Geometry Workbook Solutions

Triangles

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INTERIOR ANGLES OF TRIANGLES

■ 1. $\triangle LMN$ is a right, isosceles triangle where $\angle M$ is the vertex angle. Find $m\angle L$, $m\angle M$, and $m\angle N$.

Solution:

$m\angle L = 45$, $m\angle M = 90$, and $m\angle N = 45$. If M is the vertex angle, it's where the legs of the isosceles triangle intersect. This must be our 90° angle. Because it's isosceles, the two base angles must be congruent. They must both be 45°.

■ 2. $\triangle ABC$ has $m\angle A = 3x + 5$, $m\angle B = 10x + 5$, and $m\angle C = 4x$. Find the value of x and determine whether this is an obtuse, acute, or right triangle.

Solution:

$x = 10$ such that $m\angle A = 35$, $m\angle B = 105$, and $m\angle C = 40$. $\triangle ABC$ is an obtuse triangle because it has one obtuse angle.

$$m\angle A + m\angle B + m\angle C = 180$$

$$3x + 5 + 10x + 5 + 4x = 180$$

$$17x + 10 = 180$$

$$17x = 170$$

$$x = 10$$

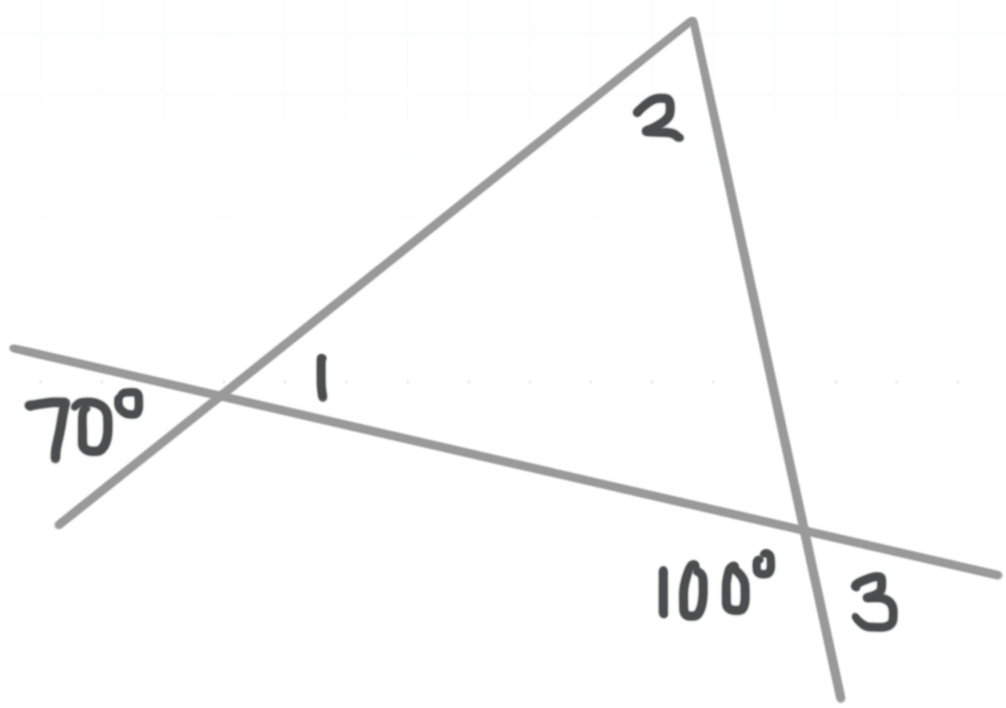
Substitute x to get the actual angle measures.

$$m\angle A = 3x + 5 = 3(10) + 5 = 30 + 5 = 35^\circ$$

$$m\angle B = 10x + 5 = 10(10) + 5 = 100 + 5 = 105^\circ$$

$$m\angle C = 4x = 4(10) = 40^\circ$$

■ 3. Find $m\angle 1$, $m\angle 2$, and $m\angle 3$ from the figure.



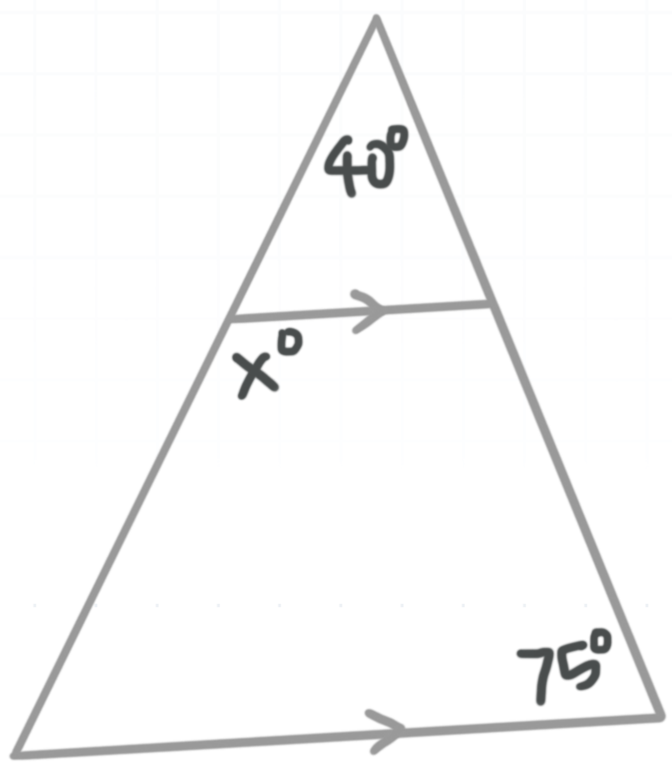
Solution:

$m\angle 1 = 70$, $m\angle 2 = 30$, and $m\angle 80$. $m\angle 1 = 70$ because vertical angles are formed. $m\angle 3 = 80$ because a linear pair is formed. The sum of the angles in the triangle is 180, so

$$m\angle 2 = 180 - 70 - 80$$

$$m\angle 2 = 30$$

■ 4. Find the value of x from the figure.



Solution:

$x = 115$. The larger triangle and the smaller triangle share the measure 40°. The smaller triangle has a sum of angles of 180° and we know the corresponding angles in the figure are congruent, making one of the angles of this small triangle 75°. So the third angle in the small triangle is

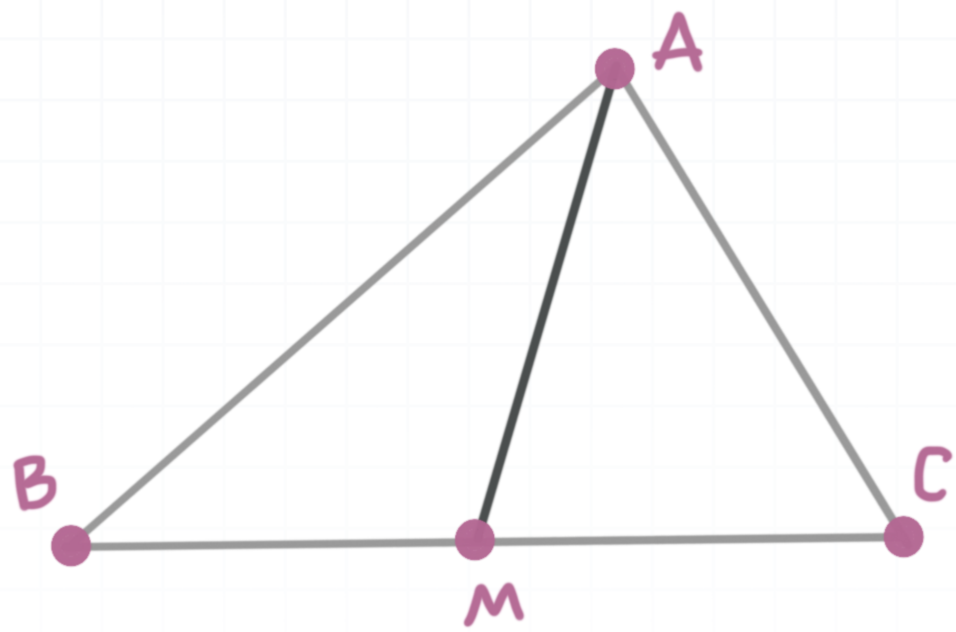
$$180 - 40 - 75 = 65^\circ$$

This angle and x form a linear pair, which means

$$x = 180 = 65 = 115^\circ$$

PERPENDICULAR AND ANGLE BISECTORS

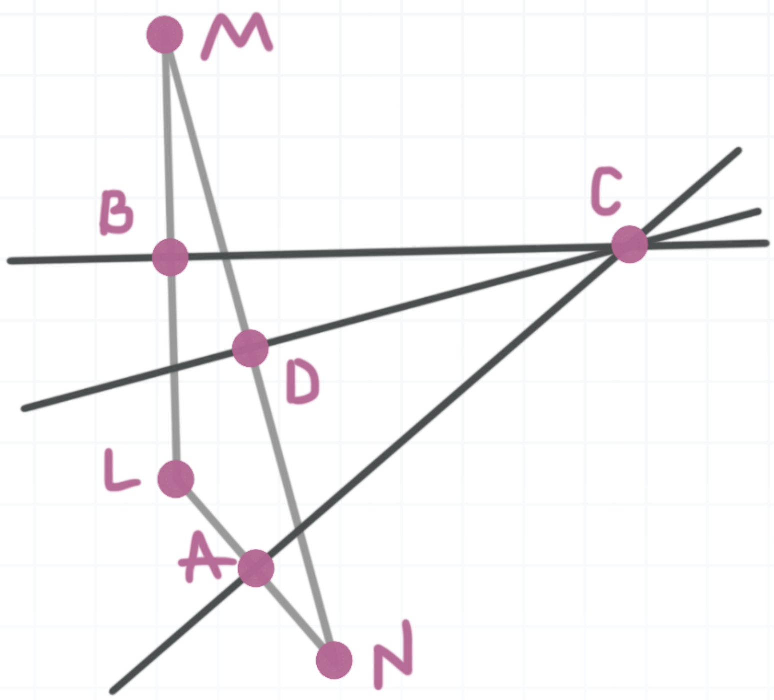
■ 1. $\overline{AM}$ is an angle bisector of $\triangle ABC$. $m\angle BMA = 108$ and $m\angle MBA = 40$. Find x if $m\angle CAM = 2x + 12$.



Solution:

$x = 10$. $\overline{AM}$ is an angle bisector, therefore $m\angle BAM = m\angle CAM$. The interior angles of a triangle always sum to 180°. We can find $m\angle BAM = 32$ and because $m\angle BAM = m\angle CAM$, $2x + 12 = 32$ and $x = 10$.

■ 2. $\overline{AC}$, $\overline{DC}$, and $\overline{BC}$ are perpendicular bisectors of $\triangle NLM$. Give the special name for C and find the length of ND if $NM = 14x - 22$ and $DM = 3x + 1$.



Solution:

C is called a circumcenter. If $\overline{DC}$ is a perpendicular bisector of $\overline{NM}$, then $ND = DM$, and $ND + DM = NM$.

$$3x + 1 + 3x + 1 = 14x - 22$$

$$6x + 2 = 14x - 22$$

$$x = 3$$

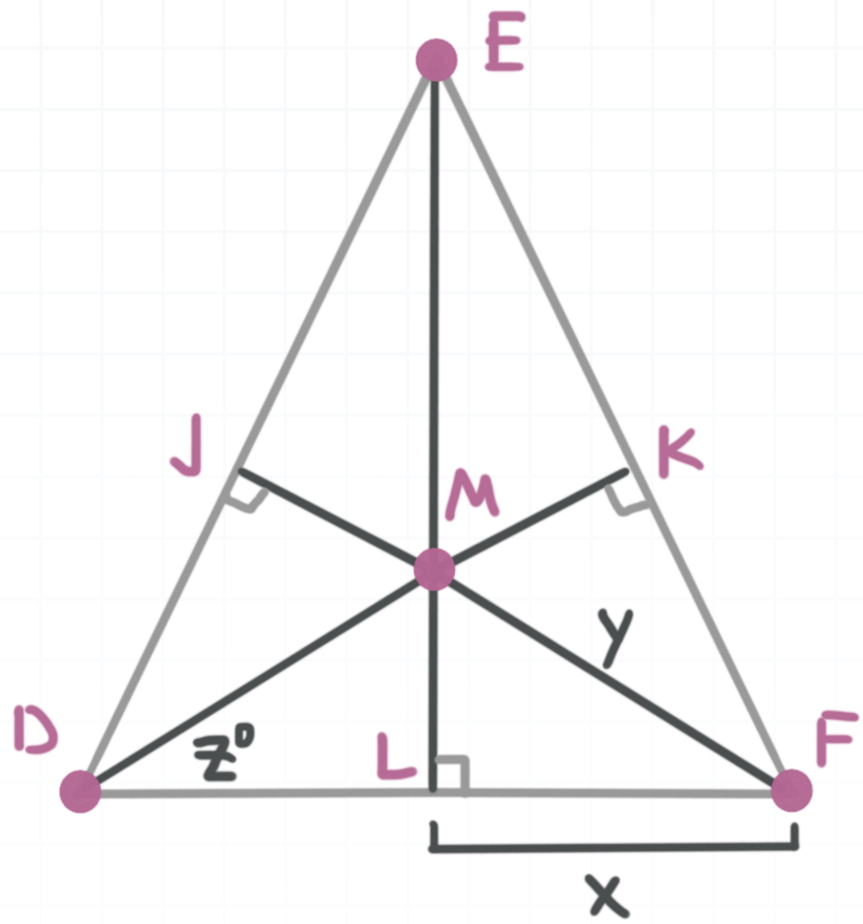
Substitute $x = 3$ and get

$$ND = 3x + 1$$

$$ND = 3(3) + 1$$

$$ND = 10$$

3. Find the values of x, y, and z, given M is an incenter, $MK = 6$, $FK = 8$, and $m\angle EDF = 80$.



Solution:

$x = 8$, $y = 10$, and $z = 40$. $FL = FK$ because $\triangle LFM \cong \triangle KFM$. So $x = FK = FL = 8$ and $KM = LM = 6$. Using the Pythagorean theorem,

$$FK^2 + KM^2 = FM^2$$

$$8^2 + 6^2 = y^2$$

$$100 = y^2$$

$$10 = y$$

Because M is an incenter, we know that DM is a perpendicular bisector of $\angle EDF$. And because we were told that $m\angle EDF = 80$, we can say

$$z = \frac{80^\circ}{2} = 40^\circ$$

■ 4. $\triangle ABC$ has coordinates $A(-3,1)$, $B(3,3)$, and $C(2,-2)$. Write the equation for the perpendicular bisector of $\overline{AB}$.

Solution:

$y = -3x + 2$. The slope of $\overline{AB}$ is

$$m = \frac{3-1}{3-(-3)} = \frac{1}{3}$$

The midpoint of $\overline{AB}$ is

$$\left(\frac{-3+3}{2}, \frac{1+3}{2}\right) = \left(\frac{0}{2}, \frac{4}{2}\right) = (0,2)$$

The perpendicular bisector of $\overline{AB}$ passes through $(0,2)$ and has a slope of -3. The equation for the line must be $y = -3x + 2$.

CIRCUMSCRIBED AND INSCRIBED CIRCLES OF A TRIANGLE

■ 1. Equilateral triangle ABC is inscribed in $\odot D$. Find $m\angle ADC$.

Solution:

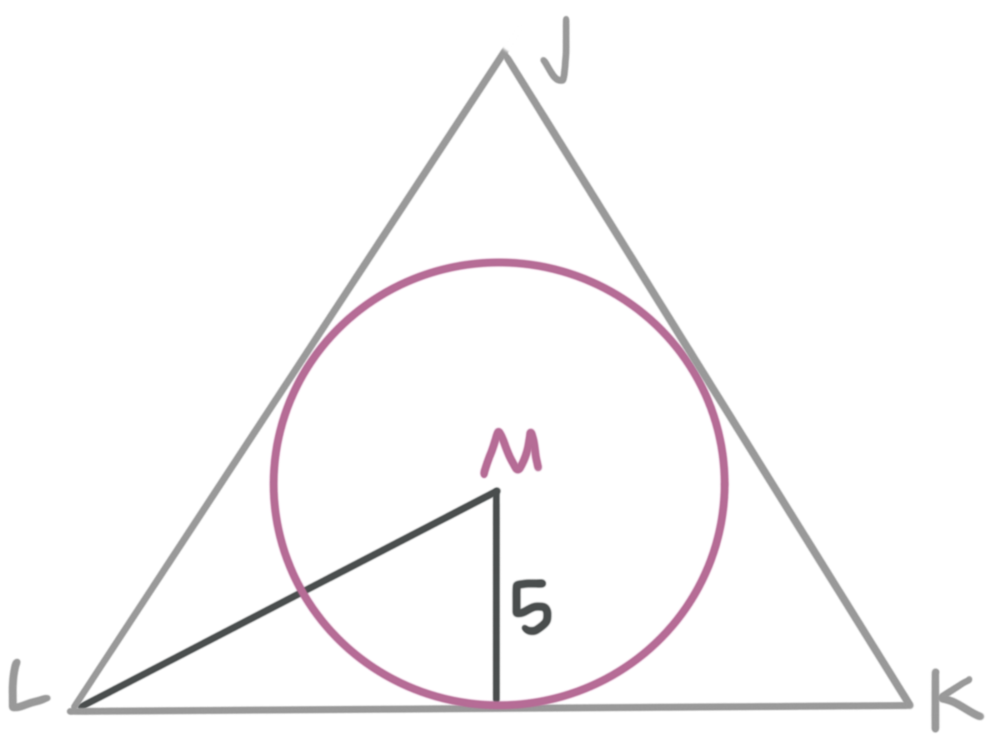
120. $m\angle ABC = 60$ because the triangle is equilateral.

$$m\angle ADC = 2(m\angle ABC)$$

$$m\angle ADC = 2(60)$$

$$m\angle ADC = 120$$

■ 2. $\triangle JKL$ is equilateral and is circumscribed about $\odot M$. The radius of $\odot M$ is 5. Find the perimeter of $\triangle JKL$.

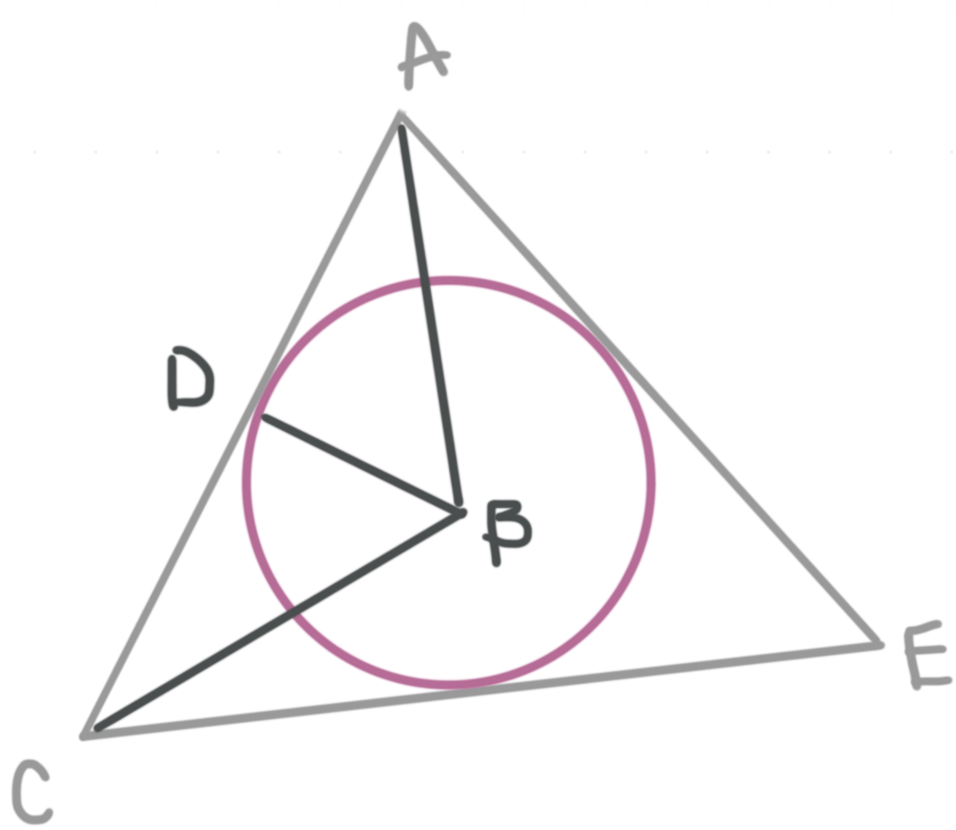


Solution:

$30\sqrt{3}$. The angles of the triangle are 60. Draw a line segment from M to L which bisects $\angle L$. Use the $30-60-90$ rule to find half the length of one of the sides of the triangle to be $5\sqrt{3}$. Double this to find the length of one side and get $10\sqrt{3}$. The perimeter is

$$3(10\sqrt{3}) = 30\sqrt{3}$$

■ 3. If $\triangle ACE$ is an equilateral triangle, if $\odot B$ is inscribed in $\triangle ACE$, and if $\overline{AB} = 12$, find the length of the radius of $\odot B$. Hint: any triangle with three interior angles 30°, 60°, and 90° have a side length ratio of x, $\sqrt{3}x$, and $2x$, respectively.



Solution:

Because $\triangle ACE$ is an equilateral triangle, D must be a midpoint of $\overline{AC}$, and the two interior triangles $\triangle ABD$ and $\triangle BCD$ are both $30-60-90$.

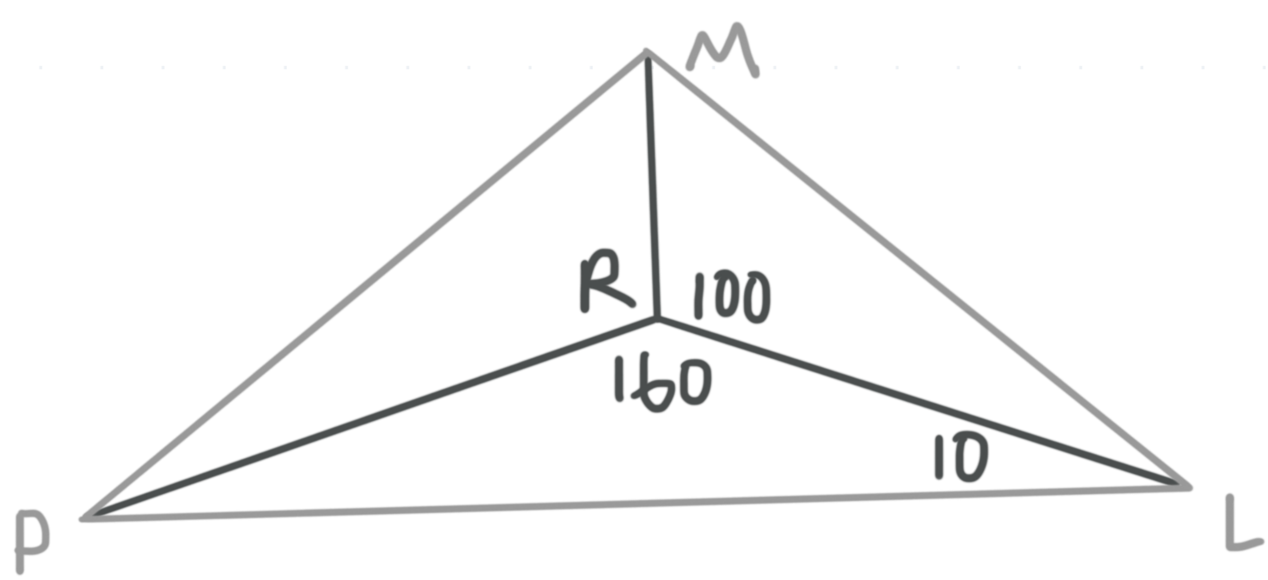
The sides lengths of a $30-60-90$ triangle are x, $\sqrt{3}x$, and $2x$, respectively. Since we're told $\overline{AB}=12$, we can set

$$2x=12$$

$$x=6$$

If $x=6$, then the side of $\triangle ABD$ opposite $\angle DAB$ must have a length of 6. This is side $\overline{BD}$, and $\overline{BD}$ is also the radius of $\odot B$, so the radius of $\odot B$ is $\overline{BD}=6$.

■ 4. R is the incenter of $\triangle PML$. Find $m\angle PMR$.



Solution:

70. $\overline{RP}$, $\overline{RM}$, and $\overline{RL}$ bisect $\angle P$, $\angle M$, and $\angle L$ respectively. Use the Triangle Sum Theorem to find that $m\angle PMR = 70$.

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