PRESSURE DROP FOR TWO PHASE CARBON DIOXIDE FLOWING IN PIPELINES

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Whenever a saturated or boiling liquid flows through a pipeline, the drop in pressure caused by the flow friction loss is accompanied by a drop in temperature and the boiling or evaporation of the liquid to convert a fraction of it to vapor in order to maintain thermodynamic equilibrium.

This formation of vapor results in a two phase mixture of liquid and vapor which causes a decrease in average density, viscosity, and temperature. For a constant mass flow, this also results in an increase in velocity of flow. This decrease in density and increase in velocity results in an increasing rate of pressure drop per unit length of pipe as the mixture of liquid and vapor flows down the pipeline.

Such a flow of a two phase mixture of vapor and liquid usually occurs when liquid carbon dioxide is used for fire extinguishing purposes. In this use of CO_2 it may be stored as a saturated liquid in refrigerated tanks holding several tons or more of the liquid CO_2. A common storage pressure is three hundred pounds per square inch gauge pressure. The saturated liquid is conducted to the discharge points by means of piping connected to the liquid phase of the tank. The starting and stopping of the discharge is controlled by means of shut off valves which may be manually or automatically actuated.

Since the quantity of CO_2 required to extinguish a fire in a given space is usually predetermined, and since a rapid discharge of the CO_2 is usually desirable, it is important to be able to determine the flow rate for a proposed piping system. Because of the changes in density and velocity, which cause a variable rate of pressure drop per unit length of pipeline, the usual formulae and tables for determining pressure loss and flow rates are not usable.

Considerable experimental and theoretical work has been done on the variation of the friction factor f with Reynolds' number. In general, this work is fairly complete and indicates that the friction factor is dependent upon Reynolds' number, regardless of the fluid involved. A number of charts are available showing this relationship. Some work has been done on the flow of saturated liquids (5).

In fire extinguishing work the CO_2 flow is well in the turbulent region and the mixture of liquid and vapor can usually be assumed to be in thermodynamic equilibrium and well dispersed.

When the CO_2 for fire extinguishing is stored at low pressure, or at about 0° and 300 PSIG, it is at a temperature considerably below ambient. Thus when the discharge is started, the piping system is at ambient temperature and is considerably warmer than the CO_2. The warmer piping causes a high rate of heat input into the CO_2 introduced into the pipeline, and thus a high rate of evaporation of it. In general, all liquid CO_2 in the pipeline will be evaporated before it reaches the points of discharge, and only vapor will be discharged until the piping system has been cooled to near the liquid CO_2 temperature. The time required for liquid CO_2 to reach the discharge nozzles is important because of the difference in rates of discharge as well as the characteristics of the discharge.

When the CO_2 is stored at 300 PSIG, the storage vessel is usually sized to hold at least twice the amount of CO_2 required for one extinguishing operation. Thus a reserve supply is provided, and less than one half of the vessel contents is discharged at one time. During the discharge from the storage vessel, the pressure in the vessel drops somewhat, due to the removal of the liquid and the formation of vapor to replace it. An average vessel pressure of about 285 PSIG or 300 PSIA during discharge is normal.

Normally the vessels are designed to be filled to only about 90% by volume with liquid in order to allow a vapor space to avoid the possibility of overfilling

An equation is developed for the flow of two phase liquid-vapor CO_2 in pipelines. Graphs and tables are prepared to facilitate the use of the equation.

An equation is also developed for the flow of saturated CO_2 vapor in pipelines.

A formula is developed for determining the time required for the liquid CO_2 to reach the discharge nozzles.

Discharge rates for liquid CO_2, at a tank pressure of 300 PSIA, and for saturated vapor CO_2 through square edge orifices at various orifice pressures are given:

Equations for CO_2 Flow

The equation

$$dz + \frac{144\,dP}{\rho} + \frac{w\,U\,dU}{g} + \frac{12 f U^2}{2gD}\,dL = 0 \qquad (1)$$

is the mechanical energy balance of Bernoulli's equation for flow only of a fluid in a pipe.

The first term represents the change in elevation of the fluid and can ordinarily be considered as 0.

The second term represents the change in pressure head of the fluid.

The third term represents the change in velocity head of the fluid.

The fourth term represents the friction head loss of the fluid in the elementary pipe length dL.

For geometrically similar pipes or ducts the friction factor f in general is a function of the Reynolds' number or of $\frac{DU\rho}{12\mu}$ and may become constant for large values of Re. However, various sizes of pipe are ordinarily not geometrically similar since the internal roughness is nearly constant for the various diameters. Thus for large values of Re, f may be a function of the diameter only.

The plot by Moody, Trans. ASME, Nov. 1944[1] shows friction factors based on Colebrook's equation. For values of $\frac{U\rho}{\mu}$ above 3×10^6, which corresponds to CO_2 flow rates of above 100 pounds per

square inch per minute and for values of D from 0.18 to 18 inches for steel pipe, the following equation holds quite closely,

$$f = \frac{0.0227}{D^{0.25}} \quad . \tag{2}$$

The factor w converts $\frac{U^2}{2}$

to the true kinetic energy. It decreases slightly for increasing values of Re, however for values of Re above 10^6 it is nearly 1.1.[2]

Substituting

$$w = 1.1 \tag{3}$$

$$144G = \rho U \tag{4}$$

$$-\frac{d\rho}{\rho} = \frac{dU}{U} \tag{5}$$

and equation (2) into equation (1) and rearranging yields,

$$\frac{g}{144G^2}\rho dP - 1.1\frac{d\rho}{\rho} + \frac{0.136}{D^{1.25}}dL = 0 \quad . \tag{6}$$

In equation (6) for a given flow case g, G, and D are constant while ρ and P are variables, the relationships of which must be known to solve the equation. dL is the length variable.

In normal cases of liquid CO_2 flow the flow can be considered to be very nearly isoenthalpic or $\Delta H = 0$, since the conversion of pressure head to velocity head is small compared to that lost in friction head.

For isoenthalpic changes of a saturated liquid in equilibrium with the vapor,

$$X = \frac{h_g - H_f}{h_g - h_f} \tag{7}$$

and

$$\rho = \frac{1}{v_t} = \frac{1}{X v_f + (1-X) v_g} \quad . \tag{8}$$

The values of hf, hg, v_f, and g are obtainable from thermodynamic tables as functions of P.

Hf is the value of hf for P equal to the storage pressure Po.

From equations (7) and (8) and thermodynamic tables, values of ρ can be calculated for various values of P for desired values of Po.

As an example the following are values of ρ for Po = 300.

Po	P	ρ
300	300	63.9
	250	33.9
	200	19.5
	150	11.23
	100	5.82

Equation (6) can be solved by graphical integration of $\int \rho \, dP$ or if ρ can be expressed as a suitable function of P, the expression can be integrated.

Several types of equations can be used to express ρ accurately as a function of P, however the following is among the more simple,

$$\rho = i \, j^{\frac{P}{100}} \tag{9}$$

where i and j are constants dependent upon Po.

The following are values of i and j for various values of Po.

Po	i	j	ln j
150	0.705	20.00	5.298
200	1.300	7.00	1.946
250	1.580	4.40	1.482
300	1.720	3.34	1.206
350	1.810	2.74	1.080
400	1.875	2.40	.713

Instead of the mass flow G, the total flow Q in pounds per second or q in pounds per minute is often the desired quantity.

In this event,

$$Q = \frac{G D^2 \pi}{4} \tag{10}$$

$$q = 15 G D^2 \pi \quad . \tag{11}$$

Substituting equations (9), (10), and (11) into equation (6), integrating and simplifying yields,

$$\frac{L - L_1}{D^{1.25}} = \frac{1.645}{G^2} Y - 8.08 Z \tag{12}$$

$$L - L_1 = 1.013 \frac{D^{5.25}}{Q^2} Y - 8.08 D^{1.25} Z \tag{13}$$

$$L - L_1 = 3647 \frac{D^{5.25}}{q^2} Y - 8.08 D^{1.25} Z \tag{14}$$

$$Q^2 = \frac{1.013 D^{5.25} Y}{L - L_1 + 8.08 D^{1.25} Z} \tag{15}$$

$$q^2 = \frac{3647 D^{5.25} Y}{L - L_1 + 8.08 D^{1.25} Z} \tag{16}$$

where $$Y = -\int_{P_1}^{P} \rho\, dP = \frac{100 i}{\ln j}\left[j^{\frac{P_1}{100}} - j^{\frac{P}{100}}\right] \tag{17}$$

$$Z = -\int_{\rho_1}^{\rho} \frac{d\rho}{\rho} = \ln\frac{\rho_1}{\rho} = \ln j\left[\frac{P_1}{100} - \frac{P}{100}\right] . \tag{18}$$

The values of Y and Z for Po = 300, i = 7.20 and j = 3.34, for various values of P are given in Table 1.

L_1 is the length at P_1 and L is the length at P. If $P_1 = P_0$ then $L_1 = 0$.

L is the equivalent length of the piping in feet. The effects of pipe fittings and valves are handled by considering them as equal to an equivalent length of pipe. Manufacturers of the fittings and handbooks give values for these equivalent lengths.(3)

Table 1

Values of Y and Z for Po = 300, i = 7.20 and j = 3.34.

P	Y	Z
300	0	0
275	1385	0.301
250	2410	.602
225	3165	.903
200	3735	1.205
175	4145	1.506
150	4450	1.807
125	4670	2.108
100	4845	2.410
75	4970	2.711

Appendix A shows values of Y and Z for various values of Po and P for P_1 = Po, determined by graphical integration.

Effects of Heat Input

Since the piping during discharge will usually be colder than the ambient air, it is, in some cases of low flow rate and long pipe runs, desirable to correct for the small fraction of carbon dioxide evaporated due to heat input from the surrounding air.

Due to the high velocity of the liquid within the pipe, the inside film heat transfer coefficient will be large, and the heat transfer will be controlled by the air to pipe film coefficient. For this purpose natural convection can be assumed and a coefficient of 3 BTU per square foot per hour per degree F will be used.[4]

The enthalpy of CO_2 saturated vapor is nearly constant from 40° F to -50°F or 567 PSIA to 95 PSIA, hence the latent heat of evaporation of the liquid based on one pound of the liquid at initial conditions is nearly constant and is about 120 BTU per pound for Po = 300.

The temperature of the CO_2 in the pipeline decreases with the pressure towards the discharge nozzles. By graphical integration, assuming a nozzle pressure of P = 100 and Po = 300, the average CO_2 temperature is found to be -5°F.

The pounds of CO_2 evaporated per minute due to heat input from the surrounding air for Po = 300 is approximately,

$$E = \frac{S(\theta + 5)}{2400}, \tag{19}$$

where S is the total surface area of the piping in square feet and θ is the ambient temperature in degrees F.

The corrected flow rate is designated Cq where C is a correction factor.

By neglecting the velocity or Z quantity in equation (16), we have,

$$q = \left[\frac{3647\, D^{5.25}}{L - L_1} Y\right]^{\frac{1}{2}}, \tag{20}$$

likewise,

$$C_q = \left[\frac{3647\, D^{5.25}}{L - L_1} \int_P^{P_1} \rho_c \, dP\right]^{\frac{1}{2}} \tag{21}$$

where the ρ_c are values of the liquid-vapor mixture density corrected for the evaporation due to the heat input. These values are obtained by using modified values of x in equation (8). The modified values of x are assumed on a trial and error basis as functions of P. The graphical integration of

$$\int \rho_c \, dP$$

for the length is then checked against the assumed values of x.

Then

$$C = \left[\frac{1}{\gamma} \int_P^{P_1} \rho_c \, dP\right]^{\frac{1}{2}} \tag{22}$$

Table 2 shows such approximate correction factors to be used with q or Q for Po = 300.

Table 2
Values of C for Po = 300

Superficial Fraction Evaporated $\frac{E}{Q}$	Correction Factor C	Actual Fraction Evaporated $\frac{E}{Cq}$
0.00	1.00	0.00
.01	.975	.0102
.02	.95	.0211
.03	.925	.0324
.04	.90	.0445
.05	.88	.0569
.10	.765	.1308
.15	.66	.227
.20	.585	.342
.25	.52	.481
.30	.465	.645
.35	.42	.835
.385	.385	1.000

The Equation to Calculate Pipe Diameter

[illegible] for the [illegible] to [illegible] the gas source pressure in relation to the CO_2 vapor line is required. This equation is given in [illegible] and [illegible].

For the flow of CO_2 vapor, equations (2) and (3) will [illegible] values of

$$\frac{U\rho}{\mu}$$

[illegible] value corresponds to [illegible] [illegible] and [illegible] [illegible]

For a [illegible] CO_2 vapor the following [illegible] [illegible] for [illegible] [illegible]

$$\rho = \frac{1}{v_g} = 1.12\,\frac{P}{100} \qquad (\text{[illegible]})$$

Substituting [illegible] (9), (11) and (12) into equation (8), integrating, and simplifying [illegible]

$$\frac{L-L_1}{D^{1.25}} = \frac{1.695}{G^2}\,Y_a - 8.08\,Z_a \qquad (\text{[illegible]})$$

$$L-L_1 = 1.013\,\frac{D^{5.25}}{Q^2}\,Y_a - 8.08\,D^{1.25}\,Z_a \qquad (\text{[illegible]})$$

$$L-L_1 = 3647\,\frac{D^{5.25}}{q^2}\,Y_a - 8.08\,D^{1.25}\,Z_a \qquad (\text{[illegible]})$$

$$Q^2 = \frac{1.013\,D^{5.25}\,Y_a}{L-L_1 + 8.08\,D^{1.25}\,Z_a} \qquad (\text{[illegible]})$$

$$q^2 = \frac{3647\,D^{5.25}\,Y_a}{L-L_1 + 8.08\,D^{1.25}\,Z_a} \qquad (\text{[illegible]})$$

Where
$$Y_a = -\int_{P_1}^{P} \rho\, dP = 56\left[\left(\frac{P_1}{100}\right)^2 - \left(\frac{P}{100}\right)^2\right] \tag{29}$$

$$Z_a = -\int_{\rho_1}^{\rho} \frac{d\rho}{\rho} = \ln\frac{\rho_1}{\rho} = \ln\frac{P_1}{P} \quad . \tag{30}$$

During the early part of the discharge before liquid CO_2 reaches the nozzles, it is assumed that the flow up to a certain moving zone is the usual liquid flow and that the flow from this zone to the nozzles is saturated vapor flow. The vaporization of liquid in the moving zone is assumed to cool the piping material from ambient temperature to the temperature of the liquid CO_2 at the zone.

The heat to be removed from the piping is

$$H = W C_p (\theta - \theta_p), \tag{31}$$

where W is the piping weight, Cp is its specific heat, θ is the ambient temperature and θ_p is the average temperature to which the piping is cooled.

The refrigerating effect of the CO_2 is the latent heat of the evaporated CO_2 plus the sensible heat change of the vapor.

An expression for the time required for liquid CO_2 to reach the discharge nozzles is obtained by simultaneously solving the equations for liquid flow (16) and for vapor flow (28) together with the advance of the moving zone, determined by equating the heat removed from the piping to the refrigerating effect of the CO_2 and by allowing for the increasing quantity of liquid in the piping.

By neglecting the second or velocity term in the denominators of equations (16) and (28) and by neglecting the sensible heat of the vapor CO_2, a general equation can be determined by graphical integration.

The following equation is thusly determined for Po = 300,

$$T = \frac{W C_p (\theta + 5)}{.913 (C_q - E)} + \frac{D^2 \pi}{576} \frac{1050}{C_q} L_a \text{ seconds} \tag{32}$$

For steel pipe Cp = .11 and

$$T = \frac{W(\theta + 5)}{8.3 (C_q - E)} + 5.5 \frac{D^2 L_a}{C_q} \text{ seconds} . \tag{33}$$

The first member on the right side of the equations reflects the time required because of cooling the piping, whereas the second member reflects the time required because of filling the piping with CO_2.

Total Quantity Discharged

The total duration of discharge is usually from one to three minutes, therefore the lesser discharge rate during the time required for liquid to reach the nozzles is important in determining the total quantity discharged.

The total quantity discharged can be determined by adding the quantities determined as follows:

1. During the time computed from the second term on the right of equation (32) or (33) the discharge rate is zero.
2. During the time computed from the first term on the right of equation (32) or (33) the discharge rate is 0.46 Cq.
3. During the remainder of the discharge time the discharge rate is Cq.