自己的笔记，浅总结，知识点少，大概 5 个推导，误人子弟，字迹差，仅供欣赏（有参么不是参考呢？因为连参考的资格都没有）。

程升彦

特别感谢：马赛尔
我的惠普打印机

感谢我的3个室友，拖延了两天的进度（没有贬的意思，都是褒）

1. 基础概念

流体性质：流动性、粘性 & 压缩性.

流体层之间这种单位面积的作用力，称为粘性应力.

粘性应力跟流体层的相对速度的或切形变率呈正比，比例系数称粘性系数

不计粘性的流体，称理想流体

*把离散分子构成的实际流体，看作是由无数流体质点没有空隙连续分布而构成，这就是连续介质假设.

流体质点：大量流体分子的集合，又称流体微团(元)，微观上够大，宏观足够小

两种描述流动的方法：L. E. 气象上通常采用欧拉方法.

E：当 $\vec{v}$ 与 (x,y,z) 无关，空间均匀流场. 与 t 无关，定常流场.

L：~~L. E~~

$$\vec{r}=\vec{r}(x_0,y_0,z_0,t).\quad \begin{cases}x=x(x_0,y_0,z_0,t)\\ y=y(x_0,y_0,z_0,t)\\ z=z(x_0,y_0,z_0,t)\end{cases}\qquad \vec{a}=\frac{d^2\vec{r}(x_0,y_0,z_0,t)}{dt^2}\rightarrow a_x=\frac{d^2x(x_0,y_0,z_0,t)}{dt^2}\ \cdots$$

E：$\vec{v}=\vec{v}(x,y,z,t)\rightarrow\begin{cases}u=u(x,y,z,t)\\ v=v(x,y,z,t)\\ w=w(x,y,z,t)\end{cases}$ E-L 可互相转换.

$E=\frac{dL}{dt}$ 后消 x_0,y_0,z_0.

$$\begin{cases}u(x_0,y_0,z_0,t)=dx(x_0,y_0,z_0,t)/dt\\ v(x_0,y_0,z_0,t)=dy(x_0,y_0,z_0,t)/dt\\ w(x_0,y_0,z_0,t)=dz(x_0,y_0,z_0,t)/dt\end{cases}$$

联立. 消 $(x_0,y_0,z_0,\cancel{t})$.

$u=u(x,y,z,t)$
$v=v(x,y,z,t)$
$w=w(x,y,z,t)$

E→L：

可通过练习PPT上的习道例题掌握

$$dx/dt=u\qquad dy/dt=v\qquad dz/dt=w$$

积分. $\begin{cases}x=x(C_1,C_2,C_3,t)\\ y=y(C_1,C_2,C_3,t)\\ z=z(C_1,C_2,C_3,t)\end{cases}$ $t=t_0,(x,y,z)=(x_0,y_0,z_0)$ 消 C_1,C_2,C_3.

$x=x(x_0,y_0,z_0,t)$
$y=y(x_0,y_0,z_0,t)$
$z=z(x_0,y_0,z_0,t)$

流体加速度(也能互转).

La：影子流点的加速度：$\vec{a}=\frac{d\vec{v}(x,y,z,t)}{dt}$

Ea：影子空间点的加速度：$\vec{a}=\frac{\partial\vec{v}(x,y,z,t)}{\partial t}$

$$\frac{d\vec{v}}{dt}=\frac{\partial\vec{v}}{\partial t}+(\vec{v}\cdot\nabla)\vec{v}$$

微商算符：$\underset{\text{个别变化}}{\frac{d}{dt}(\ )}=\underset{\text{局地变化}}{\frac{\partial}{\partial t}(\ )}+\underset{\text{平流变化}}{\vec{v}\cdot\nabla(\ )}$. 任意物理量.

①局地变化完全由平流变化引起：$\frac{d}{dt}(\ )=0\longrightarrow\frac{\partial}{\partial t}(\ )=-\vec{v}\cdot\nabla(\ )$

②. $\vec{v}\cdot\nabla=0\rightarrow\frac{\partial}{\partial t}(\ )=\frac{d}{dt}(\ )$. 要想平流变化不为0，则：

(a). $\vec{v}\neq0$ (b). 物理场空间分布不均匀. (c). $\vec{v}\cdot\nabla\neq0$. $\vec{v}$ 与 ∇ 不垂直. ($\frac{\partial[\ ]}{\partial x}+\frac{\partial[\ ]}{\partial y}+\frac{\partial[\ ]}{\partial z}$ 的 不为0).

$\frac{\partial\vec{v}}{\partial t}\equiv0$ 定常流动. 局地点的速度不随时间变化，流场与时间无关，只是空间函数. 此时流线和迹线重合.

应用：运动中加速度的产生的原因：流场非定常性和非均匀性

熟练运用：第一章PPT. 101页. 2-3. 例1.2.3.

P.103. 例3. 已知流体运动的速度场如下，分别求流体质点的加速度，并说明各种情况下产生加速度的原因.

① $\begin{cases} u=-wy \\ v=wx \end{cases}$ ② $\begin{cases} u=mt \\ v=nt \end{cases}$ ③ $\begin{cases} u=x+2t \\ v=y-t \end{cases}$ (w,m,n为const).

① $\begin{cases} a_x = \frac{du}{dt} = -w^2 x \\ a_y = \frac{dv}{dt} = -w^2 y \end{cases}$ ② $\begin{cases} a_x = \frac{du}{dt} = m \\ a_y = \frac{dv}{dt} = n \end{cases}$ ③ $\begin{cases} a_x = \frac{du}{dt} = x+2t+2 \\ a_y = \frac{dv}{dt} = y-t-1 \end{cases}$

流场的非均匀性产生流点的加速度.　流场的非定常性产生流点的加速度.　流场的非均匀性和非定常性产生流点的加速度.

迹线和流线.

迹线：是某个流点在各时刻所行路径的轨迹线　拉格朗日思想.

求法：$\begin{cases} x=x(x_0,y_0,z_0,t) \\ y=y(x_0,y_0,z_0,t) \\ z=z(x_0,y_0,z_0,t) \end{cases} \xrightarrow{\text{消去参数}t}$ 迹线方程. 练习题：PPT 111页.

若已知欧拉变量，先将其转化为拉氏变量，再求迹线.

流线：由同一时刻不同流点组成的曲线，它给出了该时刻不同流体质点的速度方向. 欧拉观点

求法：$\frac{dx}{u(x,y,z,t)} = \frac{dy}{v(x,y,z,t)} = \frac{dz}{w(x,y,z,t)} \xrightarrow{\text{积分}}$ 流线微分方程. PPT 119.

在不定常运动时，迹线和流线一般不相重合. 定常流动，二者重合.（t不存在）.

涡度、散度、环流和形变率

流点速度构成：平移、旋转、形变.　$\vec{v}(M) = \vec{v}(M_0) + \vec{v}_R + \vec{v}_D$

形变张量矩阵：$A = \begin{bmatrix} A_{xx} & A_{xy} & A_{xz} \\ A_{yx} & A_{yy} & A_{yz} \\ A_{zx} & A_{zy} & A_{zz} \end{bmatrix}$　$A = A_{ij}(A_{ij})$. $i,j=1,2,3$.

$A_{yz} = A_{zy} = \frac{1}{2}\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)$, $A_{xx} = \frac{\partial u}{\partial x}$

$A_{zx} = A_{xz} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)$　$A_{yy} = \frac{\partial v}{\partial y}$

$A_{xy} = A_{yx} = \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)$, $A_{zz} = \frac{\partial w}{\partial z}$

流体旋转角速度 $\vec{\omega} = \frac{1}{2}\nabla\times\vec{v}$

涡度 = 2倍旋转角速度

涡度：定义涡度矢为矢量微分算符∇和速度矢$\vec{v}$的矢性积. 即：$\vec{\Omega} \equiv \nabla\times\vec{v}$

物理意义：涡度>0，逆时针，气旋式涡度. 低压. [气旋？台风？].

=0，无旋运动.　<0. 顺时针，反气旋式涡度，高压.

$$\nabla\times\vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix} = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\vec{i} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\vec{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\vec{k}$$

速度环流：取任一闭合有向曲线$\vec{l}$，沿闭合曲线$\vec{l}$对该闭合曲线上的流速分量求合：

$\Gamma = \oint \vec{v}\cdot d\vec{l}$　斯托克斯：$\Gamma = \oint_l \vec{v}\cdot d\vec{l} = \iint_\sigma \nabla\times\vec{v}\cdot d\vec{\sigma} = \iint_\sigma (\nabla\times\vec{v})\cdot\vec{n}\,d\sigma$

涡度 $(\nabla\times\vec{v})_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 2\frac{\partial v}{\partial x}$.

散度. 定义散度为矢量微分算符∇和速度矢$\vec{v}$的数量积. $D \equiv \nabla\cdot\vec{v} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$.

流体通量 $F=\iint_\sigma \vec{v}\cdot d\vec{\sigma}=\iint_\sigma \vec{v}\cdot\vec{n}\,d\sigma$. 流体散度即为单位体积的流体通量. Page 3

D的物理意义: $\nabla\cdot\vec{v}>0$.: 流体净流出 源(辐散). $\nabla\cdot\vec{v}<0$. 流体净流入. 汇.(辐合)

二. 体现流体体积变化: 封闭曲面向外膨胀. 封闭曲面向内收缩.

三. 反映单位体积的流点体胀速度.

流体不可压. → $D=0$. (个人观点, 未证).

形变率: 切形变 & 法形变. 散度其实也是一种形变. 称为体形变.

法形变: $A_{xx}=\frac{\partial u}{\partial x}$. $A_{yy}=\frac{\partial v}{\partial y}$. $A_{zz}=\frac{\partial w}{\partial z}$. $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=\nabla\cdot\vec{v}$.

切形变: $A_{23}=A_{32}=\frac{1}{2}\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right)$. $A_{31}=A_{13}=\frac{1}{2}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)$. 填入形变张量即可. $A=\begin{vmatrix} \cdots \\ \cdots \\ \cdots \end{vmatrix}$

$A_{12}=A_{21}=\frac{1}{2}\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)$.

考虑: $\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y}=0$. $w=0$. 无法形变. $\frac{\partial v}{\partial x}=\frac{\partial u}{\partial y}>0$. 无旋, 有切形变.

势函数和流函数.

速度势函数: 流体运动 $\begin{cases} \text{无旋流动. } \nabla\times\vec{v}=0. \leftarrow \text{从这里入手. } \varphi(x,y,z,t). \\ \text{涡旋流动. } \nabla\times\vec{v}\neq 0. \end{cases}$

$\vec{v}=-\nabla\varphi$. 势函数的求解. $D=\nabla\cdot\vec{v}$. $D=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}$

φ $\boxed{u=-\frac{\partial\varphi}{\partial x}.\ v=\frac{\partial\varphi}{\partial y}\ w=-\frac{\partial\varphi}{\partial z}}$ ← $-D=\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}+\frac{\partial^2\varphi}{\partial z^2}$.

势函数: $\vec{v}=u\vec{i}+v\vec{j}+w\vec{k}$ $u=-\frac{\partial\varphi}{\partial x}$, ...

流函数. 流体运动 $\begin{cases} \text{无辐散流 } \nabla\cdot\vec{v}=0. \text{ 散度为0.} \\ \text{辐散流 } \nabla\cdot\vec{v}\neq 0. \end{cases}$ $\boxed{u=-\frac{\partial\psi}{\partial y}\ v=\frac{\partial\psi}{\partial x}.}$ ψ

由 $\zeta=\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)$. 则 $\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}=\zeta$.

$\boxed{\text{Summary:}}$ $\zeta=0$ → φ. 涡度为0. 势函数存在.

$D=0$, → ψ. 散度为0, 流函数存在.

一般二维流动, 既不满足无旋条件, 也不满足无辐散条件. 流动是有旋有辐散的.

$\vec{v}=\underset{(1)}{\vec{v}_\psi}+\underset{(2)}{\vec{v}_\varphi}$. (1) 无辐散的涡旋流. (2) 无涡旋的辐散流. $\vec{v}=\vec{k}\times\nabla\psi-\nabla\varphi$.

二维运动既无旋, 又无辐散 $\vec{v}=\vec{v}_\psi=\vec{v}_\varphi$.

求流函数: PPT 一. P206.

2. 流体运动方程组.

连续方程(质量守恒). 作用于流体的力. 应力张量. 运动方程(动量守恒).
能量方程(能量守恒). N-S方程

连续方程. Lagrange. 体积形状可变 但质量守恒.

$\delta\tau=\delta x\delta y\delta z$. $\delta m=\rho\delta\tau$. $\frac{d(\delta m)}{dt}=0 \to \frac{d(\rho\delta\tau)}{dt}=0$.

$$\frac{d\rho}{dt}\delta\tau+\rho\frac{d(\delta\tau)}{dt}=0 \xrightarrow{\times\frac{1}{\delta\tau\rho}} \frac{1}{\rho}\cdot\frac{d\rho}{dt}+\frac{1}{\delta\tau}\cdot\frac{d(\delta\tau)}{dt}=0.$$

∵ $\frac{1}{\delta\tau}\cdot\frac{d(\delta\tau)}{dt}=\nabla\cdot\vec v$ (体胀速度). ∴ $\frac{d\rho}{dt}+\rho\nabla\cdot\vec v=0$.

$\nabla\cdot\vec v>0$ → 流体体积增大 → $\frac{d\rho}{dt}<0$. → 流体密度减小 ……

Lagrange: 流体密度变化是由于流体体积的膨胀或收缩造成的.

$\frac{d\rho}{dt}=0$. $\rho=const$. 不可压流体. $\frac{d\rho}{dt}=0$. ρ处处为const 均质不可压流体.

$\frac{\partial\rho}{\partial t}=0$. ρ与t无关. (密度)定常流体. $\frac{d\rho}{dt}=0$. $\nabla\rho=0$. 均质不可压流体.

连续方程. Euler. 流体质量变化等于流入量与流出量之差. $\frac{\partial\delta m}{\partial t}=\delta m_{入}-\delta m_{出}$.
(好像是小体素法). 过程: PPT 二. P13.

流入: $\rho\delta\tau=\rho u\cdot\delta y\delta z\delta t$.

流出: $(\rho u+\frac{\partial\rho u}{\partial x}\delta x)\delta y\delta z\delta t$. 净流入量(x)为二者之差: $-\frac{\partial\rho u}{\partial x}\delta x\delta y\delta z\delta t$.

同理: y: $-\frac{\partial\rho v}{\partial y}\delta x\delta y\delta z\delta t$. z: $-\frac{\partial\rho w}{\partial z}\delta x\delta y\delta z\delta t$.

总流入: $\underline{-(\frac{\partial\rho u}{\partial x}+\frac{\partial\rho v}{\partial y}+\frac{\partial\rho w}{\partial z})\delta x\delta y\delta z\delta t}$. $\delta m=\rho\delta x\delta y\delta z$.

δt时间内质量变化: $\frac{\partial m}{\partial t}\delta t=\underline{\frac{\partial\rho}{\partial t}\delta x\delta y\delta z\delta t}$. → $\frac{\partial\rho}{\partial t}+(\frac{\partial\rho u}{\partial x}+\frac{\partial\rho v}{\partial y}+\frac{\partial\rho w}{\partial z})=0$.

或. $\underline{\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\vec v)=0}$. 即 E型 连续方程.

$\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\vec v)=0$. 物理意义: $\nabla\vec v$ 速度散度. 单位体积流体体积通量.

$\nabla\cdot(\rho\vec v)$. 质量散度: 单位体积的流体质量通量.

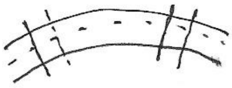
$\nabla\cdot(\rho\vec v)>0$. 净流出, 辐散. <0. 辐合. $=0$. 无辐散辐合.

E观点: 流体密度变化是由于流体质量的辐合辐散造成的.

$\frac{\partial\rho}{\partial t}=-\nabla\cdot(\rho\vec v)$. 密度定常 → $\frac{\partial\rho}{\partial t}=0$. → $\nabla\cdot(\rho\vec v)=0$. 净流入量为0. 流出=流进.

流管的定常运动: 流速与截面垂直. 密度和流速在任意截面内为定值: $\rho v\sigma=const$.

$\rho_1v_1\sigma_1=\rho_2v_2\sigma_2$



E L. 连续方程 关系：$\frac{d\rho}{dt}+\rho\nabla\cdot\vec{v}=0$.（速度散度形式，L）.

$\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\vec{v})=0$.（质量散度形式，E）.

具有自由表面的流体连续方程：$\frac{\partial h}{\partial t}+\nabla\cdot(h\vec{v})=0$.　$\frac{\partial h}{\partial t}+\vec{v}\cdot\nabla h+h\nabla\cdot\vec{v}=0$.

作用于流体的力、应力张量　流体的作用力 $\begin{cases}\text{质量力　重力 万有引力 电磁力　作用于质点.}\\\text{表面力　内部或接触面受力　粘性力 压力 摩擦力}\end{cases}$

应力张量：$\begin{cases}P_{nx}=n_xP_{xx}+n_yP_{yx}+n_zP_{zx}.\\P_{ny}=n_xP_{xy}+n_yP_{yy}+n_zP_{zy}.\\P_{nz}=n_xP_{xz}+n_yP_{yz}+n_zP_{zz}.\end{cases}$　$P=\begin{pmatrix}P_{xx}&P_{xy}&P_{xz}\\P_{yx}&P_{yy}&P_{yz}\\P_{zx}&P_{zy}&P_{zz}\end{pmatrix}$

$$\boxed{\vec{P}_n=\vec{n}\cdot P=(n_x,n_y,n_z)\cdot\begin{pmatrix}P_{xx}&P_{xy}&P_{xz}\\P_{yx}&P_{yy}&P_{yz}\\P_{zx}&P_{zy}&P_{zz}\end{pmatrix}=(P_{nx},P_{ny},P_{nz})}$$

P_{ij} 的物理含义：第一个下标表示受力面元的外法线方向，第二个下标表示面元受到的应力矢所投影的方向.

$\vec{P}_n=n_x\vec{P}_x+n_y\vec{P}_y+n_z\vec{P}_z$　　$\vec{P}_n=\vec{i}P_{nx}+\vec{j}P_{ny}+\vec{k}P_{nz}$.

沿 n,τ 方向分解：$\vec{P}_n=P_{nn}\vec{n}+P_{n\tau}\vec{\tau}=(P_{nn},P_{n\tau})$　$P_{nn}=\vec{P}_n\cdot\vec{n}$.

求某平面法应力和切应力，给定 $\vec{n}$，P.　$\boxed{\begin{aligned}P_{nn}&=\vec{P}_n\cdot\vec{n}=\vec{n}\cdot P\cdot\vec{n}.\\P_{n\tau}&=\sqrt{|\vec{P}_n|^2-P_{nn}^2}\end{aligned}}$

广义牛顿粘性假设：$P=2\mu A-[p+\frac{2}{3}\mu(\nabla\cdot\vec{v})]I$.　$I=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$.

$p=\frac{1}{3}(P_{xx}+P_{yy}+P_{zz})$.

不可压流体：$P=2\mu A-pI$.　A（形变率张量）.

不可压无粘性：$P=-pI$.

运动方程：$\frac{du}{dt}=F_x+\frac{1}{\rho}\left(\frac{\partial P_{xx}}{\partial x}+\frac{\partial P_{yx}}{\partial y}+\frac{\partial P_{zx}}{\partial z}\right)$ …… 相似 3.

矢量：$\frac{d\vec{v}}{dt}=\vec{F}+\frac{1}{\rho}\left(\frac{\partial\vec{P}_x}{\partial x}+\frac{\partial\vec{P}_y}{\partial y}+\frac{\partial\vec{P}_z}{\partial z}\right)$.　或 $\frac{d\vec{v}}{dt}=\vec{F}+\frac{1}{\rho}\nabla\cdot P$.

$\nabla\cdot P=\left(\frac{\partial}{\partial x}\ \frac{\partial}{\partial y}\ \frac{\partial}{\partial z}\right)\begin{pmatrix}P_{xx}&P_{xy}&P_{xz}\\&\cdots&\end{pmatrix}$

运动方程 $\xrightarrow{\text{广义牛顿粘性假设}}$ N-S 方程．推导：☆.

$\frac{d\vec{v}}{dt}=\vec{F}+\frac{1}{\rho}\nabla\cdot P$.　$\frac{1}{\rho}\nabla\cdot P=\frac{1}{\rho}\nabla\cdot\left(2\mu A-pI-\frac{2}{3}\mu(\nabla\cdot\vec{v})I\right)$. ← $P=2\mu A-[p+\frac{2}{3}\mu(\nabla\cdot\vec{v})]I$.

$=\frac{2\mu}{\rho}\nabla\cdot A-\frac{1}{\rho}\nabla p-\frac{2\mu}{3\rho}\nabla(\nabla\cdot\vec{v})$. ← $\nabla\cdot A=\frac{1}{2}[\nabla(\nabla\cdot\vec{v})+\nabla^2\vec{v}]$.

$=\frac{\mu}{\rho}[\nabla(\nabla\cdot\vec{v})+\nabla^2\vec{v}]-\frac{1}{\rho}\nabla p-\frac{2\mu}{3\rho}\nabla(\nabla\cdot\vec{v})$.

$=-\frac{1}{\rho}\nabla p+\frac{\mu}{3\rho}\nabla(\nabla\cdot\vec{v})+\frac{\mu}{\rho}\nabla^2\vec{v}$.　$\nu=\mu/\rho$.　不可压 $\nabla\cdot\vec{v}=0$

简化：$\underbrace{\frac{d\vec{v}}{dt}}_{\text{单位质量流体的加速度}}=\overset{\text{质量力}}{\vec{F}}-\underbrace{\frac{1}{\rho}\nabla p}_{\text{压力梯度力}}+\underbrace{\nu\nabla^2\vec{v}}_{\text{粘性摩擦力}}$

流体静止/无相对运动　理想流体 $\mu=0$．此项为0

N-S.方程 $\xrightarrow{\text{理想流体, } D=0.}$ 欧拉方程.

流体静止，速度和加速度的个别变化均为零. 静力方程 $0=\vec F-\frac{1}{\rho}\nabla p.$

流体的粘性又与流体的运动状态有关.

能量方程：(内能+动能)的变化率 = 外界对于流体所作的功率 + 吸收或释放的热量

$$\underbrace{\frac{d}{dt}\iiint_{\delta\tau}\rho(c_vT+v^2/2)d\tau}_{\text{总能量的变化项}}=\underbrace{\iiint_{\delta\tau}\rho(\vec F\cdot\vec v)d\tau}_{\text{质量力作功率}}+\underbrace{\oiint_{\delta\sigma}(\vec p_n\cdot\vec v)d\sigma}_{\text{表面力作功率}}+\underbrace{\frac{d}{dt}\iiint_{\delta\tau}\rho q d\tau}_{\text{热流量的变化率}}$$

$$\frac{d}{dt}(c_vT+v^2/2)=\vec F\cdot\vec v+\frac{1}{\rho}\nabla\cdot(p\cdot\vec v)+\frac{dq}{dt}$$

动能方程：$\frac{d}{dt}\left(\frac{v^2}{2}\right)=\vec F\cdot\vec v+\frac{1}{\rho}\nabla\cdot(p\cdot\vec v)+\frac{p}{\rho}\nabla\cdot\vec v-E.$

最终理想流体的水可写为：$\frac{d}{dt}\left(\frac{v^2}{2}\right)=\vec F\cdot\vec v-\frac{\vec v}{\rho}\cdot\nabla p.$

热流量方程 = 能量方程 − 动能方程. $\frac{dq}{dt}=\frac{d}{dt}(c_vT)+\frac{p}{\rho}\nabla\cdot\vec v\underbrace{-E}_{\text{理想流体无此项}}.$

伯努利方程：$\frac{v^2}{2}+gz+\frac{p}{\rho}=Const.$

适用条件：(1)理想流体(无粘性). (2)流不可压缩 (3)定常. (4)质量力为有势力.

3. 大气运动坐标系与方程组.

作用于大气上的力，惯性坐标系运动方程组.

惯性坐标系概念：相对于某个恒星静止或做匀速直线运动的坐标系，又称绝对坐标系.

惯性坐标系运动方程：$\underbrace{\frac{d_aV_a}{dt}}_{\text{绝对加速度}}=\underbrace{-\frac{1}{\rho}\nabla p}_{\text{气压梯度力}}\underbrace{-\frac{GM}{r^3}r}_{\text{地球引力}}+\underbrace{F}_{\text{摩擦力}}.$

绝对速度 = 相对速度 + 牵连速度.

$\frac{d_aV_a}{dt}=\frac{dv}{dt}+2\Omega\times V-\Omega^2R.$ 绝对加速度由相对、科氏、地转向心加速度构成.

推导：$\vec V_a=\vec v+\vec\Omega\times\vec r=\vec v+\vec\Omega\times\vec R.$ $\quad\frac{d_a(\ )}{dt}=\frac{d}{dt}(\ )+\vec\Omega\times(\ ).\Rightarrow$

$$\frac{d_a\vec v_a}{dt}=\frac{d\vec v_a}{dt}+\vec\Omega\times\vec v_a=\frac{d}{dt}(\vec v+\vec\Omega\times\vec r)+\vec\Omega\times(\vec v+\vec\Omega\times\vec r)$$

$$\frac{d}{dt}(\vec v+\vec\Omega\times\vec r)=\frac{d\vec v}{dt}+\underbrace{\frac{d\vec\Omega}{dt}}_{0}\times\vec r+\vec\Omega\times\frac{d\vec r}{dt}=\frac{d\vec v}{dt}+\vec\Omega\times\vec v.$$

$$\vec\Omega\times(\vec v+\vec\Omega\times\vec r)=\vec\Omega\times(\vec v+\vec\Omega\times\vec R)=\vec\Omega\times\vec v+\vec\Omega\times(\vec\Omega\times\vec R)$$

矢量三重积：$\vec A\times(\vec B\times\vec C)=(\vec A\cdot\vec C)\vec B-(\vec A\cdot\vec B)\vec C.$

令：$\vec A=\vec\Omega$, $\vec B=\vec\Omega$, $\vec C=\vec R$. 有：$\underbrace{(\vec\Omega\cdot\vec R)}_{0}\vec\Omega-(\vec\Omega\cdot\vec\Omega)\vec R.$ $\quad\vec\Omega\perp\vec R.$ $\vec\Omega\cdot\vec R=0$

则 $\vec\Omega\times(\vec v+\vec\Omega\times\vec r)=\vec\Omega\times\vec v-\Omega^2\vec R.$

$$\frac{d_a\vec v_a}{dt}=\frac{d\vec v}{dt}+2\vec\Omega\times\vec v-\Omega^2\vec R.$$

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$$\frac{d_a\vec{V}_a}{dt} = -\frac{1}{\rho}\nabla p - \frac{GM}{r^3}\vec{r} + \vec{F} \Rightarrow \frac{d\vec{v}}{dt} + 2\vec{\Omega}\times\vec{v} - \Omega^2\vec{R} = -\frac{1}{\rho}\nabla p - \frac{GM}{r^3}\vec{r} + \vec{F} \Rightarrow$$

相对加速度　　气压梯度力.　重力.　科氏力.　摩擦力.

$$\frac{d\vec{v}}{dt} = -\frac{1}{\rho}\nabla p \underline{- \frac{GM}{r^3}\vec{r}} + \vec{F} \underline{+ \Omega^2\vec{R}} - 2\vec{\Omega}\times\vec{v} = -\frac{1}{\rho}\nabla p + \vec{g} - 2\vec{\Omega}\times\vec{v} + \vec{F}.$$

局地直角坐标系的运动方程.

$$\begin{cases} \frac{du}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + 2\Omega v\sin\varphi - 2\Omega w\cos\varphi + F_x \\ \frac{dv}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - 2\Omega u\sin\varphi + F_y \\ \frac{dw}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g + 2\Omega u\cos\varphi + F_z \end{cases}$$

科氏参数 $f = 2\Omega\sin\varphi$.　$w \ll u, v$.　$F_z = 0$.　略 $2\Omega u\cos\varphi$

简化
$$\begin{cases} \frac{du}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + fv + F_x \\ \frac{dv}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - fu + F_y \\ \frac{dw}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g \end{cases}$$

大气闭合方程组.

$$\begin{cases} \frac{d\vec{v}}{dt} = g - \frac{1}{\rho}\nabla p - 2\vec{\Omega}\times\vec{v} + \vec{F} & \text{运动} \\ \frac{d\rho}{dt} + \rho\nabla\cdot\vec{v} = 0 & \text{连续} \\ p = \rho RT & \text{状态} \\ C_p\frac{dT}{dt} - \frac{1}{\rho}\frac{dp}{dt} = Q & \text{热力学} \\ \left(\frac{dq}{dt} = \frac{S}{\rho} = S.\right) & \text{水汽} \end{cases}$$

七个方程, 七个变量, 理论上可解.

$$2\vec{\Omega}\times\vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & \Omega\cos\varphi & \Omega\sin\varphi \\ v\cos\theta & v\sin\theta & 0 \end{vmatrix}$$

赤道. $60°\theta$.　$= 2\Omega v\cos\theta\,\vec{k}$

$q = \frac{\rho_w}{\rho}$ 比湿.　$\Omega = \frac{2\pi}{24\times3600}$ rad/s. 地球角速度.

$$w = u\frac{\partial h}{\partial x} + v\frac{\partial h}{\partial y}$$

z-p坐标转换, 将z换为p.

设静力平衡成立: $\frac{\partial p}{\partial z} = -\rho g$.

$$\begin{cases} \left(\frac{\partial F}{\partial x}\right)_p = \left(\frac{\partial F}{\partial x}\right)_z + \frac{\partial F}{\partial z}\cdot\left(\frac{\partial z}{\partial x}\right)_p \\ \left(\frac{\partial F}{\partial y}\right)_p = \left(\frac{\partial F}{\partial y}\right)_z + \frac{\partial F}{\partial z}\cdot\left(\frac{\partial z}{\partial y}\right)_p \\ \frac{\partial F}{\partial p} = \frac{\partial F}{\partial z}\cdot\frac{\partial z}{\partial p} \\ \left(\frac{\partial F}{\partial t}\right)_p = \left(\frac{\partial F}{\partial t}\right)_z + \frac{\partial F}{\partial z}\cdot\left(\frac{\partial z}{\partial t}\right)_p \end{cases}$$

P坐标连续方程.

$\delta\tau = \delta x\,\delta y\,\delta z$.　$\delta m = \rho\,\delta\tau = \rho\,\delta x\,\delta y\,\delta z$.

$\delta p = \cancel{\rho\delta g}\ \underline{-\rho g\,\delta z} \rightarrow \delta m = -\frac{1}{g}\delta x\,\delta y\,\delta p$.　质量守恒: $\frac{d(\delta m)}{dt} = 0$

$$\frac{d(\delta m)}{dt} = -\frac{1}{g}\left[\frac{d(\delta x)}{dt}\delta y\,\delta p + \frac{d(\delta y)}{dt}\delta x\,\delta p + \frac{d(\delta p)}{dt}\delta x\,\delta y\right].$$

"d" 与 "δ" 交换次序: $\frac{d(\delta m)}{dt} = -\frac{1}{g}\left[\delta\left(\frac{dx}{dt}\right)\delta y\,\delta p + \delta\left(\frac{dy}{dt}\right)\delta x\,\delta p + \delta\left(\frac{dp}{dt}\right)\delta x\,\delta y\right]$.

$\frac{dx}{dt} = u$, $\frac{dy}{dt} = v$, $\frac{dp}{dt} = w$.

$$\frac{d(\delta m)}{dt} = -\frac{1}{g}(\delta u\,\delta y\,\delta p + \delta v\,\delta x\,\delta p + \delta w\,\delta x\,\delta y)$$
$$= -\frac{1}{g}\delta x\,\delta y\,\delta p\left(\frac{\delta u}{\delta x} + \frac{\delta v}{\delta y} + \frac{\delta w}{\delta p}\right) = 0.$$

$\frac{\delta u}{\delta x} + \frac{\delta v}{\delta y} + \frac{\delta w}{\delta p} = 0$.　或　$\left(\frac{\partial u}{\partial x}\right)_p + \left(\frac{\partial v}{\partial y}\right)_p + \frac{\partial w}{\partial p} = 0$.

p-z 坐标转换公式：$F = F(x, y, z, t) = F[x, y, z(x_p, y_p, p, t_p), t]$. page 8.
（好像没有实际意义）

1. 自然坐标系. 2. 地转风. 3. 梯度风. 4. 摩擦风与…… 5. 热成风. 6. 地转偏差.

地转风：空气块直线运动，在水平气压梯度力和水平地转偏向力平衡的作用下，风沿等压线或等位势线吹，北半球背风而立，高压在右.

梯度风：中纬度中尺度大气运动方程的零级近似 三力平衡：水平摩擦力 + 水平科氏力 = 水平气压梯度力，水平运动为梯度风.

热成风：地转风随高度的变化定义为热成风，是由热力作用引起的.
原因：大气的斜压性.

正压大气：密度仅与气压有关，正压大气中等压面、等密度面和等温面重合在一起.
斜压大气：密度不仅与气压有关，还有温度. 三面交割. 大气一般是斜压的.

5. 尺度分析. (p74) 记住那些参量. 自己算那4个方程. 零级、一级简化. 各3种.

实在不记的话，就赌他不考

简化后物理意义：p77. p78.

罗斯贝数：$R_o = \dfrac{\frac{U^2}{L}}{f_0 U} = \dfrac{U}{f_0 L} = \dfrac{\text{水平摩擦力}}{\text{水平科里奥利力}}$

$R_o < 1$. 大尺度. 准地转. 线性过程. $R_o \geq 1$. 反之.

5个特征无量纲参数（不止5个，概念，理解）. p80.

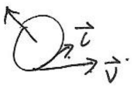
6. 量纲分析与π定理. ⟨ 方程分析. 尺度分析法.
量纲分析. π定理.

物理量 = 特征值 × 无量纲数 无量纲方程优点：没单位，可比较相对大小和相对重要性，流场相似判据.

量纲，基本量

https://www.bilibili.com/video/BV1jWtZeDEV6 瑞利法+π定理

7. 环流定理与涡度方程.

环流定理. 速度环流的定义：$\Gamma = \oint_L \vec{v} \cdot d\vec{r}$.  反映了流体沿着曲线 L 运动的总趋势，标量，但有一定的方向性.

绝对环流 = 相对~ + 牵连~

$\Gamma > 0$. 沿 L，逆时针正方向，气旋性环流. 反之则反之.

理想正压流体，在有势力的作用下，速度环流不随时间变化，这就是开尔文定理（绝对环流）. $\dfrac{d\Gamma}{dt} = 0$.

涡度方程

$$\frac{d\vec{\xi}}{dt} = \overset{(1)}{\frac{1}{\rho^2}\nabla\rho \times \nabla p} \overset{(2)}{- \vec{\xi}(\nabla \cdot \vec{v})} + \overset{(3)}{(\vec{\xi} \cdot \nabla)\vec{v}} + \overset{(4)}{\nu \nabla^2 \vec{\xi}}$$

(1). 力管项或斜压项 它表明了压力-密度变化可以引起流体涡度矢的变化，其物理实质是流体的斜压性.

(2). 散度项. 它表明了流体在运动过程中体积的收缩或膨胀，将会引起流体涡度矢的变化

(3). 流场的非均匀性，引起涡度的重新分布. 扭曲项.

(4). 粘性扩散项 涡度分布的非均匀性引起的.

Proudman - Taylor 定理. 在均质或正压旋转流体中，流体做定常和缓慢地运动，其速度在垂直方向上将不改变，即运动将趋于两维化. 条件 + 结论.

现象(实验) Taylor column. 泰勒柱.

垂直涡度矢量方程的推导 (要掌握). 写题时必须写出全过程 no ∵ ∴

$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ f 是 y 的函数. 中间令 $\frac{1}{\rho} = \alpha$. 开导!!!

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + fv + F_x \quad (1)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - fu + F_y \quad (2)$$

由 $\frac{\partial (1)}{\partial y}$: $\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial y}\right) + \frac{\partial u}{\partial y}\frac{\partial u}{\partial x} + u\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right) + \frac{\partial v}{\partial y}\frac{\partial u}{\partial y} + v\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right) + \frac{\partial w}{\partial y}\frac{\partial u}{\partial z} + w\frac{\partial}{\partial z}\left(\frac{\partial u}{\partial y}\right) = \cancel{-\frac{\partial \alpha}{\partial x}\frac{\partial p}{\partial y}} - \frac{\partial \alpha}{\partial y}\frac{\partial p}{\partial x} - \alpha\frac{\partial}{\partial y}\left(\frac{\partial p}{\partial x}\right) + f\frac{\partial v}{\partial y} + v\frac{\partial f}{\partial y} + \frac{\partial F_x}{\partial y} \quad (3)$

由 $\frac{\partial (2)}{\partial x}$: $\frac{\partial}{\partial t}\left(\frac{\partial v}{\partial x}\right) + \frac{\partial u}{\partial x}\frac{\partial v}{\partial x} + u\frac{\partial}{\partial x}\left(\frac{\partial v}{\partial x}\right) + \frac{\partial v}{\partial x}\frac{\partial v}{\partial y} + v\frac{\partial}{\partial y}\left(\frac{\partial v}{\partial x}\right) + \frac{\partial w}{\partial x}\frac{\partial v}{\partial z} + w\frac{\partial}{\partial z}\left(\frac{\partial v}{\partial x}\right) = -\frac{\partial \alpha}{\partial x}\frac{\partial p}{\partial y} - \alpha\frac{\partial}{\partial y}\left(\frac{\partial p}{\partial x}\right) - f\frac{\partial u}{\partial x} - \underbrace{u\frac{\partial f}{\partial x}}_{0} + \frac{\partial F_y}{\partial x} \quad (4)$

(4) − (3): $\frac{\partial}{\partial t}\left(\frac{\partial v}{\partial x}\right) - \frac{\partial}{\partial t}\left(\frac{\partial u}{\partial y}\right) + u\frac{\partial}{\partial x}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) + v\frac{\partial}{\partial y}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) + w\frac{\partial}{\partial z}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) + \frac{\partial u}{\partial x}\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x}\frac{\partial v}{\partial y} - \frac{\partial v}{\partial y}\frac{\partial u}{\partial y} \boxed{+ \frac{\partial w}{\partial x}\frac{\partial v}{\partial z} - \frac{\partial w}{\partial y}\frac{\partial u}{\partial z}} = \cancel{\frac{\partial \alpha}{\partial x}\frac{\partial p}{\partial y}} - \frac{\partial \alpha}{\partial x}\frac{\partial p}{\partial y} + \frac{\partial \alpha}{\partial y}\frac{\partial p}{\partial x} + \alpha\frac{\partial}{\partial y}\left(\frac{\partial p}{\partial y}\right) - \alpha\frac{\partial}{\partial y}\left(\frac{\partial p}{\partial x}\right) - f\frac{\partial u}{\partial x} - f\frac{\partial v}{\partial y} - v\frac{\partial f}{\partial y} + \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}$

$(\vec{v}\cdot\nabla)\zeta$

$\frac{\partial \zeta}{\partial t} + u\frac{\partial \zeta}{\partial x} + v\frac{\partial \zeta}{\partial y} + w\frac{\partial \zeta}{\partial z} + \left[\frac{\partial u}{\partial x}\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x}\frac{\partial v}{\partial y} - \frac{\partial v}{\partial y}\frac{\partial u}{\partial y}\right] =$

$\frac{\partial \alpha}{\partial y}\frac{\partial p}{\partial x} - \frac{\partial \alpha}{\partial x}\frac{\partial p}{\partial y} + \alpha\frac{\partial}{\partial y}\left(\frac{\partial p}{\partial x}\right) - \alpha\frac{\partial}{\partial y}\left(\frac{\partial p}{\partial x}\right) - f\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) - v\frac{\partial f}{\partial y} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)$

$$\frac{d\zeta}{dt} + \zeta\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + \frac{\partial w}{\partial x}\frac{\partial v}{\partial z} - \frac{\partial w}{\partial y}\frac{\partial u}{\partial z} = \frac{1}{\rho^2}\left(\frac{\partial \rho}{\partial x}\frac{\partial p}{\partial y} - \frac{\partial \rho}{\partial y}\frac{\partial p}{\partial x}\right) + \underbrace{\alpha\frac{\partial^2 p}{\partial x\partial y} - \alpha\frac{\partial^2 p}{\partial x\partial y}}_{0} - f\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) - v\frac{\partial f}{\partial y} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)$$

$$\frac{d\zeta}{dt} = -(f+\zeta)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + \frac{1}{\rho^2}\left(\frac{\partial \rho}{\partial x}\frac{\partial p}{\partial y} - \frac{\partial \rho}{\partial y}\frac{\partial p}{\partial x}\right) + \frac{\partial w}{\partial y}\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\frac{\partial v}{\partial z} - v\frac{\partial f}{\partial y} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)$$

f是y的函数 $\frac{d(\ )}{dt} = \frac{\partial(\ )}{\partial t} + (\nabla\cdot\vec{v})(\ )$

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + (\nabla\vec{v})f$$

$$= 0 + u\frac{\partial f}{\partial x} + v\frac{\partial f}{\partial y} + w\frac{\partial f}{\partial z}$$

$$= 0 + 0 + v\frac{\partial f}{\partial y} + 0 \qquad \therefore \frac{df}{dt} = v\frac{\partial f}{\partial y}$$

$$\therefore \frac{d(\zeta+f)}{dt} = -(f+\zeta)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + \frac{1}{\rho^2}\left(\frac{\partial\rho}{\partial x}\frac{\partial p}{\partial y} - \frac{\partial\rho}{\partial y}\frac{\partial p}{\partial x}\right) + \left(\frac{\partial w}{\partial y}\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\frac{\partial v}{\partial z}\right) + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)$$

这一项其不会略