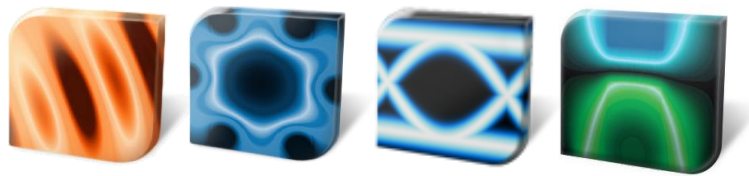


# Material Plugins

## Practical Implementation Demo

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DECEMBER, 2014



# Outline

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Plugin overview

Lorentz Example

- Numerical algorithms
- Practical Implementation on Windows

# Plugin review

The material properties are introduced via the polarization (or magnetization)

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}_{lumerical} + \vec{P}_{end-user}$$

optional

Typically

$$\vec{P}_{lumerical} = \epsilon_0 \chi(\omega) \vec{E}$$

$$\vec{D} = \epsilon(\omega)_{lumerical} \vec{E} + \vec{P}_{end-user}$$

This is very general since  $\vec{J} = -\frac{\partial \vec{P}}{\partial t}$

# Plugin review

The basic FDTD update involves solving E and H

$$\vec{E}(t) \rightarrow \vec{E}^{n\Delta t} \quad \vec{H}(t) \rightarrow \vec{H}^{(n+1/2)\Delta t}$$

$$\vec{E}^0 \longrightarrow \vec{H}^{1/2} \longrightarrow \vec{E}^1 \longrightarrow \vec{H}^{3/2} \longrightarrow \dots$$

# Plugin review

In the plugin we need a routine to solve

$$U^{n+1}E^{n+1} + \frac{P^{n+1}}{\epsilon_0} = V^{n+1}$$

for  $E^{n+1}$ , with inputs  $U^{n+1}$ ,  $V^{n+1}$ ,  $E^n$

This will add the desired polarization  $P$  onto any of Lumerical's existing material models

# Lorentz medium example

Create a Lorentz medium

$$\frac{\vec{P}_{SI}(\omega)}{\epsilon_0} = \vec{P}(\omega) = \frac{\Delta\epsilon\omega_0^2}{\omega_0^2 - 2i\delta\omega - \omega^2} \vec{E}(\omega)$$

Material has 3 parameters:

- $\Delta\epsilon$
- $\omega_0$
- $\delta$

# Lorentz medium example

Create a Lorentz medium

$$(\omega_0^2 - 2i\delta\omega - \omega^2)\vec{P}(\omega) = \Delta\epsilon\omega_0^2\vec{E}(\omega)$$

In the time domain, where  $P(t) = \text{FT}(P(\omega))$ , we have

$$(\omega_0^2\vec{P}(t) + 2\delta P'(t) + P''(t)) = \Delta\epsilon\omega_0^2\vec{E}(t)$$

# Lorentz medium example

We make  $\vec{P}(t)$  discrete in time so

$$\vec{P}(t) = \vec{P}^{n\Delta t} = \vec{P}^n$$

To calculate  $\vec{P}'(t)$  we use

$$\vec{P}'(t) = \vec{P}'^n = (\vec{P}^{n+1/2} - \vec{P}^{n-1/2})/\Delta t$$

$$\vec{P}^{n+1/2} = (\vec{P}^{n+1} + \vec{P}^n)/2$$

$$\vec{P}'(t) = \vec{P}'^n = (\vec{P}^{n+1} - \vec{P}^{n-1})/2\Delta t$$

Finally,  $\vec{P}''(t)$  is

$$\vec{P}''(t) = \vec{P}''^n = (\vec{P}^{n+1} - 2\vec{P}^n + \vec{P}^{n-1})/\Delta t^2$$



# Lorentz medium example

We substitute the discrete equations for  $E, P, P'$  and  $P''$  into

$$(\omega_0^2 \vec{P}(t) + 2\delta P'(t) + P''(t)) = \Delta \varepsilon \omega_0^2 \vec{E}(\omega)$$

To get

$$\omega_0^2 \vec{P}^n + \frac{\delta(\vec{P}^{n+1} - \vec{P}^{n-1})}{\Delta t} + \frac{\vec{P}^{n+1} - 2\vec{P}^n + \vec{P}^{n-1}}{\Delta t^2} = \Delta \varepsilon \omega_0^2 \vec{E}^n$$

We solve for  $P^{n+1}$  and find

$$\vec{P}^{n+1} = a_1 \vec{P}^n + a_2 \vec{P}^{n-1} + a_3 \vec{E}^n$$

$$a_1 = \frac{2 - \Delta t^2 \omega_0^2}{\delta \Delta t + 1}$$

$$a_2 = \frac{\delta \Delta t - 1}{\delta \Delta t + 1}$$

$$a_3 = \frac{\Delta \varepsilon \omega_0^2 \Delta t^2}{\delta \Delta t + 1}$$

# Lorentz medium example

We now can solve the equation

$$U^{n+1} E^{n+1} + \frac{P_{SI}^{n+1}}{\epsilon_0} = V^{n+1}$$

For  $E^{n+1}$  because we know  $P^{n+1}$  as a function of  $E^n$

# Lorentz medium example

Here are the update steps

1. Calculate  $a_1$ ,  $a_2$  and  $a_3$  during initialization
2. Calculate  $\vec{P}^{n+1}$  with

$$\vec{P}^{n+1} = a_1 \vec{P}^n + a_2 \vec{P}^{n-1} + a_3 \vec{E}^n$$

3. Return  $E^{n+1}$  with

$$E^{n+1} = \frac{V^{n+1} - P^{n+1}}{U^{n+1}}$$

# Demonstration of implementation

We need a material with 3 parameters

We need to store  $P^n$  and  $P^{n-1}$  (2 storage fields)

We need 3 constants ( $a_1$ ,  $a_2$ ,  $a_3$ )

We need to update all 3 axes ( $P_x$ ,  $P_y$  and  $P_z$ )

Demonstrate how to debug using storage fields

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