Geometry Workbook Solutions

Volume and surface area

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NETS/VOLUME/SURFACE AREA OF PRISMS

■ 1. Find the volume of a rectangular prism with length 14 feet, width 10 feet, and height 5 feet.

Solution:

Plugging the dimensions into the formula for volume of a rectangular prism, we get

$$V = lwh = (14)(10(5) = 700 \text{ ft}^3$$

■ 2. Find the surface area of a rectangular prism with length 14 feet, width 10 feet, and height 5 feet.

Solution:

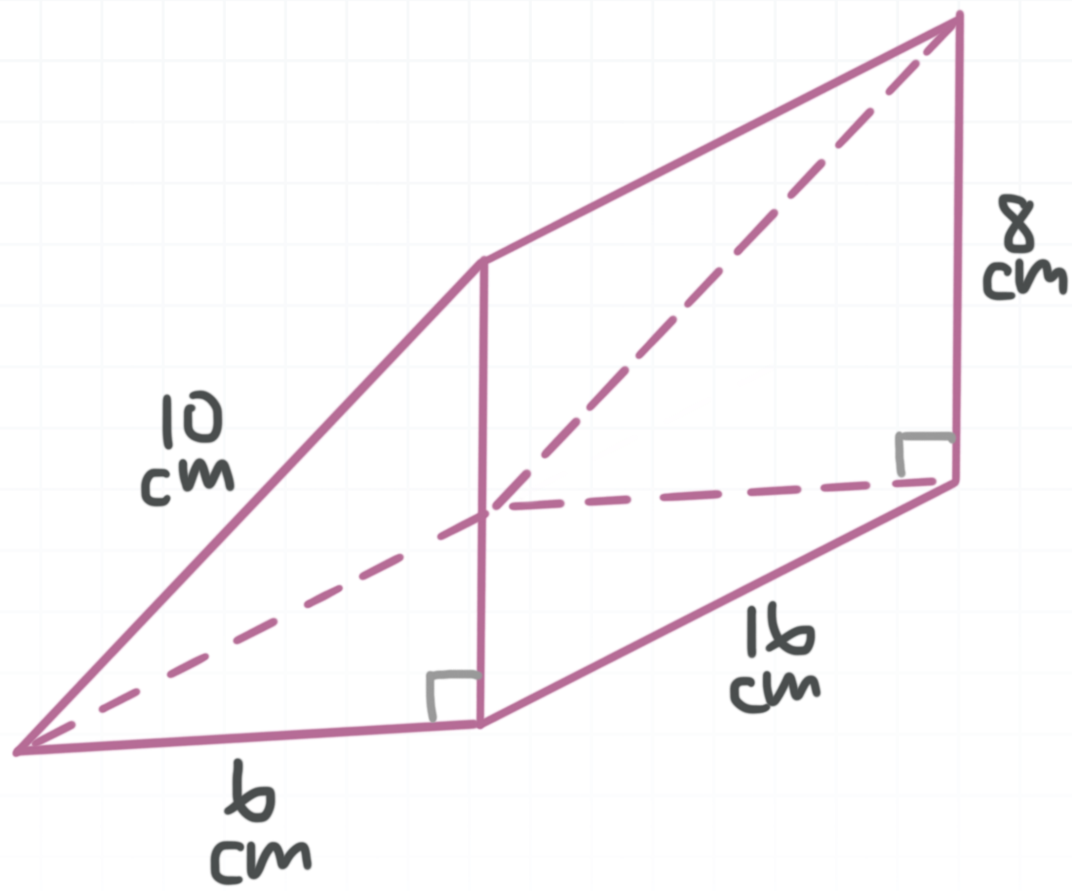
Plugging the dimensions into the formula for surface area of a rectangular prism, we get

$$SA = 2lw + 2wh + 2lh$$

$$SA = 2(14)(10) + 2(10)(5) + 2(14)(5)$$

$$SA = 520 \text{ ft}^2$$

3. Find the surface area of the triangular prism.



Solution:

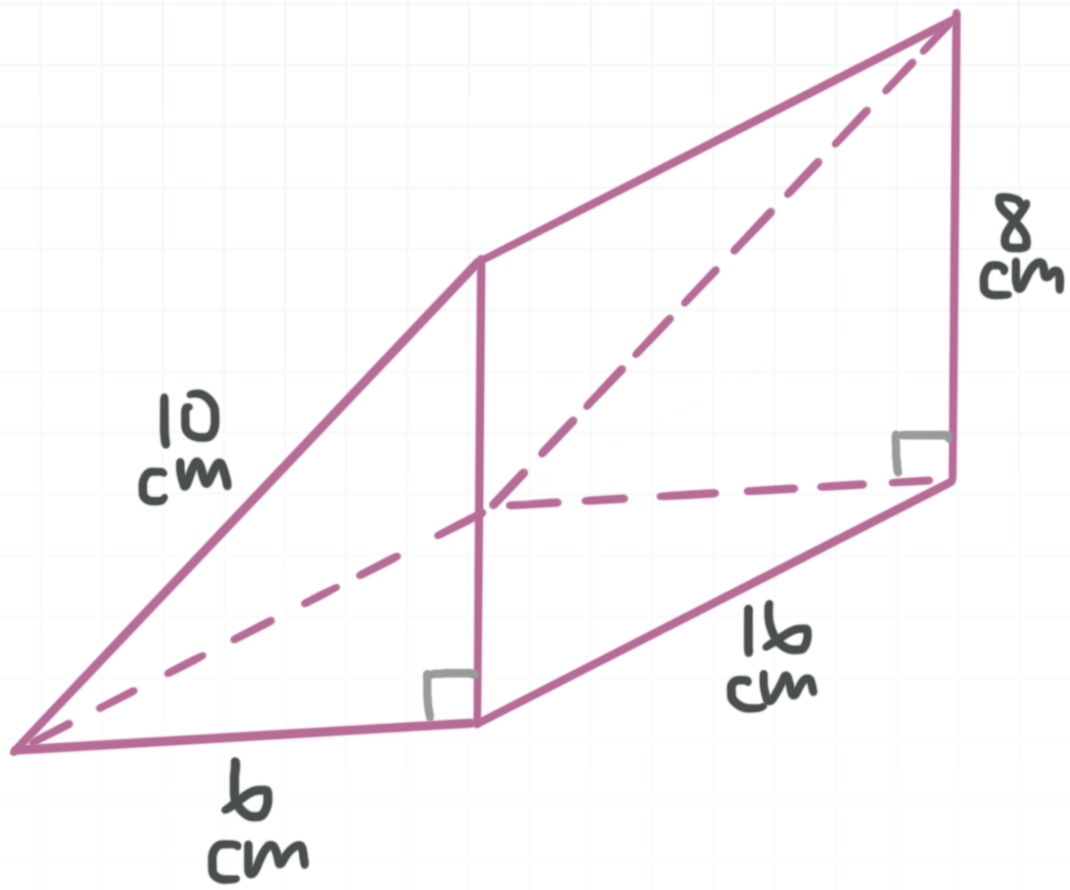
The surface area is given by

$$SA = 2\left(\frac{1}{2}\right)(6)(8) + (6)(16) + (16)(8) + (10)(16)$$

$$SA = 48 + 96 + 128 + 160$$

$$SA = 432 \text{ cm}^2$$

4. Find the volume of the triangular prism.



Solution:

The volume is given by

$$V = (\text{area of the end})(\text{length})$$

$$V = \frac{1}{2}(6)(8)(16)$$

$$V = 384 \text{ cm}^3$$

SURFACE AREA TO VOLUME RATIO OF PRISMS

■ 1. A rectangular prism has length, width, and height of 5 inches. Find the ratio of its surface area to its volume.

Solution:

The volume of the prism is

$$V = lwh = (5)(5)(5) = 125$$

and the surface area is

$$SA = 2lw + 2wh + 2lh$$

$$SA = 2(5)(5) + 2(5)(5) + 2(5)(5)$$

$$SA = 150$$

The ratio is $150/125 = 6/5$.

■ 2. A cube has a volume of 216 in^3. Suppose we double the length of each side of the cube. What is the ratio of the smaller cube to the larger cube?

Solution:

A cube has equal length, width, and height, so

$$V = lwh$$

$$V = l^3$$

$$216 = l^3$$

$$l = \sqrt[3]{216} = 6$$

Each side length is 6. Double each side length to get $6(2) = 12$. The new volume would be $(12)^3 = 1{,}728$. The ratio is

$$\frac{216}{1{,}728} = \frac{1}{8}$$

3. In lowest terms, find the ratio of volume to surface area of a cube with side length x.

Solution:

The volume of the cube would be $V = lwh = x^3$, and the surface area would be

$$SA = 2lw + 2lh + 2wh$$

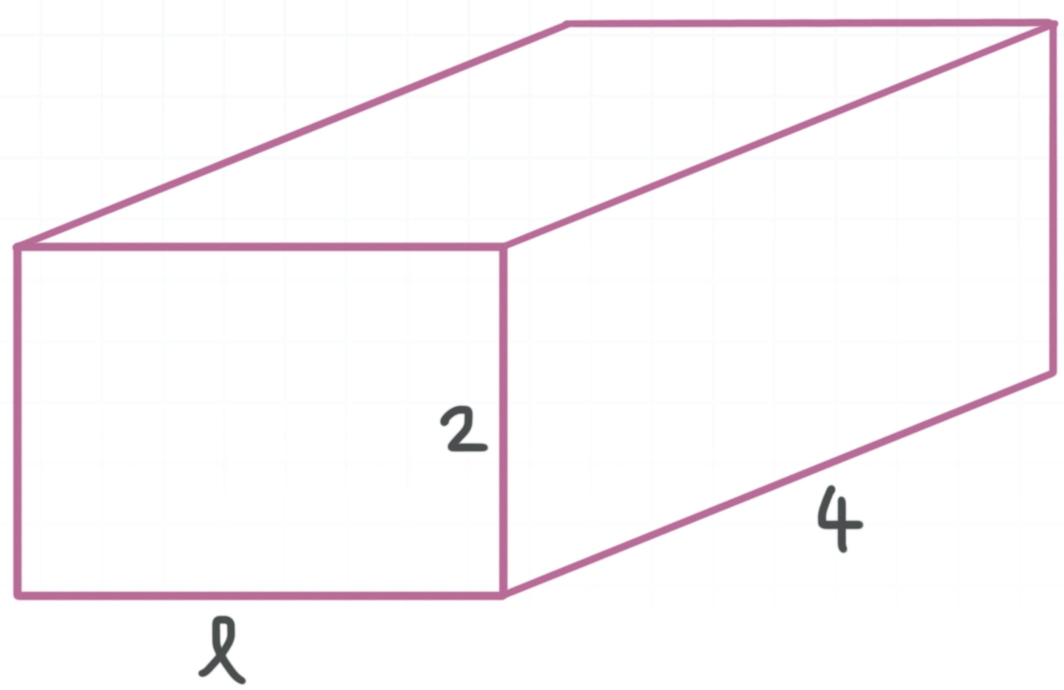
$$SA = 2(x^2) + 2(x^2) + 2(x^2)$$

$$SA = 6x^2$$

The ratio of volume to surface area is

$$\frac{x^3}{6x^2} = \frac{x}{6}$$

■ 4. The ratio of the volume to surface area for the following rectangular prism is $1:2$. Find the length of the prism.



Solution:

The volume of the rectangular prism is

$$V = lwh = (2)(4)l = 8l$$

and the surface area is

$$SA = 2(2)(4) + 2(2l) + 2(4l) = 16 + 12l$$

Setting up a proportion with the given ratio, we get

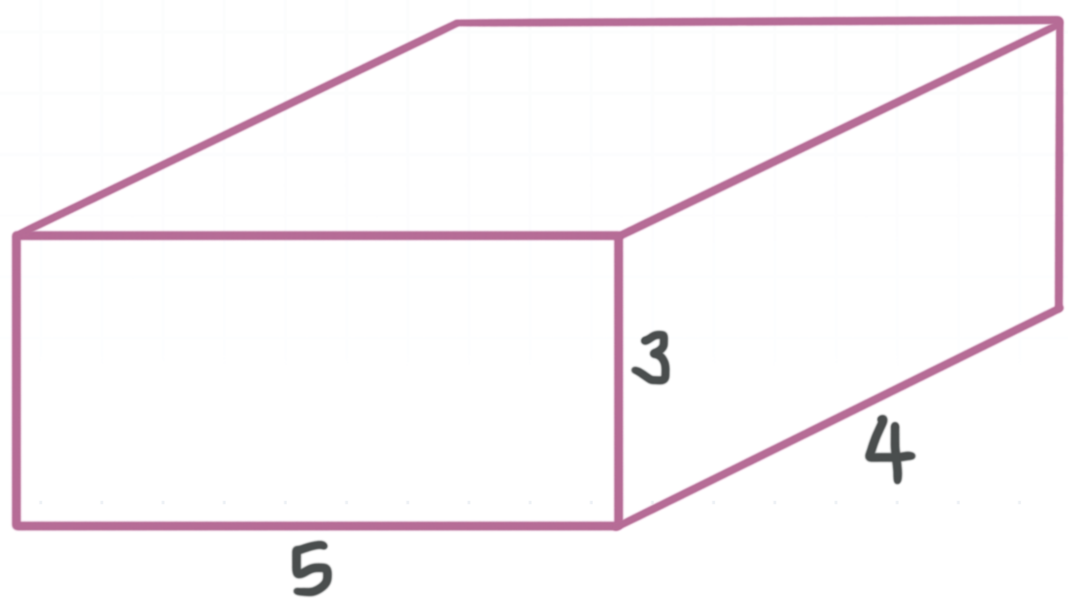
$$\frac{8l}{16 + 12l} = \frac{1}{2}$$

$$16l = 12l + 16$$

$$4l = 16$$

$$l = 4$$

■ 5. How many times greater will the surface area of this rectangular prism be if we double each side length?



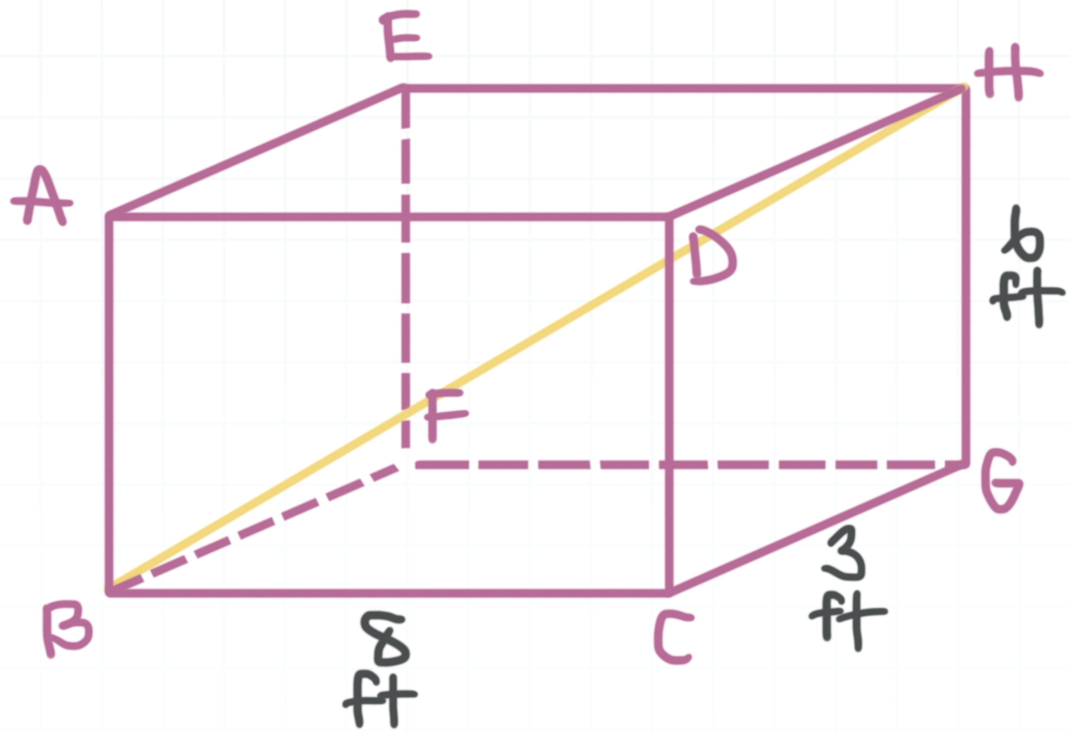
Solution:

The original surface area is $2(20) + 2(12) + 2(15) = 94$. The new surface area would be $2(80) + 2(48) + 2(60) = 376$. Therefore, the ratio gives

$$\frac{376}{94} = 4 \text{ times greater}$$

DIAGONAL OF A RIGHT RECTANGULAR PRISM

■ 1. Find the length of BH in the right rectangular prism.



Solution:

The length of the diagonal is given by

$$d = \sqrt{l^2 + w^2 + h^2}$$

$$d = \sqrt{8^2 + 3^2 + 6^2}$$

$$d = \sqrt{64 + 9 + 36}$$

$$d = \sqrt{109}$$

■ 2. Find the length of the diagonal of a cube with side length 10.

Solution:

The length of the diagonal is given by

$$d = \sqrt{l^2 + w^2 + h^2}$$

$$d = \sqrt{10^2 + 10^2 + 10^2}$$

$$d = \sqrt{300}$$

$$d = 10\sqrt{3}$$

■ 3. If the length of the diagonal of a cube is $4\sqrt{3}$, find the length of each side of the cube.

Solution:

The diagonal is given by

$$d = \sqrt{l^2 + w^2 + h^2}$$

$$4\sqrt{3} = \sqrt{x^2 + x^2 + x^2} = \sqrt{3x^2}$$

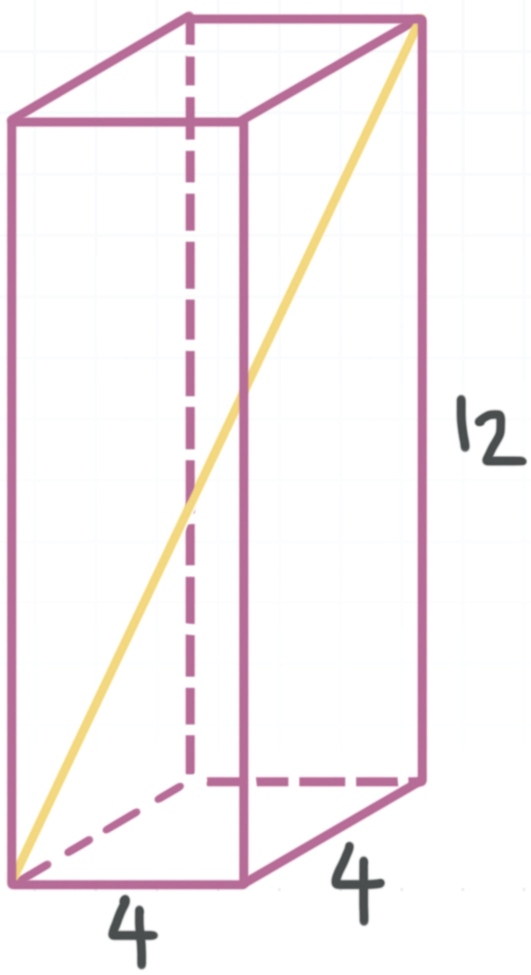
$$16(3) = 3x^2$$

$$x^2 = 16$$

$$x = 4$$

So the cube is $4 \times 4 \times 4$.

■ 4. Find the length of the diagonal of the right rectangular prism.



Solution:

The length of the diagonal is given by

$$d = \sqrt{l^2 + w^2 + h^2}$$

$$d = \sqrt{4^2 + 4^2 + 12^2}$$

$$d = \sqrt{176}$$

$$d = 4\sqrt{11}$$

■ 5. A right, rectangular prism has dimensions $4 \times 5 \times x$. Find the value of x if the diagonal is $5\sqrt{2}$.

Solution:

The length of the diagonal is given by

$$d = \sqrt{l^2 + w^2 + h^2}$$

$$5\sqrt{2} = \sqrt{4^2 + 5^2 + x^2}$$

$$5\sqrt{2} = \sqrt{41 + x^2}$$

$$50 = 41 + x^2$$

$$x^2 = 9$$

$$x = 3$$

NETS/VOLUME/SURFACE AREA OF PYRAMIDS

■ 1. A pyramid has a square base with area 25 ft^2 and height 6 feet. Find the volume of this pyramid.

Solution:

The volume is given by

$$V = \frac{1}{3}bh = \frac{1}{3}(25)(6) = 50 \text{ ft}^3$$

■ 2. A pyramid has a square base with area 25 ft^2 and height 6 feet. Find the surface area of this pyramid.

Solution:

Find the height of each triangle using the Pythagorean Theorem. The height of the pyramid is one of the legs of the right triangle formed and so is half the length of a side of the base.

$$6^2 + 2.5^2 = 42.25$$

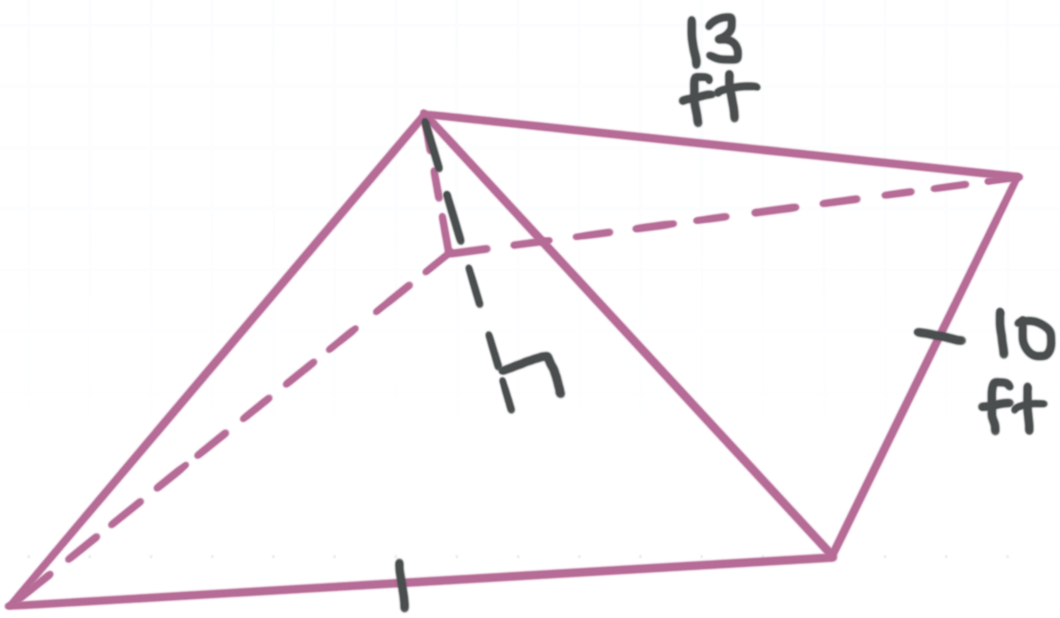
$$6^2 + 2.5^2 = 6.5^2$$

Find the area of one of the triangles of the pyramid using the formula for the area of a triangle.

$$A = \frac{1}{2}bh = \frac{1}{2}(5)(6.5) = 16.25$$

There are four triangles of equal area, so take $4(16.25) = 65$. Add on the area of the base of the pyramid to get $65 + 25 = 90$ ft^2.

■ 3. Find the surface area of the pyramid.



Solution:

Find the height of each triangle using the Pythagorean Theorem.

$$13^2 - 5^2 = 144$$

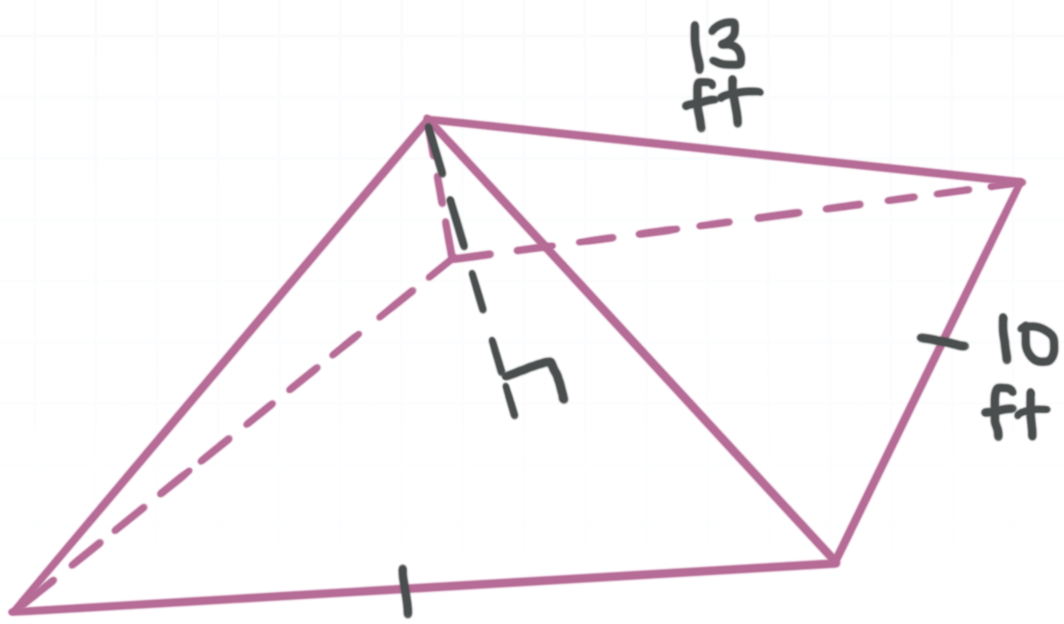
$$13^2 - 5^2 = 12^2$$

Find the area of each triangle.

$$A = \frac{1}{2}(10)(12) = 60$$

There are four triangles with this area, so take $4(60) = 240$. Now add on the base area of 100 to get $240 + 100 = 340$ ft^2.

■ 4. Find the height of the following pyramid to the nearest tenth. Then find its volume.



Solution:

The diagram indicates that this is a square pyramid. Which means each side of the base has length 10, and therefore each half side of the base has length 5.

If we imagine a triangle laying flat within the base that's formed by the corner of the pyramid, the center of the base, and the midpoint of the side of the base, then we know the two legs of that triangle each have length 5.

Using the Pythagorean Theorem to find the length of the distance from the corner of the pyramid to the center of base, we get

$$5^2 + 5^2 = c^2$$

$$c^2 = 50$$

$$c = \sqrt{50}$$

Then if we imagine the triangle formed by the corner of the pyramid, the center of the base, and the top of the pyramid, then we know the hypotenuse of this triangle is 13, and that the length of one leg is $\sqrt{50}$, so we can use the Pythagorean Theorem to find the height of the pyramid.

$$(\sqrt{50})^2 + b^2 = 13^2$$

$$50 + b^2 = 169$$

$$b^2 = 119$$

$$b \approx 10.9 \text{ ft}$$

Then the volume of the pyramid is

$$V = \frac{1}{3}lwh$$

$$V \approx \frac{1}{3}(10)(10)(10.9)$$

$$V \approx 363.3 \text{ ft}^3$$

NETS/VOLUME/SURFACE AREA OF CYLINDERS

■ 1. Find the volume of a cylinder with diameter 10 cm and height 12 cm.

Solution:

The volume of the cylinder is given by

$$V = \pi r^2 h$$

$$V = (5^2)\pi(12)$$

$$V = 942.5 \text{ cm}^3$$

■ 2. Find the height of a cylinder with volume $2{,}814.867$ in^3 and radius 8.

Solution:

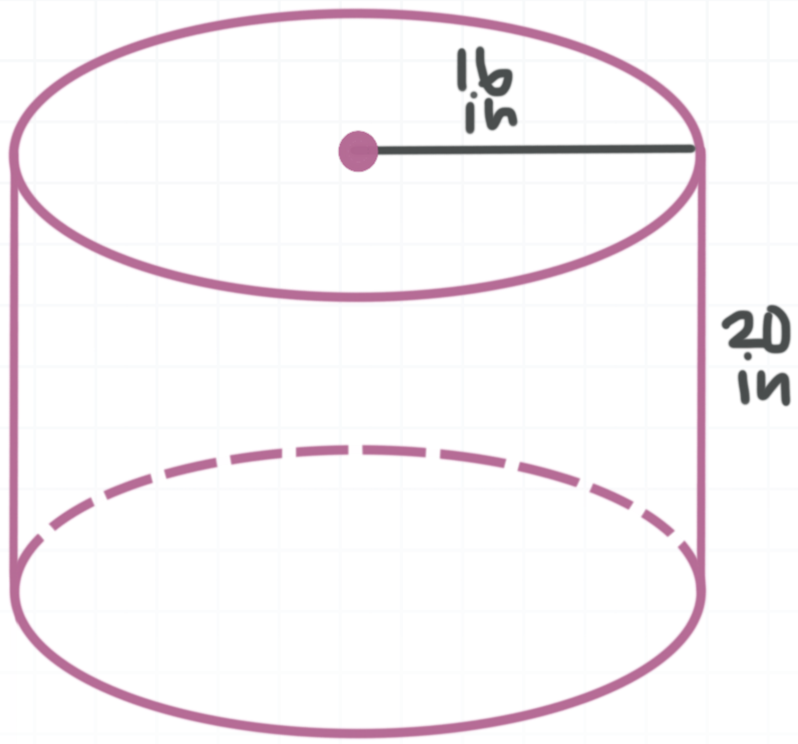
We'll plug into the volume equation, then solve it for height.

$$V = \pi r^2 h$$

$$2{,}814.867 = \pi(8^2)h$$

$$h = \frac{2{,}814.867}{64\pi} \approx 14 \text{ inches}$$

■ 3. Find the surface area of the cylinder.



Solution:

Find the area of the circular bases.

$$A = \pi r^2 = \pi(16)^2 = 804.25$$

Double this to find the combined area of the top and bottom of the cylinder to get $804.25(2) = 1{,}608.50$. Now find the area of the rectangle by finding the circumference of the circle and multiplying it by the height of the cylinder.

$$C = 2\pi r = 2\pi(16) = 100.53$$

$$A = 100.53(20) = 2{,}010.62$$

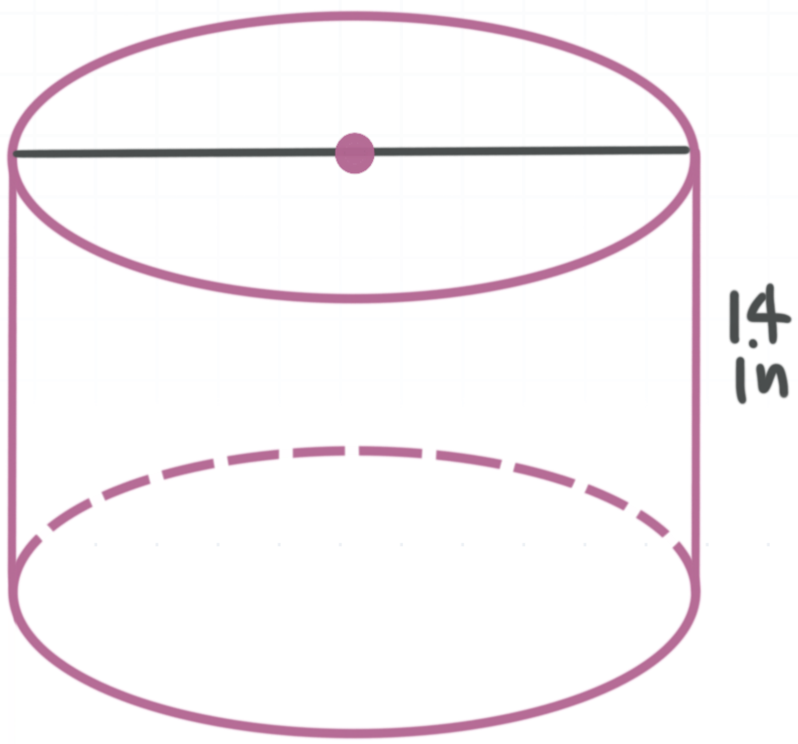
Then the total surface area of the cylinder is

$$A_{\text{Total}} = A_{\text{Bases}} + A_{\text{Side}}$$

$$A_{\text{Total}} = 1{,}608.50 + 2{,}010.62$$

$$A_{\text{Total}} = 3{,}619.12 \text{ in}^2$$

■ 4. The circumference of the base of the cylinder is 62.832 inches. Find its volume.



Solution:

The formula for the circumference of the circle gives

$$C = 2\pi r$$

$$62.832 = 2\pi r$$

$$r = 10$$

So the volume of the cylinder is

$$V = \pi r^2 h = \pi(10^2)(14) = 4{,}398.23 \text{ in}^3$$

NETS/VOLUME/SURFACE AREA OF CONES

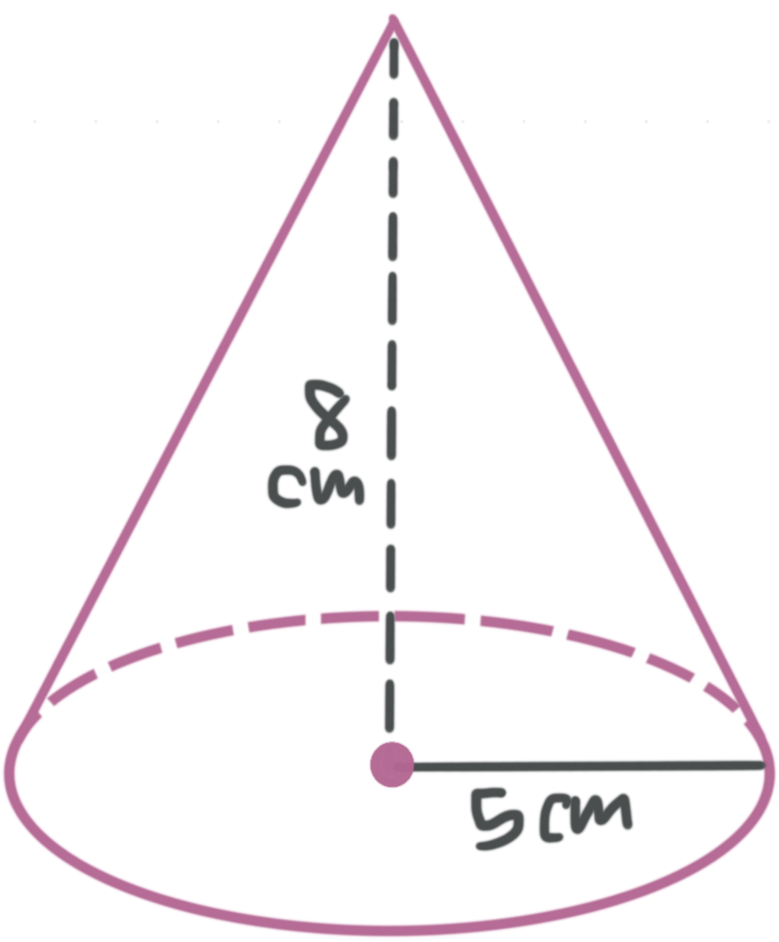
1. Find the volume of a right cone with a height of 10.5 inches and a diameter of 8 inches at its base to the nearest hundredth.

Solution:

The volume of the cone is

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(4^2)(10.5) = 175.93 \text{ in}^3$$

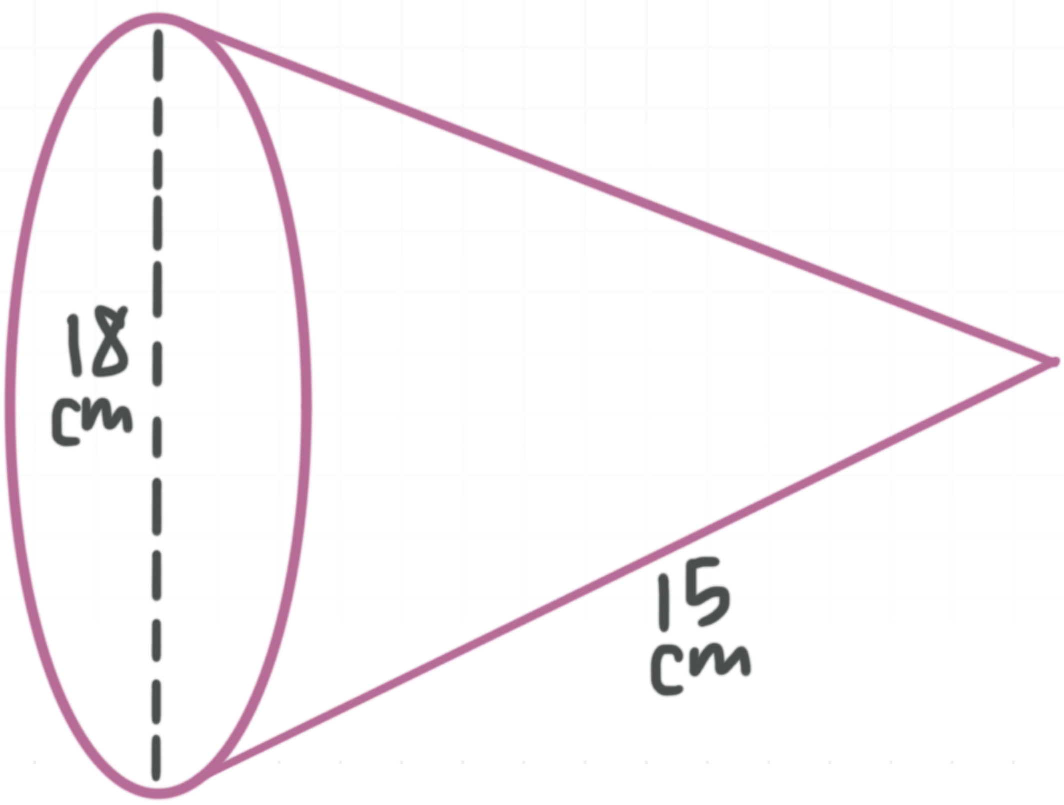
2. Find the slant height of the cone.



Solution:

The slant height is $\sqrt{89} \approx 9.43$ cm.

■ 3. Find the surface area of the cone in terms of π.



Solution:

The volume of the cone is given by $V = L + \pi r^2$. Since $L = \pi r l = \pi(9)(15) = 135\pi$, we know the volume is

$$V = 135\pi + 81\pi = 216\pi$$

4. The volume of a cone is 100π. Find the length of its radius if its height is 12.

Solution:

Using the formula for the volume of a cone, we get

$$V = \frac{1}{3}\pi r^2 h$$

$$100\pi = \frac{1}{3}\pi r^2(12)$$

$$100\pi = 4\pi r^2$$

$$25 = r^2$$

$$r = 5$$

VOLUME/SURFACE AREA OF SPHERES

■ 1. Find the volume to the nearest hundredth of a sphere with radius 15 inches.

Solution:

Plug what you know into the formula for the volume of a sphere.

$$V=\frac{4}{3}\pi r^3=\frac{4}{3}\pi(15)^3=14{,}137.17\text{ in}^3$$

■ 2. A basketball has a diameter of 9.55 inches. Find its surface area to the nearest hundredth.

Solution:

Plug what you know into the formula for the surface area of a sphere.

$$S=4\pi r^2=4\pi\left(\frac{9.55}{2}\right)^2=286.52\text{ in}^2$$

■ 3. A sphere has radius 10. How much greater is the volume than the surface area in terms of π?

Solution:

Find the volume and the surface area, then find the difference between the two.

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(10)^3 = \frac{4{,}000}{3}\pi$$

$$S = 4\pi r^2 = 4\pi(10)^2 = 400\pi$$

The difference is

$$\frac{4{,}000}{3}\pi - 400\pi = \frac{4{,}000}{3}\pi - \frac{1{,}200}{3}\pi = \frac{2{,}800}{3}\pi$$

■ 4. A sphere has a volume of 288π. Find its diameter.

Solution:

Plug what you know into the formula for the volume of a sphere.

$$V = \frac{4}{3}\pi r^3$$

$$288\pi = \frac{4}{3}\pi r^3$$

$$r^3 = 216$$

$$r = 6$$

Because the radius is 6 units long, the diameter must be 12 units long, and $d = 12$.

W W W . K R I S T A K I N G M A T H . C O M