

# Ungraded Homework Solutions

## CSC 152 – Cryptography

Please notify me of any errors you find. If you need help, ask.

1) Let's say you are using a polynomial hash function  $k^{n+1} + x_0k^n + x_1k^{n-1} + \dots + x_{n-1}k \bmod p$  to hash the three-byte data `0x 26 14 04`, and let's say that  $p = 257$ ,  $k$  is randomly chosen to be `0x55`, and that the data is broken into 8-bit chunks before hashing. What is the resulting value?

Using Python as my calculator, the result appears to be 84.

```
k=0x55
p=257
y = (k**4 + 0x26 * k**3 + 0x14 * k**2 + 0x04 * k**1) % p
print(y)
```

2) Can you find another data string (of any length) that yields the same output value?

Perhaps the fastest way to do this is to assume a one-byte data can cause an output of 84 and simply use a for-loop to find it. The following outputs 46.

```
k=0x55
p=257
for i in range(256):
    if ((k**2 + i * k) % p == 84):
        print(i)
```

There are algebraic ways to find an answer, but for such a small problem brute-force is easiest.

3) We saw that an authentication tag can be generated by combining a universal hash like the one above with a random function:  $\text{TagGen}(x, n) = h(x) \text{ op } f(n)$ . (The operation used depends on the specifics of  $h$  and  $f$ .) Because it's readily available, let's say we are using the AES S-box for  $f$ , the hash function listed above with  $k=0x55$  for  $h$ , and addition mod  $p=257$  for the  $\text{TagGen}$  operation. What authentication tag is generated for the three-byte data `0x 26 14 04` when the nonce used is `0x10`?

We know that that the hash value is 84. The S-box produces `0xCA` for input `0x10`. Since `0xCA` is 202, the tag is  $84 + 202 = 286 \bmod 257 = 29$ .