Geometry Workbook Solutions

Quadrilaterals

MEASURES OF QUADRILATERALS

■ 1. A rectangle has a width of 6 inches and diagonal with length 10 inches. Find the perimeter of the rectangle.

Solution:

28 inches. Rectangles have four right angles. When a diagonal is drawn in the rectangle, a right triangle is formed. The Pythagorean Theorem is used to find the missing side length of the triangle.

$$a^2 + b^2 = c^2$$

$$6^2 + b^2 = 10^2$$

$$b^2 = 100 - 36 = 64$$

$$b = 8$$

The missing length of the rectangle is 8 inches. Then the perimeter is the sum of the side lengths: $2(6) + 2(8) = 28$ inches.

■ 2. Classify quadrilateral $ABCD$ with vertices at $A(1, -3)$, $B(5,0)$, $C(10,0)$, and $D(6, -3)$.

Solution:

$ABCD$ is a parallelogram and a rhombus with $AB = BC = CD = DA = 5$ and $\overline{AB} \parallel \overline{DC}$ and $\overline{BC} \parallel \overline{AD}$. A parallelogram must have two sets of parallel line segments by definition, and a rhombus is a special type of parallelogram having four congruent sides.

■ 3. $CDEF$ is a rectangle with diagonals intersecting at G. $CG = 2x + 1$, $DG = x + 4$, $FG = 4y - 1$, and $EG = y + 5$. Find FD.
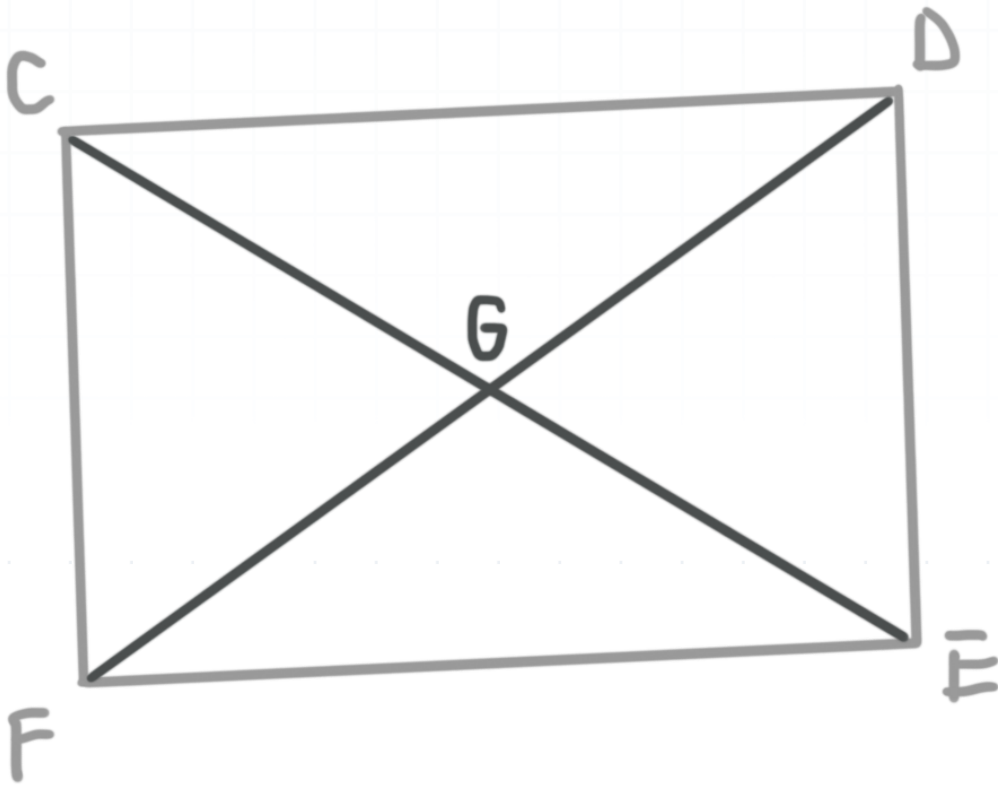
Solution:

$FD = 14$. The diagonals of a rectangle are congruent and they bisect each other. So $CE = FD$ and $CG = EG = FG = DG$. By substitution,

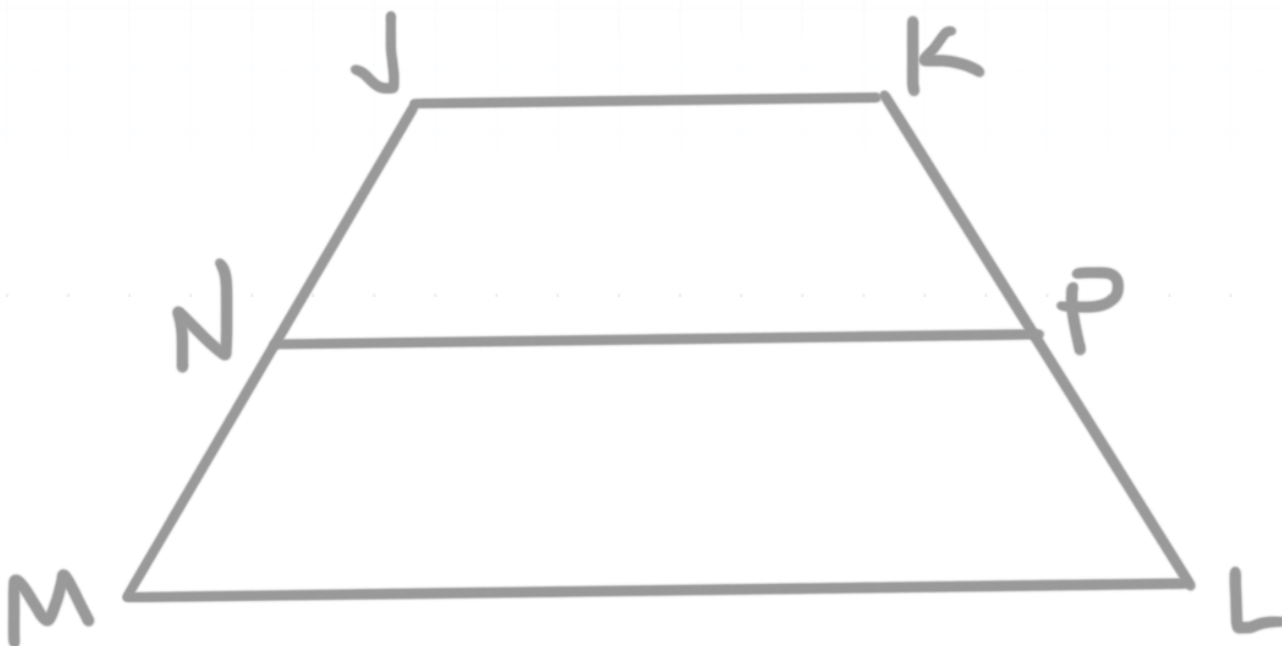$$2x + 1 = x + 4$$

$$x = 3$$

In addition,

$$4y - 1 = y + 5$$

$$3y = 6$$

$$y = 2$$

So $CG = EG = FG = DG = 7$, and therefore $FD = 14$.

■ 4. $JKLM$ is an isosceles trapezoid with median $\overline{NP}$. $MJ = 14$, $m\angle MLP = 72$, $NP = 16$, and $ML = 20$. Find KP, $m\angle MJK$, and JK.

Solution:

$KP = 7$, $m\angle MJK = 108$, and $JK = 12$. Isosceles triangles have congruent legs so $MJ = LK = 14$. The median bisects these legs, making $KP = 7$. The base

angles $\angle M$ and $\angle L$ are both 72, and $m\angle M + m\angle J = 180$, so $m\angle MJK = 108$. The median is half the sum of the bases, so

$$NP = \frac{1}{2}(JK + ML)$$

$$16 = \frac{1}{2}(JK + 20)$$

$$32 = JK + 20$$

$$JK = 12$$

MEASURES OF PARALLELOGRAMS

■ **1.** $ABCD$ is a parallelogram with $m\angle A = 2x + 10$, $m\angle B = y - 5$, and $\angle C = 100$. Find the values of x and y.

Solution:

$x = 45$ and $y = 85$. We know that $m\angle A = m\angle C$ and $m\angle B = m\angle D$ because the opposite angles of a parallelogram are congruent. $m\angle C + m\angle D = 180$ because consecutive angles of a parallelogram are supplementary. This gives the following equations.

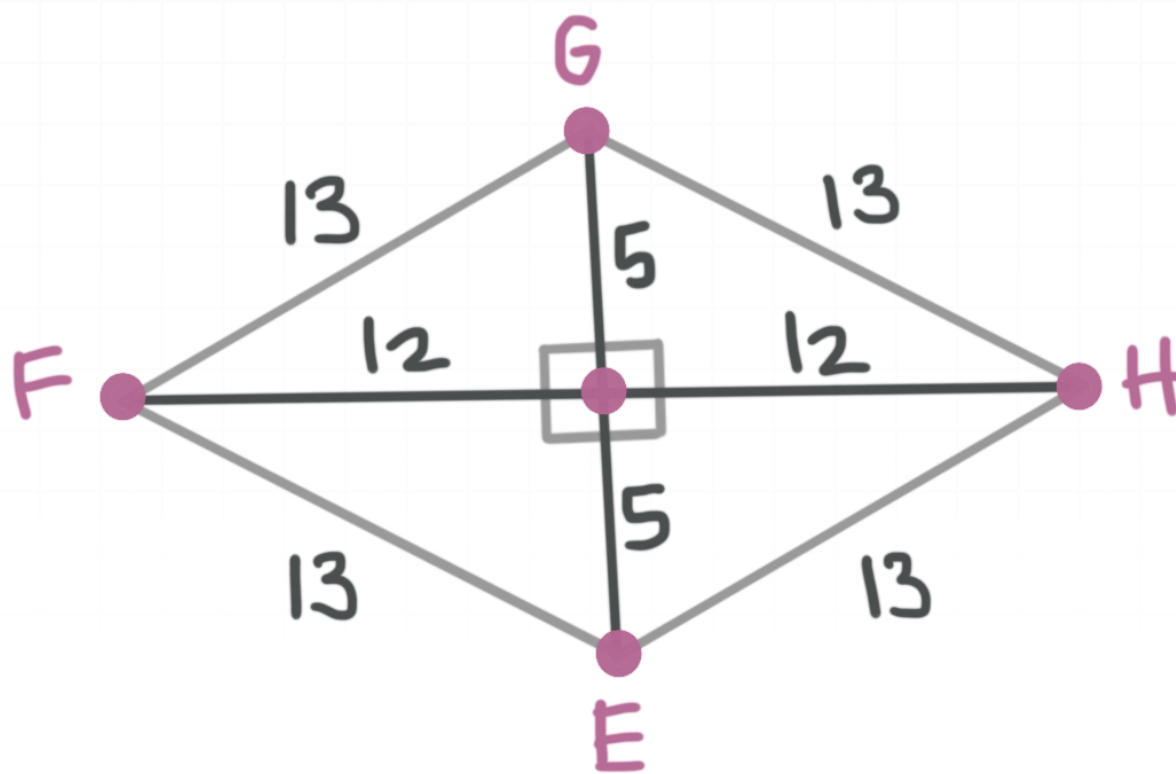$$2x + 10 = 100$$

$$x = 45$$

and

$$y - 5 = 80$$

$$y = 85$$

■ **2.** $EFGH$ is a rhombus with $FH = 24$ and $GE = 10$. Find the perimeter of $EFGH$.

Solution:

The perimeter is 52. In a rhombus, the diagonals bisect each other and are also perpendicular. Four congruent right triangles are formed, and the Pythagorean Theorem can be used to find the length of the hypotenuse of each triangle.

$$12^2 + 5^2 = 13^2$$

This gives side length of 13. The perimeter is then found by finding the sum of the side lengths of the rhombus.
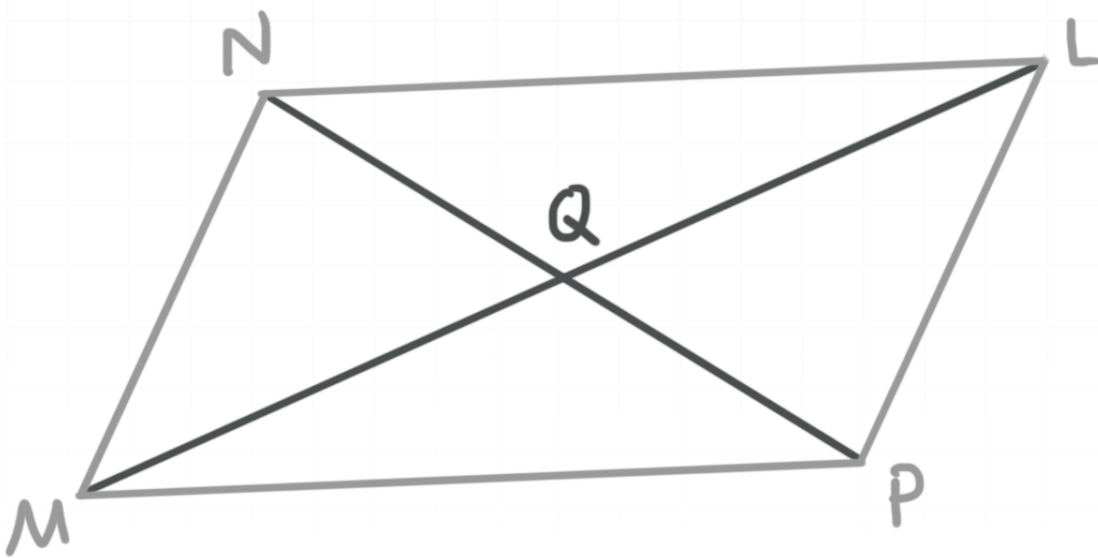
$$13 + 13 + 13 + 13 = 52$$

■ 3. *JKLM* has vertices $J(-3,2)$, $K(3,0)$, $L(3,-6)$, and $M(-3,-4)$. Determine whether *JKLM* is a parallelogram by checking if it has two sets of opposite sides that are congruent.

Solution:

$JKLM$ is a parallelogram because $JM = KL = 6$ and $JK = ML = 2\sqrt{10}$.

■ 4. $NLPM$ is a parallelogram with diagonals intersecting at point Q. $m\angle MNP = 85$, $m\angle MQP = 115$, and $m\angle MNL = 135$. Find $m\angle PML$.

Solution:

$m\angle PML = 15$. Consecutive interior angles are supplementary in a parallelogram. Triangles are also formed and we can find missing angle measures by using the Triangle Sum Theorem, vertical angles, and linear pairs within our diagram.