IB Higher Level: Mathematics
AA Internal Assessment

Title of exploration:

How accurately can the mass of a flywheel be calculated for a v8 engine using the engine specifications.

Pages: 15

Introduction

Vehicular transportation has been the backbone of this industrial era. Vehicular transportation is a broad term as there are various types of transportation such as cars, planes and boats in a range of sectors from logistics to commercial transportation but they all stem from engineering. Since the invention of the combustion engine in 1860[1] there have been many impactful uses, such as the earlier iterations of propeller planes and cars. The most impactful use of the combustion engine could arguably have been the first car by Carl Benz in 1886[2]. Cars come in various shapes and sizes as they have a multitude of uses leading to different design features and design priorities. Cars have been used in various expeditions around the world because of their versatility; to date there are approximately 1.47 billion vehicles[3] on the planet. Most of which are combustion vehicles despite the recent shift to electric vehicles.

Some types of vehicles require larger engines, however the same basic components are still commonplace in all of these combustion vehicles. One of the most important parts of an engine is the number of pistons in the engine. Eight cylinder engines (engines with 8 pistons) are one of the most recognisable and popular engines. These are commonly used in supercars, high-end luxury cars and previously even race cars such as F1 cars. When it comes to the name of an engine the amount of litres is the total displacement of the engine (the total volume of all pistons) and the general trend is that the more litres the more powerful the engine. The letter before the number, the “v” in “v8”, represents the piston arrangement of the engine so a v8 engine when looked at from the front should look like a v in the same way an i6 (inline) engine looks like an i and a w16 engine looks like a w. The last specification is the number, which represents the amount of pistons it has. So a 5 litre v8 is an engine with 8 cylinders that has 5 litres of displacement in the shape of a “v”. A flywheel is a heavy round disc usually made of metal that is connected to the crankshaft and evens out the energy variation of the crankshaft.

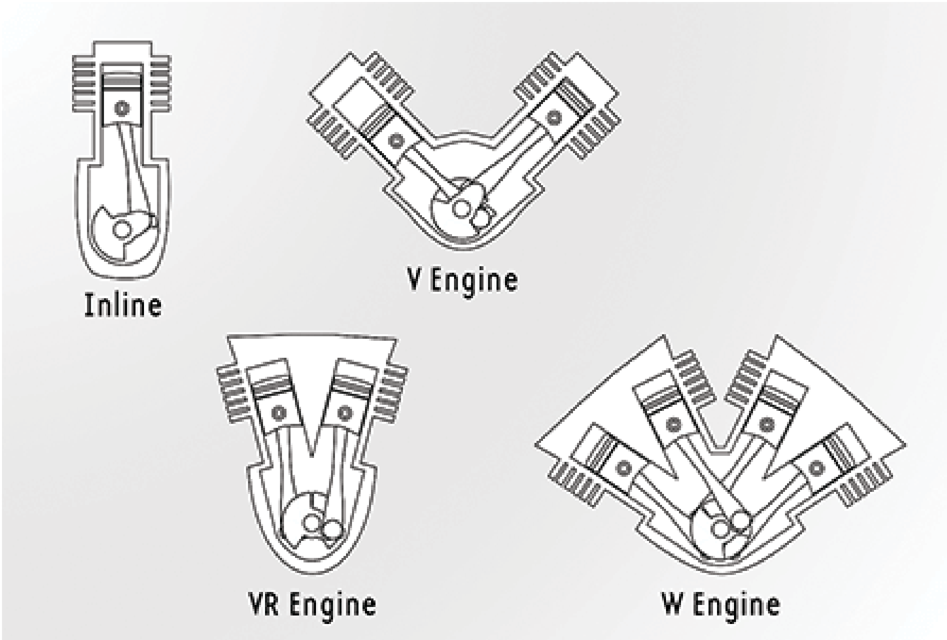

Figure 1: A diagram that shows some common engine layouts

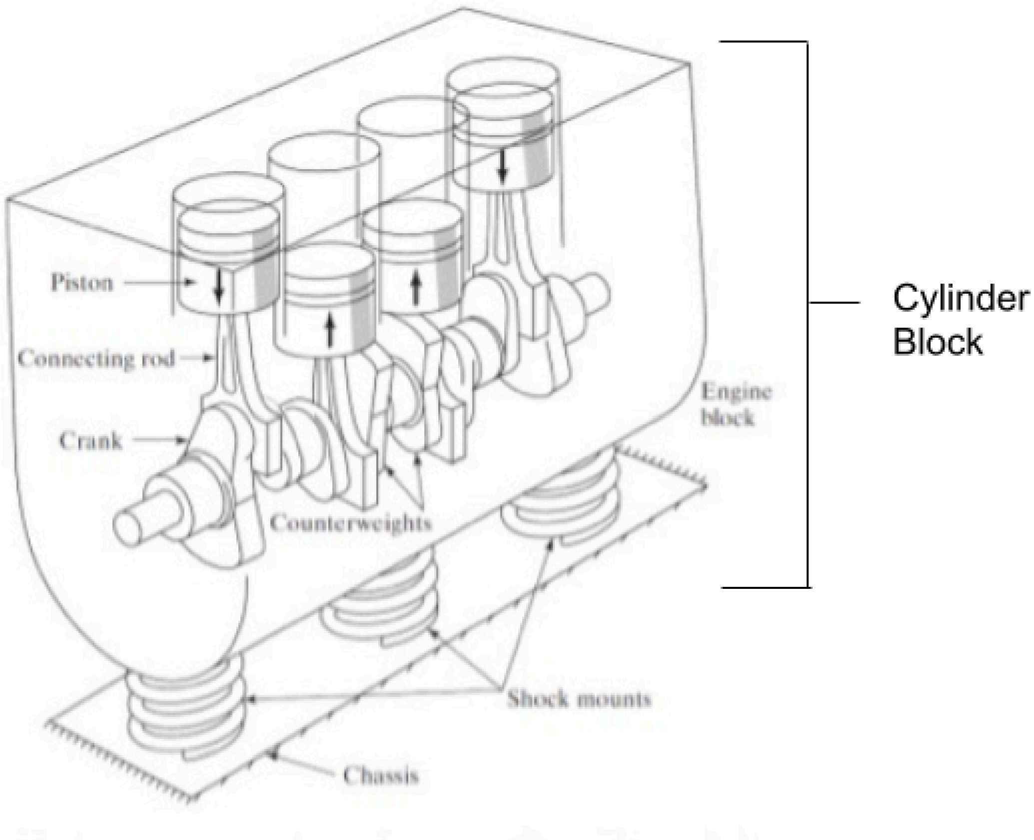

Figure 2: The cylinder block of a straight 4 engine

Aim And Methodology

The aim of this investigation is to calculate the ideal flywheel size for a 5 litre eight cylinder engine as it is the driving force of most popular sports cars such as the Ford Mustang. Each piston in the cylinder block will be identical to each other. To determine the mass of the flywheel, initially the piston firing interval will be calculated, then the angular velocity of a singular piston cycle will be plotted as a sinusoidal curve. By finding the maximum and minimum velocity of the crankshaft and then substituting into the kinetic energy variation formula to find the required inertia of the flywheel. Using this value it would then be possible to calculate the mass of the flywheel. This investigation will be based on the 302 Cid Ford v8 Engine[4] an engine that was used in the 1969-1970 Ford Mustang Boss's which are highly sought after even today and have been featured in movies such as Fast and Furious.

Finding Angular Velocities

Firing Interval

A full piston cycle is equal to two complete revolutions of the crankshaft meaning a full piston cycle is equal to 720^o, so to calculate the number of degrees the crankshaft has to turn before a piston is fired, 720^o has to be divided by the amount of pistons (in this case 8):

$$\frac{720}{8} = 90^o$$

So at every 90 degree turn of the crankshaft, a piston is fired in the order 1-3-7-2-6-5-4-8 (firing order described in the manual).

Figure 3: A v8 engine block to show how the pistons are numbered

As this is a hypothetical question the engine RPM (Rotations per minute) can not be used to calculate the mean piston speed in a piston cycle. So instead of using mean piston speed (speed over a given time) the instantaneous piston speed (speed at a given time) must be found instead. The equation for this is derived from the derivative of instantaneous piston position, the derivative of crank angle with respect to time.

The equation for instantaneous piston position is from Nigus' Kinematics and Load Formulation of Engine Crank Mechanism :

$$x = l(\lambda^2 sin^2 \theta/2) + (1 - cos\theta) \tag{1}$$

Where λ is the ratio of crank radius ($r \to 0.05m$) to connecting rod length ($l \to 0.1m$), ($\frac{r}{l}$).

This is the equation for instantaneous piston position[5] from the TDC (Top Dead Centre) as seen in figure 4. It was derived using the vector loop method which is the motion analysis of mechanisms connected by joints which form closed polygons and is expressed mathematically[6]

The closed polygon stated is similar to the one seen in Figure 4.

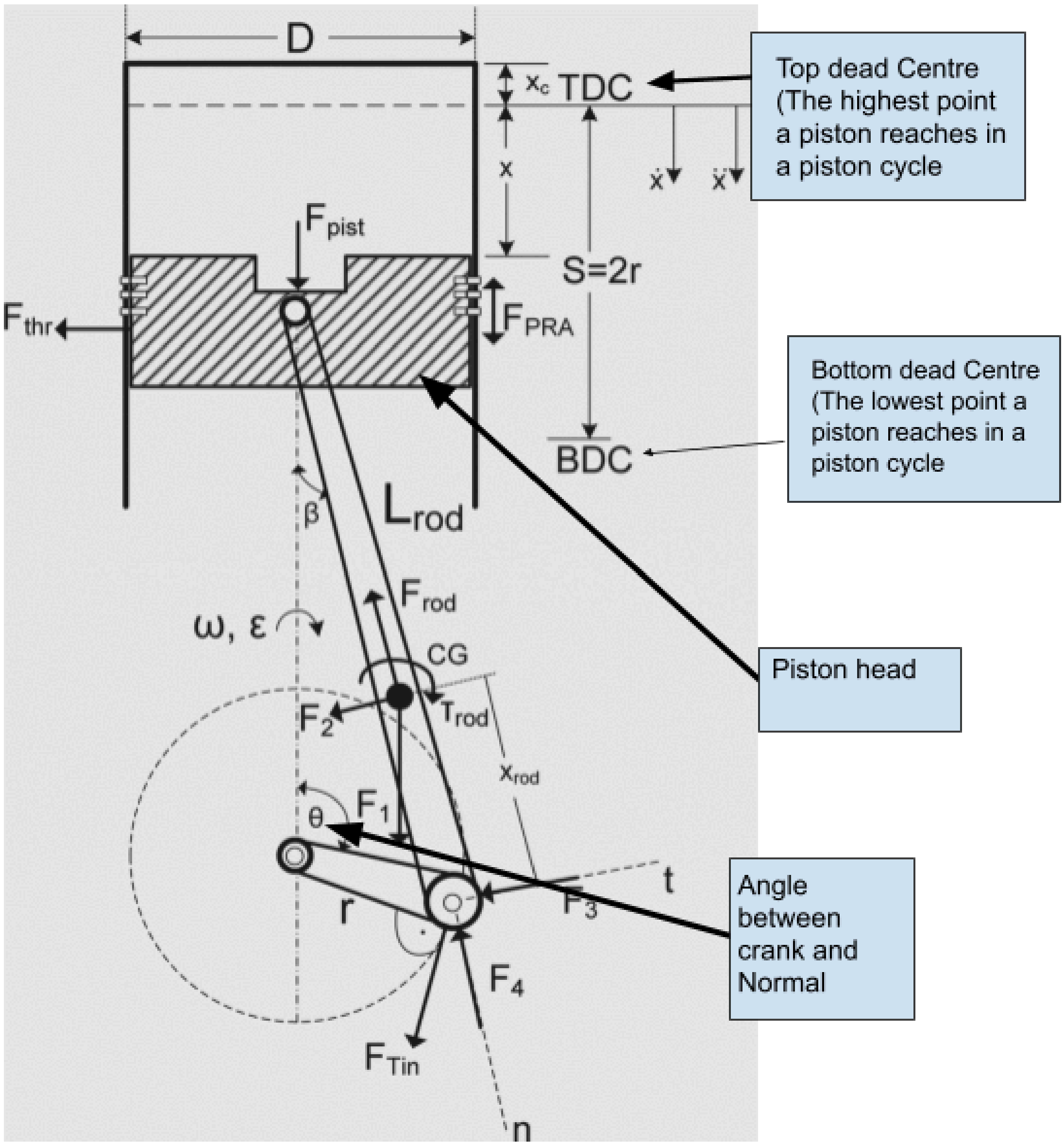

Figure 4: Schematic diagram of a slider-crank mechanism

Having equation 1 means that the instantaneous piston speed can be found easily as the basic equation for Velocity is displacement over time. In this case it will be:

$$v_{ins} = \frac{dx}{dt} = \frac{dx}{d\theta}\frac{d\theta}{dt} = \frac{dx}{d\theta}\frac{2\pi N}{60} \quad (2)$$

Where dt is time derivative, dx is the derivative of instantaneous piston position, N is the engine RPM and $d\theta$ is the crank angle derivative.

Derived by,

$$v_{ins} = \frac{dx}{dt} = \frac{dx}{d\theta}\frac{d\theta}{dt}$$

because of chain rule,

$$\frac{d\theta}{dt} = \frac{2\pi N}{60}$$

because $\frac{d\theta}{dt}$ is equivalent to angular velocity (ω) as angular velocity is displacement over time which is why it becomes $\frac{2\pi N}{60}$.

The derivation of equation 2 is substituted in for dx which leads to the final instantaneous piston velocity equation being[5]:

$$v_{ins} = r \times \omega \times (\sin\theta + \frac{\lambda \sin 2\theta}{2}) \quad (3)$$

Which can be written as:

$$v_{ins}(\theta) = r \times \omega \times (\sin\theta + \frac{r \times \sin 2\theta}{2l}) \quad (4)$$

Equation 4 will be used to graphically represent the maximum and minimum piston speed which can then be easily converted into the angular speed of the crankshaft. Before that the variables in the equation must be calculated so it can be graphed as a sinusoidal graph which would make it easier to discern the important points.

Combustion Force

The formula for combustion force is:

$$F = P_{cyl} \times A \quad (5)$$

Where P_{cyl} is peak cylinder pressure and A is the piston area.

This formula is a form of Newton's second law of motion and can be calculated from the engine parameters given. The peak cylinder pressure was approximated to be 1000psi[7] (a measure of pressure) as this theoretical engine is a high performance engine and 1000psi is approximately $6.89 \cdot 10^6$Pa. The area can be found by using the basic equation $A = \pi r^2$. Using the specification r

can be found by dividing the cylinder bore by 2 to get the radius of 5 cm which is 0.05m but pistons do not fit their cylinders perfectly so the “out-of-round” of 0.0127 cm (0.000127m) is subtracted.

$$A = \pi\,(0.05 - 0.000127)^2$$

This leads to the final equation of F:

$$F = (6.89 \times 10^6) \times (\pi \times (0.05 - 0.000127)^2)$$
$$F = 53136.11173\text{N}$$
$$F \approx 53,136.11N$$

Piston Mass

Piston mass is simple to calculate as it is just the addition of the piston components in this case the piston, piston pins and piston rings. The piston head[8] for this configuration is 559 grams, piston pins are 80g and the piston rings are 54g. This leads to the combined mass of 693 grams[8] or 0.693kg.

Piston Acceleration

With a force and a mass the acceleration of the piston can be found by using a form of the basic kinematic equation $F = ma$ by rearranging to get the equation:

$$a = \frac{F}{m} \tag{6}$$

With the piston mass and combustion force substituted into equation 6 an acceleration of 76,675.48241m/s is given or $76,675.48m/s$ when rounded to 2 decimal places.

Linear Acceleration To Angular Velocity

The equation that shows the relationship between linear acceleration is[9]:

$$a = r \times \omega^2 \times (cos\theta + \frac{cos\ 2\theta}{\lambda}) \tag{7}$$

Which when rearranged becomes:

$$a = r \times \omega^2 \times (cos\theta + \frac{r \times cos\ 2\theta}{l}) \tag{8}$$

From the piston acceleration the maximum acceleration was found to be $76,675.48m/s$ then

$$a(0) = r \times \omega^2 = 76,675.48 \tag{9}$$
$$\omega = \sqrt{\frac{a(0)}{r}} \approx 1238.35\ rad/s$$

Sinusoidal Graphs

With all the variables for equation 4 now calculated the sinusoidal graph can be plotted:

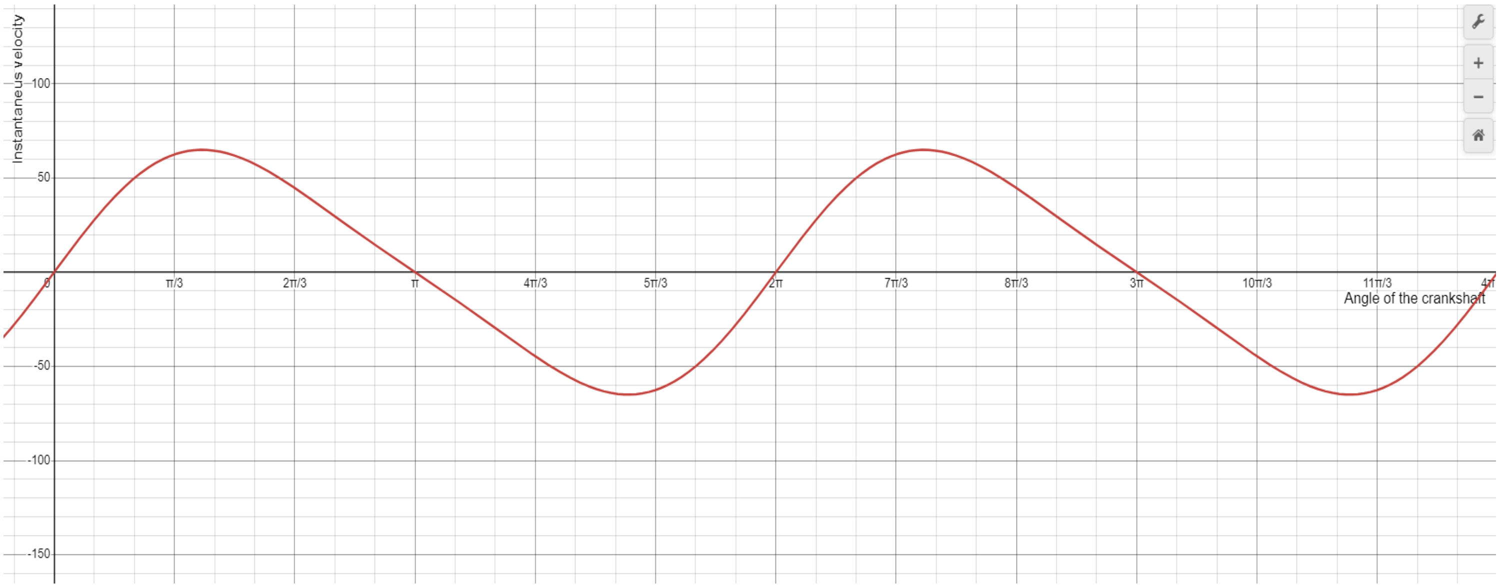

Figure 5: The Graph of the equation $v(\theta) = 1238.35 \times 0.05 \times (\sin\theta + \frac{0.05 \times \sin 2\theta}{0.3})$ with angle of the crankshaft (θ) as the x-axis and instantaneous velocity (v) as the y-axis.

Figure 5 is a depiction of the instantaneous velocity for a singular piston. However, this is a v8 engine which means that there are multiple pistons working simultaneously. So the graph to determine the maximum and minimum angular velocity of the crankshaft needs to include all the pistons on the crankshaft. Since all the pistons are identical the shape of each line is the same but the starting position is different as each piston starts out at every 90^o turn of the crankshaft. To represent this, the lines for each piston will be shifted horizontal by subtracting the equivalent of 90^o in radians (which is π/2) from the angle of each function as f(x-a) is equivalent to f(x) shifted by a to the right. This process leads to the equations below:

Piston 1: $v(\theta) = 1238.35 \times 0.05 \times (\sin\theta + (\frac{0.05 \times \sin 2\theta}{0.3})$

Piston 3 : $v(\theta - \frac{\pi}{2}) = 1238.35 \times 0.05 \times (sin(\theta - \frac{\pi}{2}) + (\frac{0.05 \times sin\, 2(\theta - \frac{\pi}{2})}{0.3})$

∘

∘

Piston 8: $v(\theta - \frac{7\pi}{2}) = 1238.35 \times 0.05 \times (sin(\theta - \frac{7\pi}{2}) + (\frac{0.05 \times sin\, 2(\theta - \frac{7\pi}{2})}{0.3})$

These equations plotted leads to the graph seen in figure 6:

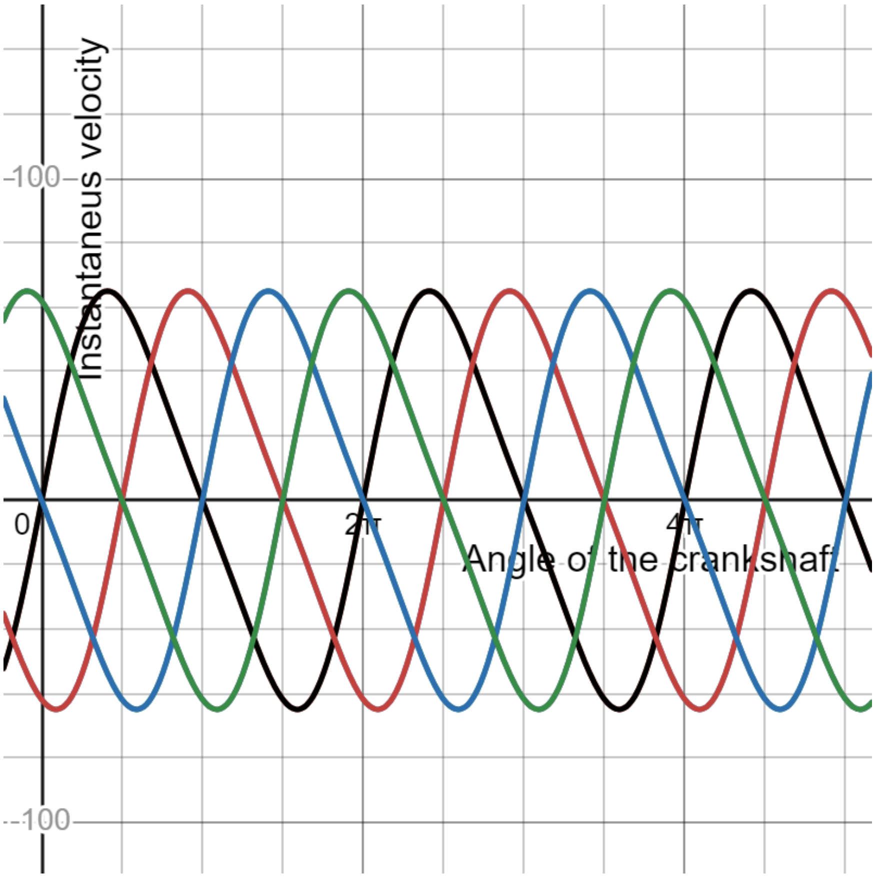

Figure 6: The Graph of the equation $v_{ins}(\theta) = r \times \omega \times (sin\, \theta + \frac{\lambda\, sin\, 2\theta}{2})$ with angle of the crankshaft (θ) as the x-axis and instantaneous velocity (v) as the y-axis.and all pistons shown.

From the graph it can be seen that the points of maximum are identical and the minimum are the points of intersection between each line. When finding the minima and maxima it would be more convenient to use the first two pistons as the maxima and minima are mirrored throughout the graph.

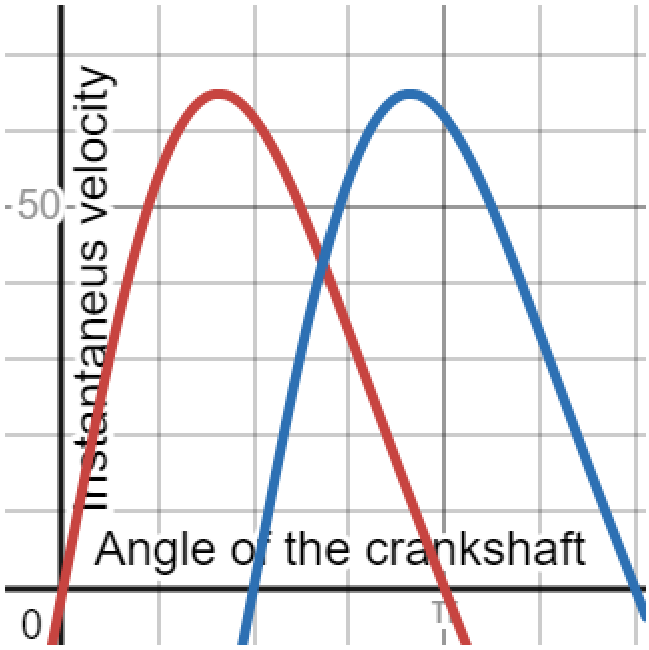

Figure 7: The Graph of the equation $v_{ins}(\theta) = r \times \omega \times (sin\,\theta + \frac{\lambda\, sin\, 2\theta}{2})$ with angle of the crankshaft (θ) as the x-axis and instantaneous velocity (v) as the y-axis and the first 2 pistons shown.

Maximum Instantaneous Velocity

The maximum is where the gradient is 0 as this will be one of the turning points.

$$v(\theta) = 1238.35 \times 0.05 \times (sin\,\theta + (\frac{0.05 \times sin\, 2\theta}{0.3}) .$$

The derivative of the equation will form an equation with $cos\theta$ as the variable.

$$\frac{24767(\frac{cos\, 2\theta}{3} + cos\,\theta)}{400} = 0$$

$$cos\, 2\theta \; + 3\, cos\,\theta = 0$$

$cos\, 2\theta$ is equal to $cos^2\,\theta - 1$

$$cos^2\,\theta - 3cos\,\theta - 1 = 0$$

$$cos\,\theta = 0.2807$$

$$\theta = 1.28619\; or\; 4.99699$$

As the graph peaks before it declines the value of θ is $1.28619\; rads$ as it comes first. This is then substituted into the equation for Piston 1 which will lead to the equation:

$$v(1.28619) = 1238.35 \times 0.05 \times (sin\,(1.28619) + (\frac{0.05 \times sin\,2(1.28619)}{0.3})$$
$$v(1.28619) = 64.98863$$
$$V_{max} = 64.98863$$
$$V_{max} \approx 64.99$$

Minimum Instantaneous Velocity

The process to find the minimum instantaneous velocity is different from the process for finding the maximum velocity. This is because of the fact that there being more than one piston comes into play making it slightly more difficult. The first step is equating the two equations as they are both equal to the same value of $v(x)$. Equating them gives us the lengthy equation:

LHS:

$$1238.35 \times 0.05 \times (sin\,\theta + (\frac{0.05 \times sin\,2\theta}{0.3})$$

RHS:

$$1238.35 \times 0.05 \times (sin(\theta - \frac{\pi}{2}) + (0.5 \times \frac{0.05}{0.15} \times sin\,2(\theta - \frac{\pi}{2}))$$

The right hand side of this equation uses $\theta - \frac{\pi}{2}$ in the trigonometric function. Solving for θ would be made harder if the right hand side was not altered first to make θ the value inside the trigonometric function. This is done by first using the double angle identity $sin\,2\,(x) = 2\,sin\,(x)\,cos\,(x)$ and then using the compound angle identities to get the right hand side's trigonometric functions to be in terms of θ. Once all of the trigonometric functions are in terms of θ then the constants have to calculated such as $1238.35 \times 0.05 = 61.1975$ and $0.5 \times \frac{0.05}{0.15} = \frac{1}{6}$. After the constants have been calculated the formula can be rearranged where the trigonometric functions are on the left hand side and the constants are on the right hand side. The inverse trigonometric function is applied to both sides which leaves the value of θ being $2.14043\,rads$. When this is then substituted into either of the given equations:

$$v(2.14043) = 1238.35 \times 0.05 \times (sin\,(2.14043) + (0.5 \times \frac{0.05}{0.15} \times sin\,2(2.14043))$$
$$v(\,2.14043) = 42.7671$$
$$V_{min} = 42.7671$$
$$V_{min} \approx 42.77$$

Linear Velocity To Angular Velocity

Now that the maximum and minimum instantaneous velocities have been found the minimum and maximum angular velocities can now also be calculated using the very simple equation :

$$\omega = \frac{v}{r} \tag{10}$$

Using equation 10 the angular velocities can be found by substituting V_{min} and V_{max} into the equation with the constant for r being 0.05.

Maximum Angular Velocity

$$\omega_{max} = \frac{V_{max}}{r}$$
$$\omega_{max} = \frac{64.99}{0.05}$$
$$\omega_{max} = 1299.8$$

Minimum Angular Velocity

$$\omega_{min} = \frac{V_{min}}{r}$$
$$\omega_{min} = \frac{42.77}{0.05}$$
$$\omega_{min} = 855.4$$

Calculating Flywheel Mass

Kinetic Energy Variation

With the maximum and minimum angular velocities calculated the kinetic energy variation can now also be calculated using the kinetic energy variation equation[10]:

$$\Delta E_k = 0.5 \times I \times (\omega_{max}^2 - \omega_{min}^2) \tag{11}$$

As I is the moment of inertia:

$$I = 0.5 \times m \times r_f^2 \tag{12}$$

Assuming a flywheel radius of 0.3 metres which would be reasonable for engines of those times and using equation 10:

$$I = 0.5 \times m \times 0.3^2$$
$$I = 0.045m$$

Substituting this value of I as well as the angular velocities calculated prior into equation 11:

$$\Delta E_k = 0.5 \times 0.045m \times (1299.8^2 - 855.4^2)$$
$$\Delta E_k = 21,549.8448$$
$$\Delta E_k \approx 21,549.84m$$

Final Flywheel Mass Calculation

With the energy requirement of the flywheel being approximately $111Wh$[11] converted to joules $399600J$.

$$\frac{\Delta E_k}{21,549.84} = m$$
$$\frac{399600}{21,549.84} = m$$
$$18.50 \approx m$$

Through newtonian laws, differentiation, substitution and trigonometry the value calculated for the flywheel was $18.5Kg$.

Conclusion

This calculation was done using engine parameters and assumptions, which means in actuality the result of 18.5kg cannot be taken as true. There are 3 main reasons for this: the energy losses in a piston cycle, the variables used in the equation and the type of flywheels used currently.

Energy Losses

There are countless points of energy losses in a combustion engine which haven't been accounted for in these calculations. These losses are part of the reason for combustion engines having such low efficiency rates. In this calculation it is assumed that there is little energy loss but in reality at various stages of the crankshaft rotation there are. It is assumed that all of the fuel in the piston is combusted leading to a consistent force being generated however in actuality that is not the case as not all the fuel ends up being combusted and some ends up being wasted. The amount of friction generated from the moving parts also leads to energy losses; this factor is particularly hard to calculate theoretically as there are hundreds of moving parts in a car. All these factors accumulate and lead to a decreased accuracy in the final calculation.

Variables Used

As this calculation is purely theoretical there were assumptions that were made as well as compromises which lead to a decreased accuracy. One of the reasons for decreased accuracy was the type of piston speed being used was instantaneous rather than mean piston speed being used which would have given a more realistic result[12]. Another assumption was the peak cylinder pressure. The value used was an estimate based on expected performance which could be higher or lower than the true value but as only engine schematics were used this can not be verified. The mass used when calculating piston acceleration did not include the upper half of the piston rod.

Modern Day flywheels

Modern day flywheels come in a variety of shapes and sizes each variation each with their pros and cons. This means that a general mass value given in this investigation cannot always be applied. In modern day vehicles there is hybrid assistance which aids the combustion engine given more power as well as maintaining engine rpm. This is because heavy flywheels decrease throttle response which affects the performance of cars noticeably.

The culmination of these factors led to the 18.5kg value. When compared to the actual flywheel mass of a 302cid engine the calculations are off by approximately 25% as in actuality the flywheel is meant to weigh 14.5kg[13] based on finding the midpoint of the range given for high performance engines. This value is not surprising as combustion engines are known for their inefficiency in comparison to electric vehicles. Overall, I believe mathematics is still a viable option for calculating flywheel mass or inertia needed, but it would be more realistic if more of the energy losses were to be taken into account.

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Figure 3 -
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Figure 4-
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Figure 5-7
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