

Simplex

①

1.73.

$$\max z = x_1 - 3x_2 + 2x_3$$

st

$$3x_1 - x_2 + 2x_3 \leq 9$$

$$-2x_1 + 4x_2 + x_3 \leq 14$$

$$-4x_1 + 4x_2 + 8x_3 \leq 10$$

$$x_1, x_2, x_3 \geq 0$$

Standardize

=>

$$\max z = x_1 - 3x_2 + 2x_3 + 0x_4 + 0x_5 + 0x_6$$

st

$$3x_1 - x_2 + 2x_3 + 4x_4 = 9$$

$$-2x_1 + 4x_2 + x_3 + 6x_5 = 14$$

$$-4x_1 + 4x_2 + 8x_3 + 6x_6 = 10$$

$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$

	-1	3	-2	0	0	0	0
w_4	3	-1	2	1	0	0	9
w_5	-2	4	1	0	1	0	14
w_6	-4	4	8	0	0	1	10

	-1	3	-2	0	0	0	0	2
w_4	3	-1	2	1	0	0	9	-2
w_5	-2	4	1	0	1	0	14	-
x_3	$-\frac{1}{2}$	$\frac{1}{2}$	1	0	0	$\frac{1}{8}$	$\frac{5}{4}$	

	-2	4	0	0	0	$\frac{1}{4}$	$\frac{5}{2}$
w_4	4	-2	0	1	0	$-\frac{1}{4}$	$\frac{23}{2}$
w_5	$-\frac{3}{2}$	$\frac{7}{2}$	0	0	1	$-\frac{1}{8}$	$\frac{5}{4}$
x_3	$-\frac{1}{2}$	$\frac{1}{2}$	1	0	0	$\frac{1}{8}$	$\frac{5}{4}$

	-2	4	0	0	0	$\frac{1}{4}$	$\frac{5}{2}$	2
x_1	1	$-\frac{1}{2}$	0	$\frac{1}{4}$	0	$-\frac{1}{16}$	$\frac{43}{8}$	
w_5	$-\frac{3}{2}$	$\frac{7}{2}$	0	0	1	$-\frac{1}{8}$	$\frac{213}{16}$	$\frac{3}{2}$
x_3	$-\frac{1}{2}$	$\frac{1}{2}$	1	0	0	$\frac{1}{8}$	$\frac{5}{4}$	$\frac{1}{2}$

	0	3	0	$\frac{1}{2}$	0	$\frac{1}{8}$	$\frac{23}{4}$
x_1	1	$-\frac{1}{2}$	0	$\frac{1}{4}$	0	$-\frac{1}{16}$	$\frac{43}{8}$
w_5	0	$\frac{1}{4}$	0	$\frac{3}{8}$	1	$-\frac{7}{32}$	$\frac{293}{16}$
x_3	0	$\frac{1}{4}$	1	$\frac{1}{8}$	0	$\frac{3}{32}$	$\frac{33}{16}$

1.2.4

②

$$\min z = 10x_1 + 8x_2 + 6x_3 + 4x_4$$

St

$$2x_1 + 4x_2 + 2x_3 + x_4 \geq 10$$

$$-4x_1 + 4x_2 - x_3 + 2x_4 \geq 12$$

$$x_1, x_2, x_3, x_4 \geq 0$$

Standardize

 \Rightarrow

$$\max z = -10x_1 + 8x_2 - 6x_3 - 4x_4$$

$$s.t. \quad x_3 + x_6 - Mx_2 - Mx_4$$

$$2x_1 + 4x_2 + 2x_3 + x_4 - x_5 + Mx_1 = 10$$

$$-4x_1 + 4x_2 - x_3 + 2x_4 - x_6 + Mx_2 = 12$$

$$\max z = 0x_1 + 0x_2 + 0x_3 + 0x_4 + 0x_5 + 0x_6 - w_1 - w_2$$

	2	-8	-1	-3	1	1				
w_1	2	(4)	2	1	-1	0	0	0	-22	
w_2	-4	4	-1	2	0	-1	0	1	12	

	2	-8	-1	-3	1	1				
x_2	$\frac{1}{2}$	(1)	$\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{4}$	0	0	0	-22	8
w_2	-4	-4	-1	2	0	-1	$\frac{1}{4}$	0	$\frac{5}{2}$	

	6	0	3	-1	-1	1	2	0	-2	+
x_2	$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{4}$	0	$\frac{1}{4}$	0	$\frac{5}{2}$	$\frac{1}{4}$
w_2	-6	0	-3	1	(1)	-1	-1	1	2	

	0	0	0	0	0	0	1	1	0	
x_2	-1	1	$-\frac{1}{4}$	$\frac{1}{2}$	0	$-\frac{1}{4}$	0	$\frac{1}{4}$	3	
x_5	-6	0	-3	1	1	-1	-1	1	2	

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$$\min z = 3x_1 + x_2 - 2x_3 - 2x_4 + x_5$$

st

$$x_1 + 2x_2 + 2x_3 + x_4 + x_5 \leq 2 \quad \text{Standardize} \Rightarrow$$

$$2x_1 + x_2 + 3x_3 + 2x_4 + 2x_5 \geq 12$$

$$\max z = -3x_1 - x_2 + 2x_3 + 2x_4 - x_5 + 0x_6 + 0x_7 - M w_1$$

$$x_1 + 2x_2 + 2x_3 + x_4 + x_5 + x_6 = 2$$

$$2x_1 + x_2 + 3x_3 + 2x_4 + 2x_5 - x_7 + M w_1 = 12$$

	-2M	-M	-3M	-2M	-2M	0	M	-M	
x_6	1	2	(2)	1	1	0	1	-1	-12M
w_1	2	1	3	2	2	1	0	0	2
						0	-1	1	12

(0, -M)

	-2M	-M	-3M	-2M	-2M	0	M	-M	
x_3	1/2	1	(1)	1/2	1/2	0	1	-1	-12M
w_1	2	1	3	2	2	1/2	0	0	1
						0	-1	1	12

	-11/2	2M	0	-4/2	-4/2	3/2	4M	2M	
x_4	1/2	1	1	(1/2)	1/2	1/2	0	0	-9M
w_1	1/2	-2	0	1/2	1/2	1/2	0	0	1
						-3/2	-1	-1	9

	0	3M	M	0	0	2M	4M	2M	
x_4	1	2	2	1	1	1	0	0	-8M
w_1	0	-3	-1	0	0	-2	-1	-1	8

optimal solution
but infeasible
since artificial var
 w_1 is in basis
with positive value

2.4

$$\max z = 10x_1 + 6x_2$$

st

$$x_1 + 2x_2 \leq 2$$

$$2x_1 + x_2 \leq 3$$

$$2x_1 + 2x_2 \leq 3$$

$$4x_1 + x_2 \leq 2$$

$$x_1, x_2 \geq 0$$

Dual
 $\leftarrow \rightarrow$

Standardize

$$\min z = 2y_1 + 3y_2 + 3y_3 + 2y_4$$

$$y_1 + 2y_2 + 2y_3 + 4y_4 \geq 10$$

$$2y_1 + y_2 + 2y_3 + y_4 \geq 6$$

$$y_1, y_2 \geq 0$$

$$\max z = -2y_1 - 3y_2 - 3y_3 - 2y_4$$

$$-y_1 - 2y_2 - 2y_3 - 4y_4 + s_1 = -10$$

$$-2y_1 + y_2 - 2y_3 - y_4 + s_2 = -6$$

	2	3	3	2			
s_1	-1	-2	-2	-4	0	0	0
s_2	-2	-1	-2	-1	1	0	-10
					0	1	-6

	2	3	3	2			
y_4	1/4	1/2	1/2	1	-1/4	0	-2
s_2	-2	-1	-2	-1	0	1	5/2
	-9/5	-4	-1/3	-2	0	1	-6

	3/2	2	2	0			
y_4	1/4	1/2	1/2	1	-1/4	0	-5
s_2	-7/4	-1/2	-3/2	0	-1/4	1	5/2

⑤

	$\frac{3}{2}$	2	2	0	$\frac{1}{2}$	0	-5	$-\frac{3}{2}$
y_4	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{2}$	1	$-\frac{1}{4}$	0	$\frac{5}{2}$	$-\frac{1}{4}$
y_2	1	$\frac{2}{7}$	$\frac{6}{7}$	0	$\frac{1}{7}$	$-\frac{4}{7}$	2	

	0	$\frac{11}{2}$	$\frac{11}{2}$	0	$\frac{2}{7}$	$\frac{6}{7}$	-8	
y_4	0	$\frac{3}{7}$	$\frac{2}{7}$	1	$-\frac{2}{7}$	$\frac{1}{7}$	2	
y_2	1	$\frac{2}{7}$	$\frac{6}{7}$	0	$\frac{1}{7}$	$-\frac{4}{7}$	2	

Final solution