Referee's report

on

Theoretical Foundations of Buffer Stock Saving

by

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The paper under review studies the properties of a canonical infinite horizon precautionary savings model. The instantaneous utility function is CRRA in consumption, $c^{1-\rho}/(1-\rho)$. Labor is supplied inelastically, with no contingent markets on future wages. Human capital evolves exogeneously, its logarithm following a random walk. At each date, earnings are equal to the level of human capital multiplied by an iid factor, which can be equal to zero with a (small) positive probability.

The above formulation is technically challenging, compared with the existing literature, because marginal utility is infinite at the origin, and the shocks to income are permanent. In particular, to apply a contraction mapping property, the state variable must be measured in reference to the growth trajectory, as in Boyd's contraction theorem. Indeed, in my opinion, the main result of the paper is Theorem 1, page 19, which gives sufficient conditions for the consumption function to satisfy a contraction mapping property.

The manuscript is long (60 pages, made of 39 pages of text, with the remainder for appendices and references). Section 2 is 22 pages long and contains the bulk of the paper in eleven subsections. Section 3 describes some further properties of the optimal consumption behavior. Section 4 finally presents long run properties of the distribution of savings.

The manuscript is very hard to penetrate. Notations are unduely extremely complicated (see Table 1, page 14 for a sample!) and make for strenuous reading. Do we gain from letting

$$\text{Þ} = (R\beta)^{1/\rho} \text{ and } \text{Þ}_\Gamma = \text{Þ}/\Gamma$$

so as to rewrite the inequality

$$\beta\Gamma^{1-\rho} < 1$$

as

$$\text{Þ}_\Gamma < (R/\Gamma)^{1/\rho}?$$

While Theorem 1 and the limit properties of the policy function are worth having, I am not sure that they are of great importance for the understanding of precautionary saving. The recursive setup is part of a theoretical exercise, but in practice lifes are of finite length, the household composition and therefore instantaneous utility functions vary, and I doubt that extensions of the contraction mapping properties be useful in applied work.

All things considered, while the paper is of interest and contains interesting material, I do not think that it is strong enough to deserve publication in *Econometrica*.

Some more detailed remarks follow :

1. Section 2.1: this is a very specific problem, with no utility for leisure, CRRA utility function, non random asset returns. Some more justification for the special income process studied here would be welcome.
   It seems that, in this section, the model has finite lifetimes (this is not stated explicitly, but seems implicit from (4)). This feature does not fit well with the discussion of the last paragraph of page 5, which claims that an important difference with the earlier formulations of Bewley and Schectman Escudero is that the current one permits permanent growth.
2. Section 2.2: a direct change of variables in the initial problem would be simpler to read and to follow, yielding

   $$\max E_t \sum_{n=0}^{T-t} \beta^n p_{t+n}^{1-\rho} \frac{c_{t+n}^{1-\rho}}{1-\rho}$$

   subject to, for all $n = 0, \ldots T-t$,

   $$m_{t+n+1} = \mathcal{R}_{t+n+1}(m_{t+n} - c_{t+n}) + \xi_{t+n+1}.$$
3. Page 7, line 6: please give the explicit definition of a 'limiting consumption function'. Can there be several optima, given the strictly concave utility index?
4. Sections 2.3 and 2.6 do not deserve to have the status of full blown sections.
5. Section 3: it would be good to have a formal theorem describing the properties of the limit consumption function, together with the assumptions that warrant them.