From R to Julia: Converting Workshop Code

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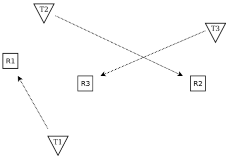
Background

- Co-organiser of Dublin R
- Give regular workshops on various topics
- Linear Dynamical Systems / Gaussian Processes
- Heavy linear algebra, ideal for translation

Before We Begin...

- Still learning this stuff myself
- Much better at R than Julia
- No benchmarking at all — unfair comparisons
- This talk an excuse to learn as much as anything

Power Control Algorithm



- Network of n transmitter/receiver pairs
- Power level: $p_i > 0$, Gain: $G_{ij} > 0$, Threshold: γ
- Signal power at receiver i: $s_i = G_{ii} p_i$.
- Noise plus interference: $q_i = \sigma + \sum_{j \neq i} G_{ij} p_j$
- SINR: $S_i = \frac{s_i}{q_i} = \alpha\gamma$, safety margin: α

Simple power update algorithm:

$$p_i(t+1) = p_i(t)\left(\frac{\alpha\gamma}{S_i(t)}\right)$$

Rearrange in matrix form:

$$\begin{bmatrix} p_1(t+1) \\ p_2(t+1) \\ p_3(t+1) \end{bmatrix} = \begin{bmatrix} 0 & \frac{\alpha\gamma G_{12}}{G_{11}} & \frac{\alpha\gamma G_{13}}{G_{11}} \\ \frac{\alpha\gamma G_{21}}{G_{22}} & 0 & \frac{\alpha\gamma G_{23}}{G_{22}} \\ \frac{\alpha\gamma G_{31}}{G_{33}} & \frac{\alpha\gamma G_{32}}{G_{33}} & 0 \end{bmatrix} \begin{bmatrix} p_1(t) \\ p_2(t) \\ p_3(t) \end{bmatrix} + \begin{bmatrix} \frac{\alpha\gamma\sigma}{G_{11}} \\ \frac{\alpha\gamma\sigma}{G_{22}} \\ \frac{\alpha\gamma\sigma}{G_{33}} \end{bmatrix}$$

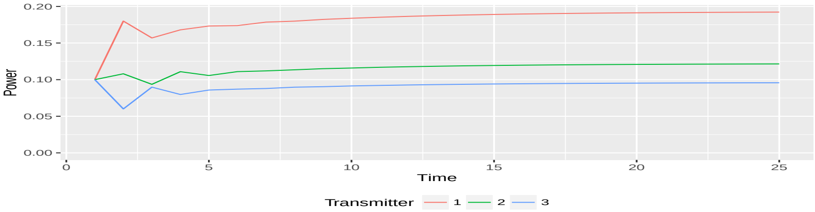
$$p_i(t+1) = Ap(t) + b$$

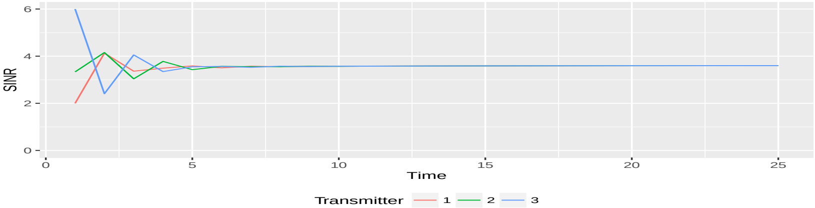
R Code

```
G <- matrix(c(1.0, 0.2, 0.2,
              0.1, 2.0, 0.4,
              0.3, 0.1, 3.0), ncol = 3, byrow = TRUE);

gamma <- 3.0;
alpha <- 1.2;
sigma <- 0.01;

N <- dim(G)[1];

mask  <- 1 - diag(N);
numer <- alpha * gamma * G;
denom <- matrix(rep(diag(G), N), ncol = N);

A <- mask * (numer / denom)

b <- alpha * gamma * sigma / diag(G)

q_mat <- mask * G;

n_iter <- 25;

pout    <- matrix(0, ncol = n_iter, nrow = N);
SINRout <- matrix(0, ncol = n_iter, nrow = N);

p0 <- rep(0.1, N);

pout[,1]    <- p0;
q           <- sigma + q_mat %*% p0;
SINRout[,1] <- (diag(G) * pout[,1]) / q;
```

R Code

```
for(i in 1:(n_iter-1)) {
    pout[,i+1] <- A %*% pout[,i] + b

    q <- sigma + q_mat %*% pout[,i+1]

    SINRout[,i+1] <- (diag(G) * pout[,i+1]) / q
}

power.plot <- qplot(Var2, value, data = melt(pout), geom = 'line', colour = as.character(Var1), size = I(0.5)) +
    xlab('Time') + ylab('Power') +
    expand_limits(y = 0) +
    theme(legend.position = 'bottom') +
    scale_colour_discrete(name = 'Transmitter')

sinr.plot <- qplot(Var2, value, data = melt(SINRout), geom = 'line', colour = as.character(Var1), size = I(0.5)) +
    xlab('Time') + ylab('SINR') +
    expand_limits(y = 0) +
    theme(legend.position = 'bottom') +
    scale_colour_discrete(name = 'Transmitter')
```


Power
0.20
0.15
0.10
0.05
0.00
0
5
10
15
20
25
Time
Transmitter 1 2 3


SINR
6
4
2
0
0
5
10
15
20
25
Time
Transmitter 1 2 3

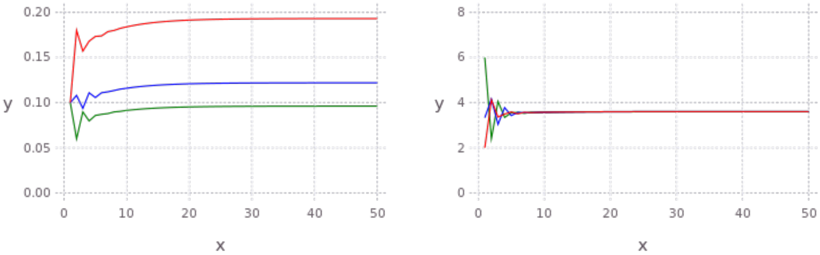
Julia Code

```
G = [1.0 0.2 0.2; 0.1 2.0 0.4; 0.3 0.1 3.0];

N = size(G)[1];
K = 50; # Number of iterations of the circuit

gamma = 3.0;
alpha = 1.2;
sigma = 0.01;

A = ((alpha * gamma * G) .* (ones(3,3) - eye(3))) ./ repmat(diag(G), 1, 3);

b = alpha * gamma * sigma ./ diag(G);

p    = zeros(N, K);
SINR = zeros(N, K);


p[:,1]     = [0.1 0.1 0.1];
q          = sigma + (G - diagm(diag(G))) * p[:,1];
SINR[:,1] = diag(G) .* p[:,1] ./ q;

for i = 2:K
    p[:,i]     = A * p[:,i-1] + b;
    q          = sigma + (G - diagm(diag(G))) * p[:,i];
    SINR[:,i] = (diag(G) .* p[:,i]) ./ q;
end
```

Julia Code

```
p1 = plot(layer(x = 1:K, y = p[1,:], Geom.line(), Theme(default_color = colorant"red"))
         ,layer(x = 1:K, y = p[2,:], Geom.line(), Theme(default_color = colorant"blue"))
         ,layer(x = 1:K, y = p[3,:], Geom.line(), Theme(default_color = colorant"green"))
         )

p2 = plot(layer(x = 1:K, y = SINR[1,:], Geom.line(), Theme(default_color = colorant"red"))
         ,layer(x = 1:K, y = SINR[2,:], Geom.line(), Theme(default_color = colorant"blue"))
         ,layer(x = 1:K, y = SINR[3,:], Geom.line(), Theme(default_color = colorant"green"))
         )
```

Julia Code



Temperatures in a Multicore Processor

Temperature of a process at two locations $T = (T_1, T_2)$

Affine functions of the power dissipated by three cores denoted $P = (P_1, P_2, P_3)$

P_1	P_2	P_3	T_1	T_2
10W	10W	10W	27C	29C
100W	10W	10W	45C	37C
10W	100W	10W	41C	49C
10W	10W	100W	35C	55C

R Code

```
C <- matrix(c( 10,  10,  10,   0,   0,   0, 1, 0
             ,  0,   0,   0,  10,  10,  10, 0, 1
             ,100,  10,  10,   0,   0,   0, 1, 0
             ,  0,   0,   0, 100,  10,  10, 0, 1
             , 10, 100,  10,   0,   0,   0, 1, 0
             ,  0,   0,   0,  10, 100,  10, 0, 1
             , 10,  10, 100,   0,   0,   0, 1, 0
             ,  0,   0,   0,  10,  10, 100, 0, 1),
            byrow = TRUE, ncol = 8, nrow = 8)

d <- c(27, 29, 45, 37, 41, 49, 35, 55)

output <- solve(C, d)

A <- matrix(output[1:6], byrow = TRUE, ncol = 3)
b <- output[7:8]
```

Julia Code

```
C = [ 10  10  10   0   0   0 1 0;
       0   0   0  10  10  10 0 1;
     100  10  10   0   0   0 1 0;
       0   0   0 100  10  10 0 1;
      10 100  10   0   0   0 1 0;
       0   0   0  10 100  10 0 1;
      10  10 100   0   0   0 1 0;
       0   0   0  10  10 100 0 1]


d = [27; 29; 45; 37; 41; 49; 35; 55]

output = C \ d

A = [output[1:3]'; output[4:6]']
b = output[7:8]

(70 - b) ./ mapslices(sum, A, 2)
```

Concentration of Chemicals in Reaction Kinetics

Reaction chain:

$$C_1 \xrightarrow{k_1} C_2 \xrightarrow{k_2} C_3$$

Model the mixture proportions as a linear system:

$$\dot{x} = \begin{bmatrix} -k_1 & 0 & 0 \\ k_1 & -k_2 & 0 \\ 0 & k_2 & 0 \end{bmatrix} x$$

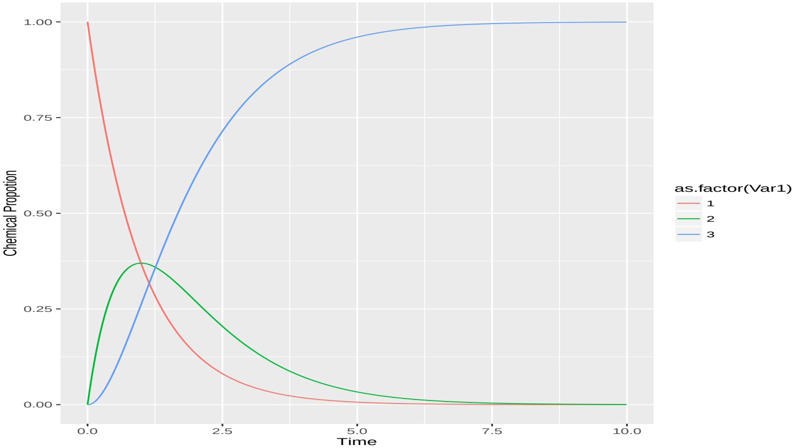
Use timestep h small to get:

$$x(t+1) = (I + hA)\, x(t)$$

R Code

```
k1 <- 1
k2 <- 1

A <- matrix(c(-k1, k1, 0, 0, -k2, k2, 0, 0, 0), ncol = 3)
h <- 0.01

A_update <- (diag(3) + h * A)
n_steps  <- 1000

x <- matrix(0, ncol = n_steps, nrow = 3)

x[, 1] <- c(1, 0, 0)


for(i in 2:n_steps) {
    x[, i] <- A_update %*% x[, i-1]
}
```

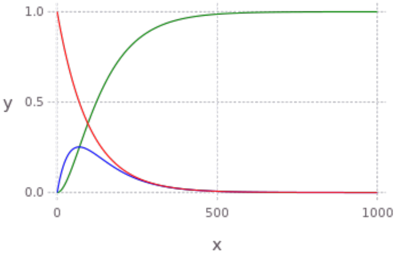
```
qplot((Var2 - 1) * h, value, data = melt(x), geom = 'line', colour = as.factor(Var1)
      ,xlab = 'Time'
      ,ylab = 'Chemical Propotion')
```



Julia Code

```
k1 = 1
k2 = 2

h = 0.01

A = [-k1   0 0;
      k1 -k2 0;
       0  k2 0]

A_update = eye(3) + h * A
n_steps  = 1000

x = zeros(3, n_steps)

x[:,1] = [1; 0; 0]

for i = 2:n_steps
    x[:,i] = A_update * x[:,i-1]
end
```

```
p1 = plot(layer(x = 1:n_steps, y = x[1,:], Geom.line(), Theme(default_color = colorant"red"))
         ,layer(x = 1:n_steps, y = x[2,:], Geom.line(), Theme(default_color = colorant"blue"))
         ,layer(x = 1:n_steps, y = x[3,:], Geom.line(), Theme(default_color = colorant"green"))
         )

draw(PNG("sec4_mixture_plot.png", 10cm, 7cm), p1)
```



Optimal Control of a Mass Unit

Optimal control problem for a force acting on a unit mass

Unit mass at position $p(t)$, velocity $\dot{p}(t)$, force $f(t)$, where $f(t) = x_i$ for $i1 < t \leq i$, for $i = 1, ..., 10$.

(a) Assume the mass has zero initial position and velocity: $p(0) = \dot{p}(0) = 0$. Minimise $\int_0^{t=10} f(t)^2 \, dt$ subject to: $p(10) = 1$, $\dot{p}(10) = 0$, and $p(5) = 0$.

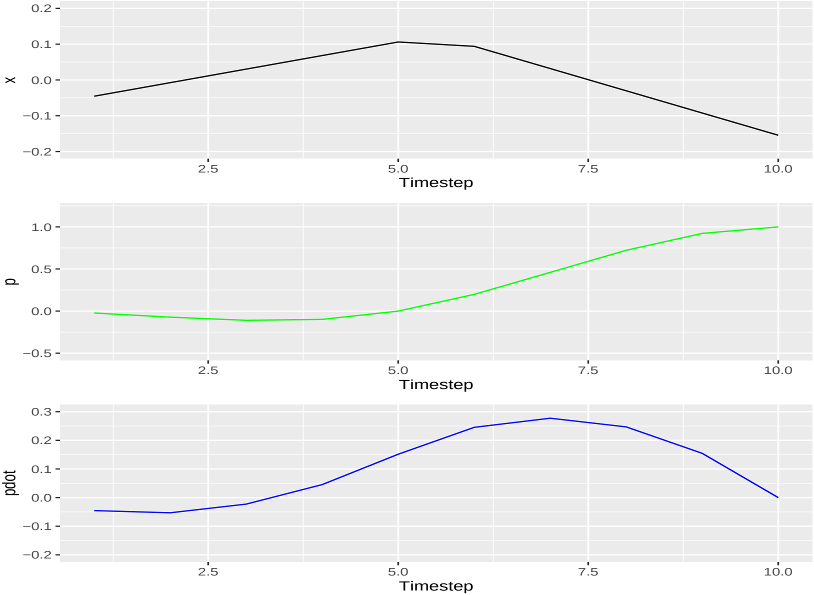
Plot the optimal force f and the resulting p and $\dot{p}$

(b) Assume the mass has initial position $p(0) = 0$ and velocity $\dot{p}(0) = 1$. Our goal is to bring the mass near or to the origin at $t = 10$, at or near rest, i.e. we want $J_1 = p(10)^2 + \dot{p}(10)^2$ small, while keeping $J_2 = \int_0^{t=10} f(t)^2 \, dt$ small, or at least not too large.

Plot the optimal trade-off curve between J_1 and J_2

R Code

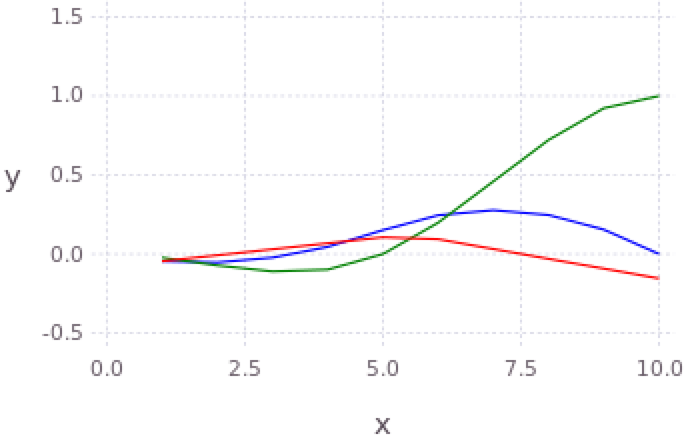
```
p10  <- seq(9.5, 0.5, by = -1);
pd10 <- rep(1, 10);
p0   <- c(seq(4.5, 0.5, by = -1), rep(0, 5));

A <- rbind(p10, pd10, p0);
y <- c(1, 0, 0);


x <- MASS::ginv(A) %*% y;
sqrt(sum(x * x))

## [1] 0.249241

x <- corpcor::pseudoinverse(A) %*% y;
sqrt(sum(x * x))

## [1] 0.249241

T1 <- pracma::Toeplitz(rep(1, 10), c(1, rep(0, 9)));
pdot <- T1 %*% x;


T2 <- pracma::Toeplitz(rev(p10), c(0.5, rep(0, 9)));
p <- T2 %*% x;
```


x
p
pdot
Timestep
Timestep
Timestep

Julia Code

```
using Gadfly
using SpecialMatrices


p10  = linspace(9.5,0.5,10)
pd10 = ones(10)
p0   = [linspace(4.5,0.5,5); zeros(5)]

A = [p10'; pd10'; p0']
y = [1; 0; 0]

x = pinv(A) * y

print(sqrt(sum(x' * x)))

T1 = full(Toeplitz([zeros(9); 1; ones(9)]))
pdot = T1 * x

T2 = full(Toeplitz([zeros(9); linspace(0.5,9.5,10)]))
p = T2 * x

plotoutput = plot(layer(x = 1:10, y = x,    Geom.line(), Theme(default_color = colorant"red"))
                 ,layer(x = 1:10, y = p,    Geom.line(), Theme(default_color = colorant"green"))
                 ,layer(x = 1:10, y = pdot, Geom.line(), Theme(default_color = colorant"blue"))
                  )

draw(PNG("sec5_plots.png", 10cm, 7cm), plotoutput)
```

Julia Code

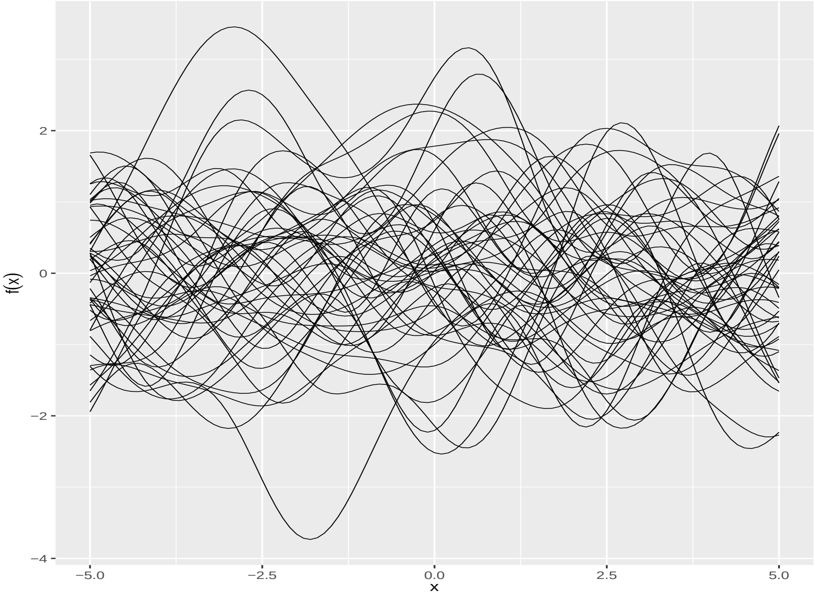


Gaussian Processes

- Gaussian Processes is a linear-algebra heavy technique
- Uses RNGs drawn from a Multivariate Normal distribution, $\mathcal{N}(\mu, \Sigma)$: mean μ, covariance Σ
- Not a huge amount of support in the languages
- Ideal topic for Julia implementation (but also done in Stan)

R Code

```
calc_covar <- function(X1, X2, l=1) {
    Sigma <- outer(X1, X2, function(a, b) exp(-0.5 * (abs(a - b) / l)^2));

    return(Sigma)
}

x_seq <- seq(-5, 5, by = 0.1);

sigma <- calc_covar(x_seq, x_seq, 1);

gp_data <- MASS::mvrnorm(50, rep(0, length(x_seq)), sigma);

plot_dt <- melt(gp_data);
setDT(plot_dt);

plot_dt[, x := x_seq[Var2]];
```


2
0
−2
−4
f(x)
−5.0
−2.5
0.0
2.5
5.0
x

Julia Code

A couple of utility functions first:

```
function calc_covar(x, y)
    N1 = length(x)
    N2 = length(y)

    sigma = zeros(N1, N2)

    for i = 1:N1
        for j = 1:N2
            sigma[i, j] = exp(-0.5 * (abs(x[i] - y[j]) / 1)^2)
        end
    end

    return sigma
end


function matplot(d::DataFrame)
    (row,col) = size(d)
    dStack = stack(d)
    dStack[:ndx] = rep(1:row,col)
    Gadfly.plot(dStack, x=:ndx, y=:value, group=:variable, Geom.line)
end
```


BOMBCAT
Will blow this bitch up!
fakeposters.com

What Went Wrong?

Multivariate Normal Distribution: Mean μ, Covariance Σ

Covariance Matrix requires Σ to be positive-definite

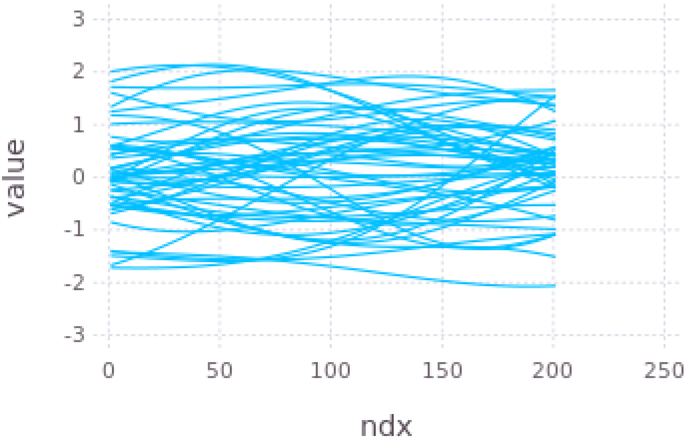
```
corpcor::is.positive.definite(sigma)

## [1] FALSE
```

Can we find PD Σ_{new} close to Σ?

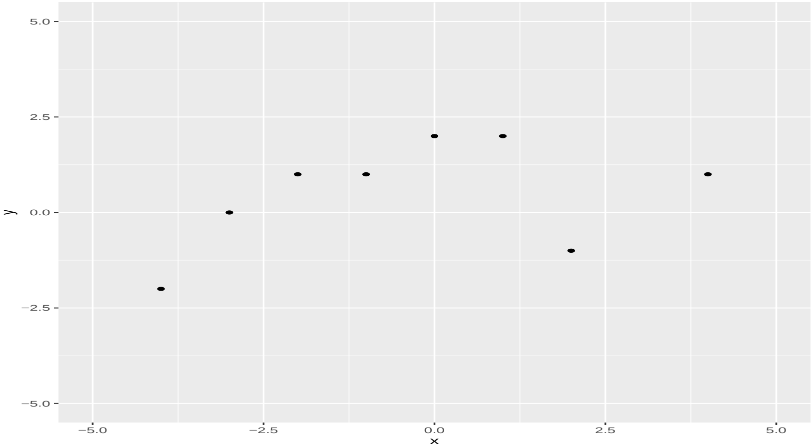
Singular Value Decomposition

Julia Code

```
using DataFrames
using Gadfly
using Distributions

include("functions.jl")



N = 201
x = linspace(-1, 1, N)

mu    = zeros(N)
sigma = calc_covar(x, x)

### Need to make sigma postive-definite
d, v = eig(sigma)

d[d .< 1e-12] = 1e-12

sigma = v * diagm(d) * v'



gp_data = rand(MvNormal(mu, sigma), 50)

gp_plot1 = gp_data |> DataFrame |> matplot

draw(PNG("sec6_gp_simple.png", 10cm, 7cm), gp_plot1)
```

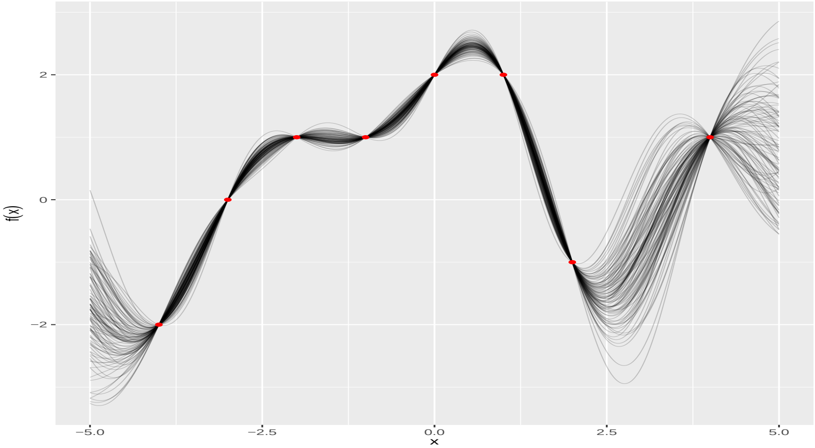

value
3
2
1
0
-1
-2
-3
0
50
100
150
200
250
ndx

Gaussian Processes Regression



```
x_seq <- seq(-5, 5, by = 0.01);

kxx_inv <- solve(calc_covar(data_dt$x, data_dt$x));

Mu    <- calc_covar(x_seq, data_dt$x) %*% kxx_inv %*% data_dt$y;
Sigma <- calc_covar(x_seq, x_seq) -
    calc_covar(x_seq, data_dt$x) %*% kxx_inv %*% calc_covar(data_dt$x, x_seq);

gp_data <- MASS::mvrnorm(100, Mu, Sigma);

plot_dt <- melt(gp_data);
setDT(plot_dt);

plot_dt[, x := x_seq[Var2]];
```

```
ggplot() +
    geom_line(aes(x, value, group = Var1), data = plot_dt, size = I(0.3), alpha = I(0.2)) +
    geom_point(aes(x, y), data = data_dt, colour = 'red') +
    xlab(expression(x)) +
    ylab(expression(f(x)));
```

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Will blow this bitch up!
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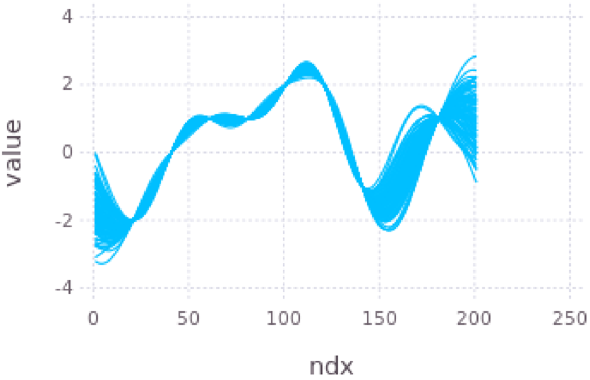
Julia Code

```
### Regression code
data_x = [-4 -3 -2 -1  0  1  2  4]
data_y = [-2  0  1  1  2  2 -1  1]

N = 201
x = linspace(-5, 5, N)


kxx_inv = inv(calc_covar(data_x, data_x))
Mu      = calc_covar(x, data_x) * kxx_inv * data_y'
Sigma   = calc_covar(x, x) - calc_covar(x, data_x) * kxx_inv * calc_covar(data_x, x)

### Need to make sigma postive-definite
Mu_vec   = Mu[:,1]
Sigma_PD = Sigma - minimum(eigvals(Symmetric(Sigma))) * I


gpreg_data = rand(MvNormal(Mu_vec, Sigma_PD), 100)

gp_plot2 = gpreg_data |> DataFrame |> matplot

draw(PNG("sec6_gpreg.png", 10cm, 7cm), gp_plot2)
```


4
2
0
-2
-4
value
0
50
100
150
200
250
ndx

Some Gotchas

- Trouble working with `knitr`
- Needed to install new ESS
- Gadfly is still pretty immature — plotting needs some improvement
- Cache of packages did need to recompile
- Could not find good introduction documentation
- Need to be more careful with linear algebra (e.g. column and row vectors are not the same)

Summary

- Julia is very powerful — Thumbs up
- Not for beginners
- Be prepared for irritation initially
- Could use some more tools and a bit more maturity
- Excellent for heavy linear-algebra problems
- Seems simple to switch from Matlab

Word

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Slides and code available on BitBucket:
`https://www.bitbucket.org/kaybenleroll/dublin_r_workshops`