

BACKGROUND: we propose the first secure federated χ^2 -test protocol FED- χ^2 . To minimize both the privacy leakage and the communication cost, we recast χ^2 -test to the second moment estimation problem and thus can take advantage of stable projection to encode the local information in a short vector. As such encodings can be aggregated with only summation, secure aggregation can be naturally applied to hide the individual updates. We formally prove the security guarantee of FED- χ^2 that the joint distribution is hidden in a subspace with exponential possible distributions. Our evaluation results show that FED- χ^2 achieves negligible accuracy drops with small client-side computation overhead. In several real-world case studies, the performance of FED- χ^2 is comparable to the centralized χ^2 -test.

BUILDING BLOCK: STABLE PROJECTION

```
1 Function ENCODE(P, u_i):
2   return P * u_i
3 Function GEOMETRICMEANESTIMATOR(e):
4   ▷ ℓ: Encoding size.
5    $\hat{d}_{(2),gm} \leftarrow \frac{\prod_{k=1}^{\ell} |e_k|^2 / \ell}{(\frac{2}{\pi} \Gamma(\frac{\ell}{2}) \Gamma(1 - \frac{\ell}{2}) \sin(\frac{\pi}{\ell}))^\ell}$ 
6   return  $\hat{d}_{(2),gm}$ 
7 Function DECODE(e):
8   return GEOMETRICMEANESTIMATOR(e)
```

METHODOLOGY

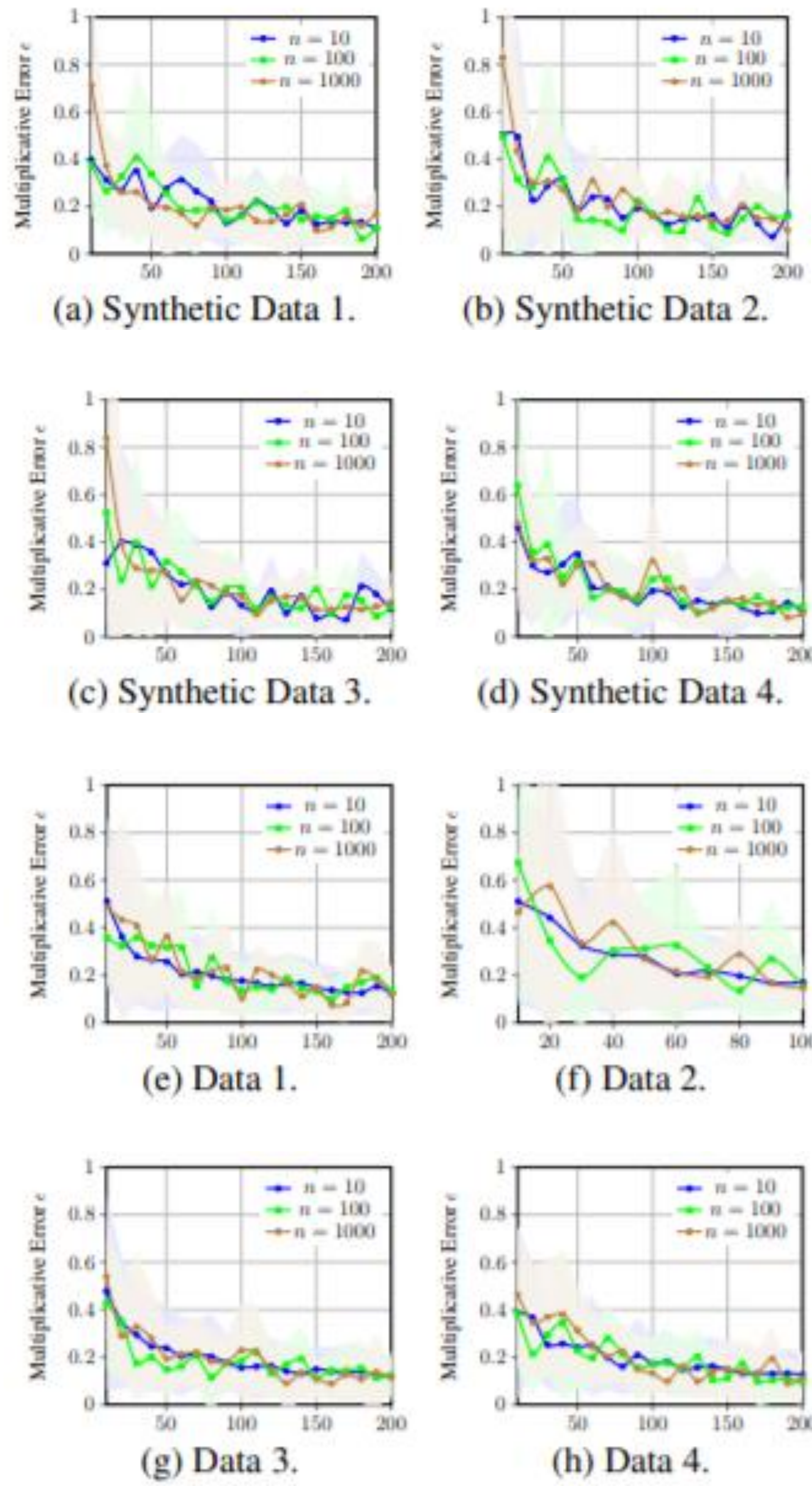
- STEP 1: Recast χ^2 -test as a second frequency moments estimation problem.
- STEP 2: Encode the second frequency moments information using stable projection.
- STEP 3: Use secure aggregation to sum the clients' updates.

we propose the first secure federated χ^2 -test protocol FED- χ^2 .

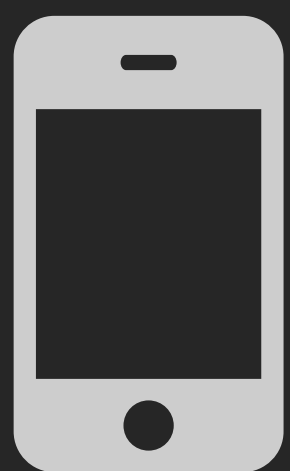
Algorithm 2: FED- χ^2 : secure federated χ^2 -test. SECUREAGG is a remote procedure that receives inputs from the clients and returns the summation to the server. INITSECUREAGG is the corresponding setup protocol deciding the communication graph and other hyper-parameters.

```
1 Round 1: Reveal the marginal distribution
2   INITSECUREAGG(n) // n is the client number.
3   for x ∈ [m_x] do v_x = SECUREAGG({v_x^{(i)}}_{i ∈ [n]})
4   for y ∈ [m_y] do v_y = SECUREAGG({v_y^{(i)}}_{i ∈ [n]})
5   Server
6   | Calculate v = ∑_x v_x and broadcast v, {v_x} and {v_y} to all the clients.
7 Round 2: Approximate the statistics
8   Server
9   | Sample the projection matrix P from Q_{2,0,1}^{ℓ × m}
10  | Broadcast the projection matrices to the clients
11  Client c_i, i ∈ [n]
12  | Calculate  $\bar{v}_{xy} = \frac{v_x v_y}{v}$ 
13  | Prepare u_i s.t.  $u_i[\mathbb{I}(x, y)] = \frac{v_{xy}^{(i)} - \bar{v}_{xy} / n}{\sqrt{\bar{v}_{xy}}}$ 
14  | Calculate e_i = ENCODE(P, u_i)
15  e = SECUREAGG({e_i}_{i ∈ [n]})
16  Server
17  |  $\hat{s}_{\chi^2} = \text{DECODE}(e)$ 
```

Experiment



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