Differentially Private
Fractional Frequency
Moments Estimation with
Polylogarithmic Space



BACKGROUND: In the present work, we make the first customized effort towards DP estimation of fractional frequency moments, i.e. $p \in (0, 1]$ with low space complexity. We show that a well-known streaming algorithm, namely Fp sketch [21], preserves differential privacy as is. With its small space complexity, Fp sketch elegantly solves the trilemma between efficiency, accuracy, and privacy.

Fp SKETCH REVIEW

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Algorithm 1: \mathbb{F}_p sketch.

Input :Data stream: \mathcal{S} = \{(k_1, v_1), \cdots, (k_n, v_n)\}

Construct:

Initialize \mathbf{a} = \{0\}^r, \mathbf{P} \sim \mathcal{D}_{p,1}^{r \times m};

Update:

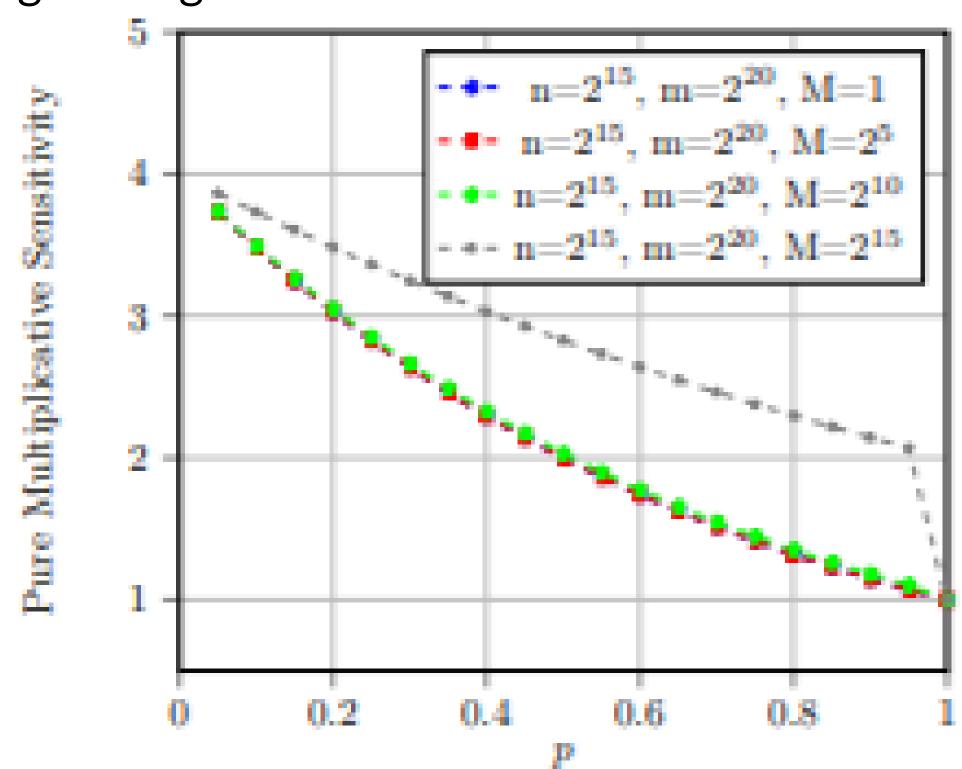
for i \in [n] do Let e_{k_i} be the one-hot encoder of k_i, \mathbf{a} = \mathbf{a} + \mathbf{P} \times v_i \mathbf{e}_{k_i};

Query:

return scale_estimator(\mathbf{a});
```

METHODOLOGY & RESULTS

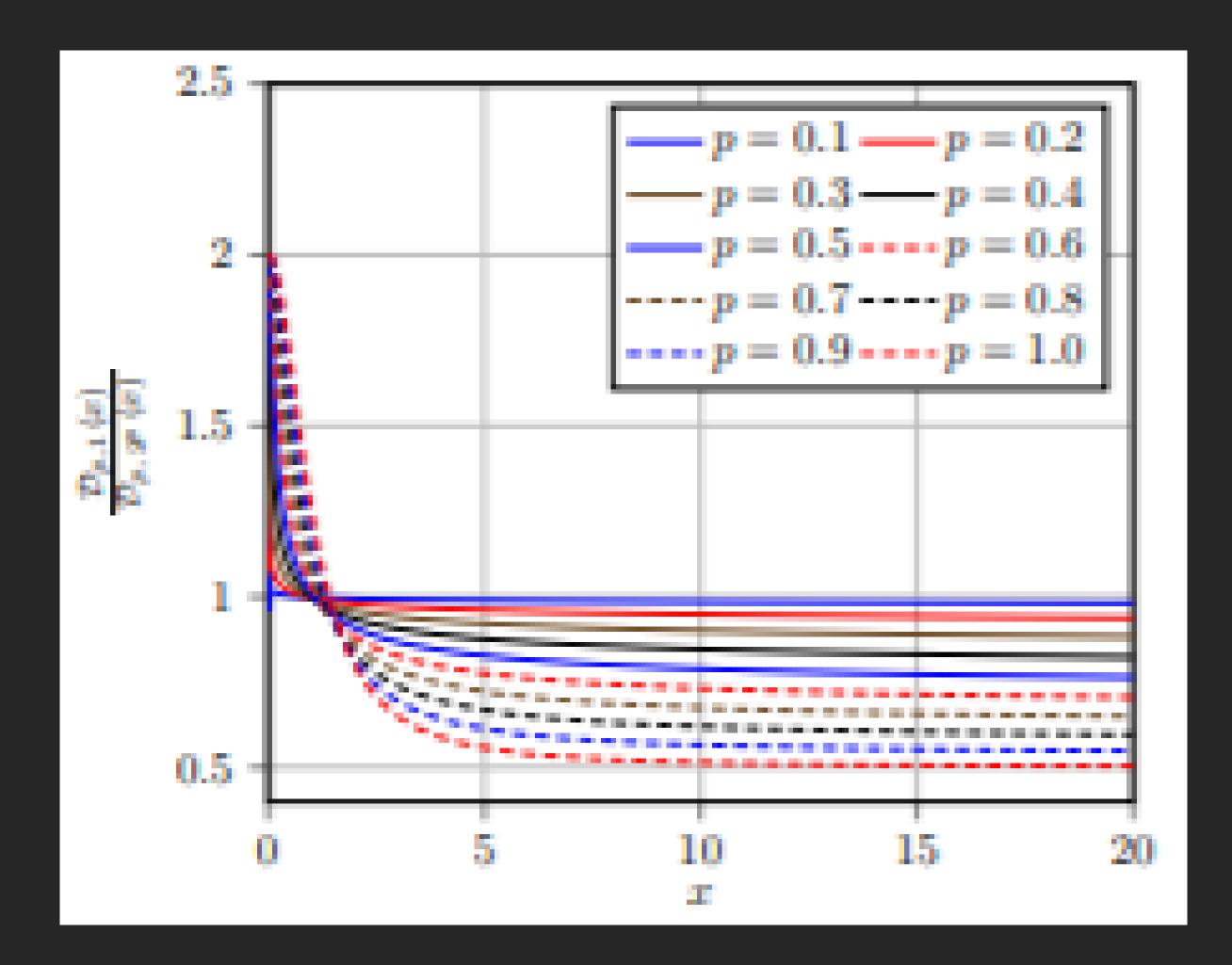
STEP 1: Bound the difference between scales of neighboring datasets



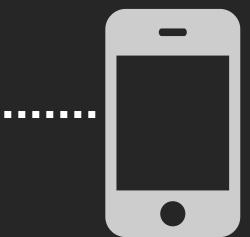
STEP 2: Bound the privacy loss

Theorem 3 (ϵ -DP for Algorithm 1). Let ρ_p represent the multiplicative sensitivity of the p-th frequency moments. When r=1 and $p \in (0,1]$, moments. When r=1 and $p \in (0,1]$, \mathbb{F}_p sketch is $\frac{1}{p} \ln \rho_p$ -differentially private.

We prove that **Fp sketch**, a streaming algorithm for **frequency moments** estimation, is differentially private when $p \in (0, 1]$.



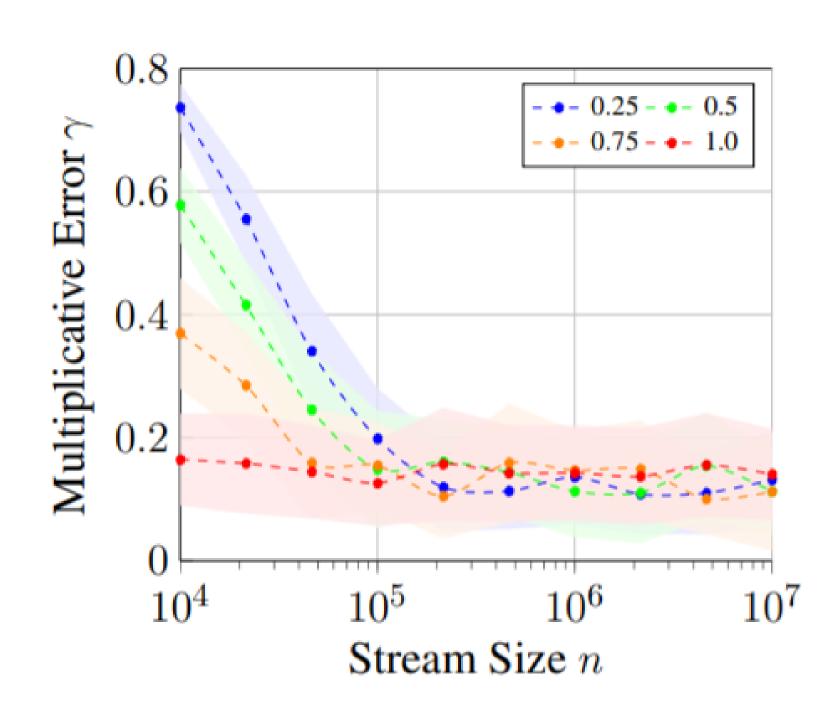




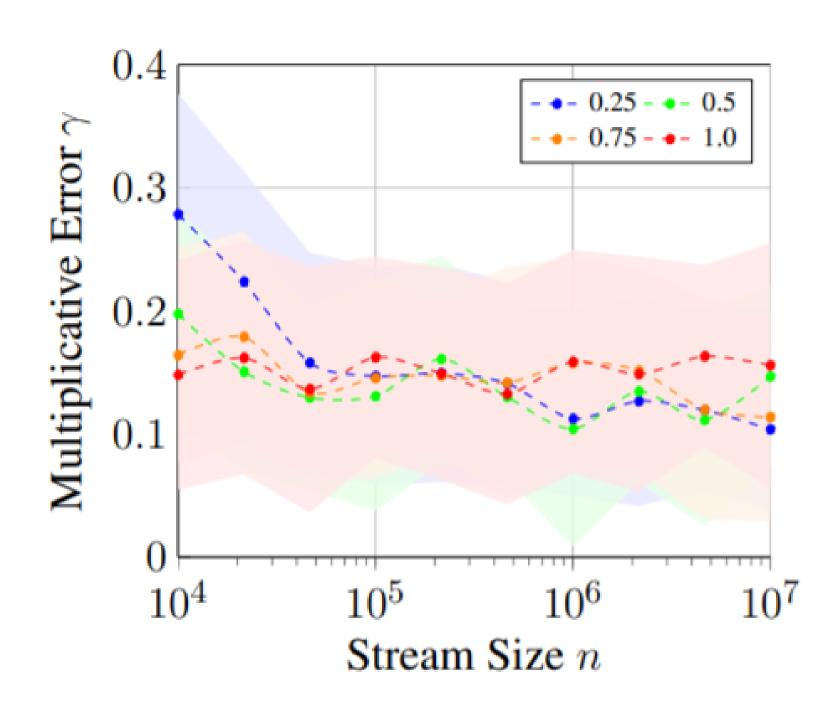
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SCAN ME

EXPERIMENT



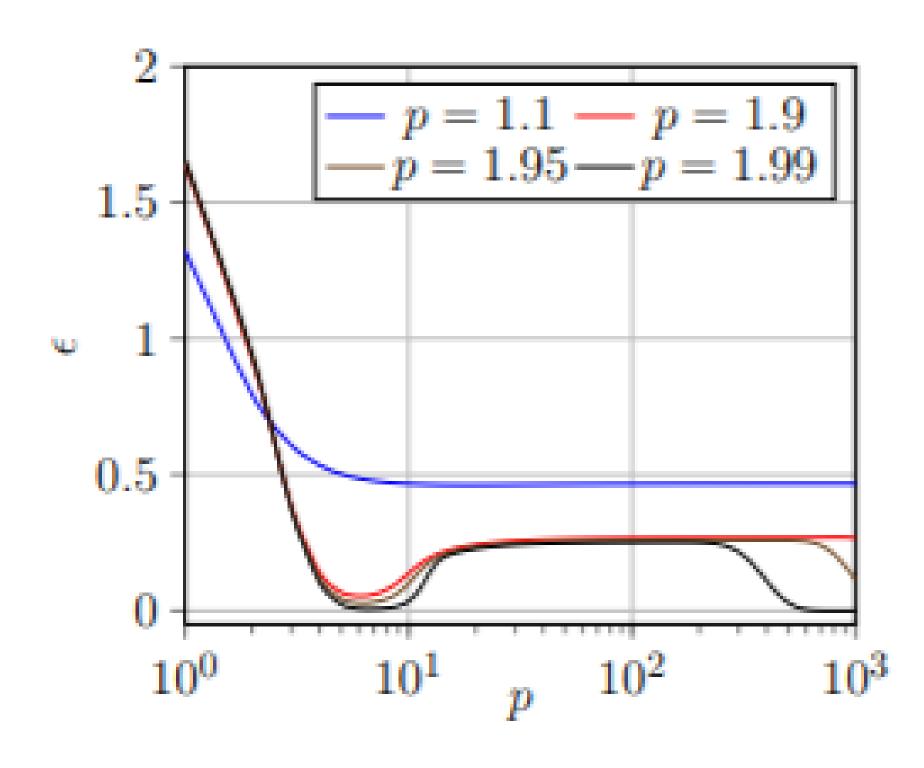
(a) Uniformly Distributed Stream.



(b) Binomially Distributed Stream.

OPEN PROBLEMS

The proof does not easily extend to $p \in (1, 2)$.



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