Material Plugins

Practical Implementation Demo

DECEMBER, 2014

Outline

Plugin overview

Lorentz Example

- Numerical algorithms
- Practical Implementation on Windows

Plugin review

The material properties are introduced via the polarization (or magnetization)

optional

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}_{lumerical} + \vec{P}_{end-user}$$

Typically

$$\vec{P}_{lumerical} = \varepsilon_0 \chi(\omega) \vec{E}$$

$$\vec{D} = \varepsilon(\omega)_{lumerical} \vec{E} + \vec{P}_{end-user}$$

This is very general since $\vec{J} = -\dfrac{\partial \vec{P}}{\partial t}$

Plugin review

The basic FDTD update involves solving E and H

$$\vec{E}(t) \rightarrow \vec{E}^{n\Delta t} \qquad \vec{H}(t) \rightarrow \vec{H}^{(n+\frac{1}{2})\Delta t}$$

$$\vec{E}^{0} \longrightarrow \vec{H}^{1/2} \longrightarrow \vec{E}^{1} \longrightarrow \vec{H}^{3/2} \longrightarrow \cdots$$

Plugin review

In the plugin we need a routine to solve

$$U^{n+1}E^{n+1} + \frac{P^{n+1}}{\varepsilon_0} = V^{n+1}$$

for E^{n+1}, with inputs U^{n+1}, V^{n+1}, E^n

This will add the desired polarization P onto any of Lumerical's existing material models

Lorentz medium example

Create a Lorentz medium

$$\frac{\vec{P}_{SI}(\omega)}{\varepsilon_0} = \vec{P}(\omega) = \frac{\Delta\varepsilon\omega_0^2}{\omega_0^2 - 2i\delta\omega - \omega^2}\vec{E}(\omega)$$

Material has 3 parameters:

- $\Delta\varepsilon$
- ω_0
- δ

Lorentz medium example

Create a Lorentz medium

$$(\omega_0^2 - 2i\delta\omega - \omega^2)\vec{P}(\omega) = \Delta\varepsilon\omega_0^2\vec{E}(\omega)$$

In the time domain, where P(t) = FT (P(ω)), we have

$$(\omega_0^2\vec{P}(t) + 2\delta P^{'}(t) + P^{''}(t)) = \Delta\varepsilon\omega_0^2\vec{E}(t)$$

Lorentz medium example

We make P(t) discrete in time so

$$\vec{P}(t) = \vec{P}^{n\Delta t} = \vec{P}^{n}$$

To calculate P'(t) we use

$$\vec{P}'(t) = \vec{P}'^{n} = (\vec{P}^{n+\frac{1}{2}} - \vec{P}^{n-\frac{1}{2}}) \big/ \Delta t$$

$$\vec{P}^{n+\frac{1}{2}} = (\vec{P}^{n+1} + \vec{P}^{n}) \big/ 2$$

$$\vec{P}'(t) = \vec{P}'^{n} = (\vec{P}^{n+1} - \vec{P}^{n-1}) \big/ 2\Delta t$$

Finally, P''(t) is

$$\vec{P}''(t) = \vec{P}''^{n} = (\vec{P}^{n+1} - 2\vec{P}^{n} + \vec{P}^{n-1}) \big/ \Delta t^{2}$$

Lorentz medium example

We substitute the discrete equations for E,P, P' and P'' into

$$(\omega_0^2 \vec{P}(t) + 2\delta P^{'}(t) + P^{''}(t)) = \Delta\varepsilon\omega_0^2 \vec{E}(\omega)$$

To get

$$\omega_0^2 \vec{P}^n + \frac{\delta(\vec{P}^{n+1} - \vec{P}^{n-1})}{\Delta t} + \frac{\vec{P}^{n+1} - 2\vec{P}^n + \vec{P}^{n-1}}{\Delta t^2} = \Delta\varepsilon\omega_0^2 \vec{E}^n$$

We solve for P^{n+1} and find

$$\vec{P}^{n+1} = a_1 \vec{P}^n + a_2 \vec{P}^{n-1} + a_3 \vec{E}^n$$

$$a_1 = \frac{2 - \Delta t^2 \omega_0^2}{\delta\Delta t + 1}$$

$$a_2 = \frac{\delta\Delta t - 1}{\delta\Delta t + 1}$$

$$a_3 = \frac{\Delta\varepsilon\omega_0^2 \Delta t^2}{\delta\Delta t + 1}$$

Lorentz medium example

We now can solve the equation

$$U^{n+1}E^{n+1} + \frac{P_{SI}^{n+1}}{\varepsilon_0} = V^{n+1}$$

For E^{n+1} because we know P^{n+1} as a function of E^n

Lorentz medium example

Here are the update steps

1. Calculate a_1, a_2 and a_3 during initialization
2. Calculate P^{n+1} with

$$\vec{P}^{n+1} = a_1 \vec{P}^{n} + a_2 \vec{P}^{n-1} + a_3 \vec{E}^{n}$$

3. Return E^{n+1} with

$$E^{n+1} = \frac{V^{n+1} - P^{n+1}}{U^{n+1}}$$

Demonstration of implementation

We need a material with 3 parameters

We need to store P^n and P^{n-1} (2 storage fields)

We need 3 constants (a_1, a_2, a_3)

We need to update all 3 axes (P_x, P_y and P_z)

Demonstrate how to debug using storage fields www.lumerical.com

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