Problem Set 6

Due November 11th 11:59pm

Problem 1

1) Write a function that will shift an array by an arbitrary amount using a convolution (yes, I know there are easier ways to do this). The function should take 2 arguments - an array, and an amount by which to shift the array. Plot a gaussian that started in the centre of the array shifted by half the array length.

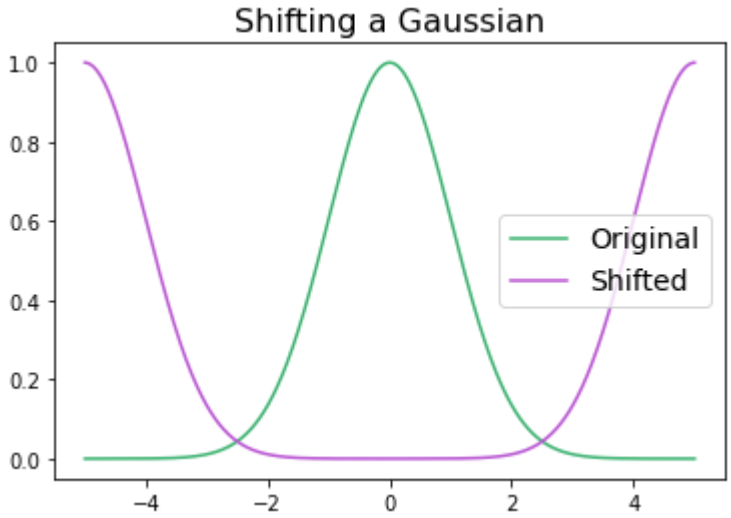
In [1]:
```
import numpy as np
import matplotlib.pyplot as plt

def shift(arr, shift):
    N = len(arr)
    yft = np.fft.fft(arr)
    k = np.arange(len(yft))
    r = np.exp(2*np.pi*1J*k*shift/N)
    yft_shift = yft*r
    y_shift = np.fft.ifft(yft_shift)
    return y_shift

n = 1000
x = np.linspace(-5, 5, n)
y = np.exp(-0.5*x**2)
s = shift(y, n/2)

plt.figure(figsize=(6,4))
plt.title("Shifting a Gaussian", fontsize=16)
plt.plot(x,y, color='mediumseagreen', label='Original')
plt.plot(x,s, color='mediumorchid', label='Shifted')
plt.legend(fontsize=14)
plt.show()
```

```
C:\Users\adesr\anaconda3\lib\site-packages\numpy\core\_asarray.py:102: ComplexW
arning: Casting complex values to real discards the imaginary part
  return array(a, dtype, copy=False, order=order)
```



This equation for r looks very similar to a DFT, except instead of x we have "shift", so when we multiply by our function in frequency space, we essentially just change our fourier transform to: $\sum f(x) \cdot exp(-2\pi ik(x + shift)/N)$, which shifts our function in frequency space, then all we have to do is take the reverse transform to get the shift that we want.

2) a)The correlation function $f \star g$ is $\int f(x)g(x+y)dx$. Through a similar proof, one can show $f \star g = ift(dft(f) * conj(dft(g)))$. Write a routine to take the correlation function of two arrays. Plot the correlation function of a Gaussian with itself.

b) using these results, write a routine to take the correlation function of a Gaussian (shifted by an arbitrary amount) with itself. How does the correlation function depend on the shift? Does this surprise you?

a)

In [2]:
```
import numpy as np
import matplotlib.pyplot as plt

def correlation(f, g):
    fft = np.fft.fft(f)
    gft = np.fft.fft(g)
    corrft = fft*np.conj(gft)
        #conjugate because we want the
        #correlation between f and g
    corr = np.fft.ifft(corrft)
    return corr

n = 1000
x = np.linspace(-5, 5, n)
y = np.exp(-0.5*x**2)
c = correlation(y, y)

plt.figure(figsize=(6,4))
plt.title("Correlation Function of Gaussian with itself", fontsize=16)
plt.plot(x,c, color='mediumorchid')
plt.show()
```

```
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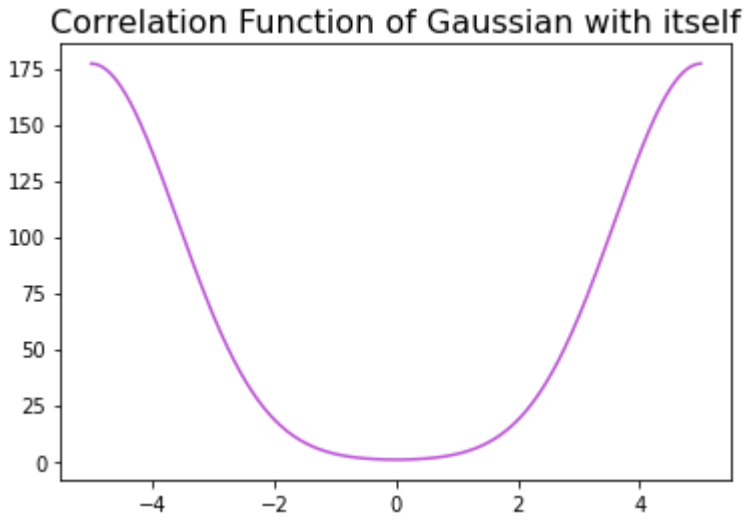

b)

In [3]:
```
#combination of the shift and correlation functions
def corr_shift(f, shift):
    N = len(f)
    yft = np.fft.fft(f)
    k = np.arange(len(yft))
    r = np.exp(2*np.pi*1J*k*shift/N)
    yft_shift = yft*r
    corrft = yft*np.conj(yft_shift)
    corr = np.fft.ifft(corrft)
    return corr

n = 1000
x = np.linspace(-5, 5, n)
y = np.exp(-0.5*x**2)
c = corr_shift(y, n/2)

plt.figure(figsize=(8,6))
plt.title("Correlation Function of Gaussian with shifted version of itself", font
for i in range(10):
    plt.plot(x, corr_shift(y, i*100), label='Shifted by {}'.format(i*100))
    plt.legend()
plt.show()
```

```
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```

How does the correlation function depend on the shift?

The correlation of a gaussian with itself will simply give another gaussian, and as we can see from the plot above, the correlation of a gaussian with a shifted version of itself just shifts the gaussian resulting from the correlation. If we have 0 shift, the resulting gaussian from the correlation is split in half on each side of the range, whereas if we have a shift of N/2, the resulting gaussian is shifted to the middle of the range.

Does this surprise you?

No, this is what we should expect since correlating a gaussian with another gaussian that is shifted will necessarily shift the resulting gaussian from the correlation, and the amount of that shift is dependant on the amount we shift our original gaussian.

3) The circulant (wrap-around) nature of the dft can sometimes be problematic. Write a routine to take an FFT-based convolution of two arrays without any danger of wrapping around. You may wish to add zeros to the end of the input arrays.

In [4]:
```
import numpy as np
import matplotlib.pyplot as plt
from math import floor

def conv(f, g):
    zeros_fft = np.fft.fft(np.append(f,np.zeros(len(f))))
                        #adding zeros to the end of arrays
    zeros_gft = np.fft.fft(np.append(g,np.zeros(len(g))))
    conv = np.fft.ifft(zeros_fft*zeros_gft) #no complex
                            #conjugate because we're taking
                            #the convolution not correlation
    c = conv[:floor(len(conv)/2)] #we then need only take
                            #half of our convolution since
                            #we doubled the length of our f
                            #and g arrays by adding 0s
    return c

n = 1000
x = np.linspace(-5, 5, int(n/2))
x2 = np.linspace(-5, 5, n)
y = np.exp(-0.5*x**2)
y2 = x**2
c = conv(y, y2)

plt.figure(figsize=(8,6))
plt.plot(x,c)
plt.title('Correlation of gaussian with quadratic', fontsize=16)
plt.show()
```

```
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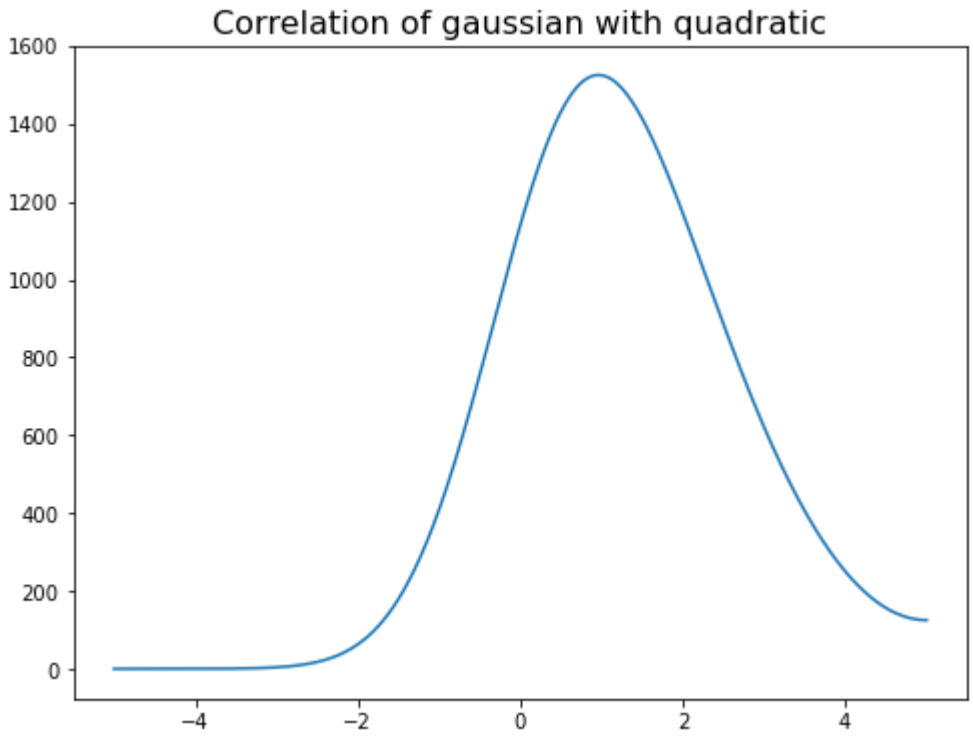

4) DFTs work very nicely out of the box when there are an integer number of periods of a wave in the region analyzed. Sadly, when we are dealing with real data, we usually are forced to analyze a finite chunk of data, and there will in general be no particular relation between the frequencies in the data and the interval we're analyzing. We'll look at the effects of this a bit now.

a) Show that:

$$\sum_{x=0}^{N-1} \exp(-2\pi i k x/N) = \frac{1-\exp(-2\pi i k)}{1-\exp(-2\pi i k/N)}$$

It may help to recognize that the sum can be re-written $\sum \alpha^x$ where $\alpha = \exp(-2\pi i k/N)$ so we can treat it as the sum of a geometric series.

b) Show that this approaches N as k approaches zero, and is zero for any integer k that is not a multiple of N.

c) We can use this to analytically write down the DFT of a non-integer sine wave. Pick a non-integer value of k and plot your analytic estimate of the DFT. Show that the FFT agrees (to within machine precision) with your analytic estimate. Normally, we think of the Fourier transform of a pure sine wave to be a delta function. Are we close to that? This phenomenon is usually known as *spectral leakage*.

d) A common tool to get around this is the use of *window functions*. The leakage essentially comes from the fact that we have a sharp jump at the edge of the interval. If we multiply our input data by a function that goes to zero at the edges, this cancels out the jump, and so prevents the leakage from the jumps at the edges. Of course, since we have multiplied by the window in real space, we have convolved by it in Fourier space. One simple window we could use is $0.5 - 0.5\cos(2\pi x/N)$ (there are many, many choices). Show that when we multiply by this window, the spectral leakage for a non-integer period sine wave drops dramatically.

e) Show that the Fourier transform of the window is $[N/2\ N/4\ 0\ ...\ 0\ N/4]$ (either numerically or analytically). Use this to show that you can get the windowed Fourier transform by appropriate combinations of each point in the unwindowed Fourier transform and its immediate neighbors (you may need to be careful with signs here, since if you work through the math, some of the transforms need to be inverse FFTs). The choice of suitable windows is as much art as science (and depends on the details of what you're most concerned about), but I hope this gives at flavor of what's going on and will be useful for matched filter results.

a)

For geometric series, we know that the solution is:

$\sum_{x=0}^{N-1} ar^k = a(\frac{1-r^N}{1-r})$ for r≠1

So to apply it to our equation ($exp(-2\pi ikx/N)$), we have:

a = 1, r = $exp(-2\pi ik/N)$ and k = x, so we can solve:

$$\sum_{x=0}^{N-1} exp(-2\pi ikx/N) = 1 \cdot (\frac{1-exp(-2\pi ik/N)^N}{1-exp(-2\pi ik/N)}) = \frac{1-exp(-2\pi ik)}{1-exp(-2\pi ik/N)}$$

b)

We can show that our sum approaches N as k approaches 0:

$$\sum_{x=0}^{N-1} exp(-2\pi ikx/N) = \sum_{x=0}^{N-1} exp(-2\pi i \cdot 0 \cdot x/N) = \sum_{x=0}^{N-1} exp(0) = \sum_{x=0}^{N-1} 1 = N$$

c)

The DFT of some function f(x) is written as:

$$\sum_{x=0}^{N-1} f(x) \cdot exp(-2\pi ikx/N)$$

In our case, the function we want to use is a non integer sine wave, which can be written as: $sin(k_{ni}x) = \frac{exp(ik_{ni}x)-exp(-ik_{ni}x)}{2i}$, where k_{ni} is a non-integer.

So our DFT becomes:

$$\sum_{x=0}^{N-1} sin(k_{ni}x) \cdot exp(-2\pi ikx/N) = \sum_{x=0}^{N-1} (\frac{exp(ik_{ni}x)-exp(-ik_{ni}x)}{2i}) \cdot exp(-2\pi ikx/N) =$$

$$\sum_{x=0}^{N-1} \frac{1}{2i} exp(ik_{ni} - 2\pi ik/N)^x - \sum_{x=0}^{N-1} \frac{1}{2i} exp(-ik_{ni} - 2\pi ik/N)^x$$

Which by what we found in a), is equal to:

$$\frac{1}{2i} \cdot \frac{1-exp(ik_{ni}N-2\pi ik)}{1-exp(ik_{ni}-2\pi ik/N)} - \frac{1}{2i} \cdot \frac{1-exp(-ik_{ni}N-2\pi ik)}{1-exp(-ik_{ni}-2\pi ik/N)}$$

In [5]:
```
import numpy as np
import matplotlib.pyplot as plt

def analytic_sine(kni, N):
    k = np.arange(N)
    analytic = (1/2J)*(1-np.exp(1J*kni*N-2*np.pi*1J*k))/(1-np.exp(1J*kni-2*np.pi*
    return analytic

k_ni = 11.5
n = 1000
x = np.arange(n)
y = analytic_sine(k_ni, n)
y2 = np.fft.fft(np.sin(k_ni*x))

plt.figure(figsize=(12,4))
plt.subplot(1,2,1)
plt.title('Analytic DFT of Sine Wave')
plt.plot(y)
plt.subplot(1,2,2)
plt.title('FFT of Sine Wave')
plt.plot(y2)
plt.show()
```

```
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```
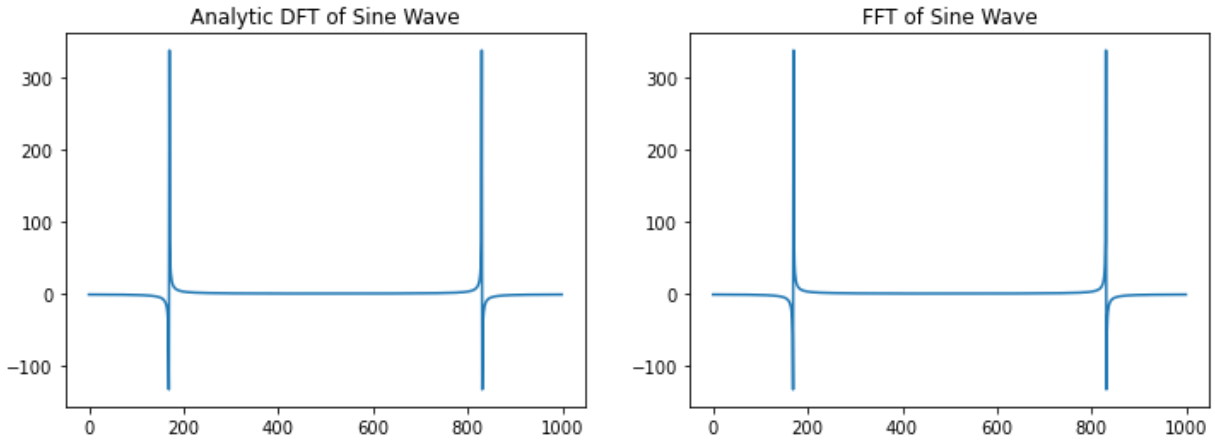
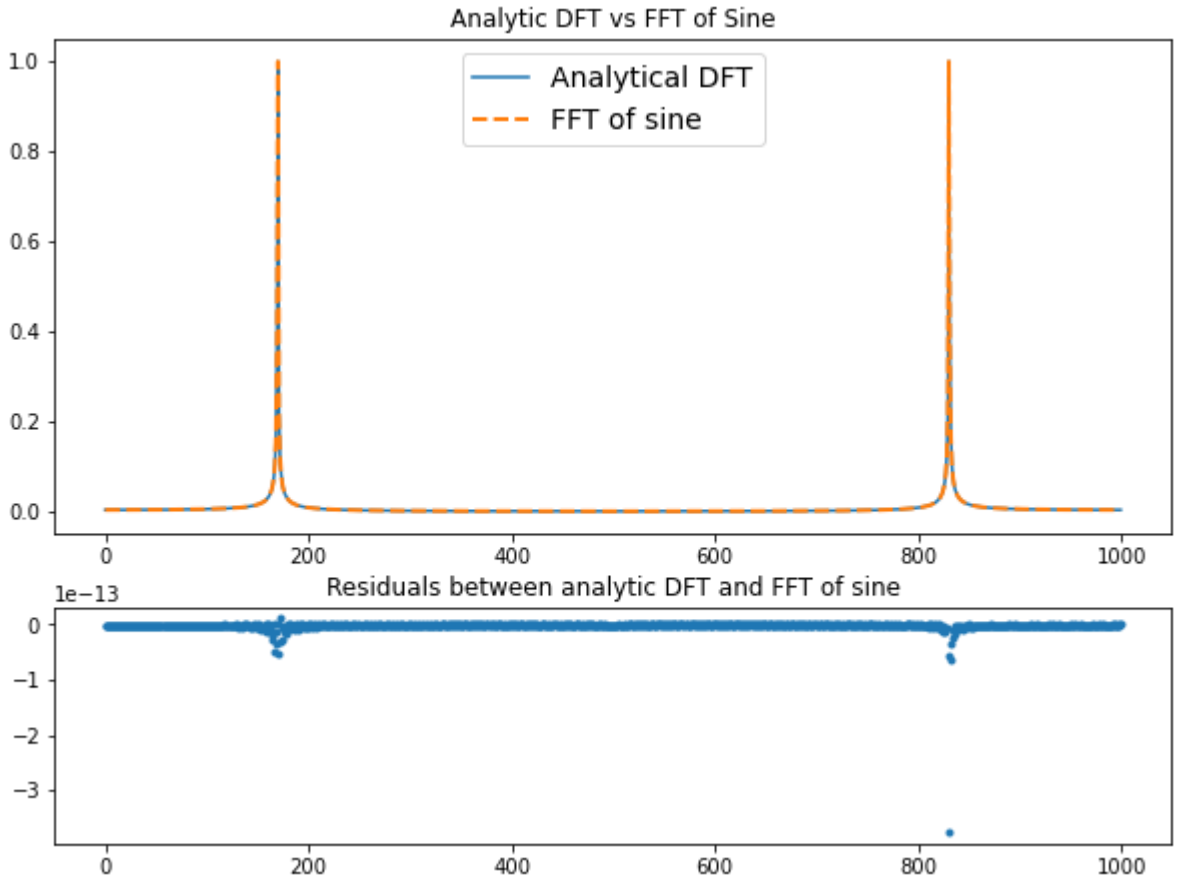



Since we know that the fourier transform of a sine wave is a delta function, we can take the absolute value of our analytic DFT and our FFT to more closely resemble a delta function.

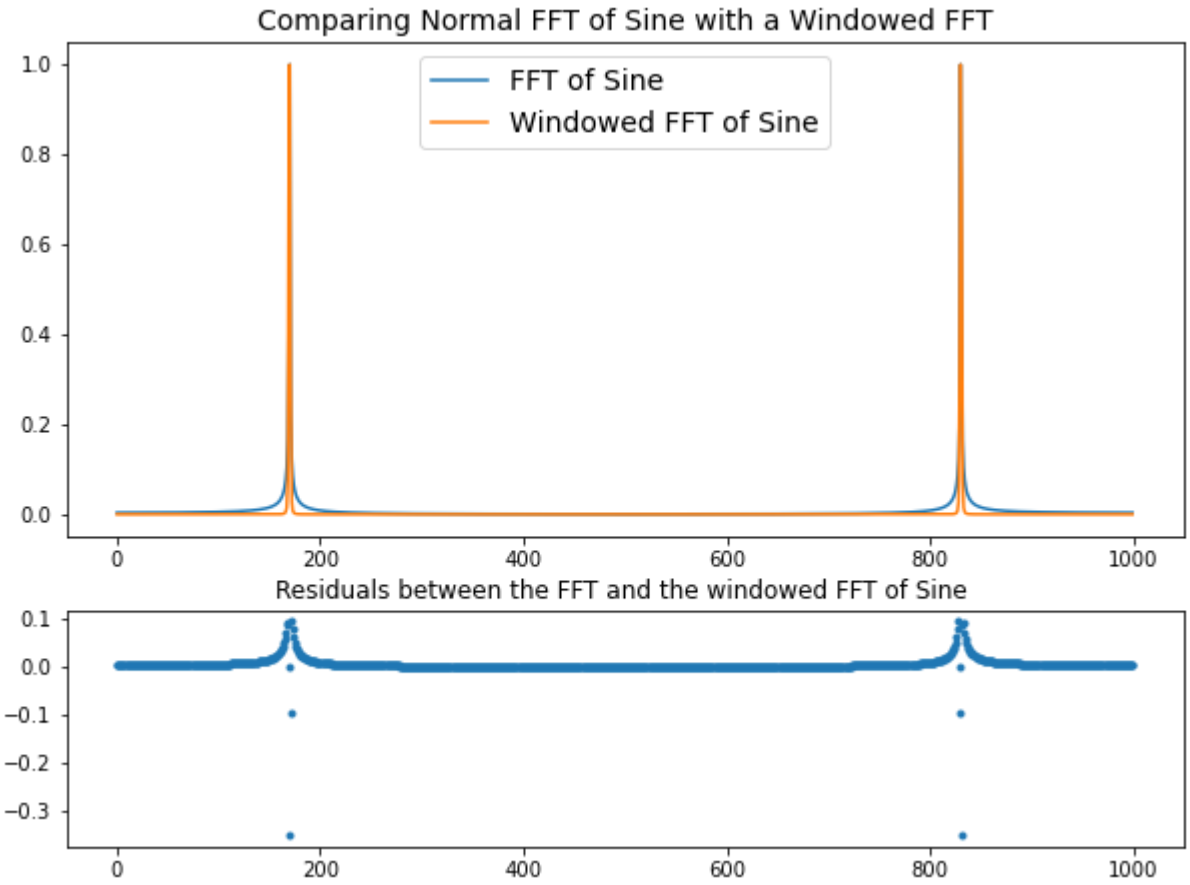
In [6]:
```
plt.figure(figsize=(10,10))
plt.subplot(2,1,1)
plt.title('Analytic DFT vs FFT of Sine')
plt.plot(abs(y)/max(abs(y)), label='Analytical DFT')
plt.plot(abs(y2)/max(abs(y2)), linestyle='--', linewidth=2, label='FFT of sine')
plt.legend(fontsize=14)
plt.subplot(4,1,3)
plt.title('Residuals between analytic DFT and FFT of sine')
plt.plot(abs(y)/max(abs(y))-abs(y2)/max(abs(y2)), '.')
plt.show()
```



Now we can see our Fourier transform resembles 2 delta functions, but the peaks are wider than a delta function, which is caused by spectral leakage. The residuals between the analytic DFT and FFT of sine show that the two methods agree to within machine precision.

d)

In [7]:
```
k_ni = 11.5
n = 1000
x = np.arange(n)
y = np.sin(k_ni*x)
window = 0.5-0.5*np.cos(2*np.pi*x/n)
y_window = y*window
yft = np.fft.fft(y)
yft_window = np.fft.fft(y_window)

plt.figure(figsize=(10,10))
plt.subplot(2,1,1)
plt.title('Comparing Normal FFT of Sine with a Windowed FFT', fontsize=14)
plt.plot(abs(yft)/max(abs(yft)), label='FFT of Sine')
plt.plot(abs(yft_window)/max(abs(yft_window)), label='Windowed FFT of Sine')
plt.legend(fontsize=14)
plt.subplot(4,1,3)
plt.plot(abs(yft)/max(abs(yft))-abs(yft_window)/max(abs(yft_window)), '.')
plt.title('Residuals between the FFT and the windowed FFT of Sine ')
plt.show()
```

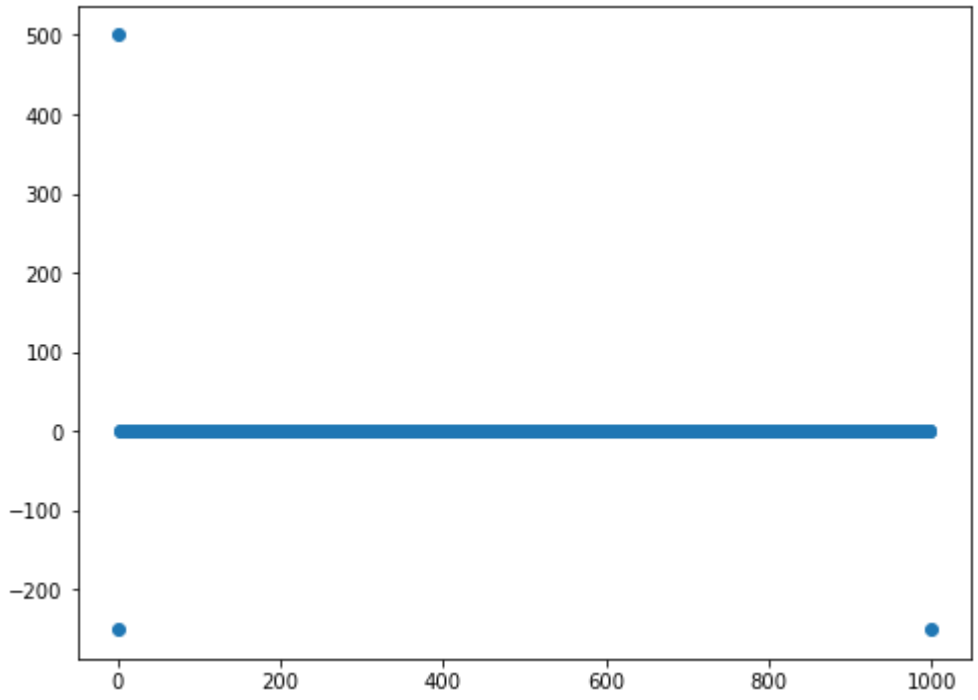


It's clear to see by the difference in widths of the peaks how much more the windowed function resembles a delta function than our normal FFT.

e)

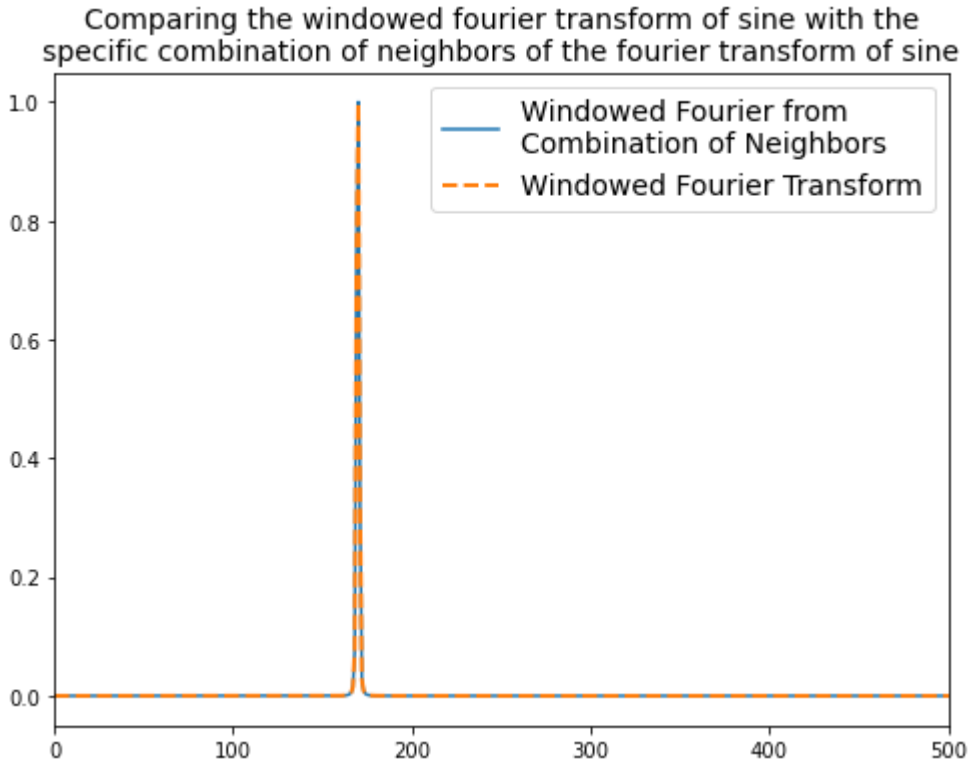
In [8]:
```
wft = np.fft.fft(window)

for i in range(len(wft)):
    if abs(wft[i].real)>1e-13:
        print('At index {} the FFT of the window function is equal to {}.'.format

plt.figure(figsize=(8,6))
plt.plot(wft.real, 'o')
plt.show()
```

```
At index 1 the FFT of the window function is equal to 500.0.
At index 2 the FFT of the window function is equal to -250.0.
At index 1000 the FFT of the window function is equal to -250.0.
```



As we can see from the plot above, the window function is equal to [500,-250,0...0,-250], which is [N/2,-N/4,0...0,-N/4] since our value for N in this case is 1000.

In [9]:
```
k_ni = 11.5
n = 1000
x = np.arange(n)
y = np.sin(k_ni*x)
yft = np.fft.fft(y)

win_four = np.copy(yft)
for i in range(n-1):
    #adding in the weights that we just determined,
    #i.e. [N/2,-N/4,0...0,-N/4]
    win_four[i] = (n/2)*yft[i]-(n/4)*yft[i+1]-(n/4)*yft[i-1]
win_four[-1] = (n/2)*yft[-1]-(n/4)*yft[0]-(n/4)*yft[-2]

plt.figure(figsize=(8,6))
plt.title('Comparing the windowed fourier transform of sine with the\nspecific co
plt.plot(abs(win_four)/max(abs(win_four)), label='Windowed Fourier from \nCombina
plt.plot(abs(yft_window)/max(abs(yft_window)), linestyle='--', linewidth=2,label=
plt.xlim(0,500)
plt.legend(fontsize=14)
plt.show()
```



?

5) Matched Filter of LIGO data

We are going to find gravitational waves! Key will be getting LIGO data from github:

`https://github.com/losc-tutorial/LOSC_Event_tutorial`

While they include code to do much of this, please don't use it (although you may look at it for inspiration) and instead write your own. You can look at/use simple_read_ligo.py that I have posted for concise code to read the hdf5 files. Feel free to have your code loop over the events and print the answer to each part for that event. In order to make our life easy, in case we have to re-run your code (which we should not have to do), please also have a variable at the top of your code that sets the directory where you have unzipped the data. LIGO has two detectors (in Livingston, Louisiana, and Hanford, Washington) and GW events need to be seen by both detectors to be considered real. Note that my read_template functon returns the templates for both the plus and cross polarizations, but for simplicity you can just pick one of them to work with.

a) Come up with a noise model for the Livingston and Hanford detectors separately. Describe in comments how you go about doing this. Please mention something about how you smooth the power spectrum and how you deal with lines (if at all). Please also explain how you window the data (you may want to use a window that has an extended flat period near the center to avoid tapering the data/template where the signal is not small).

b) Use that noise model to search the four sets of events using a matched filter. The mapping between data and templates can be found in the file BBH_events_v3.json, included in the zipfile.

c) Estimate a noise for each event, and from the output of the matched filter, give a signal-to-noise ratio for each event, both from the individual detectors, and from the combined Livingston + Hanford events.

d) Compare the signal-to-noise you get from the scatter in the matched filter to the analytic signal-to-noise you expect from your noise model. How close are they? If they disagree, can you explain why?

e) From the template and noise model, find the frequency from each event where half the weight comes from above that frequency and half below.

f) How well can you localize the time of arrival (the horizontal shift of your matched filter). The positions of gravitational wave events are inferred by comparing their arrival times at different detectors. What is the typical positional uncertainy you might expect given that the detectors area a few thousand km apart?

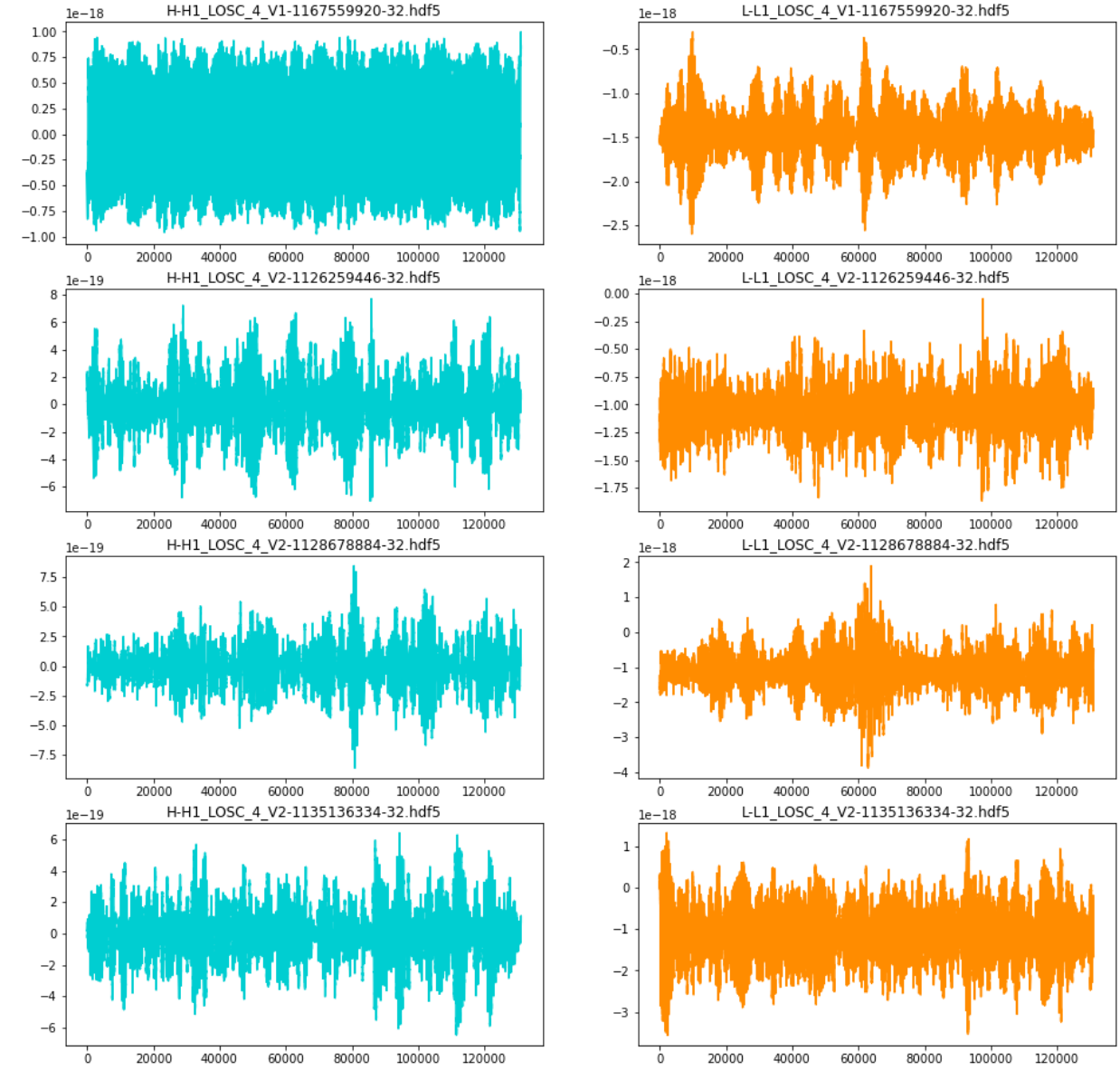
Loading all the files and templates:

In [10]:
```
import h5py

def read_template(filename):
    dataFile=h5py.File(filename,'r')
    template=dataFile['template']
    tp=template[0]
    tx=template[1]
    return tp,tx
def read_file(filename):
    dataFile=h5py.File(filename,'r')
    dqInfo = dataFile['quality']['simple']
    qmask=dqInfo['DQmask'][...]

    meta=dataFile['meta']
    #gpsStart=meta['GPSstart'].value
    gpsStart=meta['GPSstart'][()]
    #print meta.keys()
    #utc=meta['UTCstart'].value
    utc=meta['UTCstart'][()]
    #duration=meta['Duration'].value
    duration=meta['Duration'][()]
    #strain=dataFile['strain']['Strain'].value
    strain=dataFile['strain']['Strain'][()]
    dt=(1.0*duration)/len(strain)

    dataFile.close()
    return strain,dt,utc
```

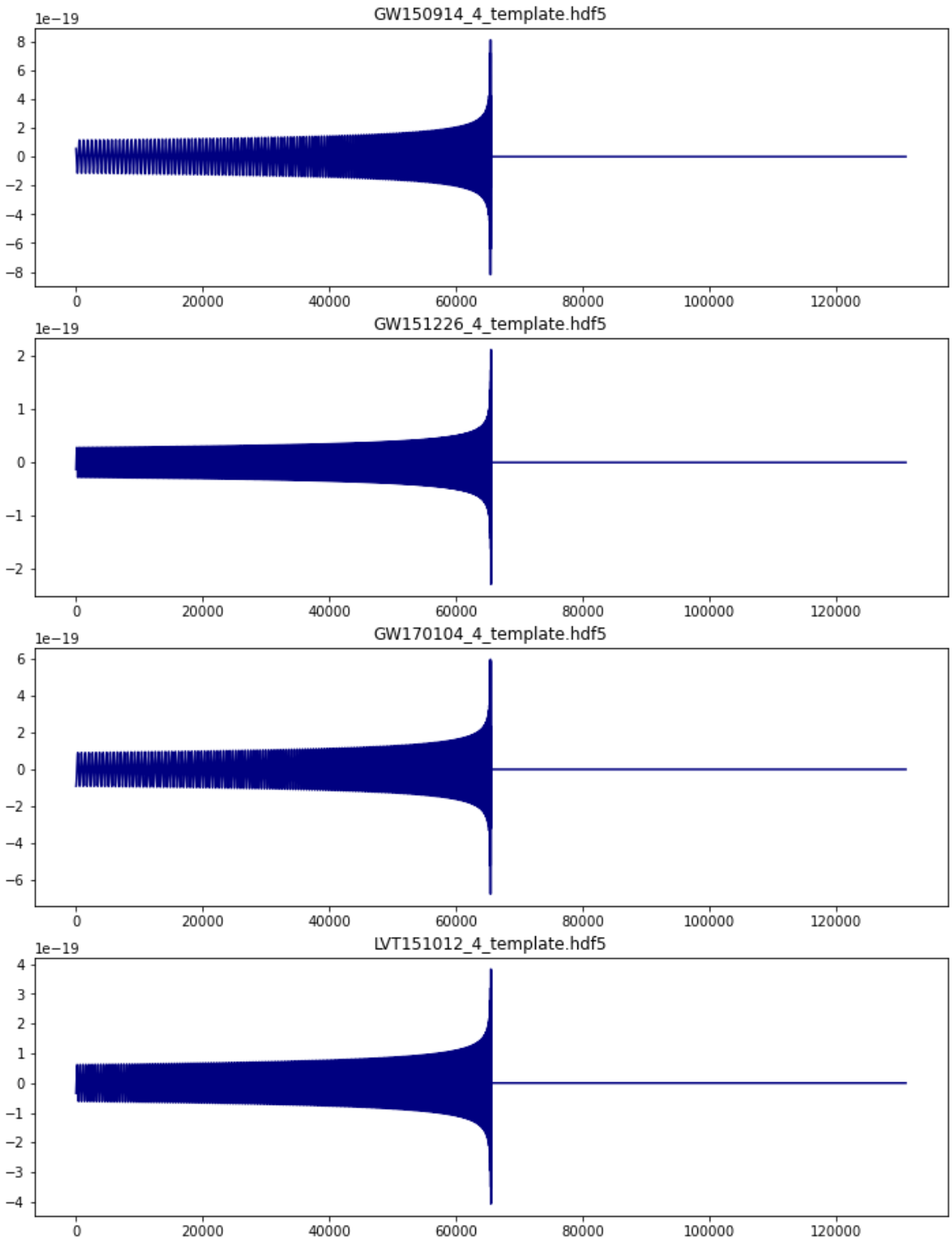
In [11]:
```
directory = 'ligo/'

Hanford_files = [0,0,0,0]
Livingston_files = [0,0,0,0]
templates = [0,0,0,0]

Hanford_files[0] = read_file('{}H-H1_LOSC_4_V1-1167559920-32.hdf5'.format(directo
Hanford_files[1] = read_file('{}H-H1_LOSC_4_V2-1126259446-32.hdf5'.format(directo
Hanford_files[2] = read_file('{}H-H1_LOSC_4_V2-1128678884-32.hdf5'.format(directo
Hanford_files[3] = read_file('{}H-H1_LOSC_4_V2-1135136334-32.hdf5'.format(directo

Livingston_files[0] = read_file('{}L-L1_LOSC_4_V1-1167559920-32.hdf5'.format(dire
Livingston_files[1] = read_file('{}L-L1_LOSC_4_V2-1126259446-32.hdf5'.format(dire
Livingston_files[2] = read_file('{}L-L1_LOSC_4_V2-1128678884-32.hdf5'.format(dire
Livingston_files[3] = read_file('{}L-L1_LOSC_4_V2-1135136334-32.hdf5'.format(dire

templates[0] = read_template('{}GW150914_4_template.hdf5'.format(directory))
templates[1] = read_template('{}GW151226_4_template.hdf5'.format(directory))
templates[2] = read_template('{}GW170104_4_template.hdf5'.format(directory))
templates[3] = read_template('{}LVT151012_4_template.hdf5'.format(directory))

print(templates[0])
```

```
(array([5.5410477e-20, 5.4439373e-20, 5.3463174e-20, ..., 0.0000000e+00,
       0.0000000e+00, 0.0000000e+00], dtype=float32), array([1.00204446e-19, 1.
00735800e-19, 1.01257744e-19, ...,
       0.00000000e+00, 0.00000000e+00, 0.00000000e+00], dtype=float32))
```

In [12]:
```
import numpy as np
import matplotlib.pyplot as plt

Hanford_names = ['H-H1_LOSC_4_V1-1167559920-32.hdf5', 'H-H1_LOSC_4_V2-1126259446-
Livingston_names = ['L-L1_LOSC_4_V1-1167559920-32.hdf5', 'L-L1_LOSC_4_V2-11262594

plt.figure(figsize=(16,16))

for i in range(4):
    strain, dt, utc = Hanford_files[i]
    plt.subplot(4,2,2*i+1)
    plt.plot(strain, color='darkturquoise')
    plt.title(Hanford_names[i])
for i in range(4):
    strain, dt, utc = Livingston_files[i]
    plt.subplot(4,2,2*i+2)
    plt.plot(strain, color='darkorange')
    plt.title(Livingston_names[i])
```

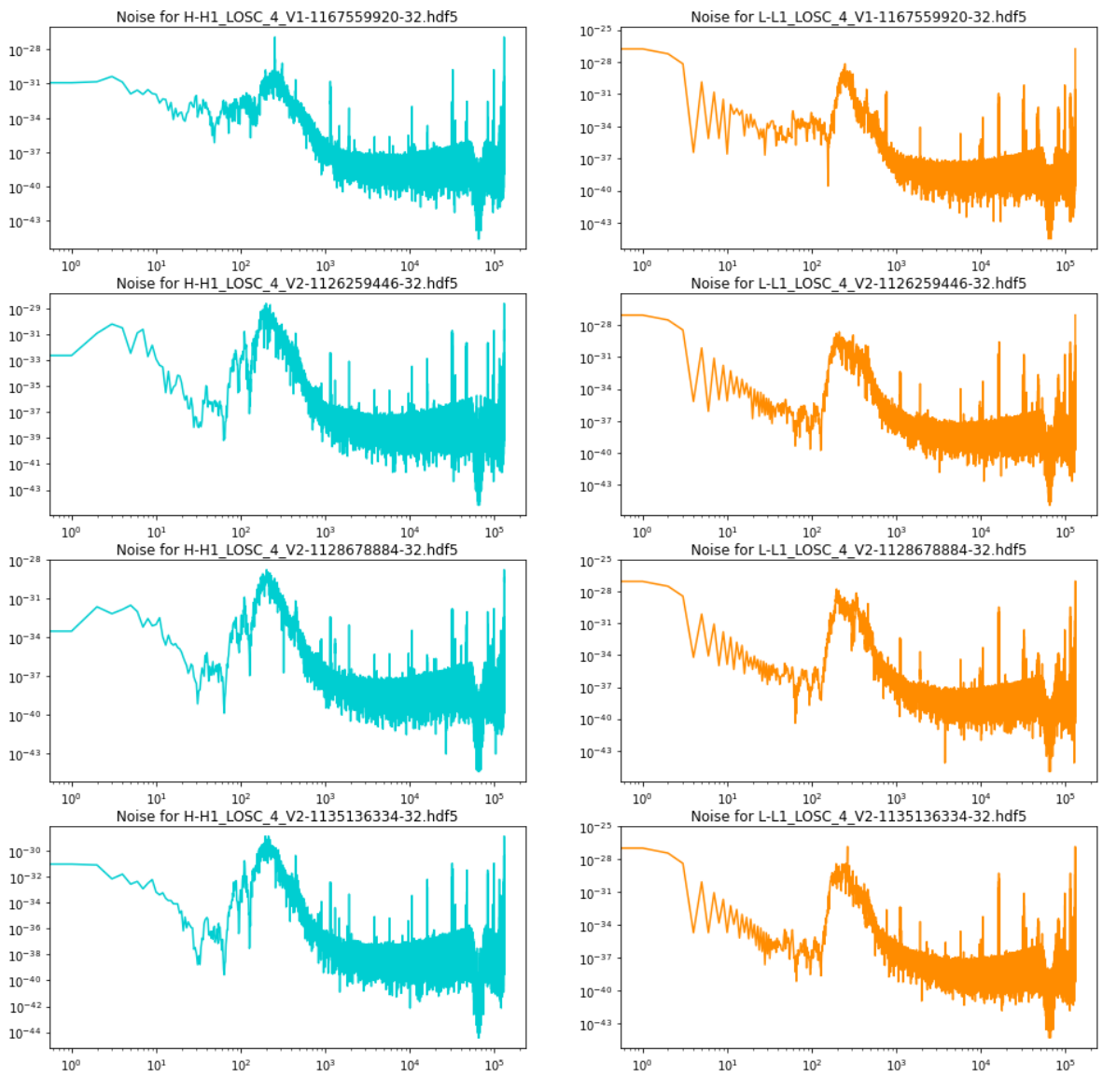


In [13]:
```
template_names = ['GW150914_4_template.hdf5', 'GW151226_4_template.hdf5', 'GW1701

plt.figure(figsize=(12,16))

for i in range(4):
    tp, tx = templates[i]
    plt.subplot(4,1,i+1)
    plt.plot(tp, color='navy')
    plt.title(template_names[i])
```



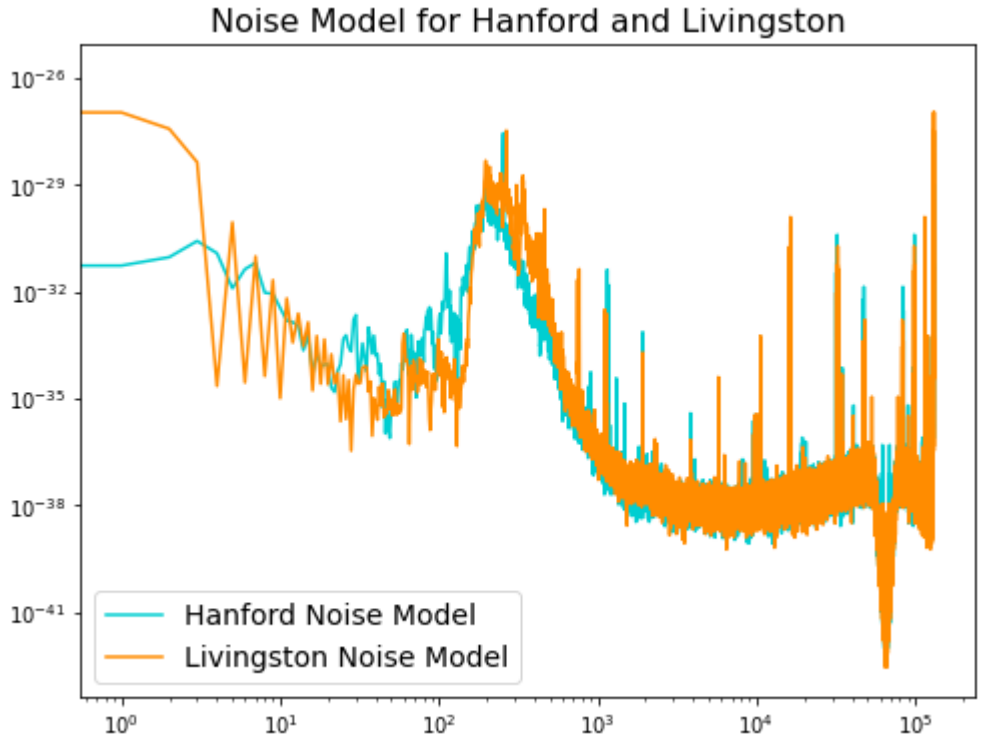
a)

In [14]:
```
import scipy.signal.windows as win

Hanford_noises = [0,0,0,0]
Livingston_noises = [0,0,0,0]

plt.figure(figsize=(16,16))
for i in range(4):
    strain, dt, utc = Hanford_files[i]
    window = win.tukey(len(strain))
    #tukey is a window function that is
    #flat in the middle
    noise_ft = np.fft.fft(window*strain)
    Hanford_noises[i] = np.abs(noise_ft)**2
    plt.subplot(4,2,2*i+1)
    plt.loglog(Hanford_noises[i], color='darkturquoise')
    plt.title('Noise for '+Hanford_names[i])
for i in range(4):
    strain, dt, utc = Livingston_files[i]
    window = win.tukey(len(strain))
    noise_ft = np.fft.fft(window*strain)
    Livingston_noises[i] = np.abs(noise_ft)**2
    plt.subplot(4,2,2*i+2)
    plt.loglog(Livingston_noises[i], color='darkorange')
    plt.title('Noise for '+Livingston_names[i])
```



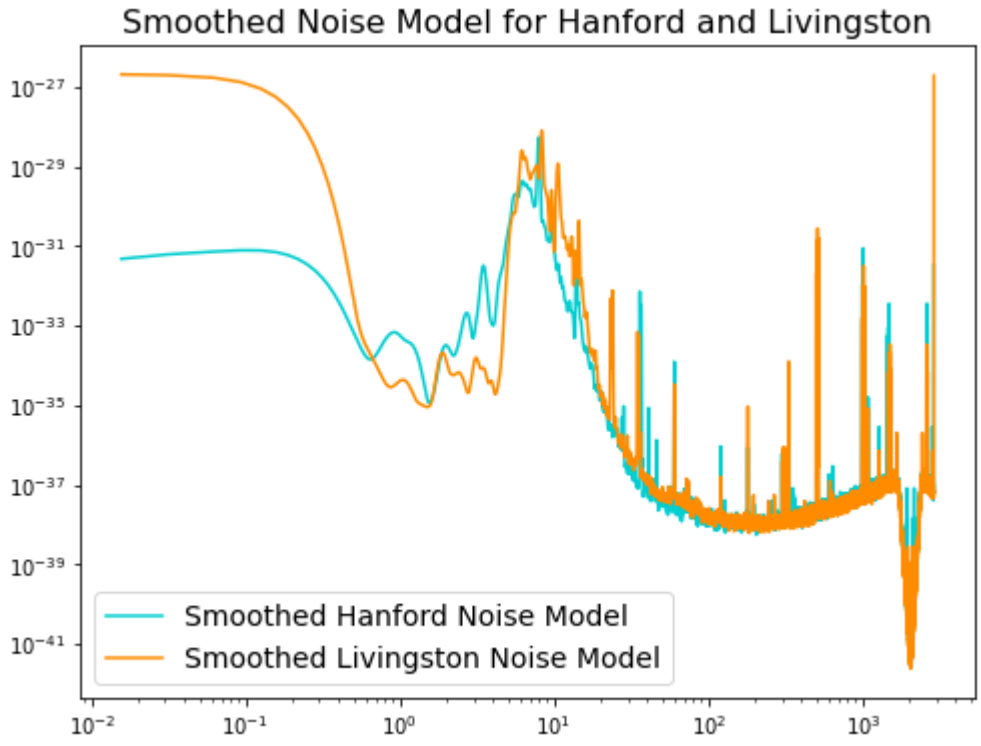
We estimate the noise to simply be the square of the input for each event. This comes from the fact that our detector is almost only outputting noise throughout the event since the gravitational wave detection is so short. We can then average over the 4 events we have for each detector to get an accurate noise model, which can be seen below.

In [15]:
```
#averaging over the noise for the 4 instances
Hanford_noise = np.mean(Hanford_noises, axis=0)
Livingston_noise = np.mean(Livingston_noises, axis=0)

plt.figure(figsize=(8,6))
plt.title('Noise Model for Hanford and Livingston', fontsize=16)
plt.loglog(Hanford_noise, color='darkturquoise', label='Hanford Noise Model')
plt.loglog(Livingston_noise, color='darkorange', label='Livingston Noise Model')
plt.legend(fontsize=14)
plt.show()
```



We then need to smooth our noise input, which can be done by convolving the noise with a gaussian kernel which helps us get rid of a lot of our spikes in the noise, effectively dampening out the small fluxuations.

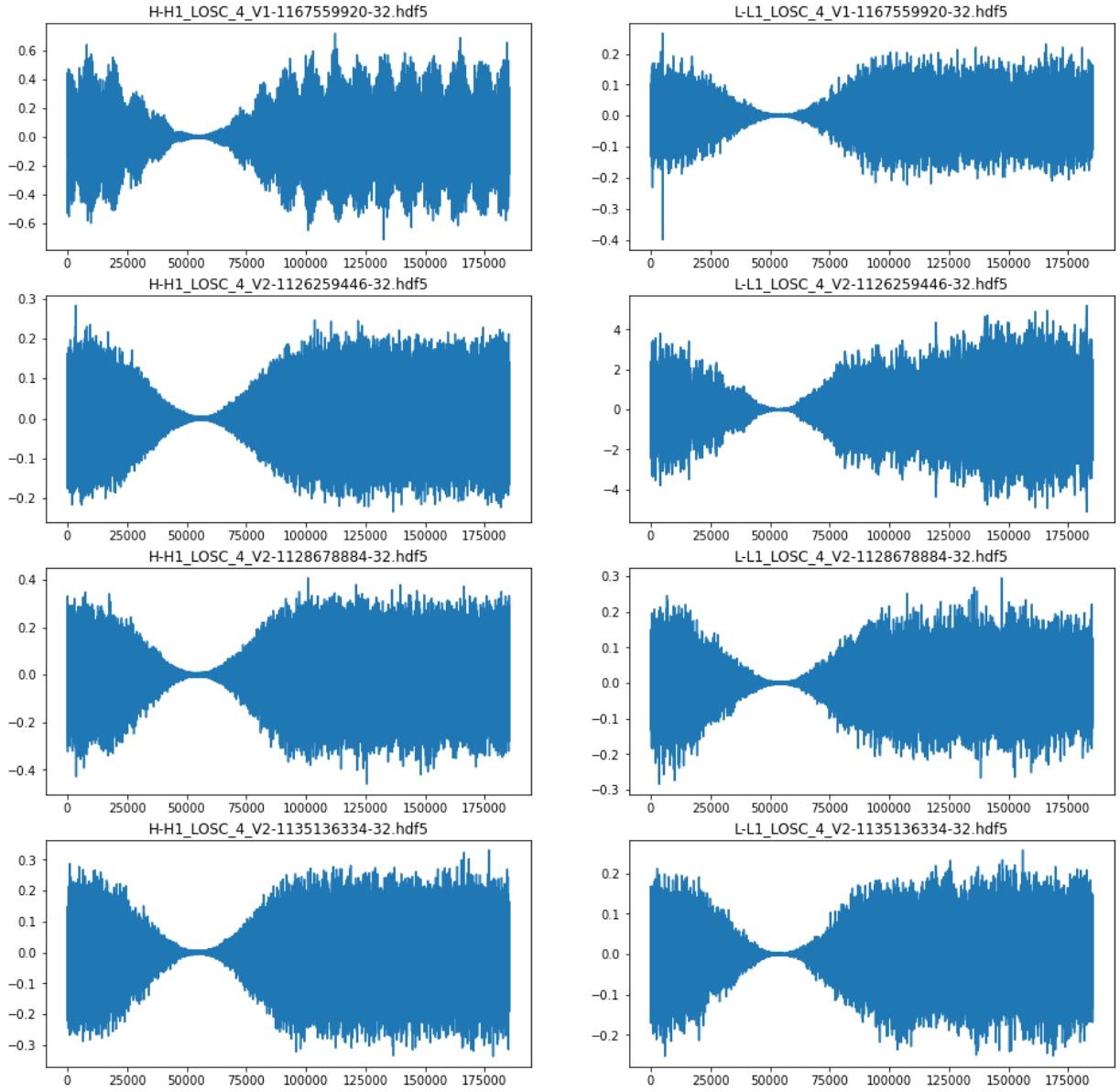
In [16]:
```
def smooth_vector(vec,sig):
    n=len(vec)
    x=np.arange(n)
    x[n//2:]=x[n//2:]-n
    kernel=np.exp(-0.5*x**2/sig**2) #make a Gaussian kernel
    kernel=kernel/kernel.sum()
    vecft=np.fft.rfft(vec)
    kernelft=np.fft.rfft(kernel)
    vec_smooth=np.fft.irfft(vecft*kernelft) #convolve the data with the kernel
    return vec_smooth

sigma = 3
Hanford_smooth = smooth_vector(np.abs(Hanford_noise)[:-38392], sigma)
                                #Have to remove some of the noise model
                                #from the end to avoid the sum in the
                                #smooth_vector function to go to inf
Livingston_smooth = smooth_vector(np.abs(Livingston_noise)[:-38392], sigma)

tobs=dt*len(strain)
dnu=1/tobs
nu=np.arange(len(Hanford_smooth))*dnu
nu[0]=0.5*nu[1]

Hanford_Ninv=1/Hanford_smooth
Hanford_Ninv[nu>1500]=0
Hanford_Ninv[nu<20]=0

Livingston_Ninv=1/Livingston_smooth
Livingston_Ninv[nu>1500]=0
Livingston_Ninv[nu<20]=0

plt.figure(figsize=(8,6))
plt.title('Smoothed Noise Model for Hanford and Livingston', fontsize=16)
plt.loglog(nu,np.abs(Hanford_smooth), color='darkturquoise', label='Smoothed Hanf
plt.loglog(nu,np.abs(Livingston_smooth), color='darkorange', label='Smoothed Livi
plt.legend(fontsize=14)
plt.show()
```


Smoothed Noise Model for Hanford and Livingston
Smoothed Hanford Noise Model
Smoothed Livingston Noise Model

b)

In [17]:

```
plt.figure(figsize=(16,16))
for i in range(4):
    strain, dt, utc = Hanford_files[i]
    tp, tx = templates[i]

    strain = strain[:-38392]
    tp = tp[:-38392]

    window = win.tukey(len(strain))
    template_ft=np.fft.fft(tp*window)
    template_filt=template_ft*Hanford_Ninv
    data_ft=np.fft.fft(strain*window)
    rhs=np.fft.irfft(data_ft*np.conj(template_filt))
    plt.subplot(4,2,2*i+1)
    plt.plot(rhs)
    plt.title(Hanford_names[i])

for i in range(4):
    strain, dt, utc = Livingston_files[i]
    tp, tx = templates[i]

    strain = strain[:-38392]
    tp = tp[:-38392]

    window = win.tukey(len(strain))
    template_ft=np.fft.fft(tp*window)
    template_filt=template_ft*Livingston_Ninv
    data_ft=np.fft.fft(strain*window)
    rhs=np.fft.irfft(data_ft*np.conj(template_filt))
    plt.subplot(4,2,2*i+2)
    plt.plot(rhs)
    plt.title(Livingston_names[i])

plt.show()
```



I think the detection isn't working because the gravitational wave was measured in the chunck of data that I got rid of, but I'm not sure how to keep that section of data without the smoothing function not working.