Equilibrium Learning in Auction Markets

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Abstract

My dissertation investigates the computation of Bayes-Nash equilibria in auctions via multiagent learning. A particular focus lies on the game-theoretic analysis of learned gradient dynamics in such markets. This requires overcoming several technical challenges like non-differentiable utility functions and infinite-dimensional strategy spaces. Positive results may open the door for wide-ranging applications in Market Design and the economic sciences.

Background and Research Questions

The field of Market Design in Economics concerns itself with designing market mechanisms that optimize certain desiderata, such as yielding efficient outcomes, or maximizing seller revenue. Auctions are mechanisms applied in markets with asymmetric information to determine an allocation of goods among participating buyers, and the corresponding prices. A key challenge is understanding the equilibria of such markets, which are modeled as Bayesian Games (BG) with continuous type and action spaces. Much theoretical progress has been made in recent decades, primarily on the mathematical analysis of equilibrium states of specific auction settings (Krishna 2009; Nobel Memorial Prize 2020). However, a general understanding of incentives in auctions remains elusive. Moreover, computing Nash equilibria is known to be PPAD-complete, even in *finite, complete-information* games, suggesting that it may be harder still in BGs with infinite-dimensional strategy spaces.

Despite these hardness results, my thesis project aims to do just that: Compute Bayesian Nash equilibria (BNE) in generic auctions numerically. My focus here lies on iterative *learning* of strategies via policy iteration methods by agents whose strategies are represented by neural networks. Unlike previous approaches, most notably Bosshard et al. (2020), a particular aim is to avoid any setting-specific optimizations but rather aim for a "plug-and-play" approach. I aim to answer some of the following research questions:

- Can equilibrium learning reliably find BNEs in auctions? Can we automate "manual" mathematical equilibrium analysis in auction theory?
- Can empirical results from equilibrium learning inform economic theory and our understanding of auctions?
- What learning rules are appropriate? Are there theoretical convergence guarantees? Is equilibrium computation in auctions "easier" than in general games?
- What constitutes the current limitations of equilibrium learning? How far, in terms of market size and complexity, can we push the envelope with current methods?

Timeline and Progress

First Year

After beginning my project in September 2018, I initially focused on single-item sealed-bid auctions, where the theory of equilibria and incentives is well understood. I quickly identified a major challenge to gradient-based learning in such games: Techniques based on backpropagation provably fail in auctions due to destructive discontinuities in individual utility functions that result from the discrete nature of allocations. With my advisor, Martin Bichler (M.B.), I developed a learning method for BGs, now called Neural Pseudogradient Ascent (NPGA), which instead relies on Evolutionary Strategies (ES) for gradient computation. In joint work with Paul Sutterer, we demonstrated convergence to BNEs in a range of auctions, and empirically validated NPGA in Normal-Form Games, where it behaves similarly to exact gradient dynamics (Heidekrüger, Sutterer, and Bichler 2019). However, naïve ES is computationally expensive, and introduces noise. To tackle larger settings, I thus began developing a software package, `bnelearn`, enabling highly performant learning simulations on the GPU.

Second Year

With my colleague Nils Kohring, I extended the `bnelearn` framework to learn equilibria in larger auctions with multiple homogenous ("multi-unit auctions") and heterogenous ("combinatorial auctions") goods, as well as correlated prior information between bidders. We presented our findings in several workshop papers. The latest of these (Heidekrüger et al. 2021b), also contains a first theoretical result: In *monotonic auction games*, NPGA provably converges to an ϵ-approximation of the unique BNE. This generalizes known results about gradient dynamics in complete-information games (Mertikopoulos and Zhou 2019) and differentiable

BGs (Ui 2016) to auctions. The proof, which I developed with Max Fichtl and M.B., relies on interpreting the ex-ante stage of the BG as an infinite-dimensional *complete-information* game, then reasoning over a finite-dimensional *proxy game* over NN parameters, and proving approximation bounds between these three. However, monotonicity is a strong assumption that does not hold for all auctions, and, even when it does, is difficult to verify.

Third Year and Current Status

In late 2020, we observed that the ex-ante views of any *symmetric* auction, which are prevalent in the literature, constitute *potential games*, where no-regret dynamics are known to converge to local equilibria. Though obvious in hindsight, this had not been described previously. I adapted our previous proof to show that, in symmetric auctions, NPGA converges to local ϵ-BNEs. These are weaker than global equilibria, but the much weaker assumptions guarantee that this result holds for many auctions of interest to economists. In August 2021, the first Journal article relating to my thesis was published in *Nature Machine Intelligence* (Bichler et al. 2021), containing this convergence result and an exhaustive empirical analysis of NPGA, including equilibrium computation in the largest auction (6 bidders, 8 items) where this has been achieved. For further scaling, many challenges remain, e.g. regarding sample efficiency and evaluation. Several of these have corresponding problems studied in Multiagent Reinforcement Learning (MARL) that have seen recent progress. I believe auction theory as a field can benefit from a deeper exchange with the MARL community and presented a position paper to this end at the AAAI-COMARL Spring Symposium (Heidekrüger et al. 2021a).

Concluding Year 3, I am currently working on two projects: Markus Ewert and I are applying equilibrium learning to a problem in Behavioral Economics, the *Overbidding Puzzle* in *all-pay auctions*. Proposed psychological explanations of this puzzle include parametrized psychological effects in players' utility functions, such as experiencing psychological regret and/or risk aversion. A direct comparison of these explanation attempts hasn't been possible thus far. In our working paper, we develop an inference scheme based on Bayesian Optimization and equilibrium computation via NPGA, that for the first time allows a quantitative goodness-of-fit comparison of proposed theories on (existing) datasets of human bidders in lab experiments.

I'm also working with Nils Kohring to improve and extend our existing simulation framework `bnelearn` into a fully-featured open-source software package for equilibrium learning in sealed-bid auctions. At an invited presentation about NPGA given by M.B. at the 2020 NBER Market Design meeting, we learned that there exists ample demand for such a tool from economics researchers who desire to understand equilibria in specific markets. On the other hand, through our research we have now accumulated the widest existing suite of performantly implemented auction mechanisms and we therefore believe that our package can also serve as an excellent benchmark suite for further research in multi-agent learning in Bayesian games. We aim to publish an expository paper about the software in a relevant journal.

Outlook

I plan to continue my work in sealed-bid auctions, tackling even larger games for which equilibria have not yet been computed. I also want to explore an idea for an alternative Actor-Critic approach, where I hope that learning a differentiable *model* of the ex-ante environment could recover our ability to use backprop, which is faster than ES. I will also continue investigating the (ex-ante) gradient dynamics in sealed-bid auctions. I conjecture that many auction games are indeed *monotonic* on a relevant subset of the strategy space. A proof (or refinement) for as wide a class of auctions as possible would constitute a strong result about the feasibility of learning *global* equilibria. On the other hand, I have recently begun to extend my scope beyond sealed-bid auctions to approach sequential *clock auction formats*, again beginning with well-understood single-item settings. Positive results could be a steppingstone to approach equilibrium learning more complex *combinatorial clock auctions* that are highly relevant in real-world applications like spectrum sales and procurement.

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