UNIT-1

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1 Ray Theory Transmission

1.1 Total internal reflection

$$\frac{\sin\phi_1}{\sin\phi_2} = \frac{n_2}{n_1} \tag{1}$$

1.2 Acceptance angle

$$NA = n_0 \sin\theta_a = (n_1^2 - n_2^2)^{\frac{1}{2}} \tag{2}$$

1.3 Numerical aperture

$$\begin{aligned} n_0 \sin\theta_1 &= n_1 \sin\theta_2 \\ &= n_1 \cos\phi \quad \because \phi = \frac{\pi}{2} - \theta_2 \\ &= n_1(1 - \sin^2\phi)^{\frac{1}{2}} \end{aligned} \tag{3}$$

refractive index difference:

$$\begin{aligned} \Delta &= \frac{n_1^2 - n_2^2}{2n_1^2} \\ &= \frac{n_1 - n_2}{n_1} \quad for \Delta \ll 1 \end{aligned} \tag{4}$$

$$\therefore NA = n_1(2\Delta)^{\frac{1}{2}} \tag{5}$$

2 EM mode Theory

$$\begin{aligned} \nabla^2\Psi &= \frac{\partial^2\Psi}{\partial x^2} + \frac{\partial^2\Psi}{\partial y^2}\frac{\partial^2\Psi}{\partial z^2} \\ &= \frac{\partial^2\Psi}{\partial r^2} + \frac{1}{r}\frac{\partial\Psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\Psi}{\partial\phi^2} + \frac{\partial^2\Psi}{\partial z^2} \end{aligned} \tag{6}$$

solution to equation 6 is given by:

$$\Psi = \Psi_0 e^{j(\omega t - \mathbf{k}\cdot\mathbf{r})} \tag{7}$$

where $k = |\mathbf{k}| = \frac{\mathbf{2\pi}}{\lambda}$

2.1 Phase and group velocity

$$v_p = \frac{\omega}{\beta} \tag{8}$$

$$v_g = \frac{\partial \omega}{\partial \beta} \tag{9}$$

$$\beta = n_1 \frac{2\pi}{\lambda} = \frac{n_1 \omega}{c} \tag{10}$$

thus eq 8 and eq 9 become:

$$\boxed{\begin{aligned} v_p &= \frac{c}{n_1} \\ v_g &= \frac{d\lambda}{d\beta} \cdot \frac{d\omega}{d\lambda} \\ &= \frac{c}{n_1 - \lambda \dfrac{dn_1}{d\lambda}} \\ &= \frac{c}{N_g} \end{aligned}} \tag{11}$$

3 Cylinderical Fiber

3.1 Modes

$$\frac{\partial^2 \Psi}{\partial r^2} + \frac{1}{r}\frac{\partial \Psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \Psi}{\partial \phi^2} + (n_1^2 k^2 - \beta^2)\Psi = 0 \tag{12}$$

where

$$n_1 k < \beta < n_2 k$$

solution of wave equation has the form

$$\Psi = E(r) \left[\frac{\cos(l\phi)}{\sin(l\phi)} e^{wt - \beta z} \right] \tag{13}$$

substitute above eq in eq 12 gives:

$$\frac{\partial^2 \mathbf{E}}{\partial r^2} + \frac{1}{r}\frac{\partial \mathbf{E}}{\partial r} + \left[(n_1^2 k^2 - \beta^2) - \frac{l^2}{r^2} \right] \mathbf{E} = \mathbf{0} \tag{14}$$

solution to the above eq is given by:

$$E(r) = \begin{cases} G J_l(UR), & \text{for } R < 1 \\ G J_l(U) \dfrac{K_i(WR)}{K_i(W)} & \text{for } R > 1 \end{cases} \tag{15}$$

where G is amplitude coefficient, $R = r/a$
J_l & K_l is Bessel's function and modified Bessel's function
U and W are eigen values in core and cladding:

$$\begin{aligned} U &= a(n_1^2 k^2 - \beta^2) \\ W &= a(\beta^2 - n_2^2 k^2) \end{aligned} \tag{16}$$

normalized frequency:

$$\begin{aligned} V &= \left(U^2 + W^2\right)^{1/2} \\ &= ka\left(n_1^2 + n_2^2\right)^{1/2} \\ &= \frac{2\pi}{\lambda} a(NA) \end{aligned} \tag{17}$$

normalized propagation constant:

$$\begin{aligned} b &= 1 - \frac{U^2}{V^2} \\ &= \frac{(\beta/k)^2 - n_2^2}{n_1^2 - n_2^2} \\ &= \frac{(\beta/k)^2 - n_2^2}{2n_1^2\Delta} \end{aligned} \tag{18}$$

3.2 Fiber Index

3.2.1 Step Index

$$n(r) = \begin{cases} n_1 & r < a \;\; \text{core} \\ n_2 & r \geq a \;\; \text{cladding} \end{cases} \tag{19}$$

No. of guided modes or mode volume

$$M_s = \frac{V^2}{2} \tag{20}$$

3.2.2 Graded Index

$$n(r) = \begin{cases} n_1(1 - 2\Delta(r/a)^{\alpha})^{1/2} & r < a \;\; \text{core} \\ n_1(1 - 2\Delta)^{1/2} = n_2 & r \geq a \;\; \text{cladding} \end{cases} \tag{21}$$

No. of guided modes or mode volume

$$M_g = \left(\frac{\alpha}{\alpha + 2}\right)\left(\frac{V^2}{2}\right) \tag{22}$$