ELECTRIGRID
Section A - Boss Report

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Introduction

ELECTRIGRID supplies electricity to a small region from 4 generators. Each day (or hour), electricity is produced and sent to the given regions with required demands(in MW). After communication 5, we have developed the following mathematical model to meet the client's objectives.

Mathematical Formulation

Sets

N	nodes 0, 1, ..., 49
G	generator nodes 12, 37, 23, 20
Arcs	transmission arcs
UnlimitedArcs	transmission arcs without limits (comm 3)
Periods	6 time periods of a day {D0,D1,D2,D3,D4,D5} (comm4)

Data

Demand_n	the amount of electricity required for node $n \in N$
Capacity_n	maximum capacity(MW) for generator node $n \in G$
Cost_n	cost($/MWh) for generator node $n \in G$
f_a, t_a	from and to nodes for $a \in \text{Arcs}$
fX_a, fY_a	location (X,Y) for f_a
tX_a, tY_a	location (X,Y) for t_a
Distance_a	distance between the nodes of arc $a \in \text{Arcs}$ which is $\sqrt{(tX_a - fX_a)^2 + (tY_a - fY_a)^2}$
Limit	electricity limits for arcs not in UnlimitedArcs (88MW)
$\text{PeriodDemand}_{n,p}$	electricity required for node $n \in N$ during the period $p \in \text{Periods}$ (comm4-5)
ChangeLimit	the limit of electricity change between 2 periods for each generator which is 185(MW) (comm5)

Variables

X_a	electricity flow for $a \in Arcs$
$X_{a,p}$	electricity flow for $a \in Arcs$ during the period $p \in$ Periods (comm4-5)

Objective

$$\min \sum_{n\in G} \sum_{\substack{a\in \text{Arcs} \\ \text{s.t. } f_a=n}} X_a \times 24 \times \text{Cost}_n \quad \text{(comm1-3)}$$

$$\min \sum_{n\in G} \sum_{p\in \text{Periods}} \sum_{\substack{a\in \text{Arcs} \\ \text{s.t. } f_a=n}} X_{a,p} \times 4 \times \text{Cost}_n \quad \text{(comm4-5)}$$

Constraints

1. Demand constraint

$$\sum_{\substack{a\in \text{Arcs} \\ \text{s.t. } t_a=n}} X_a - \sum_{\substack{a\in \text{Arcs} \\ \text{s.t. } f_a=n}} X_a = \text{Demand}_n, \quad \forall n \in N \setminus G \quad \text{(comm1)}$$

$$\sum_{\substack{a\in \text{Arcs} \\ \text{s.t. } t_a=n}} X_a(1 - 0.001 \times \text{Distance}_a) - \sum_{\substack{a\in \text{Arcs} \\ \text{s.t. } f_a=n}} X_a = \text{Demand}_n, \quad \forall n \in N \setminus G \quad \text{(comm2-3)}$$

$$\sum_{\substack{a\in \text{Arcs} \\ \text{s.t. } t_a=n}} X_{a,p}(1 - 0.001 \times \text{Distance}_a) - \sum_{\substack{a\in \text{Arcs} \\ \text{s.t. } f_a=n}} X_{a,p} = \text{PeriodDemand}_{n,p}$$

$$\forall n \in N \setminus G, p \in \text{Periods} \quad \text{(comm4-5)}$$

2. Capacity constraint

$$\sum_{\substack{a\in \text{Arcs} \\ \text{s.t. } f_a=n}} X_a \leq \text{Capacity}_n, \quad \forall n \in G$$

$$\sum_{\substack{a\in \text{Arcs} \\ \text{s.t. } f_a=n}} X_{a,p} \leq \text{Capacity}_n, \quad \forall n \in G, p \in \text{Periods} \quad \text{(comm4-5)}$$

3. **Arc limitation constraint (comm3-5)**

$$\sum_{\substack{\forall a \in \text{Arcs} \setminus \text{UnlimitedArcs} \\ \text{s.t. } f_a = n_1, t_a = n_2}} X_a \leq \text{Limit}, \quad \forall n_1, n_2 \in N$$

$$\sum_{\substack{\forall a \in \text{Arcs} \setminus \text{UnlimitedArcs} \\ \text{s.t. } f_a = n_1, t_a = n_2}} X_{a,p} \leq \text{Limit}, \quad \forall n_1, n_2 \in N, p \in \text{Periods} \quad \text{(comm4-5)}$$

4. **Change limitation constraint (comm5)**

$$\sum_{\substack{a \in \text{Arcs} \\ \text{s.t. } f_a = n}} \mathrm{X}_{a,p_k} - \sum_{\substack{a \in \text{Arcs} \\ \text{s.t. } f_a = n}} \mathrm{X}_{a,p_{k-1}} \leq \text{ChangeLimit}$$

$$\sum_{\substack{a \in \text{Arcs} \\ \text{s.t. } f_a = n}} \mathrm{X}_{a,p_k} - \sum_{\substack{a \in \text{Arcs} \\ \text{s.t. } f_a = n}} \mathrm{X}_{a,p_{k-1}} \geq -\text{ChangeLimit},$$

$$\forall n \in G, p_k \in \text{Periods}=\{\text{D0,D1,D2,D3,D4,D5}\}, \text{ where } k = 1, 2, 3, 4, 5$$

5. **Non-negative Constraint**

$$\mathrm{X}_a \geq 0, \quad \forall a \in \text{Arcs}$$

$$\mathrm{X}_{a,p} \geq 0, \quad \forall a \in \text{Arcs}, p \in \text{Periods}$$

ELECTRIGRID
Section B - Client Report

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After careful consideration of your requirements of demands, generators' capacity, cost and transmission line limits. We have come up with the following proposals for the optimal cost of producing electricity over a whole day while meeting with all the demands.

Communication 1

We are required to calculate the optimal cost for meeting the current demand over a whole day from the given generators, with different capacity limits and cost.

Since the demand does not change over time, and the loss due to the distances of Arcs (distance between nodes) are not considered. The solution is calculated by applying the constraint that the difference between the electricity among the arcs with the same destination node, as the from node of another arc, must equal to the demand of that given node (for all substation nodes). Moreover, since the generators have various capacity limits and cost, our model has also captured this constraint and produced the following solution.

The optimal cost for meeting the current demand over a whole day from your generators is $3248616. The following table showing the electricity produced and cost at each generator per hour.

Generator	Electricity Production (MW/h)	Cost ($/h)
20	549.0	34038.0
37	792.0	57816.0
23	565.0	43505.0
12	0.0	0.0

Table 1: Electricity Production & Cost (Comm1)

Since we are not considering any other factors rather than Capacity, the optimal solution utilizes the max amount of the cheapest generator and then progresses to the next cheapest one until we meet the demand.

Communication 2

We have modified the previous constraint to consider the loss of electricity estimated as 0.1% per km during transmission. This will increase our optimal cost because of the additional loss for the electricity flowing on the arcs in order to meet the demand of the substation nodes.

Thus, considering the loss during transmission, the optimal cost for meeting the current demand over a whole day from your generators is $3498981.

Generator	Electricity Production (MW/h)	Cost ($/h)
20	549.0	34038.0
37	784.14	57242.35
23	510.83	39334.11
12	187.36	15176.41

Table 2: Electricity Production & Cost (Comm2)

Communication 3

With regard to your new requirement of the limit on some transmission lines, we have developed a new constraint which limits the flow of electricity on the given lines.

Hence, considering the limit of 88 MW for some transmission lines, the optimal cost for meeting the current demand over a whole day from your generators is $3499540.

Generator	Electricity Production (MW/h)	Cost ($/h)
20	549.0	34038.0
37	770.69	56260.3
23	503.18	38744.6
12	207.05	16771.29

Table 3: Electricity Production & Cost (Comm3)

As seen in the above table, the electricity production of the cheaper Generators (37, 23) has generated less electricity than that in the previous models, but the optimal cost has only increased by less than $1000 after the inclusion of the new constraint.

Communication 4

After the forth communication, we have have modified our variable and demand data to take into account of the different electricity flows and demands of node during the six time periods.

Thus, after revision, the optimal total cost over the day for meeting the demand in each of the six time periods from your generators is $3086102.

Generator	Period	Electricity Production (MW/h)	Cost ($/h)
20	D0	549.0	34038.0
	D1	549.0	34038.0
	D2	549.0	34038.0
	D3	549.0	34038.0
	D4	549.0	34038.0
	D5	549.0	34038.0
37	D0	264.45	19305.17
	D1	423.55	30919.09
	D2	646.05	47161.44
	D3	749.51	54714.54
	D4	792.0	57816.0
	D5	768.06	56068.67
23	D0	121.58	9361.67
	D1	347.62	26766.7
	D2	473.45	36455.49
	D3	480.44	36994.02
	D4	787.35	60625.79
	D5	541.82	41720.36
12	D0	95.31	7720.5
	D1	124.61	10093.3
	D2	180.0	14579.98
	D3	203.28	16466.07
	D4	311.96	25268.77
	D5	188.39	15259.98

Table 4: Electricity Production & Cost (Comm4)

As shown in table above, some periods require less electricity than the others, so the cheaper generators take a larger proportion of the demand, as a result saving a huge amount of cost in comparison to the previous communications.

Communication 5

Based on our final communication with your company, since there is a new limitation suggesting that each generator's output should not change by more than 185 MW from one time period to the following one, we have updated our previous model and come up with our final mathematical model to incorporate this change.

In our model, we have added a new constraint to restrict the change of output of each generator from one time period to the following one within a day to be less than 185 MW. In addition, this additional constraint did not cause a huge effect to the optimal cost comparing to that in previous communication.

In which, the optimal cost for meeting the current demand in each of the six time periods from your generators calculated using the final model is $3086256.

Generator	Period	Electricity Production (MW/h)	Cost ($/h)
20	D0	549.0	34038.0
	D1	549.0	34038.0
	D2	549.0	34038.0
	D3	549.0	34038.0
	D4	549.0	34038.0
	D5	549.0	34038.0
37	D0	262.73	19179.36
	D1	441.8	32251.04
	D2	626.8	45756.04
	D3	666.22	48634.25
	D4	792.0	57816.0
	D5	742.44	54198.01
23	D0	123.22	9488.13
	D1	308.22	23733.13
	D2	491.71	37861.66
	D3	566.14	43592.42
	D4	751.14	57837.42
	D5	566.14	43592.42
12	D0	95.31	7720.5
	D1	145.71	11802.49
	D2	180.0	14579.98
	D3	197.0	15957.38
	D4	346.62	28075.9
	D5	188.39	15259.98

Table 5: Electricity Production & Cost (Comm5)

As shown in the table above, we find that other than Generator 20, the rest of the generators are never fully utilised in nearly all time periods, which may due to the various cost of the generators.

Thus, we have come up with the following adjustments which may help to further optimise your cost:

- Every 1MW increase of the Capacity of Generator 20 would reduce the cost by from \$32 to \$73 up to a total capacity of 581 MW and its average save is \$52.52.
- Any amount of the increase of the Capacity of Generators 37 (except for time period D4) , 23 and 12 will not cause any reduction of cost, as they are not the bottlenecks of the electricity supply chain. Thus, unless there are changes made to the cost prices of the Generators, or demand from substation nodes, the capacity for these Generators could be decreased if necessary.
- Every 1MW increase of ChangeLimit makes only subtle savings. In addition, With every 1MW increase of the ArcLimit, the average saving of the cost is \$ 4.1. However, due to security considerations we do not recommend this adjustment.