Bayesian Modelling of Loss Curves in Insurance

Mick Cooney
michael.cooney@applied.ai

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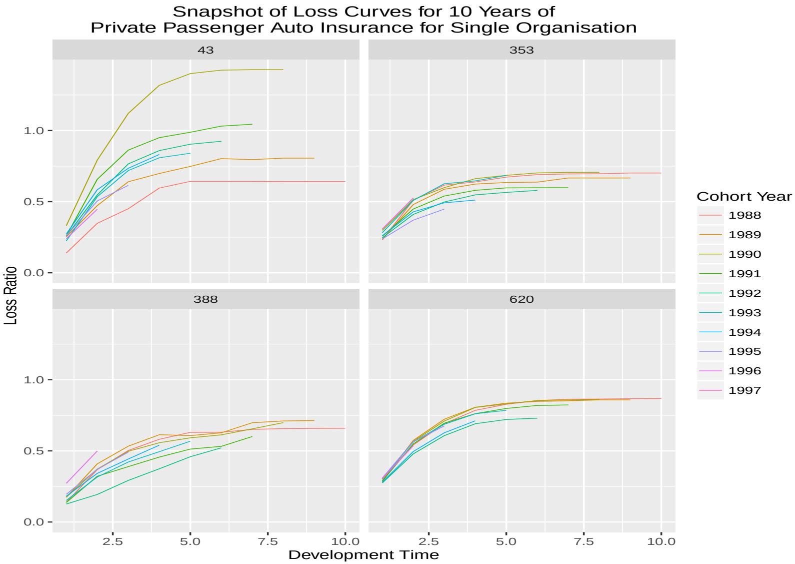
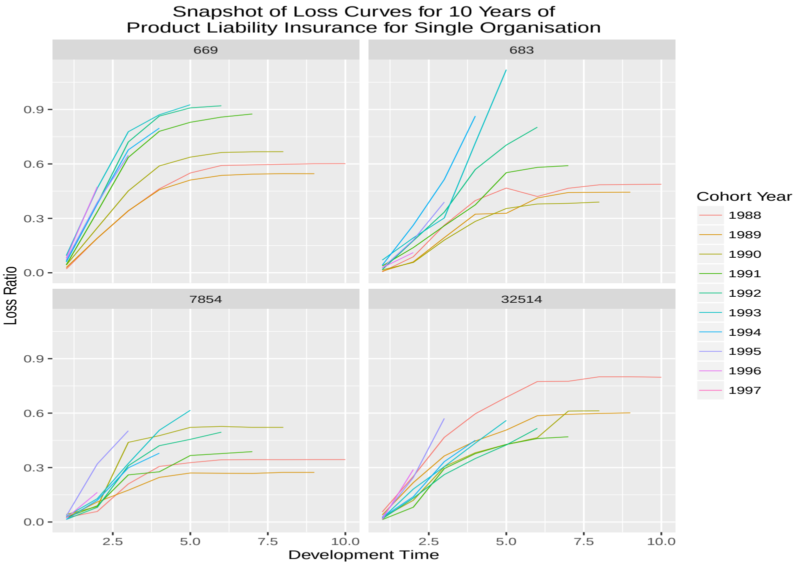
Structure of Talk

- Loss Curves
- Chain Ladder Modelling (package `ChainLadder`)
- Loss Growth Modelling
- Expanding the Model
- Posterior Predictive Checks
- Summary

Loss Curves

```
use_grcode <- c(43,353,388,620)

ppauto_ss_dt <- ppauto_dt[GRCODE %in% use_grcode
                         ][DevelopmentYear < 1998
                         ][, .(grcode      = GRCODE
                              ,accyear     = AccidentYear
                              ,devlag      = DevelopmentLag
                              ,premium     = EarnedPremDIR_B
                              ,cumloss     = CumPaidLoss_B
                              ,loss_ratio = CumPaidLoss_B / EarnedPremDIR_B)]


print(dcast(ppauto_ss_dt[grcode == 43]
           ,grcode + accyear + premium ~ devlag
           ,value.var = 'cumloss'),digits=3)

##     grcode accyear premium     1     2     3     4     5     6     7    8    9  10
##  1:     43    1988     957   133   333   431   570   615   615   615  614  614 614
##  2:     43    1989    3695   934  1746  2365  2579  2763  2966  2940 2978 2978  NA
##  3:     43    1990    6138  2030  4864  6880  8087  8595  8743  8763 8762   NA  NA
##  4:     43    1991   17533  4537 11527 15123 16656 17321 18076 18308   NA   NA  NA
##  5:     43    1992   29341  7564 16061 22465 25204 26517 27124    NA   NA   NA  NA
##  6:     43    1993   37194  8343 19900 26732 30079 31249    NA    NA   NA   NA  NA
##  7:     43    1994   46095 12565 26922 33867 38338    NA    NA    NA   NA   NA  NA
##  8:     43    1995   51512 13437 26012 31677    NA    NA    NA    NA   NA   NA  NA
##  9:     43    1996   52481 12604 23446    NA    NA    NA    NA    NA   NA   NA  NA
## 10:     43    1997   56978 12292    NA    NA    NA    NA    NA    NA   NA   NA  NA
```

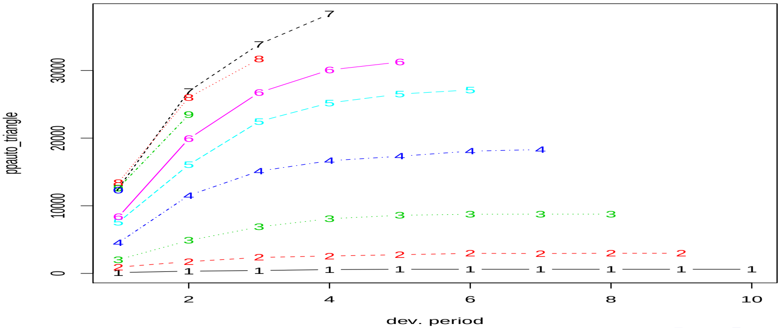

Snapshot of Loss Curves for 10 Years of
Private Passenger Auto Insurance for Single Organisation
43
353
388
620
Loss Ratio
0.0
0.5
1.0
Development Time
2.5
5.0
7.5
10.0
Cohort Year
1988
1989
1990
1991
1992
1993
1994
1995
1996
1997


Snapshot of Loss Curves for 10 Years of
Product Liability Insurance for Single Organisation
669
683
7854
32514
Loss Ratio
0.0
0.3
0.6
0.9
2.5
5.0
7.5
10.0
Development Time
Cohort Year
1988
1989
1990
1991
1992
1993
1994
1995
1996
1997

Chain Ladder

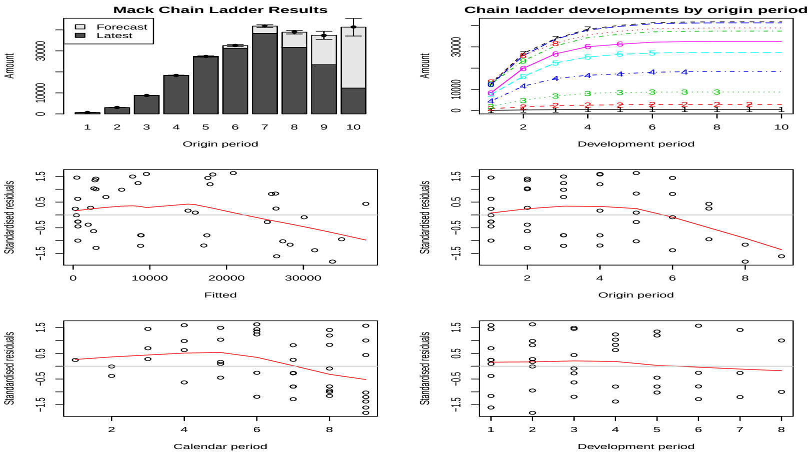
Standard R approach is `ChainLadder`

```
ppauto_mat <- as.matrix(dcast(ppauto_ss_dt[grcode == 43]
                             ,accyear ~ devlag
                             ,value.var = 'cumloss')[,-1,with=FALSE])

ppauto_triangle <- as.triangle(ppauto_mat)

plot(ppauto_triangle)
```

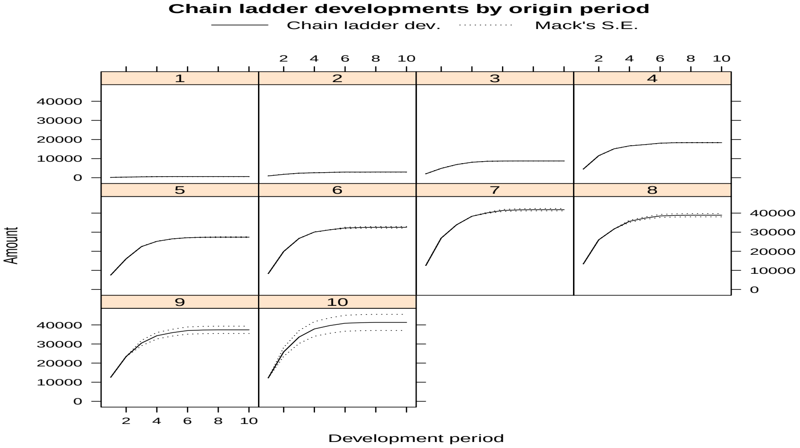


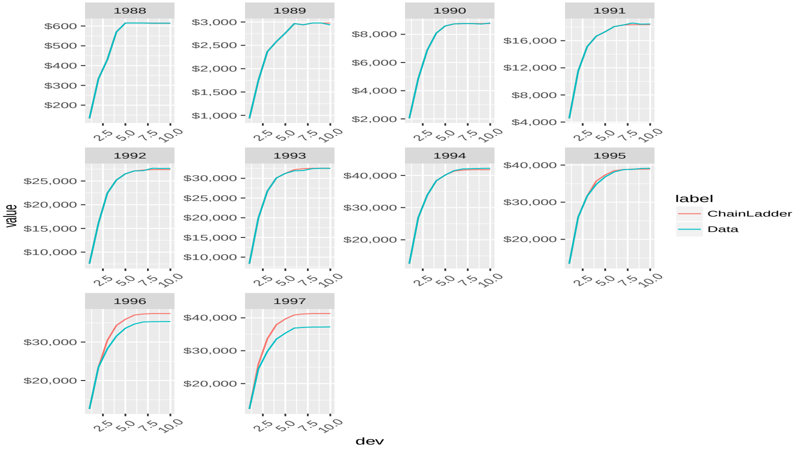
```
ppauto_mack <- MackChainLadder(ppauto_triangle, est.sigma = "Mack")

ppauto_mack$f

##  [1] 2.10486 1.29968 1.12655 1.04671 1.03069 1.00743 1.00292 1.00000 1.00000 1.00000

ppauto_mack$FullTriangle

##       dev
## origin     1     2       3       4       5       6       7       8       9      10
##      1   133   333   431.0   570.0   615.0   615.0   615.0   614.0   614.0   614.0
##      2   934  1746  2365.0  2579.0  2763.0  2966.0  2940.0  2978.0  2978.0  2978.0
##      3  2030  4864  6880.0  8087.0  8595.0  8743.0  8763.0  8762.0  8762.0  8762.0
##      4  4537 11527 15123.0 16656.0 17321.0 18076.0 18308.0 18361.5 18361.5 18361.5
##      5  7564 16061 22465.0 25204.0 26517.0 27124.0 27325.6 27405.5 27405.5 27405.5
##      6  8343 19900 26732.0 30079.0 31249.0 32208.1 32447.6 32542.4 32542.4 32542.4
##      7 12565 26922 33867.0 38338.0 40128.7 41360.4 41667.9 41789.6 41789.6 41789.6
##      8 13437 26012 31677.0 35685.7 37352.5 38499.0 38785.2 38898.6 38898.6 38898.6
##      9 12604 23446 30472.3 34328.5 35932.0 37034.8 37310.1 37419.2 37419.2 37419.2
##     10 12292 25873 33626.6 37882.0 39651.4 40868.4 41172.3 41292.6 41292.6 41292.6
```

```
plot(ppauto_mack)
```



```
plot(ppauto_mack, lattice = TRUE)
```



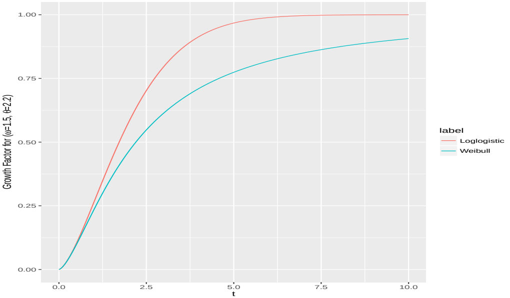

1988
1989
1990
1991
1992
1993
1994
1995
1996
1997
value
dev
label
ChainLadder
Data

Loss Growth Modelling

Model growth cumulative losses as function
Scale losses by premium

$$g(t;\ \omega, \theta) = 1 - \exp\left(-\left(\frac{t}{\theta}\right)^{\omega}\right) \qquad \text{Loglogistic Function}$$

$$g(t;\ \omega, \theta) = \frac{t^{\omega}}{t^{\omega} + \theta^{\omega}} \qquad \text{Weibull Function}$$



Start with the Loglogistic Model

$$g(t;\ \omega, \theta) = 1 - \exp\left(-\left(\frac{t}{\theta}\right)^{\omega}\right)$$

Treat as hierarchical model - group by Accident Year

$$\text{Loss}_{\text{Y},t} \sim \text{Normal}(\mu_{\text{L},\text{Y},t}, \sigma_L)$$

where

$$\begin{aligned} \mu_{\text{L},\text{Y},t} &= \text{LR}_{\text{Y}} \times \text{P}_{\text{Y}} \times g(t;\ \omega, \theta) \\ \sigma_{\text{L}} &= \text{P}_{\text{Y}} \times \sigma \\ \text{LR}_{\text{Y}} &\sim \text{Lognormal}(\mu_{\text{LR}}, \sigma_{\text{LR}}) \end{aligned}$$

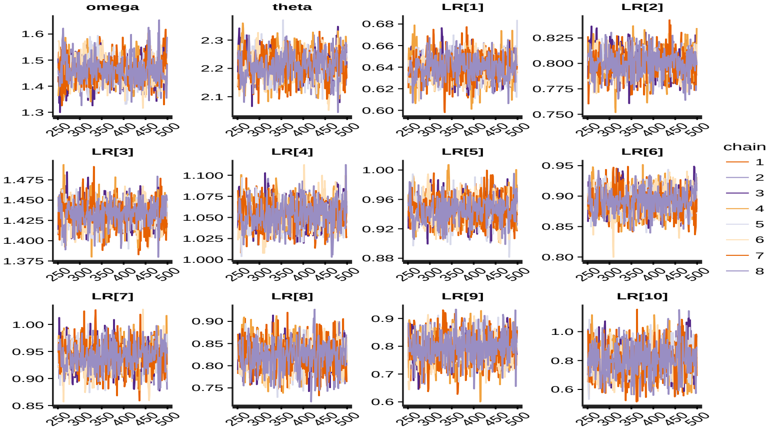
Normal prior for μ_{LR}. Lognormal prior for ω, θ, σ_{LR}, σ.

```
functions {
  real growth_factor_weibull(real t, real omega, real theta) {
    real factor;

    factor <- 1 - exp(-(t/theta)^omega);

    return(factor);
  }

  real growth_factor_loglogistic(real t, real omega, real theta) {
    real factor;

    factor <- ((t^omega) / (t^omega + theta^omega));

    return(factor);
  }
}

data {
  int<lower=0,upper=1> growthmodel_id;

  int n_data;
  int n_time;
  int n_cohort;

  int cohort_id[n_data];
  int t_idx[n_data];

  real<lower=0> t_value[n_time];

  real premium[n_cohort];
  real loss[n_data];

  int cohort_maxtime[n_cohort];
}
```

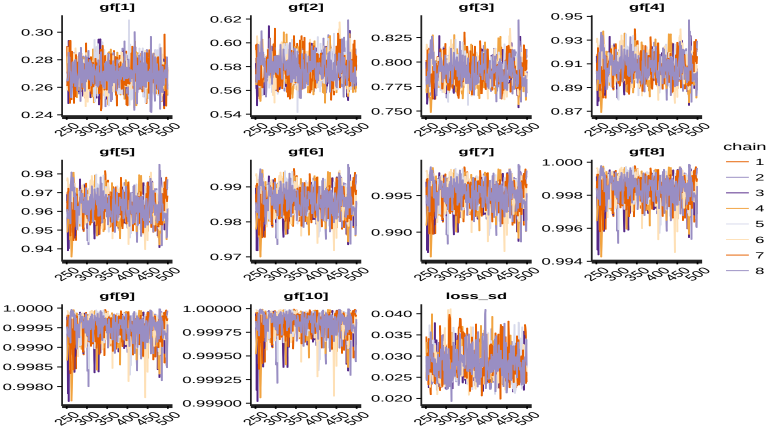
```
parameters {
  real<lower=0> omega;
  real<lower=0> theta;

  real<lower=0> LR[n_cohort];

  real mu_LR;
  real<lower=0> sd_LR;

  real<lower=0> loss_sd;
}

transformed parameters {
  real gf[n_time];
  real loss_mean[n_cohort, n_time];

  for(i in 1:n_time) {
    if(growthmodel_id == 1) {
      gf[i] <- growth_factor_weibull    (t_value[i], omega, theta);
    } else {
      gf[i] <- growth_factor_loglogistic(t_value[i], omega, theta);
    }
  }

  for(i in 1:n_data) {
    loss_mean[cohort_id[i], t_idx[i]] <- LR[cohort_id[i]] * premium[cohort_id[i]] * gf[t_idx[i]];
  }
}
```

```
model {
  mu_LR ~ normal(0, 0.5);
  sd_LR ~ lognormal(0, 0.5);

  LR ~ lognormal(mu_LR, sd_LR);

  loss_sd ~ lognormal(0, 0.7);

  omega ~ lognormal(0, 1);
  theta ~ lognormal(0, 1);

  for(i in 1:n_data) {
    loss[i] ~ normal(loss_mean[cohort_id[i], t_idx[i]], premium[cohort_id[i]] * loss_sd);
  }
}
```

Stan Output

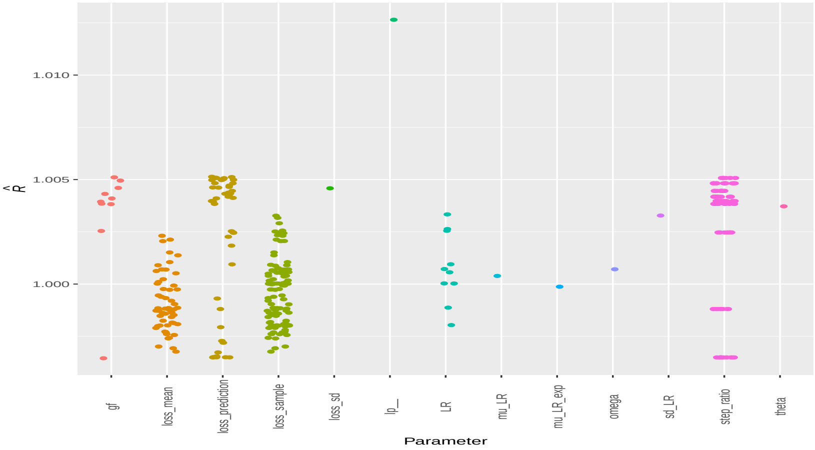


omega
theta
LR[1]
LR[2]
LR[3]
LR[4]
LR[5]
LR[6]
LR[7]
LR[8]
LR[9]
LR[10]
chain
1
2
3
4
5
6
7
8

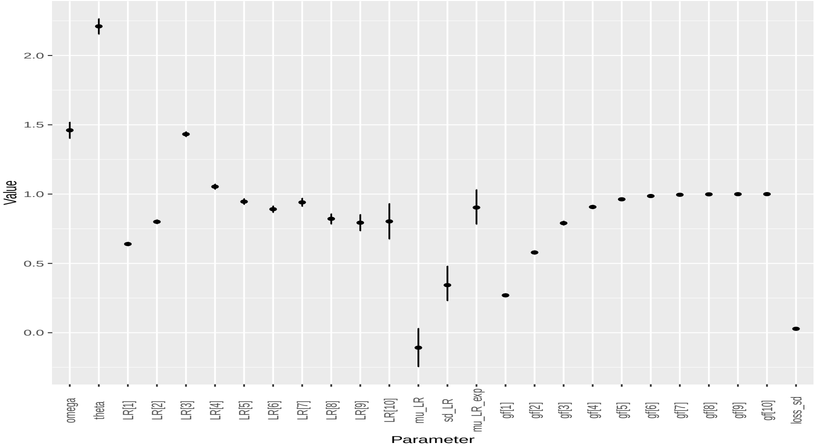

gf[1]
gf[2]
gf[3]
gf[4]
gf[5]
gf[6]
gf[7]
gf[8]
gf[9]
gf[10]
loss_sd
chain
1
2
3
4
5
6
7
8
250 300 350 400 450 500

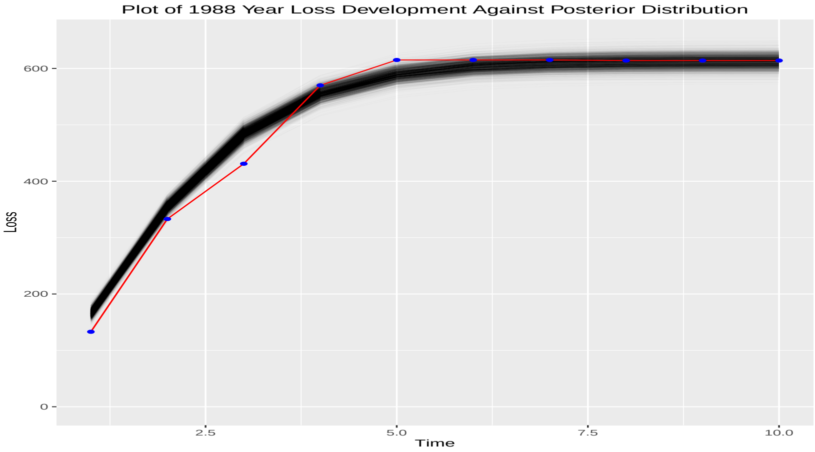
Check simple diagnostics:

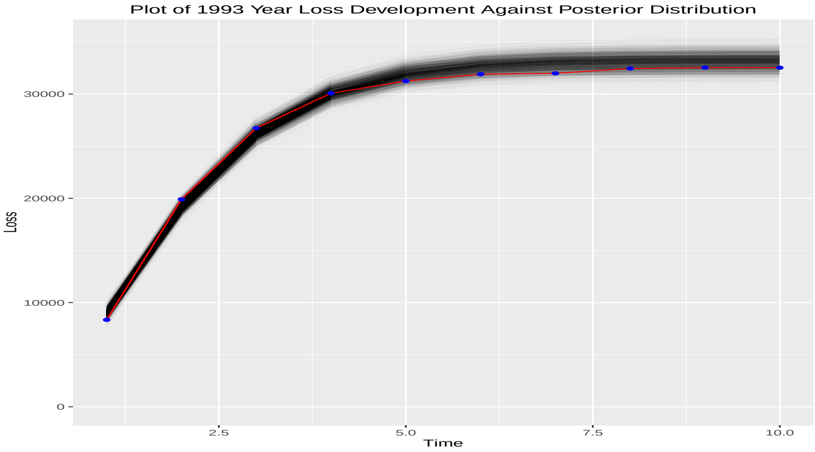
```
## Warning:  Removed 110 rows containing missing values (geom_point).
```

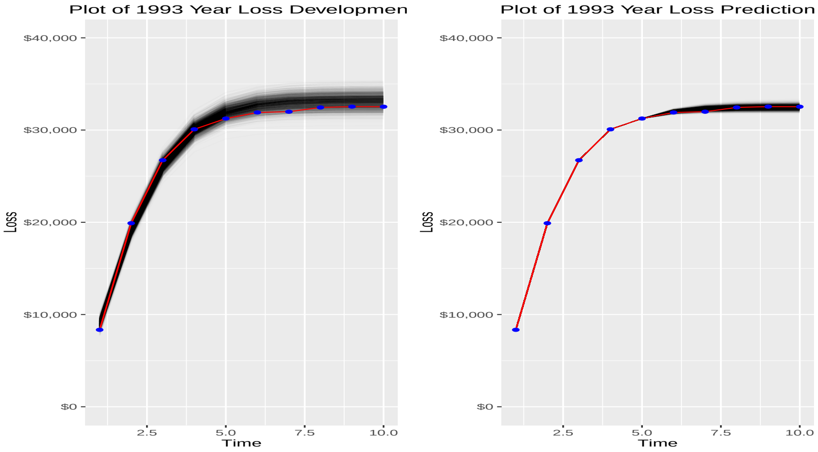


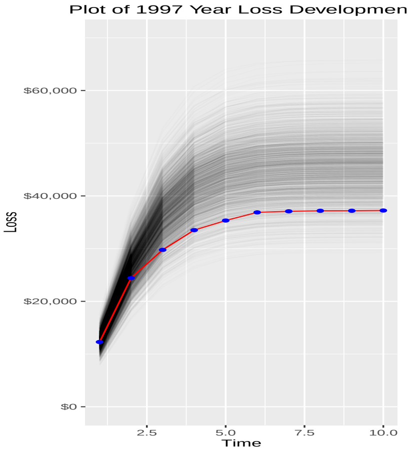
Check parameter values:

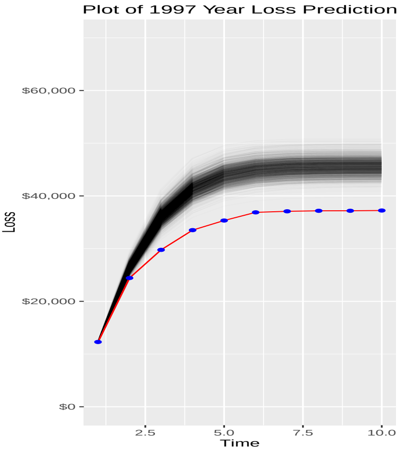



Plot of 1988 Year Loss Development Against Posterior Distribution
Loss
600
400
200
0
2.5
5.0
7.5
10.0
Time


Plot of 1993 Year Loss Development Against Posterior Distribution
Loss
30000
20000
10000
0
2.5
5.0
7.5
10.0
Time


Plot of 1993 Year Loss Developmen
Loss
$40,000
$30,000
$20,000
$10,000
$0
2.5
5.0
7.5
10.0
Time
Plot of 1993 Year Loss Prediction
Loss
$40,000
$30,000
$20,000
$10,000
$0
2.5
5.0
7.5
10.0
Time


Plot of 1997 Year Loss Developmen
$60,000
$40,000
$20,000
$0
Loss
2.5
5.0
7.5
10.0
Time


Plot of 1997 Year Loss Prediction
$60,000
$40,000
$20,000
$0
Loss
2.5
5.0
7.5
10.0
Time

Model Iteration

How might we expand this model?

Allow ω and θ to be part of the hierarchy:

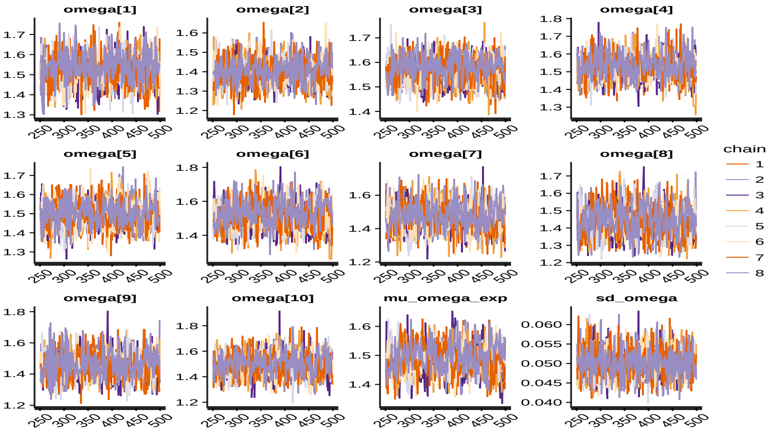
$$\begin{aligned} \omega &\rightarrow \omega_{\mathsf{Y}} \\ \theta &\rightarrow \theta_{\mathsf{Y}} \end{aligned}$$

Each Accident Year has individual $(\omega_{\mathsf{Y}}, \theta_{\mathsf{Y}})$ with

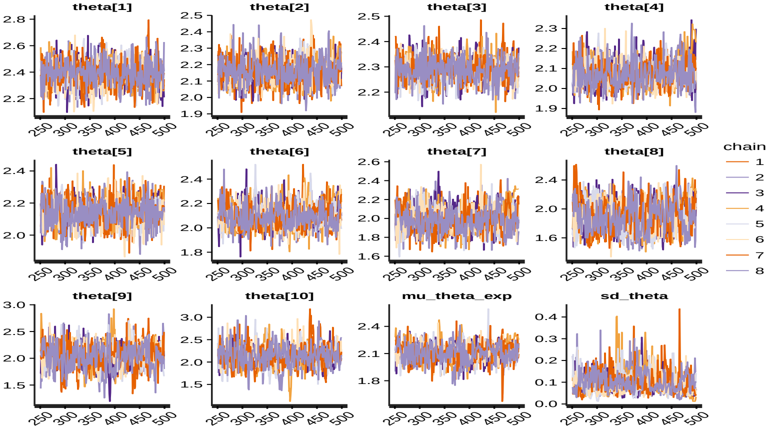
$$\begin{aligned} \omega_{\mathsf{Y}} &\sim \text{Lognormal}(\mu_\omega, \sigma_\omega) \\ \theta_{\mathsf{Y}} &\sim \text{Lognormal}(\mu_\theta, \sigma_\theta) \end{aligned}$$

$$\begin{aligned} \mu_\omega &\sim \text{Normal}(0, 1) \\ \sigma_\omega &\sim \text{Lognormal}(-3, 0.1) \\ \mu_\theta &\sim \text{Normal}(0, 1) \\ \sigma_\theta &\sim \text{Lognormal}(-3, 0.1) \end{aligned}$$

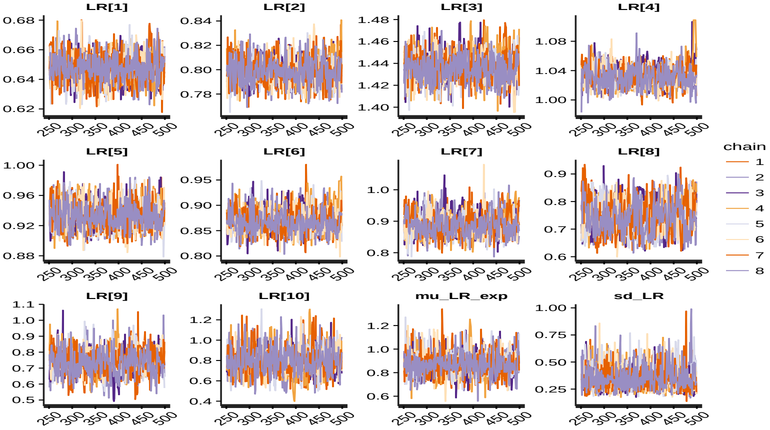
Individual Parameters - ω



Individual Parameters - θ



Individual Parameters - LR

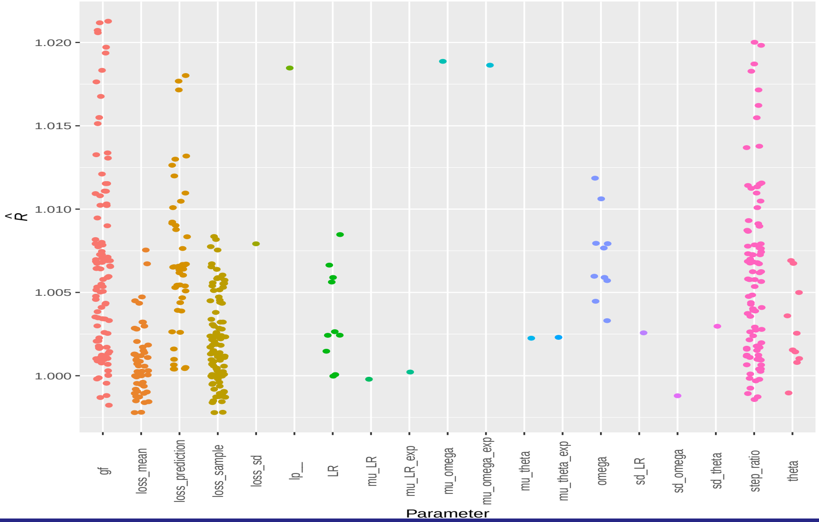


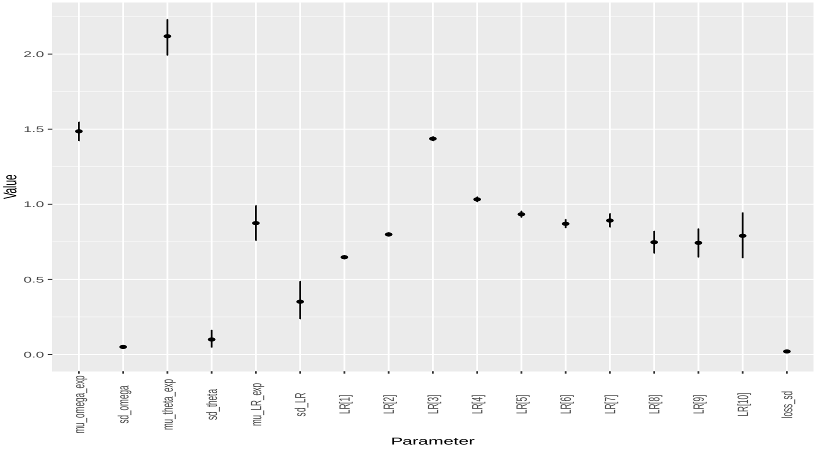
Problems with the Model

- Trouble with code
- Divergent transitions — had to raise `adapt_delta`
- Would not rely on output
- Data is very sparse for later Accident Years
- May revisit once other insurers added

Convergence Diagnostics

```
## Warning:  Removed 110 rows containing missing values (geom_point).
```





2.0
1.5
1.0
0.5
0.0
Value
mu_omega_exp
sd_omega
mu_theta_exp
sd_theta
mu_LR_exp
sd_LR
LR[1]
LR[2]
LR[3]
LR[4]
LR[5]
LR[6]
LR[7]
LR[8]
LR[9]
LR[10]
loss_sd
Parameter

Multiple Insurers

Use hierarchical model for multiple insurers

Each insurer gets LR_I

Posterior Predictive Checks

Posterior Predictive Checks

Getting more and more emphasis

Used to assess data aspects not modelled well

Use sample to generate ‘fake’ data to compare

Can also be used to generate predictions from data

No hard and fast rules

Need to think of checks for loss curves

Conclusions

Get In Touch

Mick Cooney
michael.cooney@applied.ai

Slides and code available on BitBucket:
https://www.bitbucket.org/kaybenleroll/dublin_r_workshops