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Reflections on the Relationships between Mathematics and Arts

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Abstract
This research is presenting and reflecting the philosophical analysis by Max Bense of the intellectual history of mathematics as a science of mind. Examples of important historical phases show corresponding relationships between mathematics and arts, including architecture and all areas of human creation abilities.

Keywords Philosophy · Historical treatise · History of mathematics · Art theory · Design theory

Introduction

Reflecting the relationship between mathematics and arts, including architecture, leads to more general research on the understanding of mathematics as part of the sciences of mind, introduced as 'Geisteswissenschaften' in the German philosophy. The philosopher Max Bense described in his early writings the contours of an intellectual history of mathematics related to philosophy. He pointed out strong relationships between mathematics and all areas of human creation abilities. Architecture is seen by him as part of arts. In particular, his investigations on the relationship between arts and mathematics in important historical phases show that the mathematics of an age is reflected aesthetically in the artistic styles or theories of arts. His thesis is that a high point in mathematics corresponds to a high point in arts. These relationships will be presented and explained in order to deepen the philosophical concept of mathematics as a science of mind in relation to art theory.

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The Research

The works of the philosopher, mathematician, and physicist Max Bense (1910–1990) are an extensive source for studying the relationships between mathematics and arts, including architecture. He investigated mathematics related to aesthetics through his whole work. Here, we will focus on his early publications, two volumes on the contours of an intellectual history of mathematics (Bense 1946; 1949). The topics related to the relationships between mathematics and arts can be found in volume two. Arts include for Bense all expressions of the aesthetic spirit, which manifests itself manifold in poetry, literature, painting, sculpture, architecture and in the play of dance and music.

Bense describes four historical phases of an intimate connection between arts and mathematics:

1. In pre-Greek mathematics the construction of symmetrical figures and ornaments, in particular in Egyptian mathematics and ornamentation, express the connection between arts and mathematics.
2. The formation of non-representational Gothic architectural ornament or tracery and spatial groups at Gothic cathedrals are the main connections in Gothic time.
3. Resumption of the doctrine of the golden section and the formation of perspective in Renaissance architecture and painting as well as the reception of Euclid, Archimedes and Vitruvius are seen as the characteristics of Renaissance time.
4. The connection between the Infinitesimal calculus and the emergence of the concept of Mathesis universalis as well as the spatial curves are the special features of Baroque time.

Symmetries and Ornaments in Pre-Greek Mathematics

Symmetries can be seen according Bense as the earliest relationships between aesthetics and mathematics. He refers to Paul Valéry who ascribed a fundamental importance for art to the ornament. “From this point of view, the ornamental conception is to the individual arts what mathematics is to the other sciences.” (Bense 1949: 59; Valéry 1894). This origin showed the aesthetic reduction of mathematics or the birth of mathematics out of the spirit of aesthetics. The construction of symmetrical figures including the creation of ornaments are the earliest expressions of the union of aesthetic and mathematical relationships. He refers to oriental carpet patterns and Egypt as the great source of ornamentation.

Spatial Groups and Tracery in Gothic

In Gothic times the deep expression of the relationship between mathematics and arts were seen in the spatial groups of the Gothic cathedrals and in the Gothic tracery. Spatial groups had been discussed with the Platonic and the Archimedean solids. They were characterized by special symmetries, which point out an aesthetic consciousness of form. The solids are related to space grids, describing the possible crystal systems. Bense cites the superimposed hexagonal prisms that form the helmet of Strasbourg Cathedral as an example of the occurrence of spatial groups in Gothic architecture (Fig. 1).

In the context of the formation of the Gothic in France, the tracery came into being, which already betrays the linking of the aesthetic and mathematical aim through its term. Here he refers to the work of Lottlisa Behling showing examples of basic patterns of the curvilinear style (Fig. 2).

Reception of Euclid, Perspective and Proportion Theories in Renaissance

The period of Renaissance is characterized by the reception of Euclid, starting already in Gothic time with their roots in Euclid's Optica. The developed scenography has the aim to represent a building as it appears to the eye. In terms of intellectual history, the

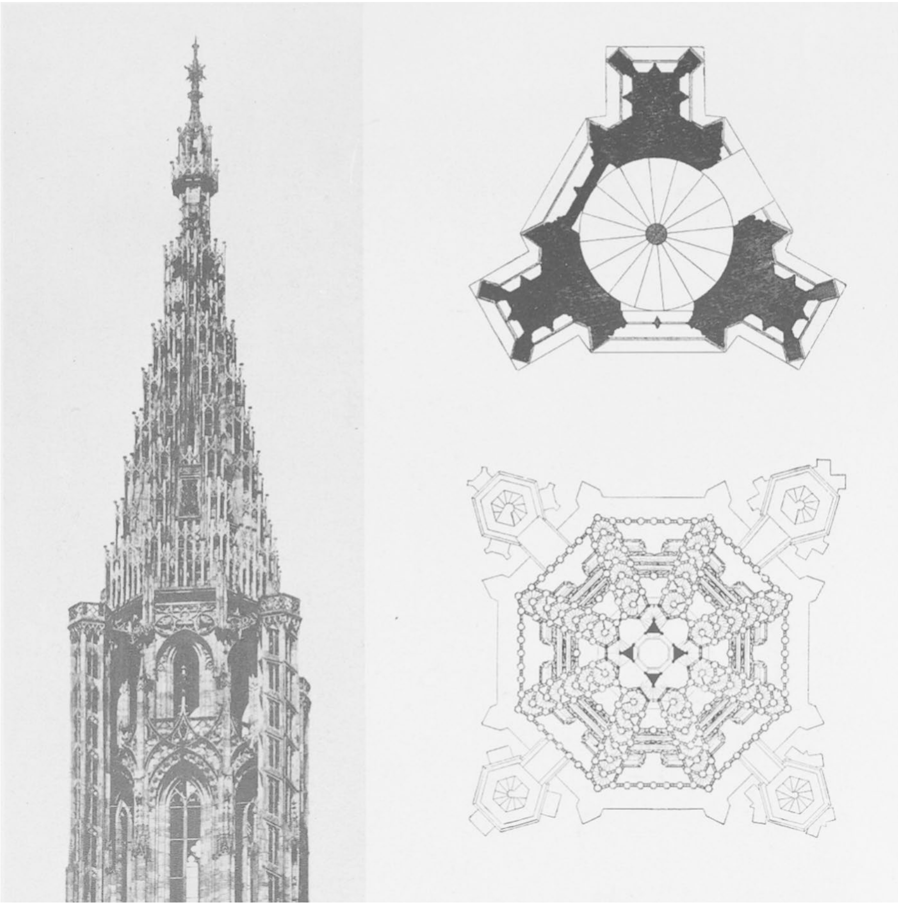

Fig. 1 The tower of the Strasbourg Cathedral, floor plans of the tower octagon and of a stairway tower. Delhio, Georg. 1922. *Das Straßburger Münster*. München: Piper: 75. https://doi.org/10.11588/diglit.12157#0078. Accessed 24 February 2023

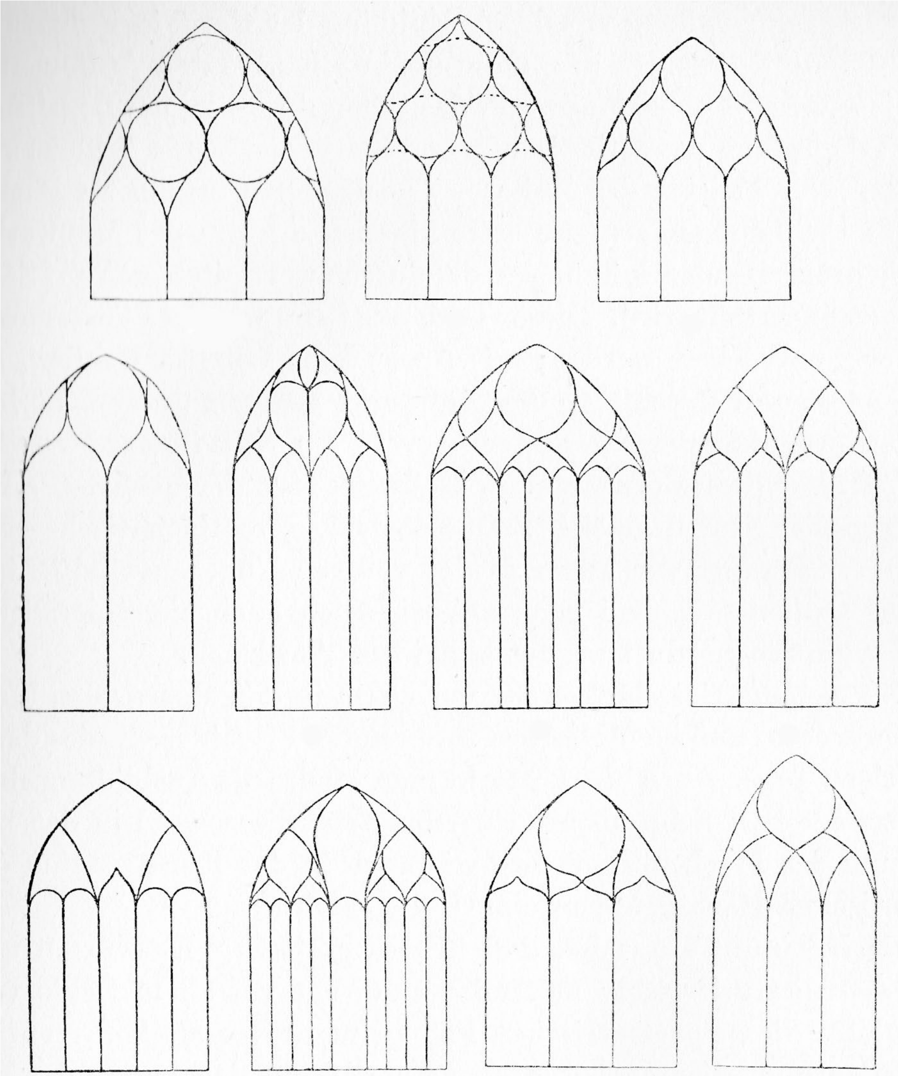

Fig. 2 Basic patterns of the curvilinear style in the Gothic period according to Behling, Lottlisa. 1944. *Gestalt und Geschichte des Maßwerks*. Halle (Saale): Max Niemeyer: 35

entire artistic rationalism, so characteristic of the Renaissance, could be understood as a resumption of tendencies that previously characterized the empirical rationalism of Late Scholasticism e.g. by Roger Bacon (1267). Bacon carried out optical experiments for receiving a true knowledge of nature. Direct experience was seen as the source of all true knowledge of the world. The perspective theories on Renaissance going back to these roots. The perspective representation of objects in paintings corresponds to the epistemological conception of the world in philosophy. The subject-object relationship is thematically presupposed in the theory of perspective as well as in the epistemology, first developed in the field of aesthetics. The work of

Michel de Montaigne and much later Kant worked on the subject-object-relationship. Sociological changes, where craft-technical skills got important besides the geometry scholars had been external reasons for the artistic rationalism in the Renaissance movement accompanied by the aspiration of exact sciences and art. As the great theorists and practitioners of the perspective as well as theory of proportions in Renaissance, Bense mentions Uccello, Piero della Francesca, Brunelleschi, Alberti, Leonardo, and Dürer (e.g. perspective drawing machine in Fig. 3) with their works.

Mathesis Universalis and Space Curves in Baroque

The art historical period of the Baroque coincides with the math historical period of a powerful mathematical production and a philosophy of mathematics with the idea of Mathesis universalis, a universal applicability of mathematics.

Descartes (1637) developed the idea of Mathesis universalis in the sense of a generalized mathematics including non-mathematical objects or a method, which mathematically controls our rational reasoning. The form of mathematics is demanded as the form of science and philosophy, characterized by axioms and theorems, following out of the axioms by logical conclusion rules.

Pascal (1658) laid down the science theory, rules for defining axioms, and proofs. Bense evaluates it as the first example of an axiomatics going beyond the program of Euclid and Aristotle.

Leibniz (1679) finally demanded the first time programmatically characteristica universalis as a universal conceptual language. The scientific-theoretical program of a Mathesis universalis corresponded with the art-theoretical program of a 'Gesamtkunstwerk' (total artwork) in the Baroque period. Architecture took the lead over painting and sculpture. Bense calls three points how the mathematical sense of form influenced the universal style in art:

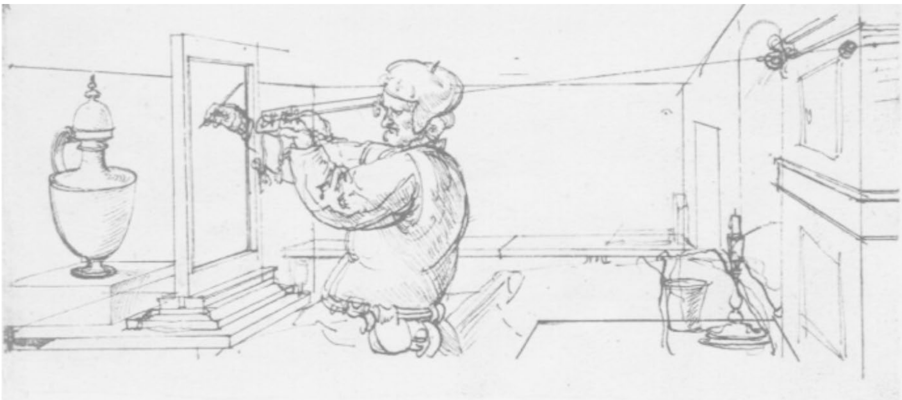

Fig. 3 Sketches of Jacob Keser's perspective machine versions by Dürer. Bruck, Robert (ed.). 1905. *Das Skizzenbuch von Albrecht Dürer*. Strassburg: Heitz: Taf. 136. https://digital.slub-dresden.de/werkansicht/dlf/71653. Accessed 24 February 2023

- The geometric content of the applied perspective theory of the Renaissance is translated into space. The development of the relief perspective shows this generalization where the two-dimensional perspective image is transformed in space (Leopold 2019).
- The deep thought of Mathesis universalis, to control everything through mathematical theory, means in the field of art that also nature can be subjected to the aesthetic system, especially expressed in the garden architecture combined with the buildings in the Baroque period.
- In his basic art-historical concepts, Wölfflin (1915) made the differentiation of the classical art of the 16th from the art of the 17th century with the conceptual pair 'multiplicity' and 'unity'. „The Baroque basically no longer reckons with a multiplicity of independent parts that interlock harmoniously, but with an absolute unity in which the individual part has lost its special right." (Wölfflin 1915: 165).

Bense describes 'representation' as one of the most important notions in the mathematics of the Baroque time. The characteristica universalis demands a bijective relation between the thought and the sign. The Baroque art could be characterized as representation art.

The concept of 'continuity' in mathematics according Leibniz and Bernoulli in their development of the infinitesimal calculus corresponds in Baroque art with moving continuous forms, the geometry of curves. Bernini, Borromini, Guarini and Juvarra are listed by Bense as representants of the Italian Baroque. In Germany especially Balthasar Neumann (Fig. 4) had been architect and mathematician at the same time. The analysis of their art means always also a mathematical analysis.

Max Bense summarizes: "As a result of this intellectual-historical study of mathematics and art, it can be stated that every emergence of a new style in European art

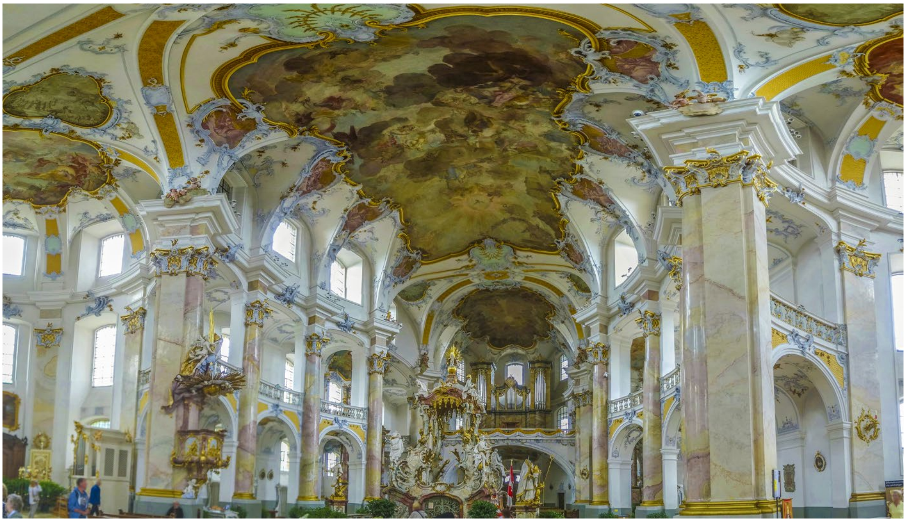

Fig. 4 Interior of the Basilica 'Vierzehnheiligen', Balthasar Neumann, 1743–1772. Erwin Meier, CC BY-SA 3.0, via Wikimedia Commons https://upload.wikimedia.org/wikipedia/commons/f/f6/Innenraum-Panorama_der_Basilika_Vierzehnheiligen.jpg. Accessed 24 February 2023

is linked to the introduction of mathematical methods and theorems, to which the formal elements of that style can be traced more or less uniformly and completely." (Bense 1949: 207). An aesthetics of pure artistic forms, which can be traced back to mathematical forms is following. But which of those forms can be called artistic and what is to be understood as aesthetics? His answer is, that the mathematical forms which affect our senses, we call aesthetic forms.

Based on these early studies, Bense developed in his later publications an exact aesthetics on the foundation of information theory and Birkhoff's definition of the aesthetic measure. These philosophical analyses and concepts had a great impact on the background of Ulm School of Design (Leopold 2013).

Conclusion

The relationships between mathematics and arts could be investigated according to the philosophical studies of Max Bense in their mutual correspondences in their intellectual history. These were described by means of four historical phases Pre-Greek, Gothic, Renaissance, and Baroque. He explained that a new style in European art was linked to new methods in mathematics. Artistic forms could be traced back to mathematical forms in his aesthetic theory. These contexts could be hinted at some examples, where the relationships became understandable. An extension to further periods would be interesting in order to prove Bense's thesis through the intellectual history of mathematics and arts. The further developments of mathematics as the thinking in structures and systems according to Felix Klein and David Hilbert provide the foundation for the general applicability of mathematics also for aesthetic structures in actual times (cf. Leopold 2013) including computerized methods like parametric design methods.

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