Problem Set 7

Due Friday November 18th at 11:59pm

Problem 1

Problem 1: As I warned (and showed) in class, there are many, many deeply flawed pseudo-random number generators out there. One widely found version is the default random number generator in the C standard library. Look at test_broken_libc.py - this shows how to wrap the C standard library in python, call its random number generator, and save the output. (Note - the numba wrapper is there for speed but is not required.) I've used it to generate random (x,y,z) positions with coordinates between 0 and 2^{31} (the max random integer value in the standard library). The type of PRNG used in the library is notorious for introducing correlations between sequential points, with sets of points in $n-$dimensional space lying on a surprisingly small number of planes.

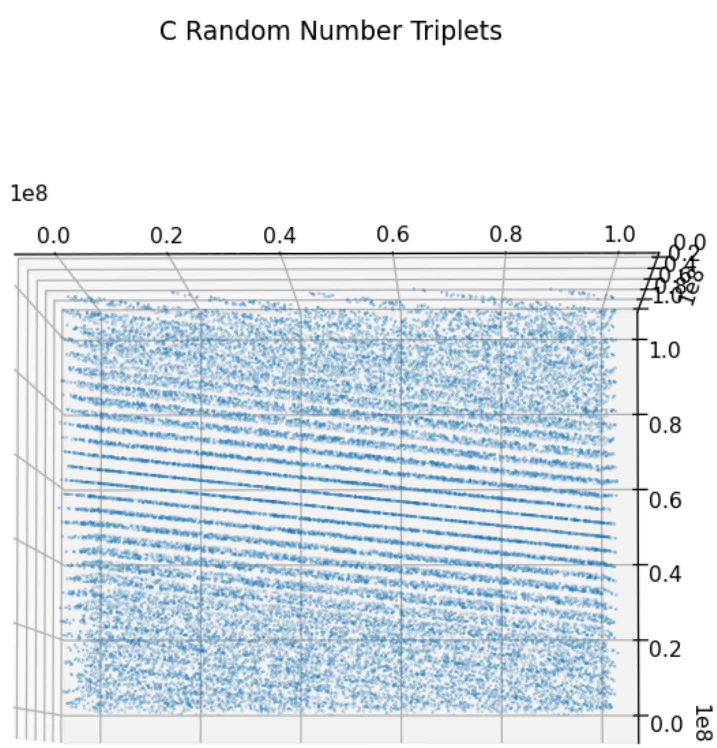
To make this effect easier to see, I've pulled out all the (x,y,z) triples with $0 < x, y, z < 10^8$ (so about 5% of the total span) and put them in the text file rand_points.txt. Show that when correctly viewed, these triples lie along a set of planes (I get about 30) and so are very much not randomly distributed in 3D space. You can either do this by changing the view angle on a 3D plot, or plotting $ax + by, z$ for suitably chosen a and b. Do you see the same thing happen with python's random number generator? If possible, can you see the same effect on your local machine? You may need to change the name of the library in the line:

```
mylib=ctypes.cdll.LoadLibrary(''libc.dylib'')
```

where libc.so would be standard under a Linux system, and you can google for Windows. If you can't get this part to work, that's OK - just state so and you won't lose points.

In [48]:
```
import numpy as np
import matplotlib.pyplot as plt
%matplotlib notebook

data = np.loadtxt('rand_points.txt')
x = data[:,0]
y = data[:,1]
z = data[:,2]

fig = plt.figure(figsize = (8,8))
ax = plt.axes(projection='3d')
ax.scatter(x, y, z, marker ='.', s=0.2)
ax.set_title('C Random Number Triplets')

plt.show()
```

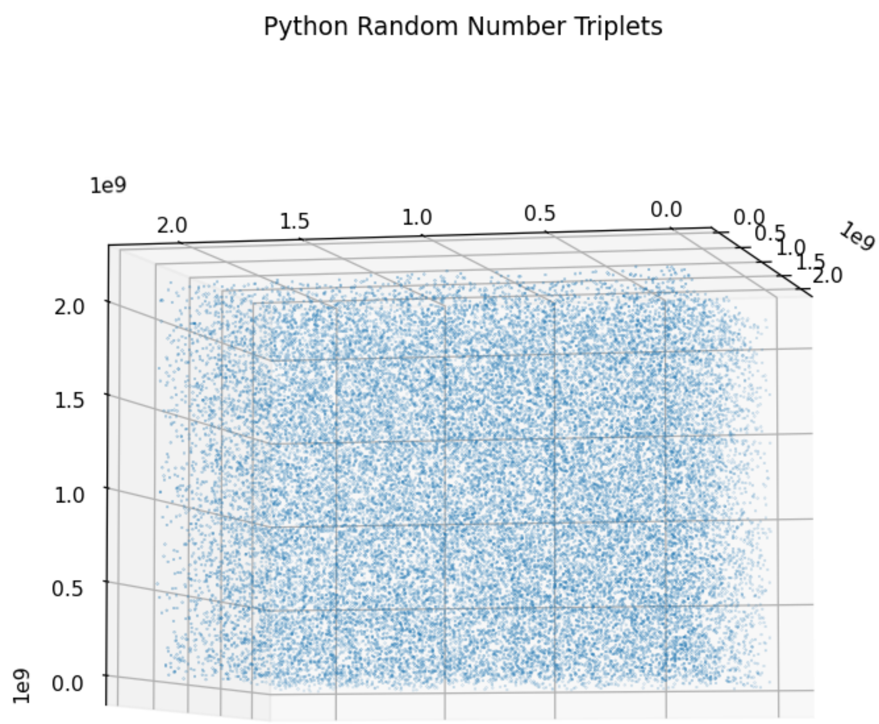
```
<IPython.core.display.Javascript object>
```



For some reason with the way I'm plotting the data in 3d, I couldn't get the planes to all be viewable from the same angle, but by simply rotating up and down from the position above, it can be seen that the random tripes all fall onto planes.

In [47]:
```
from random import randint

def get_rands_nb(vals):
    n=len(vals)
    for i in range(n):
        vals[i]=randint(0, 2**31)
    return vals

def get_rands(n):
    vec=np.empty(n,dtype='int32')
    get_rands_nb(vec)
    return vec

n=len(x)
xp = get_rands(n)
yp = get_rands(n)
zp = get_rands(n)

fig = plt.figure(figsize = (8,8))
ax = plt.axes(projection='3d')
ax.scatter(xp, yp, zp, marker ='.', s=0.2)
ax.set_title('Python Random Number Triplets')

plt.show()
```

```
<IPython.core.display.Javascript object>
```



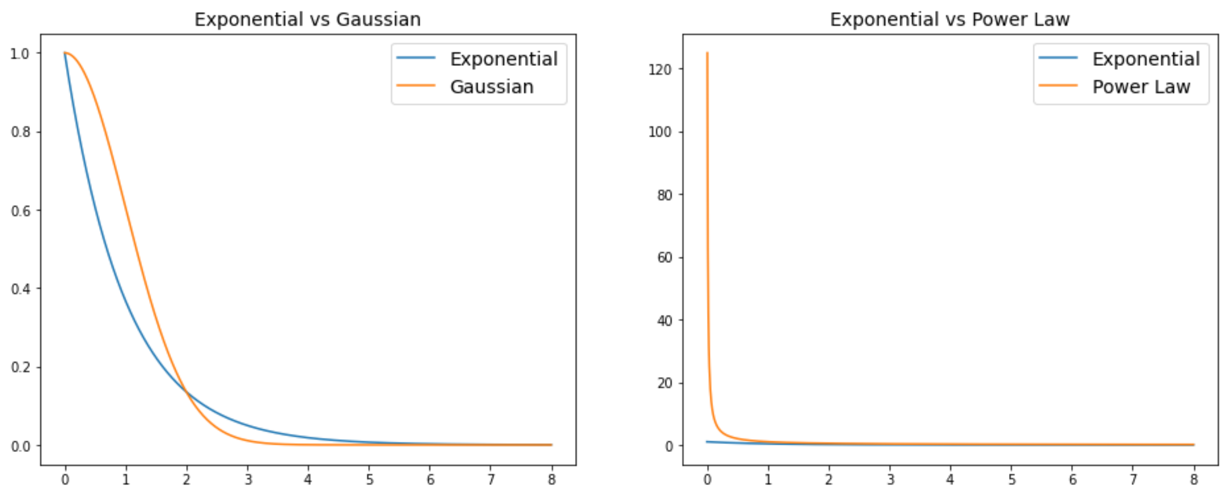
Even after rotating around 3d space, the random numbers generated with python do not show the same planes of random numbers as C, but rather it seems to be generating actual pseudo random numbers.

I couldn't figure out how to get ctypes.cdll.loadlibrary to work in calling the random number generator library from C.

Problem 2: We saw in class how to generate exponential deviates using a transformation. Now write a rejection method to generate exponential deviates from another distribution. Which of Lorentzians, Gaussians, and power laws could you use for the bounding distribution? You can assume the exponential deviates are non-negative (since you have to cut off the distribution somewhere, might as well be at zero). Show that a histogram of your deviates matches up with the expected exponential curve. How efficient can you make this generator, in terms of the fraction of uniform deviates that give rise to an exponential deviate?

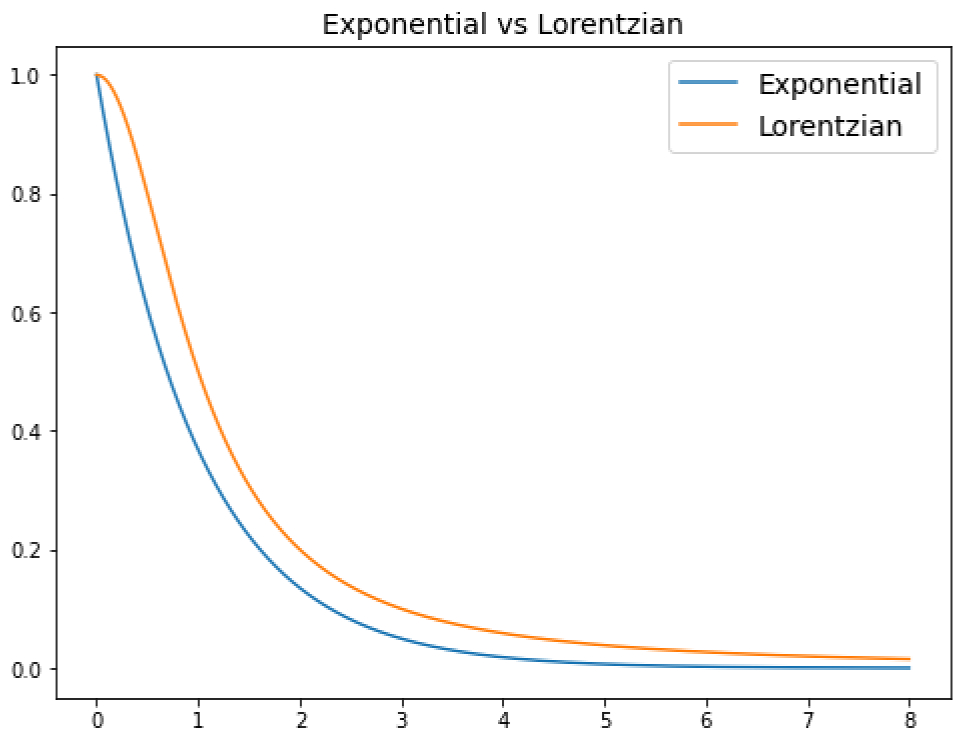
In [1]:
```
import numpy as np
import matplotlib.pyplot as plt

def gaussian(x):
    return np.exp(-0.5*x**2)
def lorentz(x):
    return 1/(1+x**2)

x = np.linspace(0,8,1001)
y = np.exp(-x)
mult = max(y)
y2 = gaussian(x)
y3 = x**(-1)

plt.figure(figsize=(16,6))
plt.subplot(1,2,1)
plt.title('Exponential vs Gaussian', fontsize=14)
plt.plot(x,y, label='Exponential')
plt.plot(x,y2, label='Gaussian')
plt.legend(fontsize=14)
plt.subplot(1,2,2)
plt.title('Exponential vs Power Law', fontsize=14)
plt.plot(x,y, label='Exponential')
plt.plot(x,y3, label='Power Law')
plt.legend(fontsize=14)
plt.show()
```

```
C:\Users\adesr\AppData\Local\Temp/ipykernel_4384/3676003091.py:13: RuntimeWarni
ng: divide by zero encountered in reciprocal
  y3 = x**(-1)
```



Can't use gaussian because it doesnt stay above exponential for all positive x. We also don't want to use a power law because they shoot up towards infinity as they approach 0 and so we would have to throw out so many random guesses that our acceptance percentage would be awful. As we can see below though, a lorentzian works well for this.

In [2]:
```
y4 = lorentz(x)

plt.figure(figsize=(8,6))
plt.title('Exponential vs Lorentzian', fontsize=14)
plt.plot(x,y, label='Exponential')
plt.plot(x,y4, label='Lorentzian')
plt.legend(fontsize=14)
plt.show()
```



We must first normalize our lorentzian as we are only evaluating positive values of x, therefore:

$$1 = \int_0^{\infty} \frac{a}{1+x^2}dx = a \cdot arctan(x)|_0^{\infty} = a \cdot arctan(\infty) = a \cdot \frac{\pi}{2} = 1 \rightarrow a = \frac{2}{\pi}$$

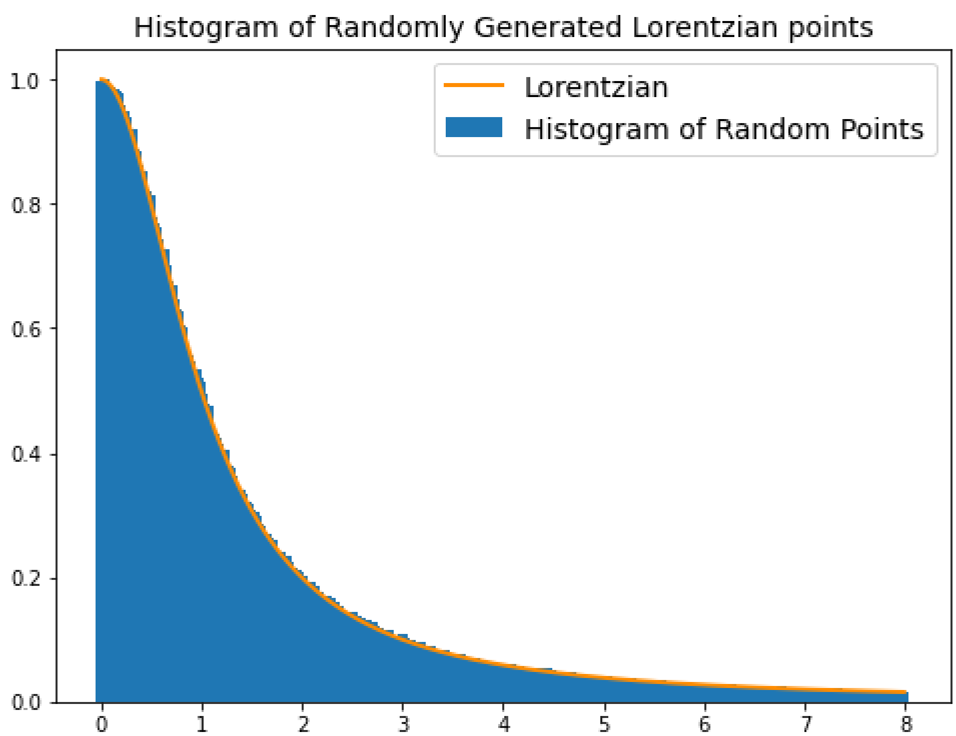
We are going to use our lorentz function to generate random numbers, so we need to take the cumulative density function of a lorentzian:

$$\int_0^x \frac{2}{\pi} \cdot \frac{1}{1+x^2}dx = \frac{2}{\pi} \cdot arctan(x)|_0^x = \frac{2}{\pi} \cdot arctan(x)$$

Now that we have our CDF, we set it equal to a random number, which we will then invert to find out what x needs to be equal to to get random numbers following a lorentzian from uniformly distributed random number between 0 and 1:

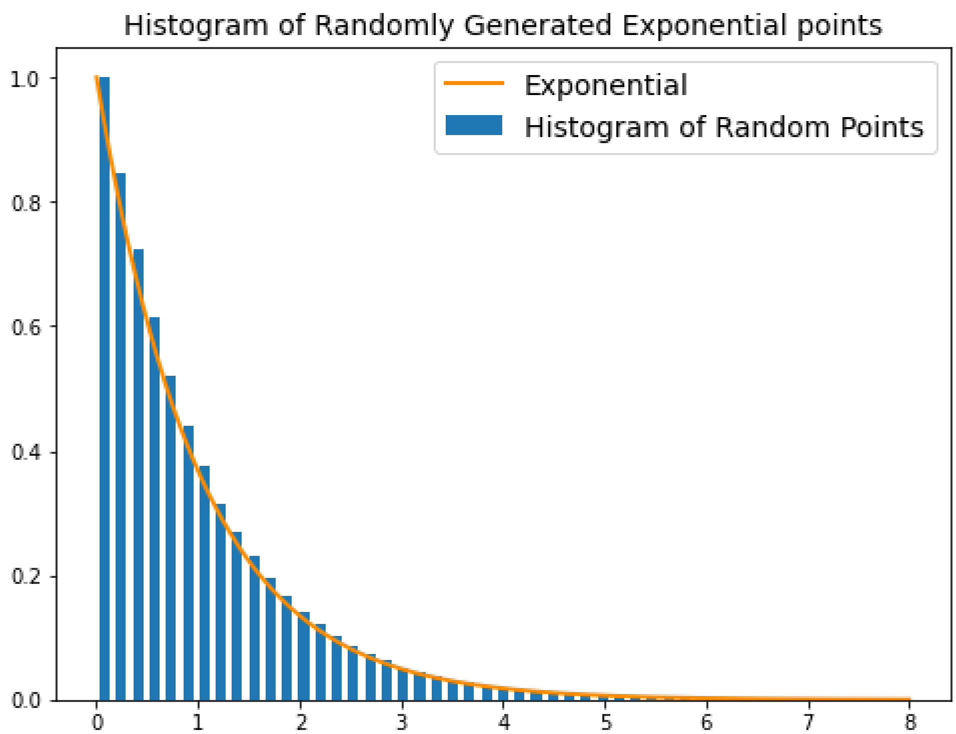
$$r = \frac{2}{\pi} \cdot arctan(x) \rightarrow x = tan(r \cdot \frac{\pi}{2})$$

In [3]:
```
def lorentz_rand(r):
    return np.tan(r*np.pi/2)

def pos_lorentz(x):
    return (2/np.pi)/(1+x**2)

n = int(1e6)
r = np.random.rand(n)
xs = lorentz_rand(r)

bins = np.linspace(0,8,501)
counts, b = np.histogram(xs, bins)
counts = counts/max(counts)
b_center = 0.5*(bins[1:]+bins[:-1])

plt.figure(figsize=(8,6))
plt.title('Histogram of Randomly Generated Lorentzian points', fontsize=14)
plt.bar(b_center,counts, width=0.1, label='Histogram of Random Points')
plt.plot(b_center, pos_lorentz(b_center)/max(pos_lorentz(b_center)), color='darko
plt.legend(fontsize=14)
plt.show()
```



Now that we have our lorentzian random number generator working, in order to create exponentially distributed random numbers, we need to get rid of points that don't fall under the eponential curve.

In [4]:
```
n = int(1e6)
r = np.random.rand(n)
xs = lorentz_rand(r)
accept = np.random.rand(n)<np.exp(-xs)/lorentz(xs)
        #Getting rid of points from our Lorentzian
        #RNG that don't fall under our exponential
x_use = xs[accept]

bins = np.linspace(0,8,50)
counts, b = np.histogram(x_use, bins)
counts = counts/max(counts)
b_center = 0.5*(bins[1:]+bins[:-1])

plt.figure(figsize=(8,6))
plt.title('Histogram of Randomly Generated Exponential points', fontsize=14)
plt.bar(b_center, counts, 0.1, label='Histogram of Random Points')
plt.plot(x, np.exp(-x), color='darkorange', linewidth=2, label='Exponential')
plt.legend(fontsize=14)
plt.show()
print('The acceptance percentage is {}%'.format(round(np.mean(accept)*100,2)))
```




```
The acceptance percentage is 63.57%
```

The most efficient I could make the exponential RNG was 63.67%, meaning that 63.67% of numbers generated using the lorentzian RNG were used for the exponential RNG and the rest fell outside the range of the exponential.

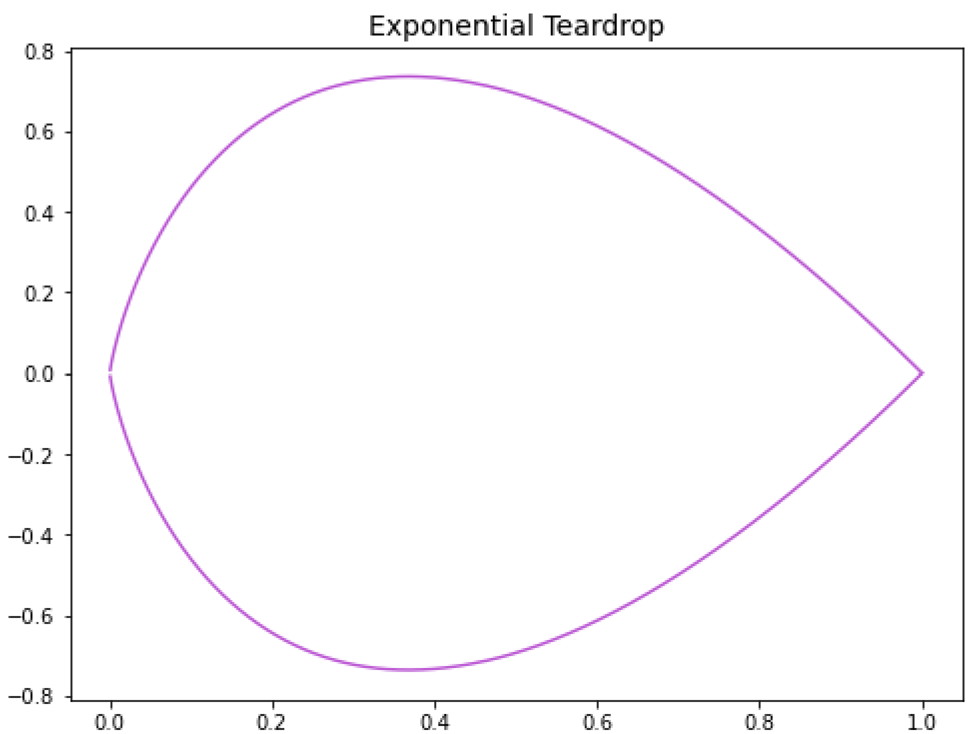
Problem 3: Repeat problem 2, but now use a ratio-of-uniforms generator. If u goes from 0 to 1, what are your limits on v? How efficient is this generator, in terms of number of exponential deviates produced per uniform deviate? Make sure to plot the histogram again and show it still produces the correct answer.

To find the limits on v, we need to use the equation $u < \sqrt{P(v/u)}$, where in our case $P(x) = e^{-x}$. This means that at our limit for v is

$$u = \sqrt{e^{-v/u}} = e^{-0.5 \cdot v/u} \rightarrow ln(u) = -0.5 \cdot v/u \rightarrow v = -2 \cdot u \cdot ln(u)$$

In [5]:

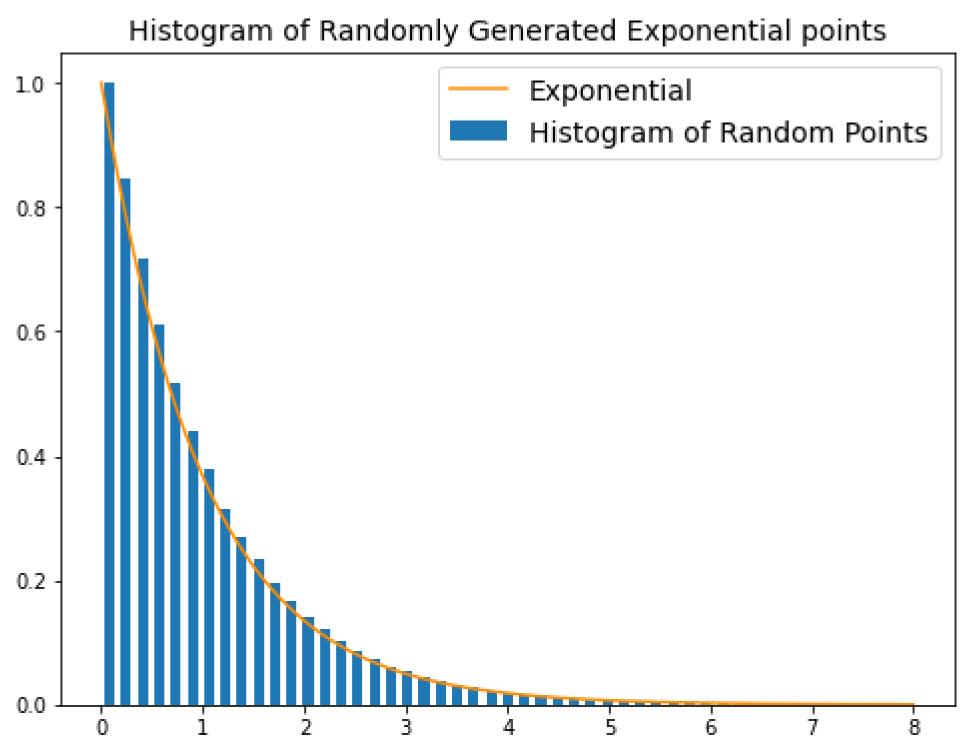
```
u = np.linspace(0,1,2001)
u = u[1:]
v=-2*u*np.log(u)
maxv = max(v)

plt.figure(figsize=(8,6))
plt.plot(u,v, color='mediumorchid')
plt.plot(u,-v, color='mediumorchid')
plt.title('Exponential Teardrop', fontsize=14)
plt.show()
print('The maximum value of v is {}'.format(maxv))
```




```
The maximum value of v is 0.7357588428385197
```

In [7]:
```
n = int(1e6)
u = np.random.rand(n)
v = (np.random.rand(n)*2-1)*maxv
        #random numbers sampled from
        #a rectangle around the above
        #teardrop shape for v
r = v/u
accept = u<np.exp(-0.5*r)
        #from the random numbers sampled
        #from the rectangle, accept only
        #those that fall in the teardrop
expo = r[accept]

bins = np.linspace(0,8,50)
counts, b = np.histogram(expo, bins)
counts = counts/max(counts)
b_center = 0.5*(bins[1:]+bins[:-1])

plt.figure(figsize=(8,6))
plt.title('Histogram of Randomly Generated Exponential points', fontsize=14)
plt.bar(b_center,counts,0.1, label='Histogram of Random Points')
plt.plot(x,np.exp(-x),'darkorange', label='Exponential')
plt.legend(fontsize=14)
plt.show()
print('The acceptance percentage is {}%'.format(round(np.mean(accept)*100,2)))
```

```
C:\Users\adesr\AppData\Local\Temp/ipykernel_4384/638754313.py:8: RuntimeWarnin
g: overflow encountered in exp
  accept = u<np.exp(-0.5*r)
```



```
The acceptance percentage is 83.98%
```

As can be seen above, using a ratio-of-uniforms generator gives us the same result as in problem 2, but the big advantage to this method is how much higher our acceptance percentage is, which has risen by over 20% to 83.98%.