Translating figures in coordinate space

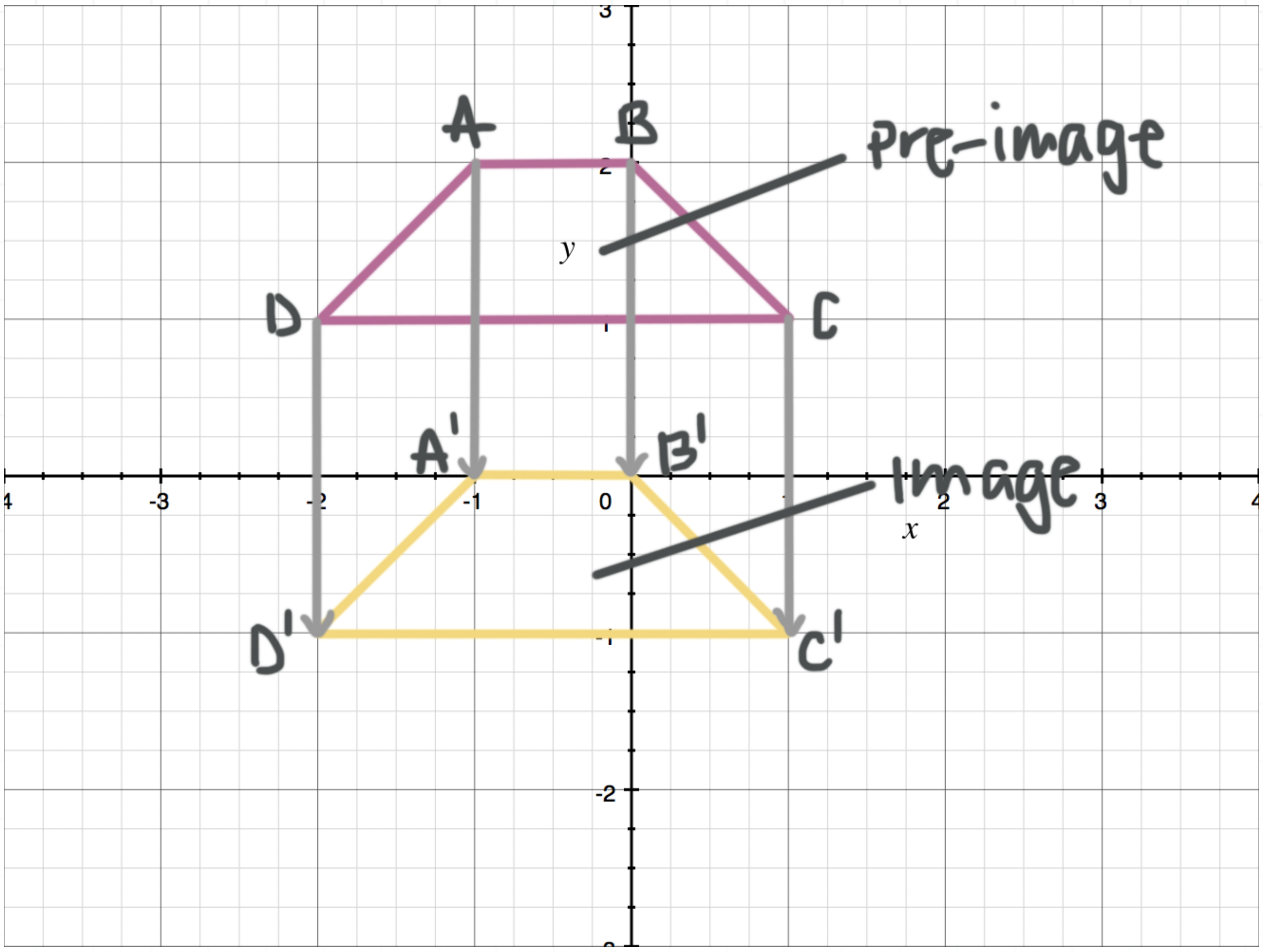
In this lesson we'll look at translation of a figure in a coordinate plane and how to determine where the figure is located after the translation takes place.

A **translation** is a transformation that moves a figure from one location to another. A translation can be thought of as a slide with no rotation. The slide won't change the shape or size of the figure, and because there's no rotation, the orientation won't change either.

The pre-image and image

Before a translation, we have the **pre-image** (the figure in its original location and orientation). Points in the pre-image are usually labeled with capital letters. After the translation, we have the **image** (the figure in its final location and orientation). Points in the image are usually labeled with the same capital letters, plus the prime symbol $'$ after each letter. So if figure $ABCD$ is translated, its image becomes figure $A'B'C'D'$.

In a translation, the image and pre-image are always congruent, because a translation never changes the measures of angles or the lengths of line segments and curves in the figure.

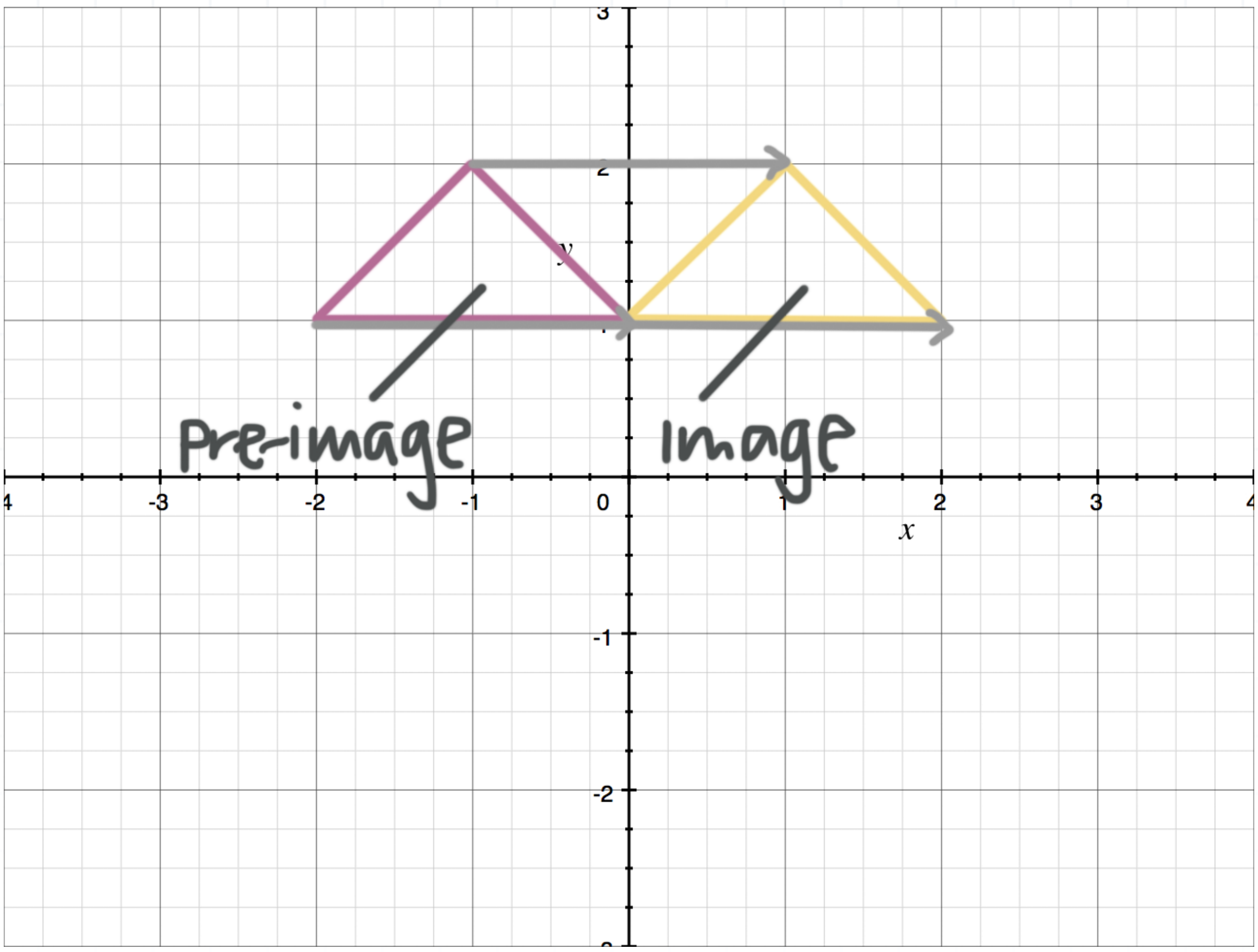


Translation notation

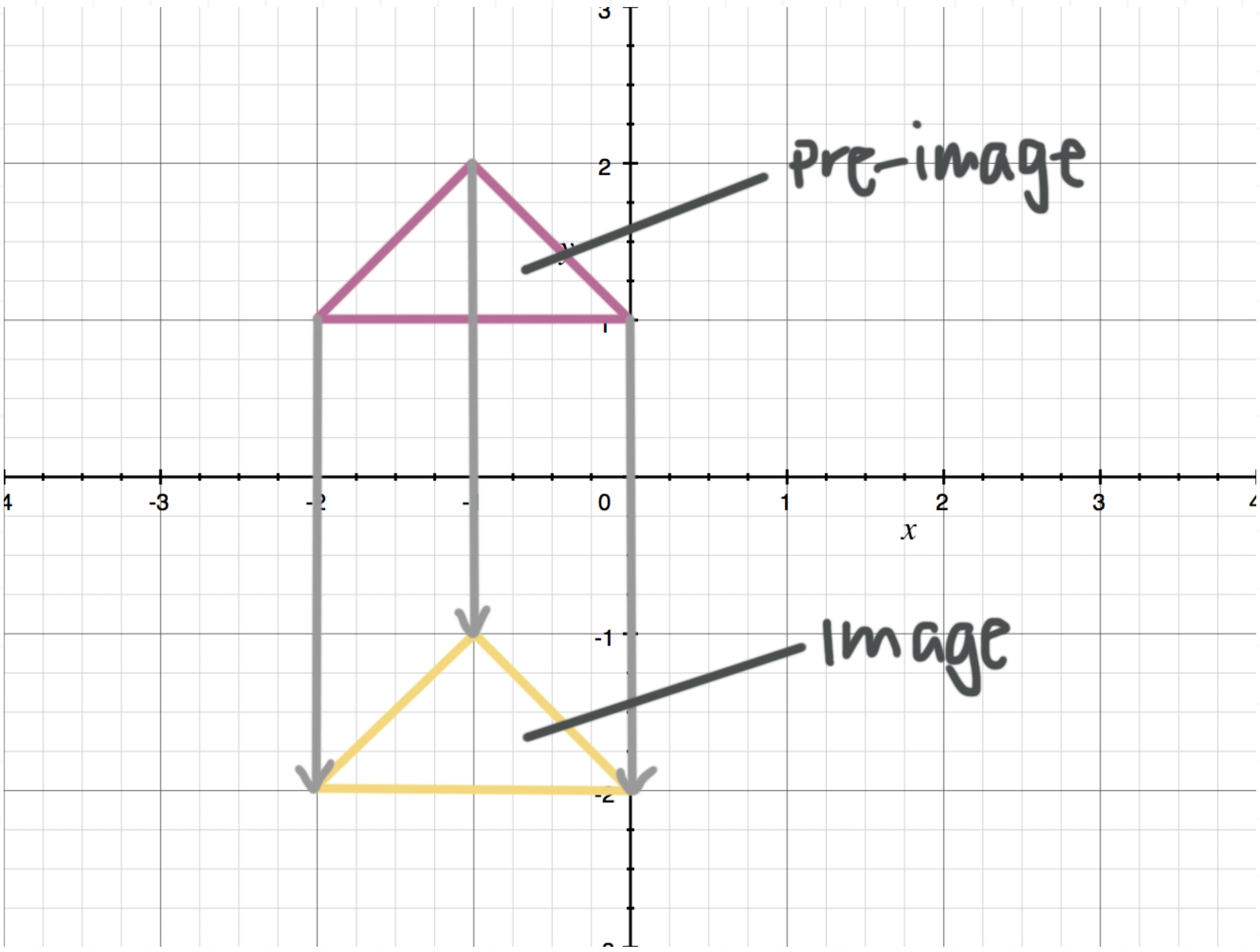
A translation can be vertical, horizontal, or both. Regardless of the direction, a translation can be written in mathematical notation. We'll use a "rule" $T(x, y)$ that express the coordinates of a point in the image in terms of the coordinates (x, y) of the corresponding point in the pre-image.

A translation 3 units to the **left**:	$T(x, y) = (x - 3, y)$
A translation 2 units to the **right**:	$T(x, y) = (x + 2, y)$
A translation 4 units **down**:	$T(x, y) = (x, y - 4)$
A translation 1 unit **up**:	$T(x, y) = (x, y + 1)$

In the translation of the triangle in this figure, the pre-image is translated 2 units to the right to get the image, so $T(x, y) = (x + 2, y)$.



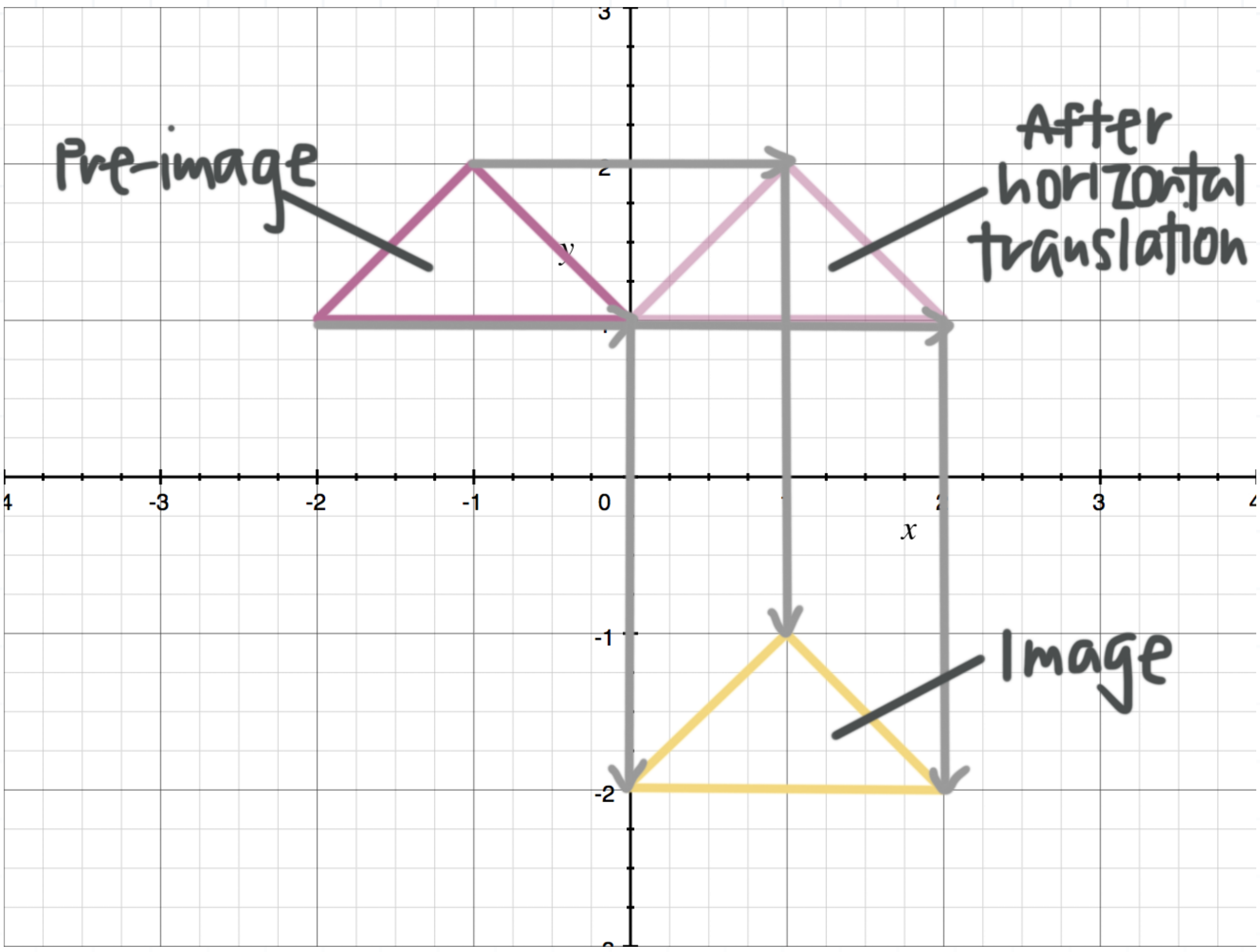
In the translation of the triangle in the next figure, the pre-image is translated 3 units down to get the image, so $T(x, y) = (x, y - 3)$.



Two translations together

We said that we can do a translation that's both horizontal and vertical. If we want to translate a figure 2 units to the right and 3 units down, then the rule for the translation is

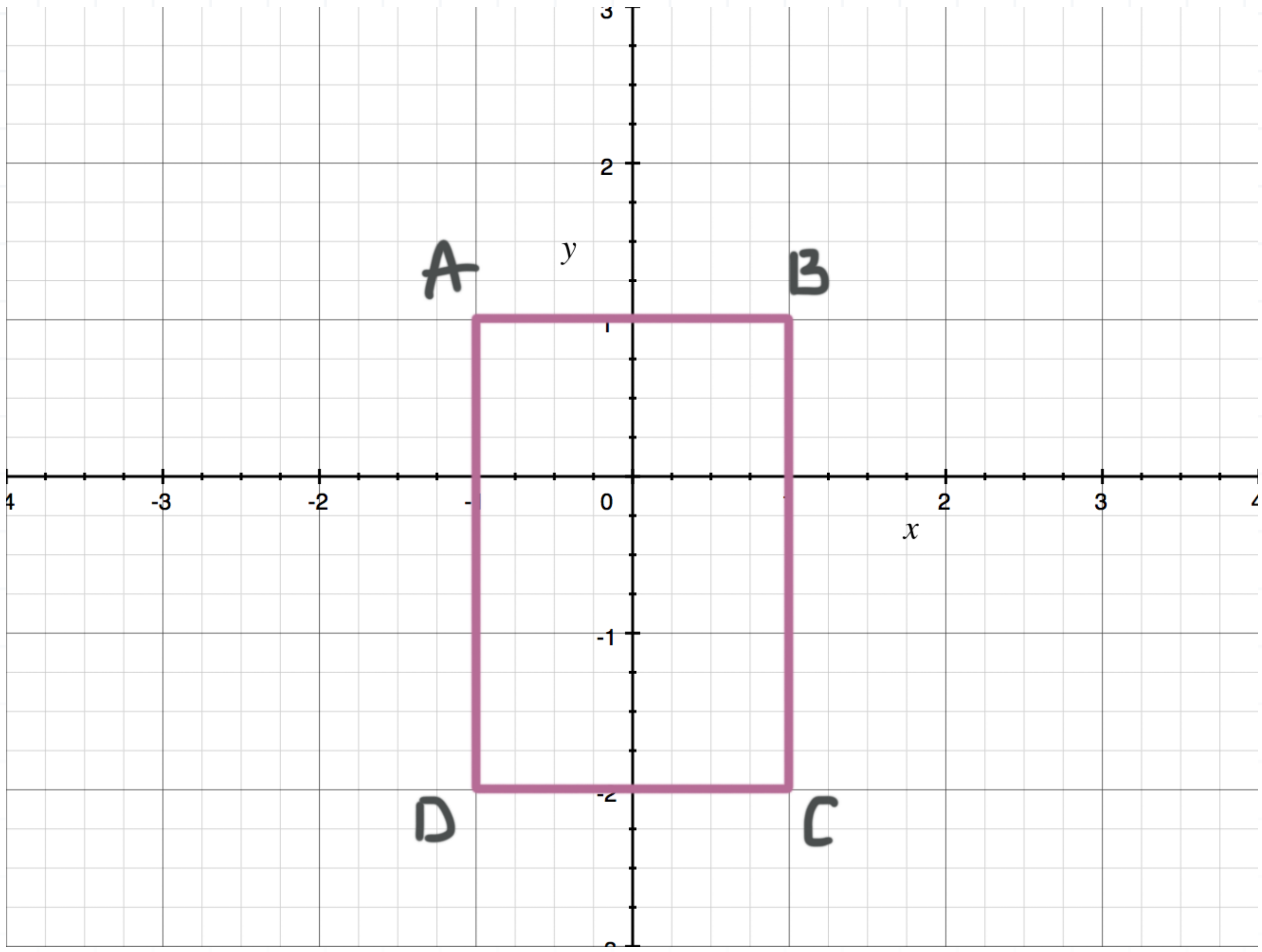
$$T(x, y) = (x + 2, y - 3)$$



Let's start by working through an example.

Example

If rectangle $ABCD$ undergoes the translation described by $T(x,y) = (x - 2, y + 3)$, where will point D' be located?



The translation is $T(x, y) = (x - 2, y + 3)$. The $x - 2$ tells you that the x-coordinate of any point in the image will be 2 less than the x-coordinate of the corresponding point in the pre-image, and that the y-coordinate of any point in the image will be 3 more than the y-coordinate of the corresponding point in the pre-image.

In other words, after the translation the figure will be located 2 units to the left of and 3 units above, its original location. The coordinates of point D are $(-1, -2)$, so the coordinates of point D' are given by

$$T(-1, -2) = (-1 - 2, -2 + 3) = (-3, 1)$$

So $D' = (-3, 1)$.

Let's try another translation problem.

Example

If a translation moves a point A to a point A', write the rule for the translation if $A = (-3,7)$ and $A' = (5, -2)$.

Let's look at what happens to each coordinate.

The x-coordinate: $-3 \to 5$

This means we added 8 because $-3 + 8 = 5$.

The y-coordinate: $7 \to -2$

This means we subtracted 9 because $7 - 9 = -2$.

Now we can put this all together to write the rule for the translation.

$$T(x, y) = (x + 8, y - 9)$$