Bayesian Modelling of Loss Curves in Insurance

Mick Cooney
michael.cooney@applied.ai

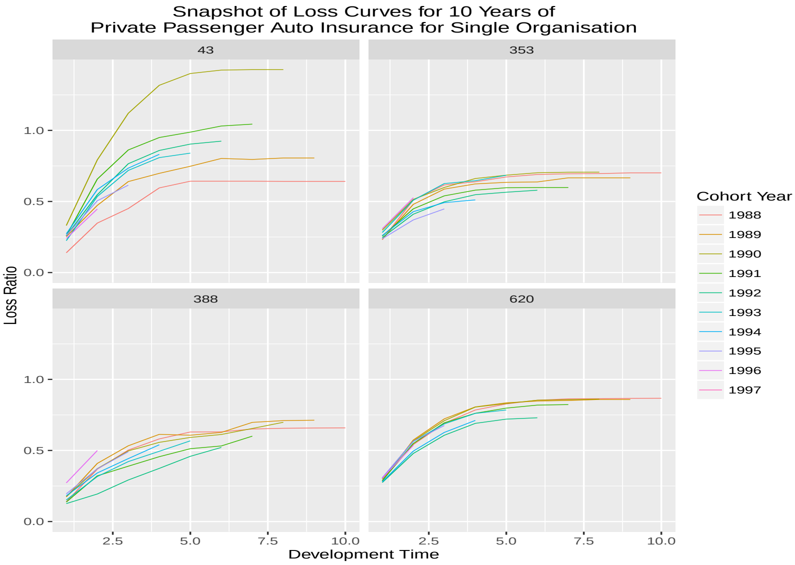
15 April 2016

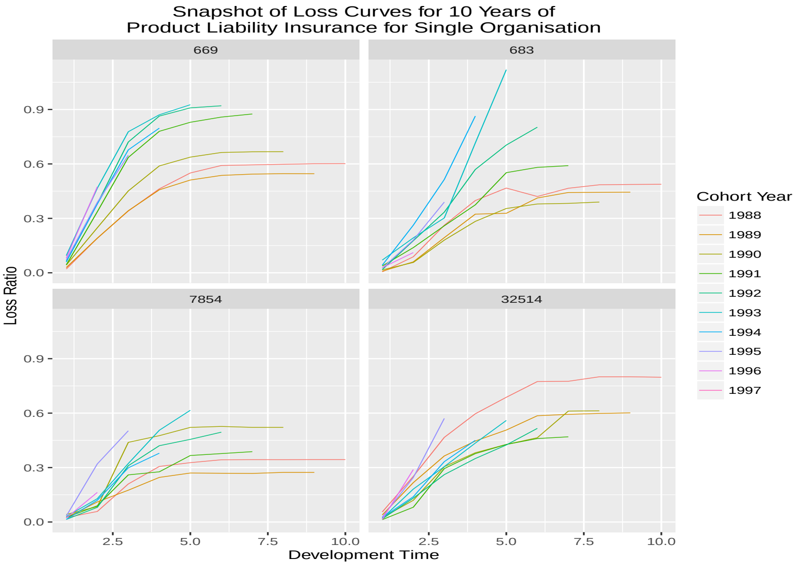
Structure of Talk

- Loss Curves
- Chain Ladder Modelling (package `ChainLadder`)
- Loss Growth Modelling
- Expanding the Model
- Posterior Predictive Checks
- Summary

Loss Curves

```
use_grcode <- c(43,353,388,620)

ppauto_ss_dt <- ppauto_dt[GRCODE %in% use_grcode
                        ][DevelopmentYear < 1998
                        ][, .(grcode     = GRCODE
                             ,accyear    = AccidentYear
                             ,devlag     = DevelopmentLag
                             ,premium    = EarnedPremDIR_B
                             ,cumloss    = CumPaidLoss_B
                             ,loss_ratio = CumPaidLoss_B / EarnedPremDIR_B)]


print(dcast(ppauto_ss_dt[grcode == 43]
           ,grcode + accyear + premium ~ devlag
           ,value.var = 'cumloss'),digits=3)

##     grcode accyear premium     1     2     3     4     5     6     7    8    9  10
##  1:     43    1988     957   133   333   431   570   615   615   615  614  614 614
##  2:     43    1989    3695   934  1746  2365  2579  2763  2966  2940 2978 2978  NA
##  3:     43    1990    6138  2030  4864  6880  8087  8595  8743  8763 8762   NA  NA
##  4:     43    1991   17533  4537 11527 15123 16656 17321 18076 18308   NA   NA  NA
##  5:     43    1992   29341  7564 16061 22465 25204 26517 27124    NA   NA   NA  NA
##  6:     43    1993   37194  8343 19900 26732 30079 31249    NA    NA   NA   NA  NA
##  7:     43    1994   46095 12565 26922 33867 38338    NA    NA    NA   NA   NA  NA
##  8:     43    1995   51512 13437 26012 31677    NA    NA    NA    NA   NA   NA  NA
##  9:     43    1996   52481 12604 23446    NA    NA    NA    NA    NA   NA   NA  NA
## 10:     43    1997   56978 12292    NA    NA    NA    NA    NA    NA   NA   NA  NA
```

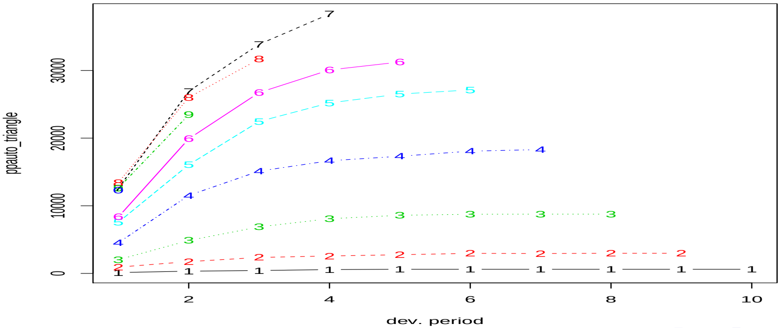

Snapshot of Loss Curves for 10 Years of
Private Passenger Auto Insurance for Single Organisation
43
353
388
620
Loss Ratio
0.0
0.5
1.0
Development Time
2.5
5.0
7.5
10.0
Cohort Year
1988
1989
1990
1991
1992
1993
1994
1995
1996
1997


Snapshot of Loss Curves for 10 Years of
Product Liability Insurance for Single Organisation
669
683
7854
32514
Loss Ratio
0.0
0.3
0.6
0.9
2.5
5.0
7.5
10.0
Development Time
Cohort Year
1988
1989
1990
1991
1992
1993
1994
1995
1996
1997

Chain Ladder

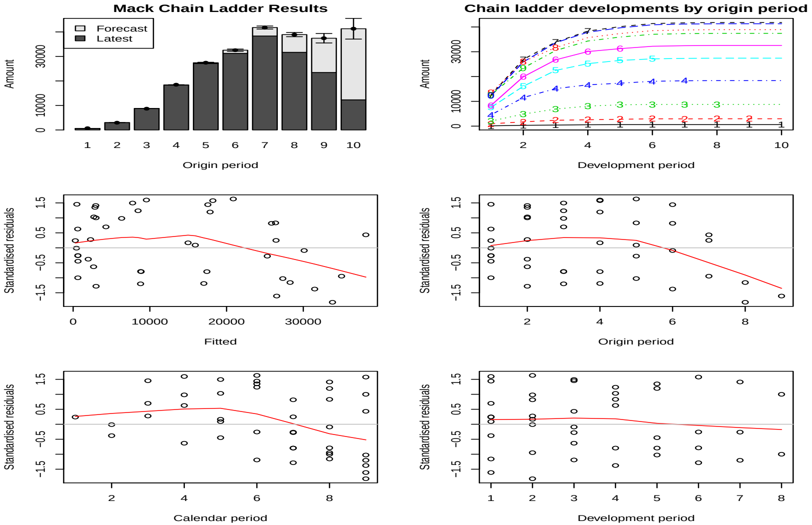
Standard R approach is ChainLadder

```
ppauto_mat <- as.matrix(dcast(ppauto_ss_dt[grcode == 43]
                            ,accyear ~ devlag
                            ,value.var = 'cumloss')[,-1,with=FALSE])

ppauto_triangle <- as.triangle(ppauto_mat)

plot(ppauto_triangle)
```

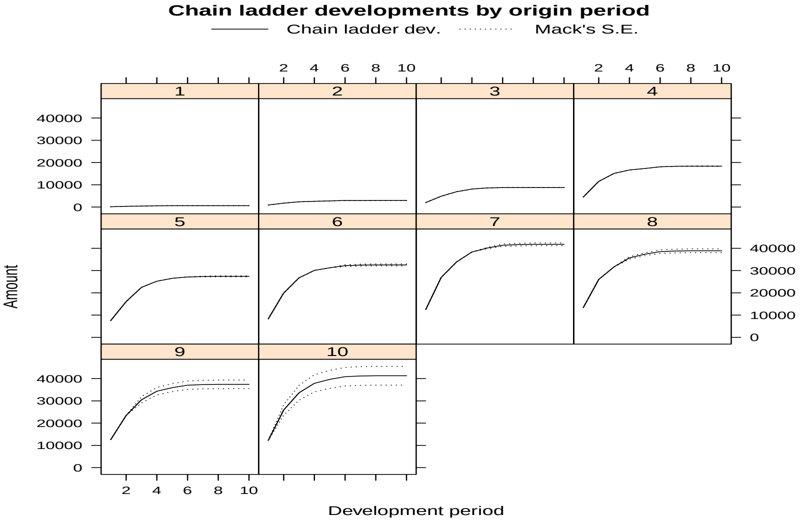


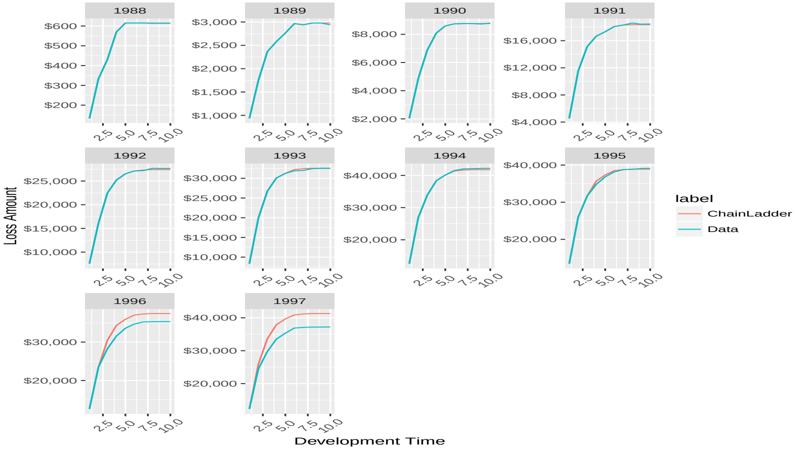
```
ppauto_mack <- MackChainLadder(ppauto_triangle, est.sigma = "Mack")

ppauto_mack$f

##  [1] 2.10486 1.29968 1.12655 1.04671 1.03069 1.00743 1.00292 1.00000 1.00000 1.00000

ppauto_mack$FullTriangle

##       dev
## origin     1     2       3       4       5       6       7       8       9      10
##      1   133   333   431.0   570.0   615.0   615.0   615.0   614.0   614.0   614.0
##      2   934  1746  2365.0  2579.0  2763.0  2966.0  2940.0  2978.0  2978.0  2978.0
##      3  2030  4864  6880.0  8087.0  8595.0  8743.0  8763.0  8762.0  8762.0  8762.0
##      4  4537 11527 15123.0 16656.0 17321.0 18076.0 18308.0 18361.5 18361.5 18361.5
##      5  7564 16061 22465.0 25204.0 26517.0 27124.0 27325.6 27405.5 27405.5 27405.5
##      6  8343 19900 26732.0 30079.0 31249.0 32208.1 32447.6 32542.4 32542.4 32542.4
##      7 12565 26922 33867.0 38338.0 40128.7 41360.4 41667.9 41789.6 41789.6 41789.6
##      8 13437 26012 31677.0 35685.7 37352.5 38499.0 38785.2 38898.6 38898.6 38898.6
##      9 12604 23446 30472.3 34328.5 35932.0 37034.8 37310.1 37419.2 37419.2 37419.2
##     10 12292 25873 33626.6 37882.0 39651.4 40868.4 41172.3 41292.6 41292.6 41292.6
```

```r
plot(ppauto_mack)
```



```r
plot(ppauto_mack, lattice = TRUE)
```



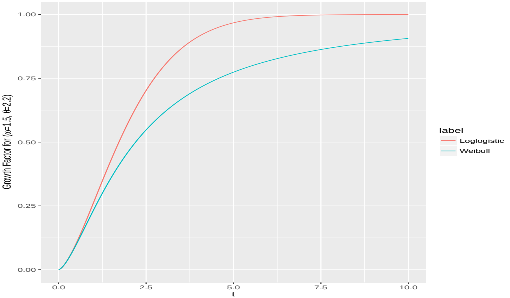

1988
1989
1990
1991
1992
1993
1994
1995
1996
1997
Loss Amount
Development Time
label
ChainLadder
Data

Loss Growth Modelling

Model growth cumulative losses as function
Scale losses by premium

$$g(t;\ \omega, \theta) = 1 - \exp\left(-\left(\frac{t}{\theta}\right)^{\omega}\right) \qquad \text{Loglogistic Function}$$

$$g(t;\ \omega, \theta) = \frac{t^{\omega}}{t^{\omega} + \theta^{\omega}} \qquad \text{Weibull Function}$$



Start with the Weibull model

$$g(t;\ \omega, \theta) = \frac{t^{\omega}}{t^{\omega} + \theta^{\omega}}$$

Treat as hierarchical model - group by Accident Year

$$\mathrm{Loss}_{\mathrm{Y},t} \sim \mathrm{Normal}(\mu_{\mathrm{L,Y},t}, \sigma_{L})$$

where

$$\begin{aligned}
\mu_{\mathrm{L,Y},t} &= \mathrm{LR}_{\mathrm{Y}} \times \mathrm{P}_{\mathrm{Y}} \times g(t;\, \omega, \theta) \\
\sigma_{\mathrm{L}} &= \mathrm{P}_{\mathrm{Y}} \times \sigma \\
\mathrm{LR}_{\mathrm{Y}} &\sim \mathrm{Lognormal}(\mu_{\mathrm{LR}}, \sigma_{\mathrm{LR}})
\end{aligned}$$

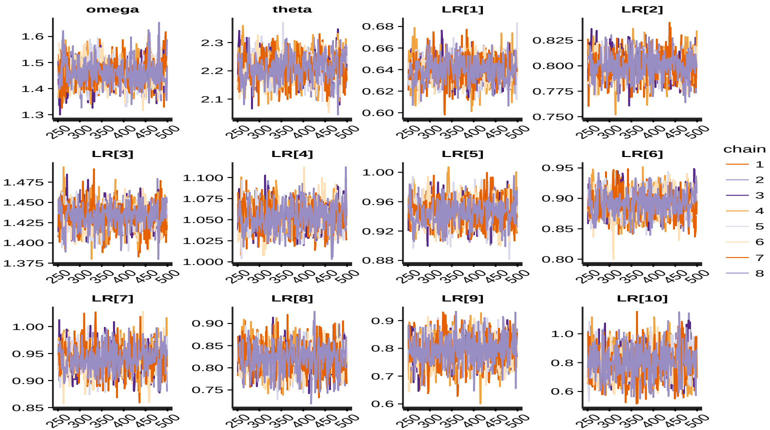
Normal prior for μ_{LR}. Lognormal prior for ω, θ, σ_{LR}, σ.

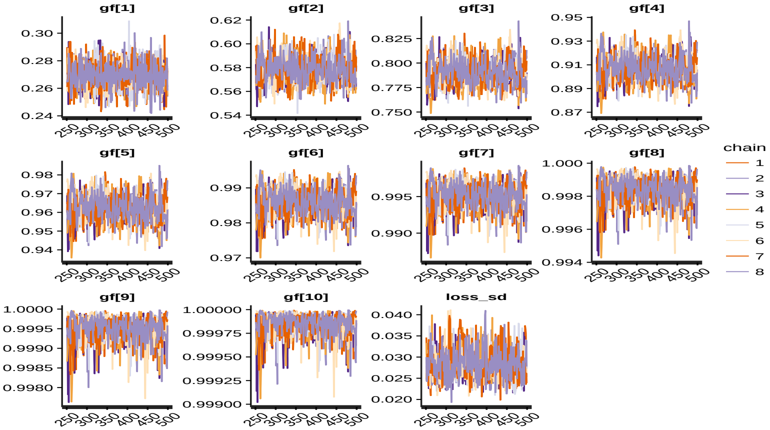
```
functions {
  real growth_factor_weibull(real t, real omega, real theta) {
    real factor;

    factor <- 1 - exp(-(t/theta)^omega);

    return(factor);
  }

  real growth_factor_loglogistic(real t, real omega, real theta) {
    real factor;

    factor <- ((t^omega) / (t^omega + theta^omega));

    return(factor);
  }
}

data {
  int<lower=0,upper=1> growthmodel_id;

  int n_data;
  int n_time;
  int n_cohort;

  int cohort_id[n_data];
  int t_idx[n_data];

  real<lower=0> t_value[n_time];

  real premium[n_cohort];
  real loss[n_data];

  int cohort_maxtime[n_cohort];
}
```

```
parameters {
  real<lower=0> omega;
  real<lower=0> theta;

  real<lower=0> LR[n_cohort];

  real mu_LR;
  real<lower=0> sd_LR;

  real<lower=0> loss_sd;
}

transformed parameters {
  real gf[n_time];
  real loss_mean[n_cohort, n_time];

  for(i in 1:n_time) {
    if(growthmodel_id == 1) {
      gf[i] <- growth_factor_weibull    (t_value[i], omega, theta);
    } else {
      gf[i] <- growth_factor_loglogistic(t_value[i], omega, theta);
    }
  }

  for(i in 1:n_data) {
    loss_mean[cohort_id[i], t_idx[i]] <- LR[cohort_id[i]] * premium[cohort_id[i]] * gf[t_idx[i]];
  }
}
```

```
model {
  mu_LR ~ normal(0, 0.5);
  sd_LR ~ lognormal(0, 0.5);

  LR ~ lognormal(mu_LR, sd_LR);

  loss_sd ~ lognormal(0, 0.7);

  omega ~ lognormal(0, 1);
  theta ~ lognormal(0, 1);

  for(i in 1:n_data) {
    loss[i] ~ normal(loss_mean[cohort_id[i], t_idx[i]], premium[cohort_id[i]] * loss_sd);
  }
}
```

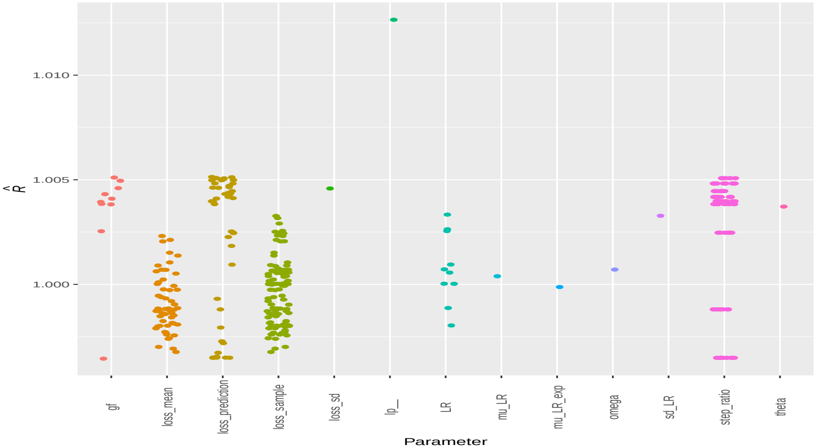
Stan Output


omega
1.6
1.5
1.4
1.3
theta
2.3
2.2
2.1
LR[1]
0.68
0.66
0.64
0.62
0.60
LR[2]
0.825
0.800
0.775
0.750
LR[3]
1.475
1.450
1.425
1.400
1.375
LR[4]
1.100
1.075
1.050
1.025
1.000
LR[5]
1.00
0.96
0.92
0.88
LR[6]
0.95
0.90
0.85
0.80
LR[7]
1.00
0.95
0.90
0.85
LR[8]
0.90
0.85
0.80
0.75
LR[9]
0.9
0.8
0.7
0.6
LR[10]
1.0
0.8
0.6
250 300 350 400 450 500
chain
1
2
3
4
5
6
7
8

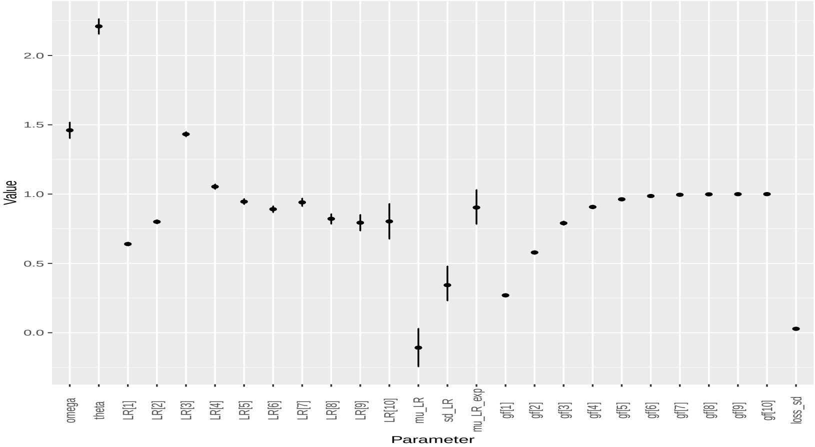

gf[1]
gf[2]
gf[3]
gf[4]
gf[5]
gf[6]
gf[7]
gf[8]
gf[9]
gf[10]
loss_sd
chain
1
2
3
4
5
6
7
8

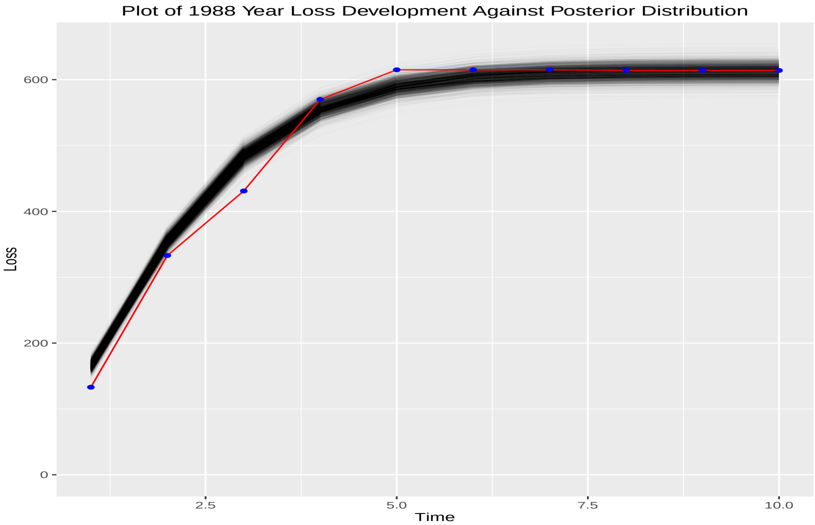
Check simple diagnostics:

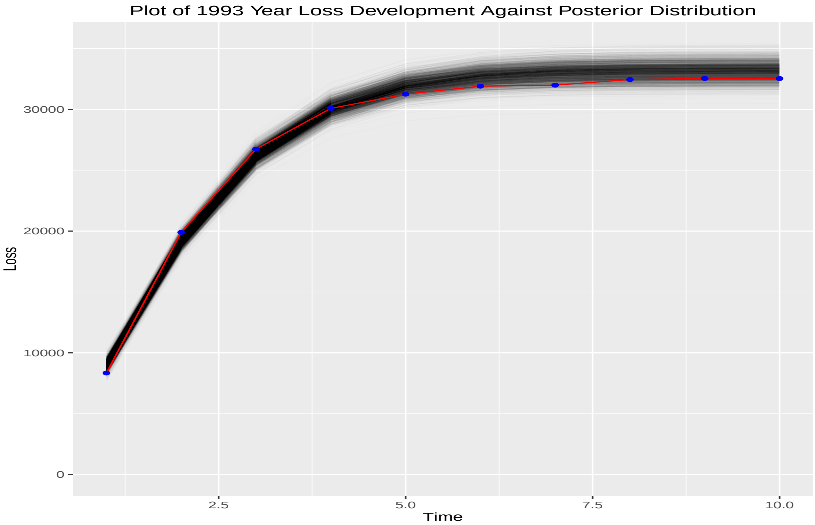
```
## Warning:  Removed 110 rows containing missing values (geom_point).
```

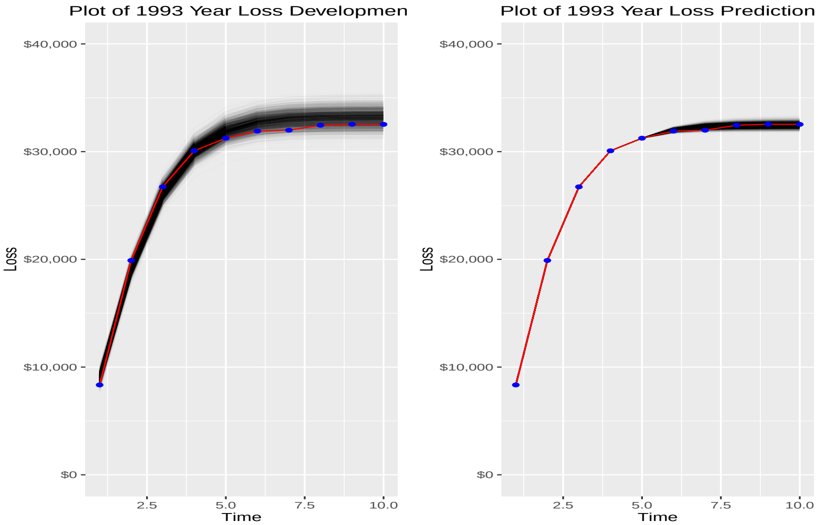


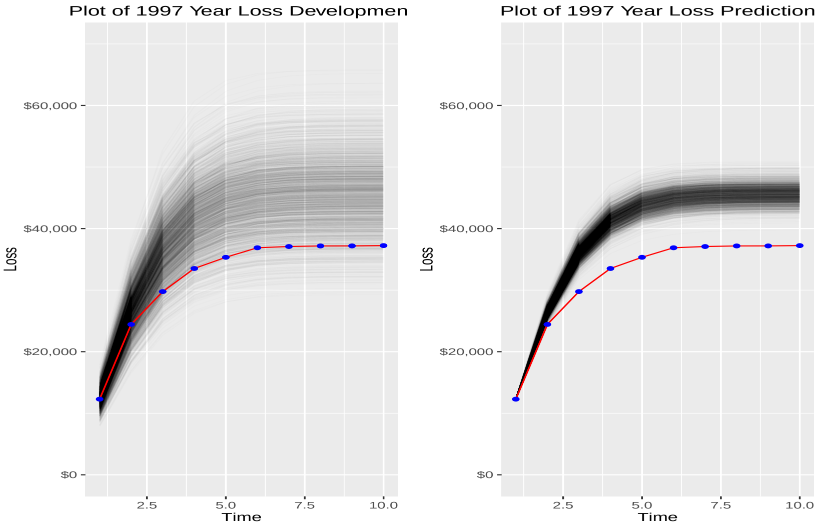
Check parameter values:




Plot of 1988 Year Loss Development Against Posterior Distribution
600
400
200
0
Loss
2.5
5.0
7.5
10.0
Time


Plot of 1993 Year Loss Development Against Posterior Distribution
Loss
30000
20000
10000
0
2.5
5.0
7.5
10.0
Time


Plot of 1993 Year Loss Developmen
$40,000
$30,000
$20,000
$10,000
$0
Loss
2.5
5.0
7.5
10.0
Time
Plot of 1993 Year Loss Prediction
$40,000
$30,000
$20,000
$10,000
$0
Loss
2.5
5.0
7.5
10.0
Time


Plot of 1997 Year Loss Developmen
Loss
$60,000
$40,000
$20,000
$0
2.5
5.0
7.5
10.0
Time
Plot of 1997 Year Loss Prediction
Loss
$60,000
$40,000
$20,000
$0
2.5
5.0
7.5
10.0
Time

Model Iteration

How might we expand this model?

Allow ω and θ to be part of the hierarchy:

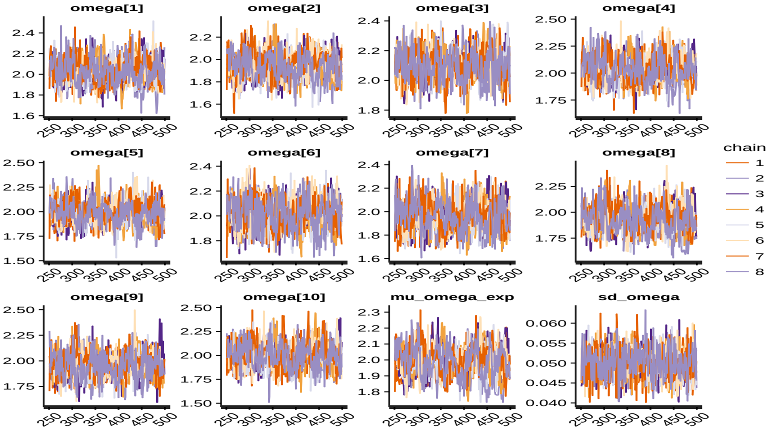
$$\begin{aligned} \omega &\rightarrow \omega_{\mathsf{Y}} \\ \theta &\rightarrow \theta_{\mathsf{Y}} \end{aligned}$$

Each Accident Year has individual $(\omega_{\mathsf{Y}}, \theta_{\mathsf{Y}})$ with

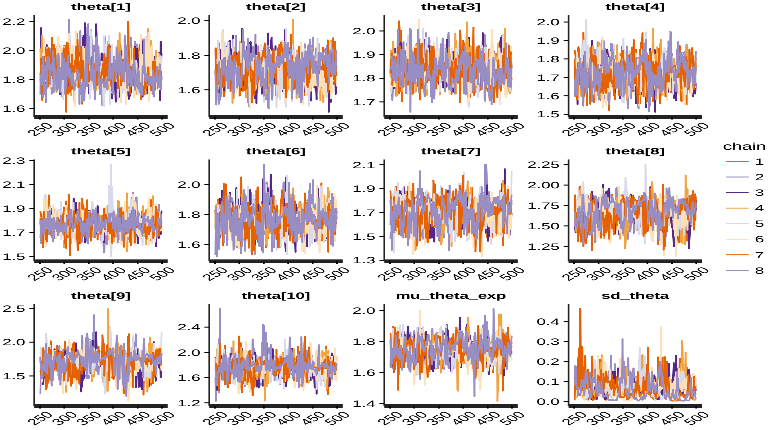
$$\begin{aligned} \omega_{\mathsf{Y}} &\sim \text{Lognormal}(\mu_\omega, \sigma_\omega) \\ \theta_{\mathsf{Y}} &\sim \text{Lognormal}(\mu_\theta, \sigma_\theta) \end{aligned}$$

$$\begin{aligned} \mu_\omega &\sim \text{Normal}(0, 1) \\ \sigma_\omega &\sim \text{Lognormal}(-3, 0.1) \\ \mu_\theta &\sim \text{Normal}(0, 1) \\ \sigma_\theta &\sim \text{Lognormal}(-3, 0.1) \end{aligned}$$

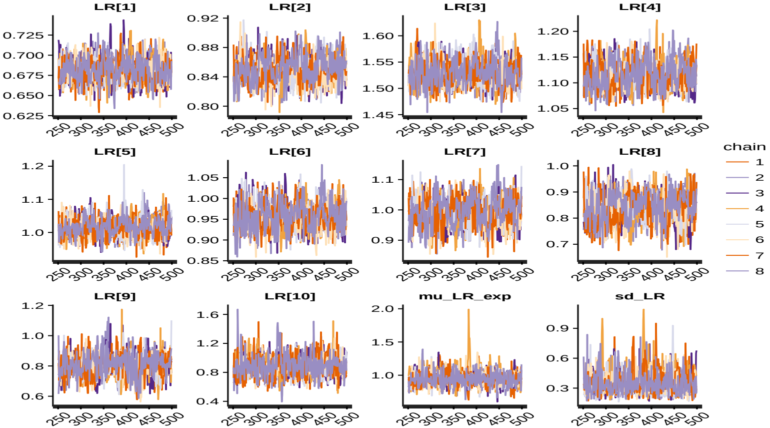
Individual Parameters - ω



Individual Parameters - θ

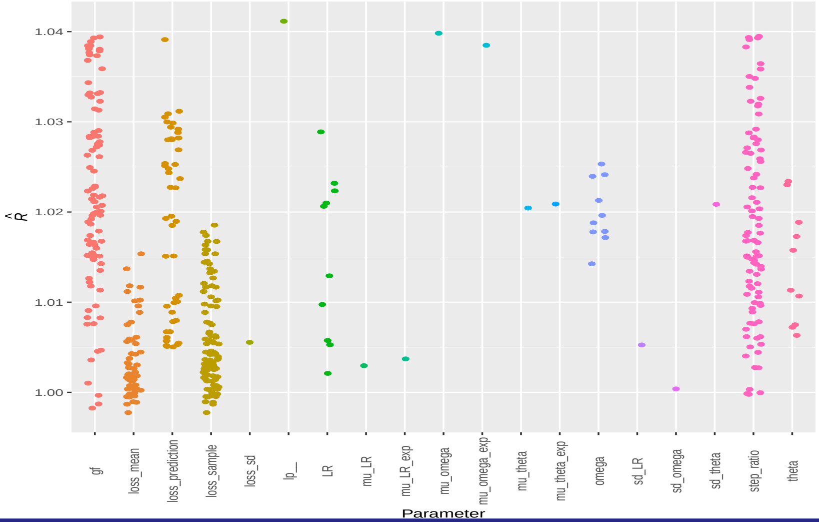


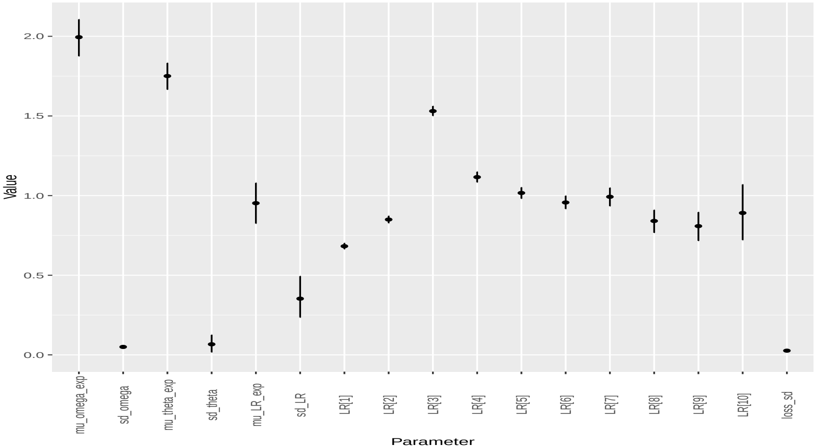
Individual Parameters - LR



Convergence Diagnostics

```
## Warning:  Removed 110 rows containing missing values (geom_point).
```





2.0
1.5
1.0
0.5
0.0
Value
mu_omega_exp
sd_omega
mu_theta_exp
sd_theta
mu_LR_exp
sd_LR
LR[1]
LR[2]
LR[3]
LR[4]
LR[5]
LR[6]
LR[7]
LR[8]
LR[9]
LR[10]
loss_sd
Parameter

Problems with the Model

- Trouble with code
- Divergent transitions — had to raise `adapt_delta`
- Would not rely on output
- Data is very sparse for later Accident Years
- May revisit once other insurers added

Multiple Insurers

Use hierarchical model for multiple insurers

Each insurer gets own set of loss ratios and growth curves:

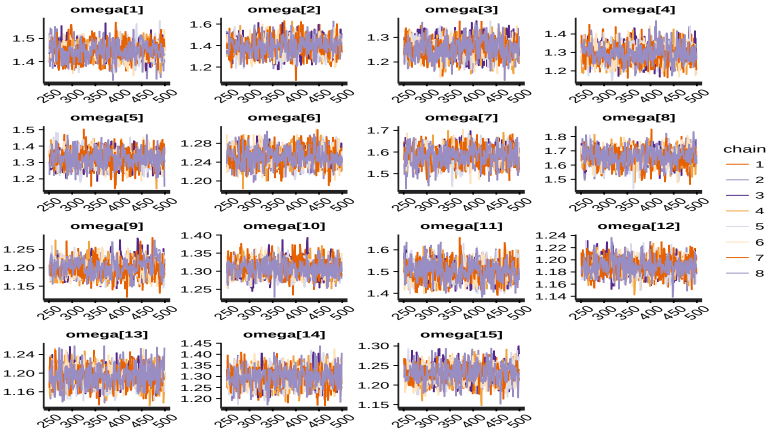
$$\begin{aligned} \mathsf{LR} &\rightarrow \mathsf{LR}_{\mathsf{I,Y}} \\ \omega &\rightarrow \omega_{\mathsf{I}} \\ \theta &\rightarrow \theta_{\mathsf{I}} \end{aligned}$$

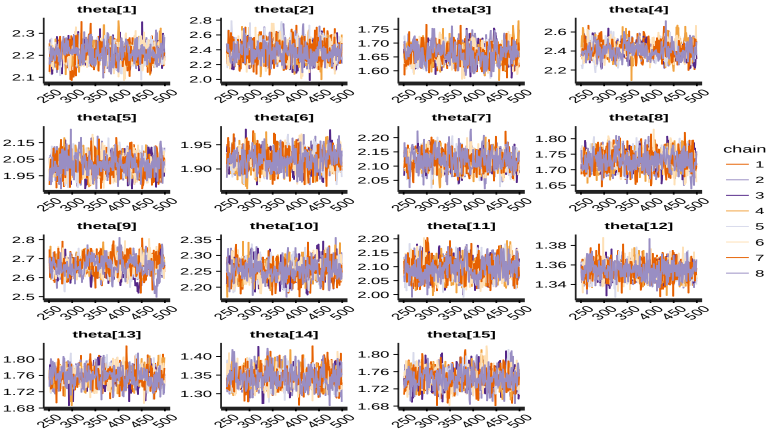
Put hierarchy on top of this

Start with 15 insurers

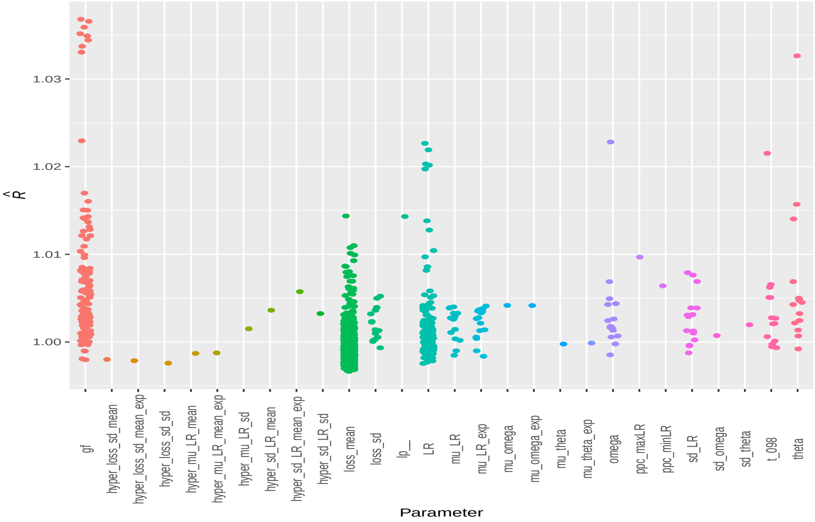
Multiple Insurers

```
model {
  mu_LR ~ normal   (hyper_mu_LR_mean, hyper_mu_LR_sd);
  sd_LR ~ lognormal(hyper_sd_LR_mean, hyper_sd_LR_sd);

  loss_sd ~ lognormal(hyper_loss_sd_mean, hyper_loss_sd_sd);

  omega ~ lognormal(mu_omega, sd_omega);
  theta ~ lognormal(mu_theta, sd_theta);

  mu_omega ~ normal(0, 1);
  sd_omega ~ lognormal(-3, 0.1);
  mu_theta ~ normal(0, 1);
  sd_theta ~ lognormal(-3, 0.1);

  hyper_mu_LR_mean ~ normal(0, 1);
  hyper_mu_LR_sd   ~ lognormal(0, 1);
  hyper_sd_LR_mean ~ normal(0, 1);
  hyper_sd_LR_sd   ~ lognormal(0, 1);

  hyper_loss_sd_mean ~ normal(0, 1);
  hyper_loss_sd_sd   ~ lognormal(0, 0.1);

  for(i in 1:n_data) {
    loss[i] ~ normal(loss_mean[org_id[i], cohort_id[i], t_idx[i]], premium[i] * loss_sd[org_id[i]]);
  }

  for(j in 1:n_org) {
    LR[j] ~ lognormal(mu_LR[j], sd_LR[j]);
  }
}
```

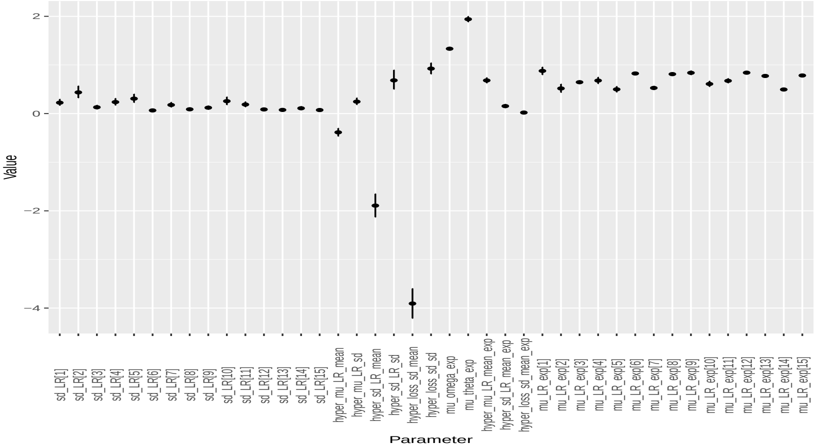

omega[1]
omega[2]
omega[3]
omega[4]
omega[5]
omega[6]
omega[7]
omega[8]
omega[9]
omega[10]
omega[11]
omega[12]
omega[13]
omega[14]
omega[15]
250 300 350 400 450 500
chain
1
2
3
4
5
6
7
8


theta[1]
theta[2]
theta[3]
theta[4]
theta[5]
theta[6]
theta[7]
theta[8]
theta[9]
theta[10]
theta[11]
theta[12]
theta[13]
theta[14]
theta[15]
chain
1
2
3
4
5
6
7
8
250 300 350 400 450 500

```
## Warning:  Removed 676 rows containing missing values (geom_point).
```



Huge amount of parameters, so check interesting subset



Model Checking

Promising on first pass

Lots of things going on

How do we check and understand model?

Posterior Predictive Checks

Posterior Predictive Checks

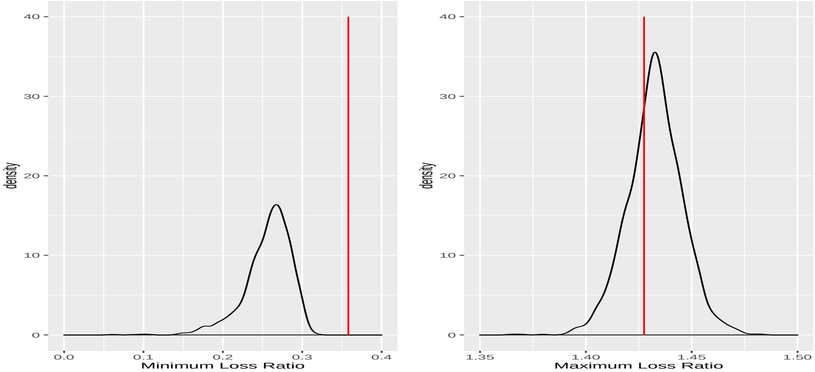
- Getting more and more emphasis
- Used to assess data aspects not modelled well
- Use sample to generate 'fake' data to compare
- Can also be used to generate predictions from data (clunky)
- No hard and fast rules
- How can we check our loss curve output?

LR Range

Question: Does model capture LR range well?

For each sample, track min/max of LR

Compare actual min/max LR with distributions (devlag $\geq$ 8 years)



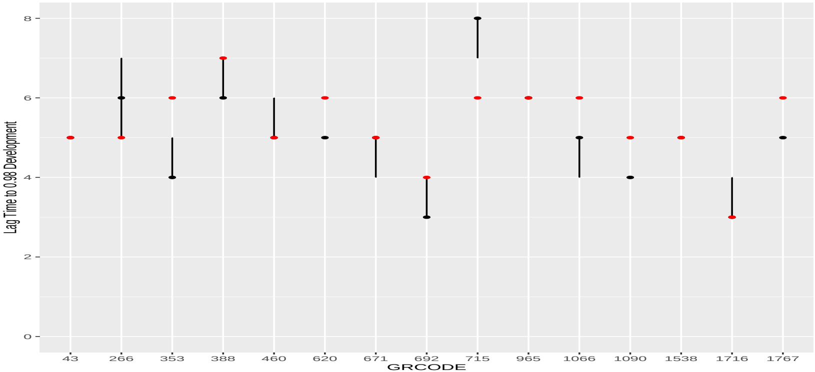
LR Range

- Better than expected
- Max/Min very sample-dependent
- May be worth considering quantiles
- Data a little too aggregated perhaps

Question: Does model capture time to final development well?

For each sample, observe time at which `gf` exceeds 0.98

Take 25%/75% intervals of time for each insurer, compare to data



Further Iterations

- Need better PPCs
- Further nesting for Insurer and Accident Year
- Look across product lines
- Try ADVI to help with iteration

Conclusions

- Alternative to Chain Ladder
- Allows interesting views into data
- Data source used is crude
- More work required!

Further Work

- Try out ADVI on the models
- Incorporate different ω and θ priors
- Generate fake data to try new approaches (change-point for example)
- Add hierarchy of product lines to model
- Write-up and contribute as Stan Case Study

Get In Touch

Mick Cooney
michael.cooney@applied.ai

Slides and code available on BitBucket:
https://www.bitbucket.org/kaybenleroll/dublin_r_workshops