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ROWLAND GHOSTS

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ABSTRACT

A number of small diffraction gratings have been ruled containing arbitrary errors. It is shown that a small sine error $a_1 \sin \theta$ produces ghosts the intensity of which depends upon the ratio of a_1 to a_0, the average grating space, and the order N, in complete agreement with Rowland's theory. Small periodic errors may be applied simultaneously, and each will produce a ghost corresponding to its period. When the errors are small, the ghosts appear at the expected places, and the shape of the error curve may be interpreted from the positions and strength of the ghosts.

The new ruling engine at the Ryerson Physical Laboratory has been described briefly, and some mention made of its performance, in the *Journal of Scientific Instruments*, **12**, 32, 1935, in the *Zeeman Verhandelingen*, p. 280, The Hague, 1935, and in *Science*, 85, 25, 1937.

Papers have been read before the joint meeting of the American Physical Society and Section B of the American Association for the Advancement of Science at the St. Louis meeting, December, 1935; at the joint meeting of the American Physical Society and the Optical Society of America, in New York, February, 1936; and at the Chicago meeting of the National Academy of Science, November, 1936.

The interest shown by spectroscopists in the results on Rowland ghosts seems to call for further publication.

A grating ruled with uniform spacing, a_0, will give in any order

N, a single sharp image of any monochromatic spectrum line of wave-length λ, in accordance with the relation,

$$a_0 (\sin i + \sin \theta) = N\lambda ,$$

where i is the angle of incidence and θ the angle of diffraction. If $\lambda = 2\pi/b$, we have

$$\mu = \frac{2\pi N}{ba_0} = \sin i + \sin \theta .$$

If periodic errors are present in the ruling, false lines, or "ghosts," appear. The theory defining their position and intensity has been given by Rowland.[1] If simple sine errors occur for each group of m lines, the distance y of the r^{th} ruling from one end of the grating will be given by

$$y = a_0 r + a_1 \sin e_1 r + a_2 \sin e_2 r + \dots ,$$

where $e_k = 2\pi/m_k$ and a_k is the maximum displacement of any ruling from the correct position, owing to the error e_k.

In the simple case where all values of a_k are zero except a_0 and a_1, Rowland's theory gives the position of the spectrum line and its ghosts as shown in Table 1.

$J_n(u)$ is a Bessel function of u of order n and is determined by the following equation:

$$J_n(u) = \frac{u^n}{2^n n!}\left(1 - \frac{u^2}{2^2(n+1)} + \frac{u^4}{2^4 2!\,(n+1)\,(n+2)} - \frac{u^6}{2^6 3!\,(n+1)\,(n+2)\,(n+3)} + \dots\right) .$$

Since here $u = b\mu_p a_1 = (2\pi[N + (p/m)]\, a_1/a_0)$, when m is large, usually several hundred in the case of the Rowland ghosts, $b\mu_p a_1$ does not differ from $b\mu a_1$ by more than about 1 per cent, and the values of $J_n^2(b\mu_p a_1)$ will differ from the values of $J_n^2(b\mu a_1)$ by much less than 1 per cent, as is shown by the curves of Figure 1, which are graphs of $J_0^2(u)$, $J_1^2(u)$, $J_2^2(u)$, and $J_3^2(u)$. It is therefore sufficiently accurate to

[1] *Phil. Mag.*, **35** (5), 397, 1893; *Astronomy and Astrophysics*, **12**, 129, 1893; *Rowland's Physical Papers*, p. 525.

use $u = 2\pi N(a_1/a_0)$ instead of $u = (2\pi[N + (p/m)]\, a_1/a_0)$ in calculating the intensities of the ghosts. Table 2 gives the values of $J_n^2(u)$ for different values of n and of $u = 2\pi N(a_1/a_0)$.

TABLE 1

Position	Intensity	Description
$\mu = \frac{2\pi N}{ba_0}$	$J_0^2\,(b\mu a_1)$	Principal line
$\mu_1 = r \pm \frac{e_1}{ba_0}$	$J_1^2\,(b\mu_1 a_1)$	First ghost
$\mu_2 = r \pm \frac{2e_1}{ba_0}$	$J_2^2\,(b\mu_2 a_1)$	Second ghost
$\mu_3 = r \pm \frac{3e_1}{ba_0}$	$J_3^2\,(b\mu_3 a_1)$	Third ghost
....	....	....

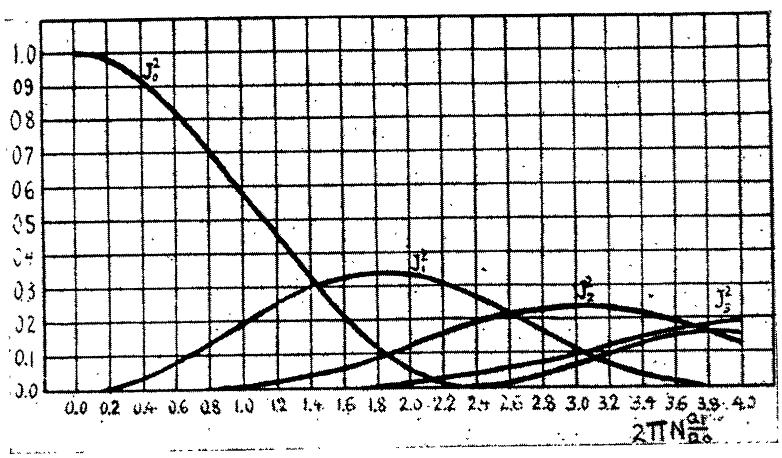

FIG. 1

When $a_1 = 0$, the case of no periodic error, $J_0^2(u)$ has its maximum value and all the other J's are equal to 0; the main line has its maximum value and there are no ghosts. As a_1 is increased, u increases and J_0 decreases. The other functions J_n begin to have finite values, and the ghosts appear.

Table 2 and the curves show that the principal line itself will be reduced to 0 in intensity when $2\pi N a_1/a_0' = 2.405$. This will occur in the third order, $N = 3$, when $a_1/a_0 = 0.1274$, i.e., when the maximum displacement, a_1, in each period is about one-eighth of the grating space, a_0, when there are 600 rulings per millimeter. The distance, a_1, is most conveniently measured by an interferometer, the back mirror of which is mounted on the grating carriage. When

TABLE 2

$2\pi N\frac{a_1}{a_0}$	J_0^2	J_1^2	J_2^2	J_3^2	J_4^2	J_5^2	J_6^2	J_7^2
0.0	1.000							
0.2	0.980	0.010						
0.4	0.922	.038	0.000					
0.6	0.832	.082	.002					
0.8	0.716	.136	.005					
1.0	0.586	.194	.013	0.000				
1.2	0.450	.248	.025	.001				
1.4	0.321	.294	.043	.003				
1.6	0.207	.325	.066	.006				
1.8	0.116	.338	.094	.010	0.001			
2.0	0.050	.333	.124	.017	.001			
2.2	0.012	.309	.157	.027	.002			
2.405	0.000	.269	.186	.040	.003			
2.6	0.009	.222	.212	.055	.007	0.001		
2.8	0.034	.168	.228	.075	.011	.001		
3.0	0.068	.115	.236	.096	.017	.002		
3.25	0.111	.058	.231	.123	.028	.005		
3.5	0.144	.019	.210	.150	.042	.006	0.001	
3.75	0.161	.001	.176	.171	.059	.011	.001	
3.832	0.162	.000	.162	.176	.065	.013	.002	
4.0	0.158	.004	.133	.185	.079	.017	.002	
4.25	0.136	.020	.088	.188	.100	.026	.004	0.000
4.5	0.103	.053	.047	.180	.121	.038	.007	.001
4.75	0.065	.084	.018	.161	.140	.052	.011	.002
5.0	0.031	0.107	0.002	0.133	0.153	0.068	0.017	0.003

λ 5460 is used, $a_0 = 1/600$ mm $= 6.1$ fringes; and when $a_1/a_0 = 0.1274$, a_1 is about three-quarters of a fringe. For this value of a_1 the first ghosts (one on each side), given by $J_1^2(u)$, have already passed their maximum intensities and are 26.9 per cent as strong as the original line. The second ghosts, given by $J_2^2(u)$, have an intensity 16.6 per cent; the third ghosts, given by $J_3^2(u)$, are 4 per cent; $J_4^2(u)$ equals 0.3 per cent; and higher ghosts are too faint to consider. Further, since we know that $1 = J_0^2 + 2J_1^2 + 2J_2^2 + \ldots$, we should expect the sum of all the intensities to be 1; and we have, from the curves, $2 \times 26.9 + 2 \times 18.6 + 2 \times 4.0 + 2 \times 0.3 = 99.6$, when the intensity of the central line without ghosts is taken as unity. The small residual, 0.4 per cent, is distributed among ghosts of higher order. Table 2 shows further that when

$$u = \frac{2\pi N a_1}{a_0} = 3.832, \text{ i.e., } \frac{a_1}{a_0} = 0.203,$$

$J_1^2(u)$ is 0 and the first ghost disappears. The main line is 16.2 per cent, the second ghost 16.2 per cent, the third ghost 17.6 per cent,

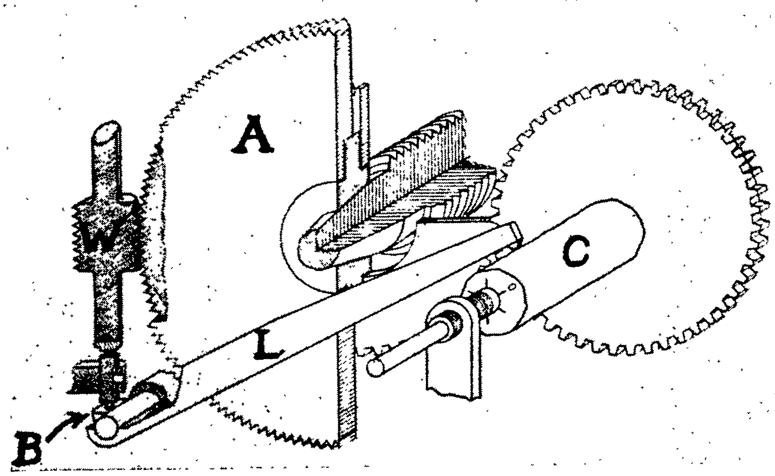

FIG. 2

the fourth ghost 6.5 per cent, the fifth ghost 1.3 per cent, and the sixth ghost 0.2 per cent. The sum is 100 per cent, since here again $16.2 + 2 \times 16.2 + 2 \times 17.4 + 2 \times 6.5 + 2 \times 0.4 = 100$.

The ruling engine at the Ryerson Physical Laboratory is provided with a compensating cylinder, C, Figure 2, which carries the long end of the lever, L. The short end of the lever carries the end butt, B, of the worm, W, which drives the worm wheel, A. A motion of 3.50 mm at C produces a motion of one fringe at the ruling carriage.

The cylinder was first shaped as carefully as possible to eliminate all original periodic errors due to the screw, the end butt, and the worm wheel. A grating of 600 lines per millimeter, ruled with the compensator in this condition, gave in the third order the spectrum of the green mercury line as shown in Figure 3 (a), a main line only with no ghosts. When an error $a_1 \sin \theta$ was impressed on the compensator with $a_1/a_0 = 0.1274$, the result in the third order was as shown in Figure 3 (*b*). The main line is reduced to 0, and the first and second ghosts show on each side. When a_1/a_0 was increased to 0.203, thus making $(2\pi N a_1/a_0) = 3.832$, the result was as shown in

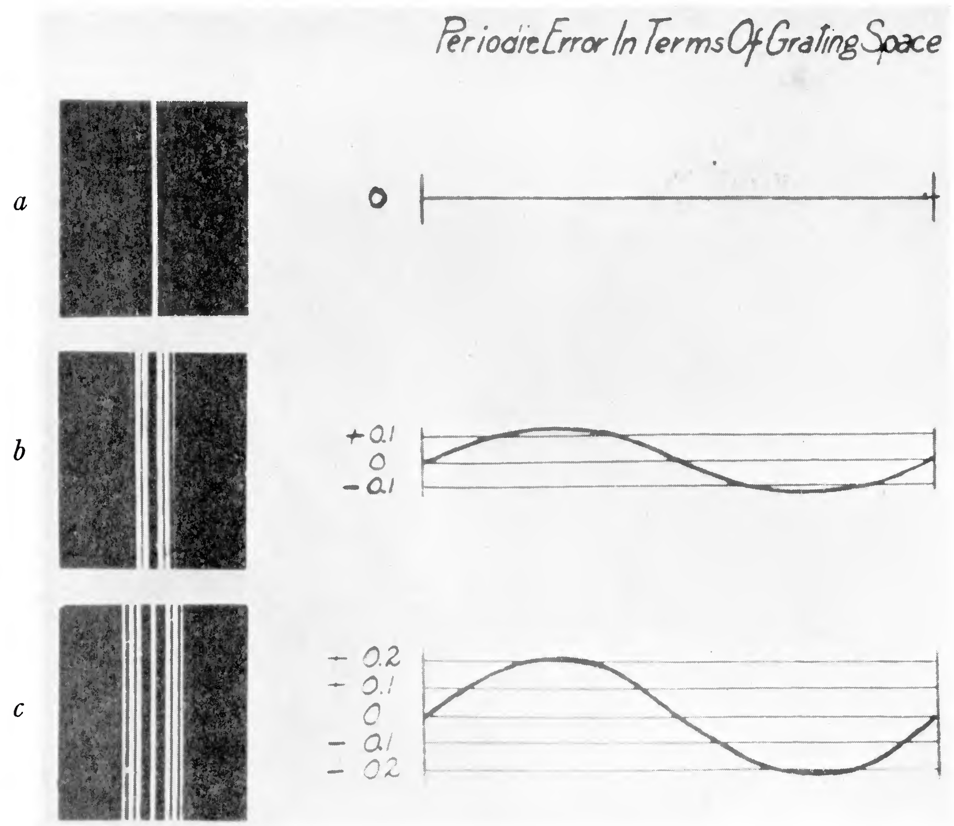

Fig. 3

Figure 3 (*c*). The first ghost has disappeared, and the main line and second ghost are of equal intensity, in complete agreement with theory. To secure the photographs, the gratings were placed on a spectrometer table and the eyepiece and telescope carefully focused on the lines and ghosts in question. When a small camera, focused for parallel rays, was set on the axis of the telescope, the pattern was photographed very readily.

To demonstrate the effect of the order, N, on the relative intensities of the ghosts, the spectrum was photographed in the second, third, and fourth orders, with the grating which produced Figure 3 (*b*) above. The corresponding values of $u = 2\pi N(a_1/a_0)$ are 1.604, 2.405, and 3.208. A comparison of Figures 4 (*a*), (*b*), (*c*), with the table or the curves, shows how faithfully the results follow the the-

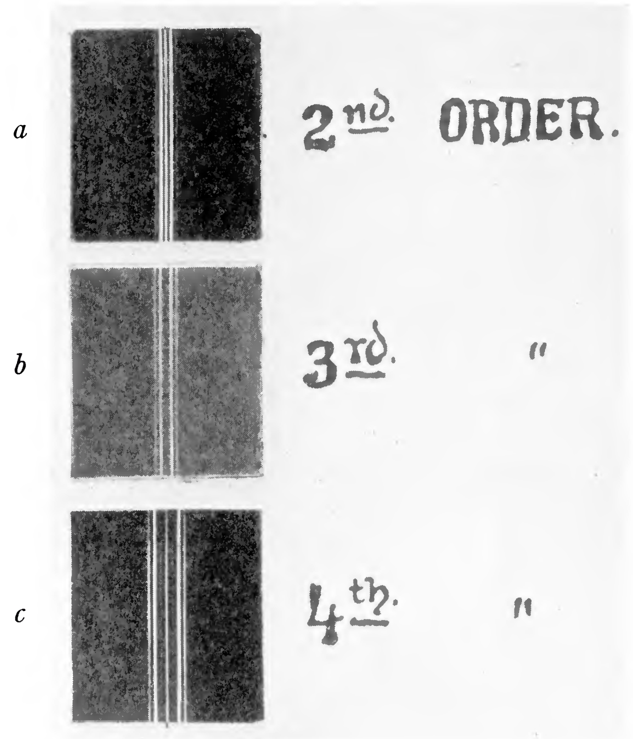

FIG. 4

ory and how badly we might be led astray in the case of strong ghosts if we followed the commonly accepted theory that the first ghost was due to an error $a_1 \sin \theta$, the second ghost to an error $a_2 \sin 2\theta$, the third ghost to an error $a_3 \sin 3\theta$, etc., since the patterns are so completely different in the different orders and only one error $a_1 \sin \theta$ is present.

Any ghost, however—for example, the third, fifth, eighth, or twentieth ghost—may be produced by impressing on the compensator an error $a \sin 3\theta$, or $a \sin 5\theta$, or $a \sin 8\theta$, etc. Errors $a \sin 8\theta$ and $a \sin 7\frac{1}{2}\theta$ impressed on the compensator gave a ghost at the

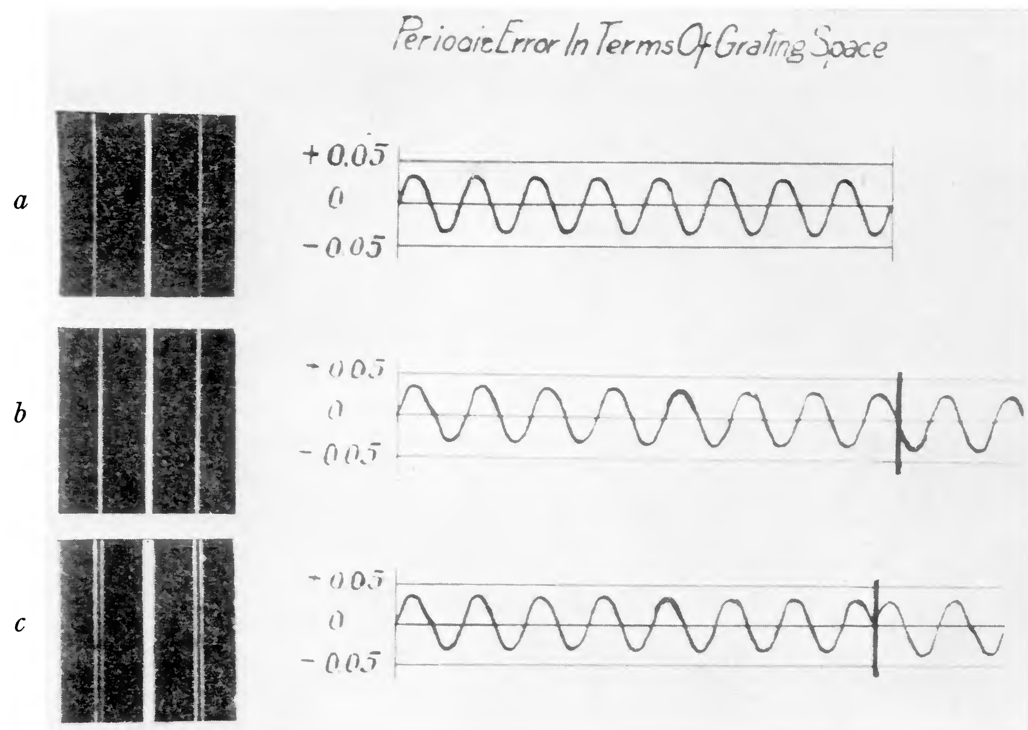
Periodic Error In Terms Of Grating Space
a
+0.05
0
-0.05
b
+0.05
0
-0.05
c
+0.05
0
-0.05

FIG. 5

position of the eighth Rowland ghosts in the first case and one at the position $7\frac{1}{2}$, midway between the seventh and eighth Rowland ghosts in the second case, as shown in Figures 5 (*a*) and 5 (*b*). However, when the error corresponding to sin $7\frac{1}{2}\theta$ was stopped at the end of each turn, and recommenced in the original phase, the error did not have a period corresponding to sin $7\frac{1}{2}\theta$ but was nearest to sin 7θ +

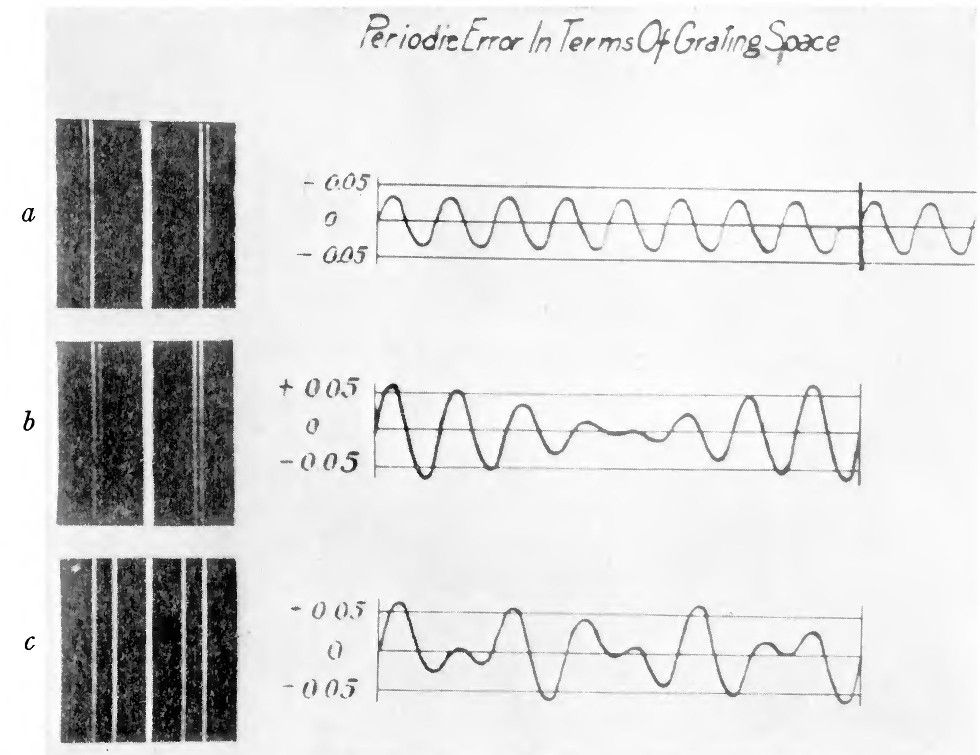

FIG. 6

sin 8θ. Ghosts of equal intensity at the positions of the seventh and eighth Rowland ghosts appeared as shown in Figure 5 (*c*). When an error corresponding to *a* sin $8\frac{1}{3}\theta$ was used, and started over in the original phase after each turn, ghosts appeared in the eighth and ninth positions; but the one at 8 had twice the amplitude, and therefore four times the intensity of the one at 9, as shown in Figure 6 (*a*). When errors corresponding to *a* sin 7θ and *a* sin 8θ were applied simultaneously on the compensator, equal ghosts appeared at the seventh and eighth positions, and a similar result was obtained for errors a sin 5θ and a sin 8θ applied simultaneously, as shown in Figures 6 (b) and 6 (c).

As a further test, an error a sin 20θ was used, and the twentieth Rowland ghost appeared, as shown in Figure 7 (a). When, however, the error a sin 20θ was used on the compensator for the first half of a revolution, and no error was present for the second half, the ghosts appeared as in Figure 7 (b), a strong ghost at the position 20, with fainter ones at 19 and 21. Since the ghosts at 7 and 8 and at 5 and 8 corresponded to curves $a \sin 7\theta + a \sin 8\theta$ on the compensator in the first case, and $a \sin 5\theta + a \sin 8\theta$ in the second case, it seemed worth while to trace with the harmonic analyzer the curves $a/3 \sin 19\theta + a \sin 20\theta + a/3 \sin 21\theta$. The result is shown in Figure 8 (a), which is a fairly good first approximation to the shape of the compensator used in Figure 7 (b). Additional faint ghosts were undoubtedly present which would have been shown by longer exposure, and additional sine terms with correspondingly small amplitude would have given a closer approximation to the original curve.

Finally the error a sin 20θ on the compensator was modified by omitting alternate loops of the curve, as in Figure 7 (c). A strong ghost appeared at position 20, with fainter ones at 10 and 30. Figure 8 (b) shows a tracing with the harmonic analyzer, using $a/2 \sin 10\theta + a \sin 20\theta + a/2 \sin 30\theta$. The reproduction of the original error curve, Figure 7 (c) is remarkably good.

The case of two periodic errors,

$$y = a_0 r + a_1 \sin e_1 r + a_2 \sin e_2 r \ ,$$

is to be interpreted as above, a ghost appearing at the positions corresponding to e_1 and e_2, when the errors are small. If the errors are large, the second, third, etc., ghosts due to each error will appear and will also act as principal lines for the production of ghosts by the other error, as shown in Rowland's theory. As soon as the ruling engine is available for the purpose, small gratings will be ruled with sufficiently large double periodic errors to reduce the principal line to zero and throw all the light into the ghosts. Smaller values of m will also be used to produce Lyman ghosts.

It should be borne in mind that the errors $e_1 = 2\pi/m_1$, $e_2 = 2\pi/m_2$ etc., are, in general, quite independent of the pitch of the screw. The

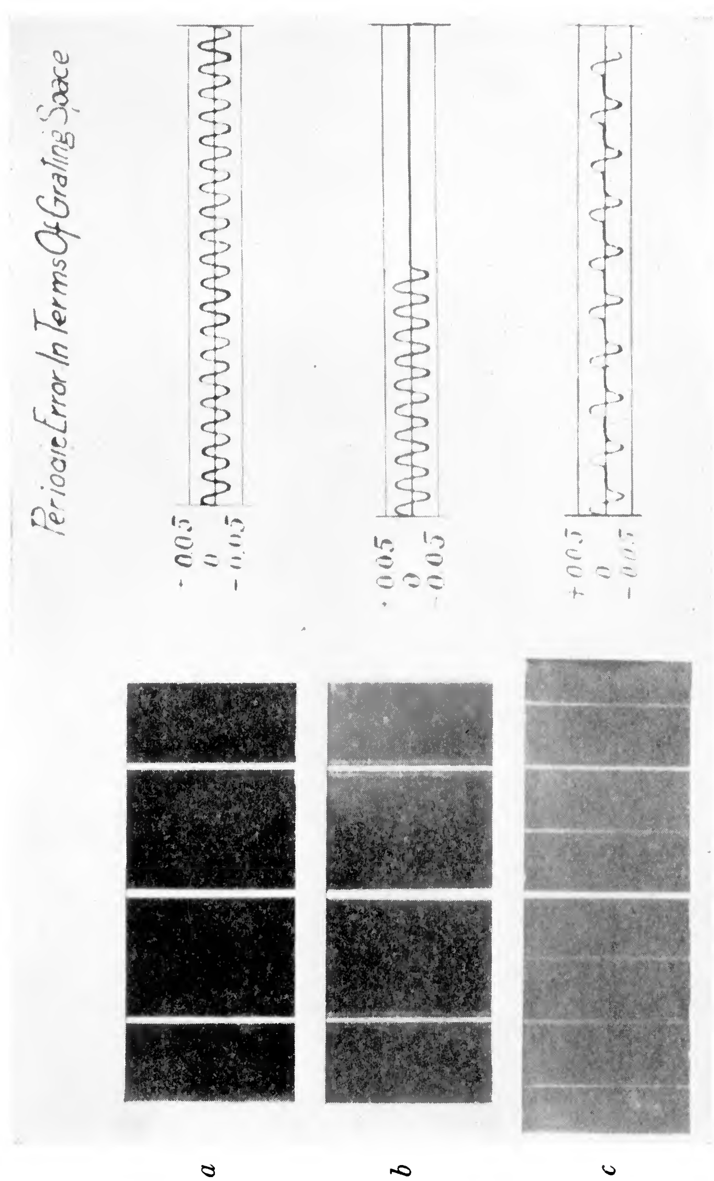

Fig. 7

number of lines, m, in a period may be fractional, and m_1 and m_2 may be incommensurable. In fact, if they are not (e.g., if $e_2 = 2e_1$), the situation becomes quite complicated, especially in the very important case of a single periodic error which is not sinusoidal. Here the natural procedure is to analyze the curve into Fourier components, and put $y = a_0 r + a_1 \sin e_1 r + a_2 \sin 2e_1 r + a_3 \sin 3e_1 r$, etc.; but even this represents the special case of a curve which is symmetrical about some point on it. The general case would demand $y = a_0 r + \Sigma a_t \sin t\theta + \Sigma b_t \cos t\theta$, or its equivalent $y = a_0 r + \Sigma a_t \sin (t\theta + \delta_t)$, where the δ's are arbitrary phase constants. The case of such a periodic error is complicated, and still more so when several such errors

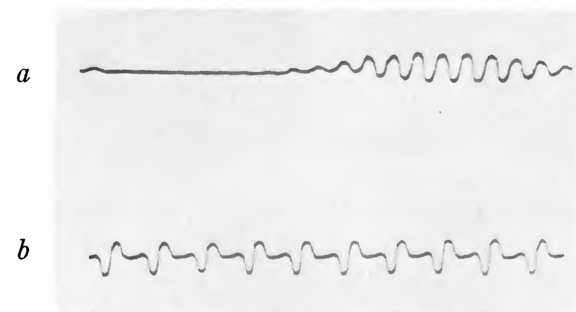

FIG. 8

are present simultaneously. Fortunately in good gratings the errors are small enough so that the first terms only in the Bessel series may be used as a sufficient approximation. Some gratings will be ruled with rather large errors of a few commensurable terms.

When numerous minor errors (probably largely typographical) are corrected, and the equations possibly extended somewhat, it is believed that Rowland's theoretical predictions will be verified.

In the Anniversary Volume dedicated to Professor K. Honda, Sendai, 1936, Siegbahn has recently published an article along lines similar to those of this article. He points out that Rowland's theory indicates that when there is an error a_1 in spacing equal to one-fifth of the wave-length of sodium light in the fourth order, and there are 500 rulings per millimeter, the first ghost will be as strong as the main line. He concludes, therefore, that interferometer methods are not sufficiently accurate to correct the compensator. We have found that we can rely on the interferometer readings to 0.1 fringe when the readings are recorded automatically and the mean is taken for several turns. With sufficient patience there is very little doubt that the errors could be determined to one-half this amount. We have used, for photographic recording of the fringes, the blue mercury line λ 4358. When the extreme range of the error curve is 1/10 fringe, $a_1 = 1/20$ fringe and $a_1/a_0 = 1/(20 \times 7.6)$ since there are about 7.6 fringes to a ruling at λ 4358 when ruling 6000 lines per centimeter. Under these conditions $2\pi N(a_1/a_0) = 0.124$.

$$J_1(0.124) = 0.0624 \,,$$

$$J_1^2(0.124) = 0.0039 = 0.39 \text{ per cent} \,.$$

We have regularly found the ghosts less than $\frac{1}{2}$ per cent in the third order. We have ruled a number of gratings in which the first ghost in the fourth order was less than $\frac{1}{2}$ per cent of the main line.

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