1、

1.1

a) $\tilde{x} = \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$ $\begin{cases} \tilde{I}_1^T \tilde{x} = 0 \\ \tilde{I}_2^T \tilde{x} = 0 \end{cases}$ 即 $\begin{cases} ax+by+c=0 \\ dx+ey+f=0 \end{cases}$

$$\Rightarrow \begin{cases} x = \frac{bf-ce}{ae-bd} \\ y = \frac{cd-af}{ae-bd} \end{cases} \quad \therefore \tilde{x} = \begin{pmatrix} bf-ce \\ cd-af \\ ae-bd \end{pmatrix}$$

$$\tilde{I}_1 \times \tilde{I}_2 = \begin{pmatrix} bf-ce \\ cd-af \\ ae-bd \end{pmatrix} = \tilde{x}$$

b) $\tilde{I} = (a,b,c)^T = (\frac{a}{c}, \frac{b}{c}, 1)^T$

$\tilde{x}_1 = \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix}$ $\tilde{x}_2 = \begin{pmatrix} x_2 \\ y_2 \\ 1 \end{pmatrix}$

$\because \tilde{I}\tilde{x} = 0 \quad \therefore \begin{cases} ax_1+by_1+c=0 \\ ax_2+by_2+c=0 \end{cases}$

$$\begin{cases} \frac{a}{c} = \frac{y_1-y_2}{x_1y_2-x_2y_1} \\ \frac{b}{c} = \frac{x_2-x_1}{x_1y_2-x_2y_1} \end{cases}$$

$$\tilde{x}_1 \times \tilde{x}_2 = \begin{pmatrix} y_1-y_2 \\ x_2-x_1 \\ x_1y_2-x_2y_1 \end{pmatrix} = \begin{pmatrix} \frac{a}{c} \\ \frac{b}{c} \\ 1 \end{pmatrix} = \tilde{I}$$

c) $\begin{cases} x+y+3=0 \\ -x-2y+7=0 \end{cases} \Rightarrow \begin{cases} x=-13 \\ y=10 \end{cases}$

$\tilde{I}_1 = (1, 1, 3)^T$ $\tilde{I}_2 = (-1,-2,7)^T$

$\tilde{I}_1 \times \tilde{I}_2 = (13, -10, -1)^T = (-13, 10, 1)^T$

same intersection point.

d) $\hat{n} = (\frac{3}{5}, \frac{4}{5})$ $d=3$

$\Rightarrow \frac{3}{5}x + \frac{4}{5}y = 3$

$\therefore \tilde{I} = (3, 4, -15)^T$

e) $\tilde{I} = (2, 5, \frac{\sqrt{29}}{5})$

$\sqrt{(2)^2+(5)^2} = \sqrt{29}$

$\tilde{I} = (\frac{2}{\sqrt{29}}, \frac{5}{\sqrt{29}}, \frac{1}{5}) = (\hat{n}, -d)$

$$\therefore \begin{cases} \hat{n} = (\frac{2}{\sqrt{29}}, \frac{5}{\sqrt{29}})^T \\ d = -\frac{1}{5} \end{cases}$$

1.2

a) $\begin{cases} 0-1=-1 \\ 3-2=1 \end{cases}$

$$T = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}$$

test: $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$

b) 令 $T = \begin{bmatrix} 1 & 0 & t_1 \\ 0 & 1 & t_2 \end{bmatrix}$

$$E(T) = E(t_1, t_2) = \sum_{i=1}^{N} (x_1^i + t_1 - y_1^i)^2 + (x_2^i + t_2 - y_2^i)^2$$

$$\frac{\partial E}{\partial t_1} = \sum_{i=1}^{N} 2(x_1^i + t_1 - y_1^i)$$

$$= 2\left[\sum_{i=1}^{N}(x_1^i - y_1^i) + Nt_1\right] = 0$$

$$\Rightarrow t_1 = \frac{1}{N}\sum_{i=1}^{N}(y_1^i - x_1^i)$$

同理 $t_2 = \frac{1}{N}\sum_{i=1}^{N}(y_2^i - x_2^i)$

$$\therefore \vec{t} = \frac{1}{N}\sum_{i=1}^{N}(\vec{y}_i - \vec{x}_i)$$

$$= \frac{1}{N}\sum_{i=1}^{N}\vec{y}_i - \frac{1}{N}\sum_{i=1}^{N}\vec{x}_i$$

∴ 最佳变换就是两组点重心对齐

c) $t_1 = \frac{1}{3}(3-0+7-5+5-4) = 2$

$t_2 = \frac{1}{3}(-5-1+6-7+(-4)-1) = -4$

$$T^* = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -4 \end{bmatrix}$$

1.3

a) $$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$$t = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$

$$[R|t] = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$K = \begin{bmatrix} 100 & 0 & 25 & 0 \\ 0 & 100 & 25 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\tilde{P} = K[R|t]$$

$$= \begin{bmatrix} 100 & 25 & 0 & 150 \\ 0 & 25 & -100 & 50 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b) $\tilde{x}_s = \tilde{P}\tilde{x}_w \Rightarrow \tilde{x}_w = \tilde{P}^{-1}x_s$

设 $\tilde{x}_w = (x, y, z, w)^T$

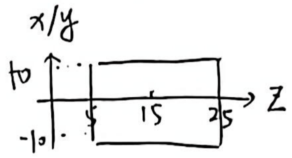
$$\tilde{x}_s = \tilde{P}\tilde{x}_w \Rightarrow \begin{cases} 100x + 25y + 150w = 25 \\ 25y - 100z + 50w = 50 \\ y + 2w = 1 \\ w = 0.25 \end{cases}$$

$$\Rightarrow \begin{cases} x = -0.25 \\ y = 0.5 \\ z = -0.25 \\ w = 0.25 \end{cases} \quad \tilde{x}_w = \left(-\frac{1}{4}, \frac{1}{2}, -\frac{1}{4}, \frac{1}{4}\right)$$

$$x_w = \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}$$

c)

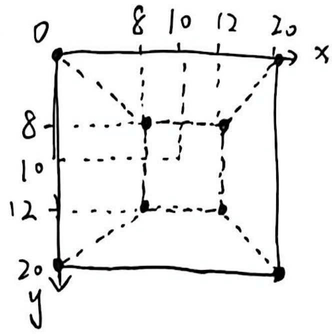
i) $$K = \begin{bmatrix} 5 & 0 & 10 \\ 0 & 5 & 10 \\ 0 & 0 & 1 \end{bmatrix}$$



$$K\begin{bmatrix} -10 \\ -10 \\ 5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix} \quad K\begin{bmatrix} 10 \\ 10 \\ 5 \end{bmatrix} = \begin{bmatrix} 100 \\ 100 \\ 5 \end{bmatrix}$$

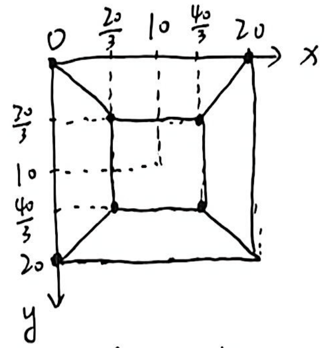
$$K\begin{bmatrix} -10 \\ -10 \\ 25 \end{bmatrix} = \begin{bmatrix} 200 \\ 200 \\ 25 \end{bmatrix} \quad K\begin{bmatrix} +10 \\ +10 \\ 25 \end{bmatrix} = \begin{bmatrix} 300 \\ 300 \\ 25 \end{bmatrix}$$

∴ 成像四点 $(0,0)$、$(20,20)$、$(8,8)$、$(12,12)$



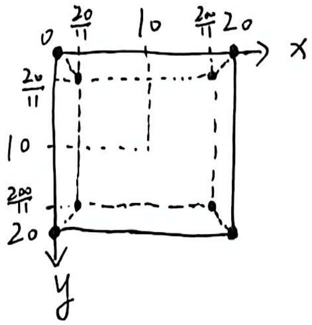
ii) $$K = \begin{bmatrix} 10 & 0 & 10 \\ 0 & 10 & 10 \\ 0 & 0 & 1 \end{bmatrix}$$

同理点为 $(0,0)$、$(20,20)$、$(\frac{20}{3}, \frac{20}{3})$、$(\frac{40}{3}, \frac{40}{3})$



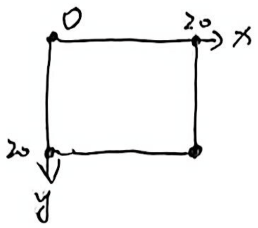
iii) $$K = \begin{bmatrix} 90 & 0 & 10 \\ 0 & 90 & 10 \\ 0 & 0 & 1 \end{bmatrix}$$

点为 $(0,0)$、$(20,20)$、$(\frac{20}{11}, \frac{20}{11})$、$(\frac{200}{11}, \frac{200}{11})$



iv) $k = \begin{bmatrix} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

对应点 $(0,0)$、$(20,20)$



v) 当 focal length / distance from camera

足够大时、

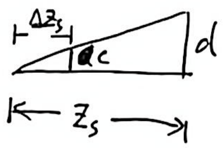
1.4

a) $\dfrac{1}{Z_s} + \dfrac{1}{Z_c} = \dfrac{1}{f}$

$\dfrac{1}{Z_c} = \dfrac{1}{f} - \dfrac{1}{Z_s}$

$Z_c = \dfrac{1}{\frac{1}{f} - \frac{1}{Z_s}} = 2600\,mm$

b)



$\dfrac{c}{d} = \dfrac{\Delta Z_s}{Z_s}$

$C = \dfrac{\Delta Z_s}{Z_s} \cdot d$

$= \dfrac{\Delta Z_s}{Z_s} \cdot \dfrac{f}{N}$

c) $C_1 = \dfrac{0.1}{40} \cdot \dfrac{35}{1.4}$

$= 0.0625\,mm$

$C_2 = \dfrac{0.03}{40} \cdot \dfrac{35}{1.4}$

$= 0.01875\,mm$

单位像素大小 $S_p = \dfrac{64}{400 \times 400}\,mm^2 / \cancel{pixel}$

$= 0.0004\,mm^2$

单位长度：$S_p = \sqrt{0.0004\,mm^2}$

$= 0.02\,mm^2$

$C_1 > S_p \Rightarrow$ not sharp

$C_2 < S_p \Rightarrow$ sharp