2.2 *Entropy of functions.* Let X be a random variable taking on a finite number of values. What is the (general) inequality relationship of $H(X)$ and $H(Y)$ if

(a) $Y = 2^X$?

(b) $Y = \cos X$?

2.4 *Entropy of functions of a random variable.* Let X be a discrete random variable. Show that the entropy of a function of X is less than or equal to the entropy of X by justifying the following steps:

$$H(X, g(X)) \overset{(a)}{=} H(X) + H(g(X) \mid X) \qquad (2.168)$$

$$\overset{(b)}{=} H(X), \qquad (2.169)$$

$$H(X, g(X)) \overset{(c)}{=} H(g(X)) + H(X \mid g(X)) \qquad (2.170)$$

$$\overset{(d)}{\geq} H(g(X)). \qquad (2.171)$$

Thus, $H(g(X)) \leq H(X)$.

2.6 *Conditional mutual information vs. unconditional mutual information.* Give examples of joint random variables X, Y, and Z such that

(a) $I(X; Y \mid Z) < I(X; Y)$.

(b) $I(X; Y \mid Z) > I(X; Y)$.