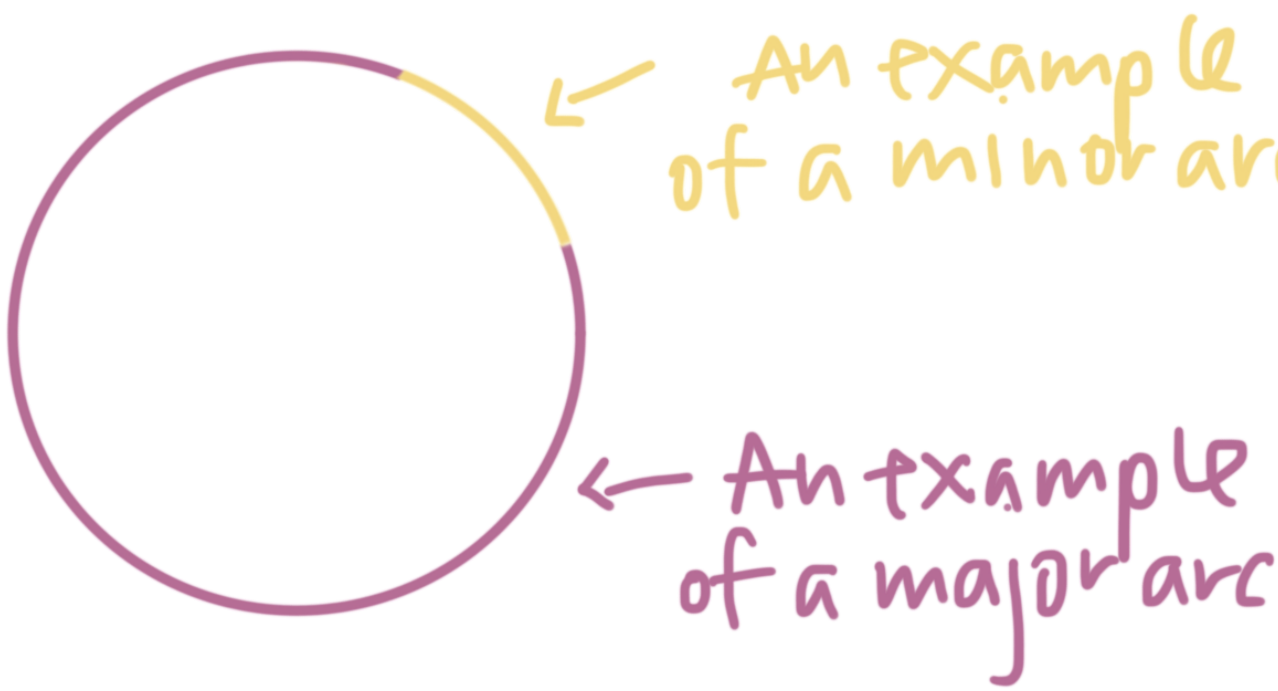
Degree measure of an arc

In this lesson we'll look at arcs of circles and how to find their degree measure. Arcs also have length, but we won't consider that in this lesson, so we'll use "measure" to mean "degree measure."

Arcs

An **arc** is a continuous part of a circle (a part that has no holes in it). There are three types of arcs.

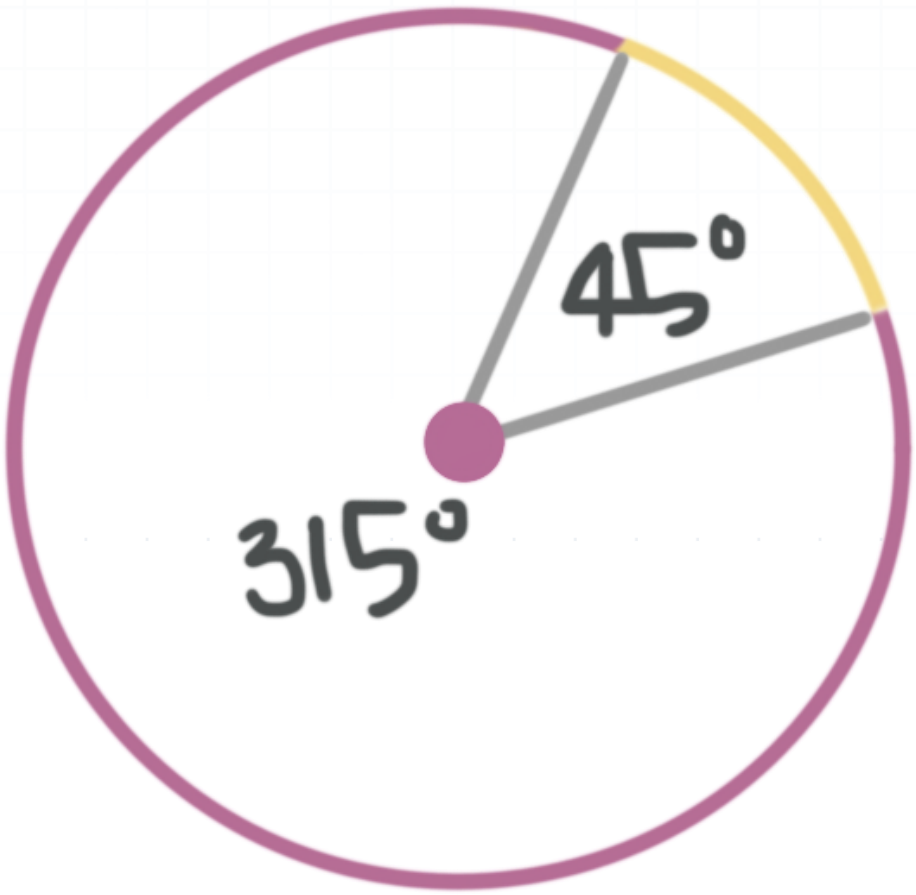
1. **Minor arcs** have measure less than 180°.
2. **Major arcs** have measure more than 180°.
3. **Semicircular arcs** (also called **semicircles**) consist of half a circle and have measure of exactly 180°.



Measures of arcs and central angles

A **central angle** of a circle is an angle whose vertex is at the center of the circle. The central angle and the corresponding arc (the arc whose endpoints are the points of intersection of the sides of the central angle with the circle) have the same measure.

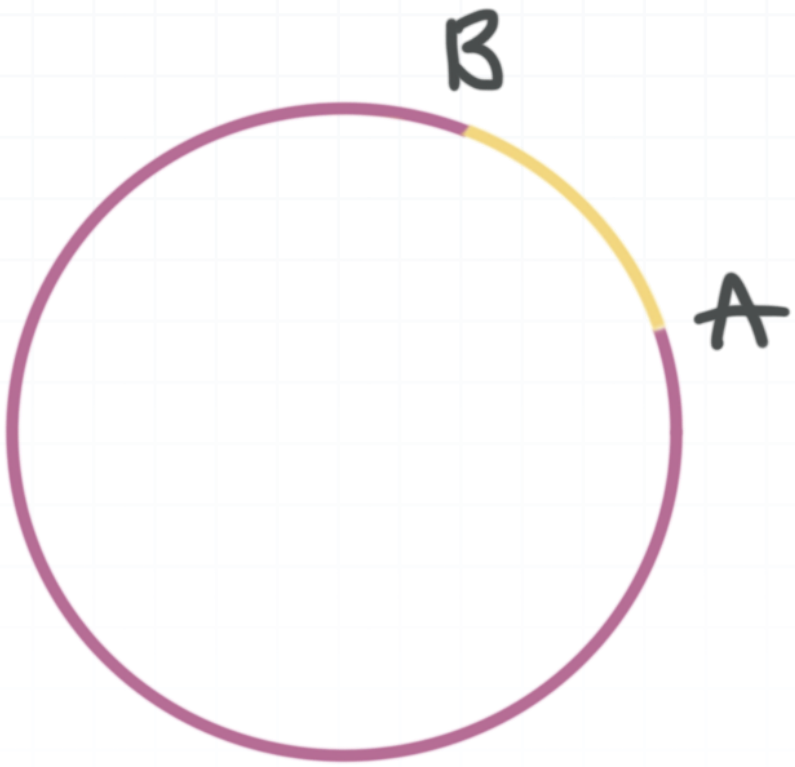
Here the measure of the central angle that corresponds to the minor arc is 45°, so the measure of the minor arc is 45°.



The measure of the central angle that corresponds to the major arc is 315°, so the measure of the major arc is 315°. Notice that the sum of the measures of a pair of central angles of a circle that correspond to a pair of arcs which (together) make up the entire circle (but intersect only at their endpoints) is 360°.

Naming arcs and their measures

We name arcs by their endpoints, so in the circle below, we can identify $\overset{\frown}{AB}$, which is the same as $\overset{\frown}{BA}$.



But the question becomes, how do we know whether we're referring to the big purple arc or the small yellow arc, when we just name the arc with two letters?

To identify the correct arc, we name major arcs differently than we name minor arcs. We'll always name major arcs with three letters and minor arcs with two letters. So in the circle above, identifying $\overset{\frown}{AB}$ or $\overset{\frown}{BA}$, because we've named the arc with two letters, means that we're naming the small yellow arc.

If we wanted to identify the big purple arc instead, we'd need to add a third point to the circle, C, that would sit anywhere else on the purple arc,

and then we could use that third point to identify the major arc as $A\overset{\frown}{C}B$ or $B\overset{\frown}{C}A$.

To indicate the measure of an arc, we use an m or the word "measure," or we can just give the name of the arc itself. So if we want to say that the minor arc measures $45°$, we could use any of these:

$$\overset{\frown}{AB} = 45°$$

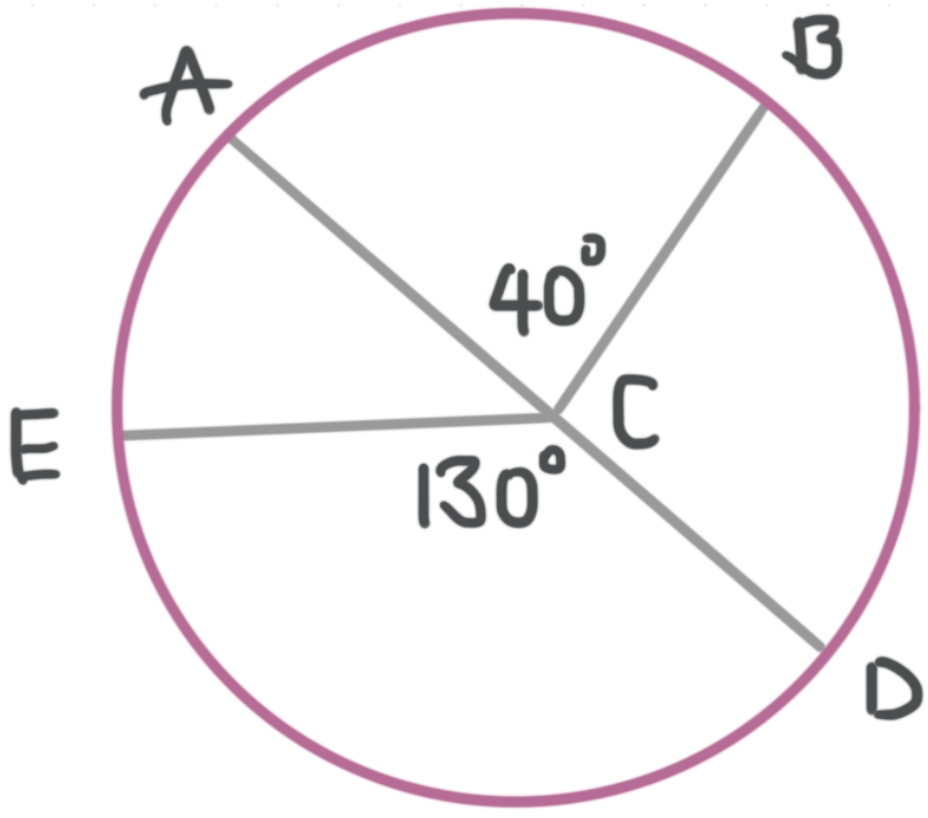
$$m\overset{\frown}{AB} = 45°$$

measure $\overset{\frown}{AB} = 45°$

Let's look at a few examples.

Example

If $\overline{AD}$ is a diameter of the circle (with center at C) in the figure, what is the difference between the measures of $\overset{\frown}{DB}$ and arc $\overset{\frown}{AE}$?



$\overline{AD}$ is a diameter of the circle, which means that the sum of the measures of $\overset{\frown}{DB}$ and $\overset{\frown}{BA}$ is $180°$.

$$m\overset{\frown}{DB} + m\overset{\frown}{BA} = 180°$$

$$m\overset{\frown}{DB} + 40° = 180°$$

$$m\overset{\frown}{DB} = 140°$$

Likewise, the sum of the measures of $\overset{\frown}{AE}$ and $\overset{\frown}{ED}$ is $180°$.

$$m\overset{\frown}{AE} + m\overset{\frown}{ED} = 180°$$

$$m\overset{\frown}{AE} + 130° = 180°$$

$$m\overset{\frown}{AE} = 50°$$

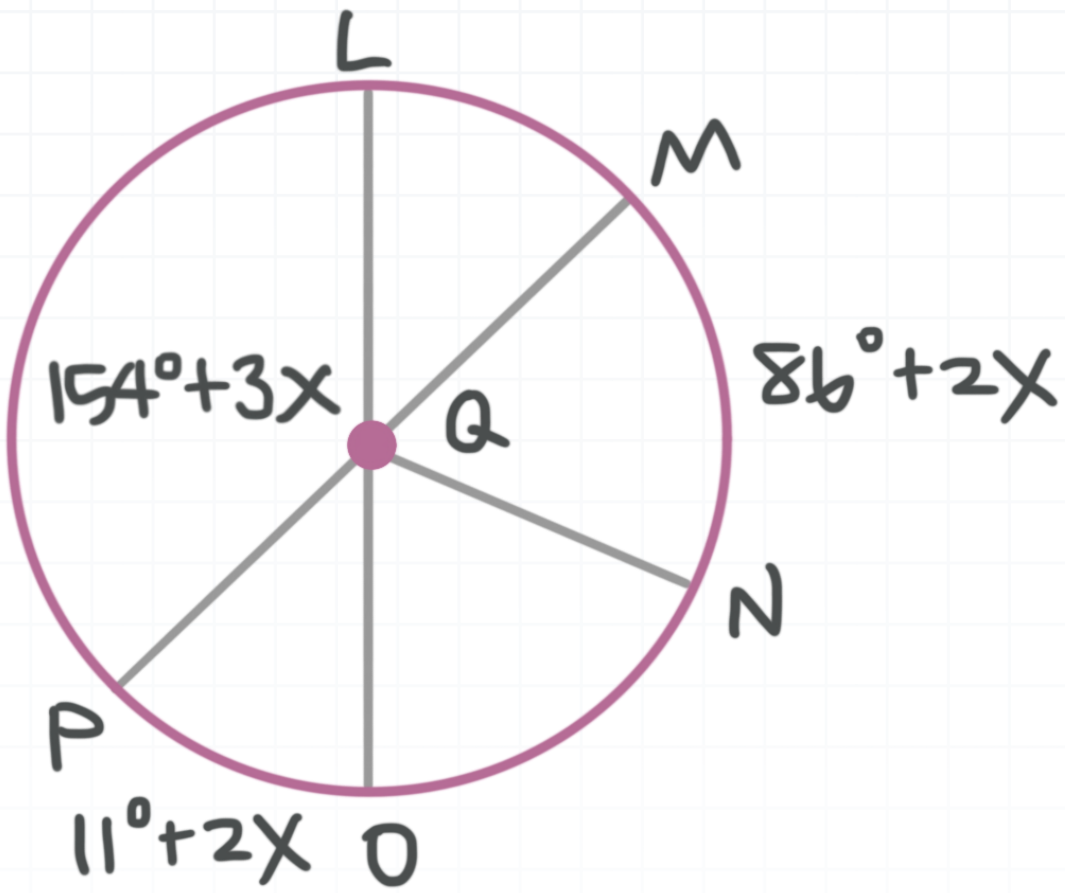
The difference between the measures of arcs $\overset{\frown}{DB}$ and $\overset{\frown}{AE}$ is

$$140° - 50° = 90°$$

Let's look at one more example.

Example

What is the measure of $\overset{\frown}{NM}$, given that $\overline{LO}$ is a diameter of the circle centered at Q?



We need to solve for x and then plug it back into the expression for the measure of $\overset{\frown}{NM}$. We know that $\overline{LO}$ is a diameter of the circle, so the sum of the measures of $\overset{\frown}{LP}$ and $\overset{\frown}{PO}$ is $180°$.

We can use this information to find the value of x.

$$m\overset{\frown}{LP} + m\overset{\frown}{PO} = 180°$$

Angle LQP has measure $154° + 3x$, which means that $\overset{\frown}{LP}$ also has measure $154° + 3x$. Therefore,

$$(154° + 3x) + (11° + 2x) = 180°$$

$$165° + 5x = 180°$$

$$5x = 15°$$

$$x = 3°$$

Now we can find the measure of $\overset{\frown}{NM}$.

$$m\overset{\frown}{NM} = 86° + 2x$$

$$m\overset{\frown}{NM} = 86° + 2(3°)$$

$$m\overset{\frown}{NM} = 92°$$
