Cálculo

Ficha 2 2011/2012

Primitivas imediatas

1. Determine a primitiva das seguintes funções:

a) $a(x) = x^2 ch\left(x^3\right) + x.4^{x^2}$

b) $d(x) = \frac{sh(5x)}{\sqrt[3]{ch^4(5x)}}$

c) $e(x) = \frac{1}{\sqrt{4-9x^2}}$

d) $j(x) = \frac{(\ln x + e)^4}{x},\ x > 0$

e) $q(x) = tgx$

f) $m(x) = \frac{5x}{4+4x^2}$

g) $m(x) = \frac{3x}{\sqrt{1+5x^2}}$

h) $m(x) = \sqrt{2x+3}$

i) $f(x) = 5k^2x^6$, com $k \in \mathbb{R}$

j) $f(x) = \sqrt[3]{x^2} + 7x + 8$

k) $h(x) = \frac{1}{x^5} + \frac{2}{\sqrt{x}}$

l) $h(x) = \frac{x^3+3\sqrt{x}+4}{x^2}$

2. Determine a função f que verifica a condição,

a) $f'(x) = \frac{x}{(1+x^2)^2}$ e tal que $f(0) = 2$.

b) O gráfico de f passa pelo ponto $(1,1)$, a tangente ao gráfico de f nesse ponto tem a equação $x + 2y = 3$ e f verifica a condição $f''(x) = x^2 + 1$.

3. Calcule a primitiva das seguintes funções:

a) $c(x) = sen(2x).e^{\cos^2 x}$

b) $c(x) = \frac{2a}{(a-x)^2}$, com $a \in \mathbb{R}$

c) $f(x) = \frac{e^x}{\sqrt{1-e^{2x}}}$

d) $g(x) = \frac{x-1}{x^2+1}$

e) $h(x) = \frac{x}{\sqrt{x^4-4}}$

f) $i(x) = \frac{\cos(7x)}{sen^3(7x)}$

g) $l(x) = x\sqrt{4-x^2}$

h) $n(x) = \frac{x^2+1}{\sqrt{x^3+3x-4}}$

i) $m(x) = x^2\left(x^3+e\right)^4$

j) $m(x) = \sin^2(4x)$

Soluções:

1.

a) $\dfrac{\mathrm{sh}(x^3)}{3} + \dfrac{4^{x^2}}{2\ln 4} + \mathcal{C}$ b) $\dfrac{-3}{5\sqrt[3]{\mathrm{ch}(5x)}} + \mathcal{C}$

c) $\dfrac{1}{3}\arcsin(\dfrac{3x}{2}) + \mathcal{C}$ d) $\dfrac{(\ln x + e)^5}{5} + \mathcal{C}$

e) $-\ln|\cos x| + \mathcal{C}$ f) $\dfrac{5}{8}\ln(1+x^2) + \mathcal{C}$

g) $\dfrac{3}{5}\sqrt{1+5x^2} + \mathcal{C}$ h) $\dfrac{(2x+3)^{\frac{3}{2}}}{3} + \mathcal{C}$

i) $5k^2\,\dfrac{x^7}{7} + \mathcal{C}$ j) $\dfrac{3}{5}x^{5/3} + \dfrac{7}{2}x^2 + 8x + \mathcal{C}$

k) $-\dfrac{1}{4x^4} + 4\sqrt{x} + \mathcal{C}$ l) $\dfrac{x^2}{2} - \dfrac{6}{\sqrt{x}} - \dfrac{4}{x} + \mathcal{C}$

2.

a) $f(x) = -\dfrac{1}{2(1+x^2)} + \dfrac{5}{2}$

b) $f(x) = \dfrac{x^4}{12} + \dfrac{x^2}{2} - \dfrac{11x}{6} + \dfrac{27}{12}$

3.

a) $-\mathrm{e}^{\cos^2(x)} + \mathcal{C}$ b) $\dfrac{2a}{a-x} + \mathcal{C}$

c) $\arcsin(\mathrm{e}^x) + \mathcal{C}$ d) $\ln(\sqrt{x^2+1}) - \mathrm{arctg}(x) + \mathcal{C}$

e) $\dfrac{1}{2}\mathrm{argch}(\dfrac{x^2}{2}) + \mathcal{C}$ f) $-\dfrac{\sin^{-2}(7x)}{14} + \mathcal{C}$

g) $-\dfrac{1}{3}(4-x^2)^{3/2} + \mathcal{C}$ h) $\dfrac{2}{3}\sqrt{x^3+3x^2-4} + \mathcal{C}$

i) $\dfrac{1}{15}(x^3+\mathrm{e})^5 + \mathcal{C}$ j) $\dfrac{x}{2} - \dfrac{\sin(8x)}{16} + \mathcal{C}$