ELECTRIGRID
Section A - Boss Report

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Mathematical Formulation

Sets

N	nodes 0, 1, ..., 49
G	generator nodes 12, 37, 23, 20
A	transmission arcs
Highs	transmission arcs without limits
T	6 time periods of a day {D0,D1,D2,D3,D4,D5}

Data

Supply_n	maximum supply(MW) for generator node $n \in G$	
Cost_n	cost($/MWh) for generator node $n \in G$	
Loss	electricity loss per km during transmission which is 0.001	
$\text{Node1}_a, \text{Node2}_a$	from node and to node on arc $a \in A$	
Distance_a	distance between the nodes on arc $a \in A$	
LowLimit	electricity limits for arcs not in UnlimitedArcs which is 88MW	
$D_{n,t}$	electricity demand for node $n \in N$ in time period $t \in T$	
Hours	number of hours per period which is 4	
LimitIncrease	increase the capacity of the three transmission lines by 50 MW	(comm6)
NewCost	cost for the new generator which is $69/MWh	(comm7)
NewSupply	capacity of the new generator which is 200MW	(comm7)
SolarSupply_t	supply of solar electricity during time period $t \in T$	(comm10)
SolarCost	cost of the solar electricity which is $42/MWh	(comm10)
ThresholdRatio	the generators operate efficiently up to 60% of their capacity	(comm11)
CostIncrease	increase 30% if generators operate over threshold	(comm11)
DemandReduction	reduce 10% demand for selected nodes and time periods	(comm12)

Variables

$X_{a,t}$ electricity flow on arc $a \in A$ in time period $t \in T$

$Y_{n,t}$ electricity generated at node $n \in N$ in time period $t \in T$

$$\mathrm{I}_a = \begin{cases} 1, & \text{if we increase the limit of arc } a \in A \\ 0, & \text{if not} \end{cases} \quad \text{(comm6)}$$

$$\mathrm{New}_n = \begin{cases} 1, & \text{if we pick } n \in N \setminus G \text{ as the small generator} \\ 0, & \text{if not} \end{cases} \quad \text{(comm7)}$$

$$\mathrm{R}_t = \begin{cases} 1, & \text{if we run the new small generator during the time period } t \in T \\ 0, & \text{if not} \end{cases} \quad \text{(comm9)}$$

$$\mathrm{Solar}_n = \begin{cases} 1, & \text{if we pick } n \in N \text{ as the solar farm} \\ 0, & \text{if not} \end{cases} \quad \text{(comm10)}$$

$S_{n,t}$ solar electricity generated at node $n \in N$ in time period $t \in T$ (comm10)

$U_{n,t}$ electricity generated at node $n \in N$ in time period $t \in T$ under threshold (comm11)

$O_{n,t}$ electricity generated at node $n \in N$ in time period $t \in T$ over threshold (comm11)

$$\mathrm{RD}_{n,t} = \begin{cases} 1, & \text{if we pick period } t \in T \text{ for node } n \in N \text{ to be reduced} \\ 0, & \text{if not} \end{cases} \quad \text{(comm12)}$$

Objective

$$\min \sum_{t \in T} \sum_{n \in G} \text{Hours} \times \text{Cost}_n \times U_{n,t} + \sum_{t \in T} \sum_{n \in G} \text{Hours} \times \text{Cost}_n \times (1 + \text{CostIncrease}) \times O_{n,t}$$
$$+ \sum_{t \in T} \sum_{n \in N \setminus G} \text{Hours} \times \text{NewCost} \times Y_{n,t} + \sum_{t \in T} \sum_{n \in N} \text{Hours} \times \text{SolarCost} \times S_{n,t}$$

Constraints

1. Variable constraint

$$X_{a,t} \geq 0, \forall a \in A, t \in T$$

$$Y_{n,t} \geq 0, \forall n \in N, t \in T$$

$$\sum_{a \in \text{Arcs}} I_a = 3 \quad \text{(comm6)}$$

$$\sum_{n \in N \setminus G} \text{New}_n = 1 \quad \text{(comm7)}$$

$$\sum_{t \in T} R_t = 4 \quad \text{(comm9)}$$

$$\sum_{n \in N} \text{Solar}_n = 1 \quad \text{(comm10)}$$

$$\sum_{n\in N} \mathrm{RD}_{n,t} \leq 9, \forall t \in T \quad \text{(comm12)}$$

$$\sum_{t\in T} \mathrm{RD}_{n,t} = 1, \forall n \in N \quad \text{(comm12)}$$

2. Demand constraint

$$Y_{n,t} + S_{n,t} + \sum_{\substack{a\in A \text{ s.t.} \\ \mathrm{Node2}_a = n}} X_{a,t}(1 - \mathrm{Loss} \times \mathrm{Distance}_a)$$

$$= \mathrm{Demand}_{n,t} \times (1 - \mathrm{DemandReduction} \times \mathrm{RD}_{n,t}) + \sum_{\substack{a\in A \text{ s.t.} \\ \mathrm{Node1}_a = n}} X_{a,t},$$

$$\forall n \in N, t \in T$$

3. Supply constraint

$$Y_{n,t} \leq \mathrm{Supply}_n, \forall n \in G, t \in T$$

$$Y_{10,t} = 0, \forall t \in T \quad \text{(comm8)}$$

$$Y_{n,t} \leq \mathrm{NewSupply} \times \mathrm{New}_n \times R_t, \forall n \in N \setminus G, t \in T \quad \text{(comm9)}$$

$$S_{n,t} \leq \mathrm{SolarSupply} \times \mathrm{Solar}_n, \forall n \in N, t \in T \quad \text{(comm10)}$$

$$U_{n,t} \leq \mathrm{Supply}_n \times \mathrm{ThresholdRatio}, \forall n \in G, t \in T \quad \text{(comm11)}$$

$$O_{n,t} \leq \mathrm{Supply}_n \times (1 - \mathrm{ThresholdRatio}), \forall n \in G, t \in T \quad \text{(comm11)}$$

4. Arc limitation constraint

$$X_{a,t} \leq \mathrm{LowLimit} + \mathrm{LimitIncrease} \times I_a, \forall a \in A \setminus \mathrm{Highs}, t \in T \quad \text{(comm6)}$$

ELECTRIGRID
Section B - Client Report

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We have come up with the following proposals for the optimal cost of producing electricity over a whole day while meeting with all the demands.

Comm 6

Based on the model for Communication 4 (results in a cost of $3086102) , with the funds to increase the capacity of three of your transmission lines by 50 MW (to 138 MW), Arcs 122, 174 and 203 should be increased.

The optimal total cost over the day for meeting the demand in each of the six time periods from your generators is $3085113.

With the increase in capacity of the three suggested arcs, it has managed to decrease the overall cost by nearly $1000 over the whole day. Hence it is worthy to note that having to increase the capacity of more arcs may result in further reduction of cost.

Comm 7

After considering your requirement of building a small gas generator at one of the existing nodes except the generator nodes on the network, Node 10 should be chosen as the new generator location for the optimal cost. In addition, this constraint has also resulted in the change of the proposed upgrade transmission liens to Arc 174, 179 and 203.

The optimal total cost over the day for meeting the demand in each of the six time periods from your generators is $3016565.

It is clear that with the new generator built on Node 10, which can supply up to 200 MW at a cost of $69/hr, it will help to decrease the optimal cost. Although in comparison to the original generators it has less capacity, its cost is relatively cheap. Thus, with the new constraint and update of transmission lines, it has significantly reduced the cost by approximately $68000.

Comm 8

Since the local government has declined your application to build the new generator at Node 10, we have modified our model by excluding Node 10 in the procedure of choosing new generator. Based on our updated model, we propose the alternative site to be Node 4.

Moreover, with the inclusion of this new constraint, it has not caused a change in our proposed upgrade transmission arcs from last communication (174, 179, 203). Which indicates that the transmission lines are currently the most suitable and optimal lines to be upgraded.

The optimal total cost over the day for meeting the demand in each of the six time periods from our generators is $3017136. As expected, since the optimal solution for our generator node has been excluded, the cost has slightly increased from last communication. However, it has not increased significantly since Node 4 is also the most optimal solution whilst considering this restriction.

Comm 9

With regard to your new requirement of running the new generator during four time periods (16 hours) each day, we propose that it should be operated during D2-D5 (8:00 - 24:00) based on your demand.

Taking the restriction into account, the proposed generator node location has not been affected (Node 4). However, the optimal transmission lines has now been updated to Arcs 122, 174 and 203.

Since the run time of the new generator has been affected, the optimal cost would as a result increase. Thus, the optimal total cost over the day for meeting the demand in each of the six time periods from our generators is $3038124. Which the cost has increased by about $21000.

Comm 10

Considering your requirement of building a solar farm with its capacity in each time period and its cost, our model has suggested to picked Node 1 as the solar farm location. With this new constraint being included, the proposed results from previous communications has been updated.

Although the location of the new generator node has not been affected (Node 4), the operation period should now be during D1, D3 - D5 (4:00-8:00, 12:00-24:00). In addition, the proposed transmission lines also remains the same as previously suggested (122, 174, 203).

Hence, the optimal total cost over the day for meeting the demand in each of the six time periods from our generators is $2993082. This result indicates that the new solar farm is important and necessary, where it has reduced the cost by over $45000 per day.

Comm 11

Noting the 60% capacity threshold in which the generators can run before the costs increases (with a 30% increase on top of the original value). We have modified our model by including a key constrain that calculates the electricity generated under and over the threshold at each station separately. Thus, allowing us to apply the different cost to each threshold amount.

Since the cost increases after the 60% threshold, the optimal cost would as a result increase, as well as affecting the decisions from previous communications. The new proposed result for the generator node location remains at Node 4, with its operation period updated to D2-D5 (8:00 - 24:00). More over, the transmission line to be upgraded should now be Arcs 2, 173 and 203, and the solar farm has been suggested to be built on Node 43.

Thus, after the series of changes applied after considering the new restriction, the optimal total cost over the day for meeting the demand in each of the six time periods from our generators is $3129662. In which the optimal cost has increased by over $135000 due to the increase in cost after generating past the threshold.

Comm 12

Based on our final communication with your company. We have modified our variables to take into account of the 10% reduction in demand for one time period each day of each node, as well as the additional requirement that no more than 9 nodes can be reduced during the same time period.

In our model, we have included new constraints to select the time periods in which the nodes will reduce the cost. Furthermore, another constraint has also been applied to restrict the total number of nodes that can be reduced at each time period. This has allowed us to produce the following results as shown in the table below.

D0	D1	D2	D3	D4	D5
0:00- 4:00	4:00 - 8:00	8:00 -12:00	12:00 -16:00	16:00 - 20:00	20:00 - 24:00
12	10	0	2	1	3
20	16	11	7	9	4
23	25	14	15	21	5
37	27	18	24	30	6
49	28	22	26	31	8
	33	29	32	39	13
	34	38	36	40	17
	35	46	42	44	19
	41	47	43	48	45

Table 1: Nodes for demand reduction for each time period

Where the given node would have a reduction in demand at the given time period as indicated.

Hence after our final communication, the proposed upgrade transmission lines are Arcs 179, 198, 203. Additionally, the new generator nodes should be built on Node 4, running during time periods D2-D5 (8:00 - 24:00), and the new solar farm is suggested to be built on Node 17.

Consequently, the optimal total cost over the day for meeting the demand in each of the six time periods from our generators is $3052747. In comparison to the previous communication, having the reduction in demand has allowed the optimal cost to be reduced by over $75000. Which has successfully countered the increased costs of peaked production.