0-1 Linear programming

2.1.1

3.1 $\max z = 8x_1 + 2x_2 + 3x_3 + x_4 + 6x_5$

s.t

$2x_1 + x_2 + 4x_3 + x_4 + 4x_5 \leq 10$

$3x_1 + 2x_2 + 2x_3 + x_4 + 5x_5 \leq 11$

$x_1, x_2, x_3, x_4, x_5 = 0$ or 1.

$\max z = y_1 + 2y_2 + 3y_3 + 6y_4 + 8y_5$

$x_1 = y_5$ $x_5 = y_4$

$x_2 = y_2$

$x_3 = y_3$

$x_4 = y_1$

$2y_5 + y_2 + y_3 + y_1 + 4y_4 \leq 10$

$3y_5 + 2y_2 + 2y_3 + y_1 + 5y_4 \leq 11$

$y_1, y_2, y_3, y_4, y_5 = 0$ or 1

$y_1 + 2y_2 + 2y_3 + 5y_4 + 3y_5 \leq 11$

$y_1 + y_2 + y_3 + 4y_4 + 2y_5 \leq 10$

Relax problem

$\max z = y_1 + 2y_2 + 3y_3 + 6y_4 + 8y_5$

$y_1, y_2, y_3, y_4, y_5 = 1$ or 0

$z(1,1,1,1,1) = 20$

Does not satisfy constrain

$z(0,1,1,1,1) = 19$

Does not satisfy Constrain

$z(0,0,1,1,1) = 17$

satisfies constrain,

$z = (1,0,1,1,1) = 18$ pruned

satisfies constrain

pruned

$y_{UB} = (0, 1, 1, 1, 1)$

$Z_{UB} = 19$

$y_1 = 0$ ← $y_1 = 1$ →

$y = (0, -, -, -, -)$

$y_{UB} = (0, 0, 1, 1, 1)$

$Z_{UB} = 17$

$y = (1, -, -, -, -)$

$y_{UB} = (1, 0, 1, 1, 1)$
$y_1\ y_2\ y_3\ y_4\ y_5$

$Z_{UB} = 18$

$(x_4, x_2, x_5, x_3, x_1)$
$1\ \ 0\ \ \Phi\ \ 1\ \ 1$

$(1, 0, 1, 1, 1)$

32- $\max z = 9x_1 + 8x_2 + 5x_3 + 2x_4 + 4x_5$

s.t

$2x_1 + 8x_2 + x_3 + x_4 + 8x_5 \leq 7$

$3x_1 + 5x_2 + 3x_3 + 2x_4 + x_5 \leq 6$

$x_1, x_2, x_3, x_4, x_5 = 1 \text{ ou } 0$

$y_1 = x_2$

$y_2 = x_4$

$y_3 = x_5$

$y_4 = x_3$

$y_5 = x_1$

$\max z = y_1 + 2y_2 + 4y_3 + 5y_4 + 9y_5$

$8y_1 + y_2 + 2y_3 + y_4 + 2y_5 \leq 7$

$5y_1 + 2y_2 + y_3 + 3y_4 + 3y_5 \leq 6$

$y_i \in (1 \ldots 5) = 100$

$z(1\ 1\ 1\ 1\ 1) = 21$

NO C

$(0\ 1\ 1\ 1\ 1) = 20$

NO C

$y_{UB} = (0, 1, 1, 1, 1)$

$z = 20$

$y_1 = 0$ | $y_1 = 1$

$y = (0, -, -, -, -)$

$y_{UB} = (0, 0, 1, 1, 1)$

$z = 18$

$y = (1, -, -, -, -)$

$y_{UB} = (1, 0, 1, 1, 1)$

$z = 19$

pruned by constraint 1 is never satisfied

$y_2 = 0$ | $y_2 = 1$

$y = (0, 0, -, -, -)$

$y_{UB} = (0, 0, 0, 1, 1)$

$z = 14$

pruned Candidate

$y = (0, 1, -, -, -)$

$y_{UB} = (0, 1, 0, 1, 1)$

$z = 16$

$y_3 = 0$ | $y_3 = 1$

$y = (0, 1, 0, -, -)$

$y_{UB} = (0, 1, 0, 0, 1)$

$z = 11$

pruned, less then lower bound

$y = (0, 1, 1, -, -)$

$y = (0, 2, 1, 0, 1)$

$x_2\ x_4\ x_5\ x_3\ x_1$

$y_1\ y_2\ y_3\ y_4\ y_5$

$z = 15$

pruned Candidate Sol

$(1, 0, 0, 1, 1)$

33- $\max z = 10x_1 + 2x_2 + 7x_3 + 6x_4 + 3x_5$

S.t

$8x_1 + x_2 + 5x_3 + 4x_4 + 2x_5 \leq 14$

$6x_1 + x_2 + 3x_3 + 6x_4 + x_5 \leq 11$

$y_1 = 2x_2$
$y_2 = 3x_5$
$y_3 = 6x_4$
$y_4 = 7x_3$
$y_5 = 10x_1$

$\max z = 2y_1 + 3y_2 + 6y_3 + 7y_4 + 10y_5$

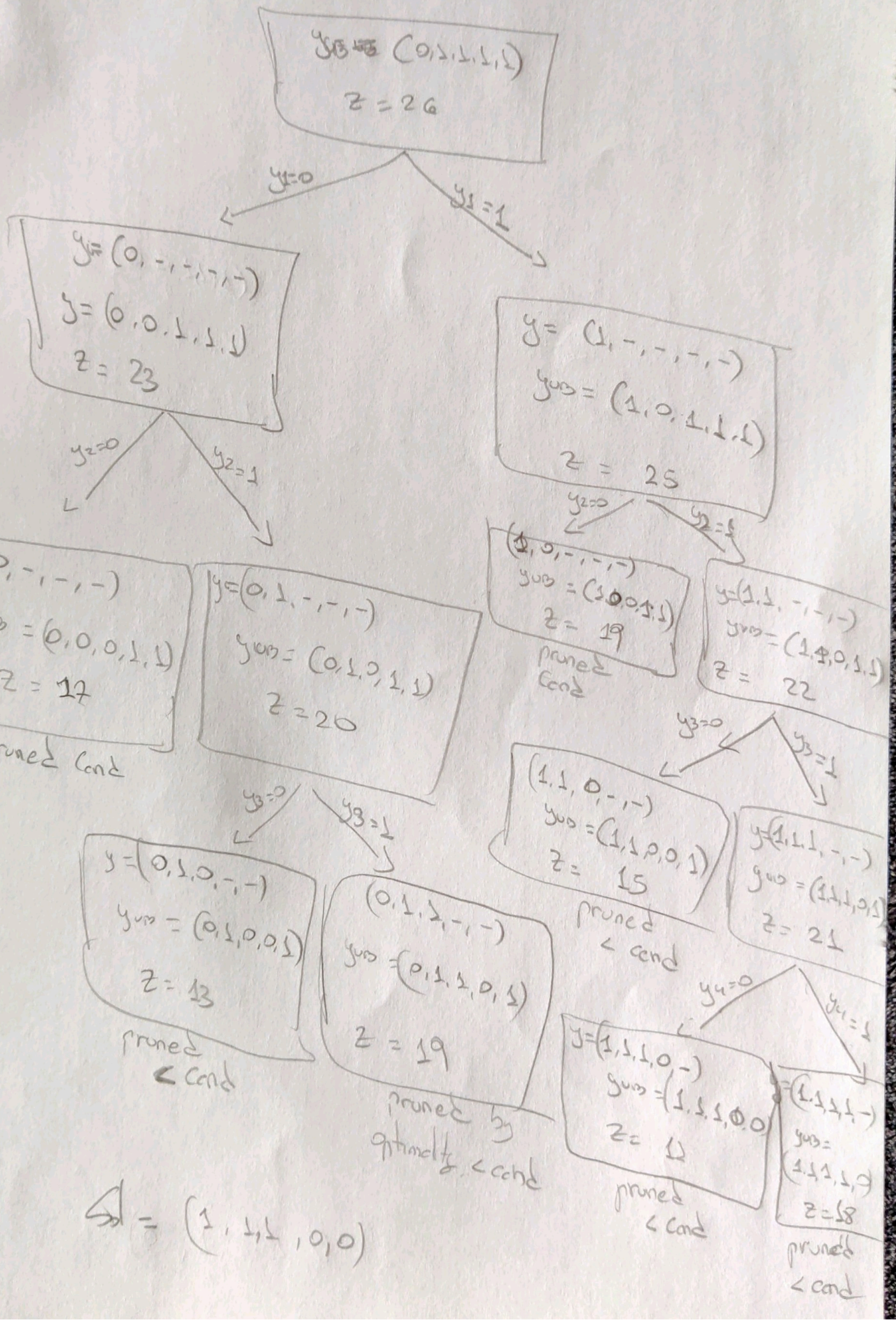
S.t

$y_1 + 2y_2 + 4y_3 + 5y_4 + 8y_5 \leq 14$

$y_1 + y_2 + 6y_3 + 3y_4 + 6y_5 \leq 11$

$z(1,1,1,1,1) = 28$
No C

$z(0,1,1,1,1) = 26$
No C


y_{UB} = (0,1,1,1,1)
z = 26
y_1=0
y_1=1
y = (0,-,-,-,-)
y_{UB} = (0,0,1,1,1)
z = 23
y = (1,-,-,-,-)
y_{UB} = (1,0,1,1,1)
z = 25
y_2=0
y_2=1
(0,0,-,-,-)
y_{UB} = (0,0,0,1,1)
z = 17
Pruned Cond
y=(0,1,-,-,-)
y_{UB} = (0,1,0,1,1)
z = 20
(1,0,-,-,-)
y_{UB} = (1,0,0,1,1)
z = 19
Pruned Cond
y=(1,1,-,-,-)
y_{UB} = (1,1,0,1,1)
z = 22
y_3=0
y_3=1
(1,1,0,-,-)
y_{UB} = (1,1,0,0,1)
z = 15
Pruned < cond
y=(1,1,1,-,-)
y_{UB} = (1,1,1,0,1)
z = 21
y_3=0
y_3=1
y=(0,1,0,-,-)
y_{UB} = (0,1,0,0,1)
z = 13
Pruned < Cond
(0,1,1,-,-)
y_{UB} = (0,1,1,0,1)
z = 19
Pruned by optimality < cond
y_4=0
y_4=1
y=(1,1,1,0,-)
y_{UB} = (1,1,1,0,0)
z = 17
Pruned < Cond
(1,1,1,1,-)
y_{UB} = (1,1,1,1,0)
z = 18
Pruned < cond
y^* = (1, 1, 1, 0, 0)

34- $\max z = -x_1 + 4x_2 - 2x_3 + 3x_4 + 7x_5 + 6x_6$

s.t

$x_1 + 4x_2 + 3x_3 + 2x_4 + 2x_5 + 4x_6 \leq 11$

$2x_1 + 5x_2 + 2x_5 + 6x_4 + 8x_3 + 4x_6 \leq 19$

$x_1 = 1 - y_1$

$x_3 = 3 - (1 - y_2)$

$x_4 = y_3$

$x_2 = y_4$

$x_6 = y_5$

$x_5 = y_6$

$y_1 \quad y_2 \quad y_3 \quad y_4 \quad y_5 \quad y_6$

$\max z = -x_1 - 2x_3 + 3x_4 + 4x_2 + 6x_6 + 7x_5$

$\max z = -1 + y_1 - 2 + 2y_2 + 3y_3 + 4y_4 + 6y_5 + 7y_6$

$1 - y_1 + 4y_4 + 3 - 3y_2 + 2y_3 + 2y_6 + 4y_5 \leq 11$

$2 - 2y_1 + 5y_4 + 2 - 2y_2 + 6y_3 + 8y_6 + 4y_5 \leq 19$

Reordenemos...

$\max z = y_1 + 2y_2 + 3y_3 + 4y_4 + 6y_5 + 7y_6 - 3$

$-y_1 - 3y_2 + 2y_3 + 4y_4 + 4y_5 + 2y_6 + 4 \leq 11$

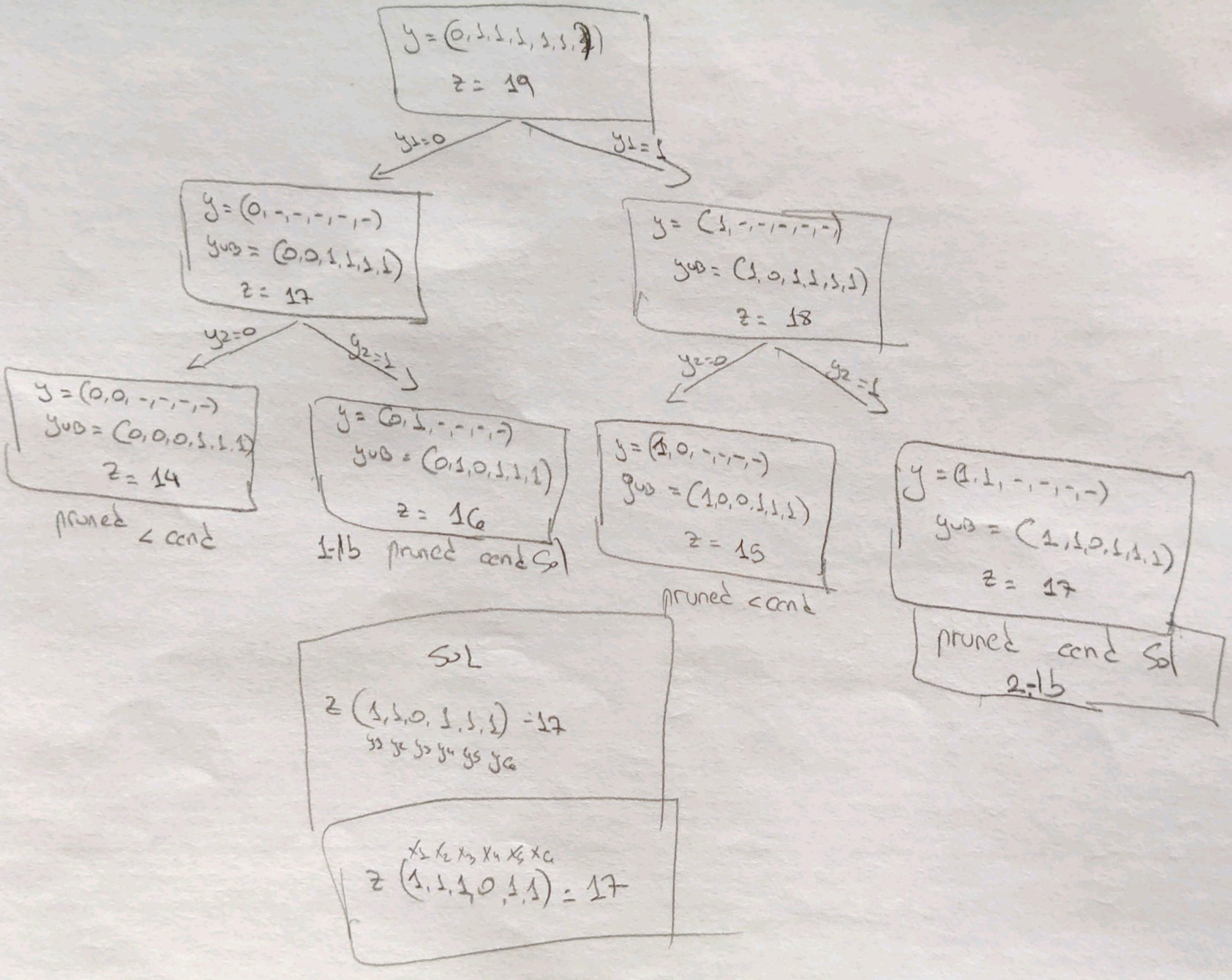
$-2y_1 - 2y_2 + 6y_3 + 5y_4 + 4y_5 + 8y_6 \leq 19$

$z(1,1,1,1,1,1) = 20$

No $\in$

$z(0,1,1,1,1,1) = 19$

No $\in$


y = (0,1,1,1,1,1,1)
z = 19
y1=0
y1=1
y = (0,-,-,-,-,-,-)
yUB = (0,0,1,1,1,1)
z = 17
y = (1,-,-,-,-,-,-)
yUB = (1,0,1,1,1,1)
z = 18
y2=0
y2=1
y2=0
y2=1
y = (0,0,-,-,-,-)
yUB = (0,0,0,1,1,1)
z = 14
pruned < cond
y = (0,1,-,-,-,-)
yUB = (0,1,0,1,1,1)
z = 16
1-1b pruned cond Sol
y = (1,0,-,-,-,-)
yUB = (1,0,0,1,1,1)
z = 15
pruned < cond
y = (1,1,-,-,-,-)
yUB = (1,1,0,1,1,1)
z = 17
pruned cond Sol
2-1b
Sol
z (1,1,0,1,1,1) = 17
y1 y2 y3 y4 y5 y6
x1 x2 x3 x4 x5 x6
z (1,1,1,0,1,1) = 17

2.2 -

$$\max z = 4x_1 + 2x_2 + x_3 + 7x_4 + 3x_5 + 6x_6$$

$$5x_1 + 8x_2 + 8x_3 + 6x_4 + x_5 + 5x_6 \leq 15$$

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \geq 3$$

$$x_i \begin{cases} 1 & \text{is in the box} \\ 0 & \text{otherwise} \end{cases}$$