信号与线性系统

第 7 讲

教材位置：第3章 傅立叶变换的性质

§3.8

内容概要：介绍傅立叶变换的性质、傅立叶变换的计算

讨论傅立叶变换的性质，旨在通过这些性质揭示信号时域特性与频域特性之间的关系，同时掌握和运用这些性质可以简化傅立叶变换对的求取。

前面学习的时域对称特性也是反应时域与频域的关系。

- 前讲回顾

1：常用信号的傅里叶变换

$$e^{-at}\varepsilon(t) \leftrightarrow \frac{1}{a+j\omega}, a>0$$

$$e^{-\alpha|t|} \leftrightarrow \frac{2\alpha}{\alpha^2+\omega^2}, a>0$$

$$f(t)=\begin{cases} e^{-\alpha t} & t>0 \\ -e^{\alpha t} & t<0 \end{cases} \qquad F(j\omega)=\frac{-2j\omega}{\alpha^2+\omega^2}$$

$$G\tau(t) \leftrightarrow \tau Sa(\frac{\omega\tau}{2})$$

$$\mathrm{sgn}(t) \leftrightarrow \frac{2}{j\omega}$$

$$\delta(t) \leftrightarrow 1$$

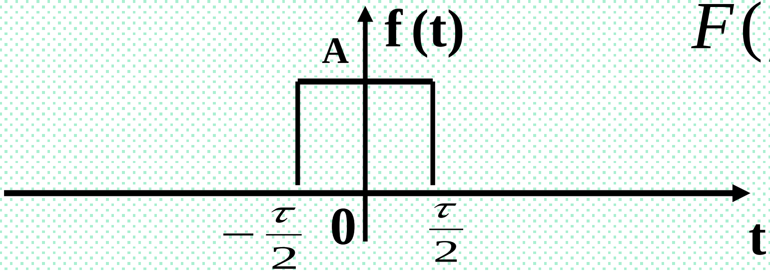
$$\boxed{\varepsilon(t) \leftrightarrow \pi\delta(\omega)+\frac{1}{j\omega}}$$

2:任意周期信号的傅里叶变换

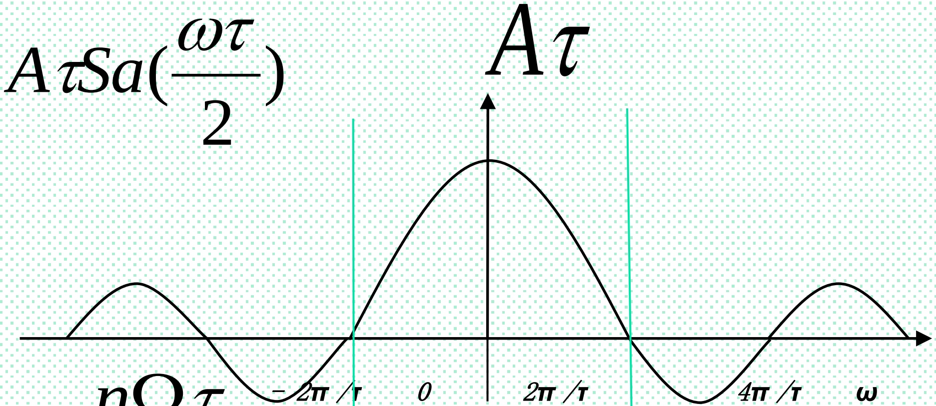
$$e^{j\omega_0 t} \leftrightarrow 2\pi\delta(\omega-\omega_0)$$

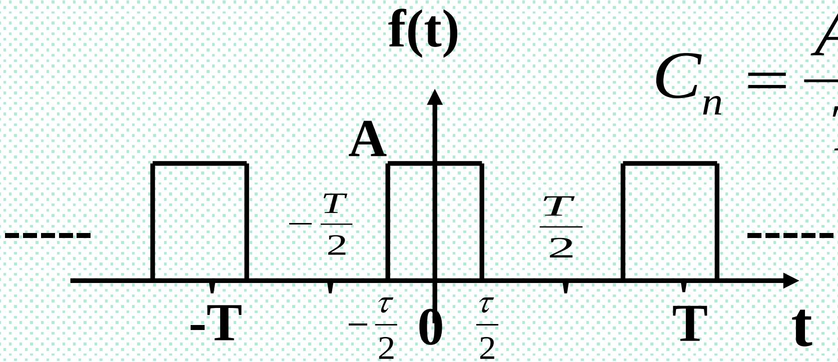
$$f_T(t) \leftrightarrow \sum_{n=-\infty}^{\infty} C_n e^{jn\Omega t}$$

$$f_T(t) \leftrightarrow 2\pi \sum_{n=-\infty}^{\infty} C_n \delta(\omega - n\Omega)$$

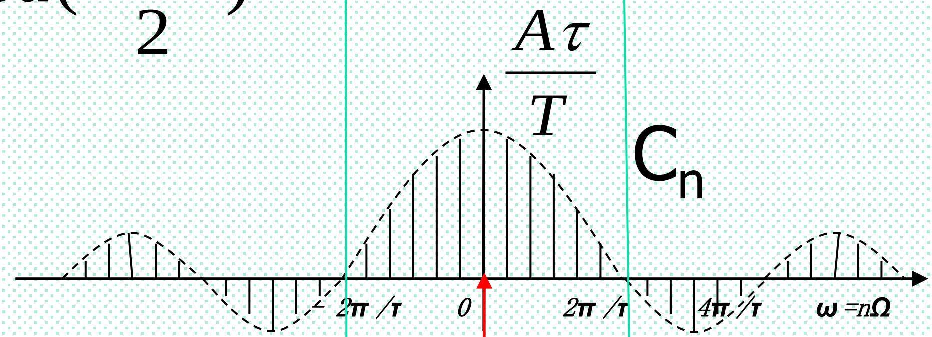


$$F(j\omega) = A\tau Sa(\frac{\omega\tau}{2})$$

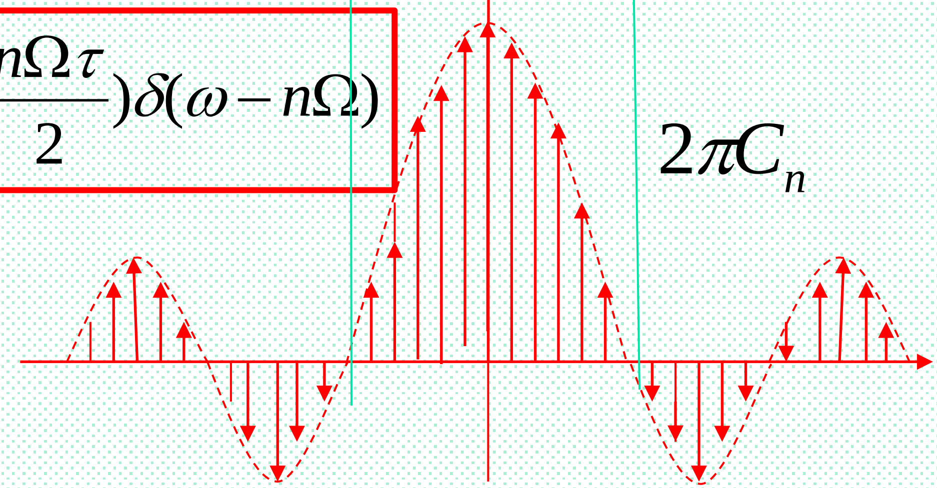




$$C_n = \frac{A\tau}{T} Sa(\frac{n\Omega\tau}{2})$$



$$\sum_{n=-\infty}^{\infty} 2\pi C_n \delta(\omega - n\Omega) \boxed{= 2\pi \sum_{n=-\infty}^{\infty} \frac{A\tau}{T} Sa(\frac{n\Omega\tau}{2})\delta(\omega - n\Omega)}$$



3：帕塞瓦尔定理与能量频谱

周期功率信号

$$P=\sum_{n=-\infty}^{\infty}\left|C_n\right|^2=C_0^2+2\sum_{n=1}^{\infty}\left|C_n\right|^2$$

$$=\left(\frac{A_0}{2}\right)^2+2\sum_{n=1}^{\infty}\left(\frac{A_n}{2}\right)^2$$

$$=\left(\frac{A_0}{2}\right)^2+\sum_{n=1}^{\infty}\frac{A_n^2}{2}$$

周期信号的平均功率等于该信号在完备正交函数集中各分量的平均功率之和。

非周期能量信号

$$E=\int_{-\infty}^{\infty}f^2(t)dt=\frac{1}{2\pi}\int_{-\infty}^{\infty}\left|F(j\omega)\right|^2d\omega$$

$$\frac{\left|F(j\omega)\right|^2}{\pi}$$ 能量密度频谱函数

开讲前言－本讲导入

- 已有傅立叶变换的知识
  - 信号的正交分解
  - 周期信号的傅立叶级数与频谱
  - 非周期信号频谱的连续与傅立叶变换的引入
    - 频谱密度函数
  - 周期信号的傅立叶变换
  - 常用信号的傅立叶变换
  - 傅立叶变换在信号能量频域分析的作用
- 为了更好的利用傅立叶变换进行信号分析需要进行傅立叶变换性质的分析，以从数学的角度提高计算傅立叶变换的能力

§3.8傅立叶变换的性质

1：线性

若 $f_1(t) \leftrightarrow F_1(j\omega),\ f_2(t) \leftrightarrow F_2(j\omega)$

则 $\boxed{af_1(t) + bf_2(t) \leftrightarrow aF_1(j\omega) + bF_2(j\omega)}$

2：延时性

若 $f(t) \leftrightarrow F(j\omega)$ 则 $\boxed{f(t-t_0) \leftrightarrow F(j\omega)e^{-j\omega t_0}}$

时域中延时 $\Longleftrightarrow$ 频域中移相

$$F(f(t-t_0)) = \int_{-\infty}^{\infty} f(t-t_0)e^{-j\omega t}dt$$

$$\boxed{(x = t - t_0, dx = dt)} = \int_{-\infty}^{\infty} f(x)e^{-j\omega(x+t_0)}dx = e^{-j\omega t_0}\int_{-\infty}^{\infty} f(x)e^{-j\omega x}dx$$

$$= F(j\omega)e^{-j\omega t_0} = |F(j\omega)|e^{j\varphi_\omega}e^{-j\omega t_0}$$

$$= |F(j\omega)|e^{j(\varphi_\omega - \omega t_0)}$$

对频率分量，幅度密度没变，相位改变量为：

$-\omega t_0$ (相移量跟频率值呈线性关系)

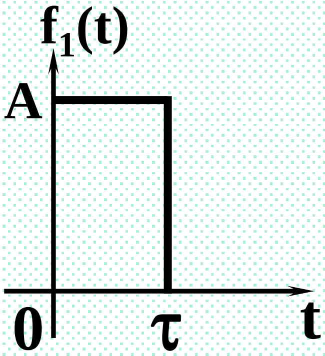
$$f_T(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} A_n Cos(n\Omega t + \varphi_n) \qquad A_n \qquad \varphi_n$$

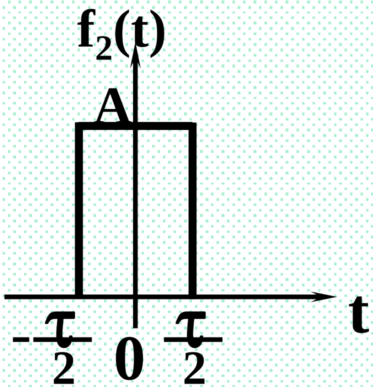
$$f_T(t - t_0) = \frac{A_0}{2} + \sum_{n=1}^{\infty} A_n Cos(n\Omega(t - t_0) + \varphi_n)$$

$$= \frac{A_0}{2} + \sum_{n=1}^{\infty} A_n Cos(n\Omega t + (\varphi_n - n\Omega t_0)) \qquad \varphi_n - n\Omega t_0$$

含义：　　信号在时域中延时和在频域中移相对应。

如正弦波在时间轴上的起点不同则相角随之变化。





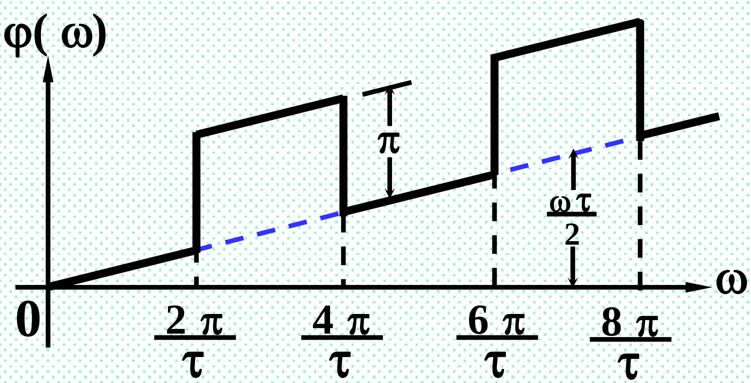
因为: $f_2(t)=AG_{\tau}(t)\leftrightarrow A\tau Sa(\frac{\omega\tau}{2})=F_2(j\omega)$

所以: $f_1(t)=f_2(t-\frac{\tau}{2})\leftrightarrow F_2(j\omega)e^{-j\omega\frac{\tau}{2}}=F_1(j\omega)$

$$F_1(j\omega)=F_2(j\omega)e^{-\frac{j\omega\tau}{2}}$$

$$=A\tau Sa(\frac{\omega\tau}{2})e^{-\frac{j\omega\tau}{2}}$$

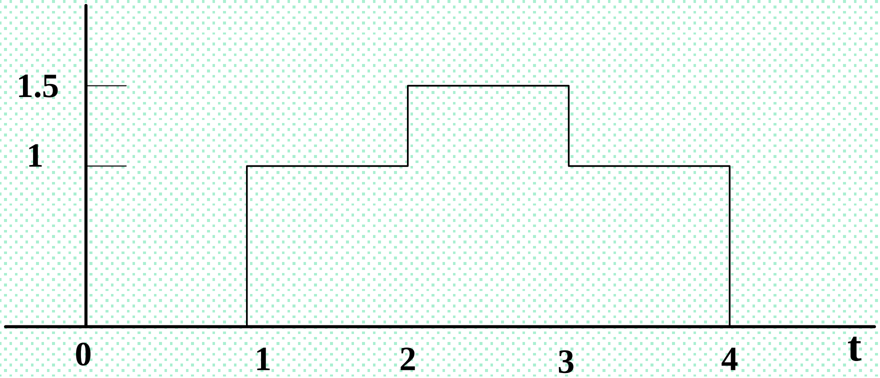
$$|F_1(j\omega)|=|F_2(j\omega)|$$



相位谱

$$\varphi_1(\omega)=\varphi_2(\omega)+\omega t_0=\varphi_2(\omega)+\frac{\omega\tau}{2}$$

- 课堂练习
  - 对图示波形进行傅立叶变换



  - 答案

$$G_3(t-\frac{5}{2})+0.5G_1(t-\frac{5}{2})$$

3:移频性质

若 $f(t) \leftrightarrow F(j\omega)$

则 $\boxed{f(t)e^{j\omega_c t} \leftrightarrow F(j(\omega-\omega_c))}$

同理 $\boxed{f(t)e^{-j\omega_c t} \leftrightarrow F(j(\omega+\omega_c))}$

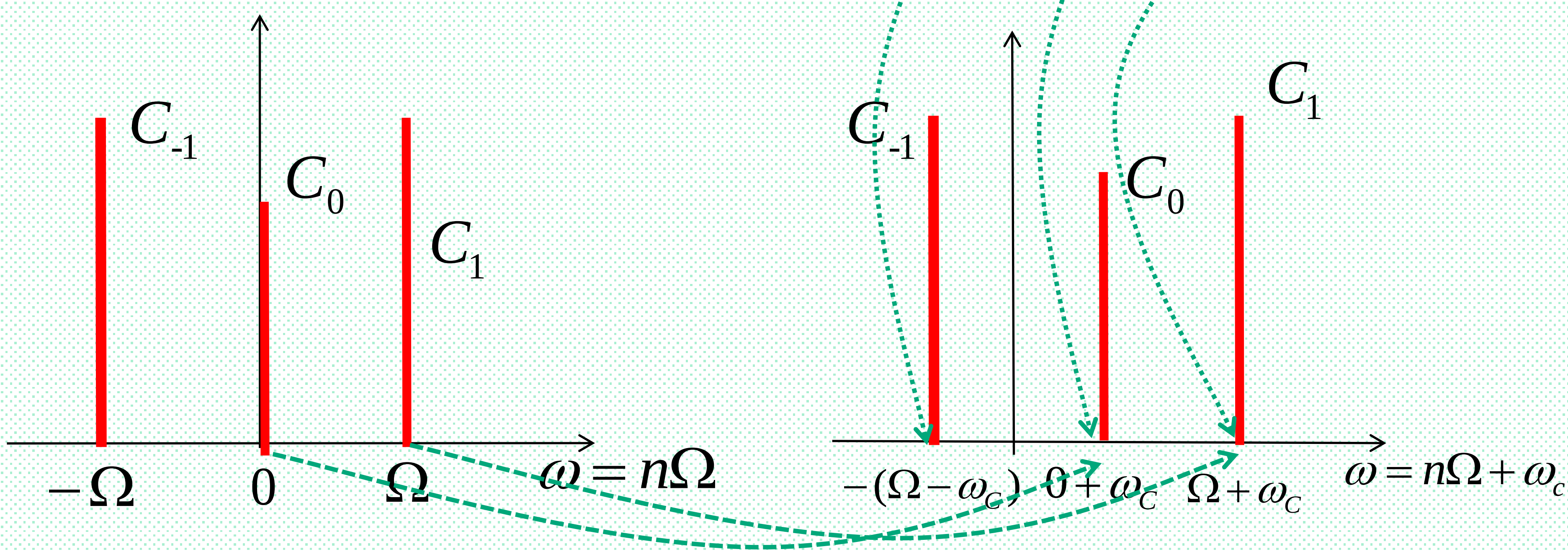
表明：信号在时域中与因子 $e^{j\omega_c t}$ 相乘，

等效于频域中频率的转移

$$f_T(t) = \sum_{-\infty}^{\infty} C_n e^{jn\Omega t} = \ldots\ldots + C_{-1}e^{-j\Omega t} + C_0 + C_1 e^{j\Omega t} + \ldots\ldots$$

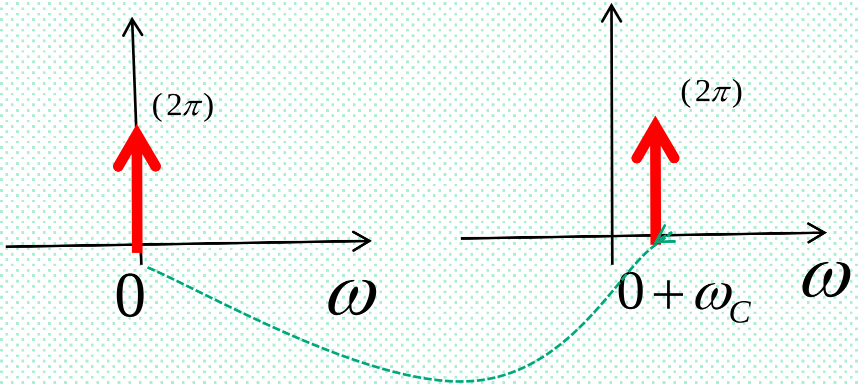
$$f_T(t)e^{j\omega_C t} = \sum_{-\infty}^{\infty} C_n e^{jn\Omega t} e^{j\omega_C t} = \ldots\ldots + C_{-1}e^{-j\Omega t} e^{j\omega_C t} + C_0 e^{j\omega_C t} + C_1 e^{j\Omega t} e^{j\omega_C t} + \ldots$$

$$= \sum_{-\infty}^{\infty} C_n e^{j(n\Omega + \omega_C)t} = \ldots\ldots + C_{-1}e^{-j(\Omega - \omega_C)t} + C_0 e^{j\omega_C t} + C_1 e^{j(\Omega + \omega_C)t} + \ldots\ldots$$



$$1 \leftrightarrow 2\pi\delta(\omega)$$

$$1 \bullet e^{j\omega_c t} \leftrightarrow 2\pi\delta(\omega-\omega_c)$$



$$\cos\omega_c t = \frac{e^{j\omega_c t}+e^{-j\omega_c t}}{2} \leftrightarrow \frac{1}{2}(2\pi\delta(\omega-\omega_c)+2\pi\delta(\omega+\omega_c))$$

$$\sin\omega_c t = \frac{e^{j\omega_c t}-e^{-j\omega_c t}}{2j} \leftrightarrow \frac{1}{2j}(2\pi\delta(\omega-\omega_c)-2\pi\delta(\omega+\omega_c))$$

$$\mathscr{F}\left[f(t)\cos\omega_c t\right]=\mathscr{F}\left[f(t)\frac{e^{j\omega_c t}+e^{-j\omega_c t}}{2}\right]$$

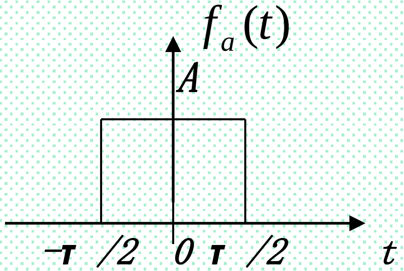
$$=\frac{1}{2}\mathscr{F}\left[f(t)e^{j\omega_c t}\right]+\frac{1}{2}\mathscr{F}\left[f(t)e^{-j\omega_c t}\right]$$

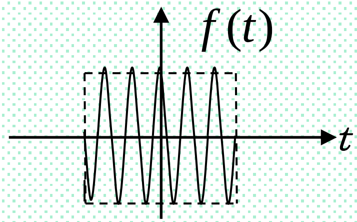
$$=\frac{1}{2}\left[F(j\omega-j\omega_c)+F(j\omega+j\omega_c)\right]$$

$$\boxed{f(t)Cos\,\omega_c t\leftrightarrow\frac{1}{2}[F(j\omega-j\omega_c)+F(j\omega+j\omega_c)]}$$ ——调幅过程

$$\boxed{f(t)\sin\omega_c t\leftrightarrow\frac{1}{2j}[F(j\omega-j\omega_c)-F(j\omega+j\omega_c)]}$$

例 求 $f_a(t)Cos\omega_c t = f(t)$ ——幅度调制信号的频谱函数



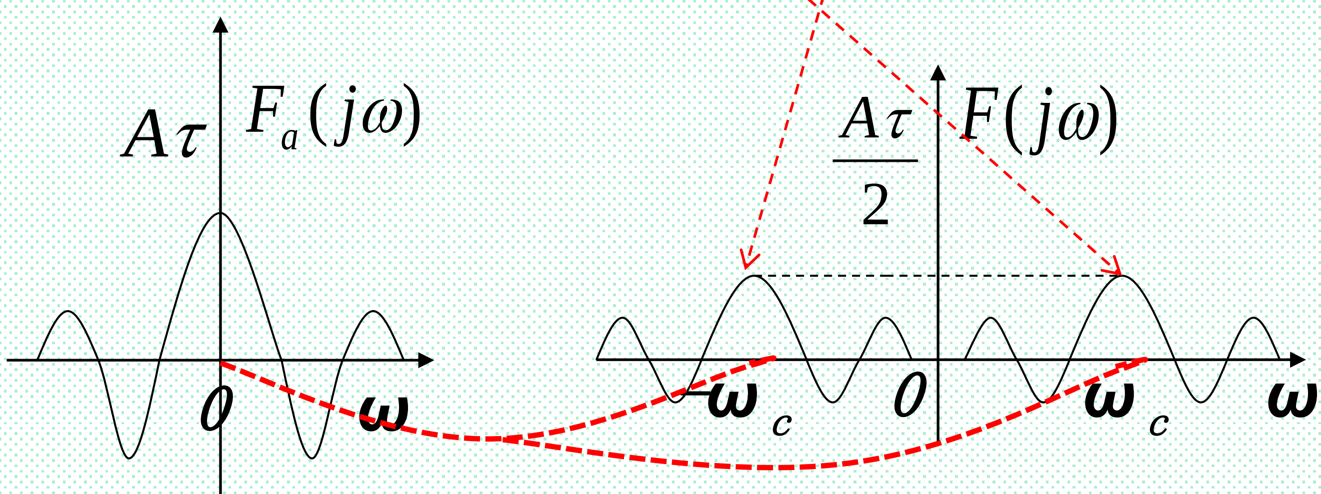


解：$f(t) = f_a(t)Cos\omega_c t = \dfrac{1}{2} f_a(t)(e^{j\omega_c t} + e^{-j\omega_c t})$

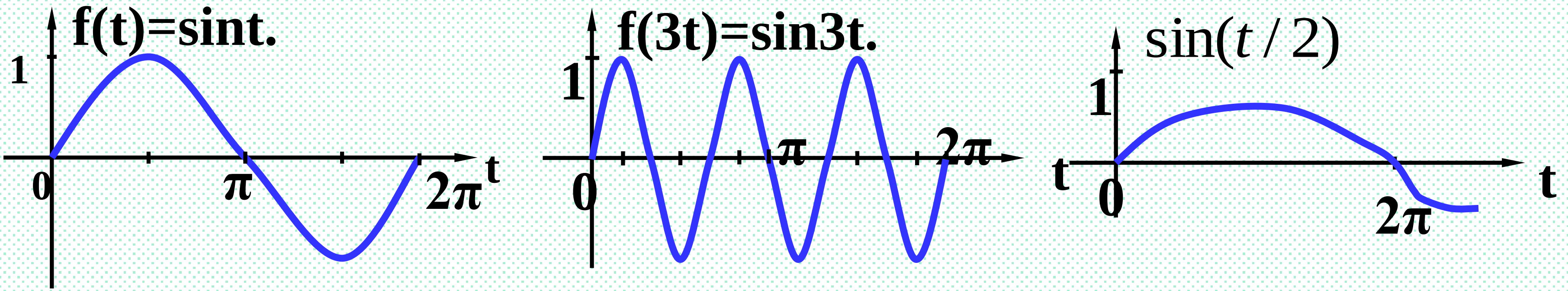
$$F(j\omega) = \frac{1}{2}F_a(j\omega - j\omega_c) + \frac{1}{2}F_a(j\omega + j\omega_c)$$

$$= \frac{A\tau}{2} Sa[\frac{(\omega-\omega_c)\tau}{2}] + \frac{A\tau}{2} Sa[\frac{(\omega+\omega_c)\tau}{2}]$$

频谱左右搬移
幅度谱减半



4、比例性（尺度变换特性）



当 **a** 是大于**1**的正实数时，表示信号压缩了**a** 倍,

当 **a** 是小于**1** 的正实数时，表示信号扩展了**1/a** 倍

若 $f(t) \leftrightarrow F(j\omega)$ 则 $\boxed{f(at) \leftrightarrow \frac{1}{|a|} F(j\frac{\omega}{a})}$

含义：信号沿时间轴压缩至原来的 $\frac{1}{|a|}$，

频域中频谱函数展宽 $|a|$倍。

即信号的脉宽与频宽成反比。

证明：（1） $a>0$

$$F[f(at)]=\int_{-\infty}^{\infty}f(at)e^{-j\omega t}dt$$

$$\overset{t'=at}{=}\int_{-\infty}^{\infty}f(t')e^{-j\omega\frac{t'}{a}}\frac{1}{a}dt'=\frac{1}{a}F(j\frac{\omega}{a})$$

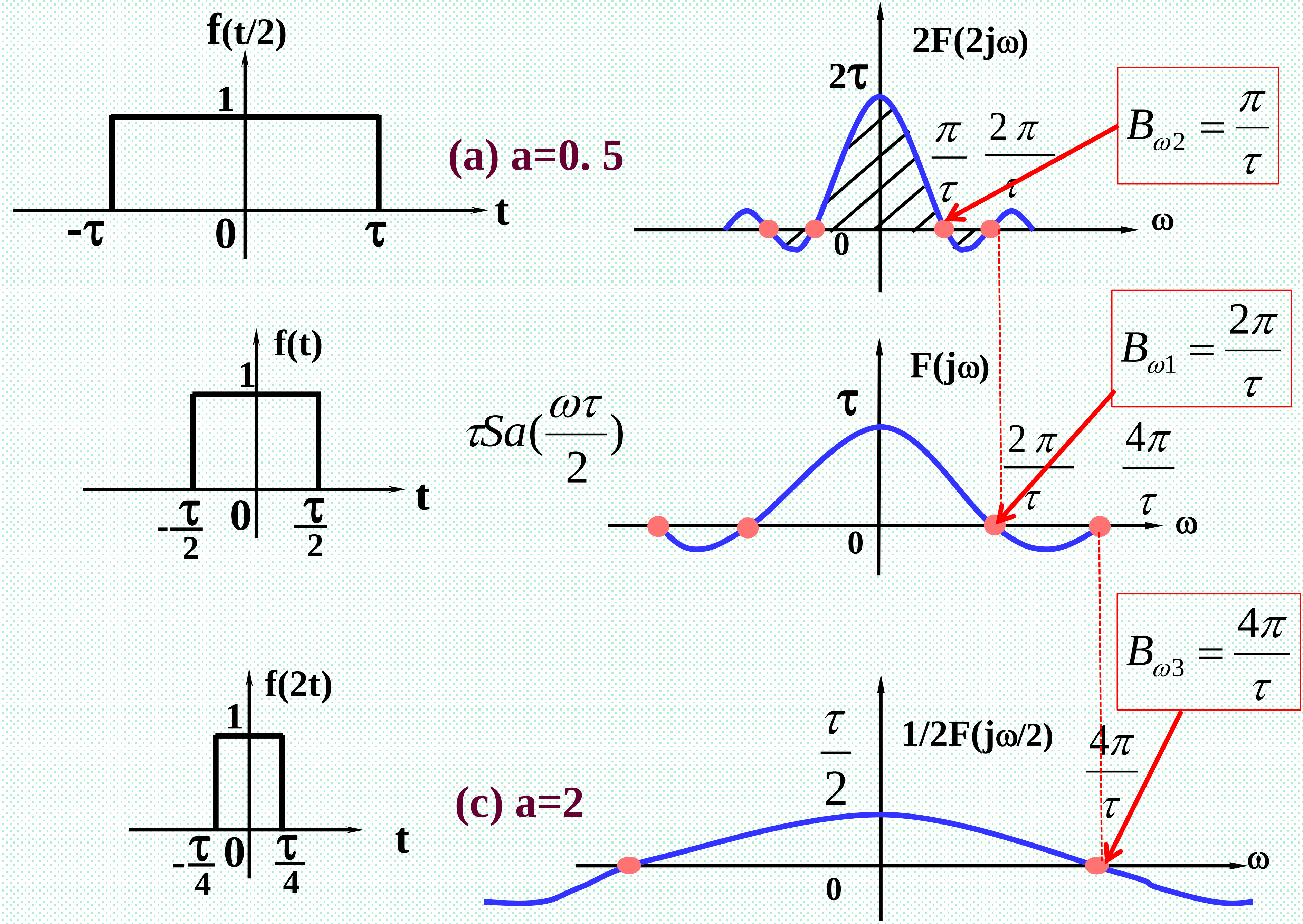
(2) $a<0$

令 $t'=at=-|a|t$ $\begin{cases}t=-\infty\to\infty\\t'=\infty\to-\infty\end{cases}$

$$F[f(at)]=\int_{\infty}^{\infty}f(t')e^{-j\omega\frac{t'}{a}}\frac{1}{a}dt'$$

$$=\int_{-\infty}^{\infty}f(t')e^{-j\omega(\frac{t'}{a})}(-\frac{1}{a})dt'=-\frac{1}{a}F(j\frac{\omega}{a})$$

令 $a=-1$ 则 $\boxed{f(-t)\leftrightarrow F(-j\omega)}$

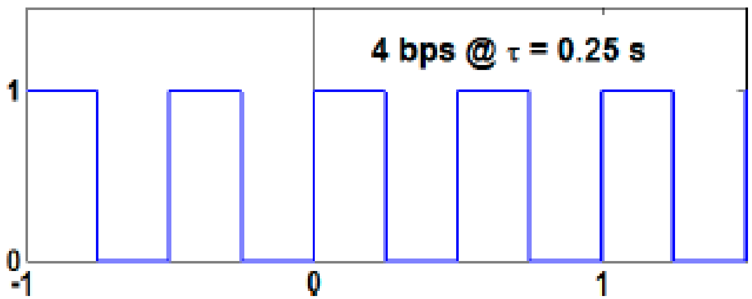


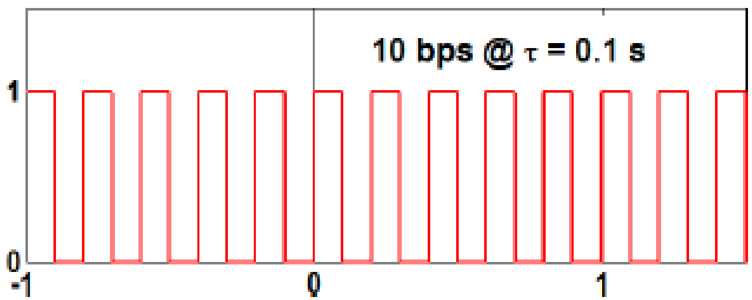
f(t/2)
1
-τ
0
τ
t
(a) a=0. 5
2F(2jω)
2τ
π/τ
2π/τ
0
ω
$B_{\omega 2}=\frac{\pi}{\tau}$
f(t)
1
-τ/2
0
τ/2
t
$\tau Sa(\frac{\omega\tau}{2})$
F(jω)
τ
2π/τ
4π/τ
0
ω
$B_{\omega 1}=\frac{2\pi}{\tau}$
f(2t)
1
-τ/4
0
τ/4
t
(c) a=2
τ/2
1/2F(jω/2)
4π/τ
0
ω
$B_{\omega 3}=\frac{4\pi}{\tau}$

尺度变换性质在实际应用中例子

通信系统中，通信速度和系统带宽是一对矛盾。

$$B_\omega = \frac{2\pi}{\tau}$$





- 速度越高，单位时间传输的比特脉冲越多
- 要求脉冲宽度越窄
- 使得信号带宽越宽：有更多的能量分布在高频分量
- 要求传输系统对输入信号高频分量的衰减更小，即系统响应带宽越宽
- 对传输媒质、器件、模块提出了更高的要求

已知 $f(t) \leftrightarrow F(j\omega)$

$$f_1(t) = f(at - b) \leftrightarrow F_1(j\omega) = ?$$

$$f_2(t) = f(\text{-}at + b) \leftrightarrow F_2(j\omega) = ?$$

$$f_1(t) = f(a(t - \frac{b}{a}))$$

$$f(at) = \frac{1}{|a|}F(\frac{j\omega}{a})$$

$$f(a(t - \frac{b}{a})) \leftrightarrow \frac{1}{|a|}F(\frac{j\omega}{a})e^{-j\omega\frac{b}{a}}$$

$$\boxed{F_1(j\omega) = \frac{1}{|a|}F(\frac{j\omega}{a})e^{-j\omega\frac{b}{a}}}$$

$$f(t\text{-}b) \leftrightarrow F(j\omega)e^{-j\omega b}$$

$$\boxed{\omega \to \frac{\omega}{a}}$$

$$f(at\text{-}b) \leftrightarrow \frac{1}{|a|}F(\frac{j\omega}{a})e^{-j\frac{\omega}{a}b}$$

$$f_2(t) = f(\text{-}a(t - \frac{b}{a}))$$

$$\boxed{F_1(j\omega) = \frac{1}{|a|}F(\frac{\text{-}j\omega}{a})e^{-j\omega\frac{b}{a}}}$$

5、奇偶性

$$F(j\omega)=\int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt=\int_{-\infty}^{\infty} f(t)\cos\omega t dt - j\int_{-\infty}^{\infty} f(t)\sin\omega t dt$$

$$=R(\omega)-jX(\omega)=|F(j\omega)|e^{-j\varphi(\omega)}$$

f(t)是实函数

$$\begin{cases} R(\omega)=\int_{-\infty}^{\infty} f(t)\cos\omega t dt \\ X(\omega)=\int_{-\infty}^{\infty} f(t)\sin\omega t dt \end{cases} \qquad \begin{cases} |F(j\omega)|=\sqrt{R^2(\omega)+X^2(\omega)} & \omega \text{ 偶函数} \\ \varphi(\omega)=arctg\dfrac{X(\omega)}{R(\omega)} & \omega \text{ 奇函数} \end{cases}$$

f(t) = f(-t) 则频谱函数只有实部,X(w)=0

$$F(j\omega)=R(\omega)=\int_{-\infty}^{\infty} f(t)\cos\omega t dt = 2\int_{0}^{\infty} f(t)\cos\omega t dt$$

f(t) = -f(-t)，则频谱函数只有虚部，R(w)=0

$$F(j\omega)=-jX(\omega)=-j\int_{-\infty}^{\infty} f(t)\sin\omega t dt=-j2\int_{0}^{\infty} f(t)\sin\omega t dt$$

- **6、微积分性质**

$$\frac{df(t)}{dt} \leftrightarrow j\omega F(j\omega)$$

$$\frac{d^n f(t)}{dt^n} \leftrightarrow (j\omega)^n F(j\omega)$$

$$(-jt)f(t) \leftrightarrow \frac{dF(j\omega)}{d\omega}$$

$$(-jt)^n f(t) \leftrightarrow \frac{d^n F(j\omega)}{d\omega^n}$$

$$\int_{-\infty}^{t} f(\tau)d\tau \leftrightarrow \pi F(0)\delta(\omega) + \frac{1}{j\omega}F(j\omega)$$

$$\pi f(o)\delta(t) + j\frac{f(t)}{t} \leftrightarrow \int_{-\infty}^{\omega} F(\Omega)d\Omega$$

$$\boxed{\frac{df(t)}{dt} \leftrightarrow j\omega F(j\omega)}$$

$$\boxed{\int_{-\infty}^{t} f(\tau)d\tau \leftrightarrow \pi F(0)\delta(\omega) + \frac{1}{j\omega}F(j\omega)}$$

$$u(t) \leftrightarrow \frac{1}{j\omega} + \pi\delta(\omega)$$

$$\delta(t) = \frac{d}{dt}u(t)$$

$$\Delta(j\omega) = j\omega U(j\omega)$$

$$= j\omega\left[\frac{1}{j\omega} + \pi\delta(\omega)\right]$$

$$= 1 + j\omega|_{\omega=0}\,\delta(\omega)$$

$$= 1$$

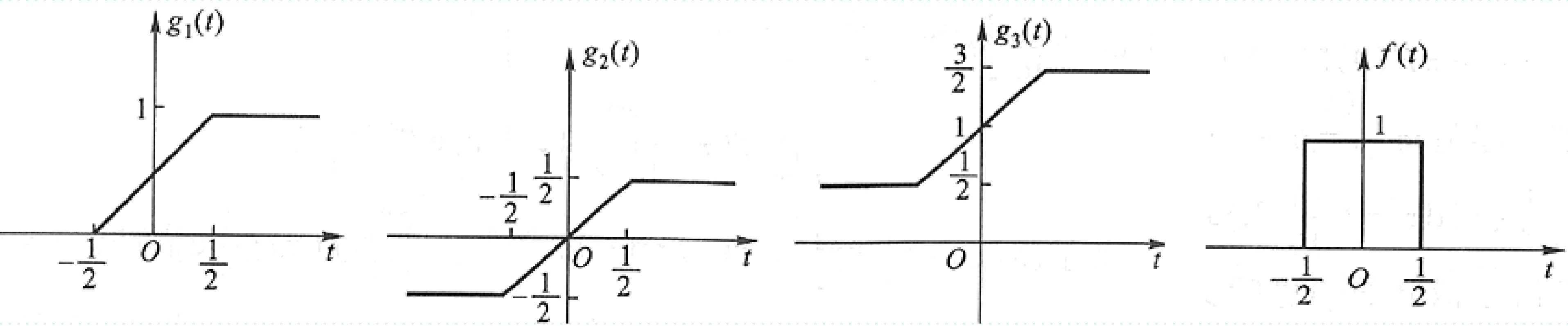
$$\delta(t) \leftrightarrow \Delta(j\omega) = 1$$

$$u(t) = \int_{-\infty}^{t} \delta(t)dt$$

$$U(j\omega) = \frac{\Delta(j\omega)}{j\omega} + \pi\Delta(0)\delta(\omega)$$

$$= \frac{1}{j\omega} + \pi\delta(\omega)$$

$$\Delta(j\omega) = j\omega U(j\omega) \Longrightarrow U(j\omega) \overset{?}{=} \frac{\Delta(j\omega)}{j\omega}$$



$$f(t)=\frac{dg_1(t)}{dt}=\frac{dg_2(t)}{dt}=\frac{dg_3(t)}{dt},\qquad f(t)\leftrightarrow Sa(\frac{\omega}{2}),\qquad F(0)=1$$

$$\int_{-\infty}^{t}\frac{dg_1(t)}{dt}dt=\int_{-\infty}^{t}f(t)dt\leftrightarrow\frac{1}{j\omega}F(j\omega)+\pi F(0)\delta(\omega)=\frac{Sa(\frac{\omega}{2})}{j\omega}+\pi\delta(\omega)$$

$$g_1(t)-g_1(-\infty)\leftrightarrow\frac{Sa(\frac{\omega}{2})}{j\omega}+\pi\delta(\omega)\qquad g_1(-\infty)=0$$

$$g_1(t)\leftrightarrow\frac{Sa(\frac{\omega}{2})}{j\omega}+\pi\delta(\omega)\qquad\boxed{\int_{-\infty}^{t}f(\tau)d\tau\leftrightarrow\pi F(0)\delta(\omega)+\frac{1}{j\omega}F(j\omega)}$$

$$\boxed{\int_{-\infty}^{t} f(\tau)d\tau \leftrightarrow \pi F(0)\delta(\omega)+\frac{1}{j\omega}F(j\omega)} \qquad \boxed{f(t)\leftrightarrow Sa(\frac{\omega}{2}),} \qquad \boxed{F(0)=1}$$

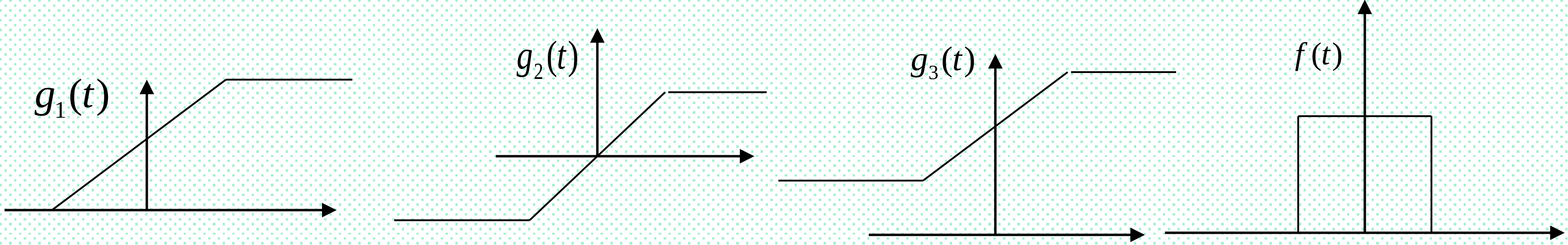
$$\int_{-\infty}^{t}\frac{dg_2(t)}{dt}dt=\int_{-\infty}^{t} f(t)dt \leftrightarrow \frac{1}{j\omega}F(j\omega)+\pi F(0)\delta(\omega)=\frac{Sa(\frac{\omega}{2})}{j\omega}+\pi\delta(\omega)$$

$$g_2(t)-g_2(-\infty)\leftrightarrow Sa(\frac{\omega}{2})/(j\omega)+\pi\delta(\omega) \qquad g_2(-\infty)=-\frac{1}{2}$$

$$g_2(t)+1/2\leftrightarrow Sa(\frac{\omega}{2})/(j\omega)+\pi\delta(\omega) \qquad \boxed{1/2\leftrightarrow \pi\delta(\omega)}$$

$$g_2(t)\leftrightarrow Sa(\frac{\omega}{2})/(j\omega)$$

同理可得 $g_3(t)\leftrightarrow Sa(\frac{\omega}{2})/(j\omega)+2\pi\delta(\omega)$



$$g_1'(t) = g_2'(t) = g_3'(t) = f(t) \qquad F(j\omega) = Sa(\frac{\omega}{2}), \qquad F(0) = 1$$

$$\boldsymbol{g_1(t) = \int_{-\infty}^{t} f(t)dt}$$

$$\boldsymbol{g_2(t) = \int_{-\infty}^{t} f(t)dt - \frac{1}{2}}$$

$$\boldsymbol{g_3(t) = \int_{-\infty}^{t} f(t)dt + \frac{1}{2}}$$

$$G_1(j\omega) = \frac{F(j\omega)}{j\omega} + \pi F(0)\delta(\omega) = \frac{1}{j\omega} Sa(\frac{\omega}{2}) + \pi\delta(\omega)$$

$$G_2(j\omega) = \frac{F(j\omega)}{j\omega} + \pi\delta(\omega) - \frac{1}{2} 2\pi\delta(\omega) = \frac{1}{j\omega} Sa(\frac{\omega}{2})$$

$$G_2(j\omega) = \frac{F(j\omega)}{j\omega} + \pi\delta(\omega) + \frac{1}{2} 2\pi\delta(\omega) = \frac{1}{j\omega} Sa(\frac{\omega}{2}) + 2\pi\delta(\omega)$$

8、对称性

若 $f(t) \leftrightarrow F(j\omega)$ $\quad F(jt) = F(j\omega)|\omega = t$

则 $F(jt) \leftrightarrow 2\pi f(-\omega)$ $\quad f(-\omega) = f(t)|t = -\omega$

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(j\omega) e^{j\omega t} d\omega$$

$$2\pi f(t) = \int_{-\infty}^{\infty} F(j\omega) e^{j\omega t} d\omega$$

$$2\pi f(-t) = \int_{-\infty}^{\infty} F(j\omega) e^{-j\omega t} d\omega$$

t, ω

互换

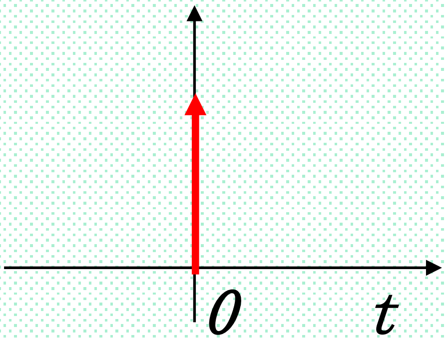
$$2\pi f(-\omega) = \int_{-\infty}^{\infty} F(jt) e^{-j\omega t} dt$$

$\delta(t) \leftrightarrow 1$

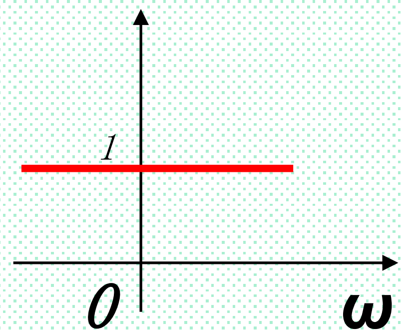
$1 \leftrightarrow 2\pi\delta(\omega)$

$$\boxed{\begin{array}{l} f(t) \leftrightarrow F(j\omega) \\ F(jt) \leftrightarrow 2\pi f(-\omega) \end{array}}$$

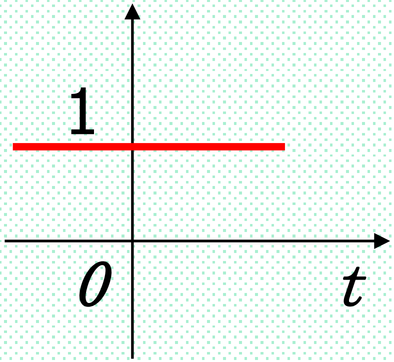
$f(t) = \delta(t)$ $\quad\quad$ $F(j\omega) = 1$



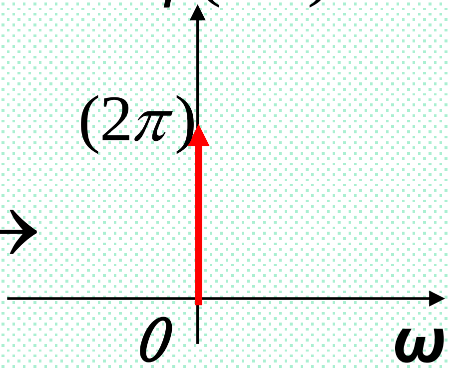
$\leftrightarrow$



$F(jt) = 1$ $\quad\quad$ $2\pi f(-\omega) = 2\pi\delta(\omega)$



$\leftrightarrow$

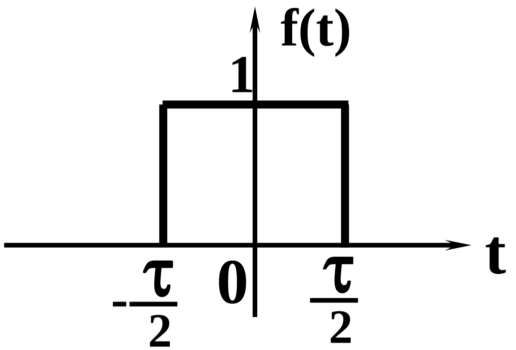


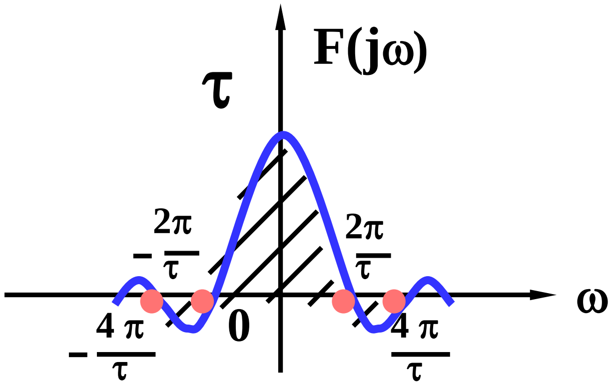
求 $\tau Sa(\frac{t\tau}{2})$ 频谱密度函数

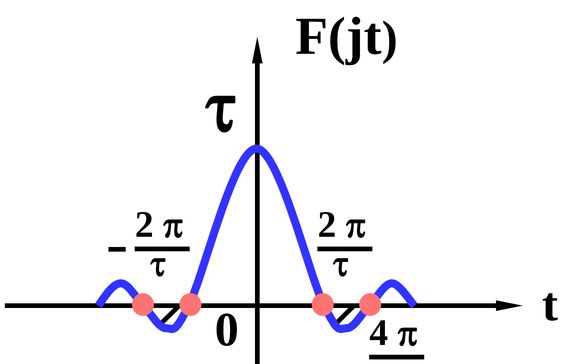
$$G_\tau(t) \leftrightarrow \tau Sa(\frac{\omega\tau}{2})$$

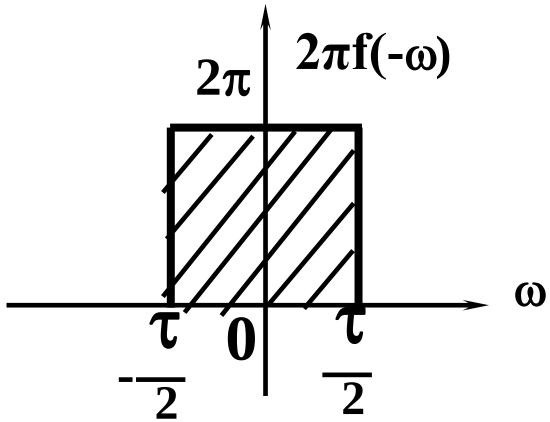
$$\tau Sa(\frac{t\tau}{2}) \leftrightarrow 2\pi G_\tau(-\omega)$$

$$\boxed{\begin{aligned} f(t) &\leftrightarrow F(j\omega) \\ F(jt) &\leftrightarrow 2\pi f(-\omega) \end{aligned}}$$









- 课堂练习
- 利用对偶性质求函数的傅立叶变换

$$g(t)=\frac{2}{1+t^2}$$

- 【解答】

$$X(j\omega)=\frac{2}{1+\omega^2}$$

- 某信号**x(t)**有傅立叶变换

$$x(t)=e^{-\alpha|t|},\alpha=1$$

- **X(t)**的函数形式
- 根据性质有：

$$\mathscr{F}\left\{\frac{2}{1+t^2}\right\}=2\pi e^{-\alpha|\omega|}$$

9卷积

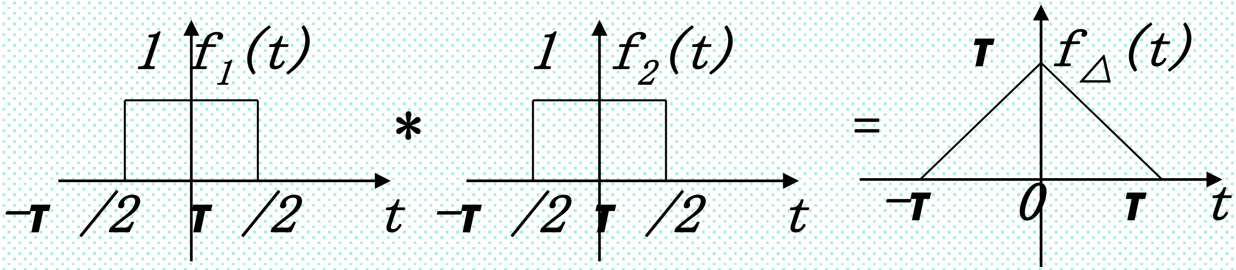
若 $f_1(t) \leftrightarrow F_1(j\omega) \quad f_2(t) \leftrightarrow F_2(j\omega)$

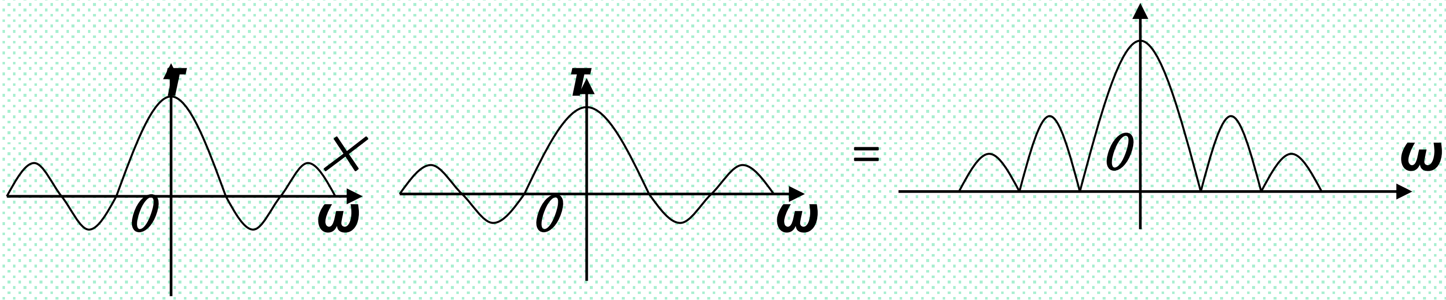
$$\boxed{f_1(t) * f_2(t) \leftrightarrow F_1(j\omega)F_2(j\omega)}$$

$$\boxed{f_1(t)f_2(t) \leftrightarrow \frac{1}{2\pi}[F_1(j\omega) * F_2(j\omega)]}$$

$$\boxed{f_1(t) * f_2(t) \leftrightarrow F_1(j\omega)F_2(j\omega)}$$

例　求三角形脉冲 $f_\Delta(t)$ 的频谱函数　$F_\Delta(j\omega)$





$$F_1(j\omega) = F_2(j\omega) = \tau Sa(\frac{\omega\tau}{2})$$

$$F_\Delta(j\omega) = F[f_1(t) * f_2(t)] = F_1(j\omega)F_2(j\omega) = \tau^2[Sa(\frac{\omega\tau}{2})]^2$$

本讲小结

- 傅里叶变换的性质
  - 线性性 $\mathscr{F}[a_1 f_1(t)+a_2 f_2(t)]=a_1F_1(j\omega)+a_2F_2(j\omega)$
  - 时延性 $\mathscr{F}[f(t-t_0)]=F(j\omega)e^{j\omega t_0}$
  - 频移性 $\mathscr{F}\left[f(t)e^{j\omega_c t}\right]=F(j\omega-j\omega_c)$
  - 比例性 $\mathscr{F}[f(at)]=\frac{1}{|a|}F(\frac{j\omega}{a})$
  - 奇偶性 $F(j\omega)=R(\omega)-jX(\omega)=|F(j\omega)|e^{-j\varphi(\omega)}$
  - 对称性 $\mathscr{F}[f(t)]=F(j\omega)$ $\quad\mathscr{F}[F(jt)]=2\pi f(-\omega)$
  - 微分性质 $\mathscr{F}\left[\frac{d^n f(t)}{dt^n}\right]=(j\omega)^n F(j\omega)$ $\quad\mathscr{F}\left[(-jt)^n f(t)\right]=\frac{d^n F(j\omega)}{d\omega^n}$
  - 积分性质 $\mathscr{F}[f(\tau)d\tau]=\frac{1}{j\omega}F(j\omega)$ $\quad\mathscr{F}\left[j\frac{f(t)}{t}\right]=\int_{-\infty}^{\omega}F(\Omega)d\Omega$
  - 卷积性质 $\mathscr{F}[x(t)*h(t)]=X(j\omega)H(j\omega)$ $\quad\mathscr{F}[f(t)g(t)]=\frac{1}{2\pi}F(j\omega)*G(j\omega)$

第 7 次课外作业

教材习题： 3.15、 3.17、 3.21