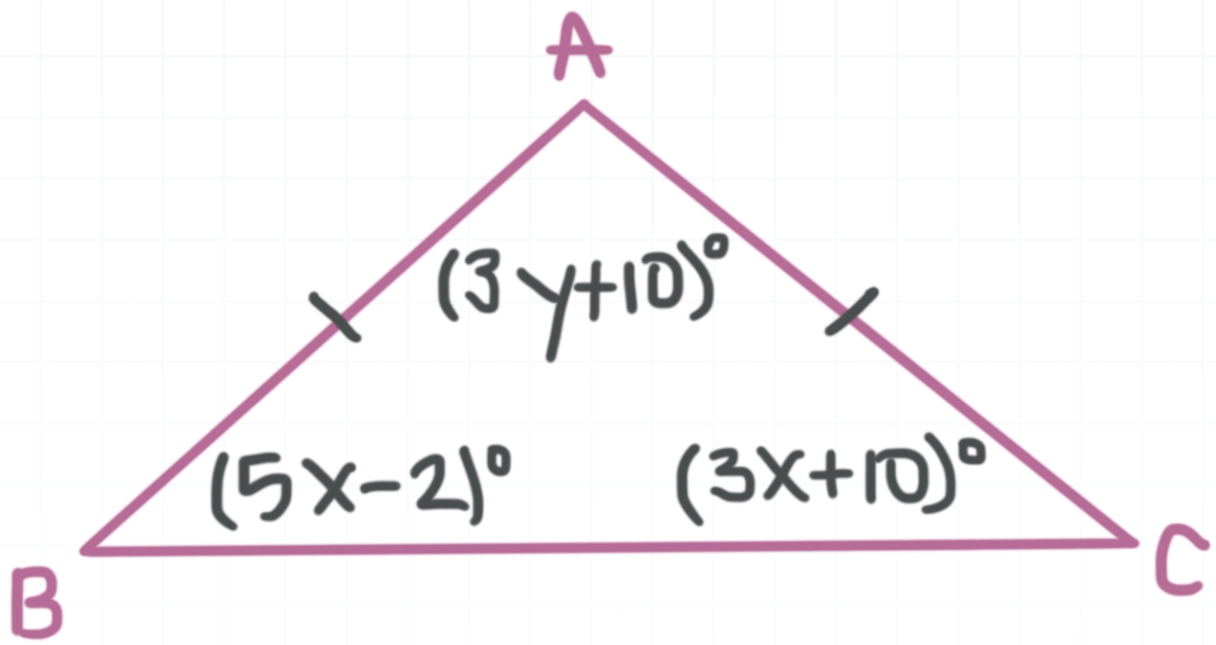


Solution:

$\angle A$ and $\angle D$ must be right angles, because $ABCD$ is a rectangle. $\triangle BAE$ and $\triangle CDE$ must be right triangles by definition of a right triangle. $\overline{AB} \cong \overline{DC}$ because opposite sides of rectangles are congruent, and $\overline{BE} \cong \overline{CE}$ because $\triangle BEC$ is an isosceles triangle. Therefore, $\triangle BAE \cong \triangle CDE$ by the HL Theorem.

ISOSCELES TRIANGLE THEOREM

■ 1. Find the values of x and y.



Solution:

$x=6$ and $y=38$. Because the triangle is isosceles, we get

$$5x-2=3x+10$$

$$2x=12$$

$$x=6$$

Therefore, the matching angles are

$$m\angle B=m\angle C=5(6)-2=28^\circ$$

Which means that $m\angle A$ must be $m\angle A = 180 - 28 = 124^\circ$, which means the value of y is

$$3y + 10 = 124$$

$$3y = 114$$

$$y = 38$$

■ 2. $\triangle JKL$ is isosceles with vertex angle K. $JK = 4x - 5$, $LK = 3x + 8$, and $m\angle J = 2x + 4$. Find $m\angle L$.

Solution:

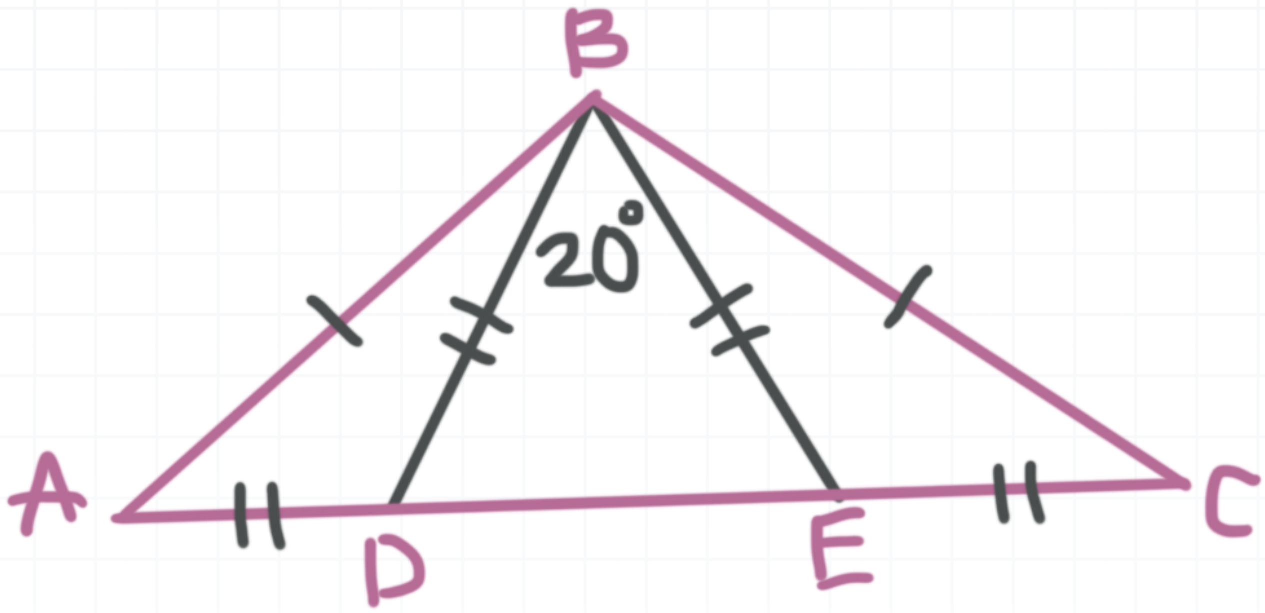
30°. $JK = LK$ because the triangle is isosceles.

$$4x - 5 = 3x + 8$$

$$x = 13$$

Then we can say $m\angle J = 2(13) + 4 = 30^\circ$, and therefore that $m\angle C = m\angle J = 30^\circ$ by the Isosceles Triangle Theorem.

■ 3. Find $m\angle ABC$.



Solution:

100°. By the Triangle Sum Theorem and the Isosceles Triangle Theorem,

$$m\angle BDE = m\angle BED = \frac{180 - 20}{2} = 80^\circ$$

Then because they form a linear pair,

$$m\angle ADB = 180 - 80 = 100^\circ$$

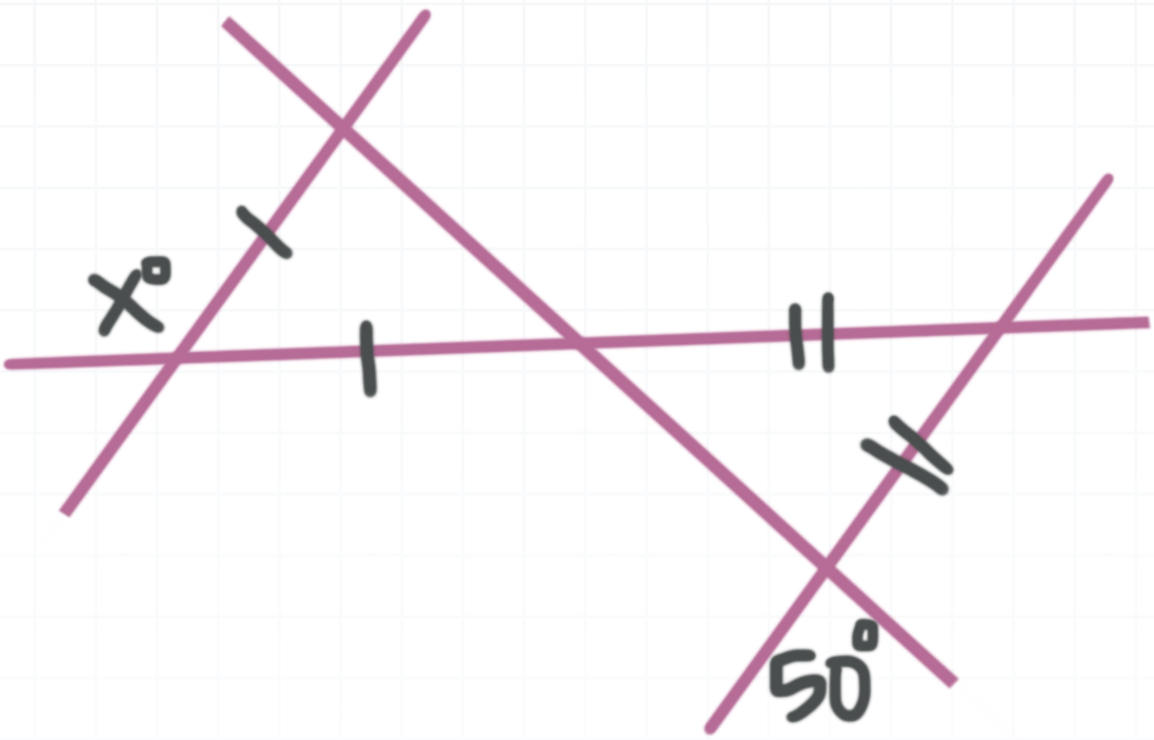
So

$$m\angle ABD = \frac{180 - 100}{2} = 40^\circ \text{ and } m\angle EBC = 40^\circ$$

Which means

$$m\angle ABC = 40 + 20 + 40 = 100^\circ$$

■ 4. Find x.



Solution:

100°. Use the Isosceles Triangle Theorem, vertical angles, and supplementary angles to find all the missing angles in the diagram.

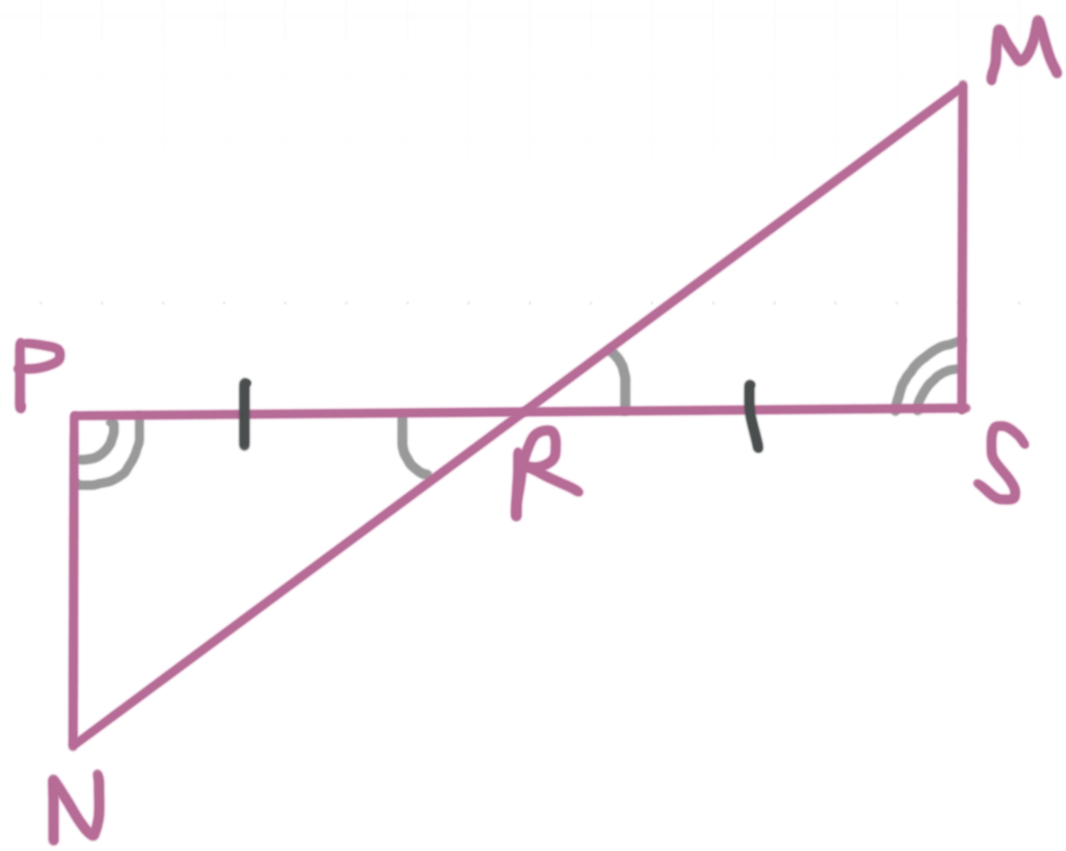
CPCTC

■ 1. Fill in the blank. Given $\triangle LMO \cong \triangle SQR$, $\overline{LO} \cong$ ________________.

Solution:

$\overline{SR}$. By CPCTC, these two line segments must be congruent if the triangles are congruent.

■ 2. Determine whether $\angle M \cong \angle N$. Justify your answer.



Solution:

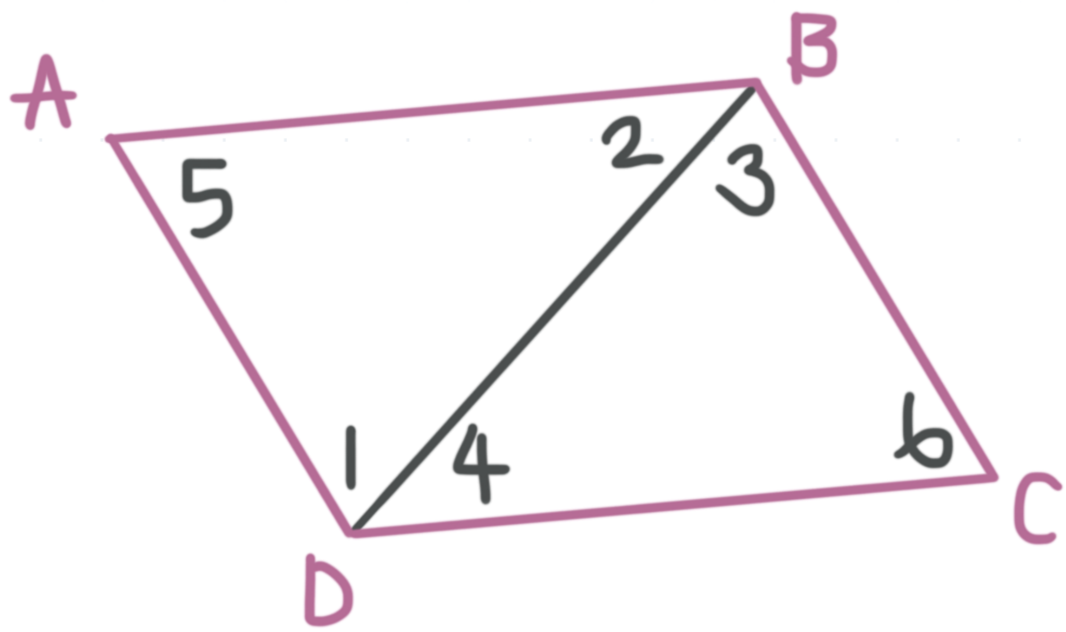
Yes, $\triangle PRN \cong \triangle SRM$ by the ASA Theorem. Therefore, $\angle M \cong \angle N$ by CPCTC.

■ 3. $\triangle DOG \cong \triangle TCA$ by SSS. What three conclusions can be drawn by CPCTC?

Solution:

$\angle D \cong \angle T$, $\angle O \cong \angle C$, and $\angle G \cong \angle A$. Congruent parts of congruent triangles are congruent (CPCTC), which makes each corresponding pair of angles congruent.

■ 4. Given $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$, prove $\overline{AB} \cong \overline{CD}$.



Solution:

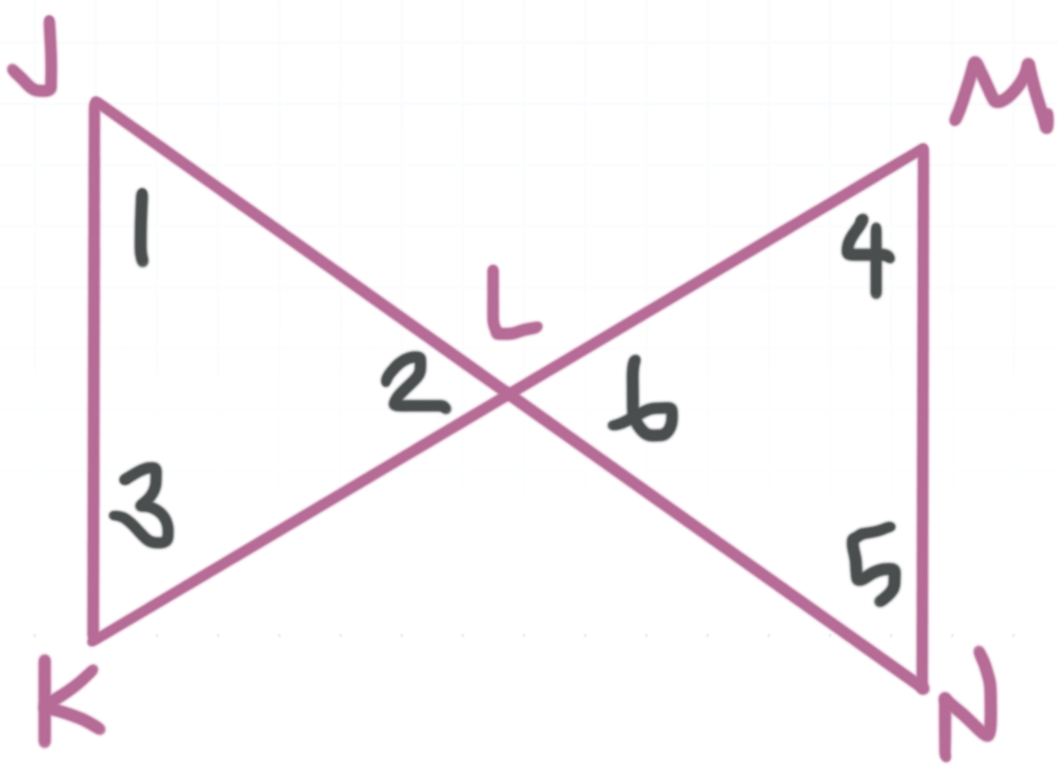
1. Given $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$.

2. $\overline{BD} \cong \overline{BD}$ by the Reflexive Property of Congruence.

3. $\triangle ABD \cong \triangle CDB$ by the ASA Theorem.

4. $\overline{AB} \cong \overline{CD}$ by CPCTC.

■ 5. Given that L is the midpoint of $\overline{JN}$ and $\overline{KM}$, prove $\overline{JK} \cong \overline{NM}$.



Solution:

1. L is the midpoint of $\overline{JN}$ and $\overline{KM}$, so by definition of midpoint $JL = NL$ and $ML = KL$.

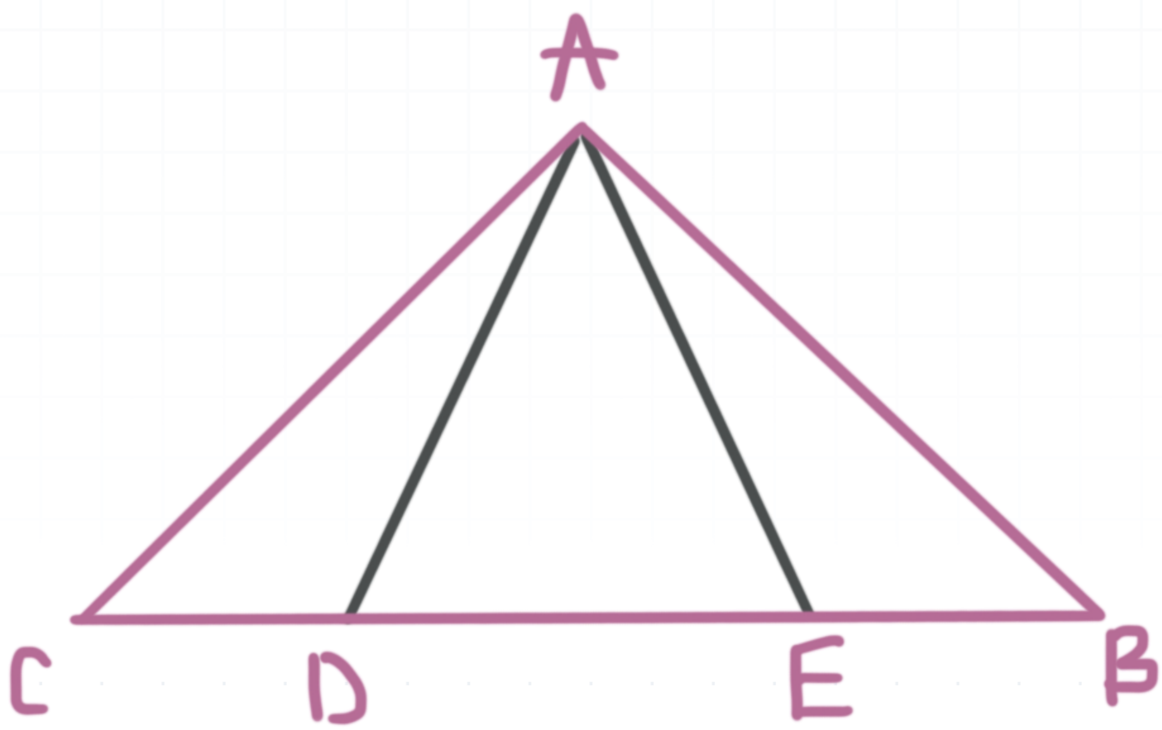
2. $\overline{JL} \cong \overline{NL}$ and $\overline{ML} \cong \overline{KL}$ by definition of congruent segments.

3. $\angle 2 \cong \angle 6$ by definition of vertical angles.

4. $\triangle JLK \cong \triangle NLM$ by SAS Theorem.

5. $\overline{JK} \cong \overline{NM}$ by CPCTC.

■ 6. Given that $\triangle CAB$ is an isosceles triangle, that D is the midpoint of $\overline{CE}$, and that E is the midpoint of $\overline{BD}$, prove that $\triangle DAE$ is isosceles.



Solution:

1. $\triangle CAB$ is an isosceles triangle, so by definition, $\overline{AC} \cong \overline{AB}$.

2. D is the midpoint of $\overline{CE}$, and E is the midpoint of $\overline{BD}$, so by definition $\overline{CD} = \overline{DE}$ and $\overline{DE} = \overline{EB}$.

3. $\overline{CD} \cong \overline{DE}$ and $\overline{DE} \cong \overline{EB}$ by definition of congruent segments.

4. $\overline{CD} \cong \overline{EB}$ by the Transitive Property of Congruence.

5. $\angle C \cong \angle B$ by the Isosceles Triangle Theorem.

6. $\triangle ACD \cong \triangle ABE$ by the SAS Theorem.

7. $\overline{AD} \cong \overline{AE}$ by CPCTC.

8. $\triangle DAE$ is isosceles by the definition of an isosceles triangle.

W W W . K R I S T A K I N G M A T H . C O M