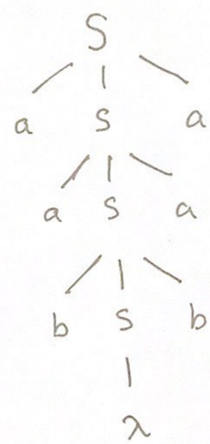
2. Draw the derivation tree corresponding to the derivation in Example 5.1.

$$S \to aSa,$$
$$S \to bSb,$$
$$S \to \lambda$$

$$S \Rightarrow aSa \Rightarrow aaSaa \Rightarrow aabSbaa \Rightarrow aabbaa$$



3. Grammar $S \to abB,$

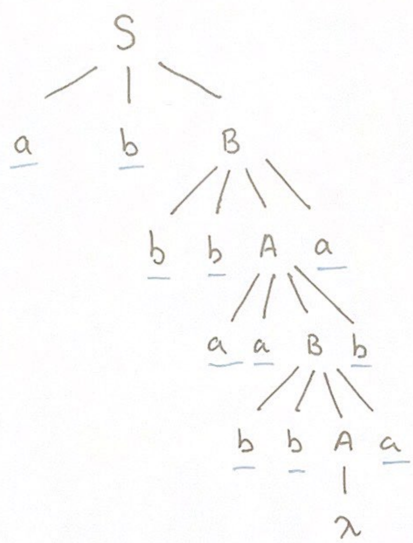
$A \to aaBb,$

$B \to bbAa,$

$A \to \lambda$

$w = abbbaabbaba$

Leftmost Derivation Tree:



Leftmost Derivation:

$S \Rightarrow abB \Rightarrow abbbAa \Rightarrow abbbaaBba \Rightarrow abbbaabbAaba$

$\Rightarrow abbbaabbaba$

7a. $L = \{a^n b^m : n \le m+3\}$

First, solve the case $n = m+3$. Then add more b's. This can be done by

$$S \to aaaA,$$
$$A \to aAb \mid B,$$
$$B \to Bb \mid \lambda$$

But this is incomplete since it creates at least three a's. To take care of the cases $n = 0, 1, 2$, we add

$$S \to \lambda \mid aA \mid aaA$$

Therefore:

$$S \to \lambda \mid aA \mid aaA,$$
$$A \to aAb \mid B,$$
$$B \to Bb \mid \lambda$$

Textbook Solution Wrong.
∴ Correct Answer is

$$S \to aSb \mid A \mid B$$
$$A \to a \mid aa \mid aaa \mid \lambda$$
$$B \to bB \mid b$$

7d. $L = \{a^n b^m : 2n \le m \le 3n\}$

$$S \to aSbb \mid aSbbb \mid \lambda$$

These productions nondeterministically produce either bb or bbb for each generated a.

7f. $L = \{ w \in \{a, b\}^* : n_a(v) \geq n_b(v)$, where v is any prefix of $w \}$

$S \to aSb \mid SS \mid S_1$

$S_1 \to aS_1 \mid \lambda$

8a. $L = \{ a^n b^m c^k : n = m \text{ or } m \leq k \}$

For the first case $n = m$ and k is arbitrary. This can be achieved by

$S_1 \to AC$,

$A \to aAb \mid \lambda$,

$C \to Cc \mid \lambda$

In the second case, n is arbitrary and $m \leq k$. Here we use

$S_2 \to BD$,

$B \to aB \mid \lambda$,

$D \to bDc \mid E$,

$E \to Ec \mid \lambda$

Finally, we start productions with $S \to S_1 \mid S_2$.

$S \to S_1 \mid S_2$

$S_1 \to AC$,

$A \to aAb \mid \lambda$,

$C \to Cc \mid \lambda$

~~$S_1 \to AC$,~~

~~$S_2 \to BD$,~~

~~$A \to aAb \mid \lambda$,~~

~~$B \to aB \mid \lambda$,~~

~~$C \to Cc \mid \lambda$,~~

~~$D \to bDc \mid E$,~~

~~$E \to Ec \mid \lambda$~~

$S_2 \to BD$,

$B \to aB \mid \lambda$

$D \to bDc \mid E$

$E \to Ec \mid \lambda$

8b. $L = \{a^n b^m c^k : n = m \text{ or } m \neq k\}$

$S \to S_1 \mid S_2$

$S_1 \to AB$

$A \to aAb \mid \lambda$

$B \to cB \mid \lambda$

$S_2 \to CD$

$C \to aC \mid \lambda$

$D \to bDc \mid E \mid F$

$E \to bE \mid b$

$F \to cF \mid c$

8d. $L = \{a^n b^m c^k : n + 2m = k\}$

$\underbrace{aaa\ldots aa}_{n}\underbrace{bb\ldots bb}_{m}cc\ldots c$

$n + 2m$

Every a add one c

Every b add 2 c's

$S \to aSc \mid B$

$B \to bBcc \mid \lambda$

8h. $L = \{a^n b^m c^k : k \geq 3\}$

$S \to AB$

$A \to aAb \mid \lambda$

$B \to cB \mid ccc$

Another Solution Presented in Discussion:

$S \to ACccc$

$A \to aAb \mid \lambda$

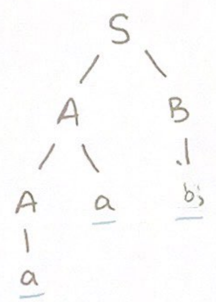
$C \to cC \mid \lambda$

6. $S \to AB \mid aaB$,

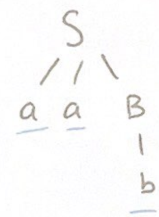
$A \to a \mid Aa$,

$B \to b$

$w = aab$



$w = aab$



String $w = aab$ shows that the above grammar is ambiguous.

Ambiguous Grammar: A CFG is ambiguous if there exists more than one derivation tree or parse tree.

10. Give an unambiguous grammar that generates the set of all regular expressions on $\Sigma = \{a, b\}$.

set of all regular expressions: $r_1 + r_2$, $r_1 \cdot r_2$, r_1^*, (r_1)

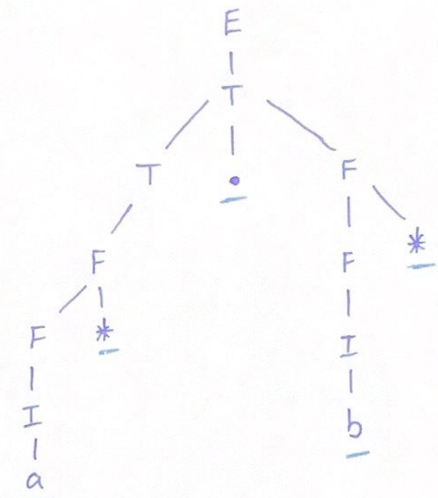
$E \longrightarrow E + T \mid T$

$T \longrightarrow T \cdot F \mid F$

$F \longrightarrow F^* \mid I$

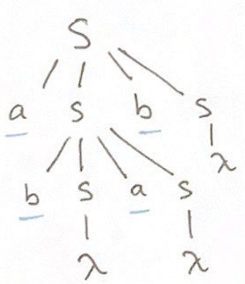
$I \longrightarrow a \mid b \mid \lambda \mid (E) \mid \phi$

$a^* \cdot b^*$

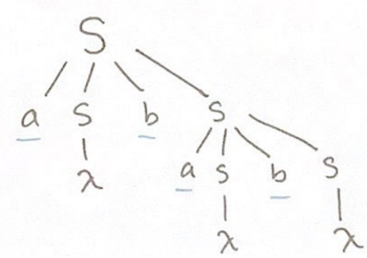


13. Show that the following grammar is ambiguous.

$S \rightarrow aSbS \mid bSaS \mid \lambda$



abab



abab

The same string "abab" can be produced more than 1 way, therefore the given grammar is ambiguous.