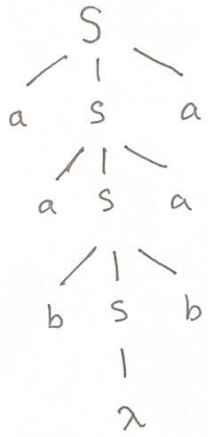
2. Draw the derivation tree corresponding to the derivation in Example 5.1.

$S \rightarrow aSa,$

$S \rightarrow bSb,$

$S \rightarrow \lambda$

$S \Rightarrow aSa \Rightarrow aaSaa \Rightarrow aabSbaa \Rightarrow aabbaa$



3. Grammar $S \rightarrow abB$,

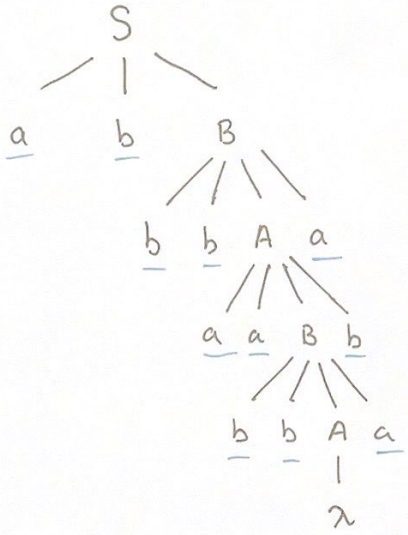
$A \rightarrow aaBb$,

$B \rightarrow bbAa$,

$A \rightarrow \lambda$

$w = abbbaabbaba$

Leftmost Derivation Tree:



7a. $L = \{a^n b^m : n \leq m+3\}$

First, solve the case $n = m+3$. Then add more b's. This can be done by

$$S \rightarrow aaaA,$$
$$A \rightarrow aAb \mid B,$$
$$B \rightarrow Bb \mid \lambda$$

But this is incomplete since it creates at least three a's. To take care of the cases $n = 0, 1, 2$, we add

$$S \rightarrow \lambda \mid aA \mid aaA$$

Therefore: $S \rightarrow \lambda \mid aA \mid aaA,$
$A \rightarrow aAb \mid B,$
$B \rightarrow Bb \mid \lambda$

Textbook Solution Wrong
∴ Correct Answer is
$S \rightarrow aSb \mid A \mid B$
$A \rightarrow a \mid aa \mid aaa \mid \lambda$
$B \rightarrow bB \mid b$

7d. $L = \{a^n b^m : 2n \leq m \leq 3n\}$

$$S \rightarrow aSbb \mid aSbbb \mid \lambda$$

These productions nondeterministically produce either bb or bbb for each generated a.

7f. $L = \{ w \in \{a, b\}^* : n_a(v) \geq n_b(v)$, where v is any prefix of $w\}$

$S \to aSb \mid SS \mid \lambda$

$S \to aSb \mid SS \mid S_1$

$S_1 = aS_1 \mid \lambda$

8a. $L = \{ a^n b^m c^k : n = m$ or $m \leq k\}$

For the first case $n = m$ and k is arbitrary. This can be achieved by

$S_1 \to AC,$

$A \to aAb \mid \lambda,$

$C \to Cc \mid \lambda$

In the second case, n is arbitrary and $m \leq k$. Here we use

$S_2 \to BD,$

$B \to aB \mid \lambda,$

$D \to bDc \mid E,$

$E \to Ec \mid \lambda$

Finally, we start productions with $S \to S_1 \mid S_2$.

$S \to S_1 \mid S_2$

$S_1 \to AC,$
$A \to aAb \mid \lambda,$
$C \to Cc \mid \lambda$

~~$S_1 \to AC,$~~
~~$S_2 \to BD,$~~
~~$A \to aAb \mid \lambda,$~~
~~$B \to aB \mid \lambda,$~~
~~$C \to Cc \mid \lambda,$~~
~~$D \to bDc \mid E,$~~
~~$E \to Ec \mid \lambda$~~

$S_2 \to BD,$
$B \to aB \mid \lambda$
$D \to bDc \mid E$
$E \to Ec \mid \lambda$

8b. $L = \{a^n b^m c^k : n = m \text{ or } m \neq k\}$

$S \to S_1 \mid S_2$

$S_1 \to AB$
$A \to aAb \mid \lambda$
$B \to cB \mid \lambda$

$S_2 \to CD$
$C \to aC \mid \lambda$
$D \to bDc \mid E \mid F$
$E \to bE \mid b$
$F \to cF \mid c$

8d. $L = \{a^n b^m c^k : n + 2m = k\}$

aaa...aa bb...bbcc...c

n m

$n + 2m$

Every a add one c

Every b add 2 c's

$S \to aSc \mid B$
$B \to bBcc \mid \lambda$

8h. $L = \{a^n b^m c^k : k \geq 3\}$

$S \to AB$
$A \to aAb \mid \lambda$
$B \to cB \mid ccc$

h. $L = \{a^n b^m c^k : k \geq 3\}$

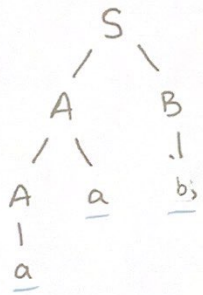
$S \to ACccc$ aAbCccc
$A \to aAb \mid \lambda$ abccc
$C \to cC \mid \lambda$

6. $S \to AB \mid aaB$,

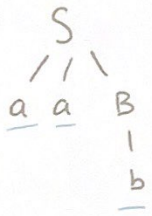
$A \to a \mid Aa$,

$B \to b$

$w = aab$



$w = aab$



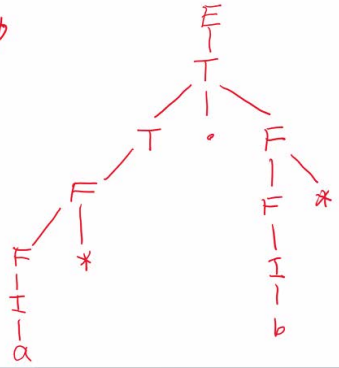
String $w = aab$ shows that the above grammar is ambiguous.

10. Give an unambiguous grammar that generates the set of all regular expressions on $\Sigma = \{a,b\}$.

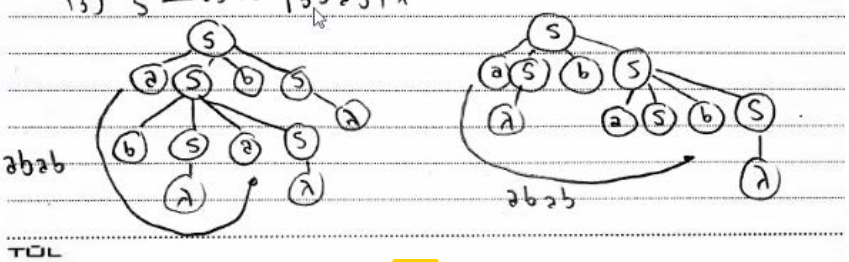
5.2 #10 r_2 of RE r_1+r_2, $r_1 \cdot r_2$, r_1^*, (r_1)

$$G \begin{cases} E \to E+T \mid T \\ T \to T \cdot F \mid F \\ F \to F^* \mid I \\ I \to a \mid b \mid \lambda \mid (E) \mid \phi \end{cases}$$

$a^* \cdot b^*$



13) $S \to aSbS \mid bSaS \mid \lambda$



TOL