

UNIT-1

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September 1, 2024

1 Ray Theory Transmission

1.1 Total internal reflection

$$\frac{\sin \phi_1}{\sin \phi_2} = \frac{n_2}{n_1} \quad (1)$$

1.2 Acceptance angle

1.3 Numerical aperture

$$\begin{aligned} n_0 \sin \theta_1 &= n_1 \sin \theta_2 \\ &= n_1 \cos \phi \quad \because \phi = \frac{\pi}{2} - \theta_2 \\ &= n_1 (1 - \sin^2 \phi)^{\frac{1}{2}} \end{aligned} \quad (2)$$

$$NA = n_0 \sin \theta_a = (n_1^2 - n_2^2)^{\frac{1}{2}} \quad (3)$$

refractive index difference:

$$\begin{aligned} \Delta &= \frac{n_1^2 - n_2^2}{2n_1^2} \\ &= \frac{n_1 - n_2}{n_1} \quad \text{for } \Delta \ll 1 \end{aligned} \quad (4)$$

$$\therefore NA = n_1 (2\Delta)^{\frac{1}{2}} \quad (5)$$

2 EM mode Theory

$$\begin{aligned} \nabla^2 \Psi &= \frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2} \\ &= \frac{\partial^2 \Psi}{\partial r^2} + \frac{1}{r} \frac{\partial \Psi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \Psi}{\partial \phi^2} + \frac{\partial^2 \Psi}{\partial z^2} \end{aligned} \quad (6)$$

solution to equation 6 is given by:

$$\Psi = \Psi_0 e^{j(\omega t - \mathbf{k} \cdot \mathbf{r})} \quad (7)$$

where $k = |\mathbf{k}| = \frac{2\pi}{\lambda}$

2.1 Phase and group velocity

$$v_p = \frac{\omega}{\beta} \quad (8)$$

$$v_g = \frac{\partial \omega}{\partial \beta} \quad (9)$$

$$\beta = n_1 \frac{2\pi}{\lambda} = \frac{n_1 \omega}{c} \quad (10)$$

thus eq 8 and eq 9 become:

$$\boxed{\begin{aligned} v_p &= \frac{c}{n_1} \\ v_g &= \frac{d\lambda}{d\beta} \cdot \frac{d\omega}{d\lambda} \\ &= \frac{c}{n_1 - \lambda \frac{dn_1}{d\lambda}} \\ &= \frac{c}{N_g} \end{aligned}} \quad (11)$$

3 Cylindrical Fiber

3.1 Modes

$$\frac{\partial^2 \Psi}{\partial r^2} + \frac{1}{r} \frac{\partial \Psi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \Psi}{\partial \phi^2} + (n_1^2 k^2 - \beta^2) \Psi = 0 \quad (12)$$

where

$$n_1 k < \beta < n_2 k$$

solution of wave equation has the form

$$\Psi = E(r) \left[\frac{\cos(l\phi)}{\sin(l\phi)} e^{wt - \beta z} \right] \quad (13)$$

substitute above eq in eq 12 gives:

$$\frac{\partial^2 \mathbf{E}}{\partial r^2} + \frac{1}{r} \frac{\partial \mathbf{E}}{\partial r} + \left[(n_1^2 k^2 - \beta^2) - \frac{l^2}{r^2} \right] \mathbf{E} = \mathbf{0} \quad (14)$$

solution to the above eq is given by:

$$E(r) = \begin{cases} G J_l(UR), & \text{for } R < 1 \\ G J_l(U) \frac{K_i(WR)}{K_i(W)} & \text{for } R > 1 \end{cases} \quad (15)$$

where G is amplitude coefficient, $R = r/a$

J_l & K_l is Bessel's function and modified Bessel's function

U and W are eigen values in core and cladding:

$$\begin{aligned} U &= a(n_1^2 k^2 - \beta^2) \\ W &= a(\beta^2 - n_2^2 k^2) \end{aligned} \quad (16)$$

normalized frequency:

$$\begin{aligned}
 V &= (U^2 + W^2)^{1/2} \\
 &= ka (n_1^2 + n_2^2)^{1/2} \\
 &= \frac{2\pi}{\lambda} a (NA)
 \end{aligned} \tag{17}$$

normalized propagation constant:

$$\begin{aligned}
 b &= 1 - \frac{U^2}{V^2} \\
 &= \frac{(\beta/k)^2 - n_2^2}{n_1^2 - n_2^2} \\
 &= \frac{(\beta/k)^2 - n_2^2}{2n_1^2\Delta}
 \end{aligned} \tag{18}$$

3.2 Fiber Index

3.2.1 Step Index

$$n(r) = \begin{cases} n_1 & r < a \text{ core} \\ n_2 & r \geq a \text{ cladding} \end{cases} \tag{19}$$

3.2.2 Graded Index

$$n(r) = \begin{cases} n_1(1 - 2\Delta(r/a)^\alpha)^{1/2} & r < a \text{ core} \\ n_1(1 - 2\Delta)^{1/2} = n_2 & r \geq a \text{ cladding} \end{cases} \tag{20}$$