Report

Task - 1: Execute bruteforce computation

读入数据后，代码中`mat`矩阵中存储文档包含的单词，`jac`中存储所有document pair的jaccard相似度，其中计算相似度代码如下：

```python
def jaccard_similarity(list1, list2):
    s1 = set(list1)
    s2 = set(list2)

    return len(s1.intersection(s2)) / len(s1.union(s2))
```

数据读取代码：

```python
with open(fileName, 'r') as f:
    lines = f.readlines()[3:]
    for line in lines:
        wordList = line.strip('\n').split()
        mat[int(wordList[1]) - 1][int(wordList[0])] = 1
```

以bruteforce的方式遍历所有document pair，时间复杂度 $\mathcal{O}(n^2)$.

其计算结果被写入文件：`Jaccard-similarities.npy`

数据集描述： `docword.kos`：文档数：3430，单词数：6906，总词量：353160

(a) The running time of your bruteforce algorithm

```
1/preprocessing(Task-1).py"
  1%|          | 28/3430 [00:14<30:24,  1.87it/s]Start calculating Jaccard similarities [6906] ...
100%|██████████| 3430/3430 [14:25<00:00,  3.96it/s]
The running time of your bruteforce algorithm: 865.9343903064728

Process finished with exit code 0
```

共用时：`865.9344 s`

(b) The average Jaccard similarity of all pairs except identical pairs

由题意，我将结果保存在`Jaccard-similarities.npy`文件内，**读取请执行Python代码：**

```python
with open('Jaccard-similarities.npy', 'rb') as f:
    jac = np.load(f)
```

Task - 2: Compute the MinHash signatures for all documents

首先根据上课所讲的计算MinHash Signature的方式，对于某个排列π，文档S_i的所有单词中在排列中第一个出现的下标作为当前排列的签名，这也是MinHash最小哈希方式的体现，即：

$$h_\pi(S) = \min(\pi(S))$$

代码实现方面我与PPT中One-pass MinHash signatures方式完全一致:

```python
    for j in range(1, docNum + 1):
        curMinHash = 9999999
        for k in range(wordNum):
            if mat[k][j] == 1 and pi[k] < curMinHash:
                curMinHash = pi[k]
        minHash[i][j] = curMinHash
    timeRes.append(time.time() - begTime)
```

这里依据上课所说，我们使用 http://www.mmds.org/ 中所述的fast method to "simulate" this randomness using different hash functions:

```python
def h0():
    return (2 * x + 1) % wordNum
def h1():
    return (3 * x + 2) % wordNum
def h2():
    return (5 * x + 2) % wordNum
def h3():
    return (7 * x + 2) % wordNum
def h4():
    return (11 * x + 2) % wordNum
```

尽量保证的哈希函数的随机性:

```python
for i in tqdm.trange(100):
    if i > 19:
        pi = ((30 * int(i / 10) + hFuncParameter[i % 10]) * x + 3) % wordNum
    else:
        exec("pi = h" + str(i) + "()\n")
    # print(pi)
    for j in range(1, docNum + 1):
```

(a) The running time of this step

```
Start computing the MinHash signatures [6906] ...
100%|██████████| 10/10 [01:59<00:00, 11.94s/it]
The running time of "computing the MinHash signatures for all documents": 119.43758749961853

Process finished with exit code 0
```

```
E:\Anaconda3\python.exe "E:/NJUCS-Course-Material-fron
Start computing the MinHash signatures [6906] ...
100%|██████████| 100/100 [21:09<00:00, 12.69s/it]
```

- d = 10 个哈希函数$h(S)$，共用时：`865.9344 s`
- d = 100 个哈希函数$h(S)$，共用时：`1269.3851` s

Task - 3: Measure the accuracy of MinHash estimators

使用Task-2的输出，首先计算MinHash的Jarcard相似度：

```python
for k in hFunList:
    print("Start calculating MinHash's Jaccard similarities, k of h = {} ...".format(k))
    jac = np.zeros((docNum + 1, docNum + 1), dtype=np.float)
    curTime = time.time()
    # print(mat)
    err = 0
    for i in tqdm.trange(1, docNum + 1):
        for j in range(i, docNum + 1):
            if i == j:
                jac[i][j] = 1
                continue

            curJac = jaccard_similarity(minHash[:k, i], minHash[:k, j])
            jac[i][j], jac[j][i] = curJac, curJac
            err += abs(curJac - jacReal[i][j]) / (docNum * docNum - docNum)

    runningTime = time.time() - curTime
```

再根据MAE计算的公式，在每个迭代步进行累加：

$$MAE = \sum_{i,j=1,i\neq j}^{n} \frac{\left|J\left(d_i, d_j\right) - \hat{J}\left(d_i, d_j\right)\right|}{n^2 - n}$$

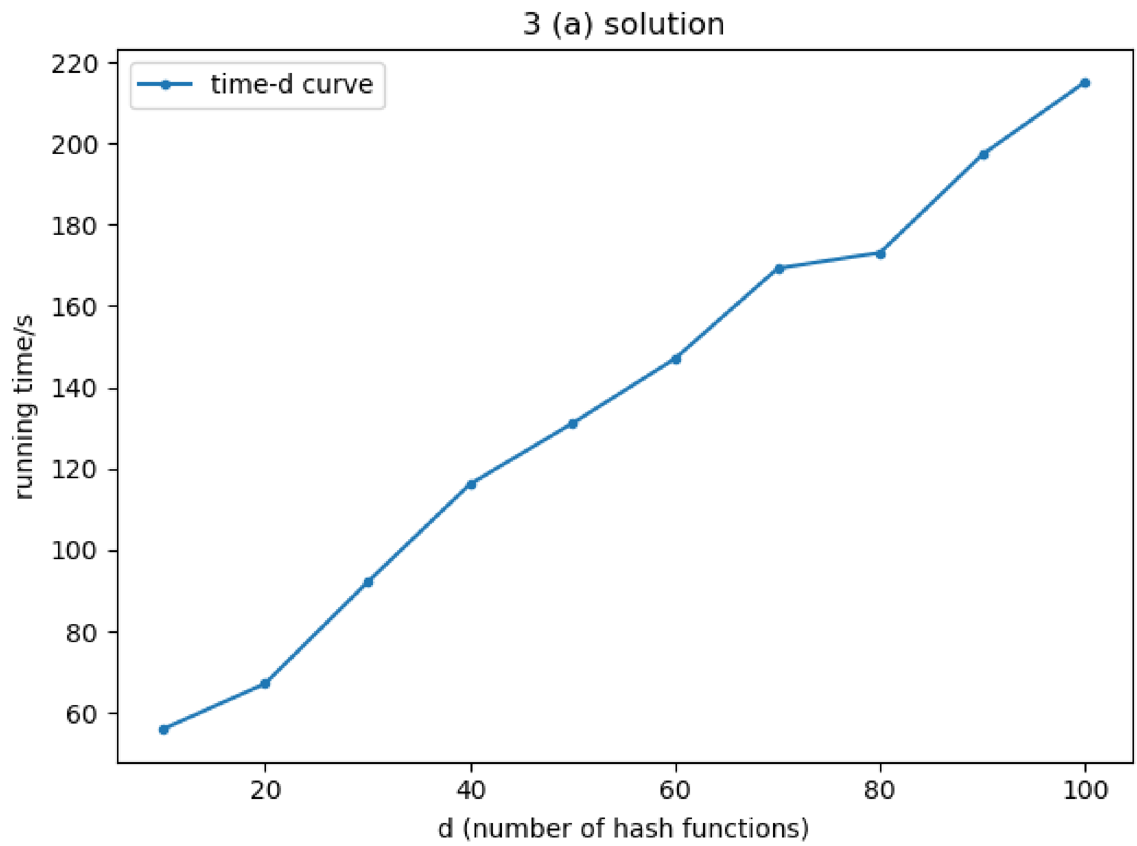
画图的代码请见： `Task-3-plot.py`

(a) The running time of estimating all pairs similarity based on Min-Hash with different values of d

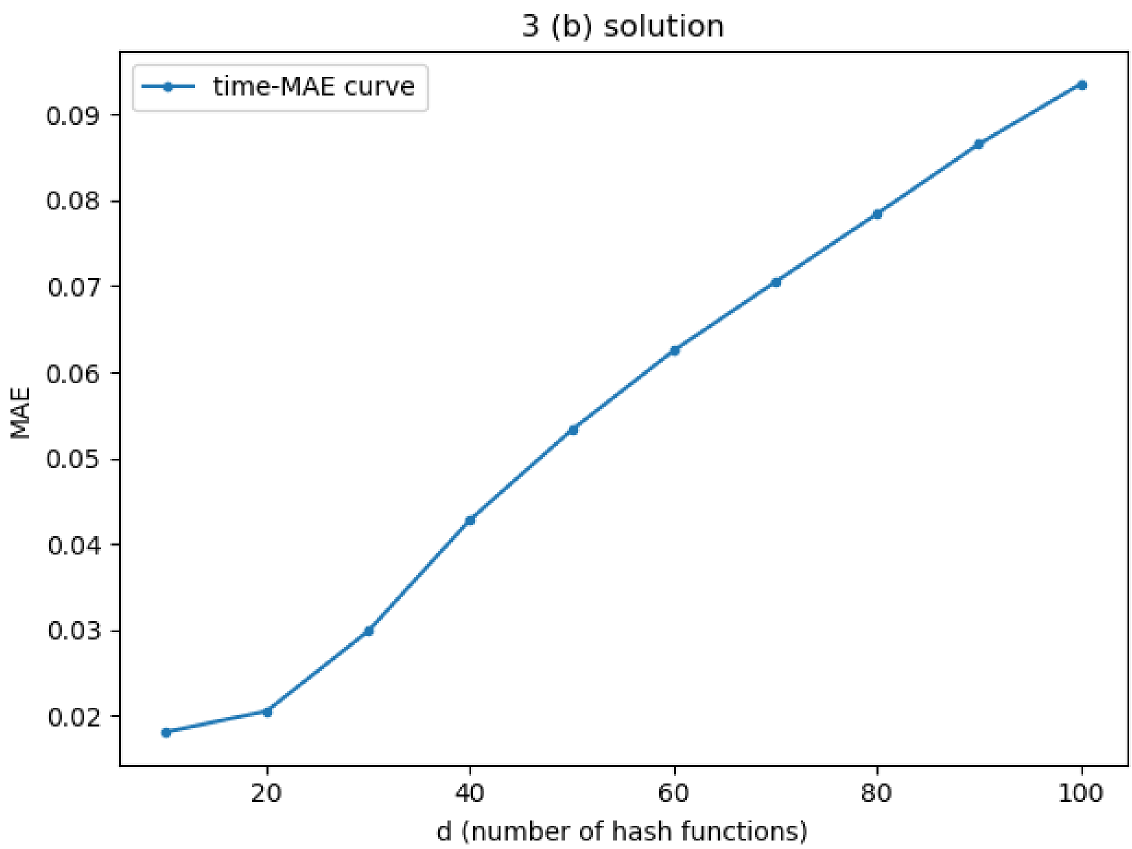
运行截图：

```
Run: Task-3
--------------------------------------------------------------------------------
Start calculating MinHash's Jaccard similarities, k of h = 89 ...
100%|██████████| 3430/3430 [03:17<00:00, 17.39it/s]
The running time: 197.30470252037048, MAE: 0.08656782831408438
--------------------------------------------------------------------------------
Start calculating MinHash's Jaccard similarities, k of h = 99 ...
100%|██████████| 3430/3430 [03:35<00:00, 15.95it/s]
The running time: 215.0799641609192, MAE: 0.0935516877538423
--------------------------------------------------------------------------------

Process finished with exit code 0
```



(b) The figure of MAEs with different values of d on x-axis and MAE values on y-axis



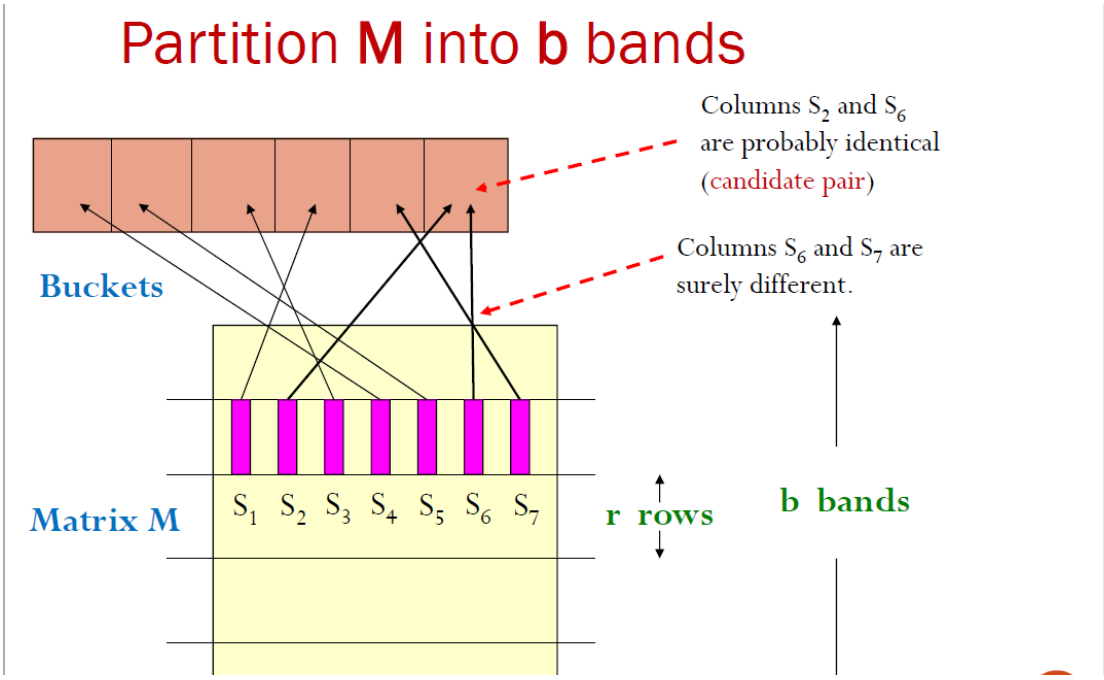
#####

（加分项）此外，在测试环节我自己构造了一个较小的数据集 `myTestData.kos.txt`，来源于Tutorial 1: Locality-sensitive Hashing中的第二题. 结果与Tutorial 2中答案一致. 可以保证正确性.

Task - 4: Exploit LSH

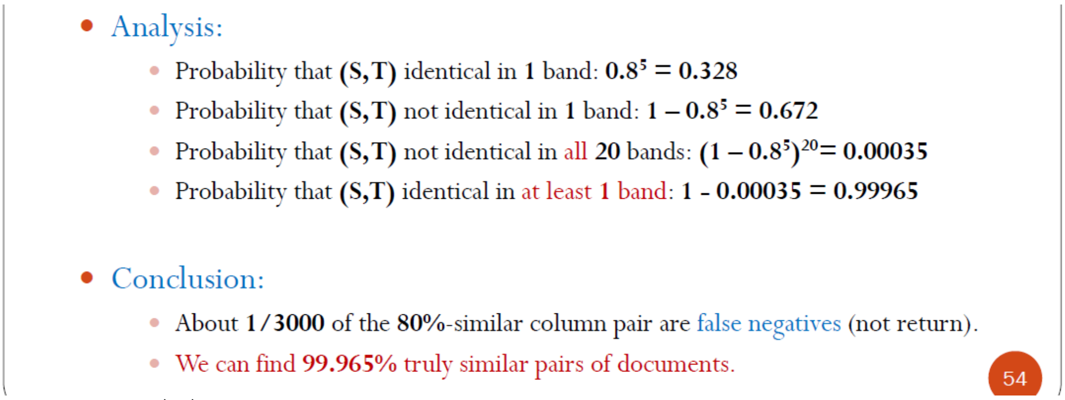
(a) How to tune the parameter b (number of bands) and r (number of rows in one band) so that we achieve the false negatives of 60%-similar pairs at most 10%

首先将Signature Matrix分成一些bands，每个bands包含一些rows，然后把每个band哈希到一些bucket中：



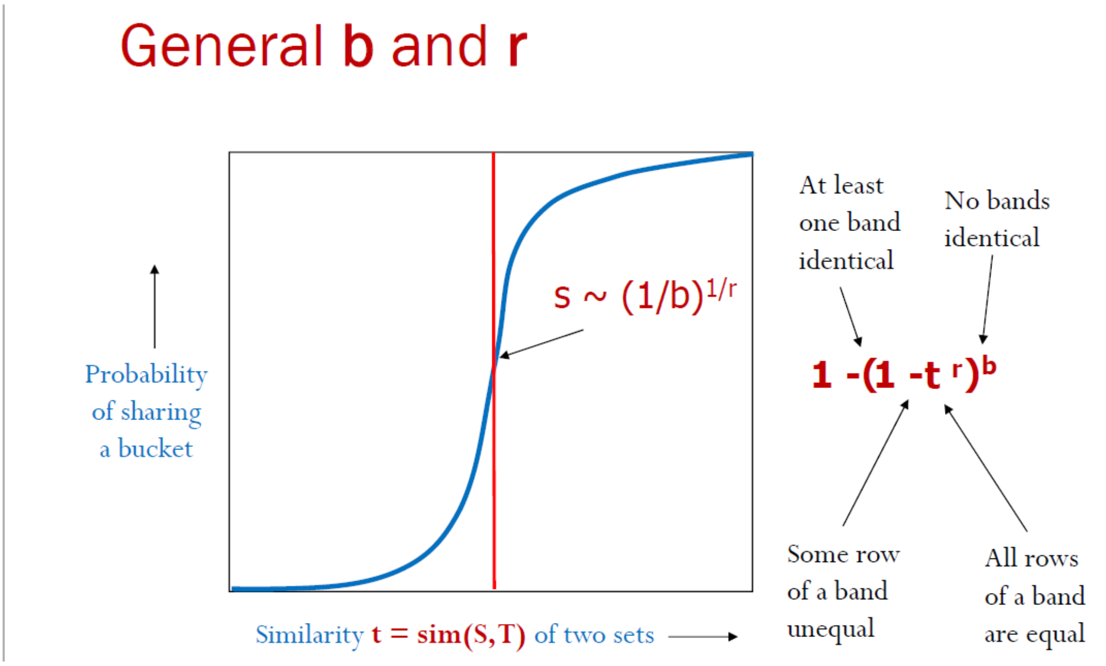
计算过程距离如下(与PPT完全一致)：

- 假设存在一个document pair相似度为80%，对应的Signature Matrix分成20个bands，每个bands有5行，那么有：



- 以上分析将用于(a)中的计算.

又因为上述计算过程具有函数单调性，如下图在每一个固定的b，r下：



所以我们可以用**Binary Search**来找到满足题目要求的点

- **Binary Search**:

  算法将在x坐标轴，即document pair的相似度空间进行二分，将$1-(1-t^r)^b$的计算结果：**上界下界中点** 与题目所需要的进行比较，如果更大，则将上下界中点赋值给下一轮迭代的下界.

  搜索部分算法可以减低到 $\mathcal{O}(log(n))$ 的时间复杂度. 其中n为相似度的值空间大小.

对b,r的搜索将采用网格搜索的方式，每一对b,r对应了唯一一个满足题目条件的$J(S_1,S_2)$，所以这种搜索方式是可以找到解的.

即有：

$$1-(1-0.6^r)^b = 1-10\%$$

又 $r \cdot b = 100$，我们可以得到解：

$$b = 20, r = 5 \Rightarrow 0.19809754616177833$$

$$b = 25, r = 4 \Rightarrow 0.03111516145458214$$

(b) The space usage affected by these parameters

由上分析我们知道，LSH算法的space usage与分成b个bands，所映射到的哈希表的数量密切相关，不妨设哈希表有n个entries，哈希表数为L(与b,r的选择密切相关)，则空间复杂度为 $\mathcal{O}(nL)$

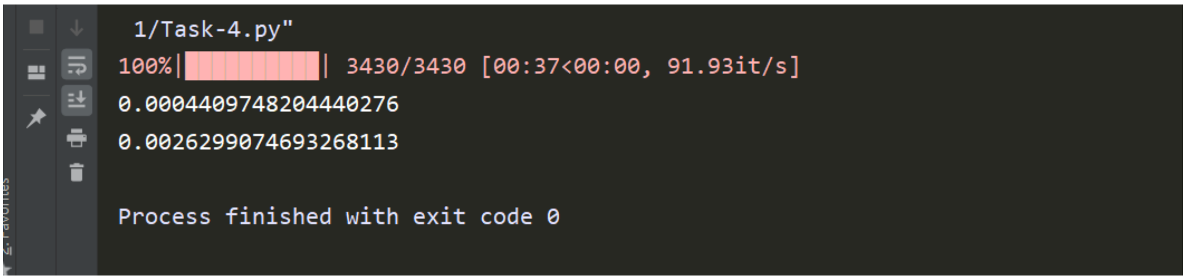
在上述计算过程中，空间复杂度为 $\mathcal{O}(\max\{docNum, wordsNum\}^2)$

(a) The false candidate ratio / (b) The probability that a dissimilar pair with Jaccard <= 0.3 is a candidate pair

如果两个document的bands中，至少有一个share了同一个bucket，那这两个document就是candidate pair，也就是很有可能是相似的.

对于每一个固定的b,r，一个pair是candidate pair的概率为$1-(1-t^r)^b$，其中t为相似度.

我们使用 $b = 20, r = 5$.



```
 1/Task-4.py"
100%|██████████| 3430/3430 [00:37<00:00, 91.93it/s]
0.0004409748204440276
0.0026299074693268113

Process finished with exit code 0
```

得到(a)题结果为0.0004409，(b)题结果为0.002629.