Cálculo

Ficha 3 — 2011/2012

Primitivação por partes

1. Utilize o método de primitivação por partes para obter as primitivas das seguintes funções:

a) $f(x) = x\ e^{-5x}$ b) $f(x) = x^3\ e^{3x^2}$

c) $f(x) = \ln(\frac{1}{x})$ d) $f(x) = \ln(5+x)$

e) $f(x) = \arcsin(x)$ f) $f(x) = x\sec^2(x)$

g) $f(x) = \operatorname{arctg}(x)$ h) $h(t) = \operatorname{ch}(t)\sin(3t)$

i) $g(x) = \dfrac{\ln(3x)}{x^2}$ j) $g(x) = \frac{\ln^2 3x}{x^3}$

k) $g(x) = \ln(1+3x^2)$ l) $g(x) = \mathrm{e}^x\cos(2x)$

m) $g(x) = 4x^2\mathrm{e}^x$ n) $g(x) = \dfrac{x^3}{\sqrt{1+x^2}}$

2. Calcule:

a) $\int (e^{-3x}\sin 5x + e^{5x}\cos 3x)dx$

b) $\int (x^2\cos 3x + x\sin 5x)dx$

c) $\int (\arcsin 5x + \operatorname{arctg}(-3x))dx$

d) $\int x\arccos(3x^2)dx$

Primitivação de potências de funções trigonométricas

3. Calcule a primitiva das seguintes funções:

a) $f(x) = \sin^2 x$ b) $f(x) = \cos^3 x$ c) $f(x) = \sin^4 x$

d) $g(x) = \sin^5(x)$ e) $g(x) = \cos^4(x)$ f) $g(x) = \sin^2(x)\cos^2(x)$

g) $g(x) = \mathrm{tg}^5(x)$ h) $g(x) = \coth^4(x)$ i) $g(x) = \dfrac{1}{\operatorname{ch}^6(x)}$

Soluções:

1.

a) $-\frac{x}{5}e^{-5x} - \frac{1}{25}e^{-5x} + \mathcal{C}$

b) $\frac{1}{6}x^2e^{3x^2} - \frac{1}{18}e^{3x^2} + \mathcal{C}$

c) $x\ln(\frac{1}{x}) + x + \mathcal{C}$

d) $5\ln(x+5) - x + x\ln(x+5) + \mathcal{C}$

e) $x\arcsin x + \sqrt{1-x^2} + \mathcal{C}$

f) $x\,\mathrm{tg}\,x + \ln|\cos x| + \mathcal{C}$

g) $x\arctan x - \frac{1}{2}\ln(x^2+1) + \mathcal{C}$

h) $\frac{1}{10}\mathrm{sh}\,t\sin(3t) - \frac{3}{10}\mathrm{ch}\,t\,\cos(3t) + \mathcal{C}$

i) $-\frac{1}{x}(1+\ln(3x)) + \mathcal{C}$

j) $-\frac{1}{2x^2}(\ln^2 3x + \ln 3x + \frac{1}{2} + \mathcal{C})$

k) $x\ln(3x^2+1) - 2x + \frac{2\sqrt{3}}{3}\mathrm{arctg}(\sqrt{3}x) + \mathcal{C}$

l) $\frac{1}{5}e^x\cos(2x) + \frac{2}{5}e^x\sin(2x) + \mathcal{C}$

m) $4e^x(x^2-2x+2) + \mathcal{C}$

n) $x^2\sqrt{x^2+1} - \frac{2}{3}(1+x^2)^{\frac{3}{2}} + \mathcal{C}$

2.

a) $e^{5x}\left(\frac{5}{34}\cos(3x) + \frac{3}{34}\sin(3x)\right) - e^{-3x}\left(\frac{5}{34}\cos(5x) + \frac{3}{34}\sin(5x)\right) + \mathcal{C}$

b) $\frac{2}{9}x\cos(3x) - \frac{2}{27}\sin(3x) + \frac{1}{3}x^2\sin(3x) + \frac{1}{25}\sin(5x) - \frac{1}{5}x\cos(5x) + \mathcal{C}$

c) $x\arcsin(5x) + x\arctan(-3x) + \frac{1}{5}\sqrt{1-25x^2} + \frac{1}{6}\ln(9x^2+1) + \mathcal{C}$

d) $\frac{x^2}{2}\arccos(3x^2) - \frac{1}{6}\sqrt{1-9x^4} + \mathcal{C}$

3.

a) $\frac{x}{2} - \frac{1}{2}\sin x\cos x + \mathcal{C}$

b) $\sin x - \frac{1}{3}\sin^3 x + \mathcal{C}$

c) $\frac{3}{8}x - \frac{1}{4}\sin(2x) + \frac{1}{32}\sin(4x) + \mathcal{C}$

d) $-\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x + \mathcal{C}$

e) $\frac{3}{8}x + \frac{1}{4}\sin(2x) + \frac{1}{32}\sin(4x) + \mathcal{C}$

f) $\frac{1}{8}x - \frac{1}{32}\sin(4x) + \mathcal{C}$

g) $\frac{1}{4}\mathrm{tg}^4 x - \frac{1}{2}\mathrm{tg}^2 x - \ln|\cos x| + \mathcal{C}$

h) $-\frac{1}{3}\coth^3 x - \coth x + x + \mathcal{C}$

i) $\mathrm{th}\,x - \frac{2}{3}\mathrm{th}^3 x + \frac{1}{5}\mathrm{th}^5 x + \mathcal{C}$