

For the Rankine oval, we have

$$\psi = V_{\infty} r \sin \theta + \frac{\Lambda}{2\pi} (\theta_1 - \theta_2) \quad \dots \textcircled{1}$$

For θ_1 and θ_2 ,

$$\theta_1 = \arctan \left(\frac{r \sin \theta}{r \cos \theta + b} \right) \quad \dots \textcircled{2}$$

$$\theta_2 = \arctan \left(\frac{r \sin \theta}{r \cos \theta - b} \right) \quad \dots \textcircled{3}$$

Let $x = r \cos \theta$, $y = r \sin \theta$, and substitute $\textcircled{2}$ $\textcircled{3}$ to $\textcircled{1}$

$$\psi = V_{\infty} y + \frac{\Lambda}{2\pi} \left(\arctan \left(\frac{y}{x+b} \right) - \arctan \left(\frac{y}{x-b} \right) \right)$$

For inviscid, irrotational, incompressible flow,

$$u = \frac{\partial \psi}{\partial y}$$

$$= V_{\infty} + \frac{\Lambda}{2\pi} \left(\frac{1}{1 + \left(\frac{y}{x+b} \right)^2} \cdot \frac{1}{x+b} - \frac{1}{1 + \left(\frac{y}{x-b} \right)^2} \cdot \frac{1}{x-b} \right)$$

$$= V_{\infty} + \frac{\Lambda}{2\pi} \left(\frac{x+b}{(x+b)^2 + y^2} - \frac{x-b}{(x-b)^2 + y^2} \right)$$

For point A, i.e. stagnation point, we have

$$\begin{cases} u = 0 \\ y = 0 \end{cases}$$

Substitute to u

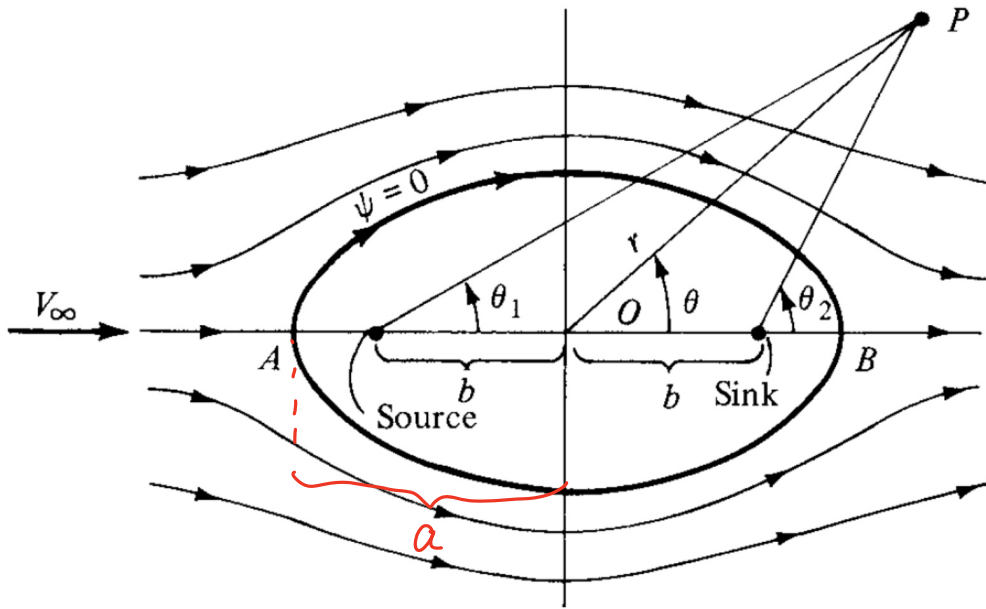
$$0 = u = V_{\infty} + \frac{\Lambda}{2\pi} \left(\frac{1}{x+b} - \frac{1}{x-b} \right)$$

$$\Rightarrow -\frac{2\pi V_{\infty}}{\Lambda} = \frac{-2b}{x^2 - b^2}$$

$$\Rightarrow x = \pm \sqrt{b^2 + \frac{\Lambda b}{\pi V_{\infty}}}$$

$$OA = OB = |x| = \sqrt{b^2 + \frac{\Lambda b}{\pi V_{\infty}}}$$

For $V_\infty = 10 \text{ m/s}$, $b = 1 \text{ m}$, $\Lambda = 20 \text{ m}^2/\text{s}$



$$\psi = 10y + \frac{10}{\pi} \left(\arctan\left(\frac{y}{x+1}\right) - \arctan\left(\frac{y}{x-1}\right) \right)$$

$$u = \frac{\partial \psi}{\partial y} = 10 + \frac{10}{\pi} \left(\frac{x+1}{(x+1)^2 + y^2} - \frac{x-1}{(x-1)^2 + y^2} \right)$$

$$\begin{aligned} v &= -\frac{\partial \psi}{\partial x} = -\frac{10}{\pi} \left(\frac{1}{1 + \left(\frac{y}{x+1}\right)^2} \cdot \frac{-y}{(x+1)^2} - \frac{1}{1 + \left(\frac{y}{x-1}\right)^2} \cdot \frac{-y}{(x-1)^2} \right) \\ &= \frac{10}{\pi} \left(\frac{y}{(x+1)^2 + y^2} - \frac{y}{(x-1)^2 + y^2} \right) \end{aligned}$$

$$V = u^2 + v^2$$

$$V(x, y) = \sqrt{\left(10 + \frac{10}{\pi} \left(\frac{x+1}{(x+1)^2 + y^2} - \frac{x-1}{(x-1)^2 + y^2} \right) \right)^2 + \left(\frac{10}{\pi} \left(\frac{y}{(x+1)^2 + y^2} - \frac{y}{(x-1)^2 + y^2} \right) \right)^2}$$

$$C_p(x, y) = 1 - \left(\frac{V(x, y)}{V_\infty} \right)^2$$

$$u = \frac{\partial \psi}{\partial y} = 10 + \frac{10}{z} \left(\frac{x+1}{(x+1)^2 + y^2} - \frac{x-1}{(x-1)^2 + y^2} \right)$$

$$v = -\frac{\partial \psi}{\partial x} = -\frac{10}{z} \left(\frac{y}{(x+1)^2 + y^2} - \frac{y}{(x-1)^2 + y^2} \right)$$

$$\nabla \times \vec{V} = 0$$

$$\Rightarrow u dy = v dx$$

$$\Rightarrow \frac{dy}{dx} = \frac{-\frac{10}{z} \left(\frac{y}{(x+1)^2 + y^2} - \frac{y}{(x-1)^2 + y^2} \right)}{10 + \frac{10}{z} \left(\frac{x+1}{(x+1)^2 + y^2} - \frac{x-1}{(x-1)^2 + y^2} \right)}$$