# Geometry Workbook Solutions

Similarity

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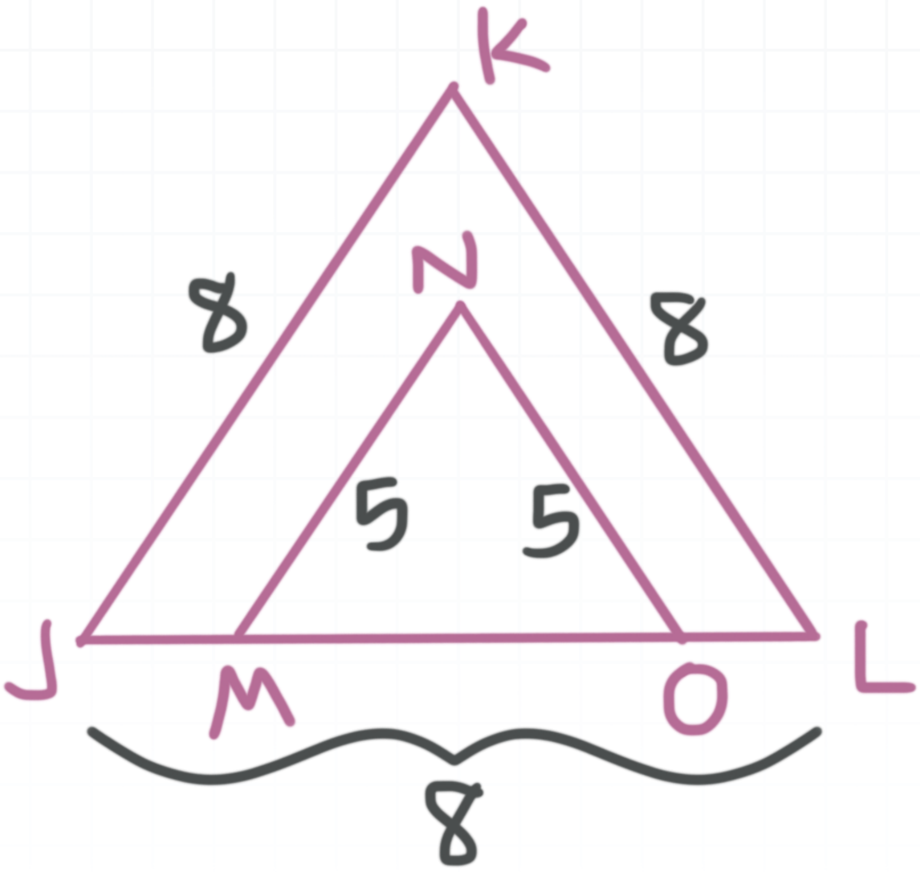
## SIMILAR TRIANGLES

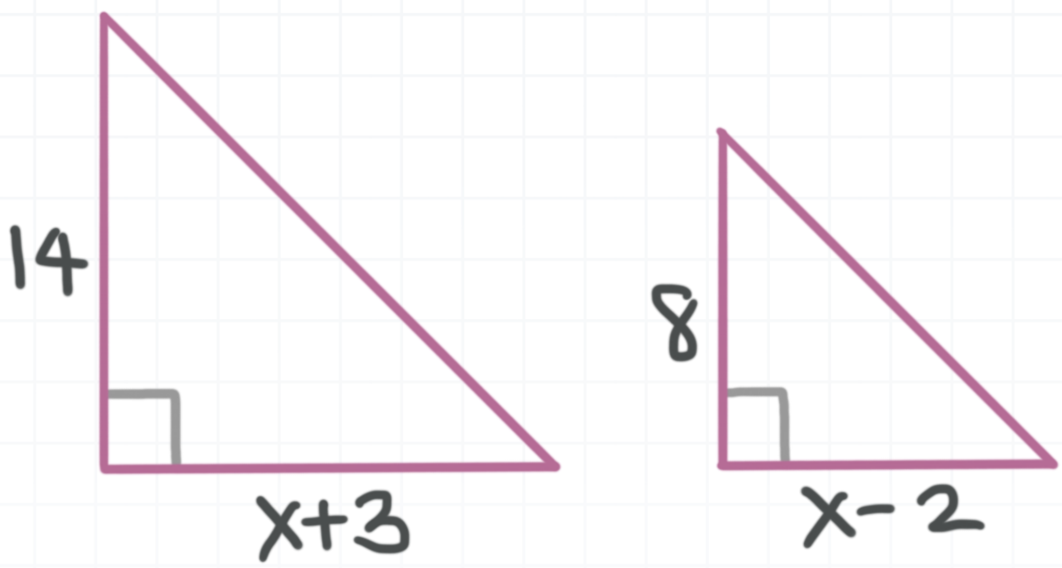
■ 1. $\triangle JKL$ is similar to $\triangle MNO$. Find $MO$.




*Solution:*

$5$. $8/5$ must be the proportion of all corresponding sides of the similar triangles. $\triangle JKL$ is equilateral and $\triangle MNO$ must also be equilateral.

■ 2. Given that the two triangles are congruent, set up a proportion to solve for $x$.




*Solution:*

$x = 26/3$. Set up a proportion, the cross multiply to solve for $x$.

$$\frac{14}{8} = \frac{x+3}{x-2}$$

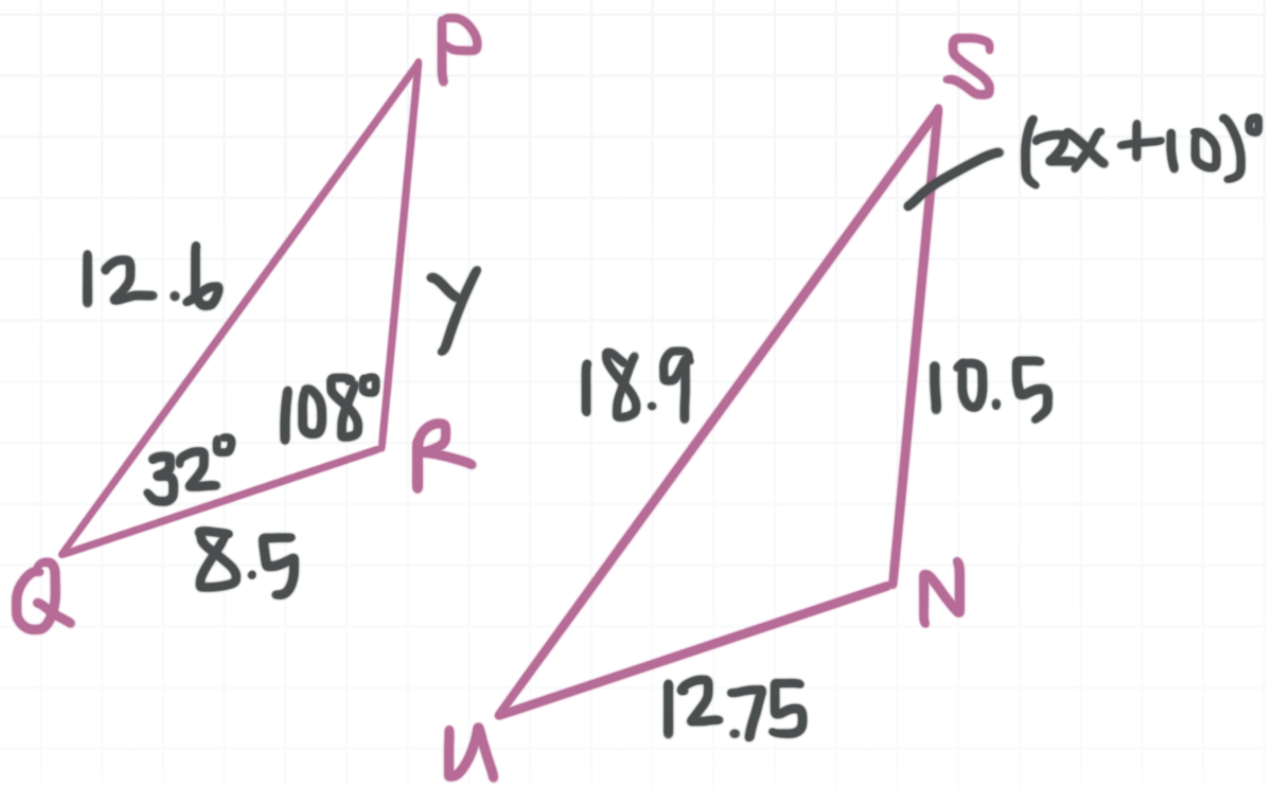
$$14(x-2) = 8(x+3)$$

$$14x - 28 = 8x + 24$$

$$6x = 52$$

$$x = \frac{26}{3}$$

■ 3. $\triangle PQR$ is similar to $\triangle SUN$. Find the values of $x$ and $y$.




*Solution:*

$x = 15$ and $y = 7$. Set up a proportion to find $y$.

$$\frac{8.5}{12.75} = \frac{y}{10.5}$$

$$(8.5)(10.5) = 12.75y$$

$$12.75y = 89.25$$

$$y = 7$$

Then we can find $m\angle P$ as $m\angle P = 180^\circ - 32^\circ - 108^\circ = 40^\circ$. And since $m\angle P = m\angle S$, we can find $x$ be setting up an equation.

$$40 = 2x + 10$$

$$30 = 2x$$

$x = 15$

■ 4. A $14$-foot tree casts a $6$-foot long shadow. A $3.5$-foot tall child would have a shadow length of how many feet?

*Solution:*

$0.75$ feet. Set up the proportion.

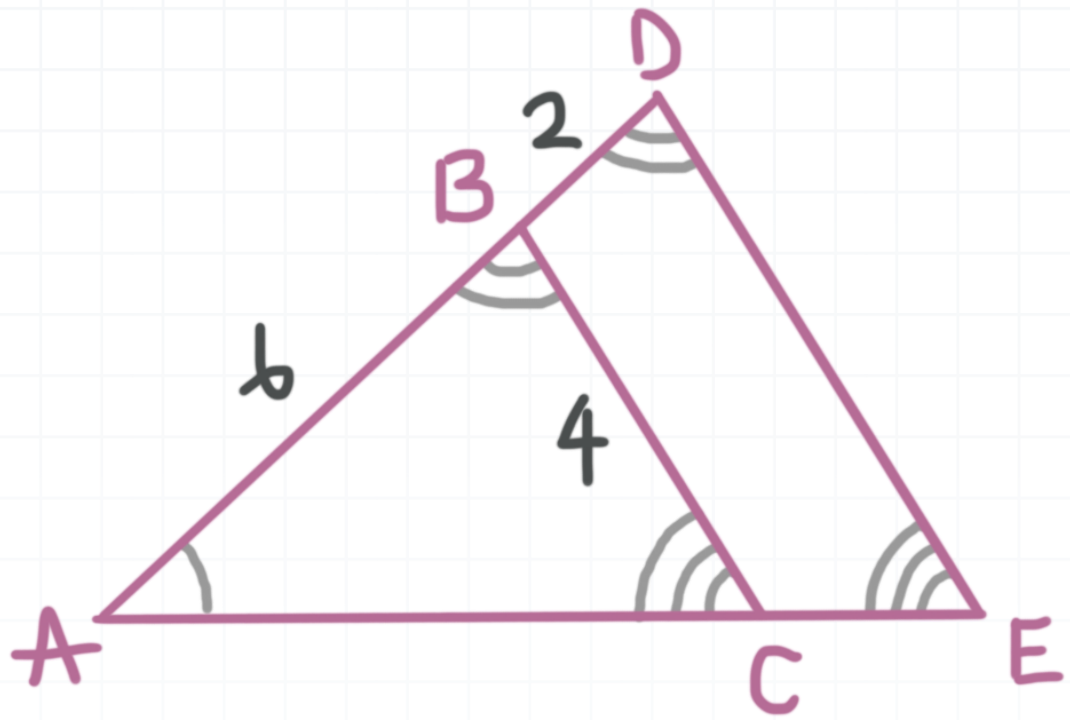
$$\frac{14}{6} = \frac{3.5}{x}$$

$$14x = 6(3.5)$$

$$14x = 21$$

$$x = 1.5$$

■ 5. Find $DE$.




*Solution:*

$DE = 16/3$. $\triangle ABC$ is similar to $\triangle ADE$, so

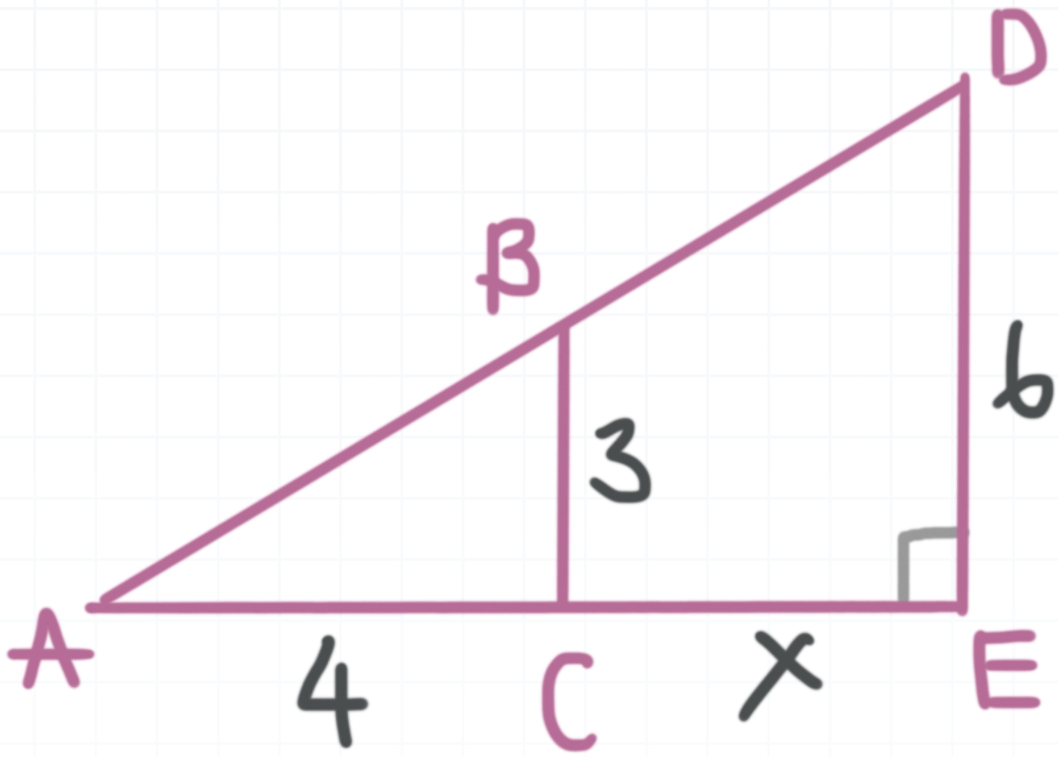
$$\frac{AB}{AD} = \frac{BC}{DE}$$

$$\frac{6}{4} = \frac{8}{DE}$$

$$6DE = 32$$

$$DE = \frac{16}{3}$$

■ 6. Find $CE$.




Solution:

$CE = 4$. $\triangle ABC$ is similar to $\triangle ADE$, so

$$\frac{AC}{AE} = \frac{BC}{DE}$$

$$\frac{4}{4+x} = \frac{3}{6}$$

$$24 = 3(4+x)$$

$$24 = 12 + 3x$$

$$12 = 3x$$

$$x = 4$$

## 45-45-90 TRIANGLES

1. $\triangle PDX$ is an isosceles right triangle with vertex $\angle D$, and $PD = 4$. Find $DX$ and $XP$.

*Solution:*

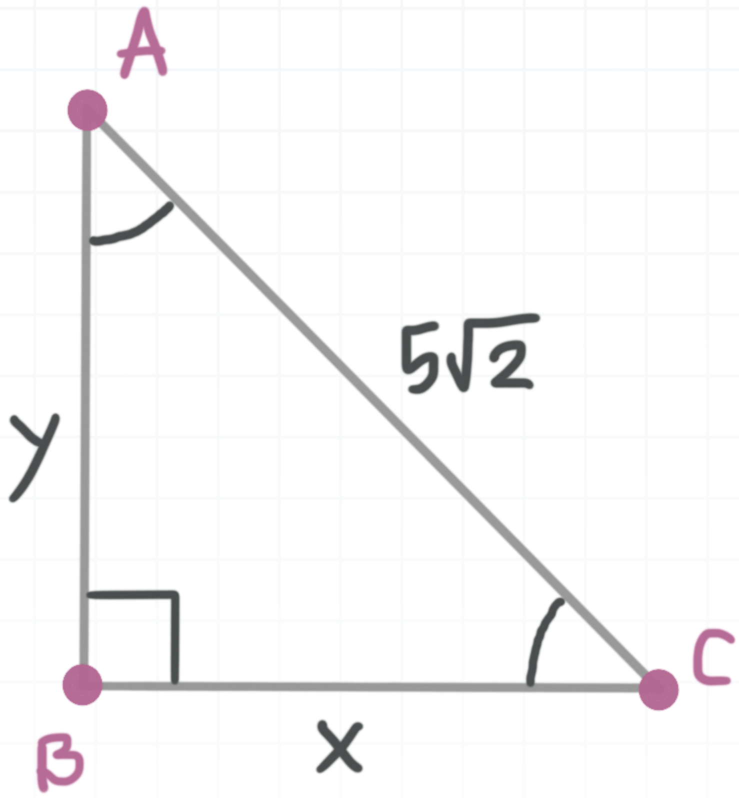
$DX = 4$ and $XP = 4\sqrt{2}$. By the $45 - 45 - 90$ rule of right triangles, legs of the triangle are congruent and the hypotenuse has a measure of $\sqrt{2}$. The legs are $PD$ and $DX$ and both have measures of $4$. The hypotenuse is $XP$ and has measure $4\sqrt{2}$.

2. A square has a perimeter of $40$ meters. Find the length of the diagonal of the square.

*Solution:*

$10\sqrt{2}$. Since the perimeter is $40$, we know the length of each side is $10$. Using the $45 - 45 - 90$ rule of right triangles, we get the length of the diagonal to be $10\sqrt{2}$.

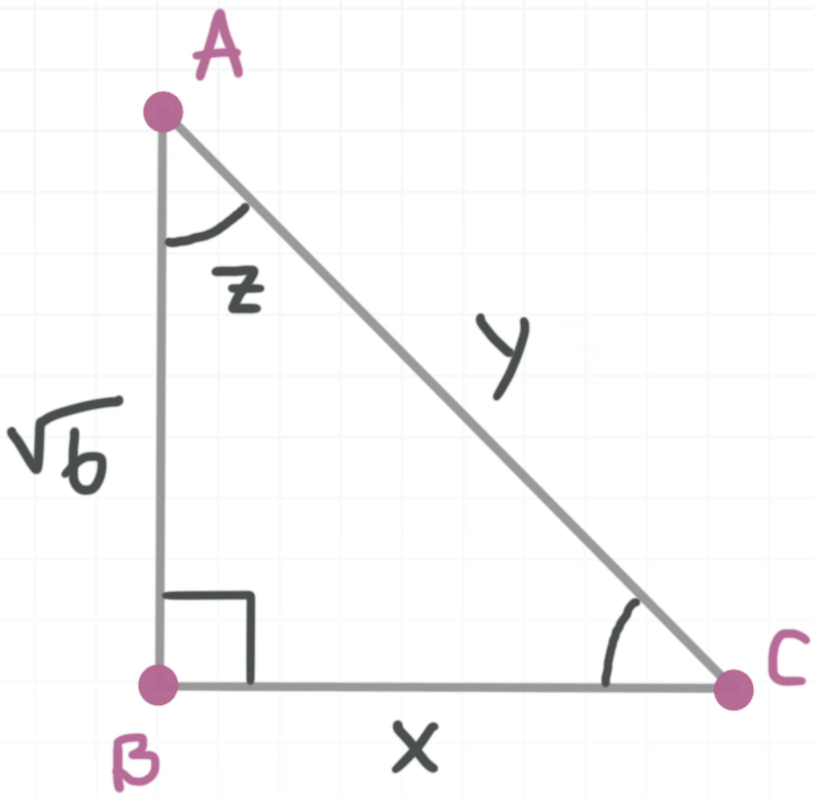
3. Find the values of $x$ and $y$.




*Solution:*

$x = 5$ and $y = 5$. By the $45-45-90$ rule of right triangles, legs of the triangle are congruent and the hypotenuse has a measure of $\sqrt{2}$. The value of each leg must equal $5$.

■ 4. Find the values of $x$, $y$, and $z$.




*Solution:*

$x = \sqrt{6}$, $y = 2\sqrt{3}$, and $z = 45$. In our $45-45-90$ special right triangle, the legs are congruent. $\overline{AB} \cong \overline{CB}$, which means they both have a measure of $\sqrt{6}$. The length of the hypotenuse can be found by taking the measure of the leg and multiplying it by $\sqrt{2}$.

$$y = \sqrt{6}\sqrt{2}$$

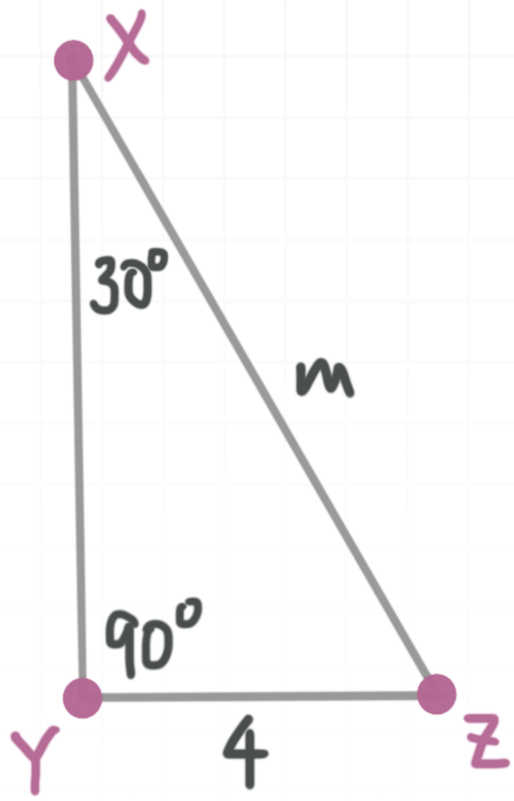
$$y = \sqrt{12}$$

$$y = \sqrt{4}\sqrt{3}$$

$$y = 2\sqrt{3}$$

## 30-60-90 TRIANGLES

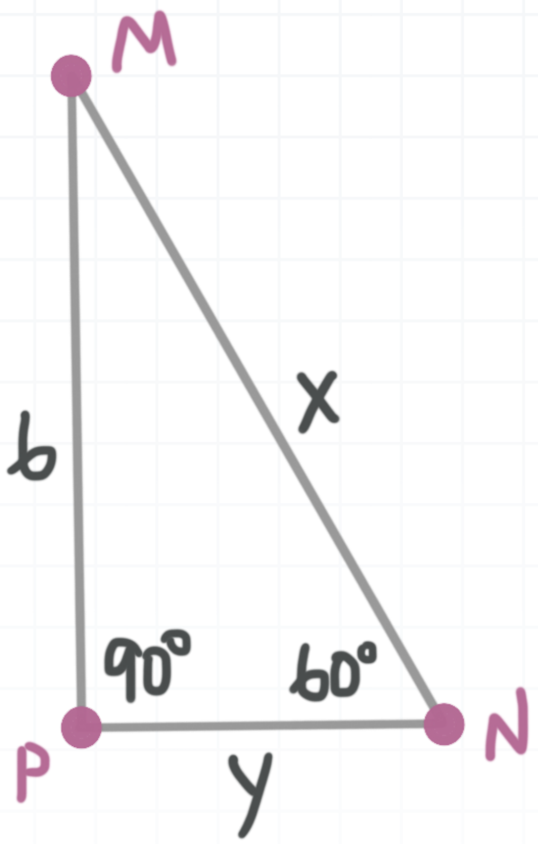
1. Find the value of $m$ in the given triangle.




*Solution:*

$m = 8$. $m$ represents the length of the hypotenuse of this $30-60-90$ triangle. The hypotenuse is always twice as long as the shortest leg, so $m = 2(4) = 8$.

2. Find the values of $x$ and $y$ in the given triangle.




*Solution:*

$x = 4\sqrt{3}$ and $y = 2\sqrt{3}$. In a $30-60-90$ triangle, the length of the longer leg is always the product of the length of the shorter leg and $\sqrt{3}$.

$$6 = y\sqrt{3}$$

$$y = \frac{6\sqrt{3}}{3}$$

$$y = 2\sqrt{3}$$

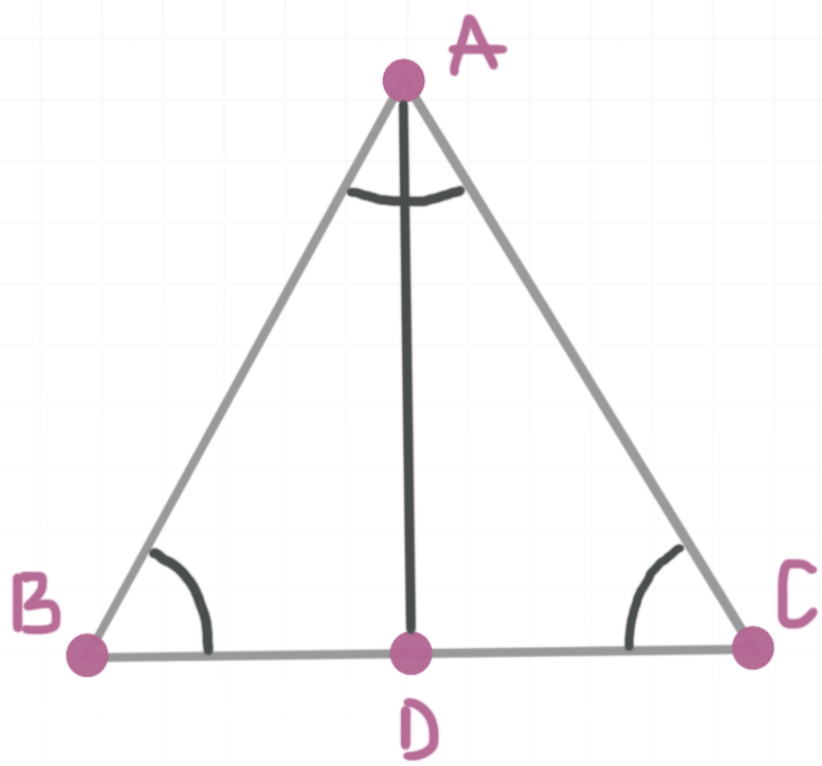
The length of the hypotenuse is twice the length of the shortest leg.

$$x = 2(y)$$

$$x = 2(2\sqrt{3})$$

$x = 4\sqrt{3}$

■ 3. $\triangle BAC$ is an equilateral triangle. The perimeter is $42$ cm and $m\angle ADC = 90$. Find $AD$.




*Solution:*

$AD = 7\sqrt{3}$. The perimeter is $42$, therefore the length of each side of the triangle is $14$. $\overline{AD}$ is an altitude of the triangle, so two $30-60-90$ triangles are formed. $BD = CD = 7$, which is the shortest leg of each of your special right triangles. The longer leg of a $30-60-90$ triangles is the product of the shortest leg and $\sqrt{3}$. So $AD = 7\sqrt{3}$.

■ 4. $\triangle XYZ$ is an equilateral triangle. $\overline{XM}$ is an altitude, median, and angle bisector of the triangle. If $XM = 9$, find the perimeter of the triangle.

*Solution:*

The perimeter is $18\sqrt{3}$. Draw an equilateral triangle and label $\overline{XM}$. Find the length of $YM$.

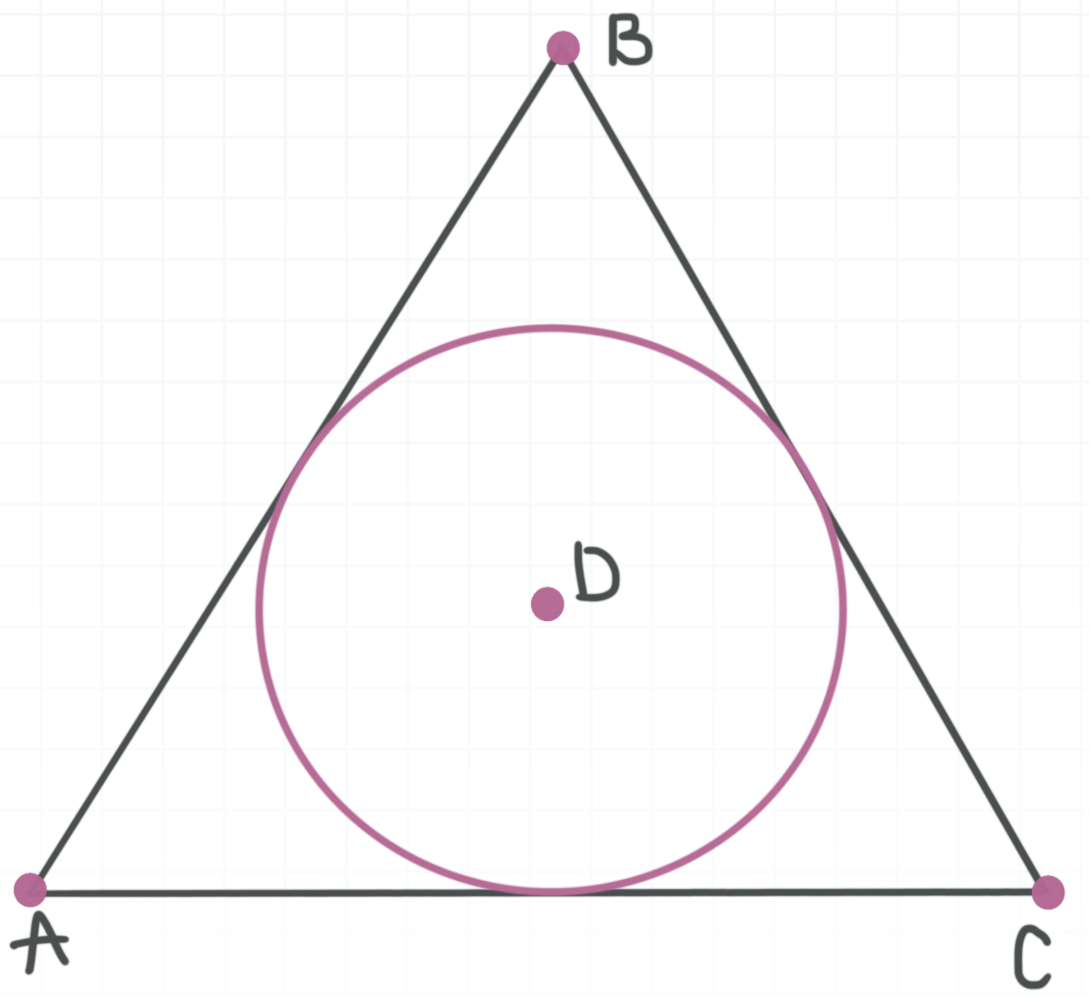
$$XM = YM\sqrt{3}$$

$$YM = \frac{XM}{\sqrt{3}} = \frac{9}{\sqrt{3}} = 3\sqrt{3}$$

Now find the length of $YZ$. $YZ = 6\sqrt{3}$. So the perimeter is

$$3(6\sqrt{3}) = 18\sqrt{3}$$

■ 5. Find the perimeter of $\triangle ABC$ if the radius of $\odot D$ is $10$ feet and $\triangle ABC$ is equilateral.




*Solution:*

We know that $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$ are congruent since $\triangle ABC$ is equilateral. Which means $\angle A = \angle B = \angle C = 60^\circ$.

Sketch a line segment from one vertex of the triangle to the center of the circle, like $\overline{BD}$, and then a line segment from $D$ to the edge of the triangle, at the center point of $\overline{AB}$, and we'll call that point $E$.

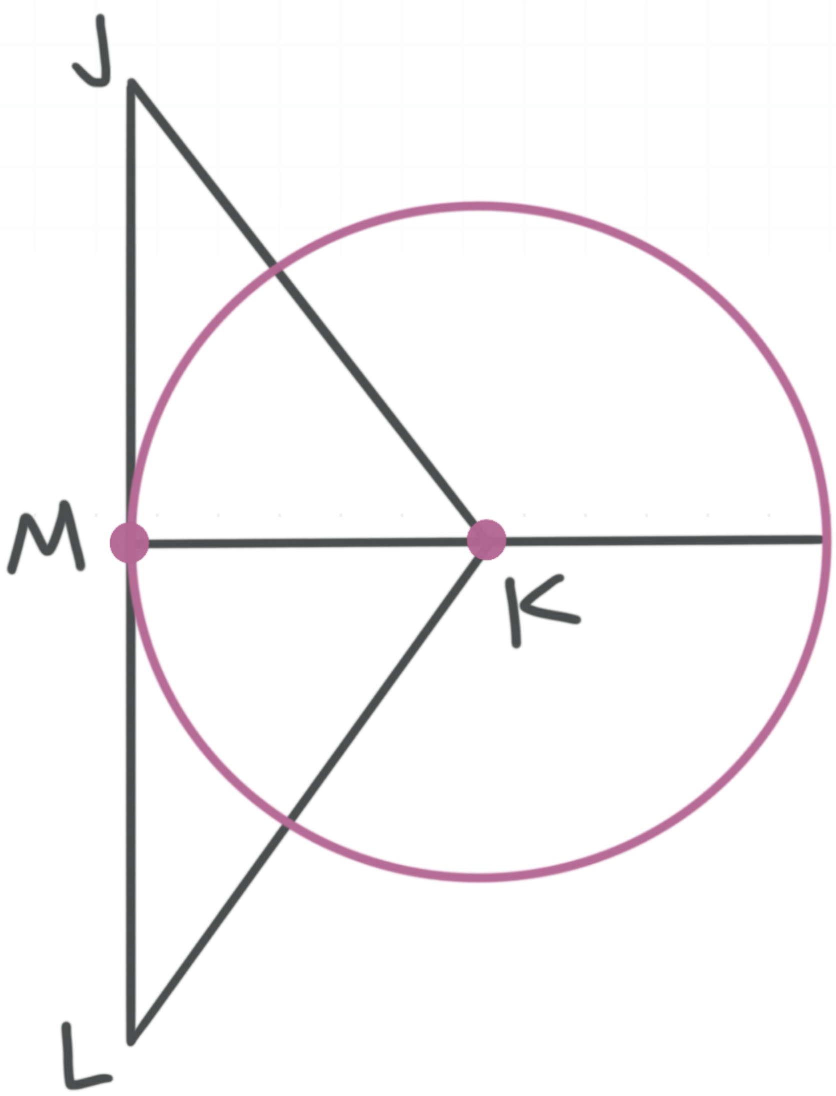
Doing so creates a 30-60-90 triangle $\triangle BDE$ inside the larger triangle. Remember that the side lengths of a 30-60-90 triangle follow the relationship $x$, $x\sqrt{3}$, and $2x$. Since the diameter of the circle is $10$, the three side lengths of our 30-60-90 triangle are $x = 10$, $x\sqrt{3} = 10\sqrt{3}$, and $2x = 20$.

So each half side of the triangle must have length $10\sqrt{3}$, which means the length of a full side of the triangle is $2(10\sqrt{3}) = 20\sqrt{3}$. Then the perimeter of the triangle is

$$P = 20\sqrt{3} + 20\sqrt{3} + 20\sqrt{3}$$

$$P = 60\sqrt{3}$$

■ 6. $\triangle JKL$ is isosceles, $\overline{JL}$ is a tangent line, $JM = LM$ and $m\angle JKL = 120^\circ$. If $MK = 8$, find $JL$.



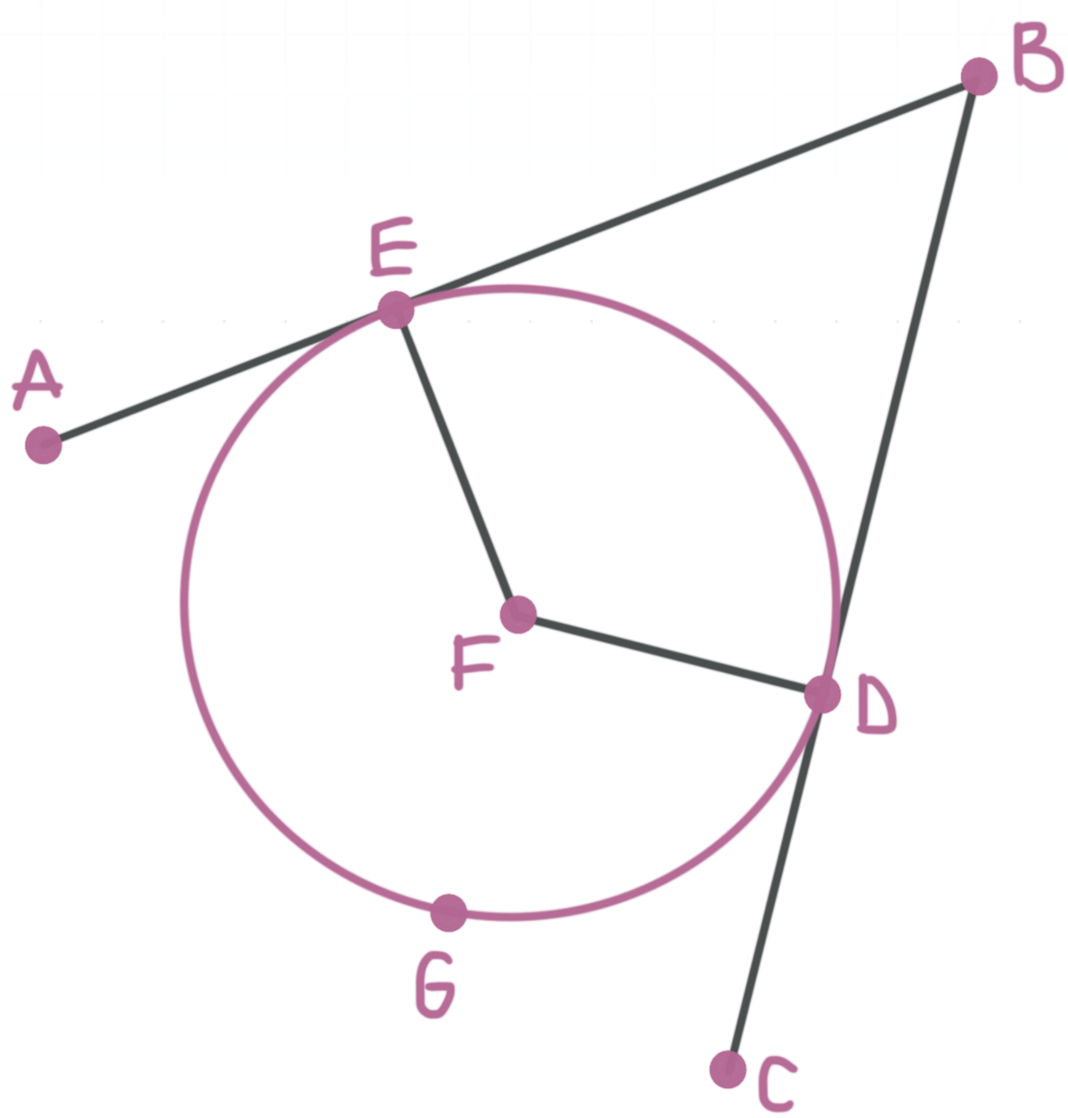
*Solution:*

The triangles $\triangle JMK$ and $\triangle LMK$ are congruent and $\overline{MK}$ bisects $\angle JKL$. We know $m\angle JKL = 120^\circ$, so $m\angle JKM = m\angle LKM = 60^\circ$. $m\angle KMJ$ must be $90^\circ$ because $\overline{MK}$ is a radius and $\overline{JL}$ is a tangent line. Use the rules for $30-60-90$ triangles to find $JM$.

$$JM = MK\sqrt{3} = 8\sqrt{3}$$

$$JL = 2JM = 2(8\sqrt{3}) = 16\sqrt{3}$$

■ 7. Arc $EGD = 240^\circ$ and $\overline{BF}$ bisects $\angle EFD$. Find the length of the radius of $\odot F$ if $FB = 14$.



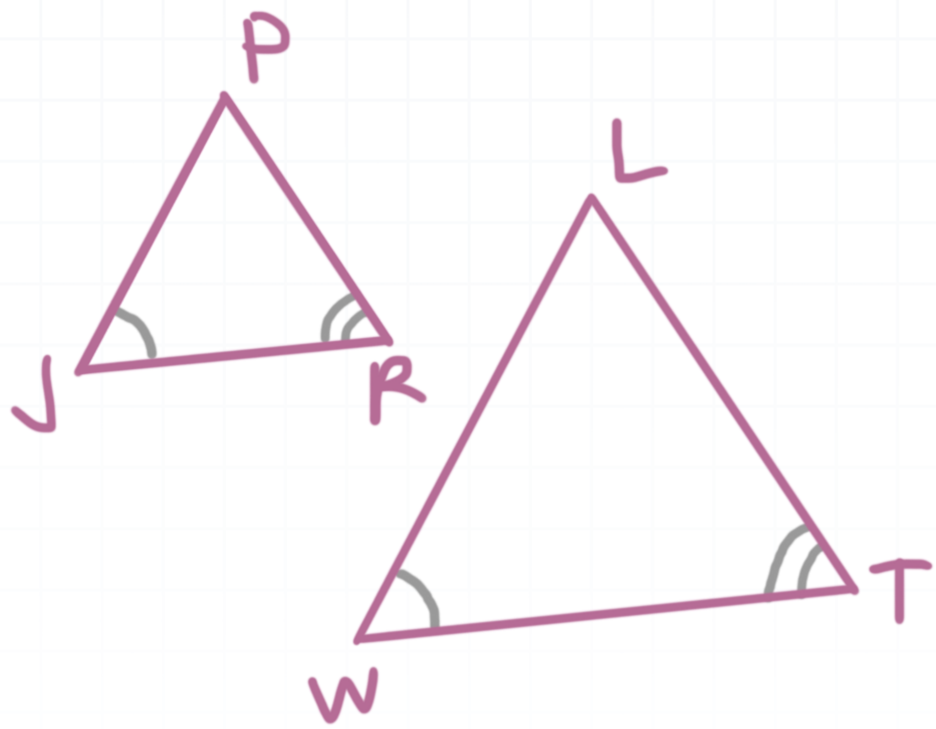
*Solution:*

We know that $m\angle EFD = 360^\circ - 240^\circ = 120^\circ$.

Draw in $\overline{FB}$ and note $\triangle FEB$ and $\triangle DFB$ are congruent $30-60-90$ triangles. $\overline{FB}$ is a hypotenuse of these triangles and has a measure of $14$. $\overline{EF}$ and $\overline{FD}$ are the shortest legs of these triangles and have a measure of $14/2 = 7$.

## TRIANGLE SIMILARITY THEOREMS

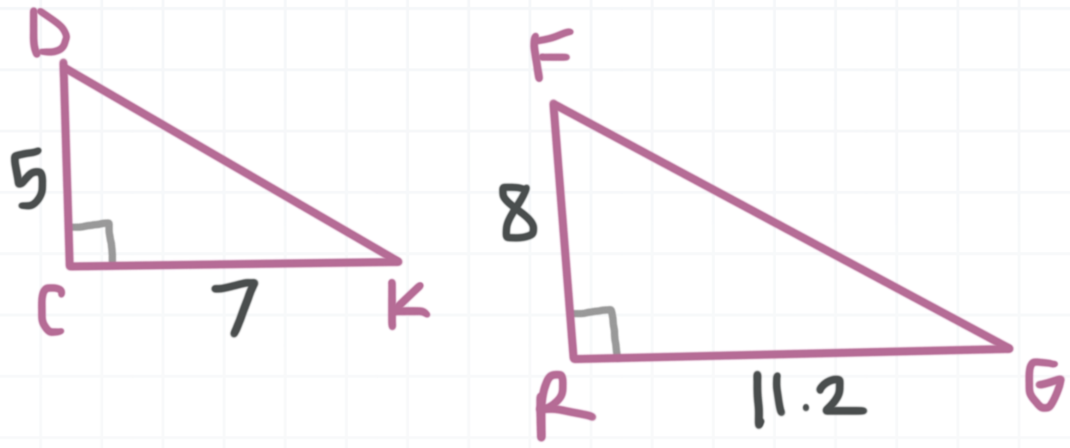
■ 1. Write a similarity statement for the triangles and provide the theorem that proves they're similar.




*Solution:*

$\triangle JPR \sim \triangle WLT$ by $AA$. If two angles of a triangle are congruent to two angles of another triangle, then the triangles must be similar.

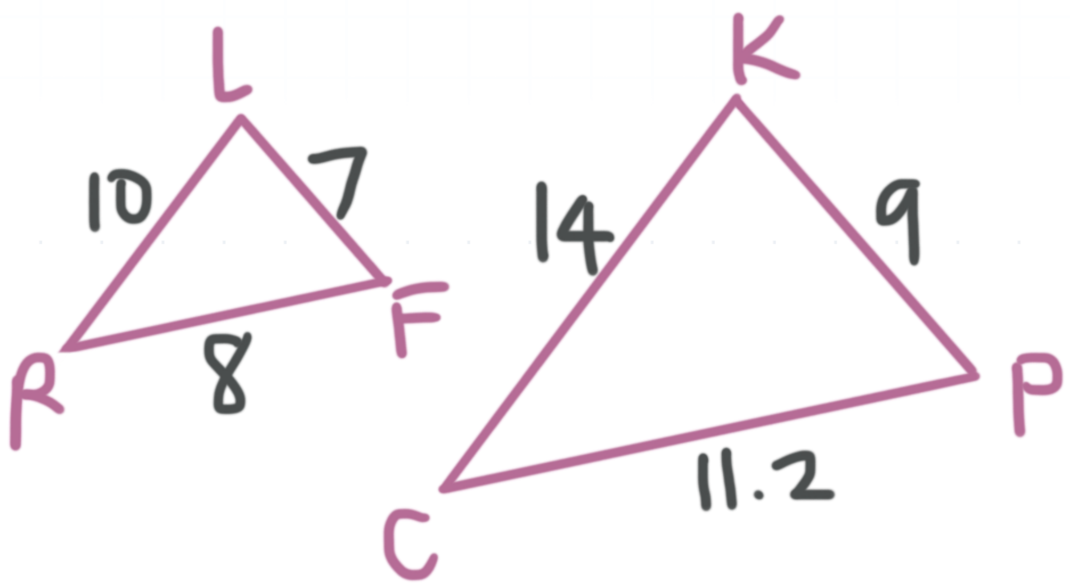
■ 2. Write a similarity statement for the triangles and provide the theorem that proves they're similar.




*Solution:*

$\triangle DCK \sim \triangle FRG$ by $SAS$. $\angle C \cong \angle R$ and $5/8 = 7/11.2$.

■ 3. Is $\triangle RLF \sim \triangle CKP$? Explain.




*Solution:*

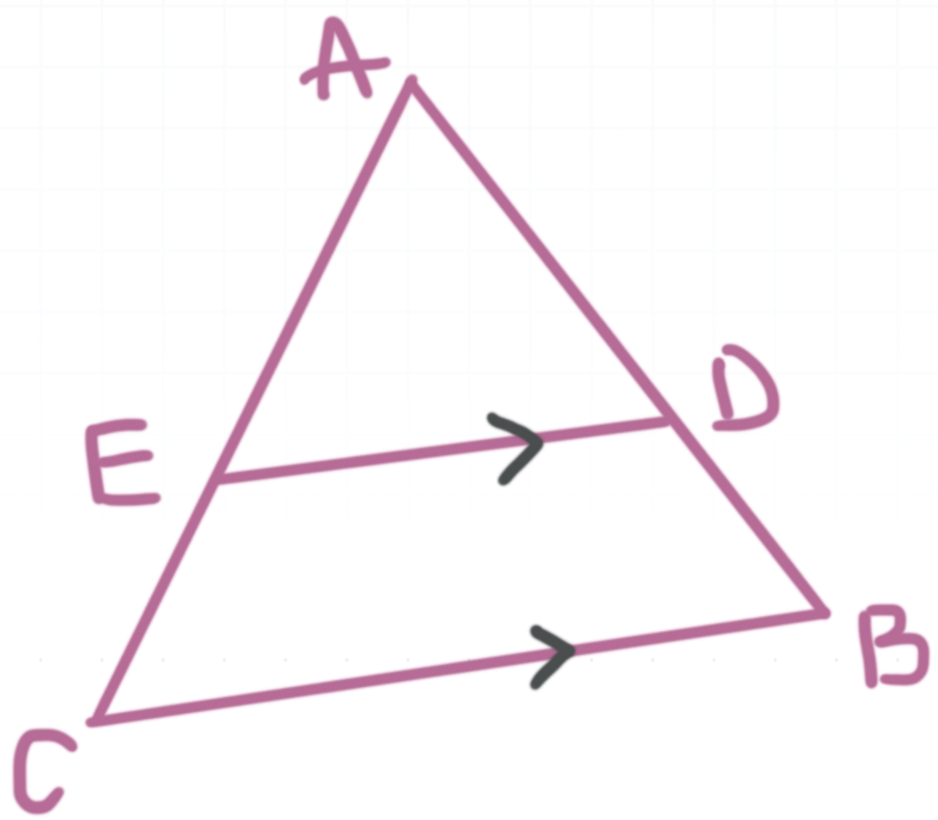
No, these triangles are not similar. Setting up proportions, it's true that

$$\frac{10}{14} = \frac{8}{11}$$

However, the triangles cannot be similar because

$$\frac{10}{14} \neq \frac{7}{9}$$

■ 4. Prove $\triangle AED \sim \triangle ACB$.

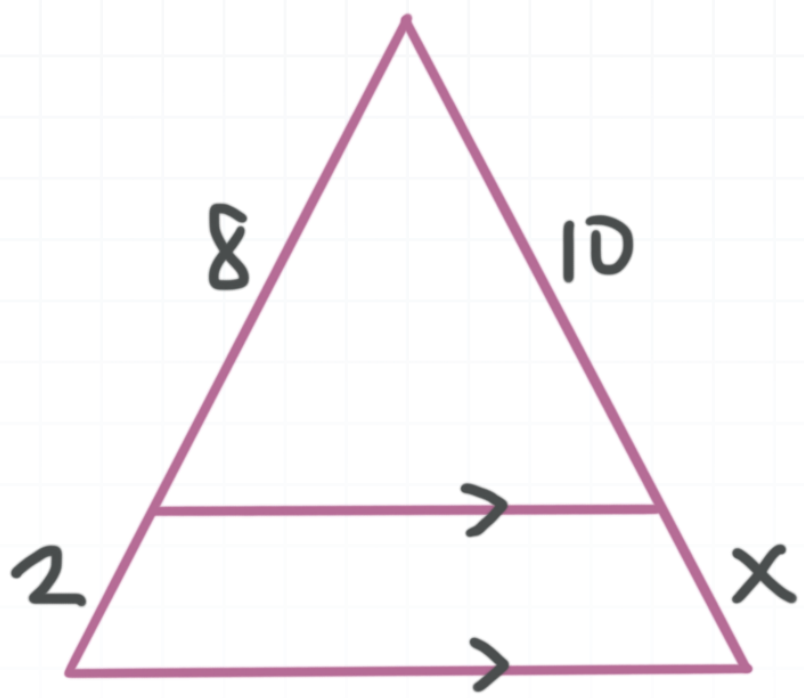



*Solution:*

$\triangle AED \sim \triangle ACB$ by $AA$. $\angle A \cong \angle A$ by the Reflexive Property of congruence, and $\angle EDA \cong \angle CBA$ because they are corresponding angles.

## TRIANGLE SIDE-SPLITTING THEOREM

■ 1. Solve for $x$.




*Solution:*

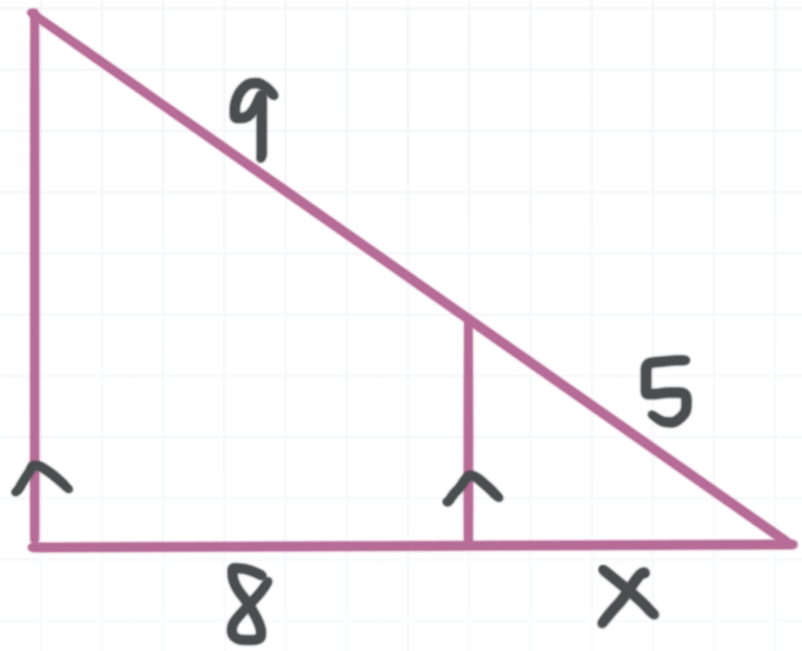
2.5. Set up a proportion.

$$\frac{8}{10}=\frac{2}{x}$$

$$8x=20$$

$$x=2.5$$

■ 2. Solve for $x$.




*Solution:*

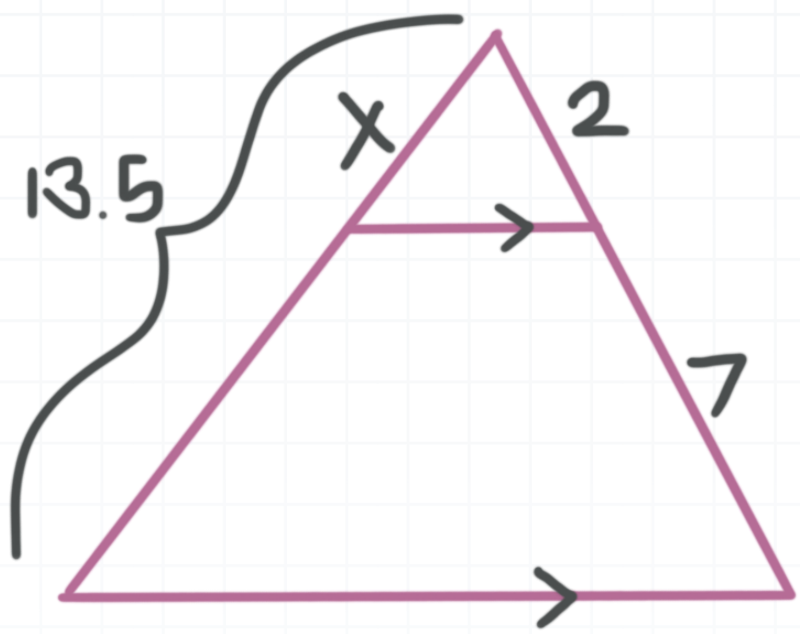
$40/9$. Set up a proportion.

$$\frac{9}{8}=\frac{5}{x}$$

$$9x=40$$

$$x=\frac{40}{9}$$

■ 3. Solve for $x$.




*Solution:*

3. Set up a proportion.

$$\frac{2}{x} = \frac{7}{13.5 - x}$$

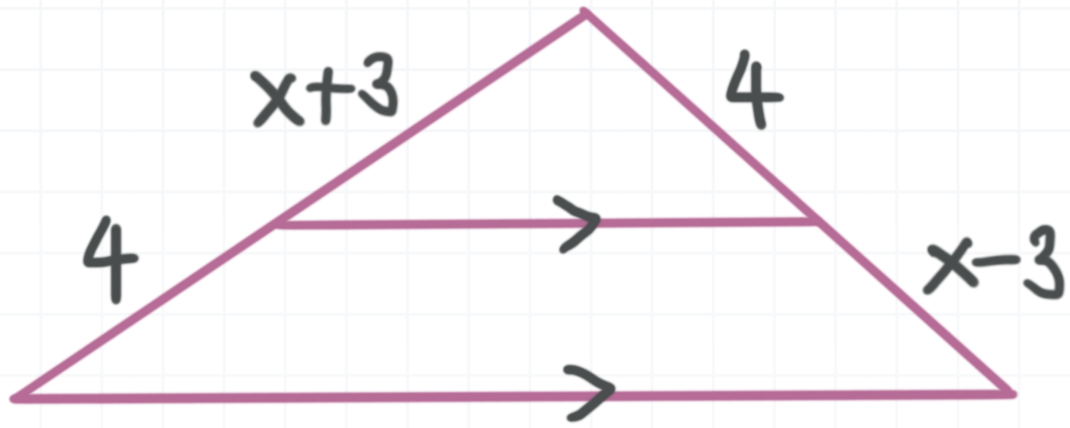
$$2(13.5 - x) = 7x$$

$$27 - 2x = 7x$$

$$27 = 9x$$

$$x = 3$$

■ 4. Solve for $x$.




*Solution:*

5. Set up a proportion.

$$\frac{x+3}{4} = \frac{4}{x-3}$$

$$(x+3)(x-3) = (4)(4)$$

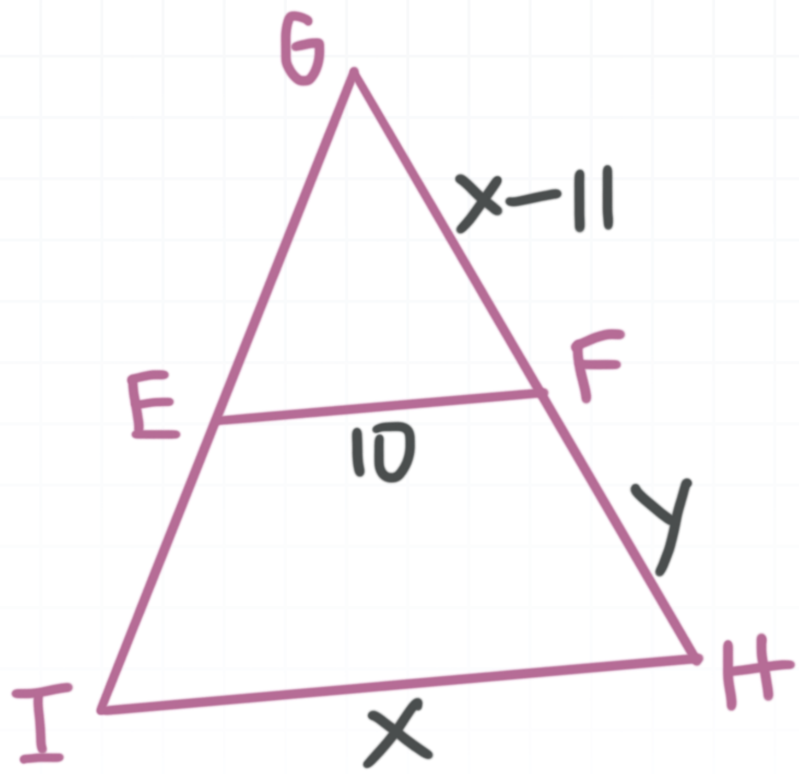
$$x^2 - 9 = 16$$

$$x^2 = 25$$

$$x = 5$$

## MIDSEGMENTS OF TRIANGLES

■ 1. $\overline{EF}$ is a midsegment of $\triangle IGH$. Find $x$ and $y$.




*Solution:*

$x = 20$ and $y = 9$. Because $EF$ is a midsegment of $\triangle IGH$,

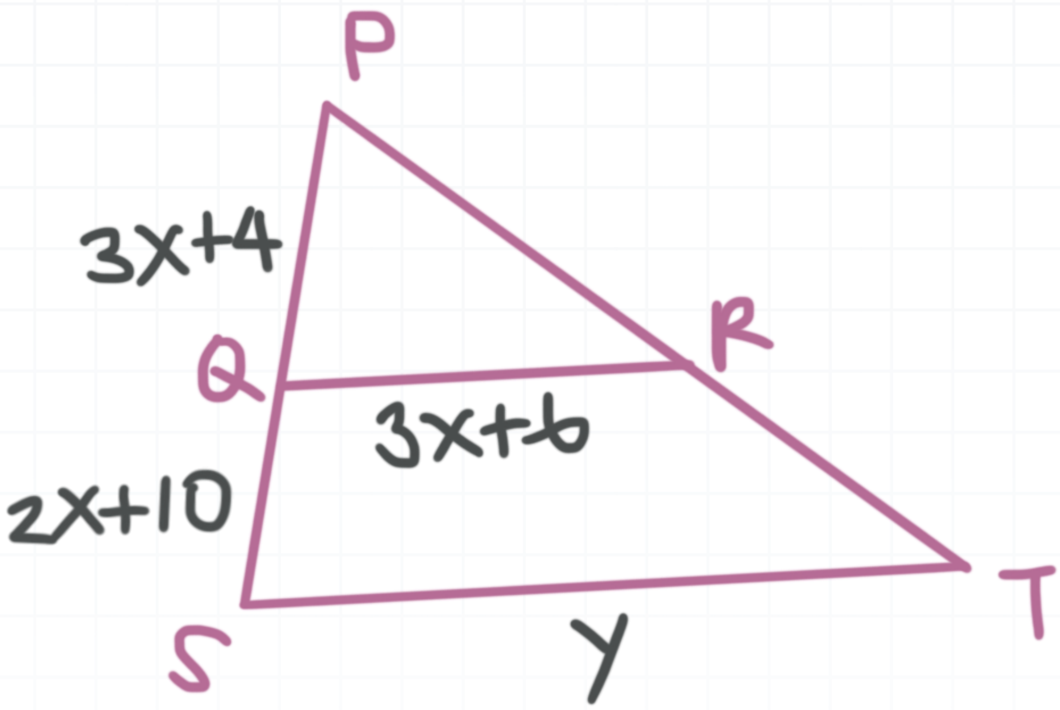
$$IH = 2(EF)$$

$$x = 2(10)$$

$$x = 20$$

Then $GF = 20 - 11 = 9$, we know that $y = 9$.

■ 2. $\overline{QR}$ is a midsegment of $\triangle SPT$. Find $x$ and $y$.




*Solution:*

$x = 6$ and $y = 48$. Because $SQ = PQ$, we get

$$2x + 10 = 3x + 4$$

$$x = 6$$

Then we can use $x = 6$ to find the length of the midsegment.

$$QR = 3x + 6$$

$$QR = 3(6) + 6$$

$$QR = 24$$

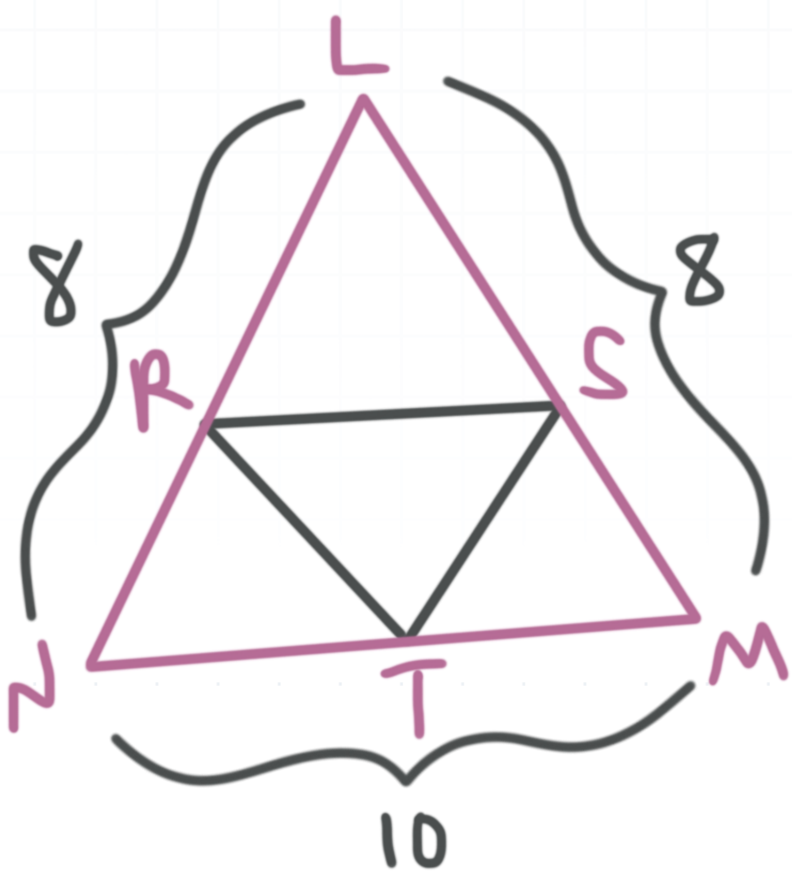
Then the value of $y$ is given by

$$ST = 2QR$$

$$y = 2(24)$$

$$y = 48$$

■ 3. $\overline{RS}$, $\overline{ST}$, and $\overline{RT}$ are midsegments of $\triangle NLM$. Find the perimeter of quadrilateral $RTMS$.




*Solution:*

18. The four side lengths are given by

$$RT = \frac{1}{2}(8) = 4$$

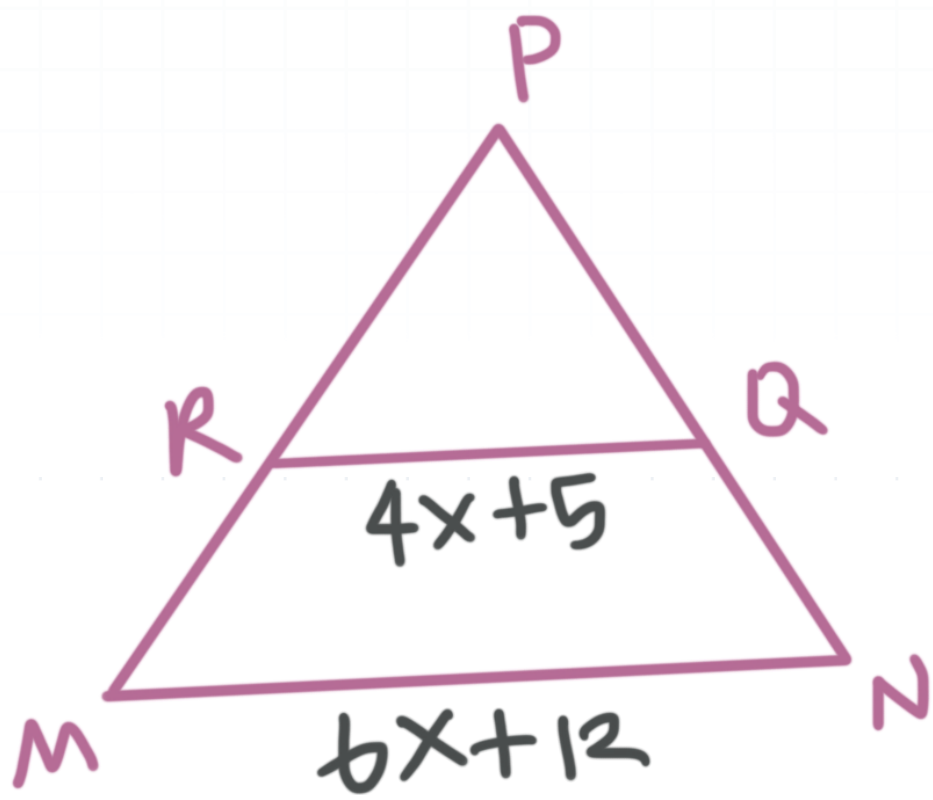
$$SM = \frac{1}{2}(8) = 4$$

$$TM = \frac{1}{2}(10) = 5$$

$$RS = \frac{1}{2}(10) = 5$$

Then the perimeter of $RTMS$ is $4 + 4 + 5 + 5 = 18$.

■ 4. $\overline{RQ}$ is a midsegment of $\triangle MPN$. Find $x$ and $MN$.




*Solution:*

$x = 1$ and $MN = 18$. From the formula for the midsegment of a triangle, we get

$$RQ = \frac{1}{2}MN$$

$$4x + 5 = \frac{1}{2}(6x + 12)$$

$$8x + 10 = 6x + 12$$

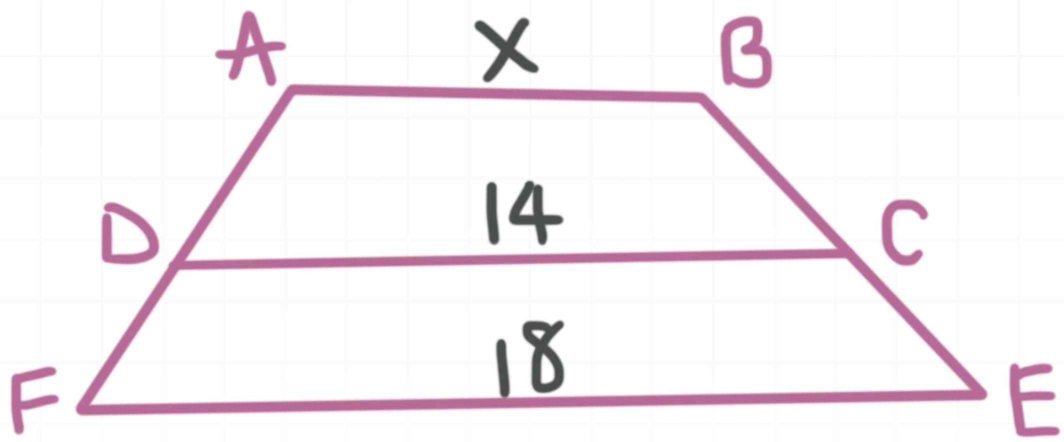
$$2x = 2$$

$$x = 1$$

Then $MN$ is $MN = 6(1) + 12 = 18$.

## MIDSEGMENTS OF TRAPEZOIDS

■ 1. The trapezoid has midsegment $\overline{DC}$. Find the value of $x$.




*Solution:*

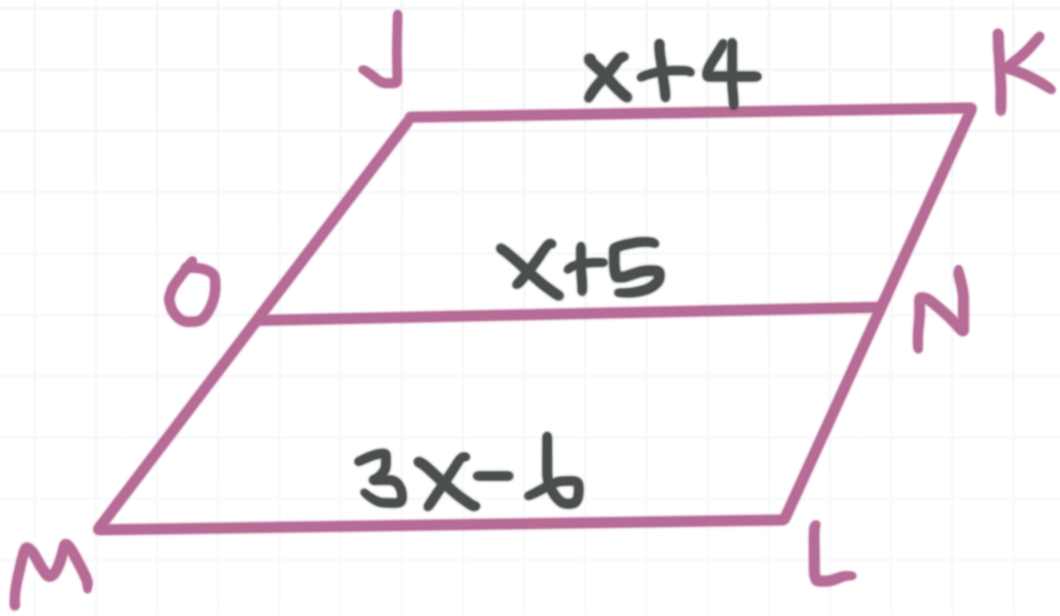
10. The length of the midsegment will be given by

$$DC = \frac{1}{2}(AB + FE)$$

$$14 = \frac{1}{2}(x + 18)$$

$$x = 10$$

■ 2. $\overline{ON}$ is a midsegment of trapezoid $JKLM$. Find $JK$, $ON$, and $ML$.




*Solution:*

10, 11, 12. The length of $ON$ will be given by

$$ON = \frac{1}{2}(JK + ML)$$

$$x + 5 = \frac{1}{2}(x + 4 + 3x - 6)$$

$$x + 5 = 2x - 1$$

$$x = 6$$

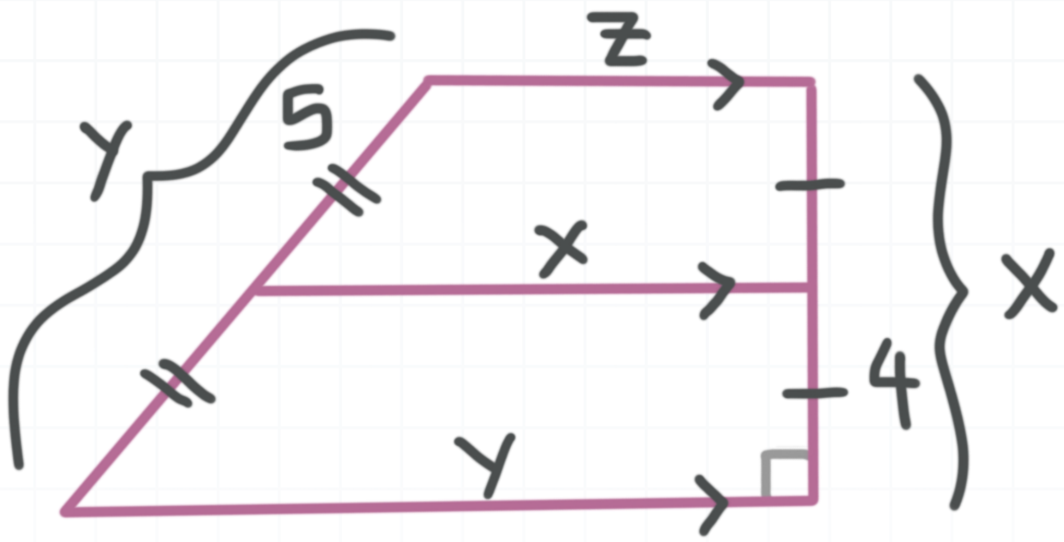
If $x = 6$, then the lengths of the line segments are

$$JK = 6 + 4 = 10$$

$$ON = 6 + 5 = 11$$

$$ML = 3(6) - 6 = 12$$

■ 3. Find $x$, $y$, and $z$.




*Solution:*

$8$, $10$, $6$. We know from the formula for the midsegment of a trapezoid, and the fact that $x=2(4)=8$ and $y=2(5)=10$, we get

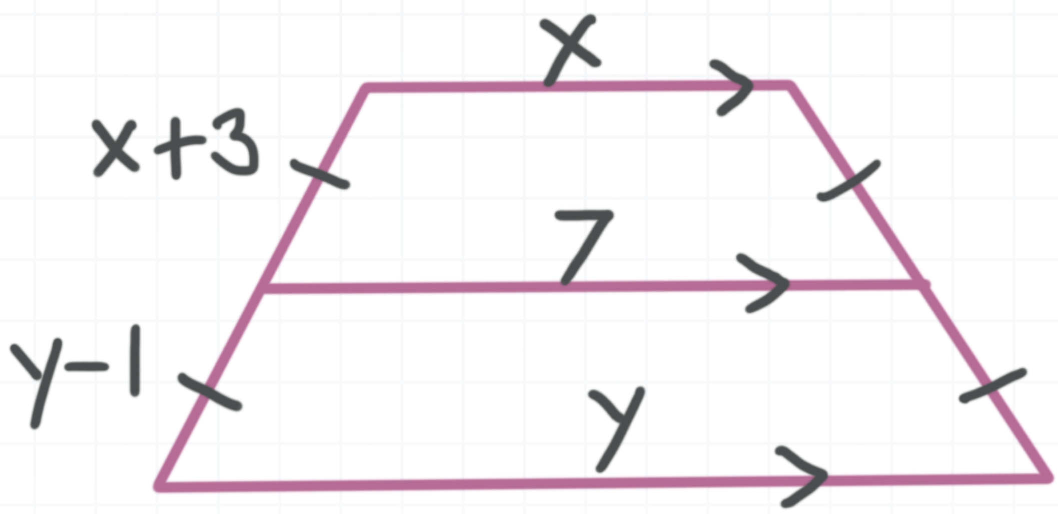
$$x=\frac{1}{2}(y+z)$$

$$8=\frac{1}{2}(10+z)$$

$$10+z=16$$

$$z=6$$

■ 4. Find $x$ and $y$.




*Solution:*

$x = 5$ and $y = 9$. Given congruent segments on the left side of the trapezoid, we get

$$x + 3 = y - 1$$

$$x - y = -4$$

And from the formula for the midsegment of a trapezoid, we get

$$\frac{1}{2}(x + y) = 7$$

$$x + y = 14$$

Use elimination to solve the system of equations by subtracting $x - y = -4$ from $x + y = 14$.

$$x + y - (x - y) = 14 - (-4)$$

$$x + y - x + y = 14 + 4$$

$$2y = 18$$

$$y = 9$$

Then

$$x + y = 14$$

$$x + 9 = 14$$

$$x = 5$$

W W W . K R I S T A K I N G M A T H . C O M