Answer ALL Questions. You can have two A4-sized cheat sheets. Total points: 115.

Question 1 [Images in the Spatial Domain] (20 points)

For the small image below with 3 bits per pixel, answer the following:

6	3	6	3	7
4	0	6	2	1
7	7	6	6	2
0	0	0	3	1

a) Find the number of gray levels. **[2]**

$2^3 = 8$ Gray levels

b) Find the image carrier and its cardinality. **[2]**

image Carrier is

$$\Omega = \{ (x, y) : 1 \leq x \leq 5 \wedge 1 \leq y \leq 4 \} \subset Z^2$$

Cardinality is :

$$4 \times 5 = 20$$

c) What is the number of all possible images that can be defined for this image carrier and number of gray levels? **[2]**

We have 20 pixel each pixel may have level of 8 gray levels so number of possible images is :

8^{20} image

d) Find the image mean and median. **[2]**

mean = 3.5, median = 3

e) Find the absolute image histogram. **[2]**

Gray Level	0	1	2	3	4	5	6	7
Histogram	4	2	2	3	1	0	5	3

f) Find the absolute image histogram after the least significant bit is set to 0. In general, what effect would setting to zero the lower-order bit planes have on the histogram of an image? **[3]**

Gray Level	0	1	2	3	4	5	6	7
Hist	4	2	2	3	1	0	5	3
LSTB 0	000 $\rightarrow$ 0	001 $\rightarrow$ 0	010 $\rightarrow$ 2	011 $\rightarrow$ 2	100 $\rightarrow$ 4	101 $\rightarrow$ 4	110 $\rightarrow$ 6	111 $\rightarrow$ 6
New H	6	0	5	0	1	0	8	0

The effect is compressing histogram of the image by clustering each two successive levels to lower one.

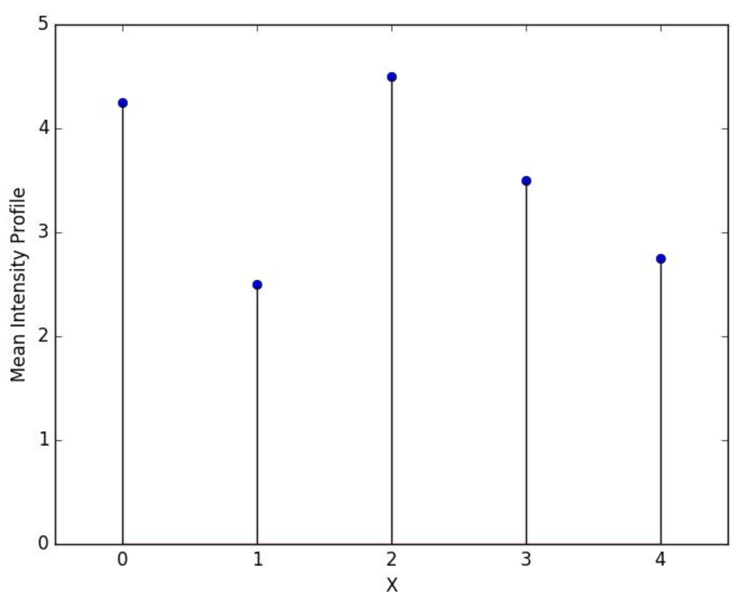
g) Find the absolute image histogram after the most significant bit is set to 0. In general, what effect would setting to zero the higher-order bit planes have on the histogram of an image? **[3]**

Level	0	1	2	3	4	5	6	7
Hist	4	2	2	3	1	0	5	3
MSTB 0	000 $\longrightarrow$ 0	001 $\longrightarrow$ 1	010 $\longrightarrow$ 2	011 $\longrightarrow$ 3	100 $\longrightarrow$ 0	101 $\longrightarrow$ 1	110 $\longrightarrow$ 2	111 $\longrightarrow$ 3
New Hist	5	2	7	6	0	0	0	0

The effect is that the image contrast will decrease and image will be darker.

h) Find and plot the means of the horizontal intensity profiles. **[2]**

X	0	1	2	3	4
Mean horizontal intensity profile	4.25	2.5	4.5	3.5	2.75



i) Find the L_1 and L_2 distances between the first and last vertical intensity profiles. **[2]**

$L_1 = \frac{1}{4} \times \sum_{0}^{3} |a - a_2|$, $L_2 = \frac{1}{4} \times \sqrt{\sum_{0}^{3} (a_1 - a_2)^2}$ and $a_1 = [6\ \ 7\ 4\ 0]^T$, $a_2 = [7\ 1\ 2\ 1]^T$

$L_1 = \frac{10}{4} = 2.5$, $L_2 = \frac{6}{4} = 1.5$

Question 2 [Images in the Frequency Domain] (25 points)

a) Use the exponential representation of the sine function to show that the DFT of the discrete function $f(x,y) = sin(2\pi u_0 x + 2\pi v_0 y)$ is

$F(u,v) = \frac{i}{2}[\delta(u + N_{cols}u_0, v + N_{rows}v_0) - \delta(u - N_{cols}u_0, v - N_{rows}v_0)]$. **[10]**

$$F(u,v) = \sum_{y=0}^{N_{rows}-1} \sum_{x=0}^{N_{cols}-1} f(x,y)\, e^{-2\pi j(\frac{ux}{N_{cols}} + \frac{vy}{N_{rows}})}$$

and $sin(\theta) = \frac{1}{2j}[\, e^{j\theta} - e^{-j\,\theta}]$

$$f(x,y) = \frac{1}{2j}[e^{j(2\pi u_0 x + 2\pi v_0 y)} - e^{-j(2\pi u_0 x + 2\pi v_0 y)}]$$

$$F(u,v) = \sum_{y=0}^{N_{rows}-1} \sum_{x=0}^{N_{cols}-1} \frac{1}{2j}[e^{j(2\pi u_0 x + 2\pi v_0 y)} - e^{-j(2\pi u_0 x + 2\pi v_0 y)}]\, e^{-2\pi j(\frac{ux}{N_{cols}} + \frac{vy}{N_{rows}})}$$

$$= \sum\sum \frac{1}{2j}[\, e^{-2\pi j(\frac{(u - u_0 N_{cols})x}{N_{cols}} + \frac{(v - v_0\, N_{rows})y}{N_{rows}})} - e^{-2\pi j(\frac{(u + u_0 N_{cols})x}{N_{cols}} + \frac{(v + v_0\, N_{rows})y}{N_{rows}})}\,]$$

$$F(u,v) = \frac{1}{2j}[\delta(u - N_{cols}u_0, v - N_{rows}v_0) - \delta(u + N_{cols}u_0, v + N_{rows}v_0)]$$

$$F(u,v) = \frac{-1}{2j}[\delta(u + N_{cols}u_0, v + N_{rows}v_0) - \delta(u - N_{cols}u_0, v - N_{rows}v_0)]$$

$$F(u,v) = \frac{j}{2}[\delta(u + N_{cols}u_0, v + N_{rows}v_0) - \delta(u - N_{cols}u_0, v - N_{rows}v_0)]$$

Where $j^2 = -1$

b) Consider a 3×3 spatial mask that averages the four closest neighbors of a point (x, y), but excludes the point itself from the average.

1. Write an expression for the filter, $h(x, y)$, in the spatial domain. **[5]**

0	$\frac{1}{4}$	0
$\frac{1}{4}$	0	$\frac{1}{4}$
0	$\frac{1}{4}$	0

2. Show that the equivalent filter, $H(u, v)$, in the frequency domain is given by
$H(u, v) = \frac{1}{2}[cos(2\pi u/N_{cols}) + cos(2\pi v/N_{rows})]$. **[5]**

$$h(x, y) = \tfrac{1}{4}[\delta(x, y-1) + \delta(x, y+1) + \delta(x-1, y) + \delta(x+1, y)]$$
$$H(u, v) = \tfrac{1}{4}[e^{-2\pi j(v/N_{rows})} + e^{2\pi j(v/N_{rows})} + e^{-2\pi j(u/N_{cols})} + e^{2\pi j(u/N_{cols})}]$$
$$H(u, v) = \tfrac{1}{2}[\tfrac{e^{-j(2\pi u/N_{cols})} + e^{j(2\pi u/N_{cols})}}{2} + \tfrac{e^{-j(2\pi v/N_{rows})} + e^{j(2\pi v/N_{rows})}}{2}]$$

$$H(u, v) = \tfrac{1}{2}[cos(2\pi u/N_{cols}) + cos(2\pi v/N_{rows})]$$

3. Show that $H(u, v)$ is a low-pass filter. **[5]**

Lets see amplitude at low frequency (central regions)

$For\ u = 0\ and\ v = 0,\ H(u, v) = 1$

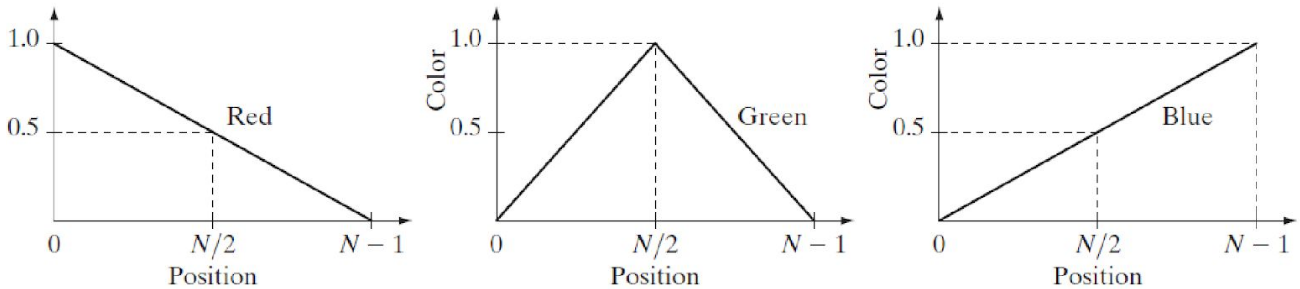
Lets see amplitude at high frequency (peripheral regions)

$For\ u = \frac{N_{cols}}{4}\ and\ v = \frac{N_{rows}}{4},\ H(u, v) = 0$

So Filter is low-pass filter that keep low frequencies and suppress high frequencies

Question 3 [Color and Color Images] (20 points)

In a simple RGB image, the R, G, and B component images have the horizontal intensity profiles shown in the following diagram.



a) What color would a person see in the first, middle, and last columns of this image? **[6]**

first [1, 0, 0] (Red) , middle [0.5, 1, 0.5] green , last [0, 0, 1] (Blue)

b) What are the cyan (C), magenta (M), and yellow (Y) components of the first, middle, and last columns of this image? **[6]**

first [0, 1, 1], middle [0.5, 0, 0.5] , last [1, 1, 0]

c) What are the hue (H), saturation (S), and intensity (I) components of the first, middle, and last columns of this image? **[8]**

first [0, 1, $\frac{1}{3}$], middle [$\frac{2\pi}{3}$, $\frac{1}{4}$, $\frac{2}{3}$] , last [$\frac{4\pi}{3}$, 1, $\frac{1}{3}$]

Question 4 [Operators](30 points)

The rectangle in the binary image below is of size 4×5 pixels.

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	**1**	**1**	**1**	**1**	**1**	0	0
0	0	**1**	**1**	**1**	**1**	**1**	0	0
0	0	**1**	**1**	**1**	**1**	**1**	0	0
0	0	**1**	**1**	**1**	**1**	**1**	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

1. What would the magnitude of the gradient of this image look like based on using the approximation

$$\| grad\, I(x,y) \|_1 = |S_x(x,y)| + |S_y(x,y)|$$

Where

S_x and S_y are obtained using the Sobel operators. Show all pixel values in the gradient image. **[10]**

Sx(x,y)

0	0	0	0	0	0	0	0	0
0	-1	-1	0	0	0	1	1	0
0	-3	-3	0	0	0	3	3	0
0	-4	-4	0	0	0	4	4	0
0	-4	-4	0	0	0	4	4	0
0	-3	-3	0	0	0	3	3	0
0	-1	-1	0	0	0	1	1	0
0	0	0	0	0	0	0	0	0

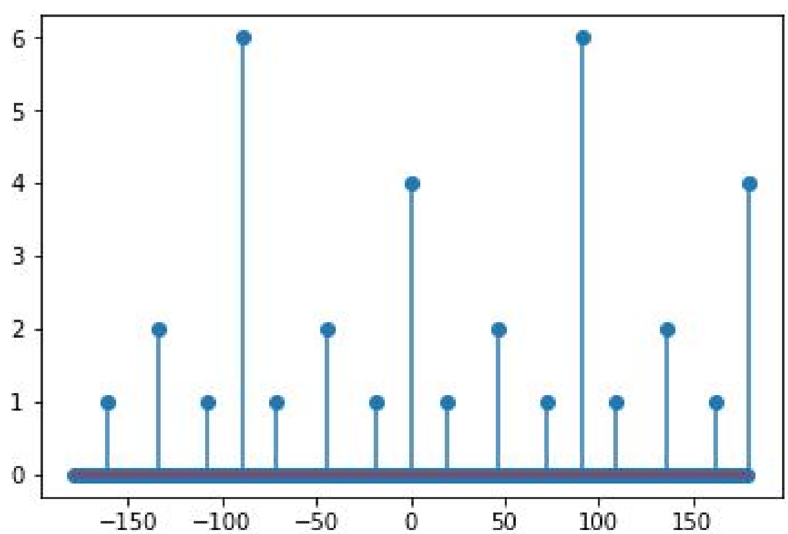
Sy(x,y)

0	0	0	0	0	0	0	0	0
0	-1	-3	-4	-4	-4	-3	-1	0
0	-1	-3	-4	-4	-4	-3	-1	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	1	3	4	4	4	3	1	0
0	1	3	4	4	4	3	1	0
0	0	0	0	0	0	0	0	0

$\| \, grad\, I(x,y) \, \|_1 = |S_x(x,y)| + |S_y(x,y)|$

0	0	0	0	0	0	0	0	0
0	2	4	4	4	4	4	2	0
0	4	6	4	4	4	6	4	0
0	4	4	0	0	0	4	4	0
0	4	4	0	0	0	4	4	0
0	4	6	4	4	4	6	4	0
0	2	4	4	4	4	4	2	0
0	0	0	0	0	0	0	0	0

2. Sketch the histogram of the gradient directions $\varphi(x,y) = tan^{-1} \frac{S_y(x,y)}{S_x(x,y)}$. Be precise in labeling the height of each component of the histogram. **[5]**



Angle in degree

3. What would the Laplacian of this image look like based on the following approximation?

$\nabla^2 I(x,y) = I(x+1,y) + I(x-1,y) + I(x,y+1) + I(x,y-1) - 4I(x,y)$ **[5]**

0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	0	0
0	1	-2	-1	-1	-1	-2	1	0
0	1	-1	0	0	0	-1	1	0
0	1	-1	0	0	0	-1	1	0
0	1	-2	-1	-1	-1	-2	1	0
0	0	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0	0

b) A biomedical engineering student is assigned the job of inspecting a certain class of images generated by an electron microscope. In order to simplify the inspection task, the student decides to use digital image enhancement techniques and, to this end, examines a set of representative images and finds the following problems:

1. bright, isolated dots that are of no interest; **[2]**

Median Filter

2. lack of sharpness; **[2]**

Sharpening Filter

3. not enough contrast in some images; and **[2]**

Histogram Equalization

4. shifts in average intensity, when this value should be K to perform correctly certain intensity measurements. **[2]**

Get zero mean image then add shift K

The student wants to correct these problems and then display in white all intensities in a band between u_1 and u_2, while keeping normal tonality in the remaining intensities. **[2]**

Set intensity of pixels for range u_1 and u_2 to G_{max} (max Intensity)

Propose a sequence of processing steps that the student can follow to achieve the desired goals.

Question 5 [True/False Statements] (20 points)

State whether the following statements are true or false and correct the false ones:

1. In a grid cell model of image pixels, a pixel is a homogeneously shaded square cell. T **[2]**

2. The 2D DFT maps a scalar image into a weighted sum of complex exponentials on the unit circle in the complex plane. T **[2]**

3. Low frequencies represent homogeneous multiplicative contributions to the input image while high frequencies represent local continuities in the image. F **[2]**

additive, discontinuities

4. Directional patterns in an input image create value distributions in the DFT of the image in an orthogonal direction. T **[2]**

5. The 2D CIE Color Space represents the brightness and colors perceived by the average person. F **[2]**

just colors

6. A 2D Gauss filter can be decomposed into two subsequent 1D Gauss filters. T **[2]**

7. Computer screens have typically less available colors than color printers. F **[2]**
more

8. Illumination artifacts between subsequent or time-synchronized images violate the intensity constancy assumption. T **[2]**

9. For corner detection using the Hessian matrix, if the magnitude of both eigenvalues is large, then we are at a low-contrast region while two small eigenvalues identify a corner. F **[2]**
small , large

10. In the edge following step of the canny edge detector, the paths of pixel locations p with gray level values exceeding the higher threshold, i.e. $g(p) > T_{high}$, are traced, and pixels on such a path are marked as being edge pixels. F **[2]**

Lower threshold, T_{low}

مع خالص تمنياتنا لكم بالتوفيق و النجاح

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